An Investigation of Structural Optimization in Crashworthiness Design Using a Stochastic Approach

Larsgunnar Nilsson[1,2] and Marcus Redhe[1]

1. *Engineering Research Nordic AB, Linkoping, Sweden, www.erab.se*
2. *Division of Solid Mechanics, Linkoping University, Linkoping, Sweden*

Abstract

In this paper the Response Surface Methodology (RSM) and the Stochastic Optimization (SO) are compared with regard to their efficiency and applicability in crashworthiness design. Optimization of simple analytic expressions and optimization of a front rail structure are application used in order to assess the respective qualities of both methods. A low detailed vehicle structure is optimized to demonstrate the applicability of the methods in engineering practice. The investigations reveal that RSM is favoured compared to SO for less than 10-15 design variables. A novel zooming method is proposed for SO, which improve its convergence behaviour. A combination of both the RSM and the SO is efficient. Stochastic Optimization can be used in order to determine an appropriate starting design for an RSM optimization, which continues the optimization. Two examples are investigated using this combined method.

Introduction

The use of structural optimization has increased rapidly during recent years, mainly due to faster computers, better algorithms and a more frequent use of Finite Element (FE) simulations. Optimization is a useful tool to improve the design in a well-structured manner. Structural optimization uses often gradients of the objective and constraints to find a search direction to the optimal solution. For transient dynamic problems, like impact problems, the responses are often noisy and it is hard and expensive to find these gradients.

Stochastic Optimization (SO) does not create these gradients therefore the amount of needed samples does not depend on the number of design parameters but simply on the precision of the desired statistical description. As opposed to traditional gradient-based optimization methods or Response Surface Methodology (RSM), the aim of SO is not to find the absolute optimum solution, but to develop a sufficiently improved design.

In SO different background variables, e.g. initial velocity or different material properties can vary stochastically during the optimization procedure. Thus, the optimum solution found includes natural changes from these background variables. A major difference between SO and traditional optimization methods is that SO does not need artificially frozen conditions. It is rather a reproduction of the real model considering uncertainties of manufacturing tolerances and material properties. However, we want to compare the convergence speed of SO and RSM, therefore all design parameters in this study are only deterministic parameters and no background variables are used.

Today, RSM is the preferred optimization method in vehicle crashworthiness design. Several attempts have been made to use optimization methods in crashworthiness design problems. Etman et al. [3,4], who were among the first using RSM in structural optimization. Forsberg [5] has studied accuracy aspects of RSM using linear polynomials and Kriging. Marklund and Nilsson [7] were among the first using RSM for an industrial application. Redhe et al. [10-13] have compared RSM with SM and applied SM on problems involving large FE vehicle models. Schramm et al. [15-18] have applied RSM in a vehicle design context. Stander et al. [1,14,20,22] were also among the first using RSM in structural optimization and have also developed the optimization package LS-OPT. Sobieszczanski-Sobieski et al. [19] have done much work in the field of multidisciplinary optimization using RSM. Yamazaki and Han [24 crashed tubes into a rigid wall for different cross sections. Finally, Yang et al. [25,26] have used RSM in several industrial applications of optimization.

As opposed to RSM, the number of evaluations per iteration for SO is user-defined, typically 10-20 evaluations per iteration. Thus, one can expect far less necessary evaluations using SO for improving a system with a large number of design variables. Marczyk [6] shows a procedure of how to use stochastic design improvement in a Simulation-Based design context. Dudeck et al. [2] uses SO to optimize a vehicle with respect to crash and Noise, Vibration and Harshness (NVH). Yang et al. [27] has used SO to optimize the crash performance of a vehicle. Their conclusions were that it was easy to use SO and it can improve the design even for many design variables. However, it does not guarantee to produce an optimal solution, other conventional methods gave better optimum solutions. The Stochastic Optimization method belongs to the group of 0-th order methods. Some other methods using a 0-th order approach are for example the Pattern Search Method, see Torczon [23] and the Downhill Simplex Method, see Nelder and Mead [9]. These methods are based on other experiment selection strategies than stochastic simulation.

This paper aims to compare the RSM with SO and determine an upper/lower limit of the number of design variables for when the convergence speed of each method are too low compared to the other method. A novel zooming in combination with the SO method is presented to improve the convergence speed. Finally, a combined method using both SO and RSM is presented and exemplified with a larger vehicle crash optimization problem

All optimization problems are solved using both RSM with linear polynomials and SO. To solve the optimization problems using RSM, the optimization package LS-OPT, see Stander et al. [21] as used. Solving the problems using the SO we used our own written MATLAB code.

Successive optimization methods

Response surface optimization

The Response Surface Methodology is a method for constructing global approximations of the objective and constraint functions based on functional evaluations at various points in the design space. The strength of the method is in applications where gradient based methods fails, i.e. when design sensitivities are difficult or impossible to evaluate, in global optimization, for exploration of design spaces and in multidisciplinary optimization.

