# ANSYS LS-DYNA®
## KEYWORD USER'S MANUAL

## VOLUME II
### Material Models

**master@098456555 (10/29/26)**

**LS-DYNA Dev**

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This file contains the code for implementing the key schedule for AES (Rijndael) for block and key sizes of 16, 24, and 32 bytes.

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# *EOS

When defining an equation of state, the type of equation of state is specified by a corresponding 3-digit number in the command name, e.g., *EOS_004, or equivalently, by it's more descriptive designation, e.g., *EOS_GRUNEISEN. The equations of state can be used with a subset of the materials that are available for solid elements; see the MATERIAL MODEL REFERENCE TABLES in the beginning of the *MAT section of this Manual. *EOS_015 is linked to the type 2 thick shell element and can be used to model engine gaskets.

The meaning associated with particular extra history variables for a subset of material models and equations of state are tabulated at https://lsdyna.ansys.com/history-variables-for-certain-material-models/. The first three extra history variables when using an equation of state are (1) internal energy, (2) pressure due to bulk viscosity, and (3) the element volume from the previous time step.

| | |
|---|---|
| TYPE 1: | *EOS_LINEAR_POLYNOMIAL |
| TYPE 2: | *EOS_JWL |
| TYPE 3: | *EOS_SACK_TUESDAY |
| TYPE 4: | *EOS_GRUNEISEN |
| TYPE 5: | *EOS_RATIO_OF_POLYNOMIALS |
| TYPE 6: | *EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK |
| TYPE 7: | *EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE |
| TYPE 8: | *EOS_TABULATED_COMPACTION |
| TYPE 9: | *EOS_TABULATED |
| TYPE 10: | *EOS_PROPELLANT_DEFLAGRATION |
| TYPE 11: | *EOS_TENSOR_PORE_COLLAPSE |
| TYPE 12: | *EOS_IDEAL_GAS |
| TYPE 13: | *EOS_PHASE_CHANGE |
| TYPE 14: | *EOS_JWLB |

TYPE 15: *EOS_GASKET

TYPE 16: *EOS_MIE_GRUNEISEN

TYPE 19: *EOS_MURNAGHAN

TYPE 21-30: *EOS_USER_DEFINED

TYPE 40: *EOS_USER_LIBRARY

TYPE 41: *EOS_HVRB

TYPE 42: *EOS_USER_LIBRARY

TYPE 43: *EOS_LEE-TARVER

TYPE 44: *EOS_AERATED_WATER

An additional option **TITLE** may be appended to all the ***EOS** keywords. If this option is used, then an additional line is read for each section in 80a format, which can be used to describe the equation of state. At present, LS-DYNA does not make use of the title. The inclusion of a title simply gives greater clarity to input decks.

### Definitions and Conventions

In order to prescribe the boundary and/or initial thermodynamic condition, manual computations are often necessary. Conventions or definitions must be established to simplify this process. Some basic variables are defined in the following. Since many of these variables have already been denoted by different symbols, the notations used here are unique to this section only! They are presented only to clarify their usage. A corresponding SI unit set is also presented as an example.

First, consider a few volumetric parameters since they are a measure of compression (or expansion).

Volume:

$$V \approx (\mathrm{m}^3)$$

Mass:

$$M \approx (\mathrm{Kg})$$

Current specific volume (per mass):

$$v = \frac{V}{M} = \frac{1}{\rho} \approx \left(\frac{m^3}{\mathrm{Kg}}\right)$$

Reference specific volume:

$$v_0 = \frac{V_0}{M} = \frac{1}{\rho_0} \approx \left(\frac{m^3}{\text{Kg}}\right)$$

Relative volume:

$$v_r = \frac{V}{V_0} = \frac{(V/M)}{(V_0/M)} = \frac{v}{v_0} = \frac{\rho_0}{\rho}$$

Current normalized volume increment:

$$\frac{dv}{v} = \frac{v - v_0}{v} = 1 - \frac{1}{v_r} = 1 - \frac{\rho}{\rho_0}$$

A frequently used volumetric parameter is:

$$\mu = \frac{1}{v_r} - 1 = \frac{v_0 - v}{v} = -\frac{dv}{v} = \frac{\rho}{\rho_0} - 1$$

Sometimes another volumetric parameter is used:

$$\eta = \frac{v_0}{v} = \frac{\rho}{\rho_0}$$

Thus, the relation between $\mu$ and $\eta$ is,

$$\mu = \frac{v_0 - v}{v} = \eta - 1$$

The following table summarizes these volumetric parameters.

| VARIABLES | COMPRESSION | NO LOAD | EXPANSION |
|---|---|---|---|
| $v_r = \frac{v}{v_0} = \frac{\rho_0}{\rho}$ | $< 1$ | 1 | $> 1$ |
| $\eta = \frac{1}{v_r} = \frac{v_0}{v} = \frac{\rho}{\rho_0}$ | $> 1$ | 1 | $< 1$ |
| $\mu = \frac{1}{v_r} - 1 = \eta - 1$ | $> 0$ | 0 | $< 0$ |

### V0 – Initial Relative Volume

There are 3 definitions of density that must be distinguished from each other:

$\rho_0 = \rho_{\text{ref}}$
= Density at nominal/reference state, usually non-stress or non-deformed state.
$\rho|_{t=0}$ = Density at time 0
$\rho$ = Current density

Recalling the current relative volume

$$v_r = \frac{\rho_0}{\rho} = \frac{v}{v_0},$$

at time = 0 the relative volume is

$$v_{r0} = v_r|_{t=0} = \frac{\rho_0}{\rho|_{t=0}} = \frac{v|_{t=0}}{v_0}.$$

Generally, the V0 input parameter in an *EOS card refers to this $v_{r0}$. $\rho_0$ is generally the density defined in the *MAT card. Hence, if a material is mechanically compressed at t = 0, V0, or $v_{r0}$, the initial relative volume, may be computed and input accordingly ($v_0 \neq V0$).

The "reference" state is a unique state with respect to which the material stress tensor is computed. Therefore $v_0$ is very critical in computing the pressure level in a material. Incorrect choice of $v_0$ would lead to incorrect pressure computed. In general, $v_0$ is chosen such that at zero compression or expansion, the material should be in equilibrium with its ambient surrounding. In many of the equations shown in the EOS section, $\mu$ is frequently used as a measure of compression (or expansion). However, the users must clearly distinguish between $\mu$ and $v_{r0}$.

### E0 – Internal Energy

Internal energy represents the thermal energy state (temperature dependent component) of a system. One definition for internal energy is

$$E = MC_vT \approx (\text{Joule})$$

Note that the capital "$E$" here is the absolute internal energy. It is not the same as that used in the subsequent *EOS keyword input, or some equations shown for each *EOS card. This internal energy is often defined with respect to a mass or volume unit.

Internal energy per unit mass (also called specific internal energy):

$$e = \frac{E}{M} = C_VT \approx \left(\frac{\text{Joule}}{\text{Kg}}\right)$$

Internal energy per unit current volume:

$$e_V = \frac{M}{V}C_VT = \rho C_VT = \frac{C_VT}{v} \approx \left(\frac{\text{Joule}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2}\right)$$

Internal energy per unit reference volume:

$$e_{V_0} = \frac{M}{V_0}C_vT = \rho_0 C_vT = \frac{C_vT}{v_0} \approx \left(\frac{\text{Joule}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2}\right)$$

$e_{V_0}$ typically refers to the capital "E" shown in some equations under this "EOS" section. Hence the initial "*internal energy per unit reference volume*", E0, a keyword input parameter in the *EOS section can be computed from

$$e_{V_0}\big|_{t=0} = \rho_0 C_V T|_{t=0}$$

To convert from $e_{V0}$ to $e_V$, simply divide $e_{V0}$ by $v_r$

$$e_V = \rho C_V T = [\rho_0 C_V T]\frac{\rho}{\rho_0} = \frac{e_{V_0}}{v_r}$$

### Equations of States (EOS)

A thermodynamic state of a homogeneous material, not undergoing any chemical reactions or phase changes, may be defined by two state variables. This relation is generally called an equation of state. For example, a few possible forms relating pressure to two other state variables are

$$P = P(\rho, T) = P(v, e) = P(v_r, e_V) = P(\mu, e_{V_0})$$

The last equation form is frequently used to compute pressure. The EOS for solid phase materials is sometimes partitioned into 2 terms, a cold pressure and a thermal pressure

$$P = P_c(\mu) + P_T(\mu, e_{V_0})$$

$P_c(\mu)$ is the cold pressure hypothetically evaluated along a 0-degree-Kelvin isotherm. This is sometimes called a 0-K pressure-volume relation or cold compression curve. $P_T(\mu, e_{V_0})$ is the thermal pressure component that depends on both volumetric compression and thermal state of the material.

Different forms of the EOS describe different types of materials and how their volumetric compression (or expansion) behaviors. The coefficients for each EOS model come from data-fitting, phenomenological descriptions, or derivations based on classical thermodynamics, etc.

### Linear Compression

In low pressure processes, pressure is not significantly affected by temperature. When volumetric compression is within an elastic linear deformation range, a linear bulk modulus may be used to relate volume changes to pressure changes. Recalling the definition of an isotropic bulk modulus is [Fung 1965],

$$\frac{\Delta v}{v} = -\frac{P}{K}.$$

This may be rewritten as

$$P = K\left[-\frac{\Delta v}{v}\right] = K\mu.$$

The bulk modulus, $K$, thus is equivalent to $C_1$ in *EOS_LINEAR_POLYNOMIAL when all other coefficients are zero. This is a simplest form of an EOS. To initialize a pressure for such a material, only $v_{r_0}$ must be defined.

### Initial Conditions

In general, a thermodynamic state must be defined by two state variables. The need to specify $v_{r0}$ and/or $e_{V_0}|_{t=0}$ depends on the form of the EOS chosen. The user should review the equation term-by-term to establish what parameters to be initialized.

For many of the EOS available, pressure is specified (given), and the user must make an assumption on either $e_{V_0}|_{t=0}$ or $v_{r0}$. Consider two possibilities (1) $T|_{t=0}$ is defined or assumed from which $e_{V_0}|_{t=0}$ may be computed, or (2) $\rho|_{t=0}$ is defined or assumed from which $v_{r0}$ may be obtained.

### When to Use EOS

For small strains considerations, a total stress tensor may be partitioned into a deviatoric stress component and a mechanical pressure.

$$\sigma_{ij} = \sigma'_{ij} + \frac{\sigma_{kk}}{3}\delta_{ij} = \sigma'_{ij} - P\delta_{ij}$$

$$P = -\frac{\sigma_{kk}}{3}$$

The pressure component may be written from the diagonal stress components.

Note that $\frac{\sigma_{kk}}{3} = \frac{[\sigma_{11}+\sigma_{22}+\sigma_{33}]}{3}$ is positive in tension while P is positive in compression.

Similarly, the total strain tensor may be partitioned into a deviatoric strain component (volume-preserving deformation) and a volumetric deformation.

$$\varepsilon_{ij} = \varepsilon'_{ij} + \frac{\varepsilon_{kk}}{3}\delta_{ij}$$

where $\frac{\varepsilon_{kk}}{3}$ is called the mean normal strain, and $\varepsilon_{kk}$ is called the dilatation or volume strain (change in volume per unit initial volume)

$$\varepsilon_{kk} = \frac{V - V_0}{V_0}$$

Roughly speaking, a typical convention may refer to the relation $\sigma'_{ij} = f(\varepsilon'_{ij})$ as a "constitutive equation", and $P = f(\mu, e_{V_0})$ as an EOS. The use of an EOS may be omitted only when volumetric deformation is very small, and $|P| \ll |\sigma'_{ij}|$.

### A Note About Contact When Using an Equation of State

When a part includes an equation of state, it is important that the initial geometry of that part not be perturbed by the contact algorithm. Such perturbation can arise due to initial penetrations in the contact surfaces but can usually be avoided by setting the variable

IGNORE to 1 or 2 in the *CONTACT input or by using a segment based contact (SOFT = 2).

## *EOS_LINEAR_POLYNOMIAL

This is Equation of State Form 1.

Purpose: Define coefficients for a linear polynomial EOS, and initialize the thermodynamic state of the material by defining E0 and V0 below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | C0 | C1 | C2 | C3 | C4 | C5 | C6 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0 | V0 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| C0 | The $0^{th}$ polynomial equation coefficient |
| C1 | The $1^{st}$ polynomial equation coefficient (when used by itself, this is the *elastic* bulk modulus, meaning it cannot be used for deformation that is beyond the elastic regime). |
| ⋮ | ⋮ |
| C6 | The $6^{th}$ polynomial equation coefficient |
| E0 | Initial internal energy per unit reference volume (see the beginning of the *EOS section) |
| V0 | Initial relative volume (see the beginning of the *EOS section) |

**Remarks:**

1. **Pressure.** The linear polynomial equation of state is linear in internal energy. The pressure is given by:

$$P = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + \left(C_4 + C_5\mu + C_6\mu^2\right)E.$$

where terms $C_2\mu^2$ and $C_6\mu^2$ are set to zero if $\mu < 0$, $\mu = \rho/\rho_0 - 1$, and $\rho/\rho_0$ is the ratio of current density to reference density. $\rho$ is a nominal or reference density defined in the *MAT_NULL card.

The linear polynomial equation of state may be used to model gas with the gamma law equation of state. This may be achieved by setting:

$$C_0 = C_1 = C_2 = C_3 = C_6 = 0$$

and

$$C_4 = C_5 = \gamma - 1$$

where

$$\gamma = \frac{C_p}{C_v}$$

is the ratio of specific heats. Pressure for a perfect gas is then given by:

$$P = (\gamma - 1)\frac{\rho}{\rho_0}E = (\gamma - 1)\frac{e_{V_0}}{v_r}$$

$E$ has the units of pressure (where $E = e_{V_0}$ and $v_r = \rho_0/\rho$).

2. **Initial Pressure.** When $C_0 = C_1 = C_2 = C_3 = C_6 = 0$, it does not necessarily mean that the initial pressure is zero ($P_0 \neq C_0$!). The initial pressure depends on the values of all the coefficients and on $\mu|_{t=0}$ and $E|_{t=0}$. The pressure in a material is computed from the whole equation above, $P = P(\mu, E)$. It is always preferable to initialize the initial condition based on $\mu|_{t=0}$ and $E|_{t=0}$. The use of $C_0 = C_1 = C_2 = C_3 = C_6 = 0$ must be done with caution as it may change the form and behavior of the material. The safest way is to use the whole EOS equation to manually check for the pressure value. For example, for an ideal gas, only $C_4$ and $C_5$ are nonzero; $C_4$ and $C_5$ are equal to $\gamma - 1$ while all other coefficients ($C_0$, $C_1$, $C_2$, $C_3$, and $C_6$) are zero to satisfy the perfect gas equation form.

3. **V0 and E0.** V0 and E0 defined in this card must be the same as the time-zero ordinates for the 2 load curves defined in the *BOUNDARY_AMBIENT_EOS card, if it is used. This is so that they would both consistently define the same initial state for a material.

***EOS_JWL**_{OPTION}

This is Equation of State Form 2.

Available options are:

<BLANK>

AFTERBURN

### Card Summary:

**Card 1.** This card is required.

| EOSID | A | B | R1 | R2 | OMEG | E0 | V0 |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if and only if the AFTERBURN keyword option is used and OPT = 1 or 2.

| OPT | QT | T1 | T2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 2b.** This card is included if and only if the AFTERBURN keyword option is used and OPT = 3.

| OPT | Q0 | QA | QM | QN | CONM | CONL | CONT |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | A | B | R1 | R2 | OMEG | E0 | V0 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| A | $A$, see Remarks. |
| B | $B$, see Remarks. |
| R1 | $R_1$, see Remarks. |

| VARIABLE | DESCRIPTION |
|---|---|
| R2 | $R_2$, see Remarks. |
| OMEG | $\omega$, see Remarks. |
| E0 | Detonation energy per unit initial volume and initial value for $e_{V_0}$. See Remarks. |
| V0 | Initial relative volume, which gives the initial value for $v_r$. See Remarks. |

**Afterburn Card.** Additional card for AFTERBURN option with OPT = 1 or 2.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OPT | QT | T1 | T2 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| OPT | Afterburn option:<br>EQ.0.0: No afterburn energy (Standard *EOS_JWL)<br>EQ.1.0: Constant rate of afterburn energy added between times T1 and T2<br>EQ.2.0: Linearly increasing rate of afterburn energy added between times T1 and T2<br>EQ.3.0: Miller's extension for afterburn energy |
| QT | Afterburn energy per unit volume for simple afterburn |
| T1 | Start time of energy addition for simple afterburn |
| T2 | End time of energy addition for simple afterburn |

**Afterburn Card.** Additional card for AFTERBURN option with OPT = 3.

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OPT | Q0 | QA | QM | QN | CONM | CONL | CONT |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | 0.5 | 1/6 | 1. | 1. | 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| OPT | Afterburn option<br>EQ.0.0: No afterburn energy (Standard *EOS_JWL)<br>EQ.1.0: Constant rate of afterburn energy added between times T1 and T2<br>EQ.2.0: Linearly increasing rate of afterburn energy added between times T1 and T2<br>EQ.3.0: Miller's extension for afterburn energy |
| Q0 | Afterburn energy per unit volume for Miller's extension |
| QA | Energy release constant *a* for Miller's extension |
| QM | Energy release exponent *m* for Miller's extension |
| QN | Pressure exponent *n* for Miller's extension |
| CONM | Mass Conversion factors:<br>GT.0.0: Mass conversion factor from model units to calibration units for Miller's extension<br>LT.0.0: Use predefined factors to convert model units to published calibration units of g, cm, μs. Choices for model units are:<br>EQ.-1.0: g, mm, ms<br>EQ.-2.0: g, cm, ms<br>EQ.-3.0: kg, m, s<br>EQ.-4.0: kg, mm, ms<br>EQ.-5.0: metric ton, mm, s |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-6.0: lbf-s$^2$/in, in, s |
| | EQ.-7.0: slug, ft, s |
| CONL | CONM.GT.0.0: Length conversion factor from model units to calibration units for Miller's extension |
| | CONM.LT.0.0: Ignored |
| CONT | CONM.GT.0.0: Time conversion factor from model units to calibration units for Miller's extension |
| | CONM.LT.0.0: Ignored |

**Remarks:**

1. **Equation of State.** The JWL equation of state defines the pressure as

$$p = A\left(1 - \frac{\omega}{R_1 v_r}\right)e^{-R_1 v_r} + B\left(1 - \frac{\omega}{R_2}\right)e^{-R_2 v_r} + \frac{\omega e_{V_0}}{v_r},$$

and is usually used for detonation products of high explosives.

A, B, and E0 have units of pressure. R1, R2, OMEG, and V0 are dimensionless. We recommend using a unit system of grams, centimeters, and microseconds when a model includes high explosive(s). In this consistent unit system, pressure is in Mbar.

2. **Afterburn.** The AFTERBURN option allows the addition of afterburn energy $Q$ to the calculation of pressure by replacing $e_{V_0}$ in the above equation with $(e_{V_0} + Q)$, that is, the last term on the right-hand side becomes

$$\frac{\omega(e_{V_0} + Q)}{v_r}$$

The simple afterburn option adds the energy at a constant rate (OPT = 1) or a linearly-increasing rate (OPT = 2) between times T1 and T2 such that the total energy added per unit volume at time T2 is the specified energy QT.

For the Miller's extension model (OPT = 3), the afterburn energy is added using a time-dependent growth term

$$\frac{d\lambda}{dt} = a(1-\lambda)^m p^n, \qquad Q = \lambda Q_0 \ .$$

Here, $m$, $n$, and $\lambda$ are dimensionless, with $\lambda$ a positive fraction less than 1.0. The parameter $a$ has units consistent with this growth equation, and $Q_0$ has units of pressure.

The values for $Q_0$, $a$, $m$, $n$ published by Miller and Guirguis (1993) are calibrated in the units of g, cm, μs, with the consistent pressure unit of Mbar, though in principle any consistent set of units may be used for calibration. The factors CONM, CONL, and CONT convert the unit system of the model being analyzed to the calibration unit system in which the Miller's extension parameters are specified, such that a mass value in model units may be multiplied by CONM to obtain the corresponding value in calibration units. These conversion factors allow consistent evaluation of the growth equation in the calibrated units. For user convenience, predefined conversion factors are provided for converting various choices for the model units system to the calibration unit system used by Miller and Guirguis.

3. **History Variables.** When this equation of state is used with *MAT_HIGH_EXPLOSIVE_BURN in which the variable BETA is set to 0 or 2, the absolute value of the history variable labeled as "effective plastic strain" is the explosive lighting time. This lighting time takes into account shadowing if invoked (see *CONTROL_EXPLOSIVE_SHADOW).

   There are four additional history variables for the JWL equation of state. Those history variables are internal energy, bulk viscosity in units of pressure, volume, and burn fraction, respectively. To output the history variables, set the variable NEIPH in *DATABASE_EXTENT_BINARY.

   The AFTERBURN option introduces an additional 5$^{th}$ history variable that records the added afterburn energy $Q$ for simple afterburn (OPT = 1 or 2) but contains the growth term $\lambda$ when using the Miller's extension model (OPT = 3).

## *EOS_SACK_TUESDAY

This is Equation of State Form 3.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | A1 | A2 | A3 | B1 | B2 | E0 | V0 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| A*i*, B*i* | Constants in the equation of state |
| E0 | Initial internal energy |
| V0 | Initial relative volume |

**Remarks:**

The Sack equation of state defines pressure as

$$p = \frac{A_3}{V^{A_1}} e^{-A_2 V} \left(1 - \frac{B_1}{V}\right) + \frac{B_2}{V} E$$

and is used for detonation products of high explosives.

### *EOS_GRUNEISEN

This is Equation of State Form 4.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | C | S1 | S2 | S3 | GAMMA0 | A | E0 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V0 | (not used) | LCID | | | | | |
| Type | F | | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| C, S*i*, GAMMA0 | Constants in the equation of state |
| A | First order volume correction coefficient |
| E0 | Initial internal energy |
| V0 | Initial relative volume |
| LCID | Load curve ID, which can be the ID of a *DEFINE_CURVE, *DEFINE_CURVE_FUNCTION, or *DEFINE_FUNCTION, defining the energy deposition rate as a function of time. If an energy leak rate is intended, do not specify a negative ordinate in LCID, rather, use the constant(s) in the equation of state, that is, set GAMMA0 and/or A to a negative value. If *DEFINE_FUNCTION is used, the input of the defined function is time. |

**Remarks:**

The Gruneisen equation of state with cubic shock-velocity as a function of particle-velocity $v_s(v_p)$ defines pressure for compressed materials as

$$p = \frac{\rho_0 C^2 \mu \left[1 + \left(1 - \frac{\gamma_0}{2}\right)\mu - \frac{a}{2}\mu^2\right]}{\left[1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^3}{(\mu + 1)^2}\right]^2} + (\gamma_0 + a\mu)E$$

and for expanded materials as

$$p = \rho_0 C^2 \mu + (\gamma_0 + a\mu)E \ .$$

Here $C$ is the intercept of the $v_s(v_p)$ curve (in velocity units); $S_1$, $S_2$, and $S_3$ are the unitless coefficients of the slope of the $v_s(v_p)$ curve; $\gamma_0$ is the unitless Gruneisen gamma; $a$ is the unitless, first order volume correction to $\gamma_{0;}$ and

$$\mu = \frac{\rho}{\rho_0} - 1 \ .$$

$E$ denotes the internal energy, which is increased according to an energy deposition rate as a function of time curve (LCID).

## *EOS_RATIO_OF_POLYNOMIALS

This is Equation of State Form 5.

### Card Summary:

**Card 1.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| EOSID | | | | | | | |

**Card 2.** This card is required.

| | | | |
|---|---|---|---|
| A10 | A11 | A12 | A13 |

**Card 3.** This card is required.

| | | | |
|---|---|---|---|
| A20 | A21 | A22 | A23 |

**Card 4.** This card is required.

| | | | |
|---|---|---|---|
| A30 | A31 | A32 | A33 |

**Card 5.** This card is required.

| | | | |
|---|---|---|---|
| A40 | A41 | A42 | A43 |

**Card 6.** This card is required.

| | | | |
|---|---|---|---|
| A50 | A51 | A52 | A53 |

**Card 7.** This card is required.

| | | | |
|---|---|---|---|
| A60 | A61 | A62 | A63 |

**Card 8.** This card is required.

| | | | |
|---|---|---|---|
| A70 | A71 | A72 | A73 |

**Card 9.** This card is required.

| | | | |
|---|---|---|---|
| A14 | A24 | | |

**Card 10.** This card is required.

| | | | |
|---|---|---|---|
| ALPHA | BETA | E0 | V0 |

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | | | | | | | |
| Type | A | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A10 | | A11 | | A12 | | A13 | |
| Type | F | | F | | F | | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A20 | | A21 | | A22 | | A23 | |
| Type | F | | F | | F | | F | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A30 | | A31 | | A32 | | A33 | |
| Type | F | | F | | F | | F | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A40 | | A41 | | A42 | | A43 | |
| Type | F | | F | | F | | F | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A50 | | A51 | | A52 | | A53 | |
| Type | F | | F | | F | | F | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A60 | | A61 | | A62 | | A63 | |
| Type | F | | F | | F | | F | |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A70 | | A71 | | A72 | | A73 | |
| Type | F | | F | | F | | F | |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A14 | | A24 | | | | | |
| Type | F | | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A*ij* | Polynomial coefficients |

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | | BETA | | E0 | | V0 | |
| Type | F | | F | | F | | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | $\alpha$ |
| BETA | $\beta$ |
| E0 | Initial internal energy |
| V0 | Initial relative volume |

**Remarks:**

The ratio of polynomials equation of state defines the pressure as

$$p = \frac{F_1 + F_2 E + F_3 E^2 + F_4 E^3}{F_5 + F_6 E + F_7 E^2}(1 + \alpha\mu) \ ,$$

where

$$F_i = \sum_{j=0}^{n} A_{ij}\mu^j \ , \qquad n = \begin{cases} 4 & i < 3 \\ 3 & i \geq 3 \end{cases}$$

$$\mu = \frac{\rho}{\rho_0} - 1$$

In expanded elements $F_1$ is replaced by $F_1' = F_1 + \beta\mu^2$. By setting coefficient $A_{10} = 1.0$, the delta-phase pressure modeling for this material will be initiated. The code will reset it to 0.0 after setting flags.

## *EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK

This is Equation of State Form 6.

Purpose: Define coefficients for a linear polynomial EOS and initialize the thermodynamic state of the material by defining E0 and V0 below. Energy deposition is prescribed using a curve.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | C0 | C1 | C2 | C3 | C4 | C5 | C6 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0 | V0 | LCID | | | | | |
| Type | F | F | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see*PART). |
| C*i* | Constants in the equation of state |
| E0 | Initial internal energy |
| V0 | Initial relative volume |
| LCID | Load curve ID, which can be the ID of *DEFINE_CURVE, *DEFINE_CURVE_FUNCTION or *DEFINE_FUNCTION, defining the specific energy deposition rate as a function of time. If a specific energy leak rate is intended, do not specify a negative ordinate in LCID, rather, use the constant(s) in the equation of state, such as setting C4 to a negative value. If *DEFINE_FUNCTION is used, the input of the defined function is time. Note that the specific energy rate has units of energy per time and volume. |

**Remarks:**

This polynomial equation of state, linear in the internal energy per initial volume, $E$, is given by

$$p = C_0 + C_1\mu + C_2\mu^2 + C_3\mu^3 + \left(C_4 + C_5\mu + C_6\mu^2\right)E$$

in which $C_1$, $C_2$, $C_3$, $C_4$, $C_5$, and $C_6$ are user defined constants and

$$\mu = \frac{1}{V} - 1 \ .$$

where $V$ is the relative volume. In expanded elements, we set the coefficients of $\mu^2$ to zero, that is,

$$C_2 = C_6 = 0$$

Internal energy, $E$, is increased according to an energy deposition rate as a function of time curve (LCID).

## *EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE

This is Equation of State Form 7.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | A | B | XP1 | XP2 | FRER | G | R1 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R2 | R3 | R5 | R6 | FMXIG | FREQ | GROW1 | EM |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AR1 | ES1 | CVP | CVR | EETAL | CCRIT | ENQ | TMP0 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GROW2 | AR2 | ES2 | EN | FMXGR | FMNGR | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| A | Product JWL constant (see second equation in Remarks) |
| B | Product JWL constant (see second equation in Remarks) |
| XP1 | Product JWL constant (see second equation in Remarks) |

| VARIABLE | DESCRIPTION |
|---|---|
| XP2 | Product JWL constant (see second equation in Remarks) |
| FRER | Constant in ignition term of reaction equation |
| G | $\omega C_v$ of product |
| R1 | Unreacted JWL constant (see first equation in Remarks) |
| R2 | Unreacted JWL constant (see first equation in Remarks) |
| R3 | $\omega C_v$ of unreacted explosive |
| R5 | Unreacted JWL constant (see first equation in Remarks) |
| R6 | Unreacted JWL constant (see first equation in Remarks) |
| FMXIG | Maximum $F$ for ignition term |
| FREQ | Constant in ignition term of reaction equation |
| GROW1 | Constant in growth term of reaction equation |
| EM | Constant in growth term of reaction equation |
| AR1 | Constant in growth term of reaction equation |
| ES1 | Constant in growth term of reaction equation |
| CVP | Heat capacity of reaction products |
| CVR | Heat capacity of unreacted HE |
| EETAL | Constant in ignition term of reaction equation |
| CCRIT | Constant in ignition term of reaction equation |
| ENQ | Heat of reaction |
| TMP0 | Initial temperature (°K) |
| GROW2 | Constant in completion term of reaction equation |
| AR2 | Constant in completion term of reaction equation |
| ES2 | Constant in completion term of reaction equation |
| EN | Constant in completion term of reaction equation |

| VARIABLE | DESCRIPTION |
|---|---|
| FMXGR | Maximum $F$ for growth term |
| FMNGR | Minimum $F$ for completion term |

**Remarks:**

Equation of State Form 7 is used to calculate the shock initiation (or failure to initiate) and detonation wave propagation of solid high explosives. It should be used instead of the ideal HE burn options whenever there is a question whether the HE will react, there is a finite time required for a shock wave to build up to detonation, and/or there is a finite thickness of the chemical reaction zone in a detonation wave. At relatively low initial pressures (<2-3 GPa), this equation of state should be used with material type 10 for accurate calculations of the unreacted HE behavior. At higher initial pressures, material type 9 can be used. A JWL equation of state defines the pressure in the unreacted explosive as

$$P_e = r_1 e^{-r_5 V_e} + r_2 e^{-r_6 V_e} + r_3 \frac{T_e}{V_e}, \qquad \left(r_3 = \omega_e \mathrm{C_{v_r}}\right)$$

where $V_e$ and $T_e$ are the relative volume and temperature, respectively, of the unreacted explosive. Another JWL equation of state defines the pressure in the reaction products as

$$P_p = a e^{-x p_1 V_p} + b e^{-x p_2 V_p} + \frac{g T_p}{V_p}, \qquad \left(g = \omega_p \mathrm{C_{v_p}}\right)$$

where $V_p$ and $T_p$ are the relative volume and temperature, respectively, of the reaction products. As the chemical reaction converts unreacted explosive to reaction products, these JWL equations of state are used to calculate the mixture of unreacted explosive and reaction products defined by the fraction reacted $F$ ($F = 0$ implies no reaction, $F = 1$ implies complete reaction). The temperatures and pressures are assumed to be equal ($T_e = T_p, p_e = p_p$) and the relative volumes are additive, that is,

$$V = (1 - F)V_e + FV_p$$

The chemical reaction rate for conversion of unreacted explosive to reaction products consists of three physically realistic terms: an ignition term in which a small amount of explosive reacts soon after the shock wave compresses it; a slow growth of reaction as this initial reaction spreads; and a rapid completion of reaction at high pressure and temperature. The form of the reaction rate equation is

$$\frac{\partial F}{\partial t} = \overbrace{\mathrm{FREQ} \times (1 - F)^{\mathrm{FRER}} (V_e^{-1} - 1 - \mathrm{CCRIT})^{\mathrm{EETAL}}}^{\text{Ignition}} + \overbrace{\mathrm{GROW1} \times (1 - F)^{\mathrm{ES1}} F^{\mathrm{AR1}} p^{\mathrm{EM}}}^{\text{Growth}} + \underbrace{\mathrm{GROW2} \times (1 - F)^{\mathrm{ES2}} F^{\mathrm{AR2}} p^{\mathrm{EN}}}_{\text{Completion}}$$

The ignition rate is set equal to zero when $F \geq$ FMXIG, the growth rate is set equal to zero when $F \geq$ FMXGR, and the completion rate is set equal to zero when $F \leq$ FMNGR.

Details of the computational methods and many examples of one and two dimensional shock initiation and detonation wave calculation can be found in the references (Cochran and Chan [1979], Lee and Tarver [1980]). Unfortunately, sufficient experimental data has been obtained for only two solid explosives to develop very reliable shock initiation models: PBX-9504 (and the related HMX-based explosives LX-14,LX-10,LX-04, etc.) and LX-17 (the insensitive TATB-based explosive). Reactive flow models have been developed for other explosives (TNT, PETN, Composition B, propellants, etc.) but are based on very limited experimental data.

When this EOS is used with *MAT_009, history variables 4, 7, 9, and 10 are temperature, burn fraction, $1/V_e$, and $1/V_p$, respectively. When used with *MAT_010, those histories variables are incremented by 1, that is, history variables 5, 8, 10, and 11 are temperature, burn fraction, $1/V_e$, and $1/V_p$, respectively. See NEIPH in *DATABASE_EXTENT_BINARY if these output variables are desired in the databases for post-processing.

## *EOS_TABULATED_COMPACTION

This is Equation of State Form 8.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | GAMMA | E0 | V0 | LCC | LCT | LCK | LCID |
| Type | A | F | F | F | I | I | I | I |

**Parameter Card Pairs.** Include one pair of the following two cards for each of VAR = $\varepsilon_{v_i}$, $C_i$, $T_i$, and $K_i$. These cards consist of four additional pairs for a total of 8 additional cards.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable | [VAR]1 | | [VAR]2 | | [VAR]3 | | [VAR]4 | | [VAR]5 | |
| Type | F | | F | | F | | F | | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable | [VAR]6 | | [VAR]7 | | [VAR]8 | | [VAR]9 | | [VAR]10 | |
| Type | F | | F | | F | | F | | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| GAMMA | $\gamma$ (unitless); see equation in Remarks. |
| E0 | Initial internal energy per unit reference volume (force per unit area) |
| V0 | Initial relative volume (unitless) |
| LCC | Load curve defining tabulated function $C$. See equation in Remarks. The abscissa values of LCC, LCT and LCK must be negative of the volumetric strain in monotonically *increasing* order, in |

| VARIABLE | DESCRIPTION |
|---|---|
| | contrast to the convention in *EOS_009. The definition can extend into the tensile regime. |
| LCT | Load curve defining tabulated function $T$. See equation in Remarks. |
| LCK | Load curve defining tabulated bulk modulus |
| LCID | Load curve ID, which can be the ID of *DEFINE_CURVE, *DEFINE_CURVE_FUNCTION, or *DEFINE_FUNCTION, defining the energy deposition rate as a function of time. If an energy leak rate is intended, do not specify a negative ordinate in LCID, rather, use the constant(s) in the equation of state, that is, set GAMMA to a negative value. If *DEFINE_FUNCTION is used, the input of the defined function is time. |
| $\varepsilon_{v1}, \varepsilon_{v2}, \ldots, \varepsilon_{v_N}$ | Volumetric strain, $\ln(V)$. The first abscissa point, EV1, must be 0.0 or positive if the curve extends into the tensile regime with subsequent points *decreasing* monotonically. |
| $C_1, C_2, \ldots, C_N$ | $C(\varepsilon_V)$ (units = force per unit area); see equation in Remarks. |
| $T_1, T_2, \ldots, T_N$ | $T(\varepsilon_V)$ (unitless) ; see equation in Remarks. |
| $K_1, K_2, \ldots, K_N$ | Bulk unloading modulus (units = force per unit area) |

**Remarks:**

The tabulated compaction model is linear in the internal energy $E$, which increases according to an energy deposition rate as a function of time curve (LCID). Pressure is defined by

$$p = C(\varepsilon_V) + \gamma\, T(\varepsilon_V) E$$

in the loading phase. The volumetric strain, $\varepsilon_V$, is given by the natural logarithm of the relative volume, $V$. Unloading occurs along the unloading bulk modulus to the pressure cutoff. The minimum bulk unloading modulus must be defined by $K_1$. The pressure cutoff, a tension limit, is defined in the material model definition. Reloading always follows the unloading path to the point where unloading began and continues on the loading path; see Figure EOS8-1. Up to 10 points and as few as 2 may be used when defining the tabulated functions. LS-DYNA will extrapolate to find the pressure if necessary.

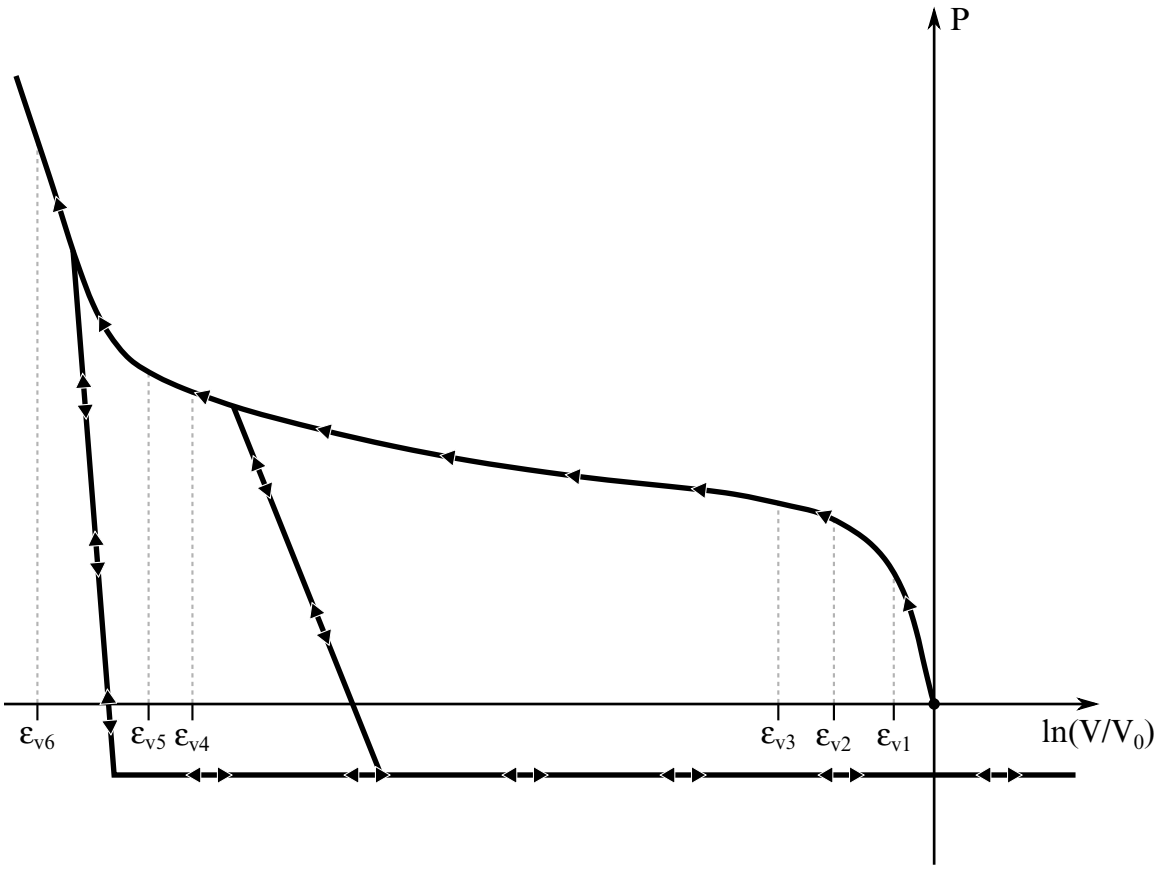



**Figure EOS8-1.** Pressure as a function of volumetric strain curve for Equation of State Form 8 with compaction. In the compacted states the bulk unloading modulus depends on the peak volumetric strain. Volumetric strain values should be input with correct sign (negative in compression) and in descending order. Pressure is positive in compression.

## *EOS_TABULATED

This is Equation of State Form 9.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | GAMA | E0 | V0 | LCC | LCT | | |
| Type | A | F | F | F | I | I | | |

**Parameter Card Pairs.** Include one pair of the following two cards for each of VAR = $\varepsilon_{V_i}$, $C_i$, $T_i$. These cards consist of three additional pairs for a total of 6 additional cards. These cards are not required if LCC and LCT are specified.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable | [VAR]1 | | [VAR]2 | | [VAR]3 | | [VAR]4 | | [VAR]5 | |
| Type | F | | F | | F | | F | | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Variable | [VAR]6 | | [VAR]7 | | [VAR]8 | | [VAR]9 | | [VAR]10 | |
| Type | F | | F | | F | | F | | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| GAMA | $\gamma$, (unitless) see equation in Remarks. |
| E0 | Initial internal energy per unit reference volume (force per unit area). |
| V0 | Initial relative volume (unitless). |

| VARIABLE | DESCRIPTION |
|---|---|
| LCC | Load curve defining tabulated function $C$. See equation in Remarks. The abscissa values of LCC and LCT must *increase* monotonically. The definition can extend into the tensile regime. |
| LCT | Load curve defining tabulated function $T$. See equation in Remarks. |
| $\varepsilon_{V_1}, \varepsilon_{V_2}, \ldots, \varepsilon_{V_N}$ | Volumetric strain, $\ln(V)$, where $V$ is the relative volume. The first abscissa point, EV1, must be 0.0 or positive if the curve extends into the tensile regime with subsequent points *decreasing* monotonically. |
| $C_1, C_2, \ldots, C_N$ | Tabulated points for function $C$ (force per unit area). |
| $T_1, T_2, \ldots, T_N$ | Tabulated points for function $T$ (unitless). |

**Remarks:**

The tabulated equation of state model is linear in internal energy. Pressure is defined by

$$P = C(\varepsilon_V) + \gamma T(\varepsilon_V)E$$

The volumetric strain, $\varepsilon_V$ is given by the natural logarithm of the relative volume $V$. Up to 10 points and as few as 2 may be used when defining the tabulated functions. LS-DYNA will extrapolate to find the pressure if necessary.

## *EOS_PROPELLANT_DEFLAGRATION

This is Equation of State Form 10. It has been added to model airbag propellants.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | A | B | XP1 | XP2 | FRER | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G | R1 | R2 | R3 | R5 | | | |
| Type | F | F | F | F | F | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R6 | FMXIG | FREQ | GROW1 | EM | | | |
| Type | F | F | F | F | F | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AR1 | ES1 | CVP | CVR | EETAL | CCRIT | ENQ | TMP0 |
| Type | F | F | F | F | F | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GROW2 | AR2 | ES2 | EN | FMXGR | FMNGR | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| A | Product JWL coefficient |
| B | Product JWL coefficient |
| XP1 | Product JWL coefficient |
| XP2 | Product JWL coefficient |
| FRER | Unreacted Co-volume |
| G | Product $\omega C_v$ |
| R1 | Unreacted JWL coefficient |
| R2 | Unreacted JWL coefficient |
| R3 | Unreacted $\omega C_v$ |
| R5 | Unreacted JWL coefficient |
| R6 | Unreacted JWL coefficient |
| FMXIG | Initial Fraction Reacted $F_0$ |
| FREQ | Initial Pressure $P_0$ |
| GROW1 | First burn rate coefficient |
| EM | Pressure Exponent ($1^{st}$ term) |
| AR1 | Exponent on $F$ ($1^{st}$ term) |
| ES1 | Exponent on $(1-F)$ ($1^{st}$ term) |
| CVP | Heat capacity $C_v$ for products |
| CVR | Heat capacity $C_v$ for unreacted material |
| EETAL | Extra, not presently used |
| CCRIT | Product co-volume |
| ENQ | Heat of Reaction |

| VARIABLE | DESCRIPTION |
|---|---|
| TMP0 | Initial Temperature (298°K) |
| GROW2 | Second burn rate coefficient |
| AR2 | Exponent on $F$ ($2^{nd}$ term) |
| ES2 | Exponent on $(1 - F)$ ($2^{nd}$ term) |
| EN | Pressure Exponent ($2^{nd}$ term) |
| FMXGR | Maximum $F$ for $1^{st}$ term |
| FMNGR | Minimum $F$ for $2^{nd}$ term |

**Remarks:**

A deflagration (burn rate) reactive flow model requires an unreacted solid equation of state, a reaction product equation of state, a reaction rate law and a mixture rule for the two (or more) species. The mixture rule for the standard ignition and growth model [Lee and Tarver 1980] assumes that both pressures and temperatures are completely equilibrated as the reaction proceeds. However, the mixture rule can be modified to allow no thermal conduction or partial heating of the solid by the reaction product gases. For this relatively slow process of airbag propellant burn, the thermal and pressure equilibrium assumptions are valid. The equations of state currently used in the burn model are the JWL, Gruneisen, the van der Waals co-volume, and the perfect gas law, but other equations of state can be easily implemented. In this propellant burn, the gaseous nitrogen produced by the burning sodium azide obeys the perfect gas law as it fills the airbag but may have to be modeled as a van der Waal's gas at the high pressures and temperatures produced in the propellant chamber. The chemical reaction rate law is pressure, particle geometry and surface area dependent, as are most high-pressure burn processes. When the temperature profile of the reacting system is well known, temperature dependent Arrhenius chemical kinetics can be used.

Since the airbag propellant composition and performance data are company private information, it is very difficult to obtain the required information for burn rate modeling. However, Imperial Chemical Industries (ICI) Corporation supplied pressure exponent, particle geometry, packing density, heat of reaction, and atmospheric pressure burn rate data which allowed us to develop the numerical model presented here for their $NaN_3$ + $Fe_2O_3$ driver airbag propellant. The deflagration model, its implementation, and the results for the ICI propellant are presented in [Hallquist, et.al., 1990].

The unreacted propellant and the reaction product equations of state are both of the form:

$$p = Ae^{-R_{1V}} + Be^{-R_{2V}} + \frac{\omega C_{vT}}{V - d} \ ,$$

where $p$ is pressure (in Mbars), $V$ is the relative specific volume (inverse of relative density), $\omega$ is the Gruneisen coefficient, $C_v$ is heat capacity (in Mbars -cc/cc°K), $T$ is the temperature in °K, $d$ is the co-volume, and $A$, $B$, $R_1$ and $R_2$ are constants. Setting $A = B = 0$ yields the van der Waal's co-volume equation of state. The JWL equation of state is generally useful at pressures above several kilobars, while the van der Waal's is useful at pressures below that range and above the range for which the perfect gas law holds. Additionally, setting $A = B = d = 0$ yields the perfect gas law. If accurate values of $\omega$ and $C_v$ plus the correct distribution between "cold" compression and internal energies are used, the calculated temperatures are very reasonable and thus can be used to check propellant performance.

The reaction rate used for the propellant deflagration process is of the form:

$$\frac{\partial F}{\partial t} = \underbrace{Z(1-F)^y F^x p^w}_{0<F<F_{\text{limit1}}} + \underbrace{V(1-F)^u F^r p^s}_{F_{\text{limit2}}<F<1}$$

where $F$ is the fraction reacted ($F = 0$ implies no reaction, $F = 1$ is complete reaction), $t$ is time, and $p$ is pressure (in Mbars), $r$, $s$, $u$, $w$, $x$, $y$, $F_{\text{limit1}}$ and $F_{\text{limit2}}$ are constants used to describe the pressure dependence and surface area dependence of the reaction rates. Two (or more) pressure dependant reaction rates are included in case the propellant is a mixture or exhibited a sharp change in reaction rate at some pressure or temperature. Burning surface area dependencies can be approximated using the $(1-F)^y F^x$ terms. Other forms of the reaction rate law, such as Arrhenius temperature dependent $e^{-E/RT}$ type rates, can be used, but these require very accurate temperatures calculations. Although the theoretical justification of pressure dependent burn rates at kilobar type pressures is not complete, a vast amount of experimental burn rate as a function of pressure data does demonstrate this effect and hydrodynamic calculations using pressure dependent burn accurately simulate such experiments.

The deflagration reactive flow model is activated by any pressure or particle velocity increase on one or more zone boundaries in the reactive material. Such an increase creates pressure in those zones and the decomposition begins. If the pressure is relieved, the reaction rate decreases and can go to zero. This feature is important for short duration, partial decomposition reactions. If the pressure is maintained, the fraction reacted eventually reaches one and the material is completely converted to product molecules. The deflagration front rates of advance through the propellant calculated by this model for several propellants are quite close to the experimentally observed burn rate versus pressure curves.

To obtain good agreement with experimental deflagration data, the model requires an accurate description of the unreacted propellant equation of state, either an analytical fit to experimental compression data or an estimated fit based on previous experience with

similar materials. This is also true for the reaction products equation of state. The more experimental burn rate, pressure production and energy delivery data available, the better the form and constants in the reaction rate equation can be determined.

Therefore, the equation used in the burn subroutine for the pressure in the unreacted propellant is

$$P_u = \mathrm{R1} \times e^{-\mathrm{R5} \times V_u} + \mathrm{R2} \times e^{-\mathrm{R6} \backslash \mathrm{times}\ V_u} + \frac{\mathrm{R3} \times T_u}{V_u - \mathrm{FRER}} \ ,$$

where $V_u$ and $T_u$ are the relative volume and temperature respectively of the unreacted propellant. The relative density is obviously the inverse of the relative volume. The pressure $P_p$ in the reaction products is given by:

$$P_p = \mathrm{A} \times e^{-\mathrm{XP1} \times V_p} + \mathrm{B} \times e^{-\mathrm{XP2} \times V_p} + \frac{G \times T_p}{V_P - \mathrm{CCRIT}} \ .$$

As the reaction proceeds, the unreacted and product pressures and temperatures are assumed to be equilibrated ($T_u = T_p = T$, $P = P_u = P_p$) while the relative volumes are additive:

$$V = (1 - F)V_u + FV_p$$

where $V$ is the total relative volume. Other mixture assumptions can and have been used in different versions of DYNA2D/3D. The reaction rate law has the form:

$$\frac{\partial F}{\partial t} = \mathrm{GROW1} \times (P + \mathrm{FREQ})^{\mathrm{EM}} (F + \mathrm{FMXIG})^{\mathrm{AR1}} (1 - F + \mathrm{FMIXG})^{\mathrm{ES1}} + \mathrm{GROW2} \times (P + \mathrm{FREQ})^{\mathrm{EN}} (F + \mathrm{FMIXG})^{\mathrm{AR2}} (1 - F + \mathrm{FMIXG})^{\mathrm{ES2}} \ .$$

If $F$ exceeds FMXGR, the GROW1 term is set equal to zero, and, if $F$ is less than FMNGR, the GROW2 term is zero. Thus, two separate (or overlapping) burn rates can be used to describe the rate at which the propellant decomposes.

This equation of state subroutine is used together with a material model to describe the propellant. In the airbag propellant case, a null material model (type #10) can be used. Material type 10 is usually used for a solid propellant or explosive when the shear modulus and yield strength are defined. The propellant material is defined by the material model and the unreacted equation of state until the reaction begins. The calculated mixture states are used until the reaction is complete and then the reaction product equation of state is used. The heat of reaction, ENQ, is assumed to be a constant and the same at all values of $F$ but more complex energy release laws could be implemented.

History variables 4 and 7 are temperature and burn fraction, respectively. See NEIPH in *DATABASE_EXTENT_BINARY if these output variables are desired in the databases for post-processing.

## *EOS_TENSOR_PORE_COLLAPSE

This is Equation of State Form 11.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | NLD | NCR | MU1 | MU2 | IE0 | EC0 | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| NLD | Virgin loading load curve ID |
| NCR | Completely crushed load curve ID |
| MU1 | Excess Compression required before any pores can collapse |
| MU2 | Excess Compression point where the Virgin Loading Curve and the Completely Crushed Curve intersect |
| IE0 | Initial Internal Energy |
| EC0 | Initial Excess Compression |

**Remarks:**

The pore collapse model described in the TENSOR manual [23] is no longer valid and has been replaced by a much simpler method. This is due in part to the lack of experimental data required for the more complex model. It is desired to have a close approximation of the TENSOR model in the LS-DYNA code to enable a quality link between them. The TENSOR model defines two curves, the virgin loading curve and the completely crushed curve, as shown in Figure EOS11-1, as well as the excess compression point required for pore collapse to begin, $\mu_1$, and the excess compression point required to completely crush the material, $\mu_2$. From this data and the maximum excess compression the material has attained, $u_{max}$, the pressure for any excess compression, $\mu$, can be determined.

Unloading occurs along the virgin loading curve until the excess compression surpasses $\mu_1$. After that, the unloading follows a path between the completely crushed curve and the virgin loading curve. Reloading will follow this curve back up to the virgin loading

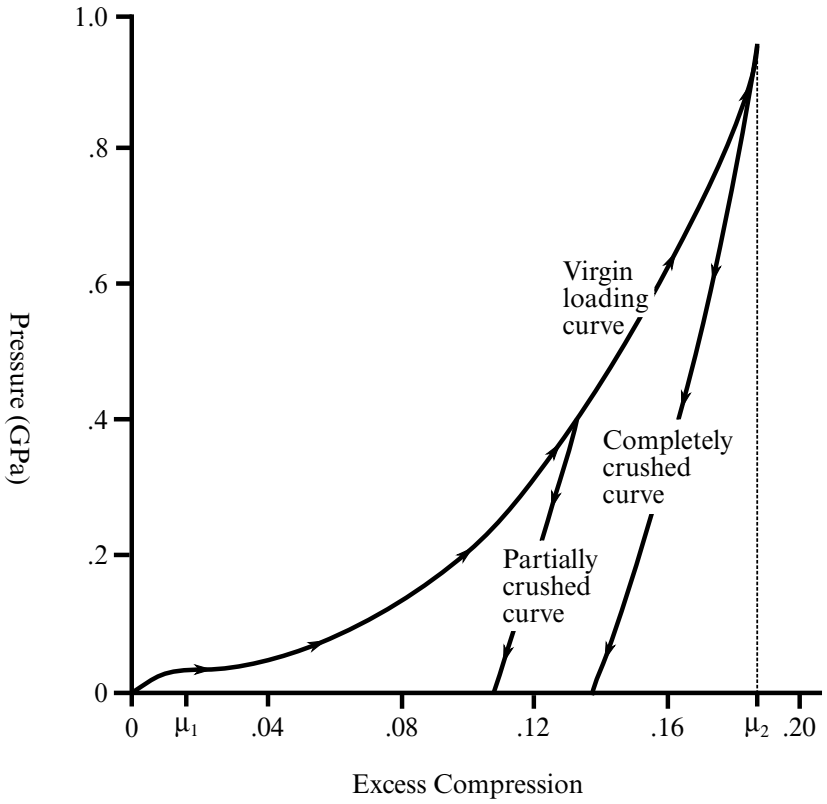



**Figure EOS11-1.** Pressure versus compaction curve

curve. Once the excess compression exceeds $\mu_2$, then all unloading will follow the completely crushed curve.

For unloading between $\mu_1$ and $\mu_2$ a partially crushed curve is determined by:

$$p_{\mathrm{pc}}(\mu) = p_{\mathrm{cc}}\left[\overbrace{\frac{(1+\mu_B)(1+\mu)}{1+\mu_{\max}}}^{\mu_a} - 1\right],$$

where

$$\mu_B = P_{\mathrm{cc}}^{-1}(P_{\max})$$

and the subscripts "pc" and "cc" refer to the partially crushed and completely crushed states, respectively. This is more readily understood in terms of the relative volume, $V$.

$$V = \frac{1}{1+\mu}$$

$$P_{\mathrm{pc}}(V) = P_{\mathrm{cc}}\left(\frac{V_B}{V_{\min}}V\right)$$

This representation suggests that for a fixed

$$V_{\min} = \frac{1}{\mu_{\max}+1}$$

the partially crushed curve will separate linearly from the completely crushed curve as $V$ increases to account for pore recovery in the material.

The bulk modulus $K$ is determined to be the slope of the current curve times one plus the excess compression

$$K = \frac{\partial P}{\partial \mu}(1+\mu) \ .$$

The slope $\frac{\partial P}{\partial \mu}$ for the partially crushed curve is obtained by differentiation as:

$$\frac{\partial p_{\rm pc}}{\partial \mu} = \frac{\partial p_{\rm cc}}{\partial x}\bigg|_{x=\frac{(1+\mu_b)(1+\mu)}{1+\mu_{\rm max}}-1} \left(\frac{1+\mu_b}{1+\mu_{\rm max}}\right) \ .$$

Simplifying,

$$K = \frac{\partial P_{\rm cc}}{\partial \mu_a}\bigg|_{\mu_{\rm a}} (1+\mu_a) \ ,$$

where

$$\mu_a = \frac{(1+\mu_B)(1+\mu)}{(1+\mu_{\rm max})} - 1 \ .$$

The bulk sound speed is determined from the slope of the completely crushed curve at the current pressure to avoid instabilities in the time step.

The virgin loading and completely crushed curves are modeled with monotonic cubic-splines. An optimized vector interpolation scheme is then used to evaluate the cubic-splines. The bulk modulus and sound speed are derived from a linear interpolation on the derivatives of the cubic-splines.

## *EOS_IDEAL_GAS

Purpose: This is Equation of State Form 12 for modeling ideal gas. It is an alternate approach to using *EOS_LINEAR_POLYNOMIAL with C4 = C5 = $(\gamma - 1)$ to model ideal gas. This has a slightly improved energy accounting algorithm.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | CVMASS | CPMASS | B | C | T0 | V0 | VCO |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ADIAB | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| CVMASS | Nominal constant-volume specific heat coefficient, $C_{v,\text{mass}}$ (see Remark 1) |
| CPMASS | Nominal constant-pressure specific heat coefficient, $C_{p,\text{mass}}$ (see Remark 1) |
| B | Linear coefficient, $B$, for the variations of $C_v$ and $C_p$ as a function of $T$ (see Remark 1) |
| C | Quadratic coefficient, $C$, for the variations of $C_v$ and $C_p$ as a function of $T$ (see Remark 1) |
| T0 | Initial temperature |
| V0 | Initial relative volume (see the beginning of the *EOS section) |
| VCO | Van der Waals covolume |

| VARIABLE | DESCRIPTION |
|---|---|
| ADIAB | Adiabatic flag:<br>EQ.0.0: Off<br>EQ.1.0: On; ideal gas follows adiabatic law |

**Remarks:**

1. **Pressure.** The pressure in the ideal gas law is defined as

$$p = \rho(C_p - C_v)T$$
$$C_p = C_{p,\text{mass}} + BT + CT^2$$
$$C_v = C_{v,\text{mass}} + BT + CT^2$$

where $C_p$ and $C_v$ are the specific heat capacities at constant pressure and at constant volume, respectively. $\rho$ is the density. The relative volume is defined as

$$v_r = \frac{V}{V_0} = \frac{(V/M)}{(V_0/M)} = \frac{v}{v_0} = \frac{\rho_0}{\rho} \ ,$$

where $\rho_0$ is a nominal or reference density defined in the *MAT_NULL card. The initial pressure can then be manually computed as

$$P|_{t=0} = \rho|_{t=0}(C_P - C_V)T|_{t=0}$$
$$\rho|_{t=0} = \left\{\frac{\rho_0}{v_r|_{t=0}}\right\}$$
$$P|_{t=0} = \left\{\frac{\rho_0}{v_r|_{t=0}}\right\}(C_P - C_V)T|_{t=0}$$

The initial relative volume, $v_r|_{t=0}$ (V0), initial temperature, $T|_{t=0}$(T0), and heat capacity information are defined in the *EOS_IDEAL_GAS input. Note that the "reference" density is typically a density at a non-stressed or nominal stress state. The initial pressure should always be checked manually against simulation result.

2. **Energy Conservation.** With adiabatic flag on, the adiabatic state is conserved, but *exact* internal energy conservation is sacrificed.

3. **Deviation from Ideal Gas Model.** The ideal gas model is good for low density gas only. Deviation from the ideal gas behavior may be indicated by the compressibility factor defined as

$$Z = \frac{Pv}{RT}$$

When $Z$ deviates from 1, the gas behavior deviates from ideal.

4. **Initial Temperature and Initial Relative Volume.** V0 and T0 defined in this card must be the same as the time-zero ordinates for the 2 load curves defined in the *BOUNDARY_AMBIENT_EOS card, if it is used. This is so that they both would consistently define the same initial state for a material.

### *EOS_PHASE_CHANGE

This is Equation of State Form 13. This EOS was designed for phase change from liquid to vapor, based on the homogeneous Schmidt model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | RHOL | RHOV | CL | CV | GAMAL | PV | KL |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0 | V0 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| RHOL | Density of liquid |
| RHOV | Density of saturated vapor |
| CL | Speed of sound of liquid |
| CV | Speed of sound of vapor |
| GAMAL | Gamma constant of liquid |
| PV | Pressure of vapor |
| KL | Bulk compressibility of liquid |
| E0 | Initial internal energy |
| V0 | Initial relative volume or initial pressure.<br>GT.0.0: Initial relative volume. Default = 1<br>LT.0.0: Initial pressure. |

**Remarks:**

This model is barotropic, so the pressure is only a function of density change. Details of the model can be found in Souli et al. [2014]. Examples of applications for this model are simulations involving water hammer or fuel injection. The ambient pressure should normally be set to atmospheric pressure.

Example input for water in the MKS system (m, kg, s):

```
*EOS_PHASE_CHANGE
$     EOSID       RHOL       RHOV         CL         CV         PV
          1 9.9742e+2  2.095e-2   1492.00     425.00      1.e+5
```

## *EOS_JWLB

This is Equation of State Form 14. The JWLB (Jones-Wilkens-Lee-Baker) equation of state, developed by Baker [1991] and further described by Baker and Orosz [1991], describes the high pressure regime produced by overdriven detonations while retaining the low pressure expansion behavior required for standard acceleration modeling. The derived form of the equation of state is based on the JWL form due to its computational robustness and asymptotic approach to an ideal gas at high expansions. Additional exponential terms and a variable Gruneisen parameter have been added to adequately describe the high-pressure region above the Chapman-Jouguet state.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | A1 | A2 | A3 | A4 | A5 | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R1 | R2 | R3 | R4 | R5 | | | |
| Type | F | F | F | F | F | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AL1 | AL2 | AL3 | AL4 | AL5 | | | |
| Type | F | F | F | F | F | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | BL1 | BL2 | BL3 | BL4 | BL5 | | | |
| Type | F | F | F | F | F | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RL1 | RL2 | RL3 | RL4 | RL5 | | | |
| Type | F | F | F | F | F | | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | OMEGA | E | V0 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state identification. A unique number or label must be specified (see *PART). |
| A*i* | Equation of state coefficient $A_i$. See below. |
| R*i* | Equation of state coefficient $R_i$. See below. |
| AL*i* | Equation of state coefficient $A_{\lambda i}$. See below. |
| BL*i* | Equation of state coefficient $B_{\lambda i}$. See below. |
| RL*i* | Equation of state coefficient $R_{\lambda i}$. See below. |
| C | Equation of state coefficient $C$. See below. |
| OMEGA | Equation of state coefficient $\omega$. See below. |
| E | Energy density per unit volume |
| V0 | Initial relative volume |

**Remarks:**

The JWLB equation-of-state defines the pressure as

$$p = \sum_{i=1}^{5} A_i \left(1 - \frac{\lambda}{R_i V}\right) e^{-R_i V} + \frac{\lambda E}{V} + C\left(1 - \frac{\lambda}{\omega}\right) V^{-(\omega+1)}$$

$$\lambda = \sum_{i=1}^{5} (A_{\lambda i} V + B_{\lambda i}) e^{-R_{\lambda i} V} + \omega$$

where $V$ is the relative volume, $E$ is the energy per unit initial volume, and $A_i$, $R_i$, $A_{\lambda i}$, $B_{\lambda i}$, $R_{\lambda i}$, $C$, and $\omega$ are input constants defined above.

JWLB input constants for some common explosives as found in Baker and Stiel [1997] are given in the following table.

| | TATB | LX-14 | PETN | TNT | Octol 70/30 |
|---|---|---|---|---|---|
| $\rho_0$ (g/cc) | 1.800 | 1.821 | 1.765 | 1.631 | 1.803 |
| $E_0$ (Mbar) | .07040 | .10205 | .10910 | .06656 | .09590 |
| $D_{CJ}$ (cm/μs) | .76794 | .86619 | .83041 | .67174 | .82994 |
| $P_{CJ}$ (Mbar) | .23740 | .31717 | .29076 | .18503 | .29369 |
| $A_1$ (Mbar) | 550.06 | 549.60 | 521.96 | 490.07 | 526.83 |
| $A_2$ (Mbar) | 22.051 | 64.066 | 71.104 | 56.868 | 60.579 |
| $A_3$ (Mbar) | .42788 | 2.0972 | 4.4774 | .82426 | .91248 |
| $A_4$ (Mbar) | .28094 | .88940 | .97725 | .00093 | .00159 |
| $R_1$ | 16.688 | 34.636 | 44.169 | 40.713 | 52.106 |
| $R_2$ | 6.8050 | 8.2176 | 8.7877 | 9.6754 | 8.3998 |
| $R_3$ | 2.0737 | 20.401 | 25.072 | 2.4350 | 2.1339 |
| $R_4$ | 2.9754 | 2.0616 | 2.2251 | .15564 | .18592 |
| $C$ (Mbar) | .00776 | .01251 | .01570 | .00710 | .00968 |
| $\omega$ | .27952 | .38375 | .32357 | .30270 | .39023 |
| $A_{\lambda 1}$ | 1423.9 | 18307. | 12.257 | .00000 | .011929 |
| $B_{\lambda 1}$ | 14387. | 1390.1 | 52.404 | 1098.0 | 18466. |
| $R_{\lambda 1}$ | 19.780 | 19.309 | 43.932 | 15.614 | 20.029 |
| $A_{\lambda 2}$ | 5.0364 | 4.4882 | 8.6351 | 11.468 | 5.4192 |
| $B_{\lambda 2}$ | -2.6332 | -2.6181 | -4.9176 | -6.5011 | -3.2394 |
| $R_{\lambda 2}$ | 1.7062 | 1.5076 | 2.1303 | 2.1593 | 1.5868 |

## *EOS_GASKET

This is Equation of State Form 15. This EOS works with solid elements and thick shell formulations ELFORM = 2, 3, 5 and 7 to model the response of gaskets. For the thick shell formulation #2 only, it is completely decoupled from the shell material, meaning in the local coordinate system of the shell, this model defines the normal stress, $\sigma_{zz}$, and does not change any of the other stress components. The model is a reduction of the *MAT_-GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | LCID1 | LCID2 | LCID3 | LCID4 | | | |
| Type | A | I | I | I | I | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UNLOAD | K | DMPF | TFS | CFS | LOFFSET | IVS | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| LCID1 | Load curve for loading |
| LCID2 | Load curve for unloading |
| LCID3 | Load curve for damping as a function of volumetric strain rate |
| LCID4 | Load curve for scaling the damping as a function of the volumetric strain |
| UNLOAD | Unloading option (see Figure EOS15-1):<br>EQ.0.0: Loading and unloading follow loading curve<br>EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve.<br>EQ.2.0: Loading follows loading curve, unloading follows |

| VARIABLE | DESCRIPTION |
|---|---|
| | unloading stiffness, K, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes. |
| | EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset. |
| K | Unloading stiffness, for UNLOAD = 2 only |
| DMPF | Damping factor for stability. Values in the neighborhood of unity are recommended. The damping factor is properly scaled to eliminate time step size dependency. |

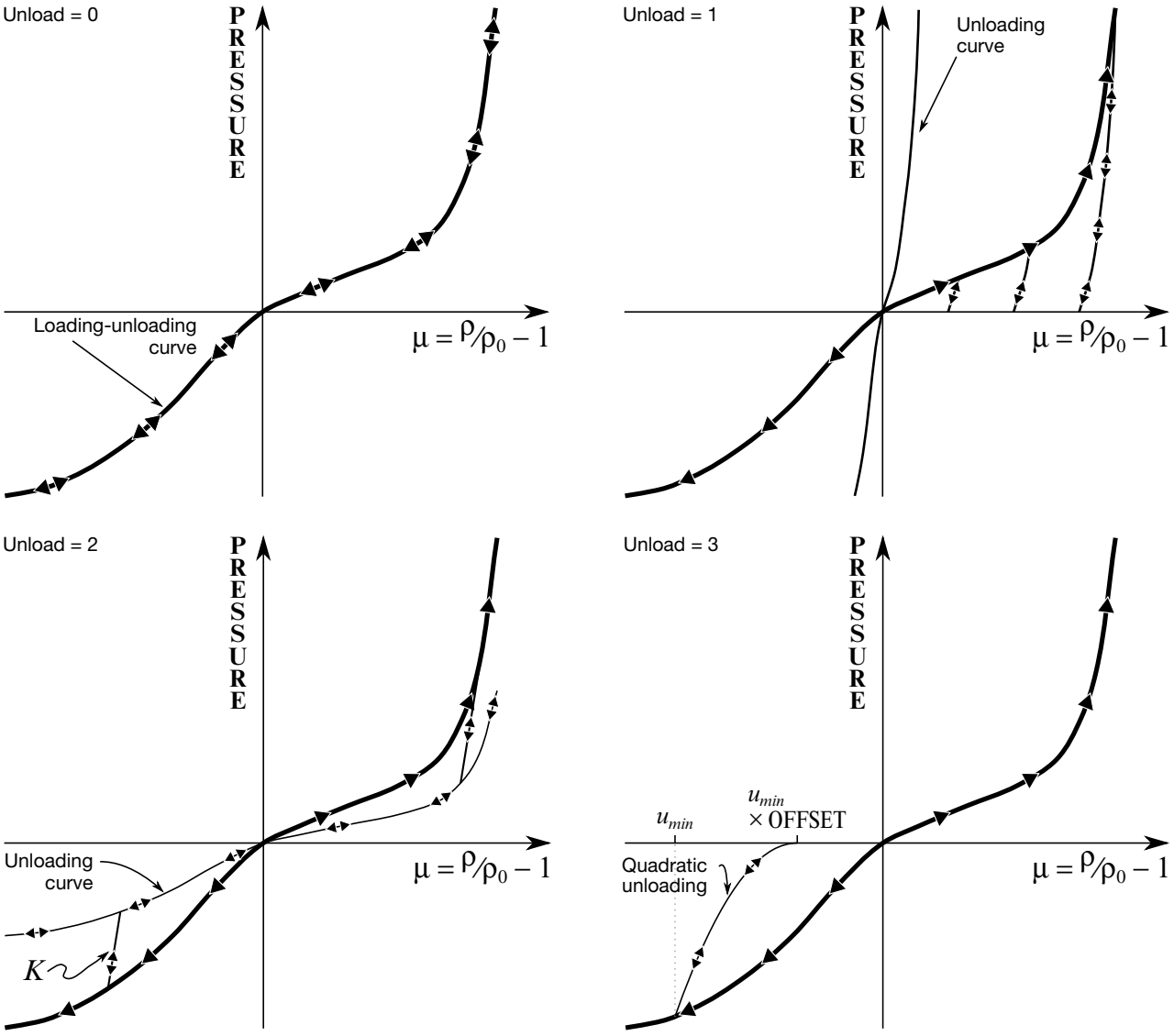



**Figure EOS15-1.** Load and unloading behavior.

| VARIABLE | DESCRIPTION |
|---|---|
| TFS | Tensile failure strain |
| CFS | Compressive failure strain |
| OFFSET | Offset factor between 0 and 1.0 to determine permanent set upon unloading if the UNLOAD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively. |
| IVS | Initial volume strain |

### *EOS_MIE_GRUNEISEN

This is Equation of State Form 16, a Mie-Gruneisen form with a $p - \alpha$ compaction model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | GAMMA | A1 | A2 | A3 | PEL | PCO | N |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA0 | E0 | V0 | | | | | |
| Type | F | F | F | | | | | |
| Default | none | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state identification. A unique number or label must be specified (see *PART). |
| GAMMA | Gruneisen gamma |
| A*i* | Hugoniot polynomial coefficient |
| PEL | Crush pressure |
| PCO | Compaction pressure |
| N | Porosity exponent |
| ALPHA0 | Initial porosity |
| E0 | Initial internal energy |
| V0 | Initial relative volume |

**Remarks:**

The equation of state is a Mie-Gruneisen form with a polynomial Hugoniot curve and a $p-\alpha$ compaction model. First, we define a history variable representing the porosity $\alpha$ that is initialised to $\alpha_0 > 1$. The evolution of this variable is given as

$$\alpha(t) = \max\left\{1, \min\left[\alpha_0, \min_{s\le t}\left(1 + (\alpha_0 - 1)\left[\frac{p_{\text{comp}} - p(s)}{p_{\text{comp}} - p_{el}}\right]^N\right)\right]\right\} ,$$

where $p(t)$ indicates the pressure at time $t$. For later use, we define the cap pressure as

$$p_c = p_{\text{comp}} - (p_{\text{comp}} - p_{el})\left[\frac{\alpha - 1}{\alpha_0 - 1}\right]^{\frac{1}{N}} .$$

The remainder of the EOS model is given by the equations

$$p(\rho, e) = \Gamma\alpha\rho e + p_H(\eta)\left[1 - \frac{1}{2}\Gamma\eta\right]$$

$$p_H(\eta) = A_1\eta + A_2\eta^2 + A_3\eta^3$$

together with

$$\eta(\rho) = \frac{\alpha\rho}{\alpha_0\rho_0} - 1 .$$

## *EOS_MURNAGHAN

This is Equation of State Form 19. This EOS was designed to model incompressible fluid flow with SPH or ALE elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | GAMMA | K0 | V0 | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| GAMMA, K0 | Constants in the equation of state |
| V0 | Initial relative volume |

**Remarks:**

The Murnaghan equation of state defines pressure as

$$p = k_0 \left[ \left( \frac{\rho}{\rho_0} \right)^{\gamma} - 1 \right] .$$

To model fluid flows accurately, $\gamma$ is often set to 7, and $k_0$ is chosen such that

$$c_0 = \sqrt{\frac{\gamma k_0}{\rho_0}} \geq 10 v_{\max} \quad ,$$

where $v_{\max}$ is the maximum expected fluid flow velocity. This will ensure low compressibility while allowing for a relatively large time step size.

## *EOS_USER_DEFINED

These are Equations of State 21-30. The user can supply his own subroutines. See also Appendix B. The keyword input must be used for the user interface with data.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | EOST | LMC | NHV | IVECT | E0 | V0 | BULK |
| Type | A | I | I | I | I | F | F | F |

Define LMC material parameters using 8 parameters per card.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| EOST | User equation of state type (21-30 inclusive). A number between 21 and 30 has to be chosen. |
| LMC | Length of material constant array which is equal to the number of material constants to be input. (LMC ≤ 48) |
| NHV | Number of history variables to be stored, see Appendix B. |
| IVECT | Vectorization flag (on = 1). A vectorized user subroutine must be supplied. |
| EO | Initial internal energy |
| V0 | Initial relative volume |
| BULK | Bulk modulus. This value is used in the calculation of the contact surface stiffness. |
| P$i$ | Material parameters $i = 1, \ldots,$LMC. |

## *EOS_HVRB

This is Equation of State Form 41. The History Variable Reaction Burn (HVRB) equation of state is used to calculate shock initiation (or failure to initiate) and detonation wave propagation of solid high explosives. It should be used instead of the ideal high explosive burn options whenever whether the high explosive will react is uncertain, a finite time is required for a shock wave to build up to detonation, or the chemical reaction zone in a detonation wave has a finite thickness. At relatively low initial pressures (< 2-3 GPa), this equation of state should be used with material type 10 for accurate calculations of the unreacted high explosive behavior. At higher initial pressures, material type 9 can be used.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | EOSURID | EOSRPID | | | | | TAO0 |
| Type | A | A | A | | | | | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PR | Z | M | PI | RMAX | RMIN | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| EOSURID | Equation of state ID of the unreacted explosive (UR). This field refers to a separately defined *EOS_GRUNEISEN card that specifies the shock behavior of the unreacted explosive. |
| EOSRPID | Equation of state ID of the reaction products (RP). This field refers to a separately defined *EOS_JWL card that specifies the afterburn behavior of the reaction products. |
| TAO0 | Parameter $\tau_0$ in the reaction fraction calculation (see Remarks). We recommend setting $\tau_0$ to 1 μs. |
| PR | Parameter $P_R$ in the reaction fraction calculation (see Remarks) |

| VARIABLE | DESCRIPTION |
|---|---|
| Z | Parameter $Z$ in the reaction fraction calculation (see Remarks) |
| M | Parameter $M$ in the reaction fraction calculation (see Remarks) |
| PI | Parameter $P_I$ in the reaction fraction calculation (see Remarks) |
| RMAX | Maximum density of the unreacted explosive. The reaction is complete for densities > RMAX. |
| RMIN | Minimum density of the unreacted explosive. The reaction is complete for densities < RMIN. |

**Remarks:**

For this equation of state, the explosive is divided into unreacted explosive (UR) and reaction products (RP). The Gruneisen equation of state models the unreacted explosive, and the JWL equation of state models the reaction product. Specifying the HVRB equation of state, thus, requires additionally defining two other equations of state, one for Gruneisen and one for JWL, referenced in EOSURID and EOSRPID, respectively. The input setup closely follows CTH.

The pressure field and internal energy are calculated in a composite manner:

$$p(\rho,T,\lambda) = (1-\lambda)p_{\mathrm{UR}}(\rho,T) + \lambda p_{\mathrm{RP}}(\rho,T)$$
$$E(\rho,T,\lambda) = (1-\lambda)E_{UR}(\rho,T) + \lambda E_{RP}(\rho,T)$$

In the above, $0 \le \lambda \le 1$ is the extent of reaction, $p$ is the pressure, $T$ is the temperature, $\rho$ is the density, and $E$ is the internal energy. The subscripts UR and RP denote the unreacted explosive and the reaction products, respectively.

The reaction fraction, $\lambda$, is defined as:

$$\lambda = \phi^M$$
$$\phi = \frac{1}{\tau_0}\int_0^t \left(\frac{P-P_I}{P_R}\right)^Z dt$$

Where $\phi$ is an integral over time of the pressure at a mass point. The parameters $P_R$ and $z$ determine the pressure dependence of time and distance to detonation, $P_I$ is the threshold pressure for initiation and $\tau_0$ is used to make $\phi$ dimensionless (default = 1.0 $\mu s$).

### *EOS_USER_LIBRARY

This is Equation of State Form 42.

Purpose: Select a material ID defined in a library called seslib, and initialize the thermodynamic state of the material by defining E0 and V0 below. seslib must be in the working directory.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | SESMID | | | | | | |
| Type | A | I | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0 | V0 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| SESMID | Material ID |
| E0 | Initial internal energy per unit reference volume (see the beginning of the *EOS section) |
| V0 | Initial relative volume (see the beginning of the *EOS section) |

## *EOS_LEE-TARVER

This is Equation of State Form 43, which implements the Lee-Tarver ignition and growth model, a phenomenological framework describing how shocked, heterogeneous explosives transition from unreacted solids to detonation products.

This EOS is intended for use with ALE and S-ALE elements and must be paired with *MAT_NULL or *MAT_ELASTIC_PLASTIC_HYDRO. Other *MAT keywords are not supported.

During shock-induced detonation, a high-pressure pulse applied to a heterogeneous explosive can trigger local deformations and subsequent ignition sites. These ignitions release energy, which accumulates until reaching a threshold that produces full detonation. The *EOS_LEE-TARVER model provides a robust alternative to *EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE, enabling continuum-scale simulations of explosive ignition and detonation. It captures how chemical energy release interacts with hydrodynamic variables, namely, pressure, internal energy, and sound speed, thus coupling reaction progression with the mechanical response of energetic materials.

### Card Summary:

**Card 1.** This card is required.

| EOSID | A | B | R1 | R2 | W | DETV | EZIG |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| PCJ | WREAC | DFMAX | RRI | RRB | RRA | RRX | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| RRG1 | RRC | RRD | RRY | RRG2 | RRE | RRG | RRZ |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| FIGMAX | FG1MAX | FG2MIN | VUMAX | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required

| EOSUNR | SHKGAM | SHKC1 | SHKS1 | SHKC2 | SHKS2 | SHKVE | SHKVB |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card if EOSUNR is set to 2.

| AUR | BUR | R1U | R2U | WU | VVNS | EZIU | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | A | B | R1 | R2 | W | DETV | EZIG |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| A | Product JWL constant $A$ (units of pressure) |
| B | Product JWL constant $B$ (units of pressure) |
| R1 | Product JWL constant $R_1$ (unitless) |
| R2 | Product JWL constant $R_2$ (unitless) |
| W | Product JWL constant $\omega$ (unitless) |
| DETV | Chapman-Jouguet (CJ) detonation velocity $u_D$ (units of speed) |
| EZIG | CJ energy per unit volume $E_{0,g}$ (units of pressure) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PCJ | WREAC | DFMAX | RRI | RRB | RRA | RRX | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| PCJ | CJ pressure (units of pressure) |
| WREAC | Reaction-front Courant number used to limit the timestep (unitless) |
| DFMAX | Maximum change in reaction ratio (unitless) |
| RRI | Reaction rate parameter $I$ (units of inverse time) |

| VARIABLE | DESCRIPTION |
|---|---|
| RRB | Reaction rate parameter $b$ (unitless) |
| RRA | Reaction rate parameter $a$ (unitless) |
| RRX | Reaction rate parameter $x$ (unitless) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RRG1 | RRC | RRD | RRY | RRG2 | RRE | RRG | RRZ |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| RRG1 | Reaction rate parameter $G_1$ (unitless) |
| RRC | Reaction rate parameter $c$ (unitless) |
| RRD | Reaction rate parameter $d$ (unitless) |
| RRY | Reaction rate parameter $y$ (unitless) |
| RRG2 | Reaction rate parameter $G_2$ (unitless) |
| RRE | Reaction rate parameter $e$ (unitless) |
| RRG | Reaction rate parameter $g$ (unitless) |
| RRZ | Reaction rate parameter $z$ (unitless) |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FIGMAX | FG1MAX | FG2MIN | VUMAX | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FIGMAX | Maximum reaction ratio of ignition (unitless) |
| FG1MAX | Maximum reaction ratio of $G_1$ (unitless) |

| VARIABLE | DESCRIPTION |
|---|---|
| FG2MIN | Minimum reaction ratio of $G_2$ (unitless) |
| VUMAX | Maximum relative volume in tension (unitless) |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSUNR | SHKGAM | SHKC1 | SHKS1 | SHKC2 | SHKS2 | SHKVE | SHKVB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSUNRE | Flag to indicate which unreacted EOS is used for the unreacted explosive:<br>EQ.1: Shock<br>EQ.2: JWL |
| SHKGAM | Grüneisen gamma for the unreacted shock EOS (unitless). This field is ignored if EOSUNRE = 2. |
| SHKC1 | Bulk sound velocity for the unreacted shock EOS (units of velocity). This field is ignored if EOSUNRE = 2. |
| SHKS1 | Unreacted shock EOS parameter (unitless). This field is ignored if EOSUNRE = 2. |
| SHKC2 | Unreacted shock EOS parameter for bilinear shock (units of velocity). This field is ignored if EOSUNRE = 2. |
| SHKS2 | Unreacted shock EOS parameter for bilinear shock (Unitless). This field is ignored if EOSUNRE = 2. |
| SHKVE | Relative volume below which SHKC2 and SHKS2 are used in bilinear shock (unitless). This field is ignored if EOSUNRE = 2. |
| SHKVB | Relative volume above which SHKC1 and SHKS1 are used in bilinear shock (unitless). This field is ignored if EOSUNRE = 2. |

**Card for JWL of unreacted solid.** Include this card if EOSUNR is set to 2. In this case, the shock EOS parameters on Card 5 are ignored.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AUR | BUR | R1U | R2U | WU | VVNS | EZIU | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| AUR | Unreacted JWL EOS constant $A$ (units of pressure) |
| BUR | Unreacted JWL EOS constant $B$ (units of pressure) |
| R1U | Unreacted JWL EOS $R_1$(unitless) |
| R2U | Unreacted JWL EOS $R_2$(unitless) |
| WU | Unreacted JWL EOS $\omega$ (unitless) |
| VVNS | von Neumann spike relative volume (unitless) |
| EZIU | CJ energy per unit volume (units of pressure) |

**Remarks:**

1. **Reactive burn models.** Equation of State (EOS) Form 43 is employed to simulate shock initiation, detonation failure, and detonation wave propagation in solid heterogeneous high explosives. This formulation implements the Lee-Tarver Ignition and Growth model originally developed within the Ansys® Autodyn solver and has been adapted for broader application. This type of equation of state is classified as a reactive burn model. Compared to *EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE, EOS Form 43 requires fewer parameters, while maintaining consistency with much of the external literature.

   Reactive burn models are preferable to ideal burn formulations (e.g., the BETA flag in *MAT_HIGH_EXPLOSIVE_BURN) in scenarios where:

   - The explosive may or may not react under applied shock loading.
   - A finite induction time is required for a shock wave to evolve into detonation.
   - The detonation front possesses a finite chemical reaction zone thickness, producing two-dimensional effects such as corner turning, detonation front curvature, and sensitivity to critical diameter.

2. **The physics of shock initiation and overview of the model.** When a pressure pulse is imposed on an explosive (for example, by impact from a flyer plate), the resulting shock mechanically deforms the material. If the shock strength is insufficient relative to the explosive's sensitivity, detonation does not occur. If the shock is sufficiently strong, the interaction typically proceeds through three phases:

   a) *Ignition phase.* Severe local deformation produces "hot spots" or micro-defects that ignite small regions behind the shock front.

   b) *Growth phase.* These reaction zones expand, releasing additional energy and amplifying the shock wave.

   c) *Transition to detonation.* Once the energy release becomes self-sustaining and supports its own shock front, a detonation is established.

   A common experimental method for assessing explosive sensitivity is the Pop plot, which relates input shock pressure to run distance to detonation. The run distance is the physical distance a shock wave travels through the explosive before transitioning from a nonreactive shock to a fully developed detonation. For a given input shock, a shorter run distance indicates a more sensitive explosive, whereas a longer run distance reflects lower sensitivity.

   Lee and Tarver demonstrated that the model:

   - Reproduces experimental shock-initiation diagnostics.
   - Captures run distance to detonation behavior (Pop plots) and reaction-zone lengths for several explosives, including PBX-9404, TATB, cast TNT, and PETN.

   The Lee-Tarver model provides a phenomenological framework for representing the transition of shocked heterogeneous explosives from unreacted solid to fully reacted detonation products. For a comprehensive theoretical background, refer to Lee and Tarver [1980] and Tarver and Hallquist [1981].

   The formulation comprises four principal components:

   a) An EOS describing the unreacted explosive.

   b) An EOS for the detonation products.

   c) A reaction rate law governing ignition and growth.

   d) A mixture model representing the partially reacted state, consisting of unreacted solid and gaseous products.

3. **Equation of state formulations for the unreacted explosive.** The unreacted material may be represented using either a shock EOS or a JWL EOS. The shock EOS is derived from shock Hugoniot parameters obtained from the shock velocity-particle velocity ($U_s$-$u_p$) relation, while the JWL EOS is the standard formulation employed in ignition and growth models.

   a) *Shock EOS (EOSUNRE = 1).* The shock EOS is expressed as:

$$P(V_u, E) = P_H(V_u) + \frac{\Gamma}{V_u}[E_u(V_u, E) - E_H(V_u)]$$

   where:

| | |
|---|---|
| $P_H(V_u)$ | Hugoniot pressure |
| $E_u(V_u, E)$ | Internal energy per unit mass of unreacted explosive, $(EV_u - E_{0,g})/\rho_0$ |
| $E$ | Current internal energy per unit volume |
| $E_{0,g}$ | Initial internal energy per unit volume |
| $E_H(V_u)$ | Hugoniot energy per unit mass |
| $V_u$ | Relative volume of the unreacted explosive, $\rho_0/\rho_u = 1/\eta_u$ |
| $\Gamma$ | Grüneisen coefficient |

   The Hugoniot pressure and energy are defined as:

$$P_H(V_u) = \frac{\rho_0 c_0^2 (1 - V_u)}{[1 - s(1 - V_u)]^2}$$

$$E_H(V_u) = \frac{1}{2}\left[\frac{c_0(1 - V_u)}{1 - s(1 - V_u)}\right]^2 = \frac{1}{2}\left[\frac{P_H(V_u) \times (1 - V_u)}{\rho_0}\right]$$

   where $c_0$ is the ambient bulk sound speed and $s$ is the slope of the $U_s$-$u_p$ fit. Note that $s$ must not take values in the range $0 < s < 1$.

   The $U_s$-$u_p$ form can be linear or bilinear:

   - The linear form is assumed if only SHKC1 and SHKS1 are supplied. In this case, SHKC1 specifies $c_0$, and SHKS1 specifies $s$. Both parameters are constants.
   - The bilinear form is assumed if additionally SHKC2 and SHKS2 input. In this case, $c_0$ and $s$ depend on $V_u$:

$$c_0 = \begin{cases} \text{SHKC1} & \text{if } V_\text{u} \geq \text{SHKVB} \\ \text{SHKC2} & \text{if } V_\text{u} \leq \text{SHKVE} \\ \text{SHKC1} + \dfrac{(\text{SHKC2} - \text{SHKC1})(V_\text{u} - \text{SHKVB})}{(\text{SHKVE} - \text{SHKVB})} & \text{if SHKVE} < V_\text{u} < \text{SHKVB} \end{cases}$$

$$s = \begin{cases} \text{SHKS1} & \text{if } V_\text{u} \geq \text{SHKVB} \\ \text{SHKS2} & \text{if } V_\text{u} \leq \text{SHKVE} \\ \text{SHKS1} + \dfrac{(\text{SHKS2} - \text{SHKS1})(V_\text{u} - \text{SHKVB})}{(\text{SHKVE} - \text{SHKVB})} & \text{if SHKVE} < V_\text{u} < \text{SHKVB} \end{cases}$$

b) *JWL EOS (EOSUNRE = 2).* The JWL EOS for the unreacted explosive is given by:

$$P_\text{u}(V_\text{u}, E) = P_{\text{AB,u}}(V_\text{u}) + \frac{\omega E_\text{u}(E)}{V_\text{u}}$$

with:

$$P_{\text{AB,u}}(V_\text{u}) = A\left(1 - \frac{\omega}{R_1 V_\text{u}}\right)e^{R_1 V_\text{u}} + B\left(1 - \frac{\omega}{R_2 V_\text{u}}\right)e^{R_2 V_\text{u}}$$

$$E_\text{u}(E) = E - \frac{E_{0,\text{g}} - E_{0,\text{u}}}{V_\text{u}}$$

Parameters $A$, $B$, $R_1$, $R_2$, and $\omega$ are required inputs. Additional inputs include the von Neumann spike volume (VVNS) and the ambient internal energy per unit volume ($E_{0,\text{u}}$).

4. **Reaction products.** The JWL EOS for the gaseous reaction products is:

$$P_\text{g}(V_\text{g}, E) = P_{\text{AB,g}}(V_\text{g}) + \frac{\omega E_\text{g}(E)}{V_\text{g}}$$

where

$$P_{\text{AB,g}}(V_\text{g}) = A\left(1 - \frac{\omega}{R_1 V_\text{g}}\right)e^{R_1 V_\text{g}} + B\left(1 - \frac{\omega}{R_2 V_\text{g}}\right)e^{R_2 V_\text{g}}$$

$$E_\text{g}(E) = E$$

Parameters $A$, $B$, $R_1$, $R_2$, and $\omega$ are required inputs. $V_\text{g}$ is the relative volume of the gaseaous reaction products. Additional required parameters include the Chapman–Jouguet pressure ($P_\text{CJ}$, the pressure behind the detonation wave), detonation velocity ($u_\text{D}$), and the ambient internal energy per unit volume ($E_{0,\text{g}}$).

5. **Reaction rate model.** The total reaction rate is the sum of an ignition term plus one or more growth terms:

$$\frac{\partial F}{\partial t} = \overbrace{I(1-F)^b(\mu - a)^x}^{\text{Ignition}} + \overbrace{G_1(1-F)^c F^d P^y}^{\text{Growth}} + \overbrace{G_2(1-F)^e F^g P^z}^{\text{Completion}}$$

where $F$ is the reaction ratio (gaseous mass/total explosive mass), $P$ is the pressure and $\mu = \rho/\rho_0 - 1$ is the compression. Constants $I$, $a$, $b$, $x$, $G_1$, $c$, $d$, $y$, $G_2$, $e$, $g$,

and $z$ are empirically determined input parameters. $I$ represents hot-spot ignition sensitivity and $G_1$ and $G_2$ the pressure-dependent growth. The exact functional forms are empirical/phenomenological (Lee & Tarver originated a practical form widely used in hydrocode suites). The reaction rate parameters and exponents are material-specific and are obtained by fitting experimental diagnostics (gauge traces, Pop plots, cylinder tests). Because of that empirical fitting, the model is predictive only within the calibrated regime.

Care must be taken with unit consistency, particularly for $G_1$and $G_2$, whose dimensions depend on $y$and $z$. $G_1$ has units $\text{Mbar}^{-y}\mu\text{s}^{-1}$, and $G_2$ has units $\text{Mbar}^{-z}\mu\text{s}^{-1}$.

6. **Partially reacted state.** The method assumes that the mixture of unreacted solid and gaseous reaction products is at the same pressure. The partially reacted state is based on the following assumption:

$$v = (1 - F)v_{\text{u}} + Fv_{\text{g}}$$

where $v$ is the combined specific volume, $F$ is the burn fraction, $v_{\text{u}}$ is the specific volume of the unreacted solid, and $v_{\text{g}}$ is the specific volume of the gaseous reaction products.

The pressure is calculated using a JWL equation of state for the detonation products (see Remark 4) and either a JWL or shock EOS for the unreacted solid state (see Remark 3). The Newton-Raphson method is used to find a relative density of the unreacted solid that minimizes the pressure difference of the solid and gas. If no pressure equalization can be found, then a more robust binary search is performed. If the second method is also unsuccessful, the pressure is set to the average of the closest found pressures.

7. **Input verification.** The code performs the following checks to validate input:

   a) *Volume check.* If the JWL EOS is used for the unreacted solids, a volume check is performed:

   $$\text{VVNS} < V_{\text{CJ}}$$

   In the above, VVNS is the von Neumann spike volume, and $V_{\text{CJ}}$ is the Chapman-Jouguet volume. $V_{\text{CJ}}$ is defined as:

   $$V_{\text{CJ}} = \frac{\rho_0 u_{\text{D}}^2 - P_{\text{CJ}}}{\rho_0 u_{\text{D}}^2}$$

   where $\rho_0$ is the ambient density and $u_{\text{D}}$ is the detonation velocity. VVNS must be greater than 0.

   b) *Pressure consistency check.* The pressure consistency check is always performed:

$$P_g(V_{CJ}, E_{CJ}) \approx P_{CJ}$$

Here, $P_g$ is the pressure evaluated using the gaseous equation of state, and $E_{CJ}$ is the internal energy consistent with $V_{CJ}$. The code accepts $P_{CJ}$ when:

$$\frac{P_g(V_{CJ}, E_{CJ}) - P_{CJ}}{P_{CJ}} < 0.01$$

c) *Shock parameter check.* If the shock option (EOSUNRE = 1) is used for the unreacted EOS, then the code checks whether the value of SHKS1 is not between 0 and 1.

d) *Reaction-front time step control.* To ensure adequate resolution of the propagating reaction zone, the Lee-Tarver model returns a time step restriction:

$$\Delta t \leq \frac{\Delta x}{\text{WREAC} \times \text{DETV}}$$

where $\Delta x$ is the characteristic cell dimension and DETV is the Chapman-Jouguet detonation velocity. Therefore, the reaction front propagates no more than 1/WREAC cell widths during a single time step. For example, with WREAC = 2, the detonation wave traverses each cell over at least two time steps, helping ensure that the reaction zone is adequately resolved. The code only allows a value of WREAC greater than or equal to 2.0.

8. **History variables.** The history variables output for this EOS are the same for *MAT_009 and *MAT_010, but their locations differ. The following table summarizes the history variables from this EOS. To request history variables set NEIPH for solid elements and NEIPS for shell elements on *DATABASE_EXTENT_BINARY to the number of extra history variables desired.

| History Variable # | | |
|---|---|---|
| *MAT_009 | *MAT_010 | Description |
| 1 | 1 | Internal energy |
| 2 | 2 | Artificial viscosity |
| 3 | 3 | Specific volume |
| 4 | 5 | Burn fraction |

## *EOS_AERATED_WATER

This is Equation of State Form 44. It is only supported for ALE and S-ALE. Water is modeled as a homogeneous air–water mixture, with pressure depending only on the compression ratio (barotropic). This model provides a stable and low-cost method for simulating phase changes between water and air. In this mixture, air is modeled with the ideal gas law and water with a linear equation of state with constant bulk modulus.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EOSID | VFAIR | ROAIR | GAMMA | BULKW | P0 | PMAX | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| EOSID | Equation of state ID. A unique number or label must be specified (see *PART). |
| VFAIR | Volume fraction of air in the air-water mixture. A tiny amount, say 1.e-4, could ensure pressure is positive under extreme expansion. |
| ROAIR | Initial air density. It is the air density in the volume occupied by air, not air density in the air-water mixture. |
| GAMMA | Heat capacity ratio of air |
| BULKW | Water bulk modulus |
| P0 | Initial pressure of the air-water mixture, $p_0$ |
| PMAX | A rough guess of the maximum pressure value of the air-water mixture during the simulation. See Remark 1. |

**Remarks:**

1. **PMAX.** *EOS_AERATED_WATER is barotropic, meaning that pressure depends solely on the compression ratio. Internally, a 2,000-point linearized load curve is generated to approximate the compression ratio-pressure relationship, with pressure values ranging from $p_0$ to $p_{max}$. Extrapolation is applied when the pressure falls outside the valid range. This user-specified $p_{max}$ is required to ensure numerical accuracy.

# *MAT

LS-DYNA has historically referenced each material model by a number. As shown below, a three digit numerical designation can still be used, e.g., *MAT_001, and is equivalent to a corresponding descriptive designation, e.g., *MAT_ELASTIC. The two equivalent commands for each material model, one numerical and the other descriptive, are listed below. The numbers in square brackets (see key below) identify the element formulations for which the material model is implemented. The number in the curly brackets, {*n*}, indicates the default number of history variables per element integration point that are stored in addition to the 7 history variables which are stored by default. Just as an example, for the type 16 fully integrated shell elements with 2 integration points through the thickness, the total number of history variables is $8 \times (n + 7)$. For the Belytschko-Tsay type 2 element the number is $2 \times (n + 7)$.

The meaning associated with particular extra history variables for a subset of material models and equations of state are tabulated at https://lsdyna.ansys.com/history-variables-for-certain-material-models/.

An additional option **TITLE** may be appended to a ***MAT** keyword in which case an additional line is read in 80a format which can be used to describe the material. At present, LS-DYNA does not make use of the title. Inclusion of titles simply gives greater clarity to input decks.

*Key to numbers in square brackets*

| | | |
|---|---|---|
| 0 | - | Solids (and 2D continuum elements, that is, shell formulations 13, 14, 15) |
| 1H | - | Hughes-Liu beam |
| 1B | - | Belytschko resultant beam |
| 1I | - | Belytschko integrated solid and tubular beams |
| 1T | - | Truss |
| 1D | - | Discrete beam |
| 1SW | - | Spotweld beam |
| 2 | - | Shells |
| 3a | - | Thick shell formulations 1, 2, 6 |
| 3c | - | Thick shell formulations 3, 5, 7 |
| 4 | - | Special airbag element |
| 5 | - | SPH element (particle) |
| 6 | - | Acoustic solid |
| 7 | - | Cohesive solid |
| 8A | - | Multi-material ALE solid (validated) |

8B - Multi-material ALE solid (implemented but not validated[1])
9 - Membrane element
10 - SPR2/SPR3 connectors
11 - Peridynamics element
12 - Incompressible SPG

*MAT_ADD_AIRBAG_POROSITY_LEAKAGE[2]
*MAT_ADD_BASIC_INCREMENTAL_FAILURE[2] [0,2]
*MAT_ADD_CHEM_SHRINKAGE[2]
*MAT_ADD_COHESIVE[2] [7] {see associated material model}
*MAT_ADD_COHESIVE_FATIGUE[2] [7]
*MAT_ADD_DAMAGE_DIEM[2] [0,2]
*MAT_ADD_DAMAGE_GISSMO[2] [0,1H,2,3a,3c,5]
*MAT_ADD_EROSION[2] [0,1H,2,3a,3c,5,7]
*MAT_ADD_EXTVAR_EXPANSION[2] [0,2]
*MAT_ADD_FATIGUE[2]
*MAT_ADD_GENERALIZED_DAMAGE[2] [0,2]
*MAT_ADD_INELASTICITY[2]
*MAT_ADD_MICRO[2] [0]
*MAT_ADD_PERMEABILTY[2]
*MAT_ADD_PORE_AIR[2]
*MAT_ADD_PROPERTY_DEPENDENCE[2]
*MAT_ADD_PZELECTRIC[2] [0,3c]
*MAT_ADD_SOC_EXPANSION[2] [0]
*MAT_ADD_THERMAL_EXPANSION[2]
*MAT_NONLOCAL[2]
*MAT_NONLOCAL_GRADIENT_DAMAGE[2] [0,2]

| | |
|---|---|
| *MAT_001: | *MAT_ELASTIC [0,1H,1B,1I,1T,2,3a,3c,5,8A] {0} |
| *MAT_001_FLUID: | *MAT_ELASTIC_FLUID [0,8A] {0} |
| *MAT_002: | *MAT_*OPTION*TROPIC_ELASTIC [0,2,3a,3c] {15} |
| *MAT_003: | *MAT_PLASTIC_KINEMATIC [0,1H,1I,1T,2,3a,3c,5,8A] {5} |
| *MAT_004: | *MAT_ELASTIC_PLASTIC_THERMAL [0,1H,1T,2,3a,3c,5,8B] {3} |
| *MAT_005: | *MAT_SOIL_AND_FOAM [0,5,3c,8A] {0} |
| *MAT_006: | *MAT_VISCOELASTIC [0,1H,2,3a,3c,5,8B] {19} |
| *MAT_007: | *MAT_BLATZ-KO_RUBBER [0,2,3ac,8B] {9} |
| *MAT_008: | *MAT_HIGH_EXPLOSIVE_BURN [0,5,3c,8A] {4} |
| *MAT_009: | *MAT_NULL [0,1,2,3c,5,8A] {3} |
| *MAT_010: | *MAT_ELASTIC_PLASTIC_HYDRO_{OPTION} [0,3c,5,8B] {4} |
| *MAT_011: | *MAT_STEINBERG [0,3c,5,8B] {5} |
| *MAT_011_LUND: | *MAT_STEINBERG_LUND [0,3c,5,8B] {5} |

[1] Error associated with advection inherently leads to state variables that may be inconsistent with nonlinear constitutive routines and thus may lead to nonphysical results, nonconservation of energy, and even numerical instability in some cases. Caution is advised, particularly when using the 2nd tier of material models implemented for ALE multi-material solids (designated by [8B]) which are largely untested as ALE materials.

[2] These commands do not, by themselves, define a material model but rather can be used in certain cases to supplement material models.