The design domain is the space spanned by the design variables, i.e. $\{x_1, x_2, ..., x_i\}$. The design domain can be further narrowed by introducing limits on the design variables separate from the global limits. This creates a sub-domain called the region of interest where the approximations are calculated. When the optimum is found, the region of interest is moved in the indicated direction during the next iteration and the optimization continues. The selection of approximation functions to represent the actual behavior is essential. For a general quadratic polynomial surface approximation the function will be,

$$y^i = \beta_0 + \sum_{j=1}^{n} \beta_j x_j^i + \sum_{j=1}^{n} \sum_{k=1}^{n} \beta_{jk} x_j^i x_k^i + \varepsilon^i$$

where β^i are the constants to be determined, x^i are the design points in the region of interest, ε^i includes both bias errors and random errors and N is the number of evaluations.

Stochastic optimization

Stochastic analysis is based on the Monte Carlo Simulation (MCS). The result of a MCS is a response cloud. It is important to distinguish between the objective response and constraint responses. Stochastic Optimization, also called Stochastic Design Improvement (SDI), aims at transporting the entire objective response cloud towards the target value in order to improve the design. However, it is not always possible to move a constellation of points to an arbitrary location in the design space due to physical limitations or boundary conditions. At each MCS constraint response clouds can be created as well. An improved design is therefore only valid, if all constraint values meet the boundary conditions to which they are subject to. If the objective response cloud is close enough to the desired location and all constraints are fulfilled, a valid and improved design is found. A response cloud consists of a user-defined number of points. These points result from different sets of randomly chosen design and background variables.

$$y_i = f(x_i)$$

where i=1,N and N is the number of evaluations. The first step is the definition of a set of design variables which generally follow a stochastic distribution, in this paper a uniform distribution is used. Then, two different types of limits have to be implemented. At first, engineering limits are required within which the design variables are allowed to be varied. Secondly, so called sampling limits have to be defined. These sampling limits must not exceed the engineering limits and can be considered as analogies of the region of interest used in RSM. They define the width of the uniform distributions of all design variables. In this way, a subregion of the design space (limited by the engineering limits) is created. Sampling limits can be described as follows

$$x_{l,k} < \mu_k < x_{r,k} \quad k = 1, 2, ..., n$$

where $x_{l,k}$ and $x_{r,k}$ define the sampling limits and n is the number of design variables.

A Monte Carlo Simulation is conducted considering the distributions of design and background variables. This simulation leads to a first response cloud with N response values of y_i. Experiment j with the minimum objective value, result and input variables N is then selected. At this stage, the uniform distributions of all design variables are redefined. Their mean values are shifted to the input variable values x_j of experiment j.

$$\mu_k = x_{j,k} \quad k = 1, 2, ..., n$$

Thus, the initial model is replaced by the best experiment j. After this redefinition a new MCS is carried out in order to find a response value even closer to the optimum value. This iterative procedure is continued convergence is reached. A working schedule of SO can be found in Figure 1.

It is important to note that the transport of response clouds is not arbitrarily fast. The “velocity” of transport of such a constellation of points is limited to approximately half of the diameter of the response cloud per iteration. Since the response clouds are generally not round in shape, this is only an estimated value. Furthermore, all sampling limits have to be defined carefully. If the width of the uniform distribution of the design variables is too small, the design variables cannot achieve the engineering limits within a given number of iterations. Therefore, either the number of iterations or the distances of the sampling limits need to be increased. One question is under which conditions the design improvement converges to the optimum value. Convergence means that in each subsequent iteration a smaller objective value can be found. Thus, it is necessary to estimate the probability of finding a value closer to the optimum value in the next iteration step. In the subsequent iteration there are N function values y_i, of which at least one should be smaller than the last iteration function value. The distribution of the y_i around the last iteration function value is supposed to be approximately symmetric. It can be shown that the probability of finding at least one y_i smaller than the y from the last iteration for a monotonic decreasing function with only feasible design points is given by,

$$p(N) = 1 - \left(\frac{1}{2}\right)^N$$

Thus, the probability of advancing towards the optimum value in the next iteration step depends on the number of experiments in each iteration. In order to guarantee convergence it is sufficient to choose about 8 to 32 experiments per iteration. This convergence rate can not be reached for practical problems due to infeasible design points and non-monotonic decreasing functions. This convergence behavior can only be a theoretical upper limit.

Zooming methods for the region of interest

Response surface optimization

The response surface optimization in LS-OPT uses a region of interest (RoI), which is a subspace of the complete design space to determine an approximate optimum. The initial RoI is given by the user and can only shrink during the iterations. The shrinking depends on three factors (γ_{pan},γ_{osc},η) and the design variable history. If the new potential optimum is located on the boundary of the RoI, there will be no shrinking of the updated RoI (if γ_{pan}=1). If the potential optimum point oscillates inside the RoI, it will shrink by a factor γ_{osc}. Finally, if the potential optimum is found inside RoI, the updated RoI will shrink with a factor η. If a combination of all above happens, a combination of all factors will shrink the RoI. All equations to calculate the new RoI follows Stander and Craig [22].