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| *MAT_012: | *MAT_ISOTROPIC_ELASTIC_PLASTIC [0,2,3a,3c,5,8B] {0} |
| *MAT_013: | *MAT_ISOTROPIC_ELASTIC_FAILURE [0,3c,5,8B] {1} |
| *MAT_014: | *MAT_SOIL_AND_FOAM_FAILURE [0,3c,5,8B] {1} |
| *MAT_015: | *MAT_JOHNSON_COOK [0,2,3a,3c,5,8A] {6} |
| *MAT_016: | *MAT_PSEUDO_TENSOR [0,3c,5,8B] {6} |
| *MAT_017: | *MAT_ORIENTED_CRACK [0,3c] {14} |
| *MAT_018: | *MAT_POWER_LAW_PLASTICITY [0,1H,2,3a,3c,5,8B] {0} |
| *MAT_019: | *MAT_STRAIN_RATE_DEPENDENT_PLASTICITY [0,2,3a,3c,5,8B] {6} |
| *MAT_020: | *MAT_RIGID [0,1H,1B,1T,2,3a] {0} |
| *MAT_021: | *MAT_ORTHOTROPIC_THERMAL [0,2,3ac] {29} |
| *MAT_022: | *MAT_COMPOSITE_DAMAGE [0,2,3a,3c,5] {12} |
| *MAT_023: | *MAT_TEMPERATURE_DEPENDENT_ORTHOTROPIC [0,2,3ac] {19} |
| *MAT_024: | *MAT_PIECEWISE_LINEAR_PLASTICITY [0,1H,2,3a,3c,5,8A] {5} |
| *MAT_025: | *MAT_GEOLOGIC_CAP_MODEL [0,3c,5] {12} |
| *MAT_026: | *MAT_HONEYCOMB [0,3c] {20} |
| *MAT_027: | *MAT_MOONEY-RIVLIN_RUBBER [0,1T,2,3c,8B] {9} |
| *MAT_028: | *MAT_RESULTANT_PLASTICITY [1B,2] {5} |
| *MAT_029: | *MAT_FORCE_LIMITED [1B] {30} |
| *MAT_030: | *MAT_SHAPE_MEMORY [0,1H,2,3ac,5] {23} |
| *MAT_031: | *MAT_FRAZER_NASH_RUBBER_MODEL [0,3c,8B] {9} |
| *MAT_032: | *MAT_LAMINATED_GLASS [2,3a] {0} |
| *MAT_033: | *MAT_BARLAT_ANISOTROPIC_PLASTICITY [0,2,3a,3c] {9} |
| *MAT_033_96: | *MAT_BARLAT_YLD96 [2,3a] {9} |
| *MAT_034: | *MAT_FABRIC [4] {29} |
| *MAT_034M: | *MAT_FABRIC_MAP [4] {17} |
| *MAT_035: | *MAT_PLASTIC_GREEN-NAGHDI_RATE [0,3c,5,8B] {22} |
| *MAT_036: | *MAT_3-PARAMETER_BARLAT [2,3a,3c] {7} |
| *MAT_036E: | *MAT_EXTENDED_3-PARAMETER_BARLAT [2,3a,3c] {7} |
| *MAT_037: | *MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC [2,3a] {9} |
| *MAT_038: | *MAT_BLATZ-KO_FOAM [0,2,3c,8B] {9} |
| *MAT_039: | *MAT_FLD_TRANSVERSELY_ANISOTROPIC [2,3a] {6} |
| *MAT_040: | *MAT_NONLINEAR_ORTHOTROPIC [0,2,3c] {17} |
| *MAT_041-050: | *MAT_USER_DEFINED_MATERIAL_MODELS [0,1H,1T,1D,2,3a,3c,5,8B] {0} |
| *MAT_051: | *MAT_BAMMAN [0,2,3a,3c,5,8B] {8} |
| *MAT_052: | *MAT_BAMMAN_DAMAGE [0,2,3a,3c,5,8B] {10} |
| *MAT_053: | *MAT_CLOSED_CELL_FOAM [0,3c,8B] {0} |
| *MAT_054-055: | *MAT_ENHANCED_COMPOSITE_DAMAGE [0,2,3a,3c] {21} |
| *MAT_057: | *MAT_LOW_DENSITY_FOAM [0,3c,5,8B] {26} |
| *MAT_058: | *MAT_LAMINATED_COMPOSITE_FABRIC [0,2,3a,3c] {15} |
| *MAT_059: | *MAT_COMPOSITE_FAILURE_{OPTION}_MODEL [0,2,3c,5] {22} |
| *MAT_060: | *MAT_ELASTIC_WITH_VISCOSITY [0,2,3a,3c,5,8B] {8} |
| *MAT_060C: | *MAT_ELASTIC_WITH_VISCOSITY_CURVE [0,2,3a,3c,5,8B] {8} |
| *MAT_061: | *MAT_KELVIN-MAXWELL_VISCOELASTIC [0,3c,5,8B] {14} |
| *MAT_062: | *MAT_VISCOUS_FOAM [0,3c,8B] {7} |
| *MAT_063: | *MAT_CRUSHABLE_FOAM [0,3c,5,8B] {8} |
| *MAT_064: | *MAT_RATE_SENSITIVE_POWERLAW_PLASTICITY [0,2,3a,3c,5,8B] {30} |
| *MAT_065: | *MAT_MODIFIED_ZERILLI_ARMSTRONG [0,2,3a,3c,5,8B] {6} |
| *MAT_066: | *MAT_LINEAR_ELASTIC_DISCRETE_BEAM [1D] {8} |
| *MAT_067: | *MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM [1D] {14} |
| *MAT_068: | *MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM [1D] {25} |
| *MAT_069: | *MAT_SID_DAMPER_DISCRETE_BEAM [1D] {13} |
| *MAT_070: | *MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM [1D] {8} |
| *MAT_071: | *MAT_CABLE_DISCRETE_BEAM [1D] {8} |
| *MAT_072: | *MAT_CONCRETE_DAMAGE [0,3c,5,8B] {6} |

*MAT_072R3: *MAT_CONCRETE_DAMAGE_REL3 [0,3c,5] {6}
*MAT_073: *MAT_LOW_DENSITY_VISCOUS_FOAM [0,3c,8B] {56}
*MAT_074: *MAT_ELASTIC_SPRING_DISCRETE_BEAM [1D] {8}
*MAT_075: *MAT_BILKHU/DUBOIS_FOAM [0,3c,5,8B] {8}
*MAT_076: *MAT_GENERAL_VISCOELASTIC [0,2,3a,3c,5,8B] {53}
*MAT_077_H: *MAT_HYPERELASTIC_RUBBER [0,2,3c,5,8B] {54}
*MAT_077_O: *MAT_OGDEN_RUBBER [0,2,3c,5,8B] {54}
*MAT_078: *MAT_SOIL_CONCRETE [0,3c,5,8B] {3}
*MAT_079: *MAT_HYSTERETIC_SOIL [0,3c,5,8B] {96}
*MAT_080: *MAT_RAMBERG-OSGOOD [0,3c,8B] {18}
*MAT_081: *MAT_PLASTICITY_WITH_DAMAGE [0,2,3a,3c] {5}
*MAT_082(_RCDC): *MAT_PLASTICITY_WITH_DAMAGE_ORTHO(_RCDC) [0,2,3a,3c] {22}
*MAT_083: *MAT_FU_CHANG_FOAM [0,3c,5,8B] {54}
*MAT_084: *MAT_WINFRITH_CONCRETE [0] {54}
*MAT_086: *MAT_ORTHOTROPIC_VISCOELASTIC [2,3a] {17}
*MAT_087: *MAT_CELLULAR_RUBBER [0,3c,5,8B] {19}
*MAT_088: *MAT_MTS [0,2,3a,3c,5,8B] {5}
*MAT_089: *MAT_PLASTICITY_POLYMER [0,2,3a,3c] {46}
*MAT_090: *MAT_ACOUSTIC [6] {25}
*MAT_091: *MAT_SOFT_TISSUE [0,2] {16}
*MAT_092: *MAT_SOFT_TISSUE_VISCO [0,2] {58}
*MAT_093: *MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM [1D] {25}
*MAT_094: *MAT_INELASTIC_SPRING_DISCRETE_BEAM [1D] {9}
*MAT_095: *MAT_INELASTC_6DOF_SPRING_DISCRETE_BEAM [1D] {25}
*MAT_096: *MAT_BRITTLE_DAMAGE [0,8B] {51}
*MAT_097: *MAT_GENERAL_JOINT_DISCRETE_BEAM [1D] {23}
*MAT_098: *MAT_SIMPLIFIED_JOHNSON_COOK [0,1H,1B,1T,2,3a,3c] {6}
*MAT_099: *MAT_SIMPLIFIED_JOHNSON_COOK_ORTHOTROPIC_DAMAGE [0,2,3a,3c] {22}
*MAT_100: *MAT_SPOTWELD_{OPTION} [0,1SW] {6}
*MAT_100_DA: *MAT_SPOTWELD_DAIMLERCHRYSLER [0] {6}
*MAT_101: *MAT_GEPLASTIC_SRATE_2000a [2,3a] {15}
*MAT_102(_T): *MAT_INV_HYPERBOLIC_SIN(_THERMAL) [0,3c,8B] {15}
*MAT_103: *MAT_ANISOTROPIC_VISCOPLASTIC [0,2,3a,3c,5] {20}
*MAT_103_P: *MAT_ANISOTROPIC_PLASTIC [2,3a,3c] {20}
*MAT_104: *MAT_DAMAGE_1 [0,2,3a,3c] {11}
*MAT_105: *MAT_DAMAGE_2 [0,2,3a,3c] {7}
*MAT_106: *MAT_ELASTIC_VISCOPLASTIC_THERMAL [0,2,3a,3c,5] {20}
*MAT_107: *MAT_MODIFIED_JOHNSON_COOK [0,2,3a,3c,5,8B] {15}
*MAT_108: *MAT_ORTHO_ELASTIC_PLASTIC [2,3a] {15}
*MAT_110: *MAT_JOHNSON_HOLMQUIST_CERAMICS [0,3c,5] {15}
*MAT_111: *MAT_JOHNSON_HOLMQUIST_CONCRETE [0,3c,5] {25}
*MAT_112: *MAT_FINITE_ELASTIC_STRAIN_PLASTICITY [0,3c,5] {22}
*MAT_113: *MAT_TRIP [2,3a] {5}
*MAT_114: *MAT_LAYERED_LINEAR_PLASTICITY [2,3a] {13}
*MAT_115: *MAT_UNIFIED_CREEP [0,2,3a,3c,5] {1}
*MAT_115_O: *MAT_UNIFIED_CREEP_ORTHO [0,3c,5] {1}
*MAT_116: *MAT_COMPOSITE_LAYUP [2] {30}
*MAT_117: *MAT_COMPOSITE_MATRIX [2] {30}
*MAT_118: *MAT_COMPOSITE_DIRECT [2] {10}
*MAT_119: *MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM [1D] {62}
*MAT_120: *MAT_GURSON [0,2,3a,3c] {12}
*MAT_120_JC: *MAT_GURSON_JC [0,2] {12}
*MAT_120_RCDC: *MAT_GURSON_RCDC [0,2] {12}
*MAT_121: *MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM [1D] {20}

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| *MAT_122: | *MAT_HILL_3R [2,3a] {8} |
| *MAT_122_3D: | *MAT_HILL_3R_3D [0] {28} |
| *MAT_122_TAB: | *MAT_HILL_3R_TABULATED [2,3a] {8} |
| *MAT_123: | *MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY [0,2,3a,3c,5] {11} |
| *MAT_124: | *MAT_PLASTICITY_COMPRESSION_TENSION [0,1H,2,3a,3c,5,8B] {7} |
| *MAT_125: | *MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC [0,2,3a,3c] {11} |
| *MAT_126: | *MAT_MODIFIED_HONEYCOMB [0,3c] {20} |
| *MAT_127: | *MAT_ARRUDA_BOYCE_RUBBER [0,3c,5] {49} |
| *MAT_128: | *MAT_HEART_TISSUE [0,3c] {15} |
| *MAT_129: | *MAT_LUNG_TISSUE [0,3c] {49} |
| *MAT_130: | *MAT_SPECIAL_ORTHOTROPIC [2] {35} |
| *MAT_131: | *MAT_ISOTROPIC_SMEARED_CRACK [0,5,8B] {15} |
| *MAT_132: | *MAT_ORTHOTROPIC_SMEARED_CRACK [0] {61} |
| *MAT_133: | *MAT_BARLAT_YLD2000 [0,2,3a,3c] {9} |
| *MAT_134: | *MAT_VISCOELASTIC_FABRIC [9] |
| *MAT_135: | *MAT_WTM_STM [2,3a,3c] {30} |
| *MAT_135_PLC: | *MAT_WTM_STM_PLC [2,3a] {30} |
| *MAT_136: | *MAT_VEGTER [2,3a] {5} |
| *MAT_136_STD: | *MAT_VEGTER_STANDARD [2,3a] {5} |
| *MAT_136_2017: | *MAT_VEGTER_2017 [2,3a] {5} |
| *MAT_138: | *MAT_COHESIVE_MIXED_MODE [7] {0} |
| *MAT_139: | *MAT_MODIFIED_FORCE_LIMITED [1B] {35} |
| *MAT_140: | *MAT_VACUUM [0,8A] {0} |
| *MAT_141: | *MAT_RATE_SENSITIVE_POLYMER [0,3c,8B] {6} |
| *MAT_142: | *MAT_TRANSVERSELY_ISOTROPIC_CRUSHABLE_FOAM [0,3c] {12} |
| *MAT_143: | *MAT_WOOD_{OPTION} [0,3c,5] {37} |
| *MAT_144: | *MAT_PITZER_CRUSHABLE_FOAM [0,3c,8B] {7} |
| *MAT_145: | *MAT_SCHWER_MURRAY_CAP_MODEL [0,5] {50} |
| *MAT_146: | *MAT_1DOF_GENERALIZED_SPRING [1D] {1} |
| *MAT_147 | *MAT_FHWA_SOIL [0,3c,5,8B] {15} |
| *MAT_147_N: | *MAT_FHWA_SOIL_NEBRASKA [0,3c,5,8B] {15} |
| *MAT_148: | *MAT_GAS_MIXTURE [0,8A] {14} |
| *MAT_151: | *MAT_EMMI [0,3c,5,8B] {23} |
| *MAT_153: | *MAT_DAMAGE_3 [0,1H,2,3a,3c] |
| *MAT_154: | *MAT_DESHPANDE_FLECK_FOAM [0,3c,8B] {10} |
| *MAT_155: | *MAT_PLASTICITY_COMPRESSION_TENSION_EOS [0,3c,5,8B] {16} |
| *MAT_156: | *MAT_MUSCLE [1T] {0} |
| *MAT_157: | *MAT_ANISOTROPIC_ELASTIC_PLASTIC [0,2,3a] {5} |
| *MAT_158: | *MAT_RATE_SENSITIVE_COMPOSITE_FABRIC [2,3a] {54} |
| *MAT_159: | *MAT_CSCM_{OPTION} [0,3c,5] {22} |
| *MAT_160: | *MAT_ALE_INCOMPRESSIBLE |
| *MAT_161: | *MAT_COMPOSITE_MSC [0] {34} |
| *MAT_162: | *MAT_COMPOSITE_DMG_MSC [0] {40} |
| *MAT_163 | *MAT_MODIFIED_CRUSHABLE_FOAM [0,3c,5,8B] {10} |
| *MAT_164: | *MAT_BRAIN_LINEAR_VISCOELASTIC [0] {14} |
| *MAT_165: | *MAT_PLASTIC_NONLINEAR_KINEMATIC [0,2,3a,3c,8B] {8} |
| *MAT_166: | *MAT_MOMENT_CURVATURE_BEAM [1B] {54} |
| *MAT_167: | *MAT_MCCORMICK [03c,,8B] {8} |
| *MAT_168: | *MAT_POLYMER [0,3c,8B] {60} |
| *MAT_169: | *MAT_ARUP_ADHESIVE [0] {30} |
| *MAT_170: | *MAT_RESULTANT_ANISOTROPIC [2,3a] {67} |
| *MAT_171: | *MAT_STEEL_CONCENTRIC_BRACE [1B] {35} |
| *MAT_172: | *MAT_CONCRETE_EC2 [1H,2,3a] {72} |
| *MAT_173: | *MAT_MOHR_COULOMB [0,3c,5] {52} |

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| *MAT_174: | *MAT_RC_BEAM [1H] {22} |
| *MAT_175: | *MAT_VISCOELASTIC_THERMAL [0,2,3a,3c,5,8B] {86} |
| *MAT_176: | *MAT_QUASILINEAR_VISCOELASTIC [0,2,3a,3c,5,8B] {81} |
| *MAT_177: | *MAT_HILL_FOAM [0,3c] {12} |
| *MAT_178: | *MAT_VISCOELASTIC_HILL_FOAM [0,3c] {92} |
| *MAT_179: | *MAT_LOW_DENSITY_SYNTHETIC_FOAM_{OPTION} [0,3c] {77} |
| *MAT_180: | *MAT_LOW_DENSITY_SYNTHETIC_FOAM_ORTHO [0,3c] |
| *MAT_181: | *MAT_SIMPLIFIED_RUBBER/FOAM_{OPTION} [0,2,3c] {39} |
| *MAT_183: | *MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE [0,2,3c] {44} |
| *MAT_184: | *MAT_COHESIVE_ELASTIC [7] {0} |
| *MAT_185: | *MAT_COHESIVE_TH [7] {0} |
| *MAT_186: | *MAT_COHESIVE_GENERAL [7] {6} |
| *MAT_187: | *MAT_SAMP-1 [0,2,3a,3c] {38} |
| *MAT_187L: | *MAT_SAMP_LIGHT [0,2,3a,3c] {7} |
| *MAT_188: | *MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP [0,2,3a,3c] {27} |
| *MAT_189: | *MAT_ANISOTROPIC_THERMOELASTIC [0,3c,8B] {21} |
| *MAT_190: | *MAT_FLD_3-PARAMETER_BARLAT [2,3a] {36} |
| *MAT_191: | *MAT_SEISMIC_BEAM [1B] {36} |
| *MAT_192: | *MAT_SOIL_BRICK [0,3c] {96} |
| *MAT_193: | *MAT_DRUCKER_PRAGER [0,3c,5] {24} |
| *MAT_194: | *MAT_RC_SHEAR_WALL [2,3a] {36} |
| *MAT_195: | *MAT_CONCRETE_BEAM [1H] {5} |
| *MAT_196: | *MAT_GENERAL_SPRING_DISCRETE_BEAM [1D] {25} |
| *MAT_197: | *MAT_SEISMIC_ISOLATOR [1D] {20} |
| *MAT_198: | *MAT_JOINTED_ROCK [0,3c] {31} |
| *MAT_199: | *MAT_BARLAT_YLD2004 [0,3c] {11} |
| *MAT_199_27P: | *MAT_BARLAT_YLD2004_27P [0,3c] {11} |
| *MAT_202: | *MAT_STEEL_EC3 [1H] {3} |
| *MAT_203: | *MAT_HYSTERETIC_REINFORCEMENT [1H,2,3a] {64} |
| *MAT_205: | *MAT_DISCRETE_BEAM_POINT_CONTACT [1D] |
| *MAT_207: | *MAT_SOIL_SANISAND [0] |
| *MAT_208: | *MAT_BOLT_BEAM [1D] {16} |
| *MAT_209: | *MAT_HYSTERETIC_BEAM [1B] {50} |
| *MAT_211: | *MAT_SPR_JLR [0] {60} |
| *MAT_213: | *MAT_COMPOSITE_TABULATED_PLASTICITY_DAMAGE [0,2,3a,3c] {54} |
| *MAT_214: | *MAT_DRY_FABRIC [9] |
| *MAT_215: | *MAT_4A_MICROMEC [0,2,3a,3c] |
| *MAT_216: | *MAT_ELASTIC_PHASE_CHANGE [0] |
| *MAT_217: | *MAT_ORTHOTROPIC_ELASTIC_PHASE_CHANGE [2] |
| *MAT_218: | *MAT_MOONEY-RIVLIN_PHASE_CHANGE [0] |
| *MAT_219: | *MAT_CODAM2 [0,2,3a,3c] |
| *MAT_220: | *MAT_RIGID_DISCRETE [0,2] |
| *MAT_221: | *MAT_ORTHOTROPIC_SIMPLIFIED_DAMAGE [0,3c,5] {17} |
| *MAT_224: | *MAT_TABULATED_JOHNSON_COOK [0,2,3a,3c,,5] {17} |
| *MAT_224_GYS: | *MAT_TABULATED_JOHNSON_COOK_GYS [0] {17} |
| *MAT_225: | *MAT_VISCOPLASTIC_MIXED_HARDENING [0,2,3a,3c,5] |
| *MAT_226: | *MAT_KINEMATIC_HARDENING_BARLAT89 [2,3a] |
| *MAT_230: | *MAT_PML_ELASTIC [0] {24} |
| *MAT_231: | *MAT_PML_ACOUSTIC [6] {35} |
| *MAT_232: | *MAT_BIOT_HYSTERETIC [0,2,3a] {30} |
| *MAT_233: | *MAT_CAZACU_BARLAT [2,3a] |
| *MAT_234: | *MAT_VISCOELASTIC_LOOSE_FABRIC [2,3a] |
| *MAT_235: | *MAT_MICROMECHANICS_DRY_FABRIC [2,3a] |
| *MAT_236: | *MAT_SCC_ON_RCC [2,3a] |

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| *MAT_237: | *MAT_PML_HYSTERETIC [0] {54} |
| *MAT_238: | *MAT_PERT_PIECEWISE_LINEAR_PLASTICITY [0,1H,2,3,5,8A] |
| *MAT_240: | *MAT_COHESIVE_MIXED_MODE_ELASTOPLASTIC_RATE [7] {17} |
| *MAT_241: | *MAT_JOHNSON_HOLMQUIST_JH1 [0,3c,5] |
| *MAT_242: | *MAT_KINEMATIC_HARDENING_BARLAT2000 [2,3a] |
| *MAT_243: | *MAT_HILL_90 [2,3a,3c] |
| *MAT_244: | *MAT_UHS_STEEL [0,2,3a,3c,5] {35} |
| *MAT_245: | *MAT_PML_{OPTION}TROPIC_ELASTIC [0] {30} |
| *MAT_246: | *MAT_PML_NULL [0] {27} |
| *MAT_248: | *MAT_PHS_BMW [2] {38} |
| *MAT_249: | *MAT_REINFORCED_THERMOPLASTIC [2] |
| *MAT_249_CRASH: | *MAT_REINFORCED_THERMOPLASTIC_CRASH [2] |
| *MAT_249_UDFIBER: | *MAT_REINFORCED_THERMOPLASTIC_UDFIBER [2] |
| *MAT_251: | *MAT_TAILORED_PROPERTIES [0,2] {6} |
| *MAT_252: | *MAT_TOUGHENED_ADHESIVE_POLYMER [0,7] {10} |
| *MAT_254: | *MAT_GENERALIZED_PHASE_CHANGE [0,2] |
| *MAT_255: | *MAT_PIECEWISE_LINEAR_PLASTIC_THERMAL [0,2,3a,3c] |
| *MAT_256: | *MAT_AMORPHOUS_SOLIDS_FINITE_STRAIN [0] |
| *MAT_258: | *MAT_NON_QUADRATIC_FAILURE [2] |
| *MAT_260A: | *MAT_STOUGHTON_NON_ASSOCIATED_FLOW [0,2] |
| *MAT_260B: | *MAT_MOHR_NON_ASSOCIATED_FLOW [2] |
| *MAT_261: | *MAT_LAMINATED_FRACTURE_DAIMLER_PINHO [0,2,3a,3c] |
| *MAT_262: | *MAT_LAMINATED_FRACTURE_DAIMLER_CAMANHO [0,2,3a,3c] |
| *MAT_263: | *MAT_LOU-YOON_ANISOTROPIC_PLASTICITY [0,2] |
| *MAT_264: | *MAT_TABULATED_JOHNSON_COOK_ORTHO_PLASTICITY [0,3c] |
| *MAT_265: | *MAT_CONSTRAINED [10] |
| *MAT_266: | *MAT_TISSUE_DISPERSED [0] |
| *MAT_267: | *MAT_EIGHT_CHAIN_RUBBER [0,5] |
| *MAT_269: | *MAT_BERGSTROM_BOYCE_RUBBER [0,5] |
| *MAT_270: | *MAT_CWM [0,2,5] |
| *MAT_271: | *MAT_POWDER [0,5] |
| *MAT_272: | *MAT_RHT [0,5] |
| *MAT_273: | *MAT_CONCRETE_DAMAGE_PLASTIC_MODEL [0] |
| *MAT_274: | *MAT_PAPER [0,2] |
| *MAT_275: | *MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC [0] |
| *MAT_276: | *MAT_CHRONOLOGICAL_VISCOELASTIC [2,3a,3c] |
| *MAT_277: | *MAT_ADHESIVE_CURING_VISCOELASTIC [0,2] |
| *MAT_278: | *MAT_CF_MICROMECHANICS [0,2] |
| *MAT_279: | *MAT_COHESIVE_PAPER [7] |
| *MAT_280: | *MAT_GLASS [2,3a] {32} |
| *MAT_291: | *MAT_SHAPE_MEMORY_ALLOY [0] {20} |
| *MAT_292: | *MAT_ELASTIC_PERI [11] |
| *MAT_292A: | *MAT_ELASTIC_PERI_LAMINATE [11] |
| *MAT_293: | *MAT_COMPRF [2] {7} |
| *MAT_295: | *MAT_ANISOTROPIC_HYPERELASTIC [0] {9} |
| *MAT_296: | *MAT_ANAND_VISCOPLASTICITY [0] |
| *MAT_303: | *MAT_DMN_COMPOSITE_FRC [0,2] |
| *MAT_305: | *MAT_HOT_PLATE_ROLLING [0]{12} |
| *MAT_307: | *MAT_GENERALIZED_ADHESIVE_CURING [0,7] |
| *MAT_317: | *MAT_RRR_POLYMER [0] {10} |
| *MAT_318: | *MAT_TNM_POLYMER [0,3c] {21} |
| *MAT_319: | *MAT_IFPD [12] |
| *MAT_326: | *MAT_COHESIVE_GASKET [7] {0} |
| *MAT_331: | *MAT_IMPACT_ORTHOTROPIC [0] {13} |

For discrete (type 6) beam elements, which are used to model complicated dampers and multi-dimensional spring-damper combinations, the following material types are available:

| | |
|---|---|
| *MAT_066: | *MAT_LINEAR_ELASTIC_DISCRETE_BEAM |
| *MAT_067: | *MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM |
| *MAT_068: | *MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM |
| *MAT_069: | *MAT_SID_DAMPER_DISCRETE_BEAM |
| *MAT_070: | *MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM |
| *MAT_071: | *MAT_CABLE_DISCRETE_BEAM |
| *MAT_074: | *MAT_ELASTIC_SPRING_DISCRETE_BEAM |
| *MAT_093: | *MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM |
| *MAT_094: | *MAT_INELASTIC_SPRING_DISCRETE_BEAM |
| *MAT_095: | *MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM |
| *MAT_119: | *MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM |
| *MAT_121: | *MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM |
| *MAT_146: | *MAT_1DOF_GENERALIZED_SPRING |
| *MAT_196: | *MAT_GENERAL_SPRING_DISCRETE_BEAM |
| *MAT_197: | *MAT_SEISMIC_ISOLATOR |
| *MAT_205: | *MAT_DISCRETE_BEAM_POINT_CONTACT |
| *MAT_208: | *MAT_BOLT_BEAM |

For discrete springs and dampers, the following material types are available:

| | |
|---|---|
| *MAT_S01: | *MAT_SPRING_ELASTIC |
| *MAT_S02: | *MAT_DAMPER_VISCOUS |
| *MAT_S03: | *MAT_SPRING_ELASTOPLASTIC |
| *MAT_S04: | *MAT_SPRING_NONLINEAR_ELASTIC |
| *MAT_S05: | *MAT_DAMPER_NONLINEAR_VISCOUS |
| *MAT_S06: | *MAT_SPRING_GENERAL_NONLINEAR |
| *MAT_S07: | *MAT_SPRING_MAXWELL |
| *MAT_S08: | *MAT_SPRING_INELASTIC |
| *MAT_S13: | *MAT_SPRING_TRILINEAR_DEGRADING |
| *MAT_S14: | *MAT_SPRING_SQUAT_SHEARWALL |
| *MAT_S15: | *MAT_SPRING_MUSCLE |

For ALE solids the following material types are available:

| | | |
|---|---|---|
| *MAT_ALE_01: | *MAT_ALE_VACUUM | (same as *MAT_140) |
| *MAT_ALE_02: | *MAT_ALE_GAS_MIXTURE | (same as *MAT_148) |
| *MAT_ALE_03: | *MAT_ALE_VISCOUS | (same as *MAT_009) |
| *MAT_ALE_04: | *MAT_ALE_MIXING_LENGTH | (same as *MAT_149) |
| *MAT_ALE_05: | *MAT_ALE_INCOMPRESSIBLE | (same as *MAT_160) |
| *MAT_ALE_06: | *MAT_ALE_HERSCHEL | |

The following material models are only available for the incompressible smoothed particle Galerkin (ISPG) method:

| | |
|---|---|
| *MAT_ISPG_01: | *MAT_ISPG_CARREAU |
| *MAT_ISPG_02: | *MAT_ISPG_CROSSMODEL |
| *MAT_ISPG_03: | *MAT_ISPG_ISO_NEWTONIAN |
| *MAT_ISPG_04: | *MAT_ISPG_CROSS_CASTRO_MACOSKO |

*MAT_ISPG_05: *MAT_ISPG_CROSS_WLF_MODEL

The following material type is only available for SPH particles:

*MAT_SPH_01: *MAT_SPH_VISCOUS (same as *MAT_009)
*MAT_SPH_02: *MAT_SPH_INCOMPRESSIBLE_FLUID
*MAT_SPH_03: *MAT_SPH_INCOMPRESSIBLE_STRUCTURE

In addition, most of the material types which are available for solids are also available for SPH. Those material models that may be used for SPH have a "5" included in square brackets in the list of materials given above. In the detailed descriptions of those materials which come later in the User's Manual, the word "solids" implies "solids and SPH".

For seat belts one material is available:

*MAT_B01: *MAT_SEATBELT

For thermal materials in a coupled structural/thermal or thermal-only analysis, the following materials are available. These materials are related to the structural material through the *PART card.

*MAT_T01: *MAT_THERMAL_ISOTROPIC
*MAT_T02: *MAT_THERMAL_ORTHOTROPIC
*MAT_T03: *MAT_THERMAL_ISOTROPIC_TD
*MAT_T04: *MAT_THERMAL_ORTHOTROPIC_TD
*MAT_T05: *MAT_THERMAL_DISCRETE_BEAM
*MAT_T06: *MAT_THERMAL_CHEMICAL_REACTION
*MAT_T07: *MAT_THERMAL_CWM
*MAT_T08 *MAT_THERMAL_ORTHOTROPIC_TD_LC
*MAT_T09 *MAT_THERMAL_ISOTROPIC_PHASE_CHANGE
*MAT_T10 *MAT_THERMAL_ISOTROPIC_TD_LC
*MAT_T11-T15: *MAT_THERMAL_USER_DEFINED DEFINED
*MAT_T17: *MAT_THERMAL_CHEMICAL_REACTION_ORTHOTROPIC
*MAT_T18: *MAT_THERMAL_ISPG

### Remarks:

Curves and tables are sometimes needed for defining material properties. An example would be a curve of effective stress as a function of effective plastic strain defined using the command *DEFINE_CURVE. In general, the following can be said about curves and tables that are referenced by material models:

1. Curves are internally rediscretized using equal increments along the $x$-axis.

2. Curve data is interpolated between rediscretized data points within the defined range of the curve and extrapolated as needed beyond the defined range of the curve.

3. Extrapolation is not employed for table value. See the manual entries for the *DEFINE_TABLE_… keywords.

1. See Remarks under *DEFINE_CURVE and *DEFINE_TABLE.

## MATERIAL MODEL REFERENCE TABLES

The tables provided on the following pages list the material models, some of their attributes, and the general classes of physical materials to which the numerical models might be applied.

If a material model, without consideration of *MAT_ADD_EROSION, *MAT_ADD_-THERMAL_EXPANSION, *MAT_ADD_SOC_EXPANSION, *MAT_ADD_DAMAGE, *MAT_ADD_GENERALIZED_DAMAGE or *MAT_ADD_INELASTICITY, includes any of the following attributes, a "Y" will appear in the respective column of the table:

**SRATE - Strain-rate effects**
**FAIL - Failure criteria**
**EOS - Equation-of-State required for 3D solids and 2D continuum elements**
**THERMAL - Thermal effects**
**ANISO - Anisotropic/orthotropic**
**DAM - Damage effects**
**TENS - Tension handled differently than compression in some manner**

Potential applications of the material models, in terms of classes of physical materials, are abbreviated in the table as follows:

**GN - General**
**CM - Composite**
**CR - Ceramic**
**FL - Fluid**
**FM - Foam**
**GL - Glass**
**HY - Hydrodynamic material**
**MT - Metal**
**PL - Plastic**
**RB - Rubber**
**SL - Soil, concrete, or rock**
**AD - Adhesive or Cohesive material**
**BIO - Biological material**
**CIV - Civil Engineering component**
**HT - Heat Transfer**
**F - Fabric**

| Material Number And Description | | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Elastic | | | | | | | | GN, FL |
| 2 | Orthotropic Elastic (Anisotropic-solids) | | | | | Y | | | CM, MT |
| 3 | Plastic Kinematic/Isotropic | Y | Y | | | | | | CM, MT, PL |
| 4 | Elastic Plastic Thermal | | | | Y | | | | MT, PL |
| 5 | Soil and Foam | | | | | | | Y | FM, SL |
| 6 | Linear Viscoelastic | Y | | | | | | | RB |
| 7 | Blatz-Ko Rubber | | | | | | | | RB |
| 8 | High Explosive Burn | | | Y | | | | | HY |
| 9 | Null Material | Y | Y | Y | | | | Y | FL, HY |
| 10 | Elastic Plastic Hydro(dynamic) | | Y | Y | | | | Y | HY, MT |
| 11 | Steinberg: Temp. Dependent Elasto-plastic | Y | Y | Y | Y | | | Y | HY, MT |
| 12 | Isotropic Elastic Plastic | | | | | | | | MT |
| 13 | Isotropic Elastic with Failure | | Y | | | | | Y | MT |
| 14 | Soil and Foam with Failure | | Y | | | | | Y | FM, SL |
| 15 | Johnson/Cook Plasticity Model | Y | Y | Y | Y | | Y | Y | HY, MT |
| 16 | Pseudo Tensor Geological Model | Y | Y | Y | | | Y | Y | SL |
| 17 | Oriented Crack (Elastoplastic w/ Fracture) | | Y | Y | | Y | | Y | HY, MT, PL, CR |
| 18 | Power Law Plasticity (Isotropic) | Y | | | | | | | MT, PL |
| 19 | Strain Rate Dependent Plasticity | Y | Y | | | | | | MT, PL |
| 20 | Rigid | | | | | | | | |
| 21 | Orthotropic Thermal (Elastic) | | | | Y | Y | | | GN |
| 22 | Composite Damage | | Y | | | Y | | Y | CM |
| 23 | Temperature Dependent Orthotropic | | | | Y | Y | | | CM |
| 24 | Piecewise Linear Plasticity (Isotropic) | Y | Y | | | | | | MT, PL |
| 25 | Inviscid Two Invariant Geologic Cap | | Y | | | | | Y | SL |
| 26 | Honeycomb | Y | Y | | | Y | | Y | CM, FM, SL |
| 27 | Mooney-Rivlin Rubber | | | | | | | Y | RB |
| 28 | Resultant Plasticity | | | | | | | | MT |
| 29 | Force Limited Resultant Formulation | | | | | | | Y | |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 30 | Shape Memory | | | | | | | | MT |
| 31 | Frazer-Nash Rubber | | | | | | | Y | RB |
| 32 | Laminated Glass (Composite) | | Y | | | | | | CM, GL |
| 33 | Barlat Anisotropic Plasticity | | | | | Y | | | CR, MT |
| 33_96 | Barlat YLD96 | Y | | | | Y | | | MT |
| 34 | Fabric | | | | | Y | | Y | F |
| 34M | Fabric Map | | | | Y | Y | | Y | F |
| 35 | Plastic-Green Naghdi Rate | Y | | | | | | | MT |
| 36 | Three-Parameter Barlat Plasticity | Y | | | Y | Y | | | MT |
| 36E | Extended Three-Parameter Barlat Plasticity | Y | | | | Y | | | MT |
| 37 | Transversely Anisotropic Elastic Plastic | | | | | Y | | | MT |
| 38 | Blatz-Ko Foam | | | | | | | | FM, PL |
| 39 | FLD Transversely Anisotropic | | | | | Y | | | MT |
| 40 | Nonlinear Orthotropic | | Y | | Y | Y | | Y | CM |
| 41-50 | User Defined Materials | Y | Y | Y | Y | Y | Y | Y | GN |
| 51 | Bamman (Temp/Rate Dependent Plasticity) | Y | | | Y | | | | GN |
| 52 | Bamman Damage | Y | Y | | Y | | Y | | MT |
| 53 | Closed cell foam (Low density polyurethane) | | | | | | | | FM |
| 54 | Composite Damage with Chang Failure | | Y | | | Y | Y | Y | CM |
| 55 | Composite Damage with Tsai-Wu Failure | | Y | | | Y | Y | Y | CM |
| 57 | Low Density Urethane Foam | Y | Y | | Y | | | Y | FM |
| 58 | Laminated Composite Fabric | | Y | | | Y | Y | Y | CM, F |
| 59 | Composite Failure (Plasticity Based) | | Y | | | Y | | Y | CM, CR |
| 60 | Elastic with Viscosity (Viscous Glass) | Y | | | Y | | | | GL |
| 61 | Kelvin-Maxwell Viscoelastic | Y | | | | | | | FM |
| 62 | Viscous Foam (Crash dummy Foam) | Y | | | | | | | FM |
| 63 | Isotropic Crushable Foam | Y | | | | | | Y | FM |
| 64 | Rate Sensitive Powerlaw Plasticity | Y | | | | | | | MT |
| 65 | Zerilli-Armstrong (Rate/Temp Plasticity) | Y | | Y | Y | | | Y | MT |

| Material Number And Description | | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 66 | Linear Elastic Discrete Beam | Y | | | | Y | | | |
| 67 | Nonlinear Elastic Discrete Beam | Y | | | | Y | | Y | |
| 68 | Nonlinear Plastic Discrete Beam | Y | Y | | | Y | | | |
| 69 | SID Damper Discrete Beam | Y | | | | | | | |
| 70 | Hydraulic Gas Damper Discrete Beam | Y | | | | | | | |
| 71 | Cable Discrete Beam (Elastic) | | | | | | | Y | Cables |
| 72 | Concrete Damage (incl. Release III) | Y | Y | Y | | | Y | Y | SL |
| 73 | Low Density Viscous Foam | Y | Y | | | | | Y | FM |
| 74 | Elastic Spring Discrete Beam | Y | Y | | | | | Y | |
| 75 | Bilkhu/Dubois Foam | | | | | | | Y | FM |
| 76 | General Viscoelastic (Maxwell Model) | Y | | | Y | | | Y | RB |
| 77 | Hyperelastic and Ogden Rubber | Y | | | | | | Y | RB |
| 78 | Soil Concrete | | Y | | | | Y | Y | SL |
| 79 | Hysteretic Soil (Elasto-Perfectly Plastic) | | Y | | | | | Y | SL |
| 80 | Ramberg-Osgood | | | | | | | | SL |
| 81 | Plasticity with Damage | Y | Y | | | | Y | | MT, PL |
| 82 | Plasticity with Damage Ortho | Y | Y | | | Y | Y | | MT |
| 83 | Fu Chang Foam | Y | Y | | | | Y | Y | FM |
| 84 | Winfrith Concrete | Y | | | | | | Y | FM, SL |
| 86 | Orthotropic Viscoelastic | Y | | | | Y | | | RB |
| 87 | Cellular Rubber | Y | | | | | | Y | RB |
| 88 | MTS | Y | | Y | Y | | | | MT |
| 89 | Plasticity Polymer | Y | | | | | | Y | PL |
| 90 | Acoustic | | | | | | | Y | FL |
| 91 | Soft Tissue | Y | Y | | | Y | | Y | BIO |
| 92 | Soft Tissue (viscous) | | | | | | | | |
| 93 | Elastic 6DOF Spring Discrete Beam | Y | Y | | | Y | | Y | |
| 94 | Inelastic Spring Discrete Beam | Y | Y | | | | | Y | |
| 95 | Inelastic 6DOF Spring Discrete Beam | Y | Y | | | Y | | Y | |
| 96 | Brittle Damage | Y | Y | | | Y | Y | Y | SL |
| 97 | General Joint Discrete Beam | | | | | | | | |
| 98 | Simplified Johnson Cook | Y | Y | | | | | | MT |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 99 | Simpl. Johnson Cook Orthotropic Damage | Y | Y | | | Y | Y | | MT |
| 100 | Spotweld | Y | Y | | | | Y | Y | MT |
| 101 | GE Plastic Strain Rate | Y | Y | | | | | Y | PL |
| 102(_T) | Inv. Hyperbolic Sin (Thermal) | Y | | | Y | | | | MT, PL |
| 103 | Anisotropic Viscoplastic | Y | Y | | | Y | | | MT |
| 103P | Anisotropic Plastic | | | | | Y | | | MT |
| 104 | Damage 1 | Y | Y | | | Y | Y | | MT |
| 105 | Damage 2 | Y | Y | | | | Y | | MT |
| 106 | Elastic Viscoplastic Thermal | Y | | | Y | | | | PL |
| 107 | Modified Johnson Cook | Y | Y | | Y | | Y | | MT |
| 108 | Ortho Elastic Plastic | | | | | Y | | | |
| 110 | Johnson Holmquist Ceramics | Y | Y | | | | Y | Y | CR, GL |
| 111 | Johnson Holmquist Concrete | Y | Y | | | | Y | Y | SL |
| 112 | Finite Elastic Strain Plasticity | Y | | | | | | | PL |
| 113 | Transformation Induced Plasticity (TRIP) | | | | Y | | | | MT |
| 114 | Layered Linear Plasticity | Y | Y | | | | | | MT, PL, CM |
| 115 | Unified Creep | | | | | | | | GN |
| 115_O | Unified Creep Ortho | | | | | Y | | | GN |
| 116 | Composite Layup | | | | | Y | | | CM |
| 117 | Composite Matrix | | | | | Y | | | CM |
| 118 | Composite Direct | | | | | Y | | | CM |
| 119 | General Nonlinear 6DOF Discrete Beam | Y | Y | | | Y | | Y | |
| 120 | Gurson | Y | Y | | | | Y | Y | MT |
| 121 | General Nonlinear 1DOF Discrete Beam | Y | Y | | | | | Y | |
| 122 | Hill 3RC | | | | | Y | | | MT |
| 122_3D | Hill 3R 3D | | | | | Y | | | MT, CM |
| 122_TAB | Hill 3R Tabulated | | | | | Y | | | MT |
| 123 | Modified Piecewise Linear Plasticity | Y | Y | | | | | | MT, PL |
| 124 | Plasticity Compression Tension | Y | Y | | | | | Y | MT, PL |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 125 | Kinematic Hardening Transversely Aniso. | | | | | Y | | | MT |
| 126 | Modified Honeycomb | Y | Y | | | Y | Y | Y | CM, FM, SL |
| 127 | Arruda Boyce Rubber | Y | | | | | | | RB |
| 128 | Heart Tissue | | | | | Y | | Y | BIO |
| 129 | Lung Tissue | Y | | | | | | Y | BIO |
| 130 | Special Orthotropic | | | | | Y | | | |
| 131 | Isotropic Smeared Crack | | Y | | | | Y | Y | MT, CM |
| 132 | Orthotropic Smeared Crack | | Y | | | Y | Y | | MT, CM |
| 133 | Barlat YLD2000 | Y | | | Y | Y | | | MT |
| 134 | Viscoelastic Fabric | | | | | | | | |
| 135 | Weak and Strong Texture Model | Y | Y | | | Y | | | MT |
| 136 | Vegter | | | | | Y | | | MT |
| 136_STD | Vegter Standard Input | Y | | | | Y | | | MT |
| 136_2017 | Vegter Simplified Input | Y | | | | Y | | | MT |
| 138 | Cohesive Mixed Mode | | Y | | | Y | Y | Y | AD |
| 139 | Modified Force Limited | | | | | | Y | Y | |
| 140 | Vacuum | | | | | | | | |
| 141 | Rate Sensitive Polymer | Y | | | | | | | PL |
| 142 | Transversely Isotropic Crushable Foam | | | | | | | Y | FM |
| 143 | Wood | Y | Y | | | Y | Y | Y | Wood |
| 144 | Pitzer Crushable Foam | Y | | | | | | Y | FM |
| 145 | Schwer Murray Cap Model | Y | Y | | | | Y | Y | SL |
| 146 | 1DOF Generalized Spring | Y | | | | | | | |
| 147 | FWHA Soil | Y | | | | | Y | Y | SL |
| 147N | FHWA Soil Nebraska | Y | | | | | Y | Y | SL |
| 148 | Gas Mixture | | | | Y | | | | FL |
| 151 | Evolving Microstructural Model of In-elast. | Y | Y | | Y | Y | Y | | MT |
| 153 | Damage 3 | Y | Y | | | | Y | | MT, PL |
| 154 | Deshpande Fleck Foam | | Y | | | | | | FM |
| 155 | Plasticity Compression Tension EOS | Y | Y | Y | | | | Y | Ice |
| 156 | Muscle | Y | | | | | | Y | BIO |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 157 | Anisotropic Elastic Plastic | | | | | Y | | | MT, CM |
| 158 | Rate-Sensitive Composite Fabric | Y | Y | | | Y | Y | Y | CM |
| 159 | CSCM | Y | Y | | | | Y | Y | SL |
| 160 | ALE incompressible | | | | | | | | |
| 161,162 | Composite MSC (Dmg) | Y | Y | | | Y | Y | Y | CM |
| 163 | Modified Crushable Foam | Y | | | | | | Y | FM |
| 164 | Brain Linear Viscoelastic | Y | | | | | | | BIO |
| 165 | Plastic Nonlinear Kinematic | | Y | | | | | | MT |
| 166 | Moment Curvature Beam | Y | Y | | | | | Y | CIV |
| 167 | McCormick | Y | | | | | | | MT |
| 168 | Polymer | | | | Y | | | Y | PL |
| 169 | Arup Adhesive | Y | Y | | | Y | | Y | AD |
| 170 | Resultant Anisotropic | | | | | Y | | | PL |
| 171 | Steel Concentric Brace | | | | | | Y | Y | CIV |
| 172 | Concrete EC2 | Y | Y | | Y | | | Y | SL, MT |
| 173 | Mohr Coulomb | | Y | | | Y | Y | Y | SL |
| 174 | RC Beam | | | | | | Y | Y | SL |
| 175 | Viscoelastic Thermal | Y | | | Y | | | Y | RB |
| 176 | Quasilinear Viscoelastic | Y | Y | | | | Y | Y | BIO |
| 177 | Hill Foam | | | | | | | Y | FM |
| 178 | Viscoelastic Hill Foam (Ortho) | Y | | | | | | Y | FM |
| 179 | Low Density Synthetic Foam | Y | Y | | | Y | Y | Y | FM |
| 181 | Simplified Rubber/Foam | Y | Y | | | | Y | Y | RB, FM |
| 183 | Simplified Rubber with Damage | Y | | | | | Y | Y | RB |
| 184 | Cohesive Elastic | | Y | | | | | Y | AD |
| 185 | Cohesive TH | | Y | | | Y | Y | Y | AD |
| 186 | Cohesive General | | Y | | | Y | Y | Y | AD |
| 187 | Semi-Analytical Model for Polymers – 1 | Y | Y | | | | Y | Y | PL |
| 187L | SAMP light | Y | | | | | | Y | PL |
| 188 | Thermo Elasto Viscoelastic Creep | Y | | | Y | | | | MT |
| 189 | Anisotropic Thermoelastic | | | | Y | Y | | | |
| 190 | Flow limit diagram 3-Parameter Bar-lat | | Y | | | Y | | Y | MT |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 191 | Seismic Beam | | | | | | | Y | CIV |
| 192 | Soil Brick | Y | | | | Y | | Y | SL |
| 193 | Drucker Prager | | | | | | | Y | SL |
| 194 | RC Shear Wall | | Y | | | | Y | Y | CIV |
| 195 | Concrete Beam | Y | Y | | | | Y | Y | CIV |
| 196 | General Spring Discrete Beam | Y | | | | | | Y | |
| 197 | Seismic Isolator | Y | Y | | | Y | | Y | CIV |
| 198 | Jointed Rock | | Y | | | Y | | Y | SL |
| 199 | Barlat YLD2004 | Y | | | | Y | | | MT |
| 199_27P | Barlat YLD2004 extended to 27 parameters by Aretz | Y | | | | Y | | | MT |
| 202 | Steel EC3 | | Y | | Y | | | | CIV |
| 203 | Hysteretic Reinforcement | | Y | | | Y | Y | Y | CIV |
| 205 | Discrete Beam Point Contact | | Y | | | | | Y | GN, CIV |
| 207 | Simple ANIsotropic SAND (SANISAND) | | | | | Y | | Y | SL |
| 208 | Bolt Beam | | Y | | | | Y | Y | MT |
| 209 | Hysteretic Beam | | Y | | | | Y | Y | CIV |
| 211 | SPR JLR | Y | Y | | | | | | MT |
| 213 | Composite tabulated plasticity and damage | Y | Y | | Y | Y | Y | Y | CM |
| 214 | Dry Fabric | Y | Y | | | Y | Y | Y | |
| 215 | 4A Micromec | Y | Y | | | Y | Y | | CM, PL |
| 216 | Elastic Phase Change | | | | | | | | GN |
| 217 | Orthotropic Elastic Phase Change | | | | | Y | | | GN |
| 218 | Mooney Rivlin Rubber Phase Change | | | | | | | Y | RB |
| 219 | CODAM2 | | Y | | | Y | Y | Y | CM |
| 220 | Rigid Discrete | | | | | | | | |
| 221 | Orthotropic Simplified Damage | | Y | | | Y | Y | Y | CM |
| 224 | Tabulated Johnson Cook | Y | Y | Y | Y | | Y | Y | HY, MT, PL |
| 224_GYS | Tabulated Johnson Cook GYS | Y | Y | Y | Y | | Y | Y | HY, MT, PL |
| 225 | Viscoplastic Mixed Hardening | Y | Y | | | | | | MT, PL |
| 226 | Kinematic hardening Barlat 89 | | | | | Y | | | MT |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 230 | Elastic Perfectly Matched Layer (PML) | Y | | | | | | | SL |
| 231 | Acoustic PML | | | | | | | | FL |
| 232 | Biot Linear Hysteretic Material | Y | | | | | | | SL |
| 233 | Cazacu Barlat | | | | | Y | | Y | MT |
| 234 | Viscoelastic Loose Fabric | Y | Y | | | Y | | Y | F |
| 235 | Micromechanic Dry Fabric | | | | | Y | | Y | F |
| 236 | SCC_on_RCC | | Y | | | Y | | Y | CM, CR |
| 237 | Biot Hysteretic PML | Y | | | | | | | SL |
| 238 | Piecewise linear plasticity (PERT) | Y | Y | | | | | | MT, PL |
| 240 | Cohesive mixed mode | Y | Y | | | Y | Y | Y | AD |
| 241 | Johnson Holmquist JH1 | Y | Y | | | | Y | Y | CR, GL |
| 242 | Kinematic hardening Barlat 2000 | | | | | Y | | | MT |
| 243 | Hill 90 | Y | | | Y | Y | | | MT |
| 244 | UHS Steel | Y | | | Y | | | | MT |
| 245 | Orthotropic/anisotropic PML | Y | | | | | | | SL |
| 246 | Null material PML | | | Y | | | | | FL |
| 248 | PHS BMW | Y | | | Y | Y | | | MT |
| 249 | Reinforced Thermoplastic | | | | Y | Y | | Y | CM, F |
| 249_CRASH | Reinforced Thermoplastic Crash | | Y | | | Y | Y | Y | CM, F |
| 249_UDfiber | Reinforced Thermoplastic UDfiber | | | | Y | Y | | Y | CM, F |
| 251 | Tailored Properties | Y | Y | | | | | | MT, PL |
| 252 | Toughened Adhesive Polymer | Y | Y | | Y | Y | Y | Y | AD |
| 254 | Generalized Phase Change | Y | | | Y | | | | MT |
| 255 | Piecewise linear plastic thermal | Y | Y | | Y | | | Y | MT |
| 256 | Amorphous solid (finite strain) | Y | | | | | | Y | GL |
| 258 | Non-quadratic failure | Y | Y | | | | Y | | MT |
| 260A | Stoughton non-associated flow | Y | | | | Y | | | MT |
| 260B | Mohr non-associated flow | Y | Y | | Y | Y | Y | | MT |
| 261 | Laminated Fracture Daimler Pinho | Y | Y | | | Y | Y | Y | CM |
| 262 | Laminated Fracture Daimler Camanho | Y | Y | | | Y | Y | Y | CM |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 263 | Anisotropic plasticity | | | | | Y | | | MT |
| 264 | Tabulated Johnson Cook Orthotropic Plasticity | Y | Y | | Y | Y | Y | Y | HY, MT, PL |
| 265 | Constrained SPR2/SPR3 | | Y | | | | Y | | MT |
| 266 | Dispersed tissue | | | | | Y | | | BIO |
| 267 | Eight chain rubber | Y | | | | Y | | | RB, PL |
| 269 | Bergström Boyce rubber | Y | | | | | | | RB |
| 270 | Welding material | | | | Y | | | | MT, PL |
| 271 | Powder compaction | | | | | | | Y | CR, SL |
| 272 | RHT concrete model | Y | Y | | | | Y | Y | SL, CIV |
| 273 | Concrete damage plastic | Y | Y | | | | Y | Y | SL |
| 274 | Paper | | | | | Y | | Y | CM, PL |
| 275 | Smooth viscoelastic viscoplastic | Y | | | | | | | MT, PL |
| 276 | Chronological viscoelastic | Y | | | Y | | | | RB |
| 277 | Adhesive curing viscoelastic | Y | | | Y | | | | AD |
| 278 | CF Micromechanics | Y | Y | | Y | Y | | | CM |
| 279 | Cohesive Paper | | Y | | | | | Y | AD |
| 280 | Glass | | | | | Y | Y | Y | GL |
| 291 | Shape Memory Alloy | | | | Y | Y | | Y | MT |
| 292 | Isotropic Elastic for Peridynamic Solids | | Y | | | | | | GL, CR, PL, SL |
| 292A | Elastic for Peridynamic Laminates | | Y | | | Y | | | CM |
| 293 | COMPRF | Y | | | | Y | | Y | CM |
| 295 | Anisotropic hyperelastic | | | | | Y | | Y | BIO, CM, RB |
| 296 | Soldering metal in semiconductor packaging | Y | | | Y | | | | MT |
| 303 | Machine-learning base multiscale material model for fiber-reinforced composites | | | | | Y | | Y | CM |
| 305 | Hot Plate Rolling | Y | | | Y | | | | MT |
| 307 | Generalized Adhesive Curing | Y | Y | | Y | Y | Y | Y | AD |
| 317 | RRR Polymer | Y | | | | | | | PL |
| 318 | TNM Polymer | Y | | | Y | | | | PL |
| 319 | Incompressible Fluids with ISPG | | | | | | | | FL |

| Material Number | And Description | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| 326 | Gaskets | | | | | | | Y | AD |
| 331 | Orthotropic composites subjected to hypervelocity impact | | Y | Y | | Y | Y | | CM |
| ALE_01 | ALE Vacuum | | | | | | | | FL |
| ALE_02 | ALE Gas Mixture | | | | Y | | | | FL |
| ALE_03 | ALE Viscous | | | Y | | | | Y | FL |
| ALE_04 | ALE Mixing Length | | | | | | | | FL |
| ALE_05 | ALE Incompressible | | | | | | | | FL |
| ALE_06 | ALE Herschel | | | Y | | | | Y | FL |
| ISPG_01 | Incompressible SPG Carreau model | Y | | | Y | | | | FL |
| ISPG_02 | Incompressible SPG Cross model | Y | | | Y | | | | FL |
| ISPG_03 | Incompressible SPG Netwonian flow behavior of an incompressible free surface flow | | | | Y | | | | FL |
| ISPG_04 | Incompressible SPG Cross Castro Macosko model | Y | | | Y | | | | FL |
| ISPG_05 | Incompressible SPG Cross WLF model | Y | | | Y | | | | FL |
| SPH_01 | SPH Viscous | | | Y | | | | Y | FL |
| SPH_02 | SPH Incompressible Fluid | | | | | | | Y | FL |
| SPH_03 | SPH Incompressible Structure | | | | | | | | FL |
| S1 | Spring Elastic (Linear) | | | | | | | | |
| S2 | Damper Viscous (Linear) | Y | | | | | | | |
| S3 | Spring Elastoplastic (Isotropic) | | | | | | | | |
| S4 | Spring Nonlinear Elastic | Y | | | | | | Y | |
| S5 | Damper Nonlinear Viscous | Y | | | | | | Y | |
| S6 | Spring General Nonlinear | | | | | | | Y | |
| S7 | Spring Maxwell (3-Parameter Viscoelastic) | Y | | | | | | | |
| S8 | Spring Inelastic (Tension or Compression) | | | | | | | Y | |
| S13 | Spring Trilinear Degrading | | Y | | | | Y | | CIV |
| S14 | Spring Squat Shearwall | | | | | | Y | | CIV |
| S15 | Spring Muscle | Y | | | | | | Y | BIO |
| B1 | Seatbelt | | | | | | | Y | |
| T01 | Thermal Isotropic | | | | Y | | | | HT |

| Material Number And Description | | SRATE | FAIL | EOS | THERMAL | ANISO | DAM | TENS | APPS |
|---|---|---|---|---|---|---|---|---|---|
| T02 | Thermal Orthotropic | | | | Y | Y | | | HT |
| T03 | Thermal Isotropic (Temp Dependent) | | | | Y | | | | HT |
| T04 | Thermal Orthotropic (Temp Dependent) | | | | Y | Y | | | HT |
| T05 | Thermal Discrete Beam | | | | Y | | | | HT |
| T06 | Thermal Chemical Reaction | | | | Y | | | | HT |
| T07 | Thermal CWM (Welding) | | | | Y | | | | HT |
| T08 | Thermal Orthotropic(Temp dep-load curve) | | | | Y | Y | | | HT |
| T09 | Thermal Isotropic (Phase Change) | | | | Y | | | | HT |
| T10 | Thermal Isotropic (Temp dep-load curve) | | | | Y | | | | HT |
| T11 | Thermal User Defined | | | | Y | | | | HT |
| T17 | Thermal Chemical Reaction Orthotropic | | | | Y | Y | | | HT |
| T18 | Thermal ISPG | | | | Y | | | | HT |

## Alphabetized Materials List

| Material Keyword | Number |
|---|---|
| *EOS_AERATED_WATER | *EOS_044 |
| *EOS_GASKET | *EOS_015 |
| *EOS_GRUNEISEN | *EOS_004 |
| *EOS_HVRB | *EOS_041 |
| *EOS_IDEAL_GAS | *EOS_012 |
| *EOS_IGNITION_AND_GROWTH_OF_REACTION_IN_HE | *EOS_007 |
| *EOS_JWL | *EOS_002 |
| *EOS_JWLB | *EOS_014 |
| *EOS_LEE-TARVER | *EOS_043 |
| *EOS_LINEAR_POLYNOMIAL | *EOS_001 |
| *EOS_LINEAR_POLYNOMIAL_WITH_ENERGY_LEAK | *EOS_006 |
| *EOS_MIE_GRUNEISEN | *EOS_016 |
| *EOS_MURNAGHAN | *EOS_019 |
| *EOS_PHASE_CHANGE | *EOS_013 |
| *EOS_PROPELLANT_DEFLAGRATION | *EOS_010 |
| *EOS_RATIO_OF_POLYNOMIALS | *EOS_005 |
| *EOS_SACK_TUESDAY | *EOS_003 |
| *EOS_TABULATED | *EOS_009 |
| *EOS_TABULATED_COMPACTION | *EOS_008 |
| *EOS_TENSOR_PORE_COLLAPSE | *EOS_011 |
| *EOS_USER_DEFINED | *EOS_021-*EOS_030 |
| *EOS_USER_LIBRARY | *EOS_042 |
| *MAT_1DOF_GENERALIZED_SPRING | *MAT_146 |
| *MAT_3-PARAMETER_BARLAT | *MAT_036 |
| *MAT_4A_MICROMEC | *MAT_215 |
| *MAT_ACOUSTIC | *MAT_090 |
| *MAT_ADD_AIRBAG_POROSITY_LEAKAGE | |

# ALPHABETIZED MATERIALS LIST

| Material Keyword | Number |
|---|---|
| *MAT_ADD_BASIC_INCREMENTAL_FAILURE | |
| *MAT_ADD_CHEM_SHRINKAGE | |
| *MAT_ADD_COHESIVE | |
| *MAT_ADD_COHESIVE_FATIGUE | |
| *MAT_ADD_DAMAGE_DIEM | |
| *MAT_ADD_DAMAGE_GISSMO | |
| *MAT_ADD_EROSION | |
| *MAT_ADD_EXTVAR_EXPANSION | |
| *MAT_ADD_FATIGUE | |
| *MAT_ADD_GENERALIZED_DAMAGE | |
| *MAT_ADD_INELASTICITY | |
| *MAT_ADD_MICRO | |
| *MAT_ADD_PERMEABILITY | |
| *MAT_ADD_PORE_AIR | |
| *MAT_ADD_SOC_EXPANSION | |
| *MAT_ADD_THERMAL_EXPANSION | |
| *MAT_ADHESIVE_CURING_VISCOELASTIC | *MAT_277 |
| *MAT_ALE_GAS_MIXTURE | *MAT_ALE_02 |
| *MAT_ALE_HERSCHEL | *MAT_ALE_06 |
| *MAT_ALE_INCOMPRESSIBLE | *MAT_160 |
| *MAT_ALE_MIXING_LENGTH | *MAT_ALE_04 |
| *MAT_ALE_VACUUM | *MAT_ALE_01 |
| *MAT_ALE_VISCOIS | *MAT_ALE_03 |
| *MAT_AMORPHOUS_SOLIDS_FINITE_STRAIN | *MAT_256 |
| *MAT_ANAND_VISCOPLASTICITY | *MAT_296 |
| *MAT_ANISOTROPIC_ELASTIC | *MAT_002_ANISO |
| *MAT_ANISOTROPIC_ELASTIC_PLASTIC | *MAT_157 |
| *MAT_ANISOTROPIC_HYPERELASTIC | *MAT_295 |
| *MAT_ANISOTROPIC_PLASTIC | *MAT_103_P |
| *MAT_ANISOTROPIC_THERMOELASTIC | *MAT_189 |

| Material Keyword | Number |
|---|---|
| *MAT_ANISOTROPIC_VISCOPLASTIC | *MAT_103 |
| *MAT_ARRUDA_BOYCE_RUBBER | *MAT_127 |
| *MAT_ARUP_ADHESIVE | *MAT_169 |
| *MAT_BAMMAN | *MAT_051 |
| *MAT_BAMMAN_DAMAGE | *MAT_052 |
| *MAT_BARLAT_ANISOTROPIC_PLASTICITY | *MAT_033 |
| *MAT_BARLAT_YLD2000 | *MAT_133 |
| *MAT_BARLAT_YLD2004 | *MAT_199 |
| *MAT_BARLAT_YLD2004_27P | *MAT_199_27P |
| *MAT_BARLAT_YLD96 | *MAT_033_96 |
| *MAT_BERGSTROM_BOYCE_RUBBER | *MAT_269 |
| *MAT_BILKHU/DUBOIS_FOAM | *MAT_075 |
| *MAT_BIOT_HYSTERETIC | *MAT_232 |
| *MAT_BLATZ-KO_FOAM | *MAT_038 |
| *MAT_BLATZ-KO_RUBBER | *MAT_007 |
| *MAT_BOLT_BEAM | *MAT_208 |
| *MAT_BRAIN_LINEAR_VISCOELASTIC | *MAT_164 |
| *MAT_BRITTLE_DAMAGE | *MAT_096 |
| *MAT_CABLE_DISCRETE_BEAM | *MAT_071 |
| *MAT_CAZACU_BARLAT | *MAT_233 |
| *MAT_CELLULAR_RUBBER | *MAT_087 |
| *MAT_CF_MICROMECHANICS | *MAT_278 |
| *MAT_CHRONOLOGICAL_VISCOELASTIC | *MAT_276 |
| *MAT_CLOSED_CELL_FOAM | *MAT_053 |
| *MAT_CODAM2 | *MAT_219 |
| *MAT_COHESIVE_ELASTIC | *MAT_184 |
| *MAT_COHESIVE_GASKET | *MAT_326 |
| *MAT_COHESIVE_GENERAL | *MAT_186 |
| *MAT_COHESIVE_MIXED_MODE | *MAT_138 |
| *MAT_COHESIVE_MIXED_MODE_ELASTOPLASTIC_RATE | *MAT_240 |

# ALPHABETIZED MATERIALS LIST

| Material Keyword | Number |
|---|---|
| *MAT_COHESIVE_PAPER | *MAT_279 |
| *MAT_COHESIVE_TH | *MAT_185 |
| *MAT_COMPOSITE_DAMAGE | *MAT_022 |
| *MAT_COMPOSITE_DIRECT | *MAT_118 |
| *MAT_COMPOSITE_DMG_MSC | *MAT_162 |
| *MAT_COMPOSITE_FAILURE_{OPTION}_MODEL | *MAT_059 |
| *MAT_COMPOSITE_LAYUP | *MAT_116 |
| *MAT_COMPOSITE_MATRIX | *MAT_117 |
| *MAT_COMPOSITE_MSC | *MAT_161 |
| *MAT_COMPOSITE_TABULATED_PLASTICITY_DAMAGE | *MAT_213 |
| *MAT_COMPRF | *MAT_293 |
| *MAT_CONCRETE_BEAM | *MAT_195 |
| *MAT_CONCRETE_DAMAGE | *MAT_072 |
| *MAT_CONCRETE_DAMAGE_PLASTIC_MODEL | *MAT_273 |
| *MAT_CONCRETE_DAMAGE_REL3 | *MAT_072R3 |
| *MAT_CONCRETE_EC2 | *MAT_172 |
| *MAT_CONSTRAINED | *MAT_265 |
| *MAT_CRUSHABLE_FOAM | *MAT_063 |
| *MAT_CSCM_{OPTION} | *MAT_159 |
| *MAT_CWM | *MAT_270 |
| *MAT_DAMAGE_1 | *MAT_104 |
| *MAT_DAMAGE_2 | *MAT_105 |
| *MAT_DAMAGE_3 | *MAT_153 |
| *MAT_DAMPER_NONLINEAR_VISCOUS | *MAT_S05 |
| *MAT_DAMPER_VISCOUS | *MAT_S02 |
| *MAT_DESHPANDE_FLECK_FOAM | *MAT_154 |
| *MAT_DISCRETE_BEAM_POINT_CONTACT | *MAT_205 |
| *MAT_DMN_COMPOSITE_FRC | *MAT_303 |
| *MAT_DRUCKER_PRAGER | *MAT_193 |
| *MAT_DRY_FABRIC | *MAT_214 |

| Material Keyword | Number |
|---|---|
| *MAT_EIGHT_CHAIN_RUBBER | *MAT_267 |
| *MAT_ELASTIC | *MAT_001 |
| *MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM | *MAT_093 |
| *MAT_ELASTIC_FLUID | *MAT_001_FLUID |
| *MAT_ELASTIC_PERI | *MAT_292 |
| *MAT_ELASTIC_PERI_LAMINATE | *MAT_292A |
| *MAT_ELASTIC_PHASE_CHANGE | *MAT_216 |
| *MAT_ELASTIC_PLASTIC_HYDRO_{OPTION} | *MAT_010 |
| *MAT_ELASTIC_PLASTIC_THERMAL | *MAT_004 |
| *MAT_ELASTIC_SPRING_DISCRETE_BEAM | *MAT_074 |
| *MAT_ELASTIC_VISCOPLASTIC_THERMAL | *MAT_106 |
| *MAT_ELASTIC_WITH_VISCOSITY | *MAT_060 |
| *MAT_ELASTIC_WITH_VISCOSITY_CURVE | *MAT_060C |
| *MAT_EMMI | *MAT_151 |
| *MAT_ENHANCED_COMPOSITE_DAMAGE | *MAT_054-055 |
| *MAT_EXTENDED_3-PARAMETER_BARLAT | *MAT_036E |
| *MAT_FABRIC | *MAT_034 |
| *MAT_FABRIC_MAP | *MAT_034M |
| *MAT_FHWA_SOIL | *MAT_147 |
| *MAT_FHWA_SOIL_NEBRASKA | *MAT_147_N |
| *MAT_FINITE_ELASTIC_STRAIN_PLASTICITY | *MAT_112 |
| *MAT_FLD_3-PARAMETER_BARLAT | *MAT_190 |
| *MAT_FLD_TRANSVERSELY_ANISOTROPIC | *MAT_039 |
| *MAT_FORCE_LIMITED | *MAT_029 |
| *MAT_FRAZER_NASH_RUBBER_MODEL | *MAT_031 |
| *MAT_FU_CHANG_FOAM | *MAT_083 |
| *MAT_GAS_MIXTURE | *MAT_148 |
| *MAT_GENERAL_JOINT_DISCRETE_BEAM | *MAT_097 |
| *MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM | *MAT_121 |
| *MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM | *MAT_119 |

# ALPHABETIZED MATERIALS LIST

| Material Keyword | Number |
|---|---|
| *MAT_GENERAL_SPRING_DISCRETE_BEAM | *MAT_196 |
| *MAT_GENERAL_VISCOELASTIC | *MAT_076 |
| *MAT_GENERALIZED_ADHESIVE_CURING | *MAT_307 |
| *MAT_GENERALIZED_PHASE_CHANGE | *MAT_254 |
| *MAT_GEOLOGIC_CAP_MODEL | *MAT_025 |
| *MAT_GEPLASTIC_SRATE_2000a | *MAT_101 |
| *MAT_GLASS | *MAT_280 |
| *MAT_GURSON | *MAT_120 |
| *MAT_GURSON_JC | *MAT_120_JC |
| *MAT_GURSON_RCDC | *MAT_120_RCDC |
| *MAT_HEART_TISSUE | *MAT_128 |
| *MAT_HIGH_EXPLOSIVE_BURN | *MAT_008 |
| *MAT_HILL_3R | *MAT_122 |
| *MAT_HILL_3R_3D | *MAT_122_3D |
| *MAT_HILL_3R_TABULATED | *MAT_122_TAB |
| *MAT_HILL_90 | *MAT_243 |
| *MAT_HILL_FOAM | *MAT_177 |
| *MAT_HONEYCOMB | *MAT_026 |
| *MAT_HOT_PLATE_ROLLING | *MAT_305 |
| *MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM | *MAT_070 |
| *MAT_HYPERELASTIC_RUBBER | *MAT_077_H |
| *MAT_HYSTERETIC_BEAM | *MAT_209 |
| *MAT_HYSTERETIC_REINFORCEMENT | *MAT_203 |
| *MAT_HYSTERETIC_SOIL | *MAT_079 |
| *MAT_IFPD | *MAT_319 |
| *MAT_IMPACT_ORTHOTROPIC | *MAT_331 |
| *MAT_INELASTC_6DOF_SPRING_DISCRETE_BEAM | *MAT_095 |
| *MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM | *MAT_095 |
| *MAT_INELASTIC_SPRING_DISCRETE_BEAM | *MAT_094 |
| *MAT_INV_HYPERBOLIC_SIN(_THERMAL) | *MAT_102(_T) |

| Material Keyword | Number |
|---|---|
| *MAT_ISOTROPIC_ELASTIC_FAILURE | *MAT_013 |
| *MAT_ISOTROPIC_ELASTIC_PLASTIC | *MAT_012 |
| *MAT_ISOTROPIC_SMEARED_CRACK | *MAT_131 |
| *MAT_ISPG_CARREAU | *MAT_ISPG_01 |
| *MAT_ISPG_CROSS_CASTRO_MACOSKO | *MAT_ISPG_04 |
| *MAT_ISPG_CROSS_WLF_MODEL | *MAT_ISPG_05 |
| *MAT_ISPG_CROSSMODEL | *MAT_ISPG_02 |
| *MAT_ISPG_ISO_NEWTONIAN | *MAT_ISPG_03 |
| *MAT_JOHNSON_COOK | *MAT_015 |
| *MAT_JOHNSON_HOLMQUIST_CERAMICS | *MAT_110 |
| *MAT_JOHNSON_HOLMQUIST_CONCRETE | *MAT_111 |
| *MAT_JOHNSON_HOLMQUIST_JH1 | *MAT_241 |
| *MAT_JOINTED_ROCK | *MAT_198 |
| *MAT_KELVIN-MAXWELL_VISCOELASTIC | *MAT_061 |
| *MAT_KINEMATIC_HARDENING_BARLAT2000 | *MAT_242 |
| *MAT_KINEMATIC_HARDENING_BARLAT89 | *MAT_226 |
| *MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC | *MAT_125 |
| *MAT_LAMINATED_COMPOSITE_FABRIC | *MAT_058 |
| *MAT_LAMINATED_FRACTURE_DAIMLER_CAMANHO | *MAT_262 |
| *MAT_LAMINATED_FRACTURE_DAIMLER_PINHO | *MAT_261 |
| *MAT_LAMINATED_GLASS | *MAT_032 |
| *MAT_LAYERED_LINEAR_PLASTICITY | *MAT_114 |
| *MAT_LINEAR_ELASTIC_DISCRETE_BEAM | *MAT_066 |
| *MAT_LOU-YOON_ANISOTROPIC_PLASTICITY | *MAT_263 |
| *MAT_LOW_DENSITY_FOAM | *MAT_057 |
| *MAT_LOW_DENSITY_SYNTHETIC_FOAM_{OPTION} | *MAT_179 |
| *MAT_LOW_DENSITY_VISCOUS_FOAM | *MAT_073 |
| *MAT_LUNG_TISSUE | *MAT_129 |
| *MAT_MCCORMICK | *MAT_167 |
| *MAT_MICROMECHANICS_DRY_FABRIC | *MAT_235 |

# ALPHABETIZED MATERIALS LIST

| Material Keyword | Number |
|---|---|
| *MAT_MODIFIED_CRUSHABLE_FOAM | *MAT_163 |
| *MAT_MODIFIED_FORCE_LIMITED | *MAT_139 |
| *MAT_MODIFIED_HONEYCOMB | *MAT_126 |
| *MAT_MODIFIED_JOHNSON_COOK | *MAT_107 |
| *MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY | *MAT_123 |
| *MAT_MODIFIED_ZERILLI_ARMSTRONG | *MAT_065 |
| *MAT_MOHR_COULOMB | *MAT_173 |
| *MAT_MOHR_NON_ASSOCIATED_FLOW | *MAT_260B |
| *MAT_MOMENT_CURVATURE_BEAM | *MAT_166 |
| *MAT_MOONEY-RIVLIN_RUBBER | *MAT_027 |
| *MAT_MOONEY-RIVLIN_PHASE_CHANGE | *MAT_218 |
| *MAT_MTS | *MAT_088 |
| *MAT_MUSCLE | *MAT_156 |
| *MAT_NON_QUADRATIC_FAILURE | *MAT_258 |
| *MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM | *MAT_067 |
| *MAT_NONLINEAR_ORTHOTROPIC | *MAT_040 |
| *MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM | *MAT_068 |
| *MAT_NONLOCAL | |
| *MAT_NONLOCAL_GRADIENT_DAMAGE | |
| *MAT_NULL | *MAT_009 |
| *MAT_OGDEN_RUBBER | *MAT_077_O |
| *MAT_ORIENTED_CRACK | *MAT_017 |
| *MAT_ORTHO_ELASTIC_PLASTIC | *MAT_108 |
| *MAT_ORTHOTROPIC_ELASTIC | *MAT_002 |
| *MAT_ORTHOTROPIC_ELASTIC_PHASE_CHANGE | *MAT_217 |
| *MAT_ORTHOTROPIC_SIMPLIFIED_DAMAGE | *MAT_221 |
| *MAT_ORTHOTROPIC_SMEARED_CRACK | *MAT_132 |
| *MAT_ORTHOTROPIC_THERMAL | *MAT_021 |
| *MAT_ORTHOTROPIC_VISCOELASTIC | *MAT_086 |
| *MAT_PAPER | *MAT_274 |

| Material Keyword | Number |
|---|---|
| *MAT_PERT_PIECEWISE_LINEAR_PLASTICITY | *MAT_238 |
| *MAT_PHS_BMW | *MAT_248 |
| *MAT_PIECEWISE_LINEAR_PLASTIC_THERMAL | *MAT_255 |
| *MAT_PIECEWISE_LINEAR_PLASTICITY | *MAT_024 |
| *MAT_PITZER_CRUSHABL_EFOAM | *MAT_144 |
| *MAT_PLASTIC_GREEN-NAGHDI_RATE | *MAT_035 |
| *MAT_PLASTIC_KINEMATIC | *MAT_003 |
| *MAT_PLASTIC_NONLINEAR_KINEMATIC | *MAT_165 |
| *MAT_PLASTICITY_COMPRESSION_TENSION | *MAT_124 |
| *MAT_PLASTICITY_COMPRESSION_TENSION_EOS | *MAT_155 |
| *MAT_PLASTICITY_POLYMER | *MAT_089 |
| *MAT_PLASTICITY_WITH_DAMAGE | *MAT_081 |
| *MAT_PLASTICITY_WITH_DAMAGE_ORTHO(_RCDC) | *MAT_082(_RCDC) |
| *MAT_PML_{OPTION}TROPIC_ELASTIC | *MAT_245 |
| *MAT_PML_ACOUSTIC | *MAT_231 |
| *MAT_PML_ELASTIC | *MAT_230 |
| *MAT_PML_ELASTIC_FLUID | *MAT_230 |
| *MAT_PML_HYSTERETIC | *MAT_237 |
| *MAT_PML_NULL | *MAT_246 |
| *MAT_POLYMER | *MAT_168 |
| *MAT_POWDER | *MAT_271 |
| *MAT_POWER_LAW_PLASTICITY | *MAT_018 |
| *MAT_PSEUDO_TENSOR | *MAT_016 |
| *MAT_QUASILINEAR_VISCOELASTIC | *MAT_176 |
| *MAT_RAMBERG-OSGOOD | *MAT_080 |
| *MAT_RATE_SENSITIVE_COMPOSITE_FABRIC | *MAT_158 |
| *MAT_RATE_SENSITIVE_POLYMER | *MAT_141 |
| *MAT_RATE_SENSITIVE_POWERLAW_PLASTICITY | *MAT_064 |
| *MAT_RC_BEAM | *MAT_174 |
| *MAT_RC_SHEAR_WALL | *MAT_194 |

# ALPHABETIZED MATERIALS LIST

| Material Keyword | Number |
|---|---|
| *MAT_REINFORCED_THERMOPLASTIC | *MAT_249 |
| *MAT_REINFORCED_THERMOPLASTIC_UDFIBER | *MAT_249_UDFIBER |
| *MAT_RESULTANT_ANISOTROPIC | *MAT_170 |
| *MAT_RESULTANT_PLASTICITY | *MAT_028 |
| *MAT_RHT | *MAT_272 |
| *MAT_RIGID | *MAT_020 |
| *MAT_RIGID_DISCRETE | *MAT_220 |
| *MAT_RRR_POLYMER | *MAT_317 |
| *MAT_SAMP-1 | *MAT_187 |
| *MAT_SAMP_LIGHT | *MAT_187L |
| *MAT_SCC_ON_RCC | *MAT_236 |
| *MAT_SCHWER_MURRAY_CAP_MODEL | *MAT_145 |
| *MAT_SEATBELT | *MAT_B01 |
| *MAT_SEISMIC_BEAM | *MAT_191 |
| *MAT_SEISMIC_ISOLATOR | *MAT_197 |
| *MAT_SHAPE_MEMORY | *MAT_030 |
| *MAT_SHAPE_MEMORY_ALLOY | *MAT_291 |
| *MAT_SID_DAMPER_DISCRETE_BEAM | *MAT_069 |
| *MAT_SIMPLIFIED_JOHNSON_COOK | *MAT_098 |
| *MAT_SIMPLIFIED_JOHNSON_COOK_ORTHOTROPIC_DAMAGE | *MAT_099 |
| *MAT_SIMPLIFIED_RUBBER/FOAM_{OPTION} | *MAT_181 |
| *MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE | *MAT_183 |
| *MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC | *MAT_275 |
| *MAT_SOFT_TISSUE | *MAT_091 |
| *MAT_SOFT_TISSUE_VISCO | *MAT_092 |
| *MAT_SOIL_AND_FOAM | *MAT_005 |
| *MAT_SOIL_AND_FOAM_FAILURE | *MAT_014 |
| *MAT_SOIL_BRICK | *MAT_192 |
| *MAT_SOIL_CONCRETE | *MAT_078 |
| *MAT_SOIL_SANISAND | *MAT_207 |

| Material Keyword | Number |
|---|---|
| *MAT_SPECIAL_ORTHOTROPIC | *MAT_130 |
| *MAT_SPH_INCOMPRESSIBLE_FLUID | *MAT_SPH_02 |
| *MAT_SPH_INCOMPRESSIBLE_STRUCTURE | *MAT_SPH_03 |
| *MAT_SPH_VISCOUS | *MAT_SPH_01 |
| *MAT_SPOTWELD_{OPTION} | *MAT_100 |
| *MAT_SPOTWELD_DAIMLERCHRYSLER | *MAT_100_DA |
| *MAT_SPR_JLR | *MAT_211 |
| *MAT_SPRING_ELASTIC | *MAT_S01 |
| *MAT_SPRING_ELASTOPLASTIC | *MAT_S03 |
| *MAT_SPRING_GENERAL_NONLINEAR | *MAT_S06 |
| *MAT_SPRING_INELASTIC | *MAT_S08 |
| *MAT_SPRING_MAXWELL | *MAT_S07 |
| *MAT_SPRING_MUSCLE | *MAT_S15 |
| *MAT_SPRING_NONLINEAR_ELASTIC | *MAT_S04 |
| *MAT_SPRING_SQUAT_SHEARWALL | *MAT_S14 |
| *MAT_SPRING_TRILINEAR_DEGRADING | *MAT_S13 |
| *MAT_STEEL_CONCENTRIC_BRACE | *MAT_171 |
| *MAT_STEEL_EC3 | *MAT_202 |
| *MAT_STEINBERG | *MAT_011 |
| *MAT_STEINBERG_LUND | *MAT_011_LUND |
| *MAT_STOUGHTON_NON_ASSOCIATED_FLOW | *MAT_260A |
| *MAT_STRAIN_RATE_DEPENDENT_PLASTICITY | *MAT_019 |
| *MAT_TABULATED_JOHNSON_COOK | *MAT_224 |
| *MAT_TABULATED_JOHNSON_COOK_GYS | *MAT_224_GYS |
| *MAT_TABULATED_JOHNSON_COOK_ORTHO_PLASTICITY | *MAT_264 |
| *MAT_TAILORED_PROPERTIES | *MAT_251 |
| *MAT_TEMPERATURE_DEPENDENT_ORTHOTROPIC | *MAT_023 |
| *MAT_THERMAL_CHEMICAL_REACTION | *MAT_T06 |
| *MAT_THERMAL_CHEMICAL_REACTION_ORTHOTROPIC | *MAT_T17 |
| *MAT_THERMAL_CWM | *MAT_T07 |

# ALPHABETIZED MATERIALS LIST

| Material Keyword | Number |
|---|---|
| *MAT_THERMAL_DISCRETE_BEAM | *MAT_T05 |
| *MAT_THERMAL_ISOTROPIC | *MAT_TO1 |
| *MAT_THERMAL_ISOTROPIC_PHASE_CHANGE | *MAT_T09 |
| *MAT_THERMAL_ISOTROPIC_TD | *MAT_T03 |
| *MAT_THERMAL_ISOTROPIC_TD_LC | *MAT_T10 |
| *MAT_THERMAL_ISPG | *MAT_T18 |
| *MAT_THERMAL_ORTHOTROPIC | *MAT_T02 |
| *MAT_THERMAL_ORTHOTROPIC_TD | *MAT_T04 |
| *MAT_THERMAL_ORTHOTROPIC_TD_LC | *MAT_T08 |
| *MAT_THERMAL_USER_DEFINED | *MAT_T11 |
| *MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP | *MAT_188 |
| *MAT_TISSUE_DISPERSED | *MAT_266 |
| *MAT_TNM_POLYMER | *MAT_318 |
| *MAT_TOUGHENED_ADHESIVE_POLYMER | *MAT_252 |
| *MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC | *MAT_037 |
| *MAT_TRANSVERSELY_ISOTROPIC_CRUSHABLE_FOAM | *MAT_142 |
| *MAT_TRIP | *MAT_113 |
| *MAT_UHS_STEEL | *MAT_244 |
| *MAT_UNIFIED_CREEP | *MAT_115 |
| *MAT_UNIFIED_CREEP_ORTHO | *MAT_115_O |
| *MAT_USER_DEFINED_MATERIAL_MODELS | *MAT_041-050 |
| *MAT_VACUUM | *MAT_140 |
| *MAT_VEGTER | *MAT_136 |
| *MAT_VEGTER_STANDARD | *MAT_136_STD |
| *MAT_VEGTER_2017 | *MAT_136_2017 |
| *MAT_VISCOELASTIC | *MAT_006 |
| *MAT_VISCOELASTIC_FABRIC | *MAT_134 |
| *MAT_VISCOELASTIC_HILL_FOAM | *MAT_178 |
| *MAT_VISCOELASTIC_LOOSE_FABRIC | *MAT_234 |
| *MAT_VISCOELASTIC_THERMAL | *MAT_175 |

| Material Keyword | Number |
|---|---|
| *MAT_VISCOPLASTIC_MIXED_HARDENING | *MAT_225 |
| *MAT_VISCOUS_FOAM | *MAT_062 |
| *MAT_WINFRITH_CONCRETE_REINFORCEMENT | *MAT_084_REINF |
| *MAT_WINFRITH_CONCRETE | *MAT_084 |
| *MAT_WOOD_{OPTION} | *MAT_143 |
| *MAT_WTM_STM | *MAT_135 |
| *MAT_WTM_STM_PLC | *MAT_135_PLC |

## *MAT_ADD_AIRBAG_POROSITY_LEAKAGE

This command enables modeling porosity leakage through non-fabric material when such material is used as part of a control volume airbag. It applies to both *AIRBAG_-HYBRID and *AIRBAG_WANG_NEFSKE.

### Card Summary:

**Card 1a.** Include this card if 0 < X0 < 1.

| MID | X2 | X3 | ELA | FVOPT | X0 | X1 | |
|---|---|---|---|---|---|---|---|

**Card 1b.** Include this card if X0 = 0 and FVOPT < 7.

| MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
|---|---|---|---|---|---|---|---|

**Card 1c.** Include this card if X0 = 0 and FVOPT ≥ 7.

| MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
|---|---|---|---|---|---|---|---|

**Card 1d.** Include this card if X0 = 1 and FVOPT < 7.

| MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
|---|---|---|---|---|---|---|---|

**Card 1e.** Include this card if X0 = 1 and FVOPT ≥ 7.

| MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

Include this card if 0 < X0 < 1.

| Card 1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | X2 | X3 | ELA | FVOPT | X0 | X1 | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | 1.0 | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the porosity leakage property applies |

| VARIABLE | DESCRIPTION |
|---|---|
| X2 | X2 is one of the coefficients of the porosity in the equation of Anagonye and Wang [1999]. (Defined below in description for X0/X1) |
| X3 | X3 is one of the coefficients of the porosity in the equation of Anagonye and Wang [1999]. (Defined below in description for X0/X1) |
| ELA | Effective leakage area for blocked fabric, ELA<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| FVOPT | Fabric venting option.<br>EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.<br>EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.<br>EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.<br>EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.<br>EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.<br>EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered.<br>EQ.7: Leakage is based on gas volume outflow as a function of pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.<br>EQ.8: Leakage is based on gas volume outflow as a function of pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC. |

| VARIABLE | DESCRIPTION |
|---|---|
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$ |

This card is included if X0 = 0 and FVOPT < 7.

| Card 1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
| Type | A | F | F | F | F | F | F | |
| Default | none | opt | 1.0 | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the porosity leakage property applies |
| FLC | Optional fabric porous leakage flow coefficient:<br>GE.0.0: Fabric porous leakage flow coefficient<br>LT.0.0: \|FLC\| is the load curve ID of the curve defining FLC as a function of time. |
| FAC | Optional fabric characteristic parameter:<br>GE.0.0: Optional fabric characteristic parameter<br>LT.0.0: \|FAC\| is the load curve ID of the curve defining FAC as a function of absolute pressure. |
| ELA | Effective leakage area for blocked fabric, ELA.<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| FVOPT | Fabric venting option.<br>EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.<br>EQ.2: Wang-Nefske formulas for venting through an orifice are |

| VARIABLE | DESCRIPTION |
|---|---|
| | used. Blockage of venting area due to contact is considered.<br>EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.<br>EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.<br>EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.<br>EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered. |
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area:<br>$A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$ |

This card is included if X0 = 0 and FVOPT ≥ 7.

| Card 1c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
| Type | A | F | F | F | F | F | F | |
| Default | none | opt | 1.0 | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the porosity leakage property applies |
| FLC | Optional fabric porous leakage flow coefficient:<br>GE.0.0: Fabric porous leakage flow coefficient<br>LT.0.0: \|FLC\| is the load curve ID of the curve defining FLC as a function of time. |
| FAC | Optional fabric characteristic parameter:<br>GE.0.0: optional fabric characteristic parameter |

| VARIABLE | DESCRIPTION |
|---|---|

LT.0.0: FAC defines leakage volume flux rate as a function of absolute pressure. The volume flux (per area) rate (per time) has the unit of velocity and it is equivalent to relative porous gas speed.

$$\left[\frac{d(\mathrm{Vol}_{\mathrm{flux}})}{dt}\right] = \frac{[\mathrm{volume}]}{[\mathrm{area}]}\frac{1}{[\mathrm{time}]} = \frac{[\mathrm{length}]}{[\mathrm{time}]} = [\mathrm{velocity}]$$

ELA: Effective leakage area for blocked fabric, ELA.

LT.0.0: |ELA| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas.

FVOPT: Fabric venting option.

EQ.7: Leakage is based on gas volume outflow as a function of pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.

EQ.8: Leakage is based on gas volume outflow as a function of pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.

X0, X1: Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area:

$$A_{\mathrm{leak}} = A_0\left(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p\right)$$

This card is included if X0 = 1 and FVOPT < 7.

| Card 1d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
| Type | A | F | F | F | F | F | F | |
| Default | none | opt | 1.0 | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the porosity leakage property applies |
| FLC | Optional fabric porous leakage flow coefficient:<br>GE.0.0: fabric porous leakage flow coefficient<br>LT.0.0: \|FLC\| is the load curve ID defining FLC as a function of the stretching ratio defined as $r_s = A/A_0$. |
| FAC | Optional fabric characteristic parameter:<br>GE.0.0: optional fabric characteristic parameter<br>LT.0.0: \|FAC\| is the load curve ID defining FAC as a function of the pressure ratio defined as $r_p = P_{\text{air}}/P_{\text{bag}}$. See Remark 2 of *MAT_FABRIC. |
| ELA | Effective leakage area for blocked fabric, ELA.<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| FVOPT | Fabric venting option.<br>EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.<br>EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.<br>EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.<br>EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.<br>EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.<br>EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered. |

| VARIABLE | DESCRIPTION |
|---|---|
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$ |

This card is included if X0 = 1 and FVOPT ≥ 7.

| Card 1e | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | FLC | FAC | ELA | FVOPT | X0 | X1 | |
| Type | A | F | F | F | F | F | F | |
| Default | none | opt | 1.0 | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the porosity leakage property applies |
| FLC | Optional fabric characteristic parameter:<br>GE.0.0: Optional fabric characteristic parameter<br>LT.0.0: \|FAC\| is the load curve ID defining FLC as a function of the stretching ratio defined as $r_s = A/A_0$. |
| FAC | Optional fabric characteristic parameter:<br>GE.0.0: Optional fabric characteristic parameter<br>LT.0.0: FAC defines leakage volume flux rate as a function of absolute pressure. The volume flux (per area) rate (per time) has the unit of velocity and it is equivalent to relative porous gas speed.<br>$\left[\frac{d(\text{Vol}_{\text{flux}})}{dt}\right] = \frac{[\text{volume}]}{[\text{area}]}\frac{1}{[\text{time}]} = \frac{[\text{length}]}{[\text{time}]} = [\text{velocity}]$ |
| ELA | Effective leakage area for blocked fabric, ELA.<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA versus time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

| VARIABLE | DESCRIPTION |
|---|---|
| FVOPT | Fabric venting option.<br>EQ.7: Leakage is based on gas volume outflow as a function of pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.<br>EQ.8: Leakage is based on gas volume outflow as a function of pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC. |
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area:<br>$A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$ |

## *MAT_ADD_BASIC_INCREMENTAL_FAILURE

Many of the implemented constitutive models do not support failure and erosion. *MAT_ADD_BASIC_INCREMENTAL_FAILURE provides a way to include load path-dependent and stress-state-dependent failure in these models. It applies to nonlinear element formulations including shells (including isogeometric shells) and solids (including isogeometric solids).

> **NOTE:** Use MAEF = 1 on *CONTROL_MAT to disable all *MAT_ADD_BASIC_INCREMENTAL_FAILURE commands in a model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | | NUMFIP | VOLFRAC | NEROD | | | |
| Type | A | | F | F | F | | | |
| Default | none | | 1.0 | 0.5 | 0.0 | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSF | LCSS | LCREGD | LCSRS | DMGEXP | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification for which this erosion definition applies |
| NUMFIP | Number or percentage of failed integration points prior to element deletion (default value is 1). NUMFIP does not apply to higher-order solid element types 24, 25, 26, 27, 28, and 29; rather, see the variable VOLFRAC. Also, when the material is a composite defined with *PART_COMPOSITE with different materials through-the-thickness, do not use NUMFIP; use *DEFINE_ELEMENT_-EROSION instead.<br>GT.0.0: Number of integration points that must fail before an |

| VARIABLE | DESCRIPTION |
|---|---|
| | element is deleted.<br>LT.0.0: Applies only to shells. \|NUMFIP\| is the percentage of layers that must fail before an element is deleted. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails. |
| VOLFRAC | Volume fraction required to fail before element deletion. The default is 0.5. It is used for higher-order solid element types 24, 25, 26, 27, 28, and 29, and all isogeometric solids and shell elements. See Remark 1. |
| NEROD | Option to turn off element erosion:<br>EQ.0.0: Elements erode according to the definition of NUMFIP.<br>EQ.1.0: Damage does not affect stresses, and elements do not erode. It could be used solely for post-processing damage. |
| EPSF | Plastic failure strain under uniaxial tension, meaning at a triaxiality of 1/3 and a Lode parameter of 1.0 |
| LCSS | Load curve ID of the related material's stress-strain curve (hardening curve). If zero, then the appropriate curve of the associated material model is automatically picked (currently supported: 3, 24, 36, 81, 120, 123, 124, 133, 187, 224, 243, 251, 324). |
| LCREGD | Load curve ID defining the failure strain scaling factor as a function of element size |
| LCSRS | Load curve ID defining the failure strain scaling factor as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. *The curve should not extrapolate to zero; otherwise, failure may occur at low strains.* |
| DMGEXP | Exponent for nonlinear damage accumulation |

**Basic Incremental Failure Model:**

The Basic Incremental Failure Model is a phenomenological formulation that considers an incremental accumulation of a damage variable dependent on the plastic strain and the stress state through the triaxiality and the Lode parameter. The failure curve (plane

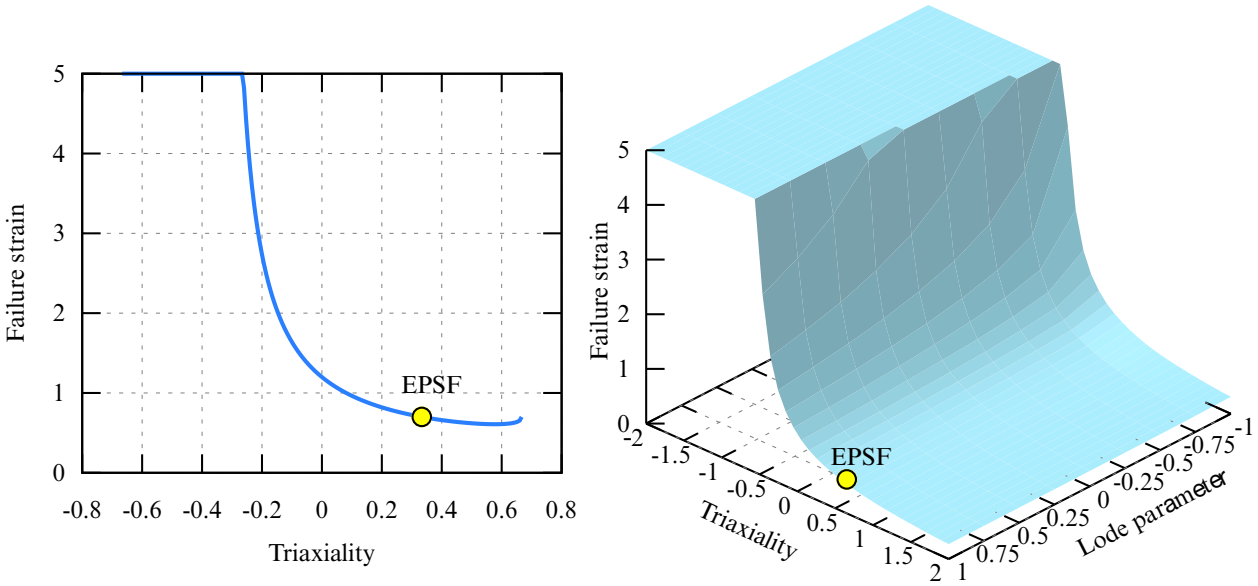



**Figure 2-1.** Examples of an internally generated failure curve (shells) and an internally generated failure surface (solids).

stress shells) or failure surface (solids) is internally generated based on the equation proposed by Cockcroft and Latham [1968].

The damage accumulation is given by:

$$\Delta D = \frac{\text{DMGEXP} \times D^{\left(1-\frac{1}{\text{DMGEXP}}\right)}}{\varepsilon_f} \Delta\varepsilon_p$$

where,

$D$ Damage value $(0 \le D \le 1)$. For numerical reasons, $D$ is initialized to a value of $10^{-20}$ for all damage types in the first time step.

$\varepsilon_f$ Equivalent plastic strain to failure. It is determined from either a curve as a function of the current triaxiality value, $\eta$, (for plane stress shell elements) or a surface as a function of both the triaxiality and the Lode parameter, $L$, (for solids). The curve or surface is automatically generated as described below.

$\Delta\varepsilon_p$ Equivalent plastic strain increment

The following equation gives the triaxiality, $\eta$, as a measure of the current stress state:

$$\eta = \frac{\sigma_H}{\sigma_M} ,$$

with hydrostatic stress, $\sigma_H$, and von Mises stress, $\sigma_M$. The Lode parameter, $L$, is defined as

$$L = \frac{27}{2}\frac{J_3}{{\sigma_M}^3} \ .$$

Here $J_3$ is the third invariant of the deviatoric stress.

The failure curve (shells) or failure surface (solids) is internally generated based on the equation proposed by Cockcroft & Latham [1968]:

$$\int_0^{\varepsilon_f} \max(\sigma_1, 0)\, d\varepsilon_p \le W_c$$

where $\sigma_1$ is the principal stress, $\varepsilon_f$ is the strain at failure, and $W_c$ the critical value at failure. The failure curve or failure surface is generated in such a way that EPSF is the plastic strain value at triaxiality 1/3 and Lode parameter 1.0 (see Figure 2-1). Generating the failure curve or surface requires a hardening curve. If specified, LCSS provides this hardening curve. Otherwise, it is determined based on the associated material model.

For shells, the failure curve is generated for the interval between $-{}^2/_3 \le \eta \le {}^2/_3$. For solids, the failure surface is generated for $-2 \le \eta \le 2$ and $-1 \le L \le 1$, and there is no extrapolation for triaxialities $\eta > 2$. In both cases, the plastic failure strain is bounded by $\varepsilon_f^p \le 5.0$. A file named mabif_crvtbl, generated at the beginning of the simulation, contains the internally generated curves/tables.

Providing optional load curve LCREGD activates an element size dependence. For this load curve, $X$-values are the element size and $Y$-values the scaling (regularization) factors. Full regularization is applied for $\eta \ge 1/3$, but no regularization for $\eta \le 0$. A linear interpolation is adopted for $0 < \eta < 1/3$. Figure 2-2 shows an example of the effect of the element size dependence for element sizes from 0.5 mm (reference) to 5.0 mm.

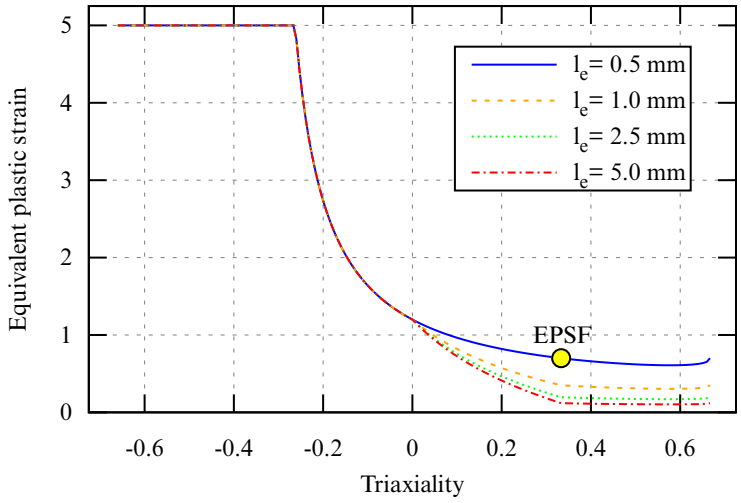



**Figure 2-2**. Example of the effect of the regularization on the failure curve

**Remarks:**

1. **VOLFRAC.** The volumes associated with individual integration points in higher-order finite elements and isogeometric elements vary widely. Thus, the number of failed integration points is not reliable for determining element failure. This failure model uses the volume fraction of the failed material for these types of elements to obtain a more stable and consistent response.

2. **History variables.** NEIPH and NEIPS must be set in *DATABASE_EXTENT_BINARY to output history data associated with this model. The damage history variables start at position ND, which is displayed in d3hsp file as, for example, "first damage history variable = 6" which means that ND = 6. For example, to view the damage parameter (first history variable) for a *MAT_024 shell element, set NEIPS = 6. In LS-PrePost, history variable #6 gives the damage parameter.

| History Variable # | Description |
|---|---|
| ND | Damage parameter $D$ ($10^{-20} < D \leq 1$) |
| ND + 1 | Equivalent plastic strain |
| ND + 2 | Regularization factor for failure strain (determined from LCREGD) |
| ND + 3 | Calculated element size, $l_e$ |
| ND + 4 | Number of IPs/layers (NUMFIP > 0/< 0) that must fail before element deletion |
| ND + 5 | Triaxiality variable, $\eta = \sigma_H/\sigma_M$ |
| ND + 6 | Lode parameter value $L$ |
| ND + 7 | Averaged triaxiality: $\eta_{n+1}^{avg} = \frac{1}{D_{n+1}}\left(D_n \times \eta_n^{avg} + (D_{n+1} - D_n) \times \eta_{n+1}\right)$ |
| ND + 8 | Averaged Lode parameter: $L_{n+1}^{avg} = \frac{1}{D_{n+1}}\left(D_n \times L_n^{avg} + (D_{n+1} - D_n) \times L_{n+1}\right)$ |
| ND + 9 | Alternative damage value: $D^{1/DMGEXP}$ |

## *MAT_ADD_CHEM_SHRINKAGE

The ADD_CHEM_SHRINKAGE option allows for adding the chemical shrinkage effect to a material model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PID | LCID | | | | | | |
| Type | I | I | | | | | | |
| Default | none | none | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PID | Part ID for which the chemical shrinkage effect applies |
| LCID | Load curve ID (see *DEFINE_CURVE) defining the chemical shrinkage coefficient, $\beta$, or a proxy in experiments for the chemical shrinkage coefficient, $\alpha$, as a function of temperature, $T$. If $\alpha$ as a function of $T$ is defined, $\alpha$ is converted to the chemical shrinkage coefficient by LS-DYNA (see Remark 2). |

**Remarks:**

1. **Chemical Shrinkage Effect on Strain.** If the chemical shrinkage effect is included, the strain rate tensor, $\dot{\boldsymbol{\varepsilon}}$, is given by

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p + \dot{\boldsymbol{\varepsilon}}^c \ .$$

   Here, $\dot{\boldsymbol{\varepsilon}}^e$ is the elastic strain rate tensor, $\dot{\boldsymbol{\varepsilon}}^p$ is the plastic strain rate tensor, and $\dot{\boldsymbol{\varepsilon}}^c$ is the chemical shrinkage strain rate tensor. $\dot{\boldsymbol{\varepsilon}}^c$ is given by

$$\dot{\boldsymbol{\varepsilon}}^c = \beta \dot{T} \mathbf{I} \ .$$

   Here $\beta$ is the chemical shrinkage coefficient and $\dot{T}$ is the rate of temperature change.

2. **Chemical Shrinkage Coefficient.** The chemical shrinkage coefficient can be defined in two ways with LCID: either directly or through the proxy variable from experiments, $\alpha$. If $\alpha$ is defined as the ordinate, LS-DYNA internally converts the ordinate of the load curve, LCID, to $\beta$:

$$\beta = (1 - \alpha)^3 - 1 \ .$$

Note that DATTYP on *DEFINE_CURVE *must be set to* -100 if LCID defines $\alpha$ as a function of temperature.

3. **Thermal Expansion with Shrinkage Effects.** If both thermal expansion and chemical shrinkage effects are modeled in one simulation, the thermal expansion should be defined with *MAT_THERMAL_ISOTROPIC_TITLE. The TITLE keyword option must be defined to distinguish between the thermal expansion and chemical shrinkage.

## *MAT_ADD_COHESIVE

The ADD_COHESIVE option offers the possibility to use a selection of three-dimensional material models in LS-DYNA in conjunction with cohesive elements. See Remark 1.

Usually the cohesive elements (ELFORM = 19 and 20 of *SECTION_SOLID) can only be used with a small number of material models (41-50, 138, 184, 185, 186, 240). But with this additional keyword, a larger number of standard three-dimensional material models can be used that would only be available for solid elements in general. Currently the following material models are supported: 1, 3, 4, 6, 15, 24, 41-50, 57, 81, 82, 83, 89, 96, 98, 103, 104, 105, 106, 107, 115, 120, 123, 124, 141, 168, 173, 187, 188, 193, 224, 225, 251, 252, 255, 277, and 307.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PID | ROFLG | INTFAIL | THICK | UNIAX | | | |
| Type | I | F | F | F | F | | | |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PID | Part ID for which the cohesive property applies. |
| ROFLG | Flag for whether density is specified per unit area or volume.<br>EQ.0.0: Density specified per unit volume (default).<br>EQ.1.0: Density specified per unit area for controlling the mass of cohesive elements with an initial volume of zero. |
| INTFAIL | The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value.<br>LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when \|INTFAIL\| integration points have failed.<br>EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion.<br>GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have |

| VARIABLE | DESCRIPTION |
|---|---|
| | failed. |
| THICK | Thickness of the adhesive layer (see Remark 3):<br>EQ.0.0: The actual thickness of the cohesive element is used.<br>GT.0.0: User specified thickness. |
| UNIAX | Flag for enforcing a uniaxial stress state (see Remark 2):<br>EQ.0.0: No modification of the three-dimensional stress state (default).<br>EQ.1.0: Stress components that are not used for the cohesive element are reset to 0.0 after each evaluation of the constitutive model. |

**Remarks:**

1. **Three-dimensional constitutive laws with cohesive elements.** Cohesive elements possess 3 kinematic variables, namely, two relative displacements $\delta_1$, $\delta_2$ in tangential directions and one relative displacement $\delta_3$ in normal direction. In a corresponding constitutive model, they are used to compute 3 associated traction stresses $t_1$, $t_2$, and $t_3$. For example, in the elastic case (*MAT_COHESIVE_ELASTIC):

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} E_T & 0 & 0 \\ 0 & E_T & 0 \\ 0 & 0 & E_N \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}.$$

Hypoelastic three-dimensional material models for standard solid elements, however, are formulated with respect to 6 independent strain rates and 6 associated stress rates. For isotropic elasticity (*MAT_ELASTIC):

$$\begin{bmatrix} \dot{\sigma}_{xx} \\ \dot{\sigma}_{yy} \\ \dot{\sigma}_{zz} \\ \dot{\sigma}_{xy} \\ \dot{\sigma}_{yz} \\ \dot{\sigma}_{zx} \end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-2\nu & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-2\nu & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-2\nu \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{zz} \\ \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{yz} \\ \dot{\varepsilon}_{zx} \end{bmatrix}.$$

To be able to use such three-dimensional material models in a cohesive element environment, an assumption is necessary to transform 3 relative displacements to 6 strain rates. Therefore, we assume that neither lateral expansion nor in-plane shearing is possible. Thus,

$$\begin{bmatrix}\delta_1\\ \delta_2\\ \delta_3\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}\dot{\varepsilon}_{xx}\\ \dot{\varepsilon}_{yy}\\ \dot{\varepsilon}_{zz}\\ \dot{\varepsilon}_{xy}\\ \dot{\varepsilon}_{yz}\\ \dot{\varepsilon}_{zx}\end{bmatrix} = \begin{bmatrix}0\\ 0\\ \dot{\delta}_3/(t+\delta_3)\\ 0\\ \dot{\delta}_2/(t+\delta_3)\\ \dot{\delta}_1/(t+\delta_3)\end{bmatrix},$$

where $t$ is the initial thickness of the adhesive layer; see parameter THICK. These strain rates are then used in a three-dimensional constitutive model to obtain new Cauchy stresses, where 3 components can finally be used for the cohesive element:

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{yz}\\ \sigma_{zx}\end{bmatrix} \quad \rightarrow \quad \begin{bmatrix}t_1\\ t_2\\ t_3\end{bmatrix} = \begin{bmatrix}\sigma_{zx}\\ \sigma_{yz}\\ \sigma_{zz}\end{bmatrix}.$$

For hyperelastic material models 57 and 83, the deformation gradient is obtained by an incremental update of the strain rates mentioned above.