Stochastic optimization

The RSM zooming method can not be adapted by SO due to that the stochastic distribution of the design variables generally not gives a sub optimum solution on the boundary of the sub domain. And due to the low number of evaluations, not all corners of the sub domain is evaluated. This might give a sub optimal solution that seems to oscillate. This is only due to that it is no design point on this side of the center, therefore cannot the same zooming/panning method as RSM be used for SO.

We introduce a novel zooming of the RoI, which we call the Stochastic Optimization Zooming Method (SOZM) and the basic idea is as follows: The initial size of the RoI remains constant as long as all subsequent iterations produce lower objective values. In case the optimization stops converging, i.e. the current iteration produces a worse result than the previous one, this indicates the necessity for zooming. In general, the topology of the objective function is unknown. Thus, there is no evidence which zooming factor should be used. One has to be aware that each optimization problem has a unique zooming factor for maximum efficiency. Consequently, an assumption has to be made for the zooming factor (between 0 and 1). However, a rather large zooming factor close to the maximum value of 1 seems to be useless, because the probability of finding a better point would only be increased marginally.

Most optimization problems are subjected to constraints. As a consequence, the result of the best experiment of each iteration is restricted to meet all constraints, otherwise the center of the region of interest of the next iteration would violate the boundary conditions. Thus, most of the subsequent evaluations would also violate the constraints and the optimization process would produce a lot of inadmissible results. In the case that no better design can be found close to a constraint, the RoI has to be scaled down. Figure 2 shows the proposed working schedule of SOZM.

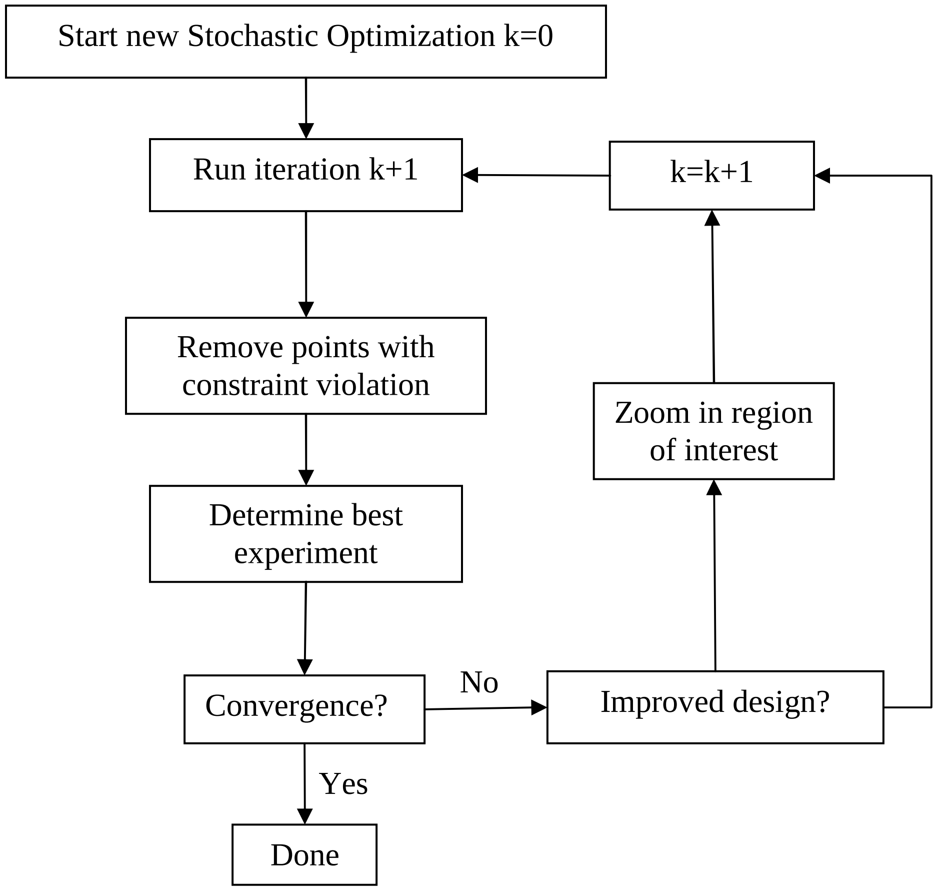

Figure 1 Proposed working schedule of Stochastic Optimization Zooming Method (SOZM)

Combination of RSM and SO

In many cases traditional RSM produces results that violate the constraints early in the optimization process. Normally, this is caused by inaccurate response surfaces and a lot of iterations are needed in order to come close to a valid design. Consequently, it is impossible to abort the optimization after a few iterations in case that only a slightly improved design is desired, or computing capacity is limited. A major quality of SOZM is that all improved designs meet the constraints, provided that at least one design point is valid. Thus, already the first iterations produce improved and valid designs. The optimization process can be stopped, if the design improvement is sufficient. However, in cases where SOZM closely approaches one of the constraints, the method generally loses its efficiency.

It is therefore proposed to switch the optimization method from SO to RSM in cases where further design improvements are demanded. The currently best design is used as starting point for RSM.

Stopping criteria

To determine if the optimization method has converged, we calculate the percentage change in the objective function value in the last two iterations. If this change is less than 1% the optimization routine is terminated.

Crashworthiness application

Front rail structure

Figure 2 shows the front rail structure that will be optimized using SOZM and RSM. The front rail structure will be parameterized for two different numbers of design variables, namely 2 and 20.

Problem description

The front rail structure of a fictitious vehicle model is subjected to a rigid wall impact. The maximum acceleration value is defined as the objective to be minimized in order to improve the crashworthiness of the vehicle. Further responses are mass and maximum intrusion after impact. Both responses are limited by predefined boundary conditions. The experimental setup is depicted in Figure 3, with initial velocity v_0=15.64 m/s and mass M = 275 kg.

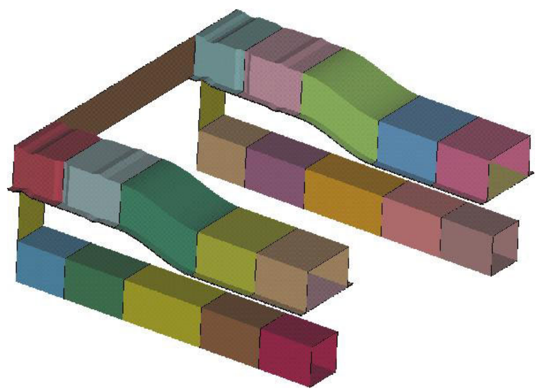

Figure 2 Front rail structure

The FE model is parameterized for a maximum number of 20 design variables, i.e. 16 thicknesses and 4 different yield stresses. For the optimizations with 2 design variables some of the variables are supposed to be constant. Thus, the optimization problem is formulated as

$$\begin{aligned}
&\min \; |a_{\max}| \\
&s.t. \qquad \text{Mass} \leq 17.5kg \\
&\qquad\quad \text{Intrusion} \leq 0.3m \\
&1\text{ mm} \leq t_n \leq 3\text{ mm} \\
&304\text{ MPa} \leq \sigma_m \leq 456\text{ MPA}
\end{aligned}$$

All optimizations are implemented with an initial size of the region of interest of 1 mm for each thickness design variable and 380 MPa for the yield stress. Twelve experiments per iteration are evaluated in SOZM and the default numbers of experiments are used for RSM optimizations, namely 5 and 32.

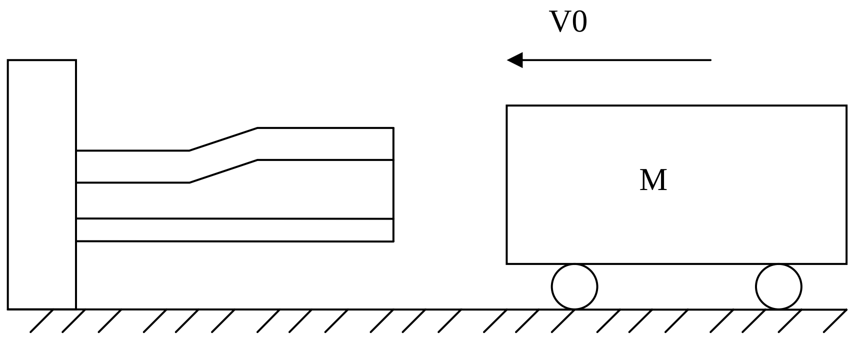

Figure 3 Experimental setup for the front rail structure

Optimization results

The discussion of the optimization results is divided into three sections with respect to the different number of design variables.

2-dimensional optimization

The design variables are the sheet thicknesses of the upper and the lower beams, respectively. All yield stress values are supposed to be constant (380 MPa). The results stated in Table 1 and Figure 4 confirm that RSM is definitely superior to SOZM for a small number of design variables. Both methodologies converge towards the same optimum solution, but RSM performs about half as many experiments as SOZM. In addition the efficiency of SOZM decreases because of constraint violations. Some of the experiments violate the intrusion constraint and therefore the RoI has to be scaled down. The optimization leads to an improved front rail structure design with a maximum acceleration that is reduced to approximately 50% of the initial value and the intrusion is at threshold value of 0.3 m. The newly determined thicknesses of upper and lower beams are TU = 1.75 mm and TL = 1.0 mm, respectively.