2. **Forcing uniaxial stress state.** As stated in Remark 1, only three values from the six stress components are used for the cohesive element. By default, the remaining stress components are not modified. Consequently, transverse normal stresses, $\sigma_{xx}$ and $\sigma_{yy}$, and in-plane shear stress, $\sigma_{xy}$, will in most cases build-up during the simulation of uniaxial loading of the cohesive zone due to Poisson's effect and the given reduced strain rate tensor. These stress components should not affect the response of the cohesive element for elastic or viscoelastic material models. They will, however, have a significant effect for most materials with plasticity If undesired, the effect can be reduced by setting the UNIAX flag which resets the unused stress components $\sigma_{xx}$, $\sigma_{yy}$ and $\sigma_{xy}$ to 0.0 after each evaluation of the three-dimensional constitutive model.

3. **Critical time step.** The critical time step size for cohesive elements depends on nodal masses (that is, element volume and density) and the stiffness of the material, $\max(E_T, E_N)$. Note that stiffness has units of stress per length$^3$, such as N/mm$^3$. With *MAT_ADD_COHESIVE, the elastic moduli (stress unit) from the corresponding 3-dimensional material model are taken and related to the thickness (length unit) of the cohesive element. Thus, the thickness of the cohesive element (either coming from geometry or THICK) changes the critical time step size. Therefore, we recommend using a reasonable value for THICK.

4. **Output to d3plot.** If this keyword is used with a three-dimensional material model, the output to d3plot or elout is organized as with other material models for cohesive elements; see for example *MAT_184. Instead of the usual six stress components, three traction stresses are written into those databases. The in-

plane shear traction along the 1-2 edge replaces the $x$ component of stress, the orthogonal in-plane shear traction replaces the $y$ component of stress, and the traction in the normal direction replaces the $z$ component of stress.

## *MAT_ADD_COHESIVE_FATIGUE

The purpose of this card is to augment a standard cohesive material with the ability to model fatigue crack growth under cyclic loading.

This keyword is under development and currently only applies to *MAT_138, *MAT_186, and *MAT_240 base cohesive materials. Only solid hexahedral and pentahedral elements are supported.

### Card Summary:

**Card 1.** This card is required.

| MID | LRATIO | LFREQ | FMIX | MSERR | FMODEL | TSTART | CRCKINIT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| C1 | M1 | C2 | M2 | CM | MM | | |
|---|---|---|---|---|---|---|---|

***Card 3.*** This card is optional.

| IMODEL | SI | SII | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

**Main Card.** Sets overall model setting.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LRATIO | LFREQ | FMIX | MSERR | FMODEL | TSTART | CRCKINIT |
| Type | I | F | F | I | I | I | F | I |
| Default | none | 0.0 | 0.0 | 1 | 1 | 1 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which cohesive fatigue is added |
| LRATIO | Load ratio, $R = \frac{F_{\min}}{F_{\max}}$:<br>GE.0.0: Constant value (default) |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: \|LRATIO\| is a load curve ID for the curve defining the load ratio as a function of pseudo time. |
| LFREQ | Loading frequency defining the number of cycles per unit of pseudo time such that $f = \frac{dN}{dt}$<br>GT.0.0: Constant value (default)<br>LT.0.0: \|LFREQ\| is a load curve ID for the curve defining the loading frequency as a function of pseudo time. |
| FMIX | Mode-mixture model for Paris Law constants:<br>EQ.1: Linear rule of mixtures by Russell & Street [1989]. This is the default for FMODEL = 1. |
| MSERR | Extraction method for peak strain energy release rate:<br>EQ.1: Integration of interface traction (default for FMODEL = 1). |
| FMODEL | Fatigue damage evolution model:<br>EQ.1: Model by Tao et al [2018] |
| TSTART | Starting pseudo time for the fatigue evaluation. It should be set to the time when the applied load is stable. |
| CRCKINIT | Flag to control which cohesive elements are initialized as crack-front elements:<br>GE.0: All free-edge cohesive elements are considered crack-front elements<br>LT.0: \|CRCKINIT\| is a solid element set containing crack-front elements |

**Fatigue Propagation Card.** This card sets the Paris Law parameters.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | M1 | C2 | M2 | CM | MM | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | C1 | M1 | C1 | M1 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| C1 | Scale of crack growth rate in Paris Law for mode I |
| M1 | Slope of crack growth rate in Paris Law for mode I |
| C2 | Scale of crack growth rate in Paris Law for mode II |
| M2 | Slope of crack growth rate in Paris Law for mode II |
| CM | Scale of crack growth rate in Paris Law for mixed mode (not needed for FMIX = 1) |
| MM | Slope of crack growth rate in Paris Law for mixed mode (not needed for FMIX = 1) |

**Fatigue Initiation Card.** Sets the initiation model parameters. This card is optional.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IMODEL | SI | SII | | | | | |
| Type | I | F | F | | | | | |
| Default | 0 | 0.0 | SI | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IMODEL | Fatigue initiation model selector:<br>EQ.0: No initiation model<br>EQ.1: Model by Tao et al [2018] |

| VARIABLE | DESCRIPTION |
|---|---|
| SI | Slope in semilogarithmic initiation (S-N) curve for mode I |
| SII | Slope in semilogarithmic initiation (S-N) curve for mode II |

**Remarks:**

The history variables associated with the fatigue model are placed after those from the base cohesive material. Let NCMHV be the number of history variables for the base material material. The history from this model are as listed in the following table.

| History Variable # | Description |
|---|---|
| NCMHV + 1 | State:<br>EQ.0: Undamaged<br>EQ.1: Damaged<br>EQ.2: Crack tip<br>EQ.3: Failed |
| NCMHV + 2 | Fatigue damage, $D_f$ ($0 \le D_f \le 1$) |
| NCMHV + 3 | Number of cycles since the element became a crack tip, $N_{tip}$ |
| NCMHV + 4 | Characteristic fatigue crack length, $L_f$ |
| NCMHV + 5 | Instantaneous mode mixity (integration point average), $\eta$ |
| NCMHV + 6 | Number of cycles to traverse element at current growth rate, $N_e$ |
| NCMHV + 7 | Initiation damage accumulator, $D_i$ |
| NCMHV + 8 | Initiation strength scale, $r_\sigma$ |
| NCMHV + 9 | Number of cycles to initiation, $N_i$ |
| NCMHV + 10 | Mode I direction cosine |

**References**

Russell, A.J., Street, K.N, "Predicting interlaminar fatigue crack growth rates in compressively loaded laminates," Delamination and Debonding of Materials, ASTM STP 1012. Philadelphia: American Society for Testing and Materials, 1989. p. 162-178.

Tao, C., Mukhopadhyay, S., Zhang, B., Kawashita, L.F., Qiu, J., Hallett, S.R. (2018), "An improved delamination fatigue cohesive interface model for complex 3D multi-interface cases," Compos Part A 2018, 107: 633–46.

## *MAT_ADD_DAMAGE_DIEM

Many of the constitutive models do not allow failure and erosion. The ADD_DAMAGE_-DIEM option provides a way of including damage and failure in these models. DIEM comprises various "damage initiation and evolution models." See remarks for details.

This keyword originates from a split out of *MAT_ADD_EROSION, where only "sudden" failure criteria without damage remain. It applies to nonlinear element formulations including 2D continuum elements, beam element formulation 1, 3D shell elements (including isogeometric shells), 3D solid elements (including isogeometric solids) and thick shells.

**NOTE:** All *MAT_ADD_DAMAGE_DIEM commands in a model can be disabled by using *CONTROL_MAT.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | NDIEMC | DINIT | DEPS | NUMFIP | VOLFRAC | | |
| Type | A | F | F | F | F | F | | |
| Default | none | 0.0 | 0.0 | 0.0 | 1.0 | 0.5 | | |

**Data Card Pairs.** Include NDIEMC pairs of Cards 2 and 3.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DITYP | P1 | P2 | P3 | P4 | P5 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DETYP | DCTYP | Q1 | Q2 | Q3 | Q4 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification for which this erosion definition applies. A unique number or label must be specified (see *PART). |
| NDIEMC | Number of damage initiation and evolution model (DIEM) criteria to be applied. NDIEMC can be at most 5. |
| DINIT | Damage initialization option:<br>EQ.0: No action is taken.<br>EQ.1: Damage history is initiated based on values of the initial plastic strains and the initial strain tensor. This is to be used in multistage analyses. |
| DEPS | Plastic strain increment between evaluation of damage instability and evolution criteria. See Remark 1. The default is zero. |
| NUMFIP | Number or percentage of failed integration points prior to element deletion (default value is 1). NUMFIP does not apply to higher order solid element types 24, 25, 26, 27, 28, and 29, rather see the variable VOLFRAC. Also, when the material is a composite defined with *PART_COMPOSITE with different materials through-the-thickness, do not use NUMFIP; use *DEFINE_ELEMENT_EROSION instead.<br>GT.0.0: Number of integration points which must fail before element is deleted.<br>LT.0.0: Applies only to shells and beams. \|NUMFIP\| is the percentage of layers which must fail before an element fails. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails. |
| VOLFRAC | Volume fraction required to fail before element deletion. The |

| VARIABLE | DESCRIPTION |
|---|---|
| | default is 0.5. It is used for higher-order solid element types 24, 25, 26, 27, 28, and 29, and all isogeometric solids and shell elements. See Remark 3. |
| DITYP | Damage initiation type (see Damage Initiation):<br>EQ.0.0: Ductile based on stress triaxiality<br>EQ.1.0: Shear<br>EQ.2.0: MSFLD<br>EQ.3.0: FLD<br>EQ.4.0: Ductile based on normalized principal stress |
| P1 | Damage initiation parameter:<br>DITYP.EQ.0.0: Load curve/table ID representing plastic strain at the onset of damage as a function of stress triaxiality ($\eta$) and optionally plastic strain rate, represented by $\varepsilon_D^p$ in the theory below. If the first strain rate value in the table is negative, it is assumed to be given with respect to logarithmic strain rate.<br>DITYP.EQ.1.0: Load curve/table ID representing plastic strain at onset of damage as a function of shear influence ($\theta$) and optionally plastic strain rate, represented by $\varepsilon_D^p$ in the theory below. If the first strain rate value in the table is negative, it is assumed to be given with respect to logarithmic strain rate.<br>DITYP.EQ.2.0: Load curve/table ID representing plastic strain at onset of damage as a function of ratio of principal plastic strain rates ($\alpha$) and optionally plastic strain rate, represented by $\varepsilon_D^p$ in the theory below. If the first strain rate value in the table is negative, it is assumed to be given with respect to logarithmic strain rate.<br>DITYP.EQ.3.0: Load curve/table ID representing plastic strain at onset of damage as a function of ratio of principal plastic strain rates ($\alpha$) and optionally plastic strain rate, represented by $\varepsilon_D^p$ in the theory below. If the first strain rate value in the table is negative, it is assumed to be given with respect |

| VARIABLE | DESCRIPTION |
|---|---|
| | to logarithmic strain rate. |
| | DITYP.EQ.4.0: Load curve/table ID representing plastic strain at onset of damage as a function of stress state parameter ($\beta$) and optionally plastic strain rate, represented by $\varepsilon_D^p$ in the theory below. If the first strain rate value in the table is negative, it is assumed to be given with respect to logarithmic strain rate. |
| P2 | Damage initiation parameter: |
| | DITYP.EQ.0.0: Not used |
| | DITYP.EQ.1.0: Pressure influence parameter, $k_s$ |
| | DITYP.EQ.2.0: Layer specification: |
| | EQ.0: Mid layer<br>EQ.1: Outer layer |
| | DITYP.EQ.3.0: Layer specification: |
| | EQ.0: Mid layer<br>EQ.1: Outer layer |
| | DITYP.EQ.4.0: Triaxiality influence parameter, $k_d$ |
| P3 | Damage initiation parameter: |
| | DITYP.EQ.0.0: Not used |
| | DITYP.EQ.1.0: Computation of maximum shear stress for shells: |
| | EQ.0: Three-dimensional<br>EQ.1: Two-dimensional |
| | DITYP.EQ.2.0: Initiation formulation: |
| | EQ.0: Direct<br>EQ.1: Incremental |
| | DITYP.EQ.3.0: Initiation formulation: |
| | EQ.0: Direct<br>EQ.1: Incremental |
| | DITYP.EQ.4.0: Not used |
| P4 | Plane stress option for shell elements: |
| | EQ.0.0: Transverse shear stresses, $\sigma_{yz}$ and $\sigma_{zx}$, are included in the computation of stress invariants, such as the |

| VARIABLE | DESCRIPTION |
|---|---|
| | triaxiality.<br>EQ.1.0: Transverse shear stresses, $\sigma_{yz}$ and $\sigma_{zx}$, are not included in the computation of stress invariants, such as the triaxiality. Useful in combination with "plane stress" material models, where the transverse shear stresses are also excluded from the yield condition, such as *MAT_024_2D or *MAT_036. |
| P5 | Load curve or table ID representing regularization factor as a function of the characteristic element size (curve) or regularization factor as a function of the characteristic element size and abscissa value of the criterion used (table). The criterion is the curve/table specified in P1. For example, for DITYP = 0.0, the regularization factor would depend on stress triaxiality. This factor scales the plastic strain at the onset of damage defined with P1. |
| DETYP | Damage evolution type:<br>EQ.0.0: Linear softening. Evolution of damage is a function of the plastic displacement after the initiation of damage.<br>EQ.1.0: Linear softening, Evolution of damage is a function of the fracture energy after the initiation of damage. |
| DCTYP | Damage composition option for multiple criteria:<br>EQ.-1.0: Damage not coupled to stress<br>EQ.0.0: Maximum<br>EQ.1.0: Multiplicative |
| Q1 | Damage evolution parameter:<br>DETYP.EQ.0.0: Plastic displacement at failure, $u_f^p$. A negative value corresponds to a table ID for $u_f^p$ as a function of triaxiality and damage.<br>DETYP.EQ.1.0: Fracture energy at failure, $G_f$ |
| Q2 | Set to 1.0 to output information to log files (messag and d3hsp) when an integration point fails. |
| Q3 | Damage evolution parameter:<br>DETYP.EQ.0.0: Exponent, $\alpha$, in nonlinear damage evolution law, activated when $u_f^p > 0$ and $\alpha > 0$. |

| VARIABLE | DESCRIPTION |
|---|---|
| | DETYP.EQ.1.0: Not used. |
| Q4 | Load curve or table ID representing regularization factor as a function of the characteristic element size (curve) or regularization factor as a function of the characteristic element size and abscissa value of the criterion used (table). The criterion is the curve/table specified in P1. For example, for DITYP = 0.0, the regularization factor would depend on stress triaxiality. If Q4 is input with a negative sign, the second table input should be plastic strain rate instead of the abscissa value. This factor scales the damage evolution parameter Q1. |

**Remarks:**

1. **DEPS.** In DIEM, you may invoke up to 5 damage initiation and evolution criteria. For the sake of efficiency, the parameter DEPS can be used to only check these criteria in quantified increments of plastic strain. In other words, the criteria are only checked when the effective plastic strain goes beyond DEPS, 2 × DEPS, 3 × DEPS, etc. For DEPS = 0 the checks are performed in each step there is plastic flow. A reasonable value of DEPS could, for instance, be DEPS = 0.0001.

2. **Damage initiation and evolution variables.** Assume that $n$ initiation/evolution types have been specified in the input deck ($n$ = NDIEMC). At each integration point a damage initiation variable, $\omega_D^i$, and an evolution history variable $D^i$ exist, such that,

$$\omega_D^i \in [0, \infty)$$

and

$$D^i \in [0,1] \ , \qquad i = 1, \ldots n \ .$$

These are initially set to zero and evolve with the deformation of the elements according to rules associated with the specific damage initiation and evolution type chosen, see below for details.

These quantities can be post-processed as ordinary material history variables and their positions in the history variables array is given in d3hsp, search for the string *Damage history listing*. The damage initiation variables do not influence the results but serve to indicate the onset of damage. As an alternative, the keyword *DEFINE_MATERIAL_HISTORIES can be used to output the instability and damage, following

| *DEFINE_MATERIAL_HISTORIES Properties | | |
|---|---|---|
| **Label** | **Attributes** | **Description** |
| Instability | - - - - | Maximum initiation variable, $\max_{i=1,\ldots,n} \omega_{\mathrm{D}}^{i}$ |
| Damage | - - - - | Effective damage $D$, see below |

The damage evolution variables govern the damage in the material and are used to form the global damage $D \in [0,1]$. Each criterion is of either of DCTYP set to maximum (DCTYP = 0) or multiplicative (DCTYP = 1), or one could choose to not couple damage to the stress by setting DCTYP = −1. This means that the damage value is calculated and stored, but it is not affecting the stress as for the other options, so if all DCTYP are set to −1 there will be no damage or failure. Letting $I_{\max}$ denote the set of evolution types with DCTYP set to maximum and $I_{\mathrm{mult}}$ denote the set of evolution types with DCTYP set to multiplicative the global damage, $D$, is defined as

$$D = \max(D_{\max}, D_{\mathrm{mult}}) \ ,$$

where

$$D_{\max} = \max_{i \in I_{\max}} D^{i} \ ,$$

and

$$D_{\mathrm{mult}} = 1 - \prod_{i \in I_{\mathrm{mult}}} \left(1 - D^{i}\right) \ .$$

The damage variable relates the macroscopic (damaged) to microscopic (true) stress by

$$\sigma = (1 - D)\tilde{\sigma} \ .$$

Once the damage has reached the level of $D_{\mathrm{erode}}$ (=0.99 by default) the stress is set to zero and the integration point is assumed failed and not processed thereafter. For NUMFIP > 0, a shell element is eroded and removed from the finite element model when NUMFIP integration points have failed. For NUMFIP < 0, a shell element is eroded and removed from the finite element model when -NUMFIP percent of the layers have failed.

3. **VOLFRAC.** The volumes associated with individual integration points in higher order finite elements and isogeometric elements varies widely. Thus, the number of failed integration points is not a reliable criterion for determining element failure. To obtain a more stable and consistent response, LS-DYNA uses the volume fraction of the failed material for these types of elements.

**Damage Initiation, $\omega_D$:**

For each evolution type $i$, $\omega_D^i$ governs the onset of damage. For $i \neq j$, the evolution of $\omega_D^i$ is independent from the evolution of $\omega_D^j$. The following list enumerates the algorithms for modelling damage initiation.

In this subsection we suppress the superscripted $i$ indexing the evolution type.

1. **Ductility based on stress triaxiality (DITYP = 0).** For the ductile initiation option, a function $\varepsilon_D^p = \varepsilon_D^p(\eta, \dot{\varepsilon}^p)$ represents the plastic strain at onset of damage (P1). This is a function of stress triaxiality defined as

$$\eta = -\frac{p}{q} \ ,$$

with $p$ being the pressure and $q$ the von Mises equivalent stress. Optionally this can be defined as a table with the second dependency being on the effective plastic strain rate $\dot{\varepsilon}^p$, where a negative sign of the first strain rate in the table means that it is in logarithmic scale. The damage initiation history variable evolves according to

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p} \ .$$

2. **Shear (DITYP = 1).** For the shear initiation option, a function $\varepsilon_D^p = \varepsilon_D^p(\theta, \dot{\varepsilon}^p)$ represents the plastic strain at onset of damage (P1). This is a function of a shear stress function defined as

$$\theta = \frac{q + k_s p}{\tau} \ .$$

Here $p$ is the pressure, $q$ is the von Mises equivalent stress and $\tau$ is the maximum shear stress defined as a function of the principal stress values:

$$\tau = \frac{(\sigma_{\text{major}} - \sigma_{\text{minor}})}{2} \ .$$

Parameter P3 allows you to select which principal stresses are used in this equation for shell elements. With P3 = 0, only the in-plane stresses are considered (2-dimensional approach), whereas with P3 = 1, they are computed from the full stress tensor (3-dimensional approach). The latter case leads to higher shear fracture risk for the range between uniaxial and biaxial loading. Introduced here is also the pressure influence parameter $k_s$ (P2). Optionally this can be defined as a table with the second dependency being on the effective plastic strain rate $\dot{\varepsilon}^p$, where a negative sign of the first strain rate in the table means that it is in logarithmic scale. The damage initiation history variable evolves according to

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p}.$$

3. **MSFLD (DITYP = 2).** For the MSFLD initiation option, a function $\varepsilon_D^p = \varepsilon_D^p(\alpha, \dot{\varepsilon}^p)$ represents the plastic strain at onset of damage (P1). The MSFLD criterion is only relevant for shells and with restrictions (discussed in the section MSFLD and FLD with solid and thick shell elements) for hexahedral solids, pentahedral solids and tshells. $\varepsilon_D^p$ can be specified as a load curve or table. In the load curve case, it only depends on $\alpha$.

$\alpha$ is the ratio of principal plastic strain rates defined as

$$\alpha = \frac{\dot{\varepsilon}^p_{\text{minor}}}{\dot{\varepsilon}^p_{\text{major}}}.$$

The principal strains should be interpreted as the in-plane principal strains. For simplicity the plastic strain evolution in this formula is assumed to stem from an associated von Mises flow rule. Hence,

$$\alpha = \frac{s_{\text{minor}}}{s_{\text{major}}}$$

with $s$ being the deviatoric stress. This ensures that the calculation of $\alpha$, is in a sense, robust at the expense of being slightly inaccurate for materials with anisotropic yield functions and/or non-associated flow rules. Note that if the material model is *MAT_036 and *MAT_133, the above simplification is not used, and the anisotropy is properly accounted for when determining $\alpha$.

Optionally, $\varepsilon_D^p$ can be defined as a table with the second dependency being on the effective plastic strain rate, $\dot{\varepsilon}^p$. If the first first strain rate in the table has a negative sign, it is assumed to be in logarithmic scale. For $\dot{\varepsilon}^p = 0$, the value of $\varepsilon_D^p$ is set to a large number to prevent the onset of damage for no plastic evolution. Furthermore, the plastic strain used in this failure criterion is a modified effective plastic strain that only evolves when the pressure is negative, meaning the material is not affected in compression. This modified plastic strain can be monitored as the second history variable of the initiation history variables in the binary output database.

For P3 = 0, the damage initiation history variable is calculated directly from the ratio of (modified) plastic strain and the critical plastic strain

$$\omega_D = \max_{t \le T} \frac{\varepsilon^p}{\varepsilon_D^p} \ .$$

This should be interpreted as the maximum value up to this point in time. If P3 = 1 the damage initiation history variable is instead incrementally updated from the ratio of (modified) plastic strain and the critical plastic strain

$$\omega_D = \int_0^{\varepsilon^p} \frac{d\varepsilon^p}{\varepsilon_D^p} \ .$$

For this initiation option with shells, P2 is used to determine the layer in the shell where the criterion is evaluated. If P2 = 0, the criterion is evaluated in the mid-layer only, whereas if P2 = 1, it is evaluated in the outer layers only (bottom and top). This can be used to distinguish between a membrane instability typically used for FLD evaluations (P2 = 0) and a bending instability (P2 = 1). For shells, as soon as $\omega_D$ reaches 1 in any of the integration points of interest, *all* integration points in the shell goes over in damage mode, meaning subsequent damage is applied to the entire element. For solids/tshells, only P2 = 0 is currently supported, and when $\omega_D$ reaches 1 in the center of the section then all elements in the section goes into damage mode. Again, more details for solids or thick shells are provided in the section titled MSFLD and FLD with solid and thick shell elements.

4. **FLD (DITYP = 3).** The FLD initiation criterion is identical to MSFLD with one subtle difference: the plastic strain used to evaluate the criteria does not account for the sign of the hydrostatic stress but is instead identical to the effective plastic strain directly from the underlying material model. In other words, it is not the modified plastic strain used in the MSFLD criterion, but apart from that it is an identical criterion.

5. **Ductile based on normalized principal stress (DITYP = 4).** For the ductile initiation option the plastic strain at the onset of damage (P1) is taken as a function of $\beta$ and $\dot{\varepsilon}^{\mathrm{p}}$, that is, $\varepsilon_{\mathrm{D}}^{\mathrm{p}} = \varepsilon_{\mathrm{D}}^{\mathrm{p}}(\beta, \dot{\varepsilon}^{\mathrm{p}})$. Here $\beta$ is the normalized principal stress

$$\beta = \frac{q + k_{\mathrm{d}} p}{\sigma_{\mathrm{major}}} \quad ,$$

where $p$ is the pressure, $q$ is the von Mises equivalent stress, $\sigma_{\mathrm{major}}$ is the major principal stress, and $k_{\mathrm{d}}$ is the pressure influence parameter specified in the P2 field. Optionally, this can be defined as a table with the second dependency being on the effective plastic strain rate $\dot{\varepsilon}^{\mathrm{p}}$, where a negative sign of the first strain rate in the table means that it is in logarithmic scale. The damage initiation history variable evolves according to

$$\omega_{\mathrm{D}} = \int_0^{\varepsilon^{\mathrm{p}}} \frac{\mathrm{d}\varepsilon^{\mathrm{p}}}{\varepsilon_{\mathrm{D}}^{\mathrm{p}}} \quad .$$

### MSFLD and FLD with solid and thick shell elements

When using MSFLD or FLD with solid or thick shell elements, the following restrictions apply:

- The part should be a thin walled section, with a well-defined "*thickness*" direction, $\boldsymbol{t}$, and associated "*plane*" indicated in blue in Figure 2-3.
- Only low order hexahedra or pentahedra may be used.

- The same number of elements in ***t***-direction must be used, essentially in the form of an extruded shell mesh. The stack of elements at any location, from bottom to top, comprises the "section".
- The element numbering scheme must in itself indicate the thickness direction, ***t***, as illustrated in Figure 2-3, for each element in the part.
- The geometry may be curved, but the mesh topology must not change. Thus, for T-intersections and similar geometries, appropriate pre-processing measures must be undertaken.

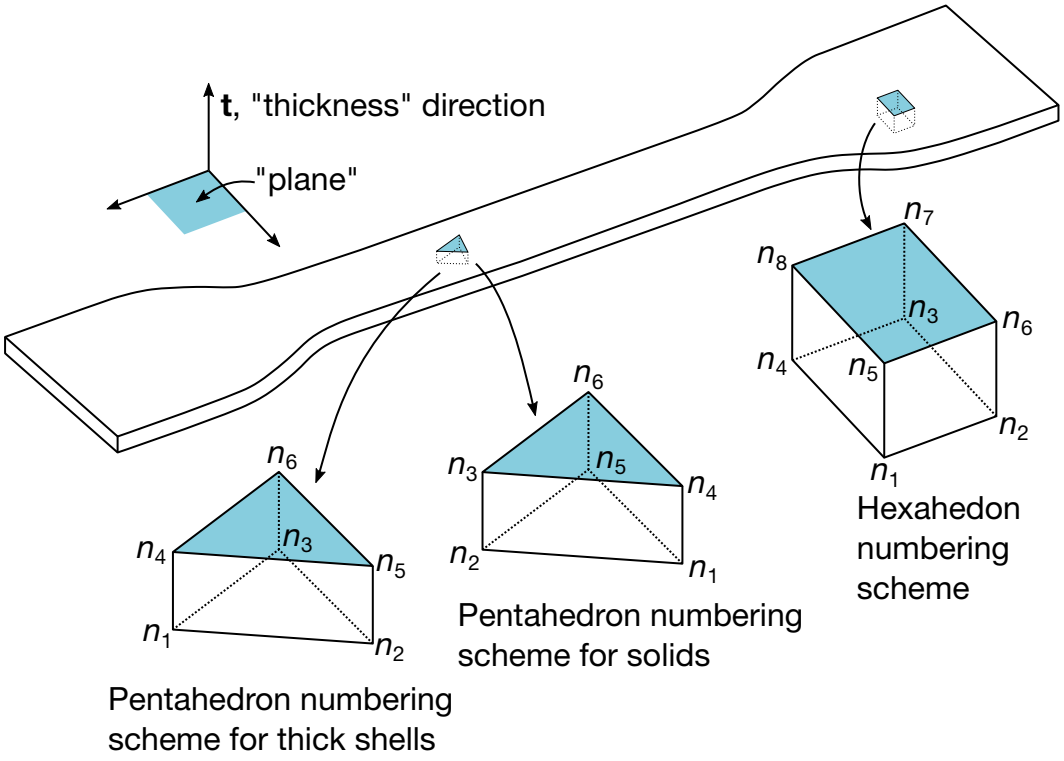



**Figure 2-3.** Solid and thick shell elements must be oriented in a part in a specific way when using MSFLD or FLD (DITYP = 2 or 3). This figure illustrates the required element numbering scheme and the thickness direction, **t**.

### Damage Evolution:

For the evolution of the associated damage variable, $D$, we introduce the plastic displacement, $u^{\mathrm{p}}$, which evolves according to

$$\dot{u}^{\mathrm{p}} = \begin{cases} 0 & \omega_{\mathrm{D}} < 1 \\ l\dot{\varepsilon}^{\mathrm{p}} & \omega_{\mathrm{D}} \geq 1 \end{cases}$$

Here $l$ is a characteristic length of the element. Fracture energy is related to plastic displacement as follows

$$G_f = \int_0^{u_f^p} \sigma_y \mathrm{d}u^p \ ,$$

where $\sigma_y$ is the yield stress. The following list enumerates the algorithms available for modelling damage.

1. **Linear (DETYP = 0).** With this option, if $\alpha = 0$ (Q3) and Q1 is positive, the damage variable evolves linearly with the plastic displacement after onset of damage:

$$\dot{D} = \frac{\dot{u}^p}{u_f^p} \ .$$

Here $u_f^p$ is the plastic displacement at failure (Q1).

If Q1 is negative, then –Q1 refers to a table that defines $u_f^p$ as a function of triaxiality and damage, that is, $u_f^p = u_f^p(\eta, D)$. In this case with $\alpha = 0$, the damage evolution law generalizes to:

$$\dot{D} = \frac{\dot{u}^p}{\frac{\partial u_f^p}{\partial D}} .$$

For a fixed triaxiality, $\eta$, $\bar{u}_f^p := u_f^p(\eta, 1)$ defines the plastic displacement at failure, and the shape of $u_f^p(\eta, D)$ as a function of $D$ determines the post-instability characteristics.

A linear curve, as illustrated by the solid line in Figure 2-4, corresponds exactly to a constant plastic displacement to failure equal to $\bar{u}_f^p$ and can be seen as a reference curve for this discussion. For simplicity assume uniaxial tension ($\eta = 1/3$). A curve with positive curvature, represented by the dash-dots in Figure 2-4, means that damage evolves quickly right after onset of instability and more slowly when approaching failure. In contrast, damage evolves slowly early and more quickly later on for a curve with negative curvature, represented by the dashes. The qualitative effect these curves have in a uniaxial tension test is also illustrated. The correlation between a damage curve and the actual behavior in tests is not straightforward, thus these curves need to be established on a trial-and-error basis.

For $\alpha > 0$ and $u_f^p > 0$, the damage evolution follows an exponential law given by

$$D = \frac{1 - e^{-\alpha \frac{u^p}{u_f^p}}}{1 - e^{-\alpha}},$$

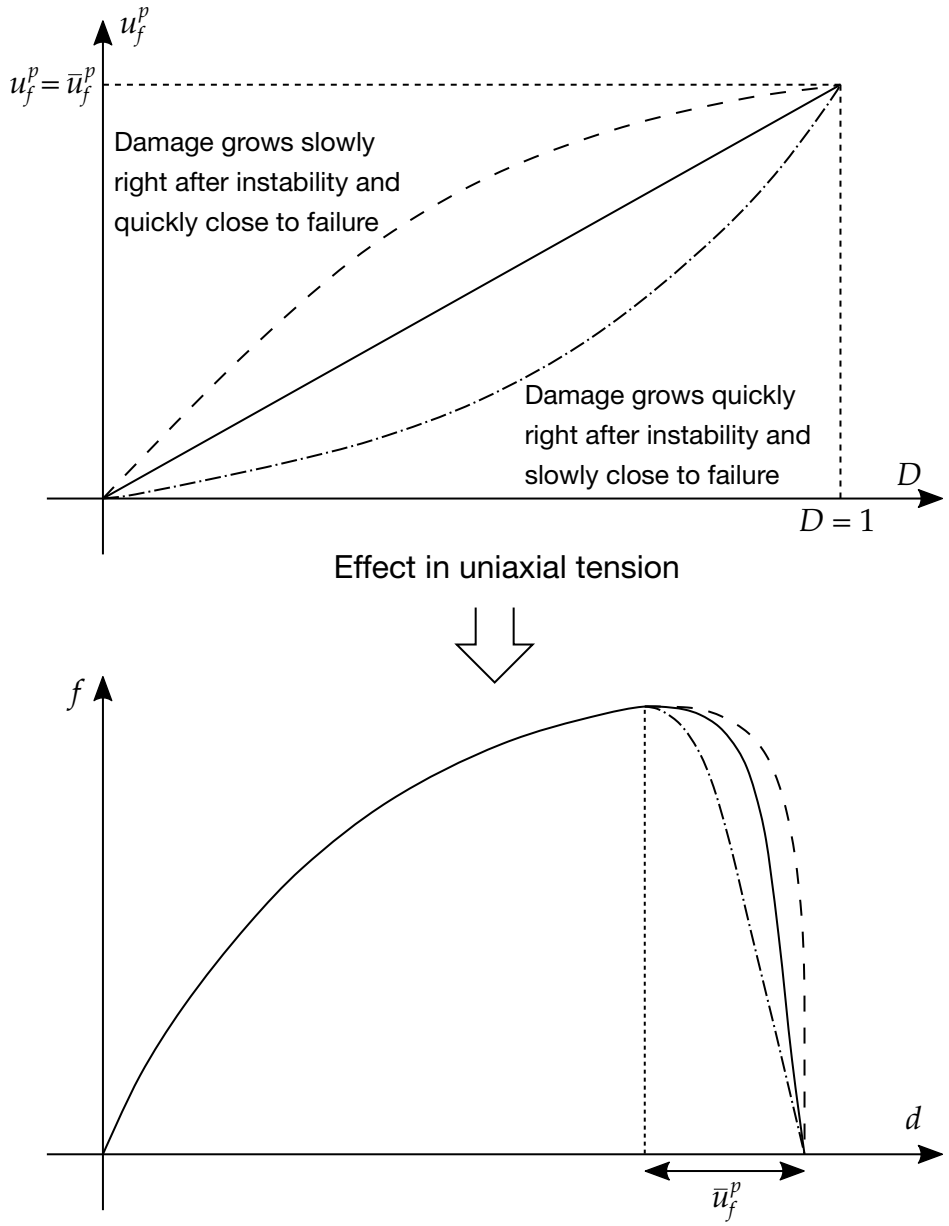



**Figure 2-4.** Top plot illustrates plastic displacement at failure as a function of damage for a given triaxiality for DETYP = 0 and $\alpha = 0$. The different curves illustrate different possible types of post-instability characteristics. The bottom plot illustrates qualitatively how these curves may interact in a tension test. $d$ is the displacement and $f$ is the force.

where $u^{\mathrm{p}} = \int \dot{u}^{\mathrm{p}}$.

2. **Linear (DETYP = 1).** With this option the damage variable evolves linearly as follows

$$\dot{D} = \frac{\dot{u}^{\mathrm{p}}}{u_{\mathrm{f}}^{\mathrm{p}}} \quad ,$$

where $u_f^p = 2G_f/\sigma_{y_0}$ and $\sigma_{y_0}$ is the yield stress when failure criterion is reached. This option is available for a specific subset of material model types: 3, 4, 10, 11, 12, 15, 16, 17, 18, 19, 24, 26, 28, 32, 33, 35, 36, 37, 39, 52, 64, 65, 72, 81, 82, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 120, 122, 123, 124, 130, 133, 135, 136, 153, 155, 157, 167, 171, 187, 188, 190, 199, and 243.

## *MAT_ADD_DAMAGE_GISSMO_{OPTION}

Available options include:

<BLANK>

STOCHASTIC

Many of the constitutive models do not allow failure and erosion. *MAT_ADD_DAMAGE_GISSMO provides a way to include damage and failure in these models. GISSMO is the "generalized incremental stress-state dependent damage model." It applies to nonlinear element formulations including 2D continuum elements, beam element formulation 1, 3D shells (including isogeometric shells), 3D thick shells, 3D solids (including isogeometric solids), and SPH. See GISSMO Damage Model for details. The STOCHASTIC option allows spatially varying failure behavior. See *DEFINE_STOCHASTIC_VARIATION and *DEFINE_HAZ_PROPERTIES for additional information.

*MAT_ADD_DAMAGE_GISSMO originates from splitting *MAT_ADD_EROSION. Only "sudden" failure criteria without damage remain in *MAT_ADD_EROSION.

**NOTE:** Use *CONTROL_MAT to disable all *MAT_ADD_DAMAGE_GISSMO commands in a model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | | DTYP | REFSZ | NUMFIP | VOLFRAC | TETSF | |
| Type | A | | F | F | F | F | F | |
| Default | none | | 0.0 | 0.0 | 1.0 | 0.5 | 0.0 | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSDG | ECRIT | DMGEXP | DCRIT | FADEXP | LCREGD | INSTF | LCSOFT |
| Type | F | F | F | F | F | F | I | I |
| Default | 0.0 | 0.0 | 1.0 | 0.0 | 1.0 | 0.0 | 0 | 0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSRS | SHRF | BIAXF | LCDLIM | MIDFAIL | HISVN | SOFT | LP2BI |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

This card is optional.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RGTR1 | RGTR2 | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

**VARIABLE** **DESCRIPTION**

MID — Material identification for which this erosion definition applies. A unique number or label must be specified (see *PART).

DTYP — DTYP is interpreted digit-wise as follows:

$$\text{DTYP} = [NM] = M + 10 \times N$$

M.EQ.0: Damage is accumulated, but there is no coupling to flow stress and no failure.

M.EQ.1: Damage is accumulated, and element failure occurs for $D = 1$. Coupling of damage to flow stress depending on parameters, see GISSMO Damage Model below.

N.EQ.0: Equivalent plastic strain is the driving quantity for the damage. (To be more precise, it's the history variable that LS-PrePost blindly labels as "plastic strain." What this history variable actually represents depends on the material model.)

N.GT.0: The $N^{th}$ additional history variable is the driving quantity for damage. These additional history variables are the same ones flagged by the *DATABASE_EXTENT_BINARY keyword's NEIPS and NEIPH fields. For example, for solid elements with *MAT_187, setting $N =$

| VARIABLE | DESCRIPTION |
|---|---|
| | 6 causes volumetric plastic strain to be the driving quantity for the GISSMO damage. |
| REFSZ | Reference element size for which an additional output of damage (and potentially plastic strain) will be generated. This is necessary to ensure the applicability of resulting damage quantities when transferred to different mesh sizes.<br>GT.0: Reference size related damage values are written to history variables ND + 9 and ND + 10. These damage values are computed in the same fashion as the actual damage, just with the given reference element size.<br>LT.0: The reference element size is \|REFSZ\|. In addition to the reference size related damage values, a corresponding plastic strain is computed and written to history variable ND + 17. See Remark 2. |
| NUMFIP | Number or percentage of failed integration points prior to element deletion (default value is 1). NUMFIP does not apply to higher order solid element types 24, 25, 26, 27, 28, and 29, rather see the variable VOLFRAC. Also, when the material is a composite defined with *PART_COMPOSITE with different materials through-the-thickness, do not use NUMFIP; use *DEFINE_ELEMENT_EROSION instead.<br>GT.0.0: Number of integration points which must fail before element is deleted.<br>LT.0.0: Applies only to shells and beams. \|NUMFIP\| is the percentage of layers which must fail before an element is deleted. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails. |
| VOLFRAC | Volume fraction required to fail before element deletion. The default is 0.5. It is used for higher-order solid element types 24, 25, 26, 27, 28, and 29, and all isogeometric solids and shell elements. See Remark 3. |
| TETSF | Scale factor applied to the element size of tetrahedral elements when obtaining the regularization factor with LCREGD. To achieve similar results with hexahedral and tetrahedral element meshes, a value of 2.0 is a good starting point. |
| LCSDG | Failure strain curve/table or function: |

| VARIABLE | DESCRIPTION |
|---|---|
| | GT.0.0: Load curve ID or table ID. As a load curve, it defines equivalent plastic strain to failure as a function of triaxiality. As a table, it defines for each Lode parameter value (between -1 and 1) a load curve ID giving the equivalent plastic strain to failure as a function of triaxiality for that Lode parameter value. With HISVN ≠ 0, a 3D table can be used, where failure strain is a function of the history variable (TABLE_3D), Lode parameter (TABLE), and triaxiality (CURVE). With HISVN = 0, a 3D table introduces thermal effects, that is, failure strain is a function of temperature (TABLE_3D), Lode parameter (TABLE), and triaxiality (CURVE). As a 4D table, failure strain is a function of strain rate (TABLE_4D), temperature (TABLE_3D), Lode parameter (TABLE), and triaxiality (CURVE).<br>LT.0.0: \|LCSDG\| is the ID of a function (*DEFINE_FUNCTION) with the arguments triaxiality $\eta$, Lode parameter $L$, plastic strain rate $\dot{\varepsilon}^p$, temperature $T$, history variable HISVN, and element size $l_e$: $f(\eta, L, \dot{\varepsilon}^p, T,$HISVN$, l_e)$. Note that the sequence of the arguments is important, not their names. |
| ECRIT | Critical plastic strain (material instability); see below.<br>LT.0.0: \|ECRIT\| is either a load curve ID defining critical equivalent plastic strain versus triaxiality or a table ID defining critical equivalent plastic strain as a function of triaxiality and Lode parameter (as in LCSDG). With HISVN ≠ 0, a 3D table can be used, where critical strain is a function of the history variable (TABLE_3D), Lode parameter (TABLE), and triaxiality (CURVE). With HISVN = 0, a 3D table introduces thermal effects, that is, critical strain is a function of temperature (TABLE_3D), Lode parameter (TABLE), and triaxiality (CURVE). As a 4D table, critical strain is a function of strain rate (TABLE_4D), temperature (TABLE_3D), Lode parameter (TABLE), and triaxiality (CURVE).<br>EQ.0.0: Fixed value DCRIT defining critical damage is read (see below).<br>GT.0.0: Fixed value for stress-state independent critical equivalent plastic strain |

| VARIABLE | DESCRIPTION |
|---|---|
| DMGEXP | Exponent for nonlinear damage accumulation; see GISSMO Damage Model and Remark 2. |
| DCRIT | Damage threshold value (critical damage). If a load curve of critical plastic strain or fixed value is given by ECRIT, input is ignored. |
| FADEXP | Exponent for damage-related stress fadeout:<br>LT.0.0: \|FADEXP\| is a load curve ID or table ID. As a load curve it gives the fading exponent as a function of element size. As a table, it specifies the fading exponent as a function triaxiality (TABLE) and element size (CURVE). For 3D tables, it specifies the fading exponent as a function Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE).<br>GT.0.0: Constant fading exponent |
| LCREGD | Load curve ID or table ID defining element size dependent regularization factors for equivalent plastic strain to failure:<br>GT.0.0: Load curve ID (regularization factor as a function of element size) or table ID (regularization factor as a function of element size curves indexed by effective strain rate)<br>LT.0.0: \|LCREGD\| is a table ID (regularization factor as a function of element size curves indexed by triaxiality) or a 3D table ID (regularization factor as function of Lode parameter, triaxiality, and element size). This table provides an alternative to the use of SHRF and BIAXF for defining the effect of triaxiality on element size regularization of equivalent plastic strain to failure. |
| INSTF | Flag for governing the behavior of instability measure, $F$, and fading exponent, FADEXP (see GISSMO Damage Model):<br>EQ.0: $F$ is incrementally updated, and FADEXP, if from a table, is allowed to vary.<br>EQ.1: $F$ is incrementally updated, and FADEXP is kept constant after $F = 1$.<br>EQ.2: $F$ is only 0 or 1 (after ECRIT is reached), and FADEXP, if from a table, is allowed to vary.<br>EQ.3: $F$ is only 0 or 1 (after ECRIT is reached), and FADEXP is kept constant after $F = 1$. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSOFT | Load curve or table with ID \|LCSOFT\| giving the soft reduction factor for failure strain in crashfront elements. Crashfront elements are elements that are direct neighbors of failed (deleted) elements. A load curve specifies the soft reduction factor as a function of triaxiality. A table gives the soft reduction factor as a function of triaxiality (curve) and element size (table). The sign of LCSOFT determines which strains are scaled:<br>EQ.0: Inactive<br>GT.0: Plastic failure strain, $\varepsilon_f$ (LCSDG), and critical plastic strain, $\varepsilon_{p,loc}$ (ECRIT), will be scaled by LCSOFT.<br>LT.0: Only plastic failure strain, $\varepsilon_f$ (LCSDG), will be scaled by LCSOFT.<br>SOFT is ignored when LCSOFT is active. |
| LCSRS | Load curve ID or table ID. Load curve ID defining failure strain scaling factor for LCSDG as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. *The curve should not extrapolate to zero or failure may occur at low strain.* Table ID defining failure strain scaling factor as a function of strain rate (TABLE) and triaxiality (CURVE).<br>GT.0: Scale ECRIT.<br>LT.0: Do not scale ECRIT. |
| SHRF | Reduction factor for regularization for shear stress states. This parameter can be defined between -1.0 and +1.0. See Remark 1. |
| BIAXF | Reduction factor for regularization for biaxial stress states. This parameter can be defined between -1.0 and +1.0. See Remark 1. |
| LCDLIM | Load curve ID defining damage limit values as a function of triaxiality. Damage can be restricted to values less than 1.0 to prevent further stress reduction and failure for certain triaxialities. |
| MIDFAIL | Mid-plane failure option for shell elements. If active, then critical strain is only checked at the mid-plane integration point (IP), meaning an odd number for NIP should be used. The other integration points compute their damage, but no coupling to the stresses is done first. As soon as the mid-plane IP reaches ECRIT/DCRIT, then all the other IPs are also checked. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Inactive.<br>EQ.1.0: Active. Those of the non-mid-plane IPs that are already above their critical value immediately start to reduce the stresses. Those still below the critical value still do not couple, only if they reach their criterion.<br>EQ.2.0: Active. All of the non-mid-plane IPs immediately start to reduce the stresses. NUMFIP is active.<br>EQ.3.0: Active. Same as 2 but when D = 1 is reached in the middle integration point, the element is eroded instantaneously. NUMFIP is disregarded.<br>EQ.4.0: Active. Damage and failure are applied only to the midpoint. When D = 1 at the midpoint, the element is eroded. NUMFIP is disregarded. Integration points away from the midplane see no stress reduction and no failure. |
| HISVN | History variable used to evaluate the 3D table LCSDG (and optionally 3D table ECRIT < 0):<br>GT.0.0: Constant value<br>LT.0.0: The constant value found at position \|HISVN\| where \|HISVN\| is the location in the history array of *INITIAL_STRESS_ SHELL/SOLID. |
| SOFT | Softening reduction factor for failure strain in crashfront elements. Crashfront elements are elements that are direct neighbors of failed (deleted) elements.<br>EQ.0.0: Inactive<br>GT.0.0: Plastic failure strain, $\varepsilon_f$ (LCSDG), and critical plastic strain, $\varepsilon_{p,loc}$ (ECRIT), will be scaled by SOFT.<br>LT.0.0: Only plastic failure strain, $\varepsilon_f$ (LCSDG), will be scaled by \|SOFT\|. |
| LP2BI | Option to use a bending indicator instead of the Lode parameter. If active (> 0), the expression "bending indicator" replaces the term "Lode parameter" everywhere in this manual page. We adopted the bending indicator from *MAT_258 (compare with variable $\Omega$). LP2BI > 0 is only available for shell elements and requires NUMFIP = 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Inactive.<br>EQ.1.0: Active. Constant regularization (LCREGD) applied.<br>EQ.2.0: Active. Regularization (LCRGED) fully applied under pure membrane loading ($\Omega = 0$) but not at all under pure bending ($\Omega = 1$). Linear interpolation in between. |
| RGTR1 | First triaxiality value for optional "tub-shaped" regularization. See Remark 1. |
| RGTR2 | Second triaxiality value for optional "tub-shaped" regularization. See Remark 1. |

**GISSMO Damage Model:**

The GISSMO damage model is a phenomenological formulation that allows for an incremental description of damage accumulation, including softening and failure. It is intended to provide a maximum in variability for the description of damage for a variety of metallic materials, such as *MAT_024, *MAT_036, and *MAT_103. The input of parameters is based on tabulated data, allowing the user to directly convert test data to numerical input. The model was originally developed by Neukamm et al. [2008] and further investigated and enhanced by Effelsberg et al. [2012] and Andrade et al. [2014, 2016].

The model is based on an incremental formulation of damage accumulation:

$$\Delta D = \frac{\mathrm{DMGEXP} \times D^{\left(1-\frac{1}{\mathrm{DMGEXP}}\right)}}{\varepsilon_f} \Delta\varepsilon_p$$

where,

$D$ Damage value $(0 \le D \le 1)$. For numerical reasons, $D$ is initialized to a value of 10-20 for all damage types in the first time step.

$\varepsilon_f$ Equivalent plastic strain to failure, determined from LCSDG as a function of the current triaxiality value $\eta$ (and Lode parameter $L$ as an option).

A typical failure curve LCSDG for metal sheet, modelled with shell elements, is shown in Figure 2-5. Triaxiality should be monotonically increasing in this curve. A reasonable range for triaxiality is -2/3 to 2/3 if shell elements are used (plane stress).

For 3-dimensional stress states (solid elements), the possible range of triaxiality goes from $-\infty$ to $+\infty$, but to get a good resolution in the internal load

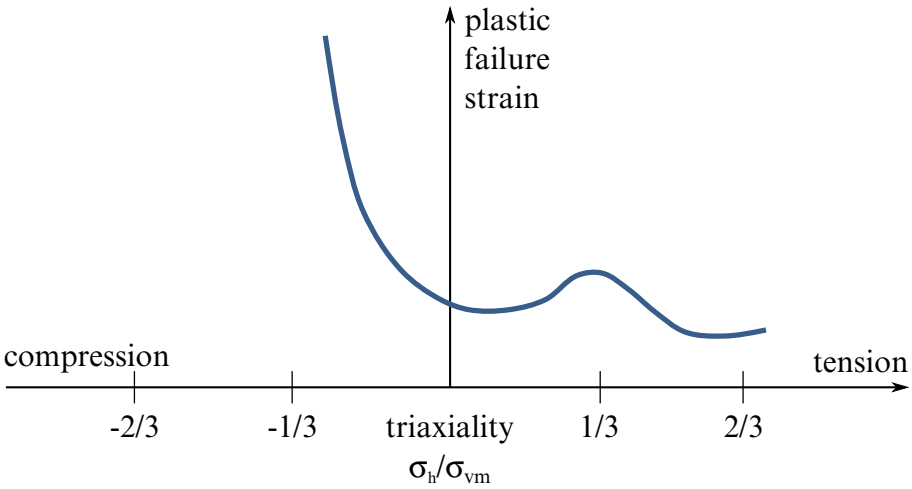



**Figure 2-5.** Typical failure curve for metal sheet, modeled with shell elements.

curve discretization (depending on parameter LCINT of *CONTROL_SOLUTION) you should define lower limits, such as -1 to 1 if LCINT = 100 (default).

$\Delta\varepsilon_p$ Equivalent plastic strain increment

For constant values of failure strain, this damage rate can be integrated to get a relation of damage and actual equivalent plastic strain:

$$D = \left(\frac{\varepsilon_p}{\varepsilon_f}\right)^{\text{DMGEXP}}, \qquad \text{for } \varepsilon_f = \text{constant}$$

Triaxiality, $\eta$, as a measure of the current stress state is defined as

$$\eta = \frac{\sigma_H}{\sigma_M},$$

with hydrostatic stress, $\sigma_H$, and von Mises stress, $\sigma_M$. Lode parameter $L$ as an additional measure of the current stress state is defined as

$$L = \frac{27}{2}\frac{J_3}{{\sigma_M}^3},$$

with third invariant of the stress deviator, $J_3$.

For DTYP = 0, damage is accumulated according to the description above, yet no softening and failure is taken into account. Thus, parameters ECRIT, DCRIT and FADEXP will not have any influence. This option can be used to calculate pre-damage in multi-stage deformations without influencing the simulation results.

For DTYPE = 1, elements will be deleted if $D \geq 1$.

Depending on the set of parameters given by ECRIT (or DCRIT) and FADEXP, a Lemaitre-type coupling of damage and stress *(effective stress concept)* can be used.

To define damage, use one of the following three principal ways:

1. Input of a fixed value of critical plastic strain (ECRIT > 0.). As soon as the magnitude of plastic strain reaches this value, the current damage parameter $D$ is stored as critical damage DCRIT and the damage coupling flag is set to unity, in order to facilitate an identification of critical elements in post-processing. From this point on, damage is coupled to the stress tensor using the following relation:

$$\sigma = \tilde{\sigma}\left[1 - \left(\frac{D - \text{DCRIT}}{1 - \text{DCRIT}}\right)^{\text{FADEXP}}\right]$$

   This leads to a continuous reduction of stress, up to the load-bearing capacrmarkrity completely vanishing as $D$ reaches unity. The fading exponent FADEXP can be element size dependent to allow for the consideration of an element-size dependent amount of energy to be dissipated during element fade-out.