Table 1 2-dimensional front rail optimization

Method	Experiments per iteration	Number of iterations	Total number of experiments	Acceleration [ms^{-2}]	Mass, [kg]	Intrusion, [m]
SOZM	12	6	72	-1050	12.3	0.291
RSM	5	6	30	-1033	12.23	0.297

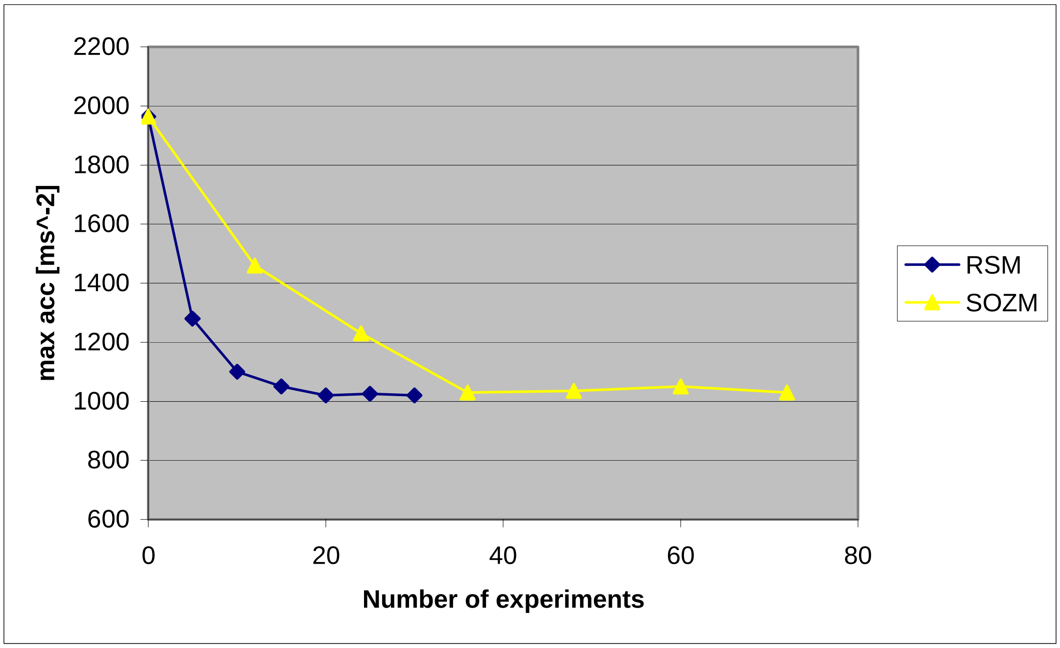

Figure 4 Convergence plot of the 2-dimensional optimization problem

20-dimensional optimization

The number of design variables is increased to 20 by adding variable yield stress values and five additional thickness values. SOZM shows a better convergence behavior in the beginning of the optimization process compared to RSM, see Table 2 and Figure 5. While SOZM determines five improved and valid front rail designs in the first 60 experiments, RSM has not even completed the second iteration. For a larger number of design variables this advantage of SOZM will be even greater, because RSM needs a lot more experiments in order to complete its iterations. As soon as the first zooming is implemented in SOZM, there is a loss of efficiency. However, it is obvious that SOZM produces much better solutions in the first few iterations compared to RSM.

Table 2 20-dimensional front rail optimization

Method	Experiments per iteration	Number of iterations	Total number of experiments	Acceleration [ms^{-2}]	Mass, [kg]	Intrusion, [m]
SOZM	12	9	108	-726.7	14.76	0.272
RSM	32	13	416	-651.1	13.06	0.311

In the beginning of the optimization process, SOZM is more efficient than RSM concerning the convergence behavior. If the number of design variables exceeds a limit of about 15 to 20. As soon as constraints are violated or the RoI has to be scaled down, SOZM loses its excellent efficiency. Further design improvement can hardly be found without lots of subsequent experiments. The results of this section indicate that for optimization problems with a large number of design variables SOZM should be used.

This front rail optimization reveals that SOZM and RSM have different qualities. On the one hand, SOZM is suitable to scan the entire design space for solutions rather close to local optimum solutions, but on the other hand, RSM is able to find exact mathematical optimum solutions. Thus, the combined method will utilize their respective qualities.

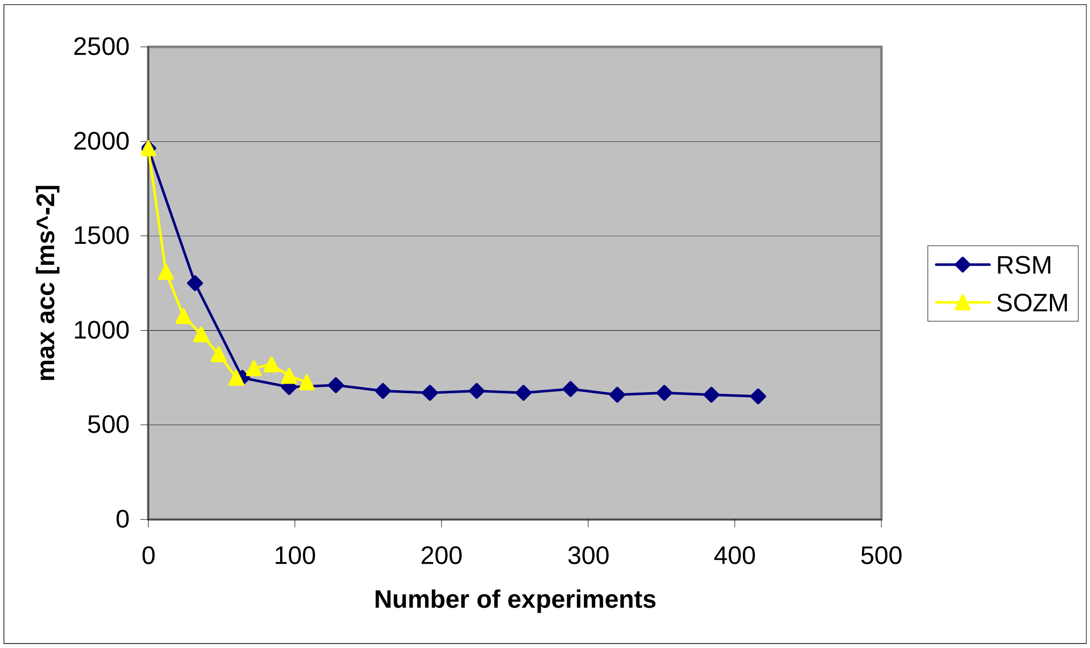

Figure 5 Convergence plot of the 20-dimensional optimization problem

Vehicle structure subjected to frontal impact

The optimization of a vehicle structure subjected to frontal impact is presented as an industrial example. The simple vehicle structure, see Figure 6, is composed of 87 parts. In order to reduce the computing time, the engine block is represented by a rigid substructure. Furthermore, the structure is symmetric and 15 design variables are used in order to improve the crashworthiness design. In consideration of all constraints and the limits of the design space, the optimization problem can be formulated as follows:

$$
\begin{aligned}
&\min \quad |a_{\max}| \\
&s.t. \qquad \text{Intrusion D1} \qquad \leq 95 \text{ mm} \\
&\qquad\qquad \text{Intrusion D2} \qquad \leq 95 \text{ mm} \\
&\qquad\qquad \text{Intrusion D3} \qquad \leq 95 \text{ mm} \\
&\qquad \text{Average rigid wall force} \leq 90 \text{ kN} \\
&\qquad 1 \text{ mm} \leq t_i \leq 3 \text{ mm}
\end{aligned}
$$

The intrusion is measured at three different points, i.e. D2, D3 and D4. All optimizations are implemented with an initial size of the RoI of 1 mm for each design variable. The default number of 25 experiments per iteration is used for the RSM optimization. In SOZM, 12 experiments per iteration are evaluated.

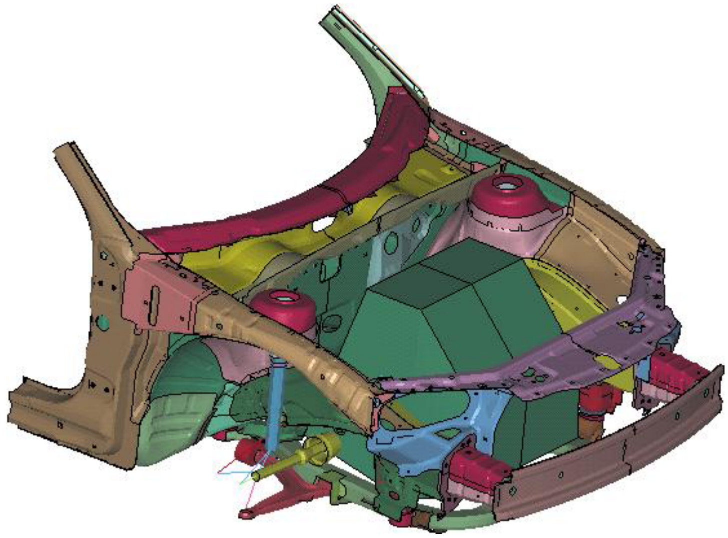

Figure 6 Vehicle structure (symmetric)

Optimization results

All relevant results are stated in Table 3 and Figure 7 shows the convergence plots of all three different optimization processes, i.e. RSM, SOZM and the combined optimization method. After six SO iterations no improved design can be found and therefore the first zooming would be implemented. The currently best design point ($|a_{max}|$ = 318.3 ms^{-1}, 72 experiments) is used as starting point for RSM. In order to avoid major constraint violations, the size of the RoI is reduced to 20 of the initial size. In general, it is very difficult to find optimization parameters leading to maximum efficiency. Thus, this reduction is rather based on experience than on predefined and established rules. Nevertheless, the first few RSM results slightly violate the constraints and no better design can be located compared to SOZM. Only after the fourth RSM iteration somewhat better results are found, but their determination is rather expensive.
Finally, it does not so much concern the comparison of the combined optimization method and SOZM, but it is more important to note that both methods are better compared to RSM concerning efficiency and result quality in this example.