2. Input of a load curve defining critical plastic strain as a function of triaxiality (ECRIT < 0.), pointing to load curve ID |ECRIT|. This allows for a definition of triaxiality-dependent material instability, which takes account of instability and localization occurring depending on the actual load case. One possibility is the use of instability curves predicted by instability models (e.g., Swift, Hill, Marciniak-Kuczynski, etc.). Another possibility is the use of a transformed Forming Limit Diagram as an input for the expected onset of softening and localization. Using this load curve, the instability measure $F$ is accumulated using the following relation, which is similar to the accumulation of damage $D$ except for the instability curve is used as an input:

$$\Delta F = \frac{\text{DMGEXP}}{\varepsilon_{p,loc}} F^{\left(1-\frac{1}{\text{DMGEXP}}\right)} \Delta\varepsilon_p$$

   with,

   $F$ Instability measure ($0 \le F \le 1$).

   $\varepsilon_{\text{p,loc}}$ Equivalent plastic strain to instability, determined from ECRIT

   $\Delta\varepsilon_p$ Equivalent plastic strain increment

   As soon as the instability measure $F$ reaches unity, the current value of damage $D$ in the respective element is stored. Damage will from this point on be coupled to the flow stress using the relation described above.

3. If no input for ECRIT is made, parameter DCRIT will be considered. Coupling of damage to the stress tensor starts if this value *(damage threshold)* is exceeded

(0 ≤ DCRIT ≤ 1). Coupling of damage to stress is done using the relation described above.

This input allows for the use of extreme values also – for example, DCRIT = 1.0 would lead to no coupling at all, and element deletion under full load (brittle fracture).

**Remarks:**

1. **Regularization.** The values of SHRF and BIAXF generally lie between 0.0 and 1.0 where 0.0 means full regularization and 1.0 means no regularization under shear (triaxiality = 0.0 for SHRF = 1.0) or biaxial tension (triaxiality = 2/3 for BIAXF = 1.0). Any other intermediate triaxiality follows a linear interpolation between triaxiality 0.0 and 1/3 and also between triaxiality 1/3 and 2/3. Notice that a full regularization is always for a one-dimensional tensile stress state (triaxiality = 1/3) according to the factors defined under LCREGD (see the next paragraph for an exception to this restriction). For the sake of generalization, both SHRF and BIAXF can also assume negative values (e.g., SHRF=-1.0 and BIAXF=-1.0). In this case, regularization is affected not at triaxialities 0.0 and 2/3 but rather at the triaxialities where the failure curve (LCSDG) crosses the instability curve (-ECRIT). The use of a triaxiality-dependent regularization approach may be necessary because simple regularization only depending on the element size can be unrealistic for certain stress states.

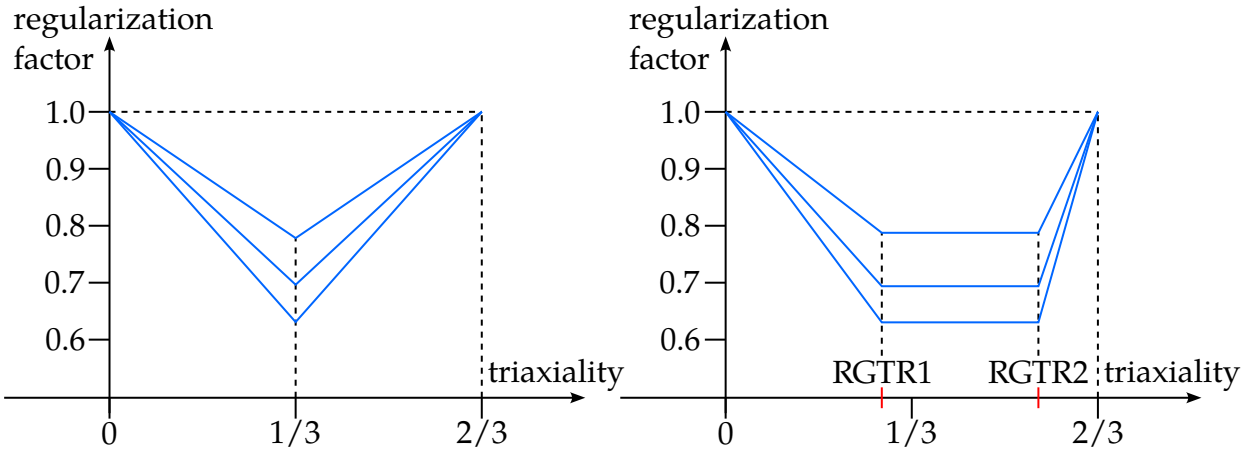



**Figure 2-6.** The left figure provides an example of the regularization curves produced with an LCREGD curve for three different element sizes. SHRF and BIAXF are both 1.0 in this case. The right figure illustrates how these curves become tub-shaped by additionally defining RGTR1 and RGTR2.

The restriction of a full regularization at triaxiality = 1/3 can be lifted with the optional parameters RGTR1 and RGTR2. As shown in Figure 2-6, full regularization starts at RGTR1 and ends at RGTR2. A linear interpolation is used

between 0.0 and RGTR1 and between RGTR2 and 2/3. Together with SHRF = BIAXF = 1 this gives a trapezoidal-(or tub-)shaped regularization. This seems to be a reasonable approach in many cases and is therefore easily accessible now.

2. **Reference element size.** If the results of a first simulation should be transferred to a second computation with potentially modified mesh size, such as mapping from forming to crash, it might be necessary to alter damage values (and maybe plastic strain) as well. For that purpose, reference element size REFSZ can be defined. With REFSZ > 0, corresponding damage is computed in the same fashion as the actual damage, just with the given reference element size instead, and written to history variable ND + 9. An alternative approach is available with the definition of REFSZ < 0. In that case, a plastic strain with regard to |REFSZ| is computed first:

$$\Delta\varepsilon_p^{|\mathrm{REFSZ}|} = \Delta\varepsilon_p \frac{\varepsilon_p^f(|\mathrm{REFSZ}|) - \varepsilon_p^{\mathrm{ECRIT}}}{\varepsilon_p^f(l_e) - \varepsilon_p^{\mathrm{ECRIT}}} \qquad (\text{if } F \geq 1)$$

The accumulated value of that is written to history variable ND + 17. Afterwards, damage with respect to the |REFSZ| is computed similarly to the standard damage accumulation, only using this new reference plastic strain:

$$\Delta D^{|\mathrm{REFSZ}|} = \frac{\mathrm{DMGEXP} \times (D^{|\mathrm{REFSZ}|})^{\left(1-\frac{1}{\mathrm{DMGEXP}}\right)}}{\varepsilon_p^f(|\mathrm{REFSZ}|)} \Delta\varepsilon_p^{|\mathrm{REFSZ}|}$$

This "reference damage" is stored on history variable ND + 9.

3. **VOLFRAC.** The volumes associated with individual integration points in higher order finite elements and isogeometric elements varies widely. Thus, the number of failed integration points is not a reliable criterion for determining element failure. To obtain a more stable and consistent response, LS-DYNA uses the volume fraction of the failed material for these types of elements.

4. **History Variable.** History variables of the GISSMO damage model are written to the post-processing database. Therefore, NEIPH and NEIPS must be set in *DATABASE_EXTENT_-BINARY. The damage history variables start at position ND, which is displayed in d3hsp file as, for example, "first damage history variable = 6" which means that ND = 6. For example, if you wish to view the damage parameter (first GISSMO history variable) for a *MAT_024 shell element, you must set NEIPS = 6. In LS-PrePost, you access the damage parameter as history variable #6.

| History Variable # | Description |
|---|---|
| ND | Damage parameter $D$ ($10^{-20} < D \leq 1$) |

| History Variable # | Description |
|---|---|
| ND + 1 | Damage threshold DCRIT |
| ND + 2 | Domain flag for damage coupling (0: no coupling, 1: coupling) |
| ND + 3 | Triaxiality variable, $\eta = \sigma_H/\sigma_M$ |
| ND + 4 | Equivalent plastic strain |
| ND + 5 | Regularization factor for failure strain (determined from LCREGD) |
| ND + 6 | Exponent for stress fading FADEXP |
| ND + 7 | Calculated element size, $l_e$ |
| ND + 8 | Instability measure $F$ |
| ND + 9 | Resultant damage parameter $D$ for element size REFSZ |
| ND + 10 | Resultant damage threshold DCRIT for element size REFSZ |
| ND + 11 | Averaged triaxiality: $\eta_{n+1}^{\text{avg}} = \frac{1}{D_{n+1}}\left(D_n \times \eta_n^{\text{avg}} + (D_{n+1} - D_n) \times \eta_{n+1}\right)$ |
| ND + 12 | Lode parameter value $L$ (only calculated if LCSDG refers to a table) |
| ND + 13 | Alternative damage value: $D^{1/\text{DMGEXP}}$ |
| ND + 14 | Averaged Lode parameter: $L_{n+1}^{\text{avg}} = \frac{1}{D_{n+1}}\left(D_n \times L_n^{\text{avg}} + (D_{n+1} - D_n) \times L_{n+1}\right)$ |
| ND + 15 | MIDFAIL control flag (set to -1 in case mid-plane IP reaches ECRIT/DCRIT) |
| ND + 16 | Number of IPs/layers (NUMFIP > 0/< 0) that must fail before an element gets deleted |
| ND + 17 | Plastic strain value related to reference element size (only if REFSZ < 0) |
| ND + 18 | Effective damage value (stress scaling factor) |
| ND + 19 | History variable for 3D table LCSDG (only if HISVN ≠ 0) |
| ND + 20 | Random scale factor on failure strain (only if STOCHASTIC option is used) |

## *MAT_ADD_EROSION

Many of the available constitutive models do not allow failure and erosion. The ADD_-EROSION option provides a way of including failure in these models. This option can also be applied to constitutive models that already include other failure/erosion criteria.

Each of the failure criteria defined here is applied independently. Deletion of the element from the calculation occurs upon satisfaction of a sufficient number of the specified criteria (see NCS on Card 1).

This keyword applies to nonlinear element formulations, including 2D continuum elements, beam formulations 1 and 11, 3D shell elements (including isogeometric shells), 3D thick shell elements, 3D solid elements (including isogeometric solids), and SPH.

Damage models GISSMO and DIEM are still available using IDAM on Card 3 for backward compatibility. The keywords *MAT_ADD_DAMAGE_DIEM and *MAT_ADD_-DAMAGE_GISSMO are preferable methods for adding damage. A combination of *MAT_ADD_EROSION failure criteria with damage from *MAT_ADD_DAMAGE_-DIEM/GISSMO is possible as long as IDAM = 0 is used.

> **NOTE:** To disable all *MAT_ADD_EROSION commands in a model, use *CONTROL_MAT.

### Card Summary:

**Card 1.** This card is required.

| MID | EXCL | MXPRES | MNEPS | EFFEPS | VOLEPS | NUMFIP | NCS |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| MNPRES | SIGP1 | SIGVM | MXEPS | EPSSH | SIGTH | IMPULSE | FAILTM |
|---|---|---|---|---|---|---|---|

***Card 3.*** This card is optional.

| IDAM | | | | | | | LCREGD |
|---|---|---|---|---|---|---|---|

***Card 4.*** This card is optional.

| LCFLD | NSFF | EPSTHIN | ENGCRT | RADCRT | LCEPS12 | LCEPS13 | LCEPSMX |
|---|---|---|---|---|---|---|---|

***Card 5.*** This card is optional.

| DTEFLT | VOLFRAC | MXTMP | DTMIN | FFUNC | DIAGR | EDGER | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | EXCL | MXPRES | MNEPS | EFFEPS | VOLEPS | NUMFIP | NCS |
| Type | A | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.0 | 1.0/0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification for which this erosion definition applies. A unique number or label must be specified (see *PART). |
| EXCL | The exclusion number (default value of 0.0 is recommended). For any failure value in *MAT_ADD_EROSION which is set to this exclusion number, the associated failure criterion is not invoked. Or in other words, only the failure values which are not set to the exclusion number are invoked. The default value of EXCL (0.0) eliminates from consideration any failure criterion whose failure value is left blank or set to 0.0.<br>As an example, to prevent a material from developing tensile pressure, you could specify an unusual value for the exclusion number, such as 1234, set MNPRES to 0.0, and set all the other failure values in *MAT_ADD_EROSION to 1234. However, use of an exclusion number in this way is nonessential since the same effect could be achieved without use of the exclusion number by setting MNPRES to a very small negative value and leaving all the other failure values blank (or set to zero). |
| MXPRES | Maximum pressure at failure, $P_{\max}$. If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files. |
| MNEPS | Minimum principal strain at failure, $\varepsilon_{\min}$. If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files. |
| EFFEPS | Maximum effective strain at failure:<br>$$\varepsilon_{\text{eff}} = \sum_{ij} \sqrt{\frac{2}{3}\varepsilon_{ij}^{\text{dev}}\varepsilon_{ij}^{\text{dev}}}.$$ |

| VARIABLE | DESCRIPTION |
|---|---|
| | If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files. If the value is negative, then \|EFFEPS\| is the effective plastic strain at failure. In combination with cohesive elements, EFFEPS is the maximum effective in-plane strain. |
| VOLEPS | Volumetric strain at failure, $\varepsilon_{\text{vol}} = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ , or $\ln(\text{relative volume})$ . VOLEPS can be a positive or negative number depending on whether the failure is in tension or compression, respectively. If the value is exactly zero, it is automatically excluded to maintain compatibility with old input files. |
| NUMFIP | Number or percentage of failed integration points prior to element deletion (default is 1). See Remark 2. NUMFIP does not apply to higher order solid element types 24, 25, 26, 27, 28, and 29, rather see the variable VOLFRAC. Also, when the material is a composite defined with *PART_COMPOSITE with different materials through-the-thickness, this field should not be used; use *DEFINE_ELEMENT_EROSION instead.<br>GT.0.0: Number of integration points which must fail before element is deleted<br>LT.0.0: Applies only to shells and beams. \|NUMFIP\| is the percentage of integration points which must exceed the failure criterion before the element fails. If NUMFIP < -100, then \|NUMFIP\|-100 is the number of failed integration points prior to element deletion. |
| NCS | Number of failure conditions to satisfy before failure occurs. For example, if SIGP1 and SIGVM are defined and if NCS = 2, both failure criteria must be met before element deletion can occur. The default is set to unity. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MNPRES | SIGP1 | SIGVM | MXEPS | EPSSH | SIGTH | IMPULSE | FAILTM |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MNPRES | Minimum pressure at failure, $P_{\min}$. |
| SIGP1 | Maximum principal stress at failure, $\sigma_{\max}$<br>LT.0: -SIGP1 is a load curve ID giving the maximum principal stress at failure as a function of the effective strain rate (the curve should not extrapolate to zero or failure may occur at low strain). A filter can be applied to the effective strain rate according to DTEFLT (see Card 5). |
| SIGVM | Equivalent stress at failure, $\overline{\sigma}_{\max}$<br>LT.0: -SIGVM is a load curve ID giving the equivalent stress at failure as a function of the effective strain rate (the curve should not extrapolate to zero or failure may occur at low strain). A filter can be applied to the effective strain rate according to DTEFLT (see Card 5). |
| MXEPS | Variable to invoke a failure criterion based on maximum principal strain.<br>GT.0.0: Maximum principal strain at failure, $\varepsilon_{\max}$<br>LT.0.0: -MXEPS is the ID of a curve giving maximum principal strain at failure as a function of effective strain rate (the curve should not extrapolate to zero or failure may occur at low strain). A filter is applied to the effective strain rate according to DTEFLT (see Card 5). |
| EPSSH | Tensorial shear strain at failure, $\gamma_{\max}/2$ |
| SIGTH | Threshold stress, $\sigma_0$ |
| IMPULSE | Stress impulse for failure, $K_{\mathrm{f}}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| FAILTM | Failure time. When the problem time exceeds the failure time, the material is removed.<br>GT.0: Failure time is active during any phase of the analysis.<br>LT.0: Failure time is set to \|FAILTM\|. This criterion in inactive during the dynamic relaxation phase. |

**Damage Model Card.** The following card is optional.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDAM | | | | | | | LCREGD |
| Type | A8 | | | | | | | F |
| Default | 0.0 | | | | | | | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| IDAM | Flag for damage model.<br>EQ.0: No damage model is used.<br>NE.0: Damage models GISSMO or DIEM, see manuals of R10 and before. Still available here for backward compatibility (see preferred keywords *MAT_ADD_DAMAGE_DIEM/GISSMO as of R11). |
| LCREGD | Load curve ID defining element size dependent regularization factors. This feature can be used with the standard failure criteria of Cards 1 (MXPRES, MNEPS, EFFEPS, VOLEPS), 2 (MNPRES, SIGP1, SIGVM, MXEPS, EPSSH, IMPULSE) and 4 (LCFLD, EPSTHIN). |

**Additional Failure Criteria Card.** This card is optional.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCFLD | NSFF | EPSTHIN | ENGCRT | RADCRT | LCEPS12 | LCEPS13 | LCEPSMX |
| Type | F | F | F | F | F | I | I | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCFLD | Load curve ID, table ID, or 3d table ID. Load curve defines the Forming Limit Diagram, where minor engineering strains in percent are defined as abscissa values and major engineering strains in percent are defined as ordinate values. Table defines for each strain rate (LCFLD > 0) or for each shell thickness (LCFLD < 0) an associated FLD curve. The 3D table defines major strain as a function of temperature (TABLE_3D), strain rate (TABLE), and minor strain (CURVE). The forming limit diagram is shown in Figure M39-1. When defining the curve, list pairs of minor and major strains, starting with the leftmost point and ending with the rightmost point. This criterion is only available for shell elements. |
| NSFF | Number of explicit time step cycles for stress fade-out used in the LCFLD criterion. The default is 10. |
| EPSTHIN | Thinning strain at failure for thin and thick shells.<br>GT.0.0: Individual thinning for each integration point from $z$-strain<br>LT.0.0: Averaged thinning strain from element thickness change |
| ENGCRT | Critical energy for nonlocal failure criterion; see Remark 1i below. |
| RADCRT | Critical radius for nonlocal failure criterion; see Remark 1i below. |
| LCEPS12 | Load curve ID defining in-plane shear strain limit $\gamma_{12}^c$ as a function of element size. See Remark 1j. |
| LCEPS13 | Load curve ID defining through-thickness shear strain limit $\gamma_{13}^c$ as a function of element size. See Remark 1j. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCEPSMX | Load curve ID defining in-plane major strain limit $\varepsilon_1^c$ as a function of element size. See Remark 1j. |

**Additional Failure Criteria Card.** This card is optional.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DTEFLT | VOLFRAC | MXTMP | DTMIN | FFUNC | DIAGR | EDGER | |
| Type | F | F | F | F | I | F | F | |
| Default | ↓ | 0.5 | optional | optional | 0 | optional | optional | |

| VARIABLE | DESCRIPTION |
|---|---|
| DTEFLT | The time period (or inverse of the cutoff frequency) for the low-pass filter applied to the effective strain rate when SIGP1, SIGVM, or MXEPS is negative. If DTEFLT is set to zero or left blank, no filtering of the effective strain rate is performed. |
| VOLFRAC | The volume fraction required to fail before the element is deleted. The default is 0.5. It is used for higher order solid element types 24, 25, 26, 27, 28, and 29, and all isogeometric solids and shell elements. See Remark 4. |
| MXTMP | Maximum temperature at failure |
| DTMIN | Minimum time step size at failure |
| FFUNC | Function ID (*DEFINE_FUNCTION) for a failure function. See Remark 5. |
| DIAGR | Maximum ratio of the largest to the smallest element diagonal in the current and original configurations. This field is used to detect extreme distortions and eliminate such elements. It is only available for solid elements. |
| EDGER | Maximum ratio of the largest to the smallest element edge when comparing the current and original configurations. This field is used to detect extreme distortions and eliminate such elements. It is only available for solid elements. |

**Remarks:**

1. **Failure criteria.** In addition to failure time, supported criteria for failure are:

   a) $P \geq P_{\max}$, where $P$ is the pressure (positive in compression), and $P_{\max}$ is the maximum pressure at failure

   b) $\varepsilon_3 \leq \varepsilon_{\min}$, where $\varepsilon_3$ is the minimum principal strain, and $\varepsilon_{\min}$ is the minimum principal strain at failure

   c) $P \leq P_{\min}$, where $P$ is the pressure (positive in compression), and $P_{\min}$ is the minimum pressure at failure

   d) $\sigma_1 \geq \sigma_{\max}$, where $\sigma_1$ is the maximum principal stress, and $\sigma_{\max}$ is the maximum principal stress at failure

   e) $\sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}} \geq \overline{\sigma}_{\max}$, where $\sigma'_{ij}$ are the deviatoric stress components, and $\overline{\sigma}_{\max}$ is the equivalent stress at failure

   f) $\varepsilon_1 \geq \varepsilon_{\max}$, where $\varepsilon_1$ is the maximum principal strain, and $\varepsilon_{\max}$ is the maximum principal strain at failure

   g) $\gamma_1 \geq \gamma_{\max}/2$, where $\gamma_1$ is the maximum tensorial shear strain = $(\varepsilon_1 - \varepsilon_3)/2$, and $\gamma_{\max}$ is the engineering shear strain at failure

   h) The Tuler-Butcher criterion,

   $$\int_0^t [\max(0, \sigma_1 - \sigma_0)]^2 \mathrm{dt} \geq \mathrm{K_f} \ ,$$

   where $\sigma_1$ is the maximum principal stress, $\sigma_0$ is a specified threshold stress, $\sigma_1 \geq \sigma_0 \geq 0$, and $\mathrm{K_f}$ is the stress impulse for failure. Stress values below the threshold value are too low to cause fracture even for very long duration loadings.

   i) A nonlocal failure criterion which is mainly intended for windshield impact can be defined using ENGCRT, RADCRT, and one additional "main" failure criterion (only SIGP1 is available at the moment). All three parameters should be defined for one part, namely, the windshield glass, and the glass should be discretized with shell elements. The course of events of this nonlocal failure model is as follows: If the main failure criterion SIGP1 is fulfilled, the corresponding element is flagged as the center of impact, but no element erosion takes place yet. Then, the internal energy of shells inside a circle, defined by RADCRT, around the center of impact is tested against the product of the given critical energy ENGCRT and the "area factor". The area factor is defined as,

$$\text{Area Factor} = \frac{\text{total area of shell elements found inside the circle}}{2\pi \times \text{RADCRT}^2}$$

The reason for having two times the circle area in the denominator is that we expect two layers of shell elements, as would typically be the case for laminated windshield glass. If this energy criterion is exceeded, all elements of the part are now allowed to be eroded by the main failure criterion.

Up through version R14.0, this nonlocal energy criterion could only be used once in a model. This was based on the assumption that one calculates the head impact on a glass pane, where both pane layers (inner and outer) were united in one part. The factor "2" in the above formula comes from this assumption.

In subsequent versions (R14.1, R15, ...). it is possible to define the energy criterion for each part separately, meaning as often as desired in a model. The criterion can be defined with either *MAT_ADD_EROSION or *MAT_280. This could be used, for example, to assign different values for ENGCRT and RADCRT to the inner and outer glass layers or in even more general cases. We kept the factor "2" in the formula for the Area Factor to not falsify old results.

j) An element size dependent mixed-mode fracture criterion (MMFC) can be defined for shell elements using load curves LCEPS12, LCEPS23, and LCEPSMX. Failure happens if NCS (see Card 1) of these three criteria are met

$$\begin{aligned}
&\text{LCEPS12:} && \gamma_{12} = \tfrac{1}{2}(\varepsilon_1 - \varepsilon_2) \geq \gamma_{12}^c(l_e) && \text{if} && -2.0 \leq \varepsilon_2/\varepsilon_1 \leq -0.5 \\
&\text{LCEPS13:} && \gamma_{13} = \tfrac{1}{2}(\varepsilon_1 - \varepsilon_3) \geq \gamma_{13}^c(l_e) && \text{if} && -0.5 \leq \varepsilon_2/\varepsilon_1 \leq 1.0 \\
&\text{LCEPSMX:} && \varepsilon_1 \geq \varepsilon_1^c(l_e) && \text{if} && -0.5 \leq \varepsilon_2/\varepsilon_1 \leq 1.0
\end{aligned}$$

where $\gamma_{12}$ and $\gamma_{13}$ are in-plane and through-thickness shear strains, $\varepsilon_1$ and $\varepsilon_2$ are in-plane major and minor strains, and $\varepsilon_3$ is the through-thickness strain. The characteristic element size is $l_e$ and it is computed as the square root of the shell element area. More details can be found in Zhu & Zhu (2011).

2. **NUMFIP.** Element erosion depends on the type of element and the value of NUMFIP.

   a) When NUMFIP > 0, elements erode when NUMFIP points fail.

   b) For shells only, when -100 ≤ NUMFIP < 0, elements erode when |NUMFIP| percent of the integration points fail.

   c) For shells only, when NUMFIP < -100, elements erode when |NUMFIP| − 100 integration points fail.

For NUMFIP > 0 and -100 ≤ NUMFIP < 0, layers retain full strength until the element is eroded. For NUMFIP < -100, the stress at an integration point immediately drops to zero when failure is detected at that integration point.

3. **Instability.** If the keyword *DEFINE_MATERIAL_HISTORIES is used to output the instability, the following table gives a summary of the output properties. Currently only failure values based on the first two cards of this keyword are supported but others can be added on request; for the unsupported options the output will be zero. The instability value is defined as the quantity of interest divided by its corresponding upper limit (restricted to be positive).

**DEFINE_MATERIAL_HISTORIES Properties*

| Label | Attributes | | | | Description |
|---|---|---|---|---|---|
| Instability | - | - | - | - | Maximum of the ones listed below |
| Instability | -1 | - | - | - | $P$/MXPRES |
| Instability | -2 | - | - | - | $\varepsilon_3$/MNEPS |
| Instability | -3 | - | - | - | $\varepsilon_p$/EFFEPS |
| Instability | -4 | - | - | - | $\varepsilon_{\text{vol}}$/VOLEPS |
| Instability | -5 | - | - | - | $P$/MNPRES |
| Instability | -6 | - | - | - | $\sigma_1$/SIGP1 |
| Instability | -7 | - | - | - | $\sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$/SIGVM |
| Instability | -8 | - | - | - | $\varepsilon_1$/MXEPS |
| Instability | -9 | - | - | - | $\gamma_1$/EPSSH |
| Instability | -10 | - | - | - | $\int_0^t [\max(0, \sigma_1 - \sigma_0)]^2 dt$/IMPULSE |
| Instability | -12 | - | - | - | $t$/FAILTM |

4. **VOLFRAC.** The volumes associated with individual integration points in higher order finite elements and isogeometric elements vary widely. Thus, the number of failed integration points is not a reliable criterion for determining element failure. Using the volume fraction of the failed material for these types of elements leads to a more stable and consistent reponse.

5. **FFUNC.** A failure function can be defined with *DEFINE_FUNCTION. Arguments to the function include stresses, strains, strain rates, and a few other quantities; see the example below. The return value of the function determines whether the integration point is marked as not failed (< 1) or failed (≥ 1). The following example leads to the same result as VOLEPS = 0.5:

```
*DEFINE_FUNCTION
100
float ffunc(
$ stress components xx, yy, zz, xy, yz, zx
  float s1, float s2, float s3, float s4, float s5, float s6,
$ strain components xx, yy, zz, xy, yz, zx
  float e1, float e2, float e3, float e4, float e5, float e6,
$ strain rate components xx, yy, zz, xy, yz, zx
  float r1, float r2, float r3, float r4, float r5, float r6,
$ time, plastic strain, effective strain rate, temperature
  float time, float eqp, float effsr, float temp,
$ element id, integration point id, characteristic element length
  float eid, float ipt, float clen,
$ internal energy
  float eng)
{
$ declarations
  float dmg,evol,evolcrit;
$ critical value
  evolcrit = 0.5;
$ volumetric strain
  evol = e1+e2+e3;
$ failure criterion
  dmg = evol/evolcrit;
$ debug output
  if (dmg >= 1.0) {
    printf("ELEMENT %d FAILED AT t=%.7e",int(eid),time);
  }
$ return value
  return dmg;
}
```

A function provides the opportunity to define an arbitrary failure criterion based on the given input arguments. The argument list for the function must include all the listed arguments in the order given in the example.

## *MAT_ADD_EXTVAR_EXPANSION

The ADD_EXTVAR_EXPANSION option adds an expansion property to an (arbitrary) material model in LS-DYNA. The expansion is controlled by the state of an external variable defined with *LOAD_EXTERNAL_VARIABLE. This option currently applies to hypoelastic material models. It is supported for solid element types -2, -1, 1, 2, and 10 and shell element types -16, 2, and 16.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PID | LCID | MULT | LCIDY | MULTY | LCIDZ | MULTZ | IDEV |
| Type | I | I | F | I | F | I | F | I |
| Default | none | 0 | 1.0 | LCID | MULT | LCID | MULT | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| PID | Part ID for which the expansion property applies |
| LCID | For isotropic material models, LCID is the load curve ID defining the expansion coefficient, $\gamma(\alpha)$, as a function of the external variable, $\alpha$. In this case, LCIDY, MULTY, LCIDZ, and MULTZ are ignored. For anisotropic material models, LCID and MULT define the expansion coefficient in the local material $a$-direction. In either case, if LCID is zero, the expansion coefficient is constant and equal to MULT. |
| MULT | Scale factor scaling load curve given by LCID |
| LCIDY | Load curve ID defining the expansion coefficient in the local material $b$-direction as a function of the external variable. If zero, the expansion coefficient in the local material $b$-direction is constant and equal to MULTY. If MULTY = 0.0 as well, LCID and MULT specify the expansion coefficient in the local material $b$-direction. |
| MULTY | Scale factor scaling load curve given by LCIDY |
| LCIDZ | Load curve ID defining the expansion coefficient in the local material $c$-direction as a function of the external variable If zero, the expansion coefficient in the local material $c$-direction is constant and equal to MULTZ. If MULTZ = 0.0 as well, LCID and MULT specify the expansion coefficient in the local material $c$-direction. |

| VARIABLE | DESCRIPTION |
|---|---|
| MULTZ | Scale factor scaling load curve given by LCIDZ |
| IDEV | External variable ID |

**Remarks:**

When invoking the isotropic external variable expansion property (no local $y$ and $z$ parameters) for a material, the stress update is based on the elastic strain rates $\dot{\varepsilon}_{ij}^{e}$. Those are calculated based on the total strain rate $\dot{\varepsilon}_{ij}$, the value $\alpha$ of the external variable and its rate $\dot{\alpha}$:

$$\dot{\varepsilon}_{ij}^{e} = \dot{\varepsilon}_{ij} - \gamma(\alpha)\dot{\alpha} \times \delta_{ij}$$

with expansion coefficient $\gamma$.

For orthotropic properties, which apply only to materials with anisotropy, this equation is generalized to

$$\dot{\varepsilon}_{ij}^{e} = \dot{\varepsilon}_{ij} - \gamma_k(\alpha)\dot{\alpha}\, q_{ik}q_{jk} \quad .$$

Here $q_{ij}$ represents the matrix with material directions with respect to the current configuration.

## *MAT_ADD_FATIGUE_{OPTION}

Available options include:

<BLANK>

EN

The ADD_FATIGUE option defines the S-N or the E-N (with option EN) fatigue property of a material model.

### Card Summary:

**Card 1a.** Include this card if no keyword option (<BLANK>) is used and LCID > 0.

| MID | LCID | LTYPE | | | | SNLIMT | SNTYPE |
|---|---|---|---|---|---|---|---|

**Card 1b.** Include this card if no keyword option (<BLANK>) is used and LCID < 0.

| MID | LCID | LTYPE | A | B | STHRES | SNLIMT | SNTYPE |
|---|---|---|---|---|---|---|---|

**Card 1c.** Include this card if the keyword option EN is used.

| MID | KP | NP | SIGMAF | EPSP | BP | CP | |
|---|---|---|---|---|---|---|---|

***Card 2a.*** This card is read if no keyword option (<BLANK>) is used and LCID < 0. Include one card for each additional S-N curve segment. Between zero and seven of these cards may be included in the deck. This input ends at the next keyword ("*") card.

| | | | A*i* | B*i* | STHRES*i* | | |
|---|---|---|---|---|---|---|---|

***Card 2b.*** This card is read if the keyword option EN is used. Card 2b is not needed if E and PR have been defined in the original material card.

| E | PR | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCID | LTYPE | | | | SNLIMT | SNTYPE |
| Type | A | I | I | | | | I | I |
| Default | none | -1 | 0 | | | | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the fatigue property applies |
| LCID | S-N fatigue curve ID:<br>GT.0: S-N fatigue curve ID |
| LTYPE | Type of S-N curve:<br>EQ.0: Semi-log interpolation (default)<br>EQ.1: Log-log interpolation<br>EQ.2: Linear-linear interpolation |
| SNLIMT | SNLIMNT determines the algorithm used when stress is lower than the lowest stress on S-N curve.<br>EQ.0: Use the life at the last point on S-N curve<br>EQ.1: Extrapolation from the last two points on S-N curve<br>EQ.2: Infinity |
| SNTYPE | Stress type of S-N curve:<br>EQ.0: Stress range (default)<br>EQ.1: Stress amplitude |

| Card 1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCID | LTYPE | A | B | STHRES | SNLIMT | SNTYPE |
| Type | A | I | I | F | F | F | I | I |
| Default | none | -1 | 0 | 0.0 | 0.0 | none | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the fatigue property applies |
| LCID | S-N fatigue curve ID:<br>EQ.-1: S-N fatigue curve uses equation $NS^b = a$<br>EQ.-2: S-N fatigue curve uses equation $\log(S) = a - b\log(N)$<br>EQ.-3: S-N fatigue curve uses equation $S = a\,N^b$<br>EQ.-4: S-N fatigue curve uses equation $S = a - b\log(N)$ |
| LTYPE | Type of S-N curve:<br>EQ.0: Semi-log interpolation (default)<br>EQ.1: Log-log interpolation<br>EQ.2: Linear-linear interpolation |
| A | Material parameter $a$ in S-N fatigue equation |
| B | Material parameter $b$ in S-N fatigue equation |
| STHRES | Fatigue threshold stress |
| SNLIMT | SNLIMIT determines the algorithm used when stress is lower than STHRES.<br>EQ.0: Use the life at STHRES<br>EQ.1: *Ignored*<br>EQ.2: Infinity |
| SNTYPE | Stress type of S-N curve.<br>EQ.0: Stress range (default)<br>EQ.1: Stress amplitude |

| Card 1c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | KP | NP | SIGMAF | EPSP | BP | CP | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification for which the fatigue property applies |
| KP | $K'$, the cyclic strength coefficient |
| NP | $N'$, the cyclic strain hardening exponent |
| SIGMAP | $\sigma_f'$, the fatigue strength coefficient |
| EPSP | $\varepsilon_f'$, the fatigue ductility coefficient |
| BP | $b'$, the fatigue strength exponent (Basquin's exponent) |
| CP | $c'$, the fatigue ductility exponent (Coffin-Manson exponent) |

**S-N Curve Segment Cards.** Include one card for each additional S-N curve segment. Between zero and seven of these cards may be included in the deck. This input ends at the next keyword ("*") card.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A*i* | B*i* | STHRES*i* | | |
| Type | | | | F | F | F | | |
| Default | | | | 0.0 | 0.0 | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A*i* | Material parameter $a$ in S-N fatigue equation for the $i^{th}$ segment |
| B*i* | Material parameter $b$ in S-N fatigue equation for the $i^{th}$ segment |

| VARIABLE | DESCRIPTION |
|---|---|
| STHRES*i* | Fatigue threshold stress for the $i^{th}$ segment which acts as the lower stress limit of that segment |

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E | PR | | | | | | |
| Type | F | F | | | | | | |
| Default | none | none | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus |
| PR | Poisson's ratio |

**Remarks:**

1. **S-N curves.** For fatigue analysis based on stress (OPTION = <BLANK>), S-N curves can be defined by *DEFINE_CURVE or by a predefined equation. When they are defined by curves, the abscissa values (the first column under *DEFINE_CURVE) represent N (number of cycles to failure) and the ordinate values (2nd column under *DEFINE_CURVE) represent S (stress). There are 4 different predefined equations:

   a) *LCID* = *-1*:

   $$NS^b = a$$

   b) *LCID* = *-2*:

   $$\log(S) = a - b\log(N)$$

   c) *LCID* = *-3*:

   $$S = a\,N^b$$

   d) *LCID* = *-4*:

   $$S = a - b\log(N)$$

   Here $N$ is the number of cycles for fatigue failure and $S$ is the stress amplitude. Note that the two equations can be converted to each other, with some minor algebraic manipulation on the constants $a$ and $b$.

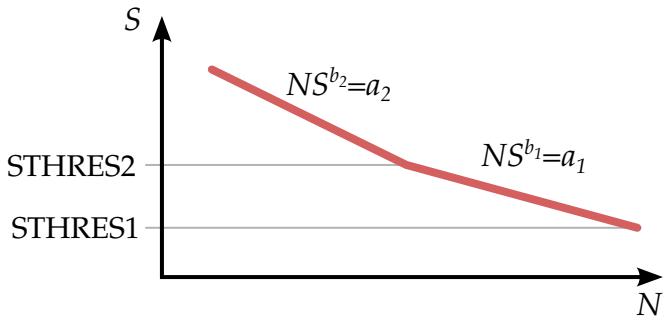



**Figure 2-7.** S-N Curve having multiple slopes

To define an S-N curve with multiple slopes, the S-N curve can be split into multiple segments with each segment defined by a set of parameters A*i*, B*i* and STHRES*i*. Up to 8 sets of the parameters (A*i*, B*i* and STHRES*i*) can be defined. The lower limit of the $i^{th}$ segment is represented by the threshold stress STHRES*i*, as shown in Figure 2-7. This only applies to the case where LCID < 0.

2. **Related keywords.** This model is applicable to frequency domain fatigue analysis, defined by the keywords: *FREQUENCY_DOMAIN_RANDOM_VIBRATION_FATIGUE and *FREQUENCY_DOMAIN_SSD_FATIGUE. It also applies to time domain fatigue analysis, defined by the keyword *FATIGUE (see these keywords for further details).

3. **Strain-based fatigue.** For fatigue analysis based on strain (OPTION = EN), the cyclic stress-strain curve is defined by

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} .$$

The relationship between true local strain amplitude and endurance is

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N)^{b'} + \varepsilon_f'(2N)^{c'} .$$

## *MAT_ADD_GENERALIZED_DAMAGE

This option provides a way of including generalized (tensor type) damage and failure in standard LS-DYNA material models. The basic idea is to apply a general damage model (e.g., GISSMO) using several history variables as damage-driving quantities at the same time. With this feature, it may be possible to obtain, for example, anisotropic damage behavior or separate stress degradation for volumetric and deviatoric deformations. A maximum of three simultaneous damage evolutions (meaning definition of 3 history variables) is possible. A detailed description of this model can be found in Erhart et al. [2017].

This option currently applies to shell element types 1, 2, 3, 4, 16, and 17 and solid element types -2, -1, 1, 2, 3, 4, 10, 13, 15, 16, and 17.

### Card Summary:

**Card 1.** This card is required.

| MID | IDAM | DTYP | REFSZ | NUMFIP | LP2BI | PDDT | NHIS |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| HIS1 | HIS2 | HIS3 | IFLG1 | IFLG2 | IFLG3 | IFLG4 | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| D11 | D22 | D33 | D44 | D55 | D66 | | |
|---|---|---|---|---|---|---|---|

**Card 4a.** Include this card for shell elements

| D12 | D21 | D24 | D42 | D14 | D41 | | |
|---|---|---|---|---|---|---|---|

**Card 4b.** Include this card for solid elements.

| D12 | D21 | D23 | D32 | D13 | D31 | | |
|---|---|---|---|---|---|---|---|

**Card 5.1.** Define NHIS sets of Cards 5.1 and 5.2 (total of 2 × NHIS cards) for each history variable (HIS*n*).

| LCSDG | ECRIT | DMGEXP | DCRIT | FADEXP | LCREG | | |
|---|---|---|---|---|---|---|---|

**Card 5.2.** Define NHIS sets of Cards 5.1 and 5.2 (total of 2 × NHIS cards) for each history variable (HIS*n*).

| LCSRS | SHRF | BIAXF | LCDLIM | MIDFAIL | NFLOC | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | IDAM | DTYP | REFSZ | NUMFIP | LP2BI | PDDT | NHIS |
| Type | A | I | I | F | F | F | I | I |
| Default | none | 0 | 0 | 0.0 | 1.0 | 0.0 | 0 | 1 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which this generalized damage definition applies |
| IDAM | Flag for damage model:<br>EQ.0: No damage model is used.<br>EQ.1: GISSMO damage model |
| DTYP | Flag for damage behavior:<br>EQ.0: Damage is accumulated; no coupling to flow stress, no failure.<br>EQ.1: Damage is accumulated; element failure occurs for D = 1. |
| REFSZ | Reference element size, for which an additional output of damage will be generated. This is necessary to ensure the applicability of resulting damage quantities when transferred to different mesh sizes. |
| NUMFIP | Number of failed integration points prior to element deletion. The default is unity.<br>LT.0: \|NUMFIP\| is the percentage of layers which must fail before the element fails. |
| LP2BI | Option to use a bending indicator instead of the Lode parameter. If active (> 0), the expression "bending indicator" replaces the term "Lode parameter" everywhere in this manual page. We adopted the bending indicator from *MAT_258 (compare with variable Ω). LP2BI > 0 is only available for shell elements and requires NUMFIP = 1.<br>EQ.0.0: Inactive. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Active. Constant regularization (LCREG) applied. |
| | EQ.2.0: Active. Regularization (LCREG) fully applied under pure membrane loading ($\Omega = 0$) but not at all under pure bending ($\Omega = 1$). Linear interpolation in between. |
| PDDT | Pre-defined damage tensors. If non-zero, damage tensor coefficients D11 to D66 on Cards 3 and 4 will be ignored. See Remark 2. |
| | EQ.0: No pre-defined damage tensor is used. |
| | EQ.1: Isotropic damage tensor |
| | EQ.2: 2-parameter isotropic damage tensor for volumetric-deviatoric split |
| | EQ.3: Anisotropic damage tensor as in MAT_104 (FLAG = -1) |
| | EQ.4: 3-parameter damage tensor associated with IFLG1 = 2 |
| NHIS | Number of history variables as driving quantities ($1 \le$ NHIS $\le 3$) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HIS1 | HIS2 | HIS3 | IFLG1 | IFLG2 | IFLG3 | IFLG4 | |
| Type | I | I | I | I | I | I | I | |
| Default | 0 | optional | optional | 0 | 0 | 0 | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| HIS*n* | Choice of variable as driving quantity for damage, called "history value" in the following: |
| | EQ.0: Equivalent plastic strain rate is the driving quantity for the damage if IFLG1 = 0. Alternatively, if IFLG1 = 1, components of the plastic strain rate tensor are driving quantities for damage (see Remarks 2 and 3). |
| | GT.0: The rate of the additional history variable HIS*n* is the driving quantity for damage. IFLG1 should be set to 0. |
| | LT.0: *DEFINE_FUNCTION IDs defining the damage driving quantities as a function of the components of the plastic strain rate tensor; IFLG1 should be set to 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| IFLG1 | Damage driving quantities:<br>EQ.0: Rates of history variables HIS*n*<br>EQ.1: Specific components of the plastic strain rate tensor; see Remarks 2 and 3.<br>EQ.2: Predefined functions of plastic strain rate components for orthotropic damage model. HIS*n* inputs will be ignored, and IFLG2 should be set to 1. This option is available for shell elements only.<br>EQ.3: Specific components of the total strain rate tensor; see Remarks 2 and 3. |
| IFLG2 | Damage strain coordinate system:<br>EQ.0: Local element system (shells) or global system (solids)<br>EQ.1: Material system, only applicable for non-isotropic material models. Supported models for shell elements: all materials with AOPT feature. Supported models for solid elements: 22, 33, 41-50, 58, 103, 122, 126, 133, 157, 199, 233.<br>EQ.2: Principal strain system (rotating)<br>EQ.3: Principal strain system (fixed when instability/coupling starts) |
| IFLG3 | Erosion criteria and damage coupling system:<br>EQ.0: Erosion occurs when one of the damage parameters computed reaches unity; the damage tensor components are based on the individual damage parameters d1 to d3.<br>EQ.1: Erosion occurs when a single damage parameter *D* reaches unity; the damage tensor components are based on this single damage parameter. Results in the isotropic limit case will only be correct if DMGEXP is set to 1.0 for all history variables.<br>EQ.2: Activation of the Domain of Shell-to-Solid Equivalence (DSSE) for shell elements, cf. Pack and Mohr (2017). Two damage variables are necessary for this model (a fracture initiation variable D1 and a localization initiation variable D2). If D1 reaches 1.0, stresses are set to zero and the integration point is no longer able to sustain any load. If D2 = 1.0, no action is taken, and the integration point is still mechanically active. Erosion occurs when at least one |

| VARIABLE | DESCRIPTION |
|---|---|
| | of the two damage variables (D1 or D2) reaches unity for all integration points. Additional required settings for this model: NUMFIP = -100, DCRIT = 1, PDDT = 1, and NFLOC = 0. |
| IFLG4 | Damage drivers' evolution flag. This option is relevant for cyclic loading when IFLG1 is set to 1 or 3. Damage cannot increase with decreasing strain or history variable, but as soon as the strain/history increase again after unloading (i.e., below the previously reached maximum), the damage also increases again (behavior with IFLG4 = 0). This can be prevented with IFLG4 = 1, where the last maximum strain/history is saved. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D11 | D22 | D33 | D44 | D55 | D66 | | |
| Type | I | I | I | I | I | I | | |

**Damage for Shell Elements Card.** This card is included for shell elements.

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D12 | D21 | D24 | D42 | D14 | D41 | | |
| Type | I | I | I | I | I | I | | |

**Damage for Solid Elements.** This card is included for solid elements.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D12 | D21 | D23 | D32 | D13 | D31 | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D*ij* | DEFINE_FUNCTION IDs for damage tensor coefficients; see Remark 2. |

**Damage Definition Cards for IDAM = 1 (GISSMO).** NHIS sets of Cards 5.1 and 5.2 (total of 2 × NHIS cards) must be defined for each history variable (HIS*n*).

| Card 5.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSDG | ECRIT | DMGEXP | DCRIT | FADEXP | LCREG | | |
| Type | I | F | F | F | F | I | | |
| Default | 0 | 0.0 | 1.0 | 0.0 | 1.0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSDG | Load curve ID defining corresponding history value to failure as a function of triaxiality. |
| ECRIT | Critical history value (material instability):<br>LT.0.0: \|ECRIT \| is load curve ID defining critical history value as a function of triaxiality.<br>EQ.0.0: Fixed value DCRIT defining critical damage is read.<br>GT.0.0: Fixed value for stress-state independent critical history value |
| DMGEXP | Exponent for nonlinear damage accumulation |
| DCRIT | Damage threshold value (critical damage). If a load curve of critical history value or fixed value is given by ECRIT, this input is ignored. |
| FADEXP | Exponent for damage-related stress fadeout.<br>LT.0.0: \|FADEXP \| is load curve ID defining element-size dependent fading exponent<br>GT.0.0: Constant fading exponent |
| LCREG | Load curve ID or table ID defining element size dependent regularization factors for history value to failure:<br>GT.0: Load curve ID (regularization factor as a function of element size) or table ID (regularization factor as a function of element size curves indexed by effective strain rate)<br>LT.0: \|LCREGD \| is a table ID for a table indexing regularization |

| VARIABLE | DESCRIPTION |
|---|---|
| | factor as a function of element size curves by triaxiality). This table provides an alternative to the use of SHRF and BIAXF for defining the effect of triaxiality on element size regularization of history value to failure. |

**Damage Definition Cards for IDAM = 1 (GISSMO).** NHIS sets of Cards 5.1 and 5.2 (total of 2 × NHIS cards) must be defined for each history variable (HIS*n*).

| Card 5.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSRS | SHRF | BIAXF | LCDLIM | MIDFAIL | NFLOC | | |
| Type | I | F | F | I | F | F | | |
| Default | 0 | 0.0 | 0.0 | 0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSRS | Load curve ID defining failure history value scaling factor for LCSDG as a function of history value rate. If the first rate value in the curve is negative, it is assumed that all rate values are given as natural logarithm of the history rate.<br>GT.0: Scale ECRIT as well.<br>LT.0: Do not scale ECRIT. |
| SHRF | Reduction factors for regularization at triaxiality = 0 (shear) |
| BIAXF | Reduction factors for regularization at triaxiality = 2/3 (biaxial) |
| LCDLIM | Load curve ID defining damage limit values as a function of triaxiality. Damage can be restricted to values less than 1.0 to prevent further stress reduction and failure for certain triaxialities. |
| MIDFAIL | Mid-plane failure option for shell elements. If active, then critical strain is only checked at the mid-plane integration point, meaning an odd number for NIP should be used. Damage is computed at the other integration points, but no coupling to the stresses is done first. As soon as the mid-plane IP reaches ECRIT/DCRIT, then all the other IPs are also checked (exception: MIDFAIL = 4).<br>EQ.0.0: Inactive |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Active. The stresses immediately begin to reduce for non-mid-plane IPs that are already above their critical value. Coupling only occurs for IPs that reach their criterion. |
| | EQ.2.0: Active. The stresses immediately begin to reduce for all the non-mid-plane IPs. NUMFIP is active. |
| | EQ.3.0: Active. Same as 2, but when D = 1 is reached in the middle integration point, the element is eroded instantaneously. NUMFIP is disregarded. |
| | EQ.4.0: Active. Damage and failure is applied only on the midpoint. When D = 1 on the midpoint, the element is eroded. NUMFIP is disregarded. Integration points away from the midplane see no stress reduction and no failure. |
| NFLOC | Optional "local" number of failed integration points prior to element deletion. Overwrites the definition of NUMFIP for history variable HIS*n*. |

**Remarks:**

1. **Comparison to GISSMO Damage Model.** The GISSMO damage model is described in detail in the remarks of *MAT_ADD_DAMAGE_GISSMO. If NHIS = 1 and HIS1 = 0 is used, this keyword behaves the same as GISSMO. The main difference between this keyword and GISSMO is that up to 3 independent but simultaneous damage evolutions are possible. Therefore, parameters LCSDG, ECRIT, DMGEXP, DCRIT, FADEXP, LCREGD, LCSRS, SHRF, BIAXF, and LCDLIM can be defined separately for each history variable.

2. **Damage Tensor.** The relation between nominal (damaged) stresses $\sigma_{ij}$ and effective (undamaged) stresses $\tilde{\sigma}_{ij}$ is now expressed as

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0\\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0\\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & D_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & D_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & D_{66}\end{bmatrix}\begin{bmatrix}\tilde{\sigma}_{11}\\ \tilde{\sigma}_{22}\\ \tilde{\sigma}_{33}\\ \tilde{\sigma}_{12}\\ \tilde{\sigma}_{23}\\ \tilde{\sigma}_{31}\end{bmatrix}$$

with damage tensor **D**. Each damage tensor coefficient $D_{ij}$ can be defined using *DEFINE_FUNCTION as a function of damage parameters $d_1$ to $d_3$. For simple

isotropic damage driven by plastic strain (NHIS = 1, HIS1 = 0, IFLG1 = IFLG2 = IFLG3 = 0) that would be

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix} = (1-d_1)\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}\begin{bmatrix}\tilde{\sigma}_{11}\\ \tilde{\sigma}_{22}\\ \tilde{\sigma}_{33}\\ \tilde{\sigma}_{12}\\ \tilde{\sigma}_{23}\\ \tilde{\sigma}_{31}\end{bmatrix}$$

That means the following function should be defined for D11 to D66 (Card 3):

```
*DEFINE_FUNCTION
1,D11toD66
func1(d1,d2,d3)=(1.0-d1)
```

and all entries in Card 4 can be left empty or equal to zero in that case.