Table 3 Results of combined vehicle structure optimization

	RSM	SOZM	COMB
Experiments per iteration	25	12	12 / 25
Number of iterations	11	14	6/7
Total number of experiments	275	168	247
Acceleration, [ms^{-2}]	-312.8	-284.3	-282.0
Intrusion D2, [m]	0.0823	0.0771	0.0917
Intrusion D3, [m]	0.0592	0.0615	0.0858
Intrusion D4, [m]	0.0921	0.0873	0.0901
Rigid wall force, [N]	86900	87080	85580
Mass, [kg]	42.7	43.45	41.75

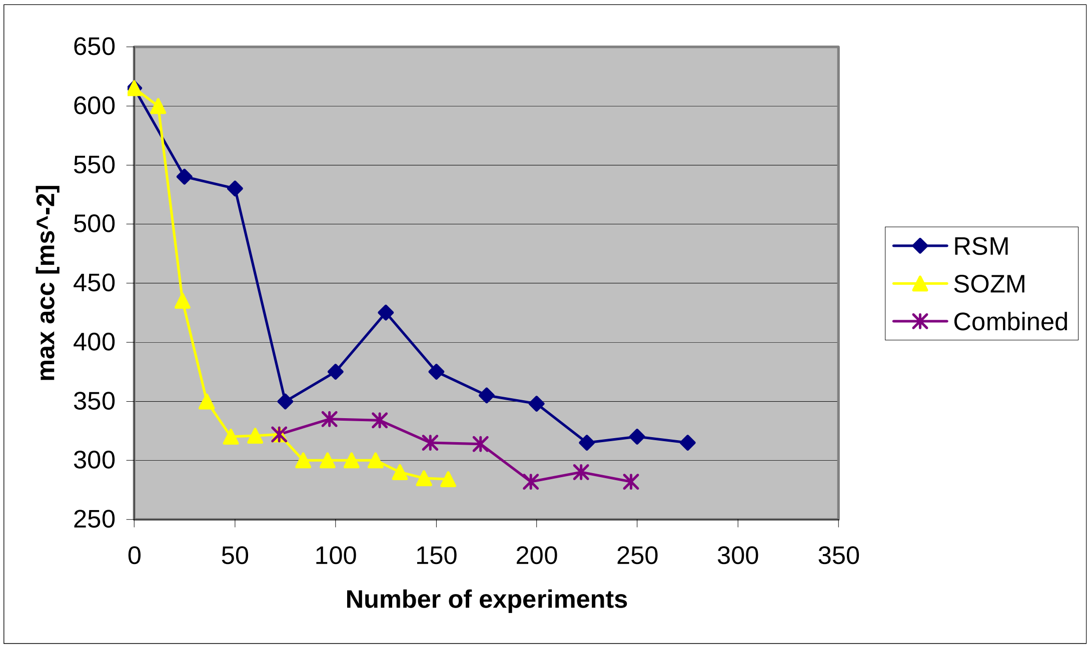

Figure 7 Convergence plot of the vehicle structure problem

Conclusions

The investigations in this paper reveal that SO and RSM show rather different qualities. On the one hand, RSM is suitable to find the mathematical optimum solution of the global or at least of a local minimum. On the other hand, for these optimization problems SO often produces a better convergence behavior in the beginning of the optimization process for a large number of design variables. It has been shown that SO should not be used for optimization problems with less than 10-15 design variables. In this case RSM is superior to SO. However, the more design variables the problem has the more efficient SO becomes.
The convergence behavior of all SO depends to a large extent on the size of the RoI. The results of this investigation indicate that SOZM has to be used instead of SO, otherwise the convergence behavior can be insufficient. SOZM combines the advantages of a large RoI with regard to the shifting capabilities and a small RoI with regard to the capability of finding accurate solutions. Thus, the optimizations should start with a relatively large RoI due to the fact that the RoI is scaled down automatically.
Finally, the combination of both methods is a promising approach. In particular, if only a slightly improved design is required or the computing capacity is limited, SOZM produces valid and improved designs within the first few iterations. In case that a further design improvement is desired, the SOZM result can be used as a starting point for a RSM optimization.

References

[1] Craig, K.J., Stander, N., Dooge, D.A. and Varadappa, S. Multidisciplinary design optimization of automotive crashworthiness and NVH using response surface methods, AIAA-2002-5507, Ed. Mistree, F., 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Atlanta, 2002.