If GISSMO (IDAM = 1) is used, the damage parameters used in those functions are internally replaced by

$$d_i \rightarrow \left(\frac{d_i - \mathrm{DCRIT}_i}{1-\mathrm{DCRIT}_i}\right)^{\mathrm{FADEXP}_i}$$

In the case of plane stress (shell) elements, coupling between normal stresses and shear stresses is implemented and the damage tensor is defined as below:

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ 0\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix} = \begin{bmatrix}D_{11}&D_{12}&0&D_{14}&0&0\\D_{21}&D_{22}&0&D_{24}&0&0\\0&0&D_{33}&0&0&0\\D_{41}&D_{42}&0&D_{44}&0&0\\0&0&0&0&D_{55}&0\\0&0&0&0&0&D_{66}\end{bmatrix}\begin{bmatrix}\tilde{\sigma}_{11}\\ \tilde{\sigma}_{22}\\ 0\\ \tilde{\sigma}_{12}\\ \tilde{\sigma}_{23}\\ \tilde{\sigma}_{31}\end{bmatrix}$$

Since the evaluation of *DEFINE_FUNCTION for variables D11 to D66 is relatively time consuming, pre-defined damage tensors (PDDT) can be used. Currently the following options are available for shell elements:

| PDDT | Damage Tensor |
|---|---|
| 1 | $(1-D_1)\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\\0&0&0&1&0&0\\0&0&0&0&1&0\\0&0&0&0&0&1\end{bmatrix}$ |

| PDDT | Damage Tensor |
|---|---|
| 2 | $\begin{bmatrix} 1-\frac{2}{3}D_1-\frac{1}{3}D_2 & \frac{1}{3}D_1-\frac{1}{3}D_2 & 0 & 0 & 0 & 0 \\ \frac{1}{3}D_1-\frac{1}{3}D_2 & 1-\frac{2}{3}D_1-\frac{1}{3}D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ |
| 3 | $\begin{bmatrix} 1-D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\frac{1}{2}(D_1+D_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-\frac{1}{2}D_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-\frac{1}{2}D_1 \end{bmatrix}$ |
| 4 | $\begin{bmatrix} 1-D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-D_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ |

and the following ones for solid elements:

| PDDT | Damage Tensor |
|---|---|
| 1 | $(1-D_1)\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ |
| 2 | $\begin{bmatrix} 1-\frac{2}{3}D_1-\frac{1}{3}D_2 & \frac{1}{3}D_1-\frac{1}{3}D_2 & \frac{1}{3}D_1-\frac{1}{3}D_2 & 0 & 0 & 0 \\ \frac{1}{3}D_1-\frac{1}{3}D_2 & 1-\frac{2}{3}D_1-\frac{1}{3}D_2 & \frac{1}{3}D_1-\frac{1}{3}D_2 & 0 & 0 & 0 \\ \frac{1}{3}D_1-\frac{1}{3}D_2 & \frac{1}{3}D_1-\frac{1}{3}D_2 & 1-\frac{2}{3}D_1-\frac{1}{3}D_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-D_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-D_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-D_1 \end{bmatrix}$ |
| 3 | $\begin{bmatrix} 1-D_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-D_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-D_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-(\frac{1}{2})(D_1+D_2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-(\frac{1}{2})(D_2+D_3) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1-(\frac{1}{2})(D_3+D_1) \end{bmatrix}$ |

3. **History Variables.** The increment of the damage parameter is computed in GISSMO based on a driving quantity that has the dimension of a strain rate:

$$\dot{d} = nd^{1-\frac{1}{n}} \frac{\dot{\text{HIS}}_\iota}{\text{epf}}$$

The history variables defined by the user through HIS*i* should thus have the dimension of a strain as the rate is computed internally by MAT_ADD_GENERALIZED_DAMAGE:

$$\dot{\text{HIS}}_\iota = \frac{\text{HIS}_i\left(t^{n+1}\right) - \text{HIS}_i(t^n)}{t^{n+1} - t^n}$$

History variables can either come directly from associated material models (IFLG1 = 0 and HIS*i* > 0), or they can be equivalent to plastic strain rate tensor components (IFLG1 = 1 and HIS*i* = 0):

$$\begin{aligned}
&\dot{\text{HIS}}_1 = \dot{\varepsilon}^p_{xx}, \quad \dot{\text{HIS}}_2 = \dot{\varepsilon}^p_{yy}, \quad \dot{\text{HIS}}_3 = \dot{\varepsilon}^p_{xy} \quad \text{for} \quad \text{IFLG2} = 0\\
&\dot{\text{HIS}}_1 = \dot{\varepsilon}^p_{aa}, \quad \dot{\text{HIS}}_2 = \dot{\varepsilon}^p_{bb}, \quad \dot{\text{HIS}}_3 = \dot{\varepsilon}^p_{ab} \quad \text{for} \quad \text{IFLG2} = 1\\
&\dot{\text{HIS}}_1 = \dot{\varepsilon}^p_{1}, \quad \dot{\text{HIS}}_2 = \dot{\varepsilon}^p_{2}, \quad \dot{\text{HIS}}_3 = 0 \quad \text{for} \quad \text{IFLG2} = 2
\end{aligned}$$

or they can be provided via *DEFINE_FUNCTIONs by the user (IFLG1 = 1 and HIS*i* < 0):

$$\begin{aligned}
&\dot{\text{HIS}}_i = f_i\left(\dot{\varepsilon}^p_{xx}, \dot{\varepsilon}^p_{yy}, \dot{\varepsilon}^p_{zz}, \dot{\varepsilon}^p_{xy}, \dot{\varepsilon}^p_{yz}, \dot{\varepsilon}^p_{zx}\right) \quad \text{for} \quad \text{IFLG2} = 0\\
&\dot{\text{HIS}}_i = f_i\left(\dot{\varepsilon}^p_{aa}, \dot{\varepsilon}^p_{bb}, \dot{\varepsilon}^p_{zz}, \dot{\varepsilon}^p_{ab}, \dot{\varepsilon}^p_{bz}, \dot{\varepsilon}^p_{za}\right) \quad \text{for} \quad \text{IFLG2} = 1\\
&\dot{\text{HIS}}_i = f_i\left(\dot{\varepsilon}^p_{1}, \dot{\varepsilon}^p_{2}\right) \quad \text{for} \quad \text{IFLG2} = 2
\end{aligned}$$

The following example defines a history variable (HIS*i* = -1234) as function of the transverse shear strains in material coordinate system $(a, b, z)$ for shells:

```
*DEFINE_FUNCTION
      1234
fhis1(eaa,ebb,ezz,eab,ebz,eza)=1.1547*sqrt(ebz**2+eza**2)
```

The plastic strain rate tensor is not always available in the material law and is estimated as:

$$\dot{\boldsymbol{\varepsilon}}^p = \frac{\dot{\varepsilon}^p_{eff}}{\dot{\varepsilon}_{eff}}\left[\dot{\boldsymbol{\varepsilon}} - \frac{\dot{\varepsilon}_{vol}}{3}\boldsymbol{\delta}\right]$$

This is a good approximation for isochoric materials with small elastic strains (such as metals) and correct for J2 plasticity.

You can also use the *total* strain rate components $\dot{\varepsilon}_{ij}$ instead of the *plastic* strain rate components $\dot{\varepsilon}^p_{ij}$ by changing IFLG1 = 1 to IFLG1 = 3. Setting IFLG4 = 1 should be considered in that case (see description for IFLG4).

The following table gives an overview of the driving quantities used for incrementing the damage in function of the input parameters (strain superscript "p" for "plastic" is omitted for convenience):

| IFLG1 | IFLG2 | HISi > 0 | HISi = 0 | HISi < 0 |
|---|---|---|---|---|
| 0 | 0 | $(\mathrm{HIS}_i)^{\cdot}$ | $\varepsilon^{\cdot}$ | – |
| 0 | 1 | $(\mathrm{HIS}_i)^{\cdot}$ | – | – |
| 0 | 2 | $(\mathrm{HIS}_i)^{\cdot}$ | – | – |
| 1/3 | 0 | – | $\varepsilon^{\cdot}_{ij}$ | $f(\varepsilon^{\cdot}_{ij})$ |
| 1/3 | 1 | – | $\varepsilon^{\cdot mat}_{ij}$ | $f(\varepsilon^{\cdot mat}_{ij})$ |
| 1/3 | 2 | – | $\varepsilon^{\cdot}_{i}$ | $f(\varepsilon^{\cdot}_{i})$ |
| 2 | 0 | – | – | – |
| 2 | 1 | Preprogrammed functions of plastic strain rate | | |
| 2 | 2 | – | – | – |

4. **Post-Processing History Variables.** History variables of the GENERALIZED_DAMAGE model are written to the post-processing database behind those already occupied by the material model which is used in combination:

| History Variable # | Description |
|---|---|
| ND | Triaxiality variable $\frac{\sigma_H}{\sigma_M}$ |
| ND + 1 | Lode parameter value |
| ND + 2 | Single damage parameter $D$", " $(10^{-20} < D \leq 1)$, only for IFLG3 = 1 |
| ND + 3 | Damage parameter $d_1$ |
| ND + 4 | Damage parameter $d_2$ |
| ND + 5 | Damage parameter $d_3$ |
| ND + 6 | Damage threshold $\mathrm{DCRIT}_1$ |
| ND + 7 | Damage threshold $\mathrm{DCRIT}_2$ |
| ND + 8 | Damage threshold $\mathrm{DCRIT}_3$ |
| ND + 12 | History variable $\mathrm{HIS}_1$ |
| ND + 13 | History variable $\mathrm{HIS}_2$ |
| ND + 14 | History variable $\mathrm{HIS}_3$ |
| ND + 15 | Angle between principal and material axes |

| History Variable # | Description |
|---|---|
| ND + 21 | Characteristic element size (used in LCREG) |

For instance, ND = 6 for *MAT_024, ND = 9 for *MAT_036, and ND = 23 for *MAT_187. Exact information of the variable locations can be found in the d3hsp section "MAGD damage history listing."

## *MAT_ADD_INELASTICITY

The purpose of this card is to add inelasticity features to an arbitrary standard material model. It may either be used as a modular concept on top of a simple elastic model or patching a more complex material model with a missing inelastic feature.

This keyword is under development and currently only applies to shell types 2, 4 and 16, and solid types -18, -2, -1, 1, 2, 10, 15, 16 and 17. Implicit as well as explicit analyses are supported, and the user should be aware of an extra cost associated with using this feature.

### Card Summary:

**Card 1.** This card is required. NIELINKS groups of Cards 4 through 6 should follow this card, possibly after input of anisotropy information in Cards 2 and 3.

| MID | NIELINKS | | G | K | AOPT | MACF | BETA |
|---|---|---|---|---|---|---|---|

**Card 2.** For AOPT > 0, define Cards 2 and 3.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 3.** For AOPT > 0, define Cards 2 and 3.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** For each link, Cards 4 through 6 are required. NIELAWS groups of Cards 5 and 6 should follow immediately after each Card 4.

| NIELAWS | WEIGHT | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** NIELAWS sets of Cards 5 and 6 are required after each Card 4.

| LAW | MODEL | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6a.** This card is required for LAW = 3 with MODEL = 1.

| P1 | P2 | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6b.** This card is required for LAW = 3 with MODEL = 2.

| P1 | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6c.** This card is included for LAW = 5 with MODEL ≤ 2.

| P1 | P2 | P3 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6d.** This card is required for LAW = 5 with MODEL = 3 and for LAW = 6 with MODEL = 4.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | ILC |
|---|---|---|---|---|---|---|---|

**Card 6e.** This card is required for LAW = 5 with MODEL = 4 and for LAW = 6 with MODEL = 5.

| P1 | P2 | P3 | P4 | | | | |
|---|---|---|---|---|---|---|---|

**Card 6f.** This card is required for LAW = 5 with MODEL = 5 and for LAW = 6 with MODEL = 6.

| P1 | P2 | P3 | P4 | ILC | | | |
|---|---|---|---|---|---|---|---|

**Card 6g.** This card is required for LAW = 6 with MODEL ≤ 3.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

**Main Card.** Only one instance of this card is needed. NIELINKS groups of Cards 4 through 6 should follow this card, possibly after input of anisotropy information in Cards 2 and 3.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | NIELINKS | | G | K | AOPT | MACF | BETA |
| Type | A | I | | F | F | F | F | F |
| Default | none | 1 | | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification for which this inelasticity definition applies. A unique number or label must be specified (see *PART). |
| NIELINKS | Number of links/networks/phases specified by the user. An additional link may be added internally if the weights below do not sum up to unity. |
| G | Characteristic shear modulus used for some of the inelasticity models. This should reflect the elastic stiffness for the material |

| VARIABLE | DESCRIPTION |
|---|---|
| | without any inelasticity effects. For instance, if *MAT_ELASTIC is used, set $G = E/(2(1+\nu))$. |
| K | Characteristic bulk modulus used for some of the inelasticity models. This should reflect the elastic stiffness for the material without any inelasticity effects. For instance, if *MAT_ELASTIC is used, set $K = E/(3(1-2\nu))$. |
| AOPT | Material axes option (see *MAT_002 for a detailed description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, $P$, defining the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID. |
| MACF | Material axes change flag for solid elements: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when to apply MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| BETA | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3 (all element types). This angle may be overriden on the element card; see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. |

**Anisotropy cards.** Include Cards 2 and 3 if AOPT > 0.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point, $p$, for AOPT = 1 and 4; see *MAT_002. |
| A1, A2, A3 | Components of vector, **a**, for AOPT = 2; see *MAT_002. |
| V1, V2, V3 | Components of vector, **v**, for AOPT = 3 and 4; see *MAT_002. |
| D1, D2, D3 | Components of vector, **d**, for AOPT = 2; see *MAT_002. |

**Link/network/phase Cards.** Include NIELINKS sets of all cards that follow; NIELAWS groups of Cards 5 and 6 should follow immediately after each Card 4.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NIELAWS | WEIGHT | | | | | | |
| Type | I | F | | | | | | |
| Default | none | 0 or 1 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| NIELAWS | Number of inelasticity laws that apply to this material model at this link, each contributing in its own way to the total inelastic strain (rate) |
| WEIGHT | Weight of this link/network/phase used when computing total stress. |

**Inelasticity model cards.** Include NIELAWS sets of Cards 5 and 6; the Card 6 determined by the law and model selected should follow immediately after Card 5.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LAW | MODEL | | | | | | |
| Type | I | I | | | | | | |
| Default | none | none | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LAW | Inelasticity law. One of the laws listed below must be chosen:<br>EQ.3: Isotropic hardening plasticity<br>EQ.5: Creep<br>EQ.6: Viscoelasticity |
| MODEL | Model definition with choice dependent on the specified law above. A valid combination of law and model must be chosen.<br>For isotropic hardening plasticity (LAW = 3), choices are<br>EQ.1: Linear hardening<br>EQ.2: Hardening from curve/table<br>For creep (LAW = 5), choices are<br>EQ.1: Norton incremental formulation<br>EQ.2: Norton total formulation<br>EQ.3: Norton-Bailey formulation<br>EQ.4: Bergström-Boyce formulation<br>EQ.5: Garofalo formulation<br>For viscoelasticity (LAW = 6), choices are<br>EQ.1: Bulk and shear decay, with optional temperature shifts, hypoelastic version<br>EQ.2: Bulk and shear decay, with optional temperature shifts, hyperelastic version #1<br>EQ.3: Bulk and shear decay, with optional temperature shifts, hyperelastic version #2 |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4: Norton-Bailey formulation |
| | EQ.5: Bergström-Boyce formulation |
| | EQ.6: Garofalo formulation |

**Inelasticity Parameters.** This card is included for LAW = 3 with MODEL = 1.

| Card 6a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Virgin yield stress, $\sigma_0$ |
| P2 | Hardening, $H$ |

**Inelasticity Parameters.** This card is included for LAW = 3 with MODEL = 2.

| Card 6b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | | | | | | | |
| Type | I | | | | | | | |
| Default | 0 | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Curve or table ID that defines the hardening |

**Inelasticity Parameters.** This card is included for LAW = 5 with MODEL ≤ 2.

| Card 6c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | 0.0 | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Norton creep parameter, $A$ |
| P2 | Norton creep parameter, $m$ |
| P3 | Norton creep parameter, $n$ |

**Inelasticity Parameters.** This card is included for LAW = 5 with MODEL = 3 and for LAW = 6 with MODEL = 4.

| Card 6d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | ILC |
| Type | F | F | F | F | F | F | F | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Norton-Bailey creep parameter, $A$ |
| P2 | Norton-Bailey creep parameter, $\sigma_0$ |
| P3 | Norton-Bailey creep parameter, $n$ |
| P4 | Norton-Bailey creep parameter, $T_0$ |
| P5 | Norton-Bailey creep parameter, $p$ |
| P6 | Norton-Bailey creep parameter, $m$ |

| VARIABLE | DESCRIPTION |
|---|---|
| P7 | Norton-Bailey creep parameter, $\varepsilon_0$ |
| ILC | Flag to determine whether P1 through P7 on this card are constants or refer to a load curve ID:<br>EQ.0: P1 through P7 are constants.<br>EQ.1: P1 through P7 refer to load curve IDs. Each curve is the respective parameter as a function temperature. |

**Inelasticity Parameters.** This card is included for LAW = 5 with MODEL = 4 and for LAW = 6 with MODEL = 5.

| Card 6e | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | | | | |
| Type | F | F | F | F | | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Bergström-Boyce creep parameter, $A$ |
| P2 | Bergström-Boyce creep parameter, $m$ |
| P3 | Bergström-Boyce creep parameter, $C$ |
| P4 | Bergström-Boyce creep parameter, $E$ |

**Inelasticity Parameters.** This card is included for LAW = 5 with MODEL = 5 and for LAW = 6 with MODEL = 6.

| Card 6f | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | ILC | | | |
| Type | F | F | F | F | I | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Garofalo creep parameter, $A$ |
| P2 | Garofalo creep parameter, $B$ |
| P3 | Garofalo creep parameter, $m$ |
| P4 | Garofalo creep parameter, $Q$ |
| ILC | Flag to determine whether P1 through P4 on this card are constants or refer to a load curve ID:<br>EQ.0: P1 through P4 are constants.<br>EQ.1: P1 through P4 refer to load curve IDs. Each curve is the respective parameter as a function temperature. |

**Inelasticity Parameters.** This card is included for LAW = 6 with MODEL = 1, 2, or 3.

| Card 6g | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | Shear decay coefficient, $\beta_G$ |
| P2 | Bulk decay coefficient, $\beta_K$ |
| P3 | Shear reference temperature, $T_G$ |
| P4 | Shear shift coefficient, $A_G$ |
| P5 | Shear shift coefficient, $B_G$ |
| P6 | Bulk reference temperature, $T_K$ |
| P7 | Bulk shift coefficient, $A_K$ |
| P8 | Bulk shift coefficient, $B_K$ |

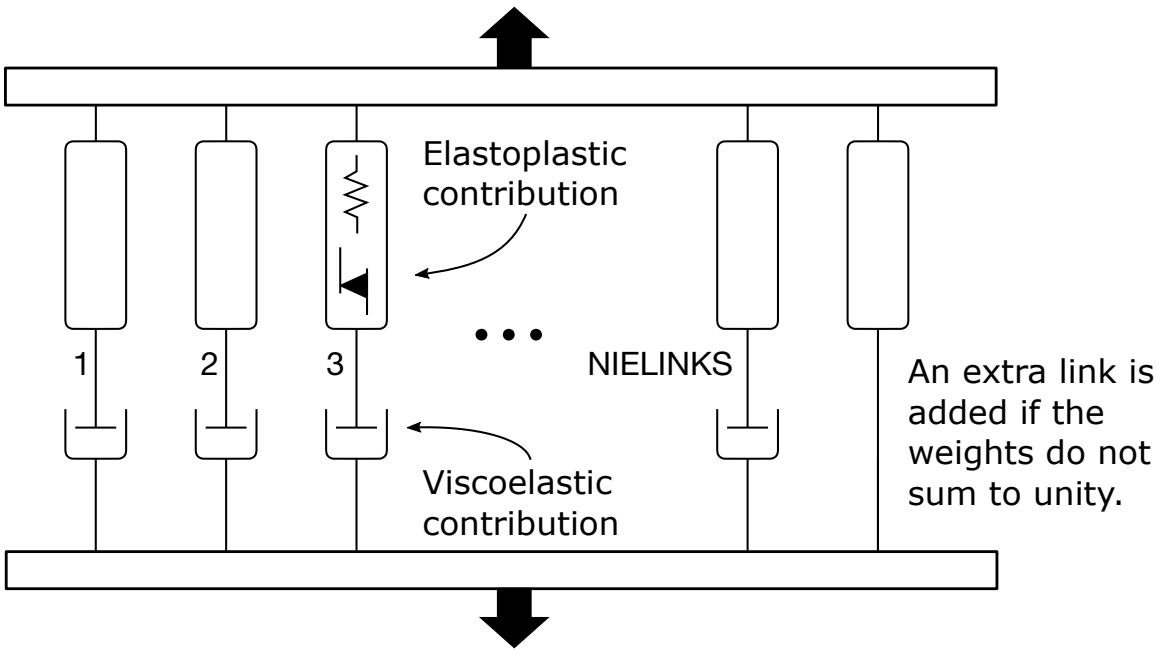



**Figure 2-8.** Schematic view of how inelasticity is added to the model.

**Remarks:**

**General**

The resulting stress from an integration point with inelasticities is the sum of the stress $\sigma_I$ from each link, weighed by its weight, $w_I$ (see WEIGHT above). A link in this context can also be referred to as a network or a phase, depending on the physical interpretation, and we use the subscript $I$ to refer to a specific one. So, the stress, $\sigma$, is in the end given by

$$\sigma = \sum_{I=1}^{\text{NIELINKS}(+1)} w_I \sigma_I \ .$$

The data for the links are specified by the user, except for a possible last one which is internally created if the weights do not sum to unity (whence the +1 in the number of terms in the sum above). This last link will get its stress $\sigma_{\text{NIELINKS}+1}$ only from the material model without any inelasticities, and its weight will be

$$w_{\text{NIELINKS}+1} = 1 - \sum_{I=1}^{\text{NIELINKS}} w_I \ ,$$

that is, just enough for the total weight to sum to 1. The stress for each link will be treated next, for which we drop the subscript $I$ for the sake of clarity, and emphasize that this first part will only treat creep and plasticity, since viscoelasticity is somewhat different and explained on its own at the end of this section.

**A single link/network/phase**

*Infinitesimal description*

The inelasticity feature assumes that the strain or strain rate is somehow decomposed into an elastic and inelastic part. This decomposition is in general not trivial and depends upon the underlying material model, but to make things simple we can begin by restricting ourselves to a small deformation context. In this case the decomposition is *additive*, so

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{\mathrm{e}} + \boldsymbol{\varepsilon}_{\mathrm{i}} \ ,$$

where $\boldsymbol{\varepsilon}$ is the *total* (given) strain, $\boldsymbol{\varepsilon}_{\mathrm{e}}$ is the *elastic* strain, and $\boldsymbol{\varepsilon}_{\mathrm{i}}$ is the *inelastic* strain. A material model then amounts to determining the stress for the elastic strain, which can be written as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}(\boldsymbol{\varepsilon}_{\mathrm{e}}) = \boldsymbol{\sigma}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{i}}).$$

The material model used as a basis for this feature, that is, the model indicated by parameter MID above, here acts as the function $\boldsymbol{\sigma}(*)$. If no inelasticity is added to the model, $\boldsymbol{\varepsilon}_{\mathrm{i}} = \mathbf{0}$ and the stress will be given by $\boldsymbol{\sigma}(\boldsymbol{\varepsilon})$. It is simply a plain evaluation of the material model in the absence of this keyword. For linear elasticity, for instance, the function would be given by Hooke's law

$$\boldsymbol{\sigma}(\boldsymbol{\varepsilon}) = \mathbf{C}\boldsymbol{\varepsilon} \ ,$$

where $\mathbf{C}$ is the Hooke elasticity tensor. Needless to say, the material model itself can deal with inelasticities of various kinds, such as plasticity, creep, thermal expansion and viscoelasticity, so the variable $\boldsymbol{\varepsilon}_i$ is restricted to the inelasticities specifically defined here and thus added to whatever is used in the material model. For the sake of generality, we allow the inelastic strain to come from many sources and be combined:

$$\boldsymbol{\varepsilon}_i = \boldsymbol{\varepsilon}_i^1 + \boldsymbol{\varepsilon}_i^2 + \boldsymbol{\varepsilon}_i^3 + \cdots \ .$$

Here each superscript on the right-hand side refers to a specific combination of LAW and MODEL (excluding viscoelastic laws).

*Large strain formulation*

For incrementally updated material models, using hypoelasticity with an objective rate of stress, the exposition above is generalized to large deformations by applying the appropriate time derivative to strains and stresses:

$$\boldsymbol{\varepsilon} \to \mathbf{D}, \qquad \boldsymbol{\varepsilon}_{\mathrm{e}} \to \mathbf{D}_{\mathrm{e}}, \qquad \boldsymbol{\varepsilon}_{\mathrm{i}} \to \mathbf{D}_{\mathrm{i}}, \qquad \boldsymbol{\sigma} \to \boldsymbol{\sigma}^{\nabla}, \ldots$$

Here $\mathbf{D}$ is the rate of deformation tensor, and $\nabla$ indicates an objective time derivative[3]. For now, we restrict the evolution of inelastic strain to be based on a von Mises stress potential:

[3] In LS-DYNA the objective rate is to be understood as the Jaumann rate for solid elements and the rate resulting from the specific co-rotational formulation for shell elements.

$$\mathbf{D}_\mathrm{i}^j = \dot{\varepsilon}_\mathrm{i}^j \frac{\partial \overline{\sigma}}{\partial \sigma} \ ,$$

where

$$\overline{\sigma} = \sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}} \quad \left(\mathbf{s} = \sigma - \frac{1}{3}\sigma:\mathbf{I}\right)$$

is the von Mises effective stress, and $\dot{\varepsilon}_i^j$ is the rate of effective inelastic strain for the MODEL and LAW corresponding to superscript $j$. The constitutive law is thus written as

$$\sigma^\nabla = \sigma^\nabla(\mathbf{D}_\mathrm{e}) = \ \sigma^\nabla(\mathbf{D} - \mathbf{D}_\mathrm{i}) \ .$$

For hyperelastic materials the role of $\mathbf{D}_\mathrm{e}$ is replaced by the *elastic deformation gradient,* $\mathbf{F}_\mathrm{e}$, and instead of a constitutive law for the rate of stress, the total stress is given as

$$\sigma = \sigma(\mathbf{F}_\mathrm{e}) \ .$$

The evolution of the elastic deformation gradient is taken as

$$\dot{\mathbf{F}}_\mathrm{e} = (\mathbf{L} - \mathbf{L}_\mathrm{i})\mathbf{F}_\mathrm{e} \ ,$$

where $\mathbf{L} = \frac{\partial \mathbf{v}}{\partial \mathbf{x}}$ is the *spatial velocity gradient* and $\mathbf{L}_\mathrm{i}$ is the inelastic part. For simplicity, we assume zero plastic spin for all involved features, thus $\mathbf{W}_\mathrm{i} = \mathbf{0}$ and $\mathbf{L}_\mathrm{i} = \mathbf{D}_\mathrm{i}$.

From here, we will give the evolution law of the effective inelastic strain for the available contributions.

**Isotropic hardening (LAW = 3)**

The current yield stress is defined as

$$\sigma_\mathrm{Y} = \begin{cases} \sigma_0 + H\varepsilon_\mathrm{p} & \text{MODEL} = 1 \\ c(\varepsilon_\mathrm{p}, \dot{\varepsilon}_\mathrm{p}) & \text{MODEL} = 2 \end{cases} ,$$

where the inelastic strain is represented by the plastic strain, $\varepsilon_\mathrm{p}$, and $c$ is the curve or table used to evaluate the yield stress. The evolution of plastic strain is given by the KKT condition

$$\overline{\sigma} - \sigma_\mathrm{Y} \le 0, \qquad \dot{\varepsilon}_\mathrm{p} \ge 0, \qquad (\overline{\sigma} - \sigma_\mathrm{Y})\dot{\varepsilon}_\mathrm{p} = 0 \ .$$

In other words, it is the classical von Mises plasticity available in many standard plasticity models; see, for instance, *MAT_PIECEWISE_LINEAR_PLASTICITY (*MAT_024). As an example, materials 1 and 2 below are equivalent.

```
*MAT_ELASTIC
$      mid        ro         e        pr
         1    7.8e-9  210000.0       0.3
*MAT_ADD_INELASTICITY
$      mid
         1
```

```
$  nielaws
         1
$      law     model
         3         2
$      cid
         1
*MAT_PIECEWISE_LINEAR_PLASTICITY
$      mid        ro         e        pr
         1    7.8e-9  210000.0       0.3
$                         lcss
                             1
```

CID/LCSS can be either a curve or table defining effective stress as a function of effective plastic strain.

**Creep (LAW = 5)**

For creep, the inelastic strain is represented by the creep strain, $\varepsilon_c$. The evolution depends on the model specified.

a) *Norton incremental formulation (MODEL = 1)*

$$\dot{\varepsilon}_c = A\bar{\sigma}^m t^n.$$

This is essentially the creep law available in *MAT_THERMO_ELASTO_-VISCOPLASTIC_CREEP (*MAT_188).

b) *Norton total formulation (MODEL = 2).*

$$\dot{\varepsilon}_c = \frac{d}{dt}(A\bar{\sigma}^m t^n) \quad .$$

This is essentially the creep law available in *MAT_UNIFIED_CREEP (*MAT_115), with some slight modifications.

c) *Norton-Bailey formulation (MODEL = 3).*

$$\dot{\varepsilon}_c = \left( A\left(\frac{\bar{\sigma}}{\sigma_0}\right)^n \left(\frac{T}{T_0}\right)^p \left((m+1)(\varepsilon_0+\varepsilon_c)\right)^m \right)^{\frac{1}{m+1}} \quad .$$

Here $T$ is the current temperature.

d) *Bergström-Boyce formulation (MODEL = 4).*

$$\dot{\varepsilon}_c = A(\lambda_c - 1 + E)^C \bar{\sigma}^m \quad ,$$

where $\lambda_c = \sqrt{\frac{1}{3}\mathbf{I}:\mathbf{B}_c} \geq 1$ and $\mathbf{B}_c = \exp\{2\boldsymbol{\varepsilon}_c\}$

e) *Garofalo formulation (MODEL = 5).*

$$\dot{\varepsilon}_c = A\sinh(B\bar{\sigma})^m e^{-Q/T}$$

**Viscoelasticity (LAW = 6)**

In the absence of viscoelasticity, we are now done with the description of the stress update, and we simply set

$$\mathbf{s}_I = \mathbf{s}$$
$$p_I = p$$

where we use $\mathbf{s}_I$ and $p_I$ to denote the final deviatoric stress and pressure in link $I$ that is used in the weighted sum at the beginning of this section. The $\mathbf{s}$ and $p$ are to be understood as the deviatoric stress and pressure resulting from treatment of creep and plasticity that we just covered, so $\sigma = \mathbf{s} - p\mathbf{I}$. For viscoelasticity the stress in link $I$ will be subject to stress decay (relaxation and creep), in that it evolves according to the specified viscoelastic law. For deviatoric and volumetric decay coefficients $\beta_\mathrm{s}$ and $\beta_\mathrm{p}$, we have for the hypoelastic laws (MODEL = 1 and MODEL = 4):

$$\mathbf{s}_I^\nabla = \mathbf{s}^\nabla - \beta_\mathrm{s}\mathbf{s}_I$$
$$\dot{p}_I = \dot{p} - \beta_\mathrm{p} p_I$$

The hyperelastic laws are formulated directly in terms of the Kirchhoff stress $\boldsymbol{\tau} = J\sigma$, where $J = \det \mathbf{F}$. More specifically, using the notation $q = -\frac{1}{3}\boldsymbol{\tau}{:}\mathbf{I}$ and $\mathbf{t} = \boldsymbol{\tau} + q\mathbf{I}$, we have for hyperelastic law #1 (MODEL = 2):

$$\mathbf{t}_I = \mathbf{t} - \mathrm{dev}\left[\beta_\mathrm{s}\int_0^t e^{-\beta_\mathrm{s}(t-s)}\overline{\mathbf{F}}_{s\to t}\mathbf{t}(s)\overline{\mathbf{F}}_{s\to t}^T \mathrm{d}s\right]$$
$$q_I = q - \beta_\mathrm{p}\int_0^t e^{-\beta_\mathrm{p}(t-s)} q(s)\mathrm{d}s$$

and for hyperelastic law #2 (MODEL = 3)

$$\mathbf{t}_I = \mathbf{t} - \mathrm{sym}\left[\beta_\mathrm{s}\int_0^t e^{-\beta_\mathrm{s}(t-s)}\mathbf{F}_{s\to t}\mathbf{t}(s)\mathbf{F}_{s\to t}^{-1} \mathrm{d}s\right]$$
$$q_I = q - \beta_\mathrm{p}\int_0^t e^{-\beta_\mathrm{p}(t-s)} q(s)\mathrm{d}s$$

The Kirchhoff stress for link $I$ is obtained as $\boldsymbol{\tau}_I = \mathbf{t}_I - q_I\mathbf{I}$. Here we use $\overline{\mathbf{F}}_{s\to t} = J_{s\to t}^{-1/3}\mathbf{F}_{s\to t}$, where $J_{s\to t} = \det \mathbf{F}_{s\to t}$, and $\mathbf{F}_{s\to t}$ is the deformation gradient between the configuration at time $s$ and time $t$. For law #1, $\overline{\mathbf{F}}_{s\to t}$ is used to push the stress forward from time $s$ to time $t$, while for law #2, $\mathbf{F}_{s\to t}$ is used to transform the stress from time $s$ to time $t$, both essential to preserve frame invariance.

The decay coefficients can be constants but can also dependent on the state of the system (stress, internal variables, temperature, etc.). Note that if the decay coefficients are equal to zero ($\beta_\mathrm{s} = \beta_\mathrm{p} = 0$), this is equivalent to not having viscoelasticity. Currently, we can

specify temperature dependent decay coefficients to affect both the deviatoric and volumetric stress, formalized in the following.

*Linear viscoelasticity (MODEL = 1)*

For viscoelasticity, the decay of stress is governed by the decay coefficients $\beta_s$ and $\beta_p$, optionally incorporating shift functions depending on the temperature $T$. In this implementation, the shear and bulk decay are given as

$$\beta_s = \beta_G \phi_G(T)$$
$$\beta_p = \beta_K \phi_K(T)$$

where $\phi_*$ ($*$ being G or K) are shift functions given by

$$\phi_*(T) = \begin{cases} e^{-A_*\left(\frac{1}{T}-\frac{1}{T_*}\right)} & \text{if } B_* = 0 \text{ (Arrhenius)} \\ e^{-A_*\left(\frac{T-T_*}{B_*+T-T_*}\right)} & \text{if } B_* \neq 0 \text{ (Williams} - \text{Landel} - \text{Ferry)} \end{cases}$$

This is essentially the viscoelastic law available in *MAT_GENERAL_VISCOELASTIC (*MAT_076), except that the driving mechanism for the stress is here $\mathbf{s}^{\nabla}$ and $\dot{p}$ rather than $2G_I\mathbf{D}_{\text{dev}}$ and $K_I D_{\text{vol}}$. Note also, that in contrast to *MAT_076, the shift coefficients are to be given for each link and for both the shear and bulk decay. This allows for using independent shifts for each link, and if the traditional usage of the shift functions is desired one needs to put the same triplet (i.e., the values of $T_*$, $A_*$ and $B_*$) on all links (parameters P3-P5 for shear, and P6-P8 for bulk). If *MAT_ELASTIC is used in combination with viscoelasticity here, the two formulations can be made (almost) equivalent after proper transformation of input data. For instance, the following two material definitions (1 and 2) are equivalent;

```
*MAT_ELASTIC
$      mid        ro         e        pr
         1    7.8e-9  210000.0       0.2
*MAT_ADD_INELASTICITY
$      mid  nielinks
         1         2
$  nielaws    weight
         1       0.3
$      law     model
         6         1
$    betag     betak
      0.05
$  nielaws    weight
         1       0.5
$      law     model
         6         1
$    betag     betak
     0.005
```

```
*MAT_GENERAL_VISCOELASTIC
$      mid        ro      bulk
         2    7.8e-9 116666.67
$ blank card

$         g     betag
   43750.0     0.005
   26250.0      0.05
   17500.0
```

In general, with

$$G = \frac{E}{2(1+\nu)}, \quad K = \frac{E}{3(1-2\nu)}$$

in *MAT_ELASTIC, and $G_I$ and $K_I$ in *MAT_GENERAL_VISCOELASTIC, we have

$$G = \sum G_I, \qquad K = \sum K_I$$

while the weights are given as

$$w_I = \frac{G_I}{G} = \frac{K_I}{K}$$

which implies that the add inelasticity approach is somewhat more restrictive than the general approach. However, an almost pure shear/bulk link can be created by setting the bulk/shear decay coefficient to a very large number compared to the simulation time. To be specific, to get a shear link set $\beta_K \gg 1/T$ and to get a bulk link set $\beta_G \gg 1/T$, where $T$ is the termination time. See also VFLAG in *MAT_GENERAL_HYPERELASTIC_RUBBER for the difference of the two approaches.

*Nonlinear viscoelasticity*

The nonlinear creep laws (LAW = 5) can be formulated as nonlinear viscoelastic laws (LAW = 6) by setting

$$\beta_s = \frac{G}{\sigma_I}\dot{\varepsilon}_c$$
$$\beta_p = 0$$

where $G$ is an estimated elastic stiffness of the base material, $\sigma_I$ is the von Mises effective stress of $\mathbf{s}_I$ and $\dot{\varepsilon}_c$ is the creep law of interest. Currently the following models are supported

a) Norton-Bailey formulation (MODEL = 4).

$$\dot{\varepsilon}_c = \left( A \left(\frac{\sigma_I}{\sigma_0}\right)^n \left(\frac{T}{T_0}\right)^p \left((m+1)(\varepsilon_0 + \varepsilon_c)\right)^m \right)^{\frac{1}{m+1}},$$

where $T$ is the current temperature.

b) Bergström-Boyce formulation (MODEL = 5).

$$\dot{\varepsilon}_c = A(\lambda_c - 1 + E)^C \sigma_I^m \ ,$$

where $\lambda_c = \sqrt{\frac{1}{3}\mathbf{I}:\mathbf{B}_c} \geq 1$ and $\mathbf{B}_c = \exp\{2\boldsymbol{\varepsilon}_c\}$

c) Garofalo formulation (MODEL = 6).

$$\dot{\varepsilon}_c = A\sinh(B\overline{\sigma})^m e^{-Q/T}$$

For linear elasticity the creep and nonlinear viscoelastic laws are equivalent. For other models they are similar assuming that a reasonable value of $G$ is used (see input field 4 on Card 1).

**History**

With *DEFINE_MATERIAL_HISTORIES you can output the effective plastic and creep strains for plastic and creep models, respectively. The presence of this keyword in the input deck will automatically move the total plastic strain to the appropriate location in the d3plot database. Its value will be

$$\varepsilon_p = \sum_{I=1}^{\text{NIELINKS}} w_I \varepsilon_p^I.$$

The creep strain can also be retrieved similarly as shown in the following table.

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Effective Creep Strain | - - - - | $\varepsilon_c = \sum_{I=1}^{\text{NIELINKS}} w_I \varepsilon_c^I$ |

## *MAT_ADD_MICRO

During hot forging, the microstructure of metal materials evolves following the material deformation and temperature history. This keyword is applied to a hot-forged part to calculate its microstructure evolution. With this keyword, the material properties of the hot-forged part do not depend on its microstructure (decoupled effect), and the material microstructure is computed as post-processing based on material plastic strain, strainn rate, and temperature.

Currently, *MAT_ADD_MICRO can only be applied to a forged part in a 3D *r*-adaptive simulation. It is activated by setting ADPMIC = 1 in *CONTROL_ADAPTIVE.

### Card Summary:

**Card 1.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| MID | MICTYP | NUMP | | | | | |

**Card 2.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Q1 | A1 | M1 | | | | | Q2 |

**Card 3.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A2 | M2 | H2 | Q3 | A3 | M3 | BETAD | D0 |

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | MICTYP | NUMP | | | | | |
| Type | A | I | I | | | | | |
| Default | none | 1 | 16 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which this keyword applies |
| MICTYP | Microstructure evolution model: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: Sommitch's Johnson-Mehl-Avrami-Kolmogorov (JMAK) model (see Remark 1) |
| NUMP | Number of material constants for the microstructure evolution model |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | A1 | M1 | | | | | Q2 |
| Type | F | F | F | | | | | F |
| Default | none | none | none | | | | | none |

| VARIABLE | DESCRIPTION |
|---|---|
| Q1 | Material constant $Q_1$ needed for determining critical strain (see Remark 1) |
| A1 | Material constant $a_1$ needed for determining critical strain (see Remark 1) |
| M1 | Material constant $m_1$ needed for determining critical strain (see Remark 1) |
| Q2 | Material constant $Q_2$ needed for determining the strain at 50% recrystallization (see Remark 1) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A2 | M2 | H2 | Q3 | A3 | M3 | BETAD | D0 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| A2 | Material constant $a_2$ needed for determining the strain at 50% recrystallization (see Remark 1) |
| M2 | Material constant $m_2$ needed for determining the strain at 50% recrystallization (see Remark 1) |
| H2 | Material constant $h_2$ needed for determining the strain at 50% recrystallization (see Remark 1) |
| Q3 | Material constant $Q_3$ needed for determining the dynamic recrystallization grain size (see Remark 1) |
| A3 | Material constant $a_3$ needed for determining the dynamic recrystallization grain size (see Remark 1) |
| M3 | Material constant $m_3$ needed for determining the dynamic recrystallization grain size (see Remark 1) |
| BETAD | Material constant $\beta_d$ for determining the volume fraction of dynamic recrystallization (see Remark 1) |
| D0 | Initial grain size, $d_0$ (see Remark 1) |

**Remarks:**

1. **JMAK model.** *MAT_ADD_MICRO uses an implementation of Sommitch's JMAK-type model (Sommitch et al. [2009]) for determining dynamic grain recrystallization in a thermo-mechanical process. This model computes the average grain size and volume fraction of dynamic recrystallization in metal and alloys.

   Dynamic recrystallization occurs when the effective plastic strain reaches the critical strain:

   $$\varepsilon_c = 0.8 a_1 \dot{\varepsilon}^{m_1} \exp\left(\frac{Q_1}{RT}\right)$$

   Here, $Q_1$ is the activation energy, $\dot{\varepsilon}$ is the strain rate, and $T$ is the temperature.

   The strain when 50% of grains are recrystallized is defined by:

   $$\varepsilon_{0.5} = a_2 {d_0}^{h_2} \dot{\varepsilon}^{m_2} \exp\left(\frac{Q_2}{RT}\right)$$

   The grain size of dynamic recrystallization is given by:

$$d_{\mathrm{drx}} = a_3 \dot{\varepsilon}^{m_3} \exp\left(\frac{Q_3}{RT}\right)$$

The volume fraction of recrystallized grains is:

$$X_{\mathrm{drx}} = \begin{cases} 1 - \exp\left[-\beta_{\mathrm{d}} \left(\dfrac{\varepsilon_{\mathrm{p}} - \varepsilon_{\mathrm{c}}}{\varepsilon_{0.5} - \varepsilon_{\mathrm{c}}}\right)^2\right], & \text{when } \varepsilon_{\mathrm{p}} > \varepsilon_{\mathrm{c}} \\ 0, & \text{otherwise} \end{cases}$$

Here, $\varepsilon_{\mathrm{p}}$ is the effective plastic strain. From these, the average grain size is obtained:

$$d_{\mathrm{ave}} = X_{\mathrm{drx}} d_{\mathrm{drx}} + (1 - X_{\mathrm{drx}}) d_0$$

## *MAT_ADD_PERMEABILITY

Add permeability to material model for consolidation calculations. See *CONTROL_-PORE_FLUID.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | PERM | PERMY | PERMZ | THEXP | LCKZ | PMTYP | |
| Type | A | F/I | F/I | F/I | F | I | I | |
| Default | none | none | PERM | PERM | 0.0 | none | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification – must be same as the structural material. |
| PERM | Permeability or load curve ID defining permeability, depending on the definition of PMTYP below. If PERMY and PERMZ are non-zero, then PERM gives the permeability in the global $X$ direction. See Remark 3. |
| PERMY | Optional permeability or load curve ID defining permeability in the global $Y$ direction, depending on the definition of PMTYP below |
| PERMZ | Optional permeability or load curve ID defining permeability in the global $Z$ direction, depending on the definition of PMTYP below |
| THEXP | Undrained volumetric thermal expansion coefficient (see Remark 2):<br>GE.0.0: Constant undrained volumetric thermal expansion coefficient<br>LT.0.0: \|THEXP\| is the ID of a load curve giving the thermal expansion coefficient ($y$-axis) as a function of temperature ($x$-axis). |
| LCKZ | Load curve giving factor on PERM as a function of $z$-coordinate |
| PMTYP | Permeability definition type:<br>EQ.0: PERM is a constant. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: PERM is a load curve ID giving permeability ($y$-axis) as a function of the volume ratio of current volume to volume in the stress-free state ($x$-axis). |
| | EQ.2: PERM is a load curve ID giving permeability ($y$-axis) as a function of effective plastic strain ($x$-axis) of materials other than MAT_072R3. For MAT_072R3, the $x$-axis is the output selector specified by NOUT; see *MAT_072R3. |
| | EQ.3: PERM is a load curve ID giving permeability ($y$-axis) as a function of effective pressure ($x$-axis) which is positive when in compression. |

**Remarks:**

1. **Permeability Units.** The units of PERM are length/time (volume flow rate of water per unit area per gradient of pore pressure head).

2. **Thermal Expansion.** THEXP represents the thermal expansion of the material caused by the pore fluid (units: 1/temperature). It should be set equal to $n\alpha_w$, where $n$ is the porosity of the soil and $\alpha_w$ is the volumetric thermal expansion coefficient of the pore fluid. If the pore fluid is water, the thermal expansion coefficient varies strongly with temperature; a curve of coefficient as a function of temperature may be input instead of a constant value. Note that this property is for *volumetric* strain increase, whereas regular thermal expansion coefficients (e.g. on *MAT or *MAT_ADD_THERMAL_EXPANSION) are linear, meaning they describe thermal expansion in one direction. The volumetric expansion coefficient is three times the linear thermal expansion coefficient. The regular thermal expansion coefficients apply to the soil skeleton and to drained parts. Pore pressure can be generated due to the difference of expansion coefficients of the soil skeleton and pore fluid, that is, if THEXP is not equal to three times the regular thermal expansion coefficient for the part.

3. **Isotropic/Orthotropic Permeability.** If only PERM is defined and PERMY and PERMZ are left blank or zero, the permeability is isotropic. To obtain orthotropic permeability, define values for PERM, PERMY and PERMZ, giving the permeability in the global $X$, $Y$ and $Z$ directions respectively.

## *MAT_ADD_PORE_AIR

For pore air pressure calculations.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | PA_RHO | PA_PRE | PORE | | | | |
| Type | A | I | F | F | | | | |
| Default | none | AIR_RO | AIR_RO | 1. | | | | |
| Remarks | 1 | | | 1, 2 | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PERM1 | PERM2 | PERM3 | CDARCY | CDF | LCPGD1 | LCPGD2 | LCPGD3 |
| Type | F | F | F | F | F | I | I | I |
| Default | 0. | PERM1 | PERM1 | 1. | 0. | none | LCPGD1 | LCPGD1 |
| Remarks | 2, 3, 4, 5 | 2, 3, 4, 5 | 2, 3, 4, 5 | 1 | 1, 5 | 6 | 6 | 6 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification which must be same as the structural material |
| PA_RHO | Initial density of pore air. The default is the atmospheric air density, AIR_RO, defined in *CONTROL_PORE_AIR |
| PA_PRE | Initial pressure of pore air. The default is the atmospheric air pressure, AIR_P, defined in *CONTROL_PORE_AIR |
| PORE | Porosity, meaning the ratio of pores to total volume (default = 1). If less than 0, PORE is taken to be a load curve ID for the curve giving the pore air volume ratio as a function of the volume ratio of the current volume to the volume in the stress free state. |

| VARIABLE | DESCRIPTION |
|---|---|
| PERM*i* | Permeability of pore air along $x$, $y$, and $z$-directions. If less than 0, PERM*i* is taken to be the curve ID defining the permeability coefficient as a function of volume ratio of current volume to volume in the stress free state. |
| CDARCY | Coefficient of Darcy's law |
| CDF | Coefficient of Dupuit-Forchheimer law |
| LCPGD*i* | Curves defining non-linear Darcy's laws along $x$, $y$ and $z$-directions |

**Remarks:**

1. **Card 1.** This card must be defined for all materials requiring consideration of pore air pressure. The pressure contribution of pore air is $(\rho - \rho_{\text{atm}})RT\times$ PORE, where $\rho$ and $\rho_{\text{atm}}$ are the current and atmospheric air densities, $R$ is air's gas constant, $T$ is atmospheric air temperature and PORE is the porosity. The values for $R$, $T$ and PORE are assumed to be constant during simulation.

2. **Permeability Model.** The unit of PERM*i* is $[\text{Length}]^3[\text{time}]/[\text{mass}]$, (air flow velocity per gradient of excess pore pressure), i.e.

$$(\text{CDARCY} + \text{CDF} \times |v_i|) \times \text{PORE} \times v_i = \text{PERM}i \times \frac{\partial P_a}{\partial x_i}, \qquad i = 1,2,3$$

   where $v_{\text{i}}$ is the pore air flow velocity along the $i^{\text{th}}$ direction, $\partial P_a / \partial x_i$ is the pore air pressure gradient along the i$^{\text{th}}$ direction, and $x_1 = x$, $x_2 = y$, $x_3 = z$.

3. **Default Values for PERM2 and PERM3.** PERM2 and PERM3 are assumed to be equal to PERM1 when they are not defined. A definition of "0" means no permeability.

4. **Local Coordinate Systems.** When MID is an orthotropic material, such as *MAT_002 or *MAT_142, $(x,y,z)$, or $(1,2,3)$, refers to the local material coordinate system $(a,b,c)$; otherwise it refers to the global coordinate system.

5. **CDF for Viscosity.** CDF can be used to consider the viscosity effect for high speed air flow.

6. **Nonlinearity.** LCPGD*i* can be used to define a non-linear Darcy's law as follows:

$$(\text{CDARCY} + \text{CDF} \times |v_i|) \times \text{PORE} \times v_i = \text{PERM}i \times f_i \frac{\partial P_a}{\partial x_i}, \qquad i = 1,2,3$$

where $f_i$ is the value of the function defined by the LCPGD*i* field. The linear version of Darcy's law (see Remark 2) can be recovered when the LCPGD*i* curves are defined as straight lines of slope of 1.

## *MAT_ADD_PROPERTY_DEPENDENCE_{OPTION}

Available options include:

FREQ

TIME

The ADD_PROPERTY_DEPENDENCE option defines dependence of a material property on frequency or time.

| Card 1 | 1 | 2 | 3 | | | | | |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | PROP | LCID | | | | | |
| Type | A | C | I | | | | | |
| Default | none | none | 0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification for which the property dependence applies |
| PROP | Name of the property (same as the variable for a material model in keyword card). For example, "E" is used for Young's modulus in *MAT_ELASTIC. See Remark 4. |
| LCID | Curve ID to define the property dependence. For the FREQ keyword option, the abscissa values define frequency; for the TIME keyword option, the abscissa values define time. The ordinate values define the property at each frequency or each time |

**Remarks:**

1. **Overview.** This keyword defines how a property (for example, the Young's modulus) of a material changes with frequency (for FREQ option) or with time (for TIME option). Particularly, *MAT_ADD_PROPERTY_DEPENDENCE_FREQ can be used in direct SSD analysis (*FREQUENCY_DOMAIN_SSD_DIRECT_FREQUENCY_DEPENDENT).

2. **Properties without Frequency/Time Dependence.** Some properties of a material model have no frequency or time dependence. A warning message will be

issued if a dependence curve is defined on a property of a material, which has no frequency or time dependence.

3. **Initial Property Values.** The original property value defined in a material card will be overridden by the property values defined at frequency or time 0 in this keyword. If the starting frequency or time of LCID in this keyword is larger than 0, then the original property value defined in the material card is used until the starting frequency or time of LCID is reached.

4. **Supported Material Models and Properties.** So far, only the Young's modulus (E) of *MAT_ELASTIC is supported by this keyword. More material models (and properties) will be supported in the future.

## *MAT_ADD_PZELECTRIC

The ADD_PZELECTRIC option is used to occupy an arbitrary material model in LS-DYNA with a piezoelectric property. This option applies to 4-node solids, 6-node solids, 8-node solids, thick shells, 2D plane strain elements and axisymmetric solids. Orthotropic properties are assumed. This feature is available in SMP since 115324/dev and MPP since 126577/dev. We recommend a double precision executable.

### Card Summary:

**Card 1.** This card is required.

| MID | DTYPE | GPT | AOPT | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| DXX | DYY | DZZ | DXY | DXZ | DYZ | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| PX11 | PX22 | PX33 | PX12 | PX13 | PX23 | PY11 | PY22 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| PY33 | PY12 | PY13 | PY23 | PZ11 | PZ22 | PZ33 | PZ12 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| PZ13 | PZ23 | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| | | | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | DTYPE | GPT | AOPT | | | | |
| Type | A | A | F | I | | | | |
| Default | none | S | 8 | 0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID for which the piezoelectric properties apply |
| DTYPE | Type of piezoelectric property definition (see remarks below)<br>EQ.S: Stress based definition<br>EQ.E: Strain based definition |
| GPT | Number of Gauss points used for integration:<br>EQ.0: Default value 8, full integration<br>EQ.1: Reduced integration |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DXX | DYY | DZZ | DXY | DXZ | DYZ | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D$\alpha\beta$ | Dielectric permittivity matrix, $d_{\alpha\beta}$. $\alpha$, $\beta = x, y, z$ (see remarks below). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PX11 | PX22 | PX33 | PX12 | PX13 | PX23 | PY11 | PY22 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PY33 | PY12 | PY13 | PY23 | PZ11 | PZ22 | PZ33 | PZ12 |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PZ13 | PZ23 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P$\alpha ij$ | Piezoelectric matrix which depends on DTYPE (see remarks below). $\alpha = x, y, z$ and $i,j = 1, 2, 3$. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | D1 | D2 | D3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

The stress-based definition for piezoelectric effects is:

$$\sigma_{ij} = k_{ijkl}\varepsilon_{kl} - p_{\alpha ij}E_\alpha$$
$$\Delta_\alpha = p_{\alpha kl}\varepsilon_{kl} + d_{\alpha\beta}E_\beta$$

Here $\sigma_{ij}$ are the mechanical stresses, $k_{ijkl}$ are the material stiffness constants, $\varepsilon_{kl}$ are the material strains, $p_{\alpha ij}$ are the stress-based piezoelectric coefficients, $\Delta_\alpha$ are the electric displacements, $E_\alpha$ are the electronic fields, and $d_{\alpha\beta}$ are the dielectric permittivity constants.

The strain-based definition for piezoelectric effects is:

$$\varepsilon_{ij} = f_{ijkl}\sigma_{kl} + P_{\alpha ij}E_\alpha$$
$$\Delta_\alpha = P_{\alpha kl}\sigma_{kl} + d_{\alpha\beta}E_\beta$$

Here $f_{ijkl}$ are the material flexibility parameters and $P_{\alpha ij}$ are the strain-based piezoelectric coefficients.