[2] Duddeck, F., Heiserer, D. and Lescheticky, J. Stochastic methods for optimization of crash and NVH problems, Proceedings of the Second MIT Conference on Computational Fluid and Solid Mechanics, Cambridge, 2003.

[3] Etman, L., Adriaens, J., van Slagmaat, M. and Schoofs, A. Crashworthiness design using multipoint sequential linear programming, Structural Optimization, volume 12, 222-228, 1997.

[4] Etman, L.F.P. Optimization of multibody system using approximation concepts, Ph.D. Thesis, Technical University Eindhoven, 1997.
[5] Forsberg, J. Simulation based crashworthiness design - Accuracy aspects of structural optimization using response surfaces, LIU-TEK-LIC-2002:27, Linköping University, Linköping, 2002.
[6] Marczyk, J., Principles of Simulation-based Computer-Aided Engineering, FIM Publication, Barcelona, 1999.
[7] Marklund, P-O and Nilsson, L. Optimization of a car body component subjected to impact, Structural and Multidisciplinary Optimization, volume 21, 383-392, 2001.
[8] Myers, R.H. and Montgomery, D.C. Response Surface Methodology, Wiley, New York, 1995.
[9] Nelder, J.A. and Mead, R., A simplex method for function minimization, The Computational Journal, volume 7, 308-313, 1965.
[10] Redhe, M. Simulation Based Design - Structural optimization at early design stages, LIU-TEK-LIC-2001:51, Linköping University, Linköping, 2001.
[11] Redhe, M and Nilsson, L. A method to determine structural sensitivities in vehicle crashworthiness design, International Journal of Crashworthiness volume 7, 179-190, 2002.
[12] Redhe, M., Forsberg J., Jansson T., Marklund P.O. and Nilsson L.Using the response surface methodology and the D-optimality criterion in crashworthiness related problems, Structural and Multidisciplinary Optimization, volume 24, 185-194, 2002.
[13] Redhe, M. and Nilsson, L. Using space mapping and surrogate models to optimize vehicle crashworthiness design, AIAA-2002-5536, Ed. Mistree, F., 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Atlanta, 2002.
[14] Roux, W., Stander, N. and Haftka, R. Response surface approximations for structural optimization. International Journal for Numerical Methods in Engineering, volume 42, 517-534, 1998.
[15] Schramm, U. and Thomas, H. Structural optimization in occupant safety and crash analysis, Design Optimization, volume 1, 374-387, 1998.
[16] Schramm, U. Multidisciplinary optimization for NVH and crashworthiness, The First MIT Conference on Computational Fluid and Solid Mechanics, Ed. Bathe, K., Boston, 2001.
[17] Schramm, U. Multidisciplinary optimization for NVH and crashworthiness, The First MIT Conference on Computational Fluid and Solid Mechanics, Ed. Bathe, K., Boston, 2001.
[18] Schramm, U. Designing with structural optimization - a practical point of view, AIAA Paper 2002-5191, Ed. Mistree, F., 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Atlanta, 2002.
[19] Sobieszczanski-Sobieski, J., Kodiyalam, S. and Yang, R-J. Optimization of car body under constraints of noise, vibration and harshness (NVH), and crash, AIAA Paper-2000-1521, 7th Symposium on Multidisciplinary Analysis and Optimization, Long Beach, 2000.
[20] Stander, N., Burger, M., Balasubramanyam, S. and Varadappa, S. Shape optimization for head and knee impact featuring, Third LS-DYNA Conference, Paris, 2001.
[21] Stander, N., Craig K. and Roux, W. LS-OPT User's manual, version 1, Livermore Software Technology Corporation, Livermore, 1999.
[22] Stander, N. and Craig K. On the robustness of a simple domain reduction scheme for simulation-based optimization, Engineering Computation, volume 16, 431-450, 2002.
[23] Torczon, V. On the convergence of pattern search methods, SIAM Journal on Optimization, volume 7, 1-26, 1997.
[24] Yamazaki, K. and Han, J. Maximization of the crushing energy absorption of tubes, Structural Optimization, volume 16, 37-46, 1998.
[25] Yang, R-J., Gu, L., Tho, C. and Sobieszczanski-Sobieski, J. Multi-disciplinary design optimization of a full vehicle with high performance computing, AIAA Paper-2001-1273, 8th Symposium on ultidisciplinary Analysis and Optimization, St. Louis, 2001.
[26] Yang, R-J., Tho, C.H. and Gu, L. Recent development in multi-disciplinary design optimization of vehicle structures, AIAA-2002-5606, Ed. Mistree, F., 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Atlanta, 2002.
[27] Yang, R-J., Gu, L., Fu, Y. and Tho, C-H. Experience with sequential stochastic design improvement methods, Proceedings of the Second MIT Conference on Computational Fluid and Solid Mechanics, Cambridge, 2003.