## *MAT_ADD_SOC_EXPANSION

The ADD_SOC_EXPANSION option adds a state of charge (SOC) expansion property to an (arbitrary) material model in LS-DYNA. The state of charge comes from the EM module during a coupled simulation. This option currently only applies to solid elements type -2, -1, 1, 2, and 10 and to hypoelastic material models.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PID | LCID | MULT | LCIDY | MULTY | LCIDZ | MULTZ | |
| Type | I | I | F | I | F | I | F | |
| Default | none | 0 | 1.0 | LCID | MULT | LCID | MULT | |

| VARIABLE | DESCRIPTION |
|---|---|
| PID | Part ID for which the SOC expansion property applies |
| LCID | For isotropic material models, LCID is the load curve ID defining the SOC expansion coefficient as a function of state of charge. In this case, LCIDY, MULTY, LCIDZ, and MULTZ are ignored. For anisotropic material models, LCID and MULT define the SOC expansion coefficient in the local material *a*-direction. In either case, if LCID is zero, the SOC expansion coefficient is constant and equal to MULT. |
| MULT | Scale factor scaling load curve given by LCID |
| LCIDY | Load curve ID defining the SOC expansion coefficient in the local material *b*-direction as a function of state of charge. If zero, the SOC expansion coefficient in the local material *b*-direction is constant and equal to MULTY. If MULTY = 0.0 as well, LCID and MULT specify the SOC expansion coefficient in the local material *b*-direction. |
| MULTY | Scale factor scaling load curve given by LCIDY |
| LCIDZ | Load curve ID defining the SOC expansion coefficient in the local material *c*-direction as a function of state of charge. If zero, the SOC expansion coefficient in the local material *c*-direction is constant and equal to MULTZ. If MULTZ = 0.0 as well, LCID and MULT specify the SOC expansion coefficient in the local material *c*- |

| VARIABLE | DESCRIPTION |
|---|---|
| | direction. |
| MULTZ | Scale factor scaling load curve given by LCIDZ |

**Remarks:**

When invoking the isotropic SOC expansion property (no local $y$ and $z$ parameters) for a material, the stress update is based on the elastic strain rates given by

$$\dot{\varepsilon}_{ij}^{e} = \dot{\varepsilon}_{ij} - \gamma(\mathrm{SOC})\dot{\mathrm{SOC}} \times \delta_{ij}$$

rather than on the total strain rates, $\dot{\varepsilon}_{ij}$. For orthotropic properties, which apply only to materials with anisotropy, this equation is generalized to

$$\dot{\varepsilon}_{ij}^{e} = \dot{\varepsilon}_{ij} - \gamma_k(\mathrm{SOC})\dot{\mathrm{SOC}}\, q_{ik}q_{jk} \quad .$$

Here $q_{ij}$ represents the matrix with material directions with respect to the current configuration.

## *MAT_ADD_THERMAL_EXPANSION

The ADD_THERMAL_EXPANSION option adds a thermal expansion property to an arbitrary material model in LS-DYNA. This option applies to all nonlinear solid, shell, thick shell, and beam elements and to all material models except those models which use resultant formulations, such as *MAT_RESULTANT_PLASTICITY and *MAT_SPECIAL_ORTHOTROPIC. Orthotropic expansion effects are supported for anisotropic materials.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ID | LCID | MULT | LCIDY | MULTY | LCIDZ | MULTZ | TREF |
| Type | I | I | F | I | F | I | F | F |
| Default | none | none | 1.0 | LCID | MULT | LCID | MULT | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ID | Part or material ID for which the thermal expansion property applies:<br>GT.0: Part ID<br>LT.0: Material ID given by \|ID\| |
| LCID | For isotropic material models, LCID is the load curve ID defining the thermal expansion coefficient as a function of temperature. In this case, LCIDY, MULTY, LCIDZ, and MULTZ are ignored. For anisotropic material models, LCID and MULT define the thermal expansion coefficient in the local material *a*-direction. In either case, if LCID is zero, the thermal expansion coefficient is constant and equal to MULT. |
| MULT | Scale factor scaling load curve given by LCID |
| LCIDY | Load curve ID defining the thermal expansion coefficient in the local material *b*-direction as a function of temperature. If zero, the thermal expansion coefficient in the local material *b*-direction is constant and equal to MULTY. If MULTY = 0 as well, LCID and MULT define the thermal expansion coefficient in the local material *b*-direction. |
| MULTY | Scale factor scaling load curve given by LCIDY |
| LCIDZ | Load curve ID defining the thermal expansion coefficient in the |

| VARIABLE | DESCRIPTION |
|---|---|
| | local material *c*-direction as a function of temperature. If zero, the thermal expansion coefficient in the local material *c*-direction is constant and equal to MULTZ. If MULTZ = 0 as well, LCID and MULT define the thermal expansion coefficient in the local material *c*-direction. |
| MULTZ | Scale factor scaling load curve given by LCIDZ |
| TREF | Reference temperature. A nonzero value activates the secant approach; see Remarks below. |

**Remarks:**

When invoking the isotropic thermal expansion property (no local $y$ and $z$ parameters) for a material, the stress update is based on the elastic strain rates given by

$$\dot{\varepsilon}^e_{ij} = \dot{\varepsilon}_{ij} - \alpha(T)\dot{T}\delta_{ij}$$

rather than on the total strain rates $\dot{\varepsilon}_{ij}$. For a material with the stress based on the deformation gradient, $F_{ij}$, the elastic part of the deformation gradient is used for the stress computations:

$$F^e_{ij} = J_T^{-1/3} F_{ij} \ ,$$

where $J_T$ is the thermal Jacobian. The thermal Jacobian is updated using the rate given by

$$\dot{J}_T = 3\alpha(T)\dot{T}J_T \ .$$

For orthotropic properties, which apply only to materials with anisotropy, these equations are generalized to

$$\dot{\varepsilon}^e_{ij} = \dot{\varepsilon}_{ij} - \alpha_k(T)\dot{T}q_{ik}q_{jk}$$

and

$$F^e_{ij} = F_{ik}\beta_l^{-1}Q_{kl}Q_{jl} \ ,$$

where the $\beta_i$ are updated as

$$\dot{\beta}_i = \alpha_i(T)\dot{T}\beta_i \ .$$

Here $q_{ij}$ represents the matrix with material directions with respect to the current configuration whereas $Q_{ij}$ are the corresponding directions with respect to the initial configuration. For (shell) materials with multiple layers of different anisotropy directions, the midsurface layer determines the orthotropy for the thermal expansion.

In the first equation above, the thermal strain increment is defined as

$$d\varepsilon_{ij}^{\text{thermal}} = \alpha(T)dT\delta_{ij} \ .$$

where $\alpha(T)$ is often referred to as the tangent coefficient of thermal expansion, representing the instantaneous change in material length per degree of temperature change. The secant coefficient of thermal expansion, on the other hand, represents the average change in length over a range of temperatures, relative to a reference temperature. This alternative approach can be enabled by defining a nonzero value for TREF. In that case, the thermal strain is given by

$$\varepsilon_{ij}^{\text{thermal}} = \big(\alpha(T)(T - T_{\text{ref}}) - \alpha(T_0)(T_0 - T_{\text{ref}})\big)\delta_{ij} \ .$$

Here, temperature $T_0$ is the initial temperature.

## *MAT_NONLOCAL

In nonlocal failure theories, the failure criterion depends on the state of the material within a radius of influence which surrounds the integration point. An advantage of nonlocal failure is that mesh size sensitivity on failure is greatly reduced leading to results which converge to a unique solution as the mesh is refined.

Without a nonlocal criterion, strains will tend to localize randomly with mesh refinement leading to results which can change significantly from mesh to mesh. The nonlocal failure treatment can be a great help in predicting the onset and the evolution of material failure. This option can be used with two and three-dimensional solid elements, three-dimensional shell elements, and thick shell elements. This option applies to a subset of elasto-plastic materials that include a damage-based failure criterion.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDNL | PID | P | Q | L | NFREQ | NHV | NHVT |
| Type | I | I | F | F | F | I | I | I |
| Default | none | none | none | none | none | none | none | none |

**History Cards.** Include as many cards as needed to identify the NHV and NHVT history variables. *One card 2 will be read, even if both NHV and NHVT are zero.* If only NHV > 0, then NL*i* are assumed to be incremental variables. If only NHVT > 0, then NL*i* are assumed to be non-incremental variables. If both NHV and NHVT are nonzero, then NHV variables will be read starting at Card 2, and NHVT variables will be read starting on a new line.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NL1 | NL2 | NL3 | NL4 | NL5 | NL6 | NL7 | NL8 |
| Type | I | I | I | I | I | I | I | I |
| Default | none | none | none | none | none | none | none | none |

**Symmetry Plane Cards.** Define one card for each symmetry plane. Up to six symmetry planes can be defined. The next keyword ("*") card terminates this input.

| Cards 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC1 | YC1 | ZC1 | XC2 | YC2 | ZC2 | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IDNL | Nonlocal material input ID |
| PID | Part ID for nonlocal material |
| P | Exponent of weighting function. A typical value might be 8 depending somewhat on the choice of L. See Remark 4. |
| Q | Exponent of weighting function. A typical value might be 2. See Remark 4. |
| L | Characteristic length. This length should span a few elements. See Remark 4. |
| NFREQ | Number of time steps between searching for integration points that lie in the neighborhood. Nonlocal smoothing will be done each cycle using these neighbors until the next search is done. The neighbor search can add significant computational time, so it is suggested that NFREQ be set to a value between 10 and 100 depending on the problem. This parameter may be somewhat problem dependent. If NFREQ = 0, a single search will be done at the start of the calculation. |
| NHV | Number of variables with nonlocal treatment of increments. See Remark 1 |
| NHVT | Number of variables with nonlocal treatment of total values. See Remark 1. |
| NL1, …, NL8 | Identifies the history variable(s) for nonlocal treatment. Define NHV + NHVT values (maximum of 8 values per line). See Remark 2. |

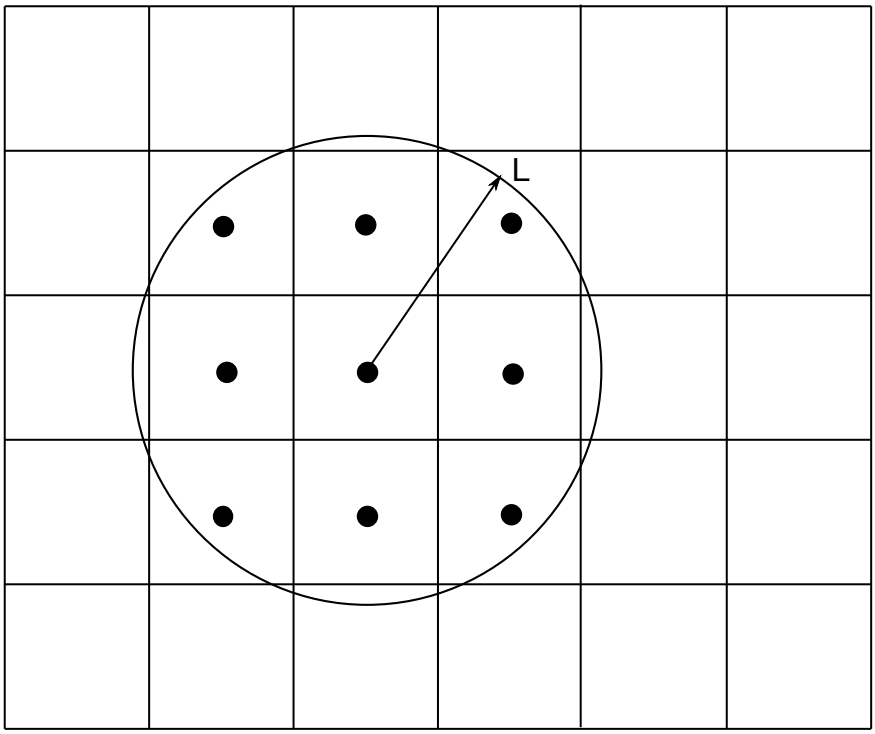



**Figure 2-9.** Here $\dot{f}_r$ and $x_r$ are respectively the nonlocal rate of increase of damage and the center of the element $e_r$, and $\dot{f}^i_{\text{local}}$, $V_i$ and $y_i$ are respectively the local rate of increase of damage, the volume and the center of element $e_i$.

| VARIABLE | DESCRIPTION |
|---|---|
| XC1, YC1, ZC1 | Coordinate of point on symmetry plane |
| XC2, YC2, ZC2 | Coordinate of a point along the normal vector |

**Remarks:**

1. **NHV and NHVT.** NHV is a count of the number of variables for which increments of the variable are used in the nonlocal function. NHVT is a count of the number of variables for which the whole value of the variable is used in the nonlocal function. NHVT type variables would be used only if the variable is itself an increment of some value which is rare. Many history variables are calculated by a sum of increments, but since the variable is the sum, one would include this variable in the NHV type variables for nonlocal treatment so that only the increments are modified.

2. **History Variables.** For elastoplastic material models in LS-DYNA which use the plastic strain as a failure criterion, setting the variable NL1 to 1 would flag plastic strain for nonlocal treatment. A sampling of other history variables that can be flagged for nonlocal treatment are listed in the table below. The value in

the third column in the table below corresponds to the history variable number as tabulated at https://lsdyna.ansys.com/history-variables-for-certain-material-models/. Note that the NL*n* value is the history variable number plus 1.

| Material Model Name | | *MAT_NONLOCAL NL*n* Value | History Variable Number |
|---|---|---|---|
| JOHNSON_COOK | 15 | 5 (shells); 7 (solids) | 4 (shells); 6 (solids) |
| PLASTICITY_WITH_DAMAGE | 81 | 2 | 1 |
| DAMAGE_1 | 104 | 4 | 3 |
| DAMAGE_2 | 105 | 2 | 1 |
| JOHNSON_HOLMQUIST_CONCRETE | 111 | 2 | 1 |
| GURSON | 120 | 2 | 1 |

3. **Integration Points and Nonlocal Equations.** When applying the nonlocal equations to shell and thick shell elements, integration points lying in the same plane within the radius determined by the characteristic length are considered. Therefore, it is important to define the connectivity of the shell elements consistently within the part ID, so that, for example, the outer integration points lie on the same surface.

4. **Nonlocal Equations.** The equations and our implementation are based on the implementation by Worswick and Lalbin [1999] of the nonlocal theory to Pijaudier-Cabot and Bazant [1987]. Let $\Omega_r$ be the neighborhood of radius, L, of element $e_r$ and $\{e_i\}_{i=1,\dots,N_r}$ the list of elements included in $\Omega_r$, then

$$\dot{f}_r = \dot{f}(x_r) = \frac{1}{W_r}\int_{\Omega_r} \dot{f}_{\text{local}} w(x_r - y)\, dy \approx \frac{1}{W_r}\sum_{i=1}^{N_r} \dot{f}^i_{\text{local}} w_{ri}\, V_i$$

where

$$W_r = W(x_r) = \int w(x_r - y)\, dy \approx \frac{1}{W_r}\sum_{i=1}^{N_r} w_{ri} V_i$$

$$w_{ri} = w(x_r - y_i) = \frac{1}{\left[1 + \left(\frac{\|x_r - y_i\|}{L}\right)^p\right]^q}$$

## *MAT_NONLOCAL_GRADIENT_DAMAGE

As an alternative to the integral approach in *MAT_NONLOCAL, a so-called gradient-enhanced damage formulation can be used to remedy mesh dependence due to stress softening. The idea of this method is to introduce non-local plastic strain as a new degree of freedom and provide an implicit second gradient equation to be solved for it. The increments of those non-local plastic strains are then used to replace the local plastic strain increments in damage/temperature evolution laws of certain material models (see Remarks). A more detailed description of the theory can be found in papers by Davaze et al. [2021, 2023].

This method is currently available for solid element types -2, -1, 1, 2, 10, 13, 15, and 115 and for (thin) shell element types 2, 4, and 16.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PID | CLEN | LEMAX | ZETA | DNRTS | DAMP | | |
| Type | I | F | F | F | F | F | | |
| Default | none | none | 0.0 | 0.0 | 0.0 | 0.2 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PID | Part ID for which the gradient damage property applies |
| CLEN | Characteristic length, $l$. This is a material dependent internal length that influences the regularization band width. |
| LEMAX | Maximum element size, $L_e^{max}$, for which results should be mesh convergent. If a nonzero value is defined, the characteristic length, $l$, is set internally based on the idea that at least one neighboring element in each direction is included in the nonlocal averaging. The input of CLEN is ignored in this case. If the default value of 0.0 is used, then CLEN is used directly. |
| ZETA | Nonlocal density, $\zeta$. If the default value of zero is used, an appropriate value is estimated based on the mechanical time step and element sizes of the part in question. This value has units of time$^2$. |
| DNRTS | Flag for the regularization of transverse strain of shell elements:<br>EQ.0.0: Yes, regularize transverse strain (see Remark 3). |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: No, do not regularize transverse strain. |
| DAMP | Damping coefficient. A damping parameter, $\gamma$, is internally computed using DAMP as a scaling factor. See Remark 3. Values around the default of 0.2 are recommended, corresponding to 20% of critical damping. This value has units of time. |

**Remarks:**

1. **Governing equation.** The following dynamic formulation is used to determine the *nonlocal* effective plastic strain, $\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{nl}}$, at each finite element node:

$$\begin{cases} l^2\nabla^2\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{nl}} + \gamma\dot{\bar{\varepsilon}}^{\mathrm{p}}_{\mathrm{nl}} + \left(\bar{\varepsilon}^{\mathrm{p}} - \bar{\varepsilon}^{\mathrm{p}}_{\mathrm{nl}}\right) = \zeta\ddot{\bar{\varepsilon}}^{\mathrm{p}}_{\mathrm{nl}} & \text{on } \Omega \\ \nabla\bar{\varepsilon}^{\mathrm{p}}_{\mathrm{nl}} \cdot \mathrm{n} = 0 & \text{on } \Gamma \end{cases}$$

   with $\bar{\varepsilon}^{\mathrm{p}}$ as the *local* effective plastic strain obtained from the integration points, $\zeta$ as the nonlocal density, and $\gamma$ as the damping parameter.

2. **Supported material/damage models.** This method is currently supported by the damage evolution laws in *MAT_ADD_DAMAGE_DIEM, *MAT_ADD_DAMAGE_GISSMO, *MAT_015, *MAT_081, *MAT_110, *MAT_111, and *MAT_272, as well as the temperature evolution in *MAT_015.

3. **Transverse strain regularization.** To avoid localization issues associated with the necking of shell elements, Davaze et al. [2023] proposed to use the nonlocal plastic multiplier, $\Delta\lambda_{\mathrm{nl}}$, to compute the transverse strain increment, $\Delta\varepsilon_{zz}$, like this:

$$\Delta\varepsilon_{zz} = -\frac{\nu}{1-\nu}\left(\Delta\varepsilon_{xx} - \Delta\lambda_{\mathrm{nl}}n_{xx} + \Delta\varepsilon_{yy} - \Delta\lambda_{\mathrm{nl}}n_{yy}\right) + \Delta\lambda_{\mathrm{nl}}n_{zz}$$

### *MAT_ELASTIC_{OPTION}

This is Material Type 1. This is an isotropic hypoelastic material and is available for beam, shell, and solid elements in LS-DYNA. A specialization of this material allows for modeling fluids.

Available options include:

<BLANK>

FLUID

such that the keyword cards appear as:

*MAT_ELASTIC or MAT_001

*MAT_ELASTIC_FLUID or MAT_001_FLUID

The fluid option is valid for solid elements only.

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | E | PR | DA | DB | K | |
|---|---|---|---|---|---|---|---|

**Card 1.1.** Include this card when E < 0.0.

| EFUNC | CNVT | ITERLM | | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** Include this card when using the FLUID keyword option.

| VC | CP | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Defintions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | DA | DB | K | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Definition of Young's modulus<br>GT.0: E is the Young's modulus.<br>LT.0: \|E\| is the ID of a curve defining Young's modulus as a function of elemental variables, EFUNC. It is supported for explicit simulation only. |
| PR | Poisson's ratio |
| DA | Axial damping factor (used for Belytschko-Schwer beam, type 2, only). |
| DB | Bending damping factor (used for Belytschko-Schwer beam, type 2, only). |
| K | Bulk modulus (define for fluid option only). |

Additional card for E < 0.

| Card 1.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFUNC | CNVT | ITERLM | | | | | |
| Type | A | F | I | | | | | |
| Default | P | $10^{-3}$ | 3 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EFUNC | Element variable used as the independent variable of curve \|E\|:<br>EQ.P: Elemental pressure |
| CNVT | Convergence tolerance. It is needed when EFUNC is a variant of stress. |
| ITERLM | Iteration limit. It is needed when EFUNC is a variant of stress. |

Additional card for FLUID keyword option.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VC | CP | | | | | | |
| Type | F | F | | | | | | |
| Default | none | $10^{20}$ | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| VC | Tensor viscosity coefficient. Values between .1 and .5 should be okay. |
| CP | Cavitation pressure (default = $10^{20}$). |

**Remarks:**

1. **Finite strains.** This hypoelastic material model may not be stable for finite (large) strains. If large strains are expected, a hyperelastic material model, such as *MAT_002, would be more appropriate.

2. **Damping factors.** The axial and bending damping factors are used to damp down numerical noise. The update of the force resultants, $F_i$, and moment resultants, $M_i$, includes the damping factors:

$$F_i^{n+1} = F_i^n + \left(1 + \frac{DA}{\Delta t}\right) \Delta F_i^{n+\frac{1}{2}}$$

$$M_i^{n+1} = M_i^n + \left(1 + \frac{DB}{\Delta t}\right) \Delta M_i^{n+\frac{1}{2}}$$

3. **Effective plastic strain.** The history variable labeled as "effective plastic strain" by LS-PrePost is volumetric strain in the case of *MAT_ELASTIC.

4. **Truss elements and damping stress.** Truss elements include a damping stress given by

$$\sigma = 0.05 \rho c L / \Delta t$$

where $\rho$ is the mass density, $c$ is the material wave speed, $L$ is the element length, and $\Delta t$ is the computation time step.

If the damping stress is undesired, it can be switched off with IRATE = 2 on *CONTROL_IMPLICIT_DYNAMICS.

5. **FLUID keyword option.** For the FLUID keyword option, the bulk modulus field, $K$, must be defined, and both the Young's modulus and Poisson's ratio fields are ignored. Fluid-like behavior is obtained where the bulk modulus, $K$, and pressure rate, $p$, are given by:

$$K = \frac{E}{3(1-2\nu)}$$

$$\dot{p} = -K\dot{\varepsilon}_{ii}$$

and the shear modulus is set to zero. A tensor viscosity is used which acts only the deviatoric stresses, $S_{ij}^{n+1}$, given in terms of the damping coefficient as:

$$S_{ij}^{n+1} = \mathrm{VC} \times \Delta L \times a \times \rho\dot{\varepsilon}'_{ij}$$

where $\Delta L$ is a characteristic element length, $a$ is the fluid bulk sound speed, $\rho$ is the fluid density, and $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate.

## *MAT_OPTIONTROPIC_ELASTIC

This is Material Type 2. This material is valid for modeling the elastic-orthotropic behavior of solids, shells, and thick shells. An anisotropic option is available for solid elements. For orthotropic solids, isotropic frictional damping is available.

Depending on the element type and solver, the implementation of this material model changes. See the theory manual for more details than the overview provided here. In the case of solids with an explicit solver or nonlinear implicit solver (meaning NSOLVR ≠ 1 on *CONTROL_IMPLICIT_SOLUTION), the model is the (hyperelastic) St. Venant-Kirchhoff model. The stress update is performed using the second Piola-Kirchhoff tensor. It is then transformed into the Cauchy stress for output. For shells (and this includes the 2D continuum elements, that is, shell types 13, 14, and 15), the model is implemented in the local coordinates of the shell as linear elasticity for explicit and nonlinear implicit. While the material response is linear, the shells themselves can undergo finite rotations consistent with applied forces. For the linear implicit solver, this material model is a linear elasticity model.

> **NOTE:** This material does not support specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

Available options include:

ORTHO

ANISO

such that the keyword cards appear:

*MAT_ORTHOTROPIC_ELASTIC or MAT_002 (4 cards follow)

*MAT_ANISOTROPIC_ELASTIC or MAT_002_ANIS (5 cards follow)

### Card Summary:

**Card 1a.1.** This card is required for the ORTHO keyword option.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 1a.2.** This card is required for the ORTHO keyword option.

| GAB | GBC | GCA | AOPT | G | SIGF | | |
|---|---|---|---|---|---|---|---|

**Card 1b.1.** This card is required for the ANISO keyword option.

| MID | RO | C11 | C12 | C22 | C13 | C23 | C33 |
|---|---|---|---|---|---|---|---|

**Card 1b.2.** This card is required for the ANISO keyword option.

| C14 | C24 | C34 | C44 | C15 | C25 | C35 | C45 |
|---|---|---|---|---|---|---|---|

**Card 1b.3.** This card is required for the ANISO keyword option.

| C55 | C16 | C26 | C36 | C46 | C56 | C66 | AOPT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MACF | IHIS |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

**Orthotropic Card 1.** Included for ORTHO keyword option.

| Card 1a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in $a$-direction |
| EB | $E_b$, Young's modulus in $b$-direction |
| EC | $E_c$, Young's modulus in $c$-direction (nonzero value required but not used for shells) |
| PRBA | $\nu_{ba}$, Poisson's ratio in the $ba$ direction |

| VARIABLE | DESCRIPTION |
|---|---|
| PRCA | $\nu_{ca}$, Poisson's ratio in the *ca* direction |
| PRCB | $\nu_{cb}$, Poisson's ratio in the *cb* direction |

**Orthotropic Card 2.** Included for ORTHO keyword option.

| Card 1a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | G | SIGF | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus in the *ab* direction |
| GBC | $G_{bc}$, shear modulus in the *bc* direction |
| GCA | $G_{ca}$, shear modulus in the *ca* direction |
| AOPT | Material axes option (see Figure M2-1 and the Material Directions section): |

EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. When this option is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work correctly. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.

EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by a

| VARIABLE | DESCRIPTION |
|---|---|
| | vector **v** and the normal vector to the plane of the element (see Figure M2-1). The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: \|AOPT\| is a coordinate system ID (see *DEFINE_COORDINATE_*OPTION*). |
| G | Shear modulus for frequency independent damping. Frequency independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF. For the best results, the value of G should be 250-1000 times greater than SIGF. This option applies only to solid elements. |
| SIGF | Limit stress for frequency independent, frictional, damping |

**Anisotropic Card 1.** Included for ANISO keyword option.

| Card 1b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | C11 | C12 | C22 | C13 | C23 | C33 |
| Type | A | F | F | F | F | F | F | F |

**Anisotropic Card 2.** Included for ANISO keyword option.

| Card 1b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C14 | C24 | C34 | C44 | C15 | C25 | C35 | C45 |
| Type | F | F | F | F | F | F | F | F |

**Anisotropic Card 3.** Included for ANISO keyword option.

| Card 1b.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C55 | C16 | C26 | C36 | C46 | C56 | C66 | AOPT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| C11 | The 1, 1 term in the 6 × 6 anisotropic constitutive matrix. Note that 1 corresponds to the *a* material direction |
| C12 | The 1, 2 term in the 6 × 6 anisotropic constitutive matrix. Note that 2 corresponds to the *b* material direction |
| ⋮ | ⋮ |
| C66 | The 6, 6 term in the 6 × 6 anisotropic constitutive matrix. |
| AOPT | Material axes option (see Figure M2-1 and the Material Directions section):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. When this option is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work correctly. For shells only, the material axes are then rotated |

| VARIABLE | DESCRIPTION |
|---|---|
| | about the normal vector to the surface of the shell by the angle BETA. |
| | EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element (see Figure M2-1). The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: \|AOPT\| is a coordinate system ID (see *DEFINE_COORDINATE_*OPTION*). |

**Local Coordinate System Card 1.** Required for all keyword options.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | IHIS |
| Type | F | F | F | F | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF when finding the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA rotates the axes for all AOPT options. Otherwise, unless AOPT = 3, the material axes will be switched as specified by MACF, but no BETA rotation will occur. If AOPT = 3, then BETA input on Card 3 rotates the axes. |
| IHIS | Flag for anisotropic stiffness terms initialization (for solid elements only):<br>EQ.0: C11, C12, … from Cards 1b.1, 1b.2, and 1.b3 are used.<br>EQ.1: C11, C12, … are initialized with history data from *INITIAL_STRESS_SOLID. |

**Local Coordinate System Card 2.** Required for all keyword options.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4. |
| D1, D2, D3 | Components of vector **d** for AOPT = 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Angle in degrees of a material rotation about the c-axis, available for AOPT = 0 (shells and tshells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |
| REF | Flag to use reference geometry specified with *INITIAL_FOAM_REFERENCE_GEOMETRY to initialize the stress tensor.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Remarks:**

1. **Stress-strain material law.** The material law that relates stresses to strains is defined as:

$$\mathbf{C} = \mathbf{T}^{\mathrm{T}}\mathbf{C}_L\mathbf{T}$$

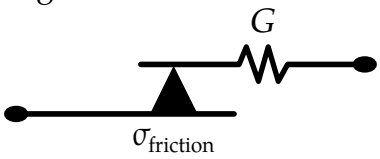
where $\mathbf{T}$ is a transformation matrix, and $\mathbf{C}_L$ is the constitutive matrix defined in terms of the material constants of the orthogonal material axes, $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$. The inverse of $\mathbf{C}_L$for the orthotropic case is defined as:

$$\mathbf{C}_L^{-1} = \begin{bmatrix} \frac{1}{E_a} & -\frac{v_{ba}}{E_b} & -\frac{v_{ca}}{E_c} & 0 & 0 & 0 \\ -\frac{v_{ab}}{E_a} & \frac{1}{E_b} & -\frac{v_{cb}}{E_c} & 0 & 0 & 0 \\ -\frac{v_{ac}}{E_a} & -\frac{v_{bc}}{E_b} & \frac{1}{E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ab}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{bc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}} \end{bmatrix}$$

where,

$$\frac{v_{ab}}{E_a} = \frac{v_{ba}}{E_b} \ , \qquad \frac{v_{ca}}{E_c} = \frac{v_{ac}}{E_a} \ , \qquad \frac{v_{cb}}{E_c} = \frac{v_{bc}}{E_b} \ .$$

2. **Frequency-independent damping.** Frequency independent damping is obtained by having a spring and slider in series as shown in the following sketch:




This option applies only to orthotropic solid elements and affects only the deviatoric stresses.

3. **Poisson's ratio.** PRBA is the minor Poisson's ratio if EA > EB, and the major Poisson's ratio will be equal to PRBA(EA/EB). If EB > EA, then PRBA is the major Poisson's ratio. PRCA and PRCB are similarly defined. They are the minor Poisson's ratio if EA > EC or EB > EC, and the major Poisson's ratio if the EC > EA or EC > EB.

Care should be taken when using material parameters from third party products regarding the directional indices $a$, $b$ and $c$, as they may differ from the definition used in LS-DYNA.

4. **History variables.** This material has the following history variables. Note that for shells and thick shells with element formulations 1, 2, and 6 the history variable labeled as "effective plastic strain" by LS-PrePost is stiffness component $C_{11}$.

| History Variable # | Description (solids and thick shells 3, 5, and 7) | Description (shells and thick shells 1, 2 and 6) |
|---|---|---|
| 1 | Deformation gradient component $F_{11}$ | Stiffness component $C_{12}$ |
| 2 | Deformation gradient component $F_{12}$ | Stiffness component $C_{13}$ |
| 3 | Deformation gradient component $F_{13}$ | Stiffness component $C_{14}$ |
| 4 | Deformation gradient component $F_{21}$ | Stiffness component $C_{22}$ |
| 5 | Deformation gradient component $F_{22}$ | Stiffness component $C_{23}$ |
| 6 | Deformation gradient component $F_{23}$ | Stiffness component $C_{24}$ |
| 7 | Deformation gradient component $F_{31}$ | Stiffness component $C_{33}$ |
| 8 | Deformation gradient component $F_{32}$ | Stiffness component $C_{34}$ |
| 9 | Deformation gradient component $F_{33}$ | Stiffness component $C_{44}$ |

| History Variable # | Description (solids and thick shells 3, 5, and 7) | Description (shells and thick shells 1, 2 and 6) |
|---|---|---|
| 10 | | Stiffness component $C_{55}$ |
| 11 | | Stiffness component $C_{56}$ |
| 12 | | Stiffness component $C_{66}$ |

**Material Directions:**

We will give an overview of how LS-DYNA finds the principal material directions for solid and shell elements for this material and other anisotropic materials based on the input. We will call the material coordinate system the {**a**, **b**, **c**} coordinate system. The AOPT options illustrated in Figure M2-1 define the preliminary {**a**, **b**, **c**} system for all elements of the parts that use the material, but this is not the final material direction. The {**a**, **b**, **c**} system defined by the AOPT options may be offset by a final rotation about the **c**-axis. The offset angle we call BETA. Note that *ELEMENT_SOLID_ORTHO and *ELEMENT_SHELL_MCID allow you to set the preliminary {**a**, **b**, **c**} coordinate system for individual solid and shell elements, instead of using the preliminary system created with AOPT. Figures M2-2 and M2-3 give the flow chart for finding the final material direction based on the input to the *MAT keyword and the *ELEMENT keyword. As indicated in the figures, the orientation of the final material axes is updated continuously through the simulation as the element moves and deforms, in accordance with the invariant numbering scheme (INN in *CONTROL_ACCURACY).

For solid elements, the BETA angle is specified in one of two ways. When using AOPT = 3, the BETA parameter defines the offset angle for all elements that use the material. The BETA parameter on *MAT has no meaning for the other AOPT options. Alternatively, a BETA angle can be defined for individual solid elements as described in Remark 5 for *ELEMENT_SOLID_ORTHO. The BETA angle set using the ORTHO option is available for all values of AOPT, and it overrides the BETA angle on the *MAT card for AOPT = 3 (see Figure M2-2). Note that when the BETA angle is applied in either case depends on the value of MACF (if available) of the *MAT keyword. With MACF two of the material directions may be switched.

There are two fundamental differences between shell and solid element orthotropic materials. (In the following discussion, tshell elements fall into the "shell" category.) First, the **c**-direction is always normal to a shell element such that the **a** and **b**-directions are within the plane of the element. Second, for some anisotropic materials, shell elements may have unique fiber directions within each layer through the thickness of the element so that a layered composite can be modeled with a single element.

As a result of the **c**-direction being defined by the element normal, MACF does not apply to shells. Similarly, AOPT = 1 and AOPT = 4 are not available for shells. Also, AOPT = 2 requires only the vector **a** be specified since **d** is not used. The shell procedure projects the inputted **a**-direction onto each element surface.

Note that when AOPT = 0 is used in two-dimensional planar and axisymmetric analysis, it is critical that the nodes in the element definition be numbered counterclockwise for this option to work correctly.

Similar to solid elements, the $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ coordinate system determined by AOPT is then modified by a rotation about the **c**-axis which we will call $\phi$. For those materials that allow a unique rotation angle for each integration point through the element thickness, the rotation angle is calculated by

$$\phi_i = \beta + \beta_i \ ,$$

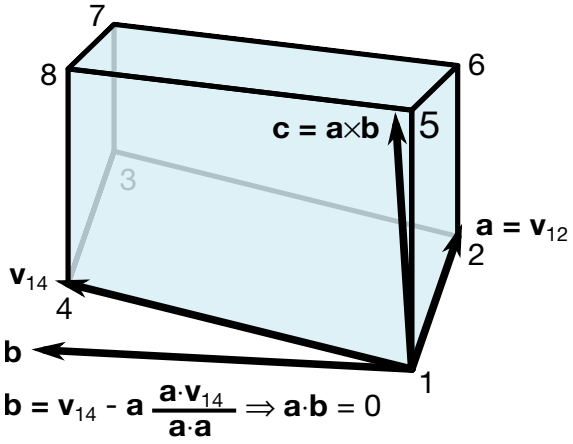
where $\beta$ is a rotation for the element, and $\beta_i$ is the rotation for the $i^{\text{th}}$ layer of the element. The $\beta$ angle can be input using the BETA parameter on the *MAT data but will be overridden for individual elements if *ELEMENT_SHELL_BETA (*ELEMENT_TSHELL_BETA) is used. The $\beta_i$ angles are input using the ICOMP = 1 option of *SECTION_SHELL (*SECTION_TSHELL) or with *PART_COMPOSITE (*PART_COMPOSITE_TSHELL). If $\beta$ or $\beta_i$ is omitted, they are assumed to be zero.

All anisotropic shell materials have the BETA parameter on the *MAT card available for both AOPT = 0 and AOPT = 3, except for materials 91 and 92 which have it available (but called FANG instead of BETA) for AOPT = 0, 2, and 3.

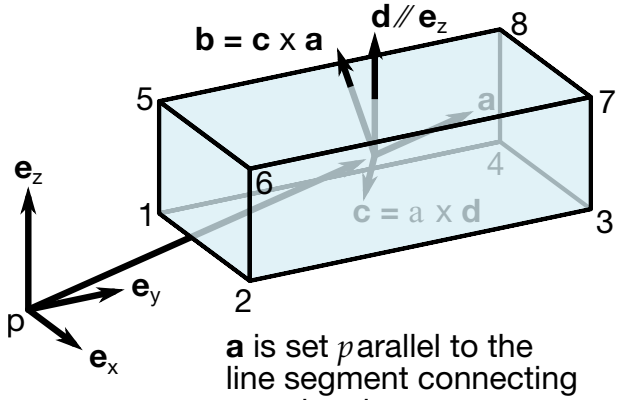
All anisotropic shell materials allow an angle for each integration point through the thickness, $\beta_i$, *except for* materials 2, 86, 91, 92, 117, 130, 170, 172, and 194.

**Illustration of AOPT: Figure M2-1**

**AOPT = 0.0**




**AOPT = 1.0 (solid only)**

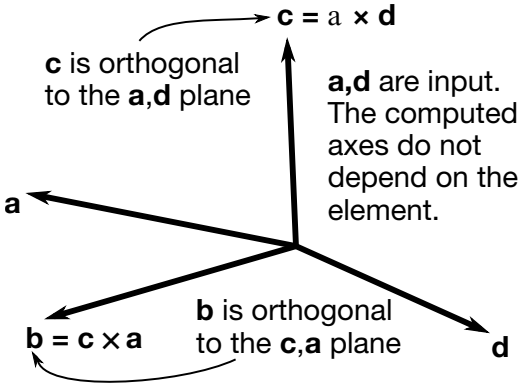



**a** is set $p$arallel to the line segment connecting $p$ to the element center.

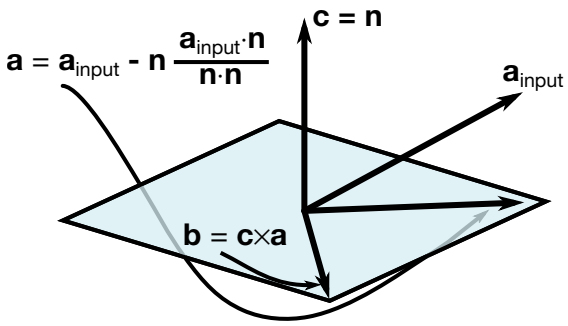
**d** is set $p$arallel to $\mathbf{e}_z$.

input($p$) → {**a**} → {**c**} → {**b**}

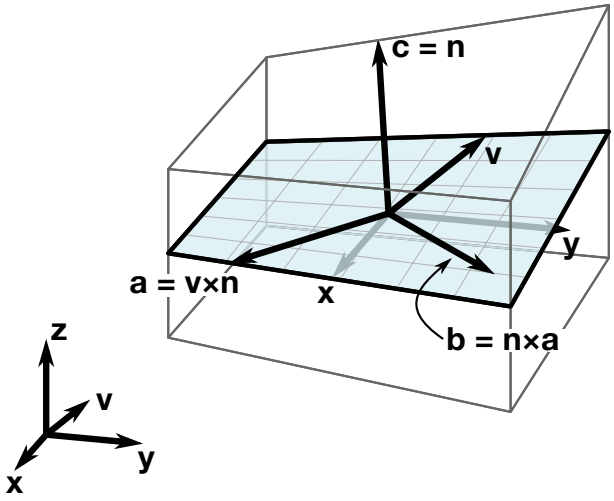
**AOPT = 2.0 (solid)**



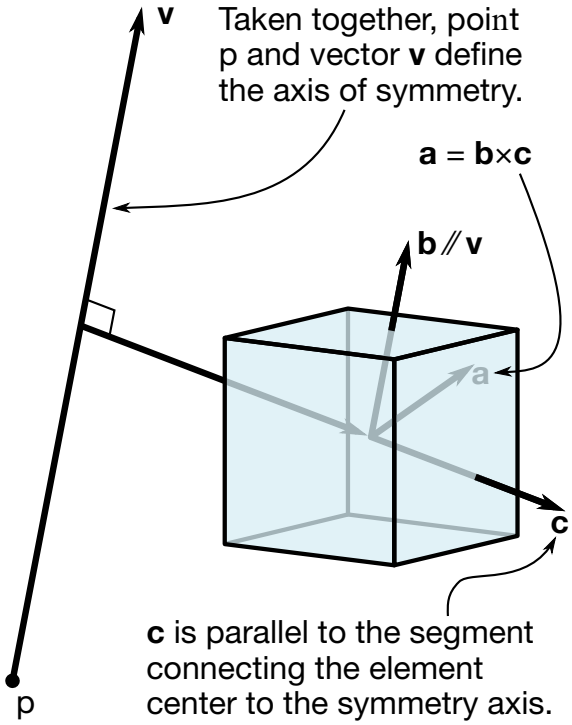

**AOPT = 2.0 (shell)**



**AOPT = 3.0**




**AOPT = 4.0 (solid only)**



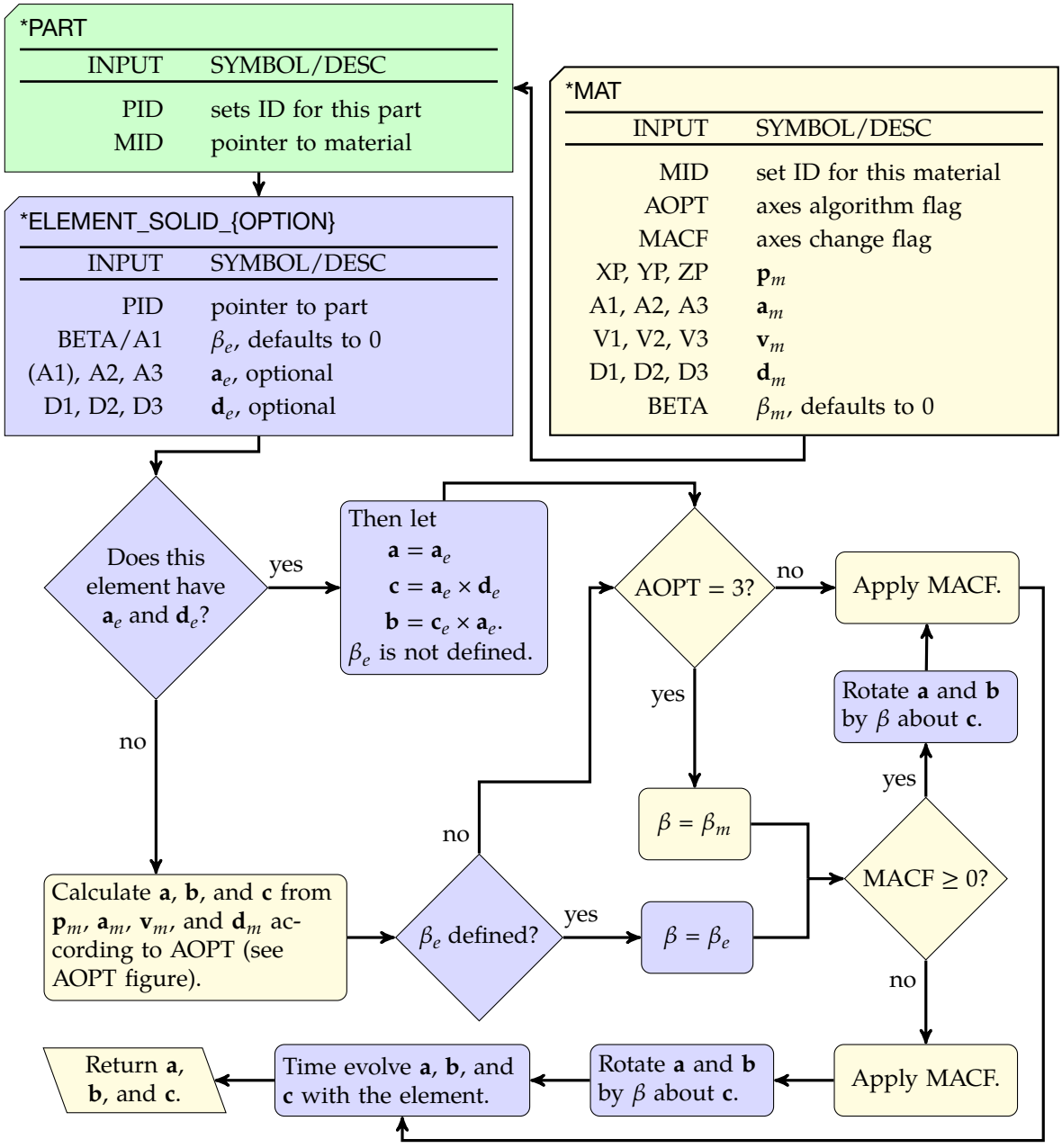



**Figure M2-2.** Flowchart showing how for each solid element LS-DYNA determines the vectors $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ from the input.

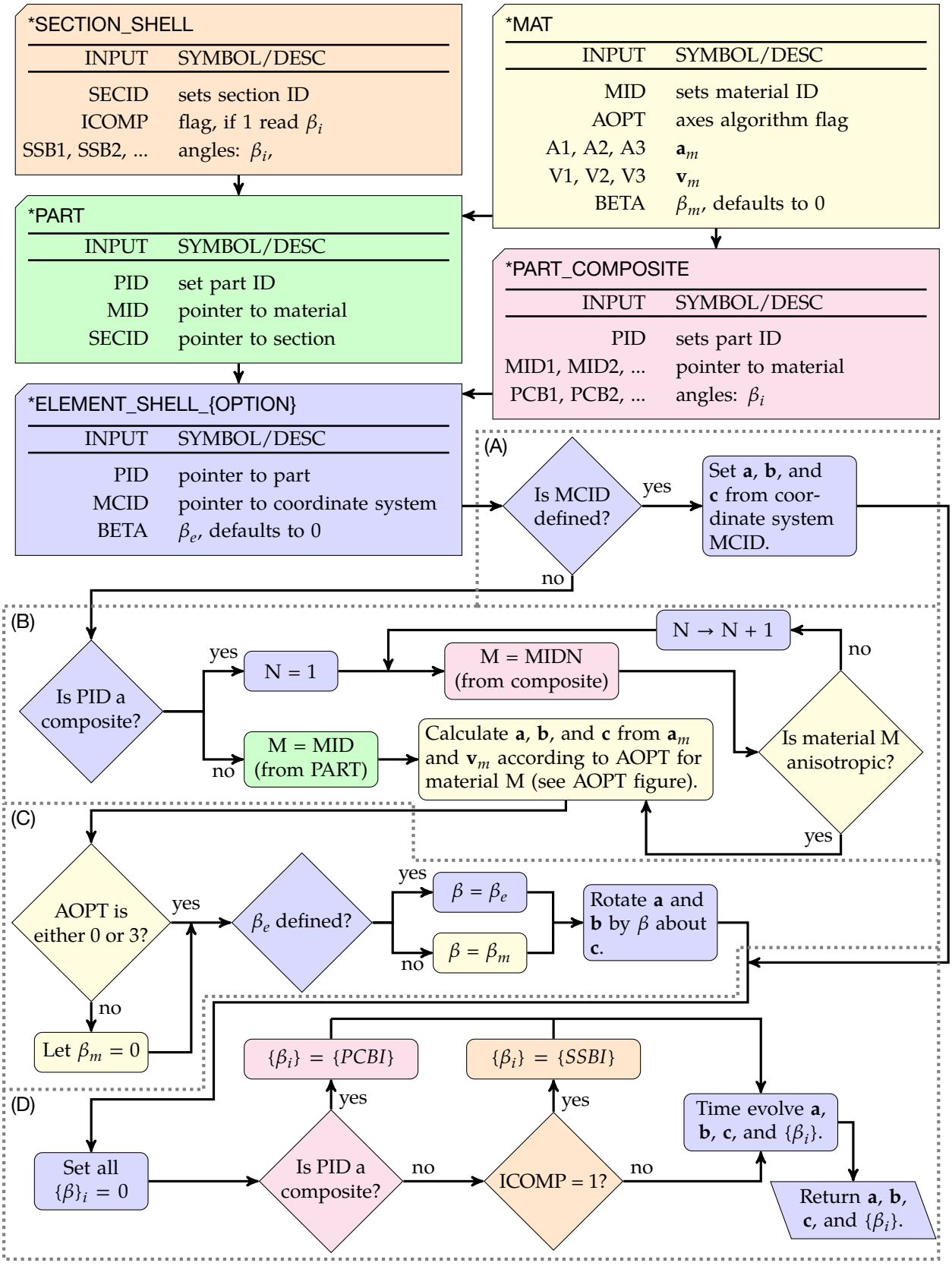



**Figure M2-3.** Flowchart for shells: (a) check for coordinate system ID; (b) process AOPT; (c) determine $\beta$; and (d) for each layer determine $\beta_i$.

## *MAT_PLASTIC_KINEMATIC

This is Material Type 3. This model is suitable for modeling isotropic and kinematic hardening plasticity with the option to include rate effects. It is a very cost-effective model and is available for beam (Hughes-Liu, truss, and Belytschko integrated solid and tubular beams), shell, thick shell, and solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | BETA | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | none | 0.0 | 0.0 | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SRC | SRP | FS | VP | | | | |
| Type | F | F | F | F | | | | |
| Default | 0.0 | 0.0 | $10^{20}$ | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus; see Figure M3-1. |
| BETA | Hardening parameter, $0 < \beta' < 1$. See Remark 2. |

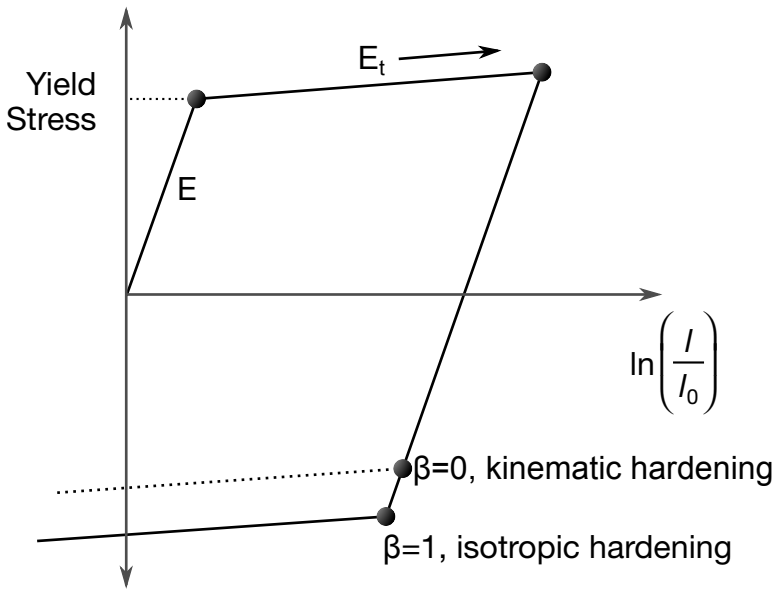



**Figure M3-1.** Elastic-plastic behavior with kinematic and isotropic hardening where $l_0$ and $l$ are undeformed and deformed lengths of uniaxial tension specimen. $E_t$ is the slope of the bilinear stress strain curve.

| VARIABLE | DESCRIPTION |
|---|---|
| SRC | Strain rate parameter, $C$, for the Cowper Symonds strain rate model; see Remark 1. If zero, rate effects are not considered. |
| SRP | Strain rate parameter, $p$, for Cowper Symonds strain rate model; see Remark 1. If zero, rate effects are not considered. |
| FS | Effective plastic strain for eroding elements |
| VP | Formulation for rate effects:<br>EQ.0.0: scale yield stress (default)<br>EQ.1.0: viscoplastic formulation (recommended) |

**Remarks:**

1. **Cowper Symonds Strain Rate Model.** Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p}$$

where $\dot{\varepsilon}$ is the strain rate. A fully viscoplastic formulation is optional which incorporates the Cowper and Symonds formulation within the yield surface. To ignore strain rate effects set both SRC and SRP to zero.

2. **Hardening Parameter.** Kinematic, isotropic, or a combination of kinematic and isotropic hardening may be specified by varying $\beta'$ between 0 and 1. For $\beta'$ equal to 0 and 1, respectively, kinematic and isotropic hardening are obtained as shown in Figure M3-1. For isotropic hardening, $\beta' = 1$, Material Model 12, *MAT_ISOTROPIC_ELASTIC_PLASTIC, requires less storage and is more efficient. Whenever possible, Material 12 is recommended for solid elements, but for shell elements, it is less accurate and thus Material 12 is not recommended in this case.

3. **History Variables.** This material has the following extra history variables.

| History Variable # | Description |
|---|---|
| 1 | Back stress component $xx$ |
| 2 | Back stress component $yy$ |
| 3 | Back stress component $xy$ |
| 4 | Back stress component $yz$ |
| 5 | Back stress component $zx$ |

## *MAT_ELASTIC_PLASTIC_THERMAL

This is Material Type 4. Temperature-dependent material coefficients can be defined with this material. This material needs at least two temperatures with their corresponding data specified but supports specifying a maximum of eight data points. This material type requires defining nodal temperatures by activating a coupled analysis or using another option, such as *LOAD_THERMAL_LOAD_CURVE or *LOAD_THERMAL_VARIABLE.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | R0 | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PR1 | PR2 | PR3 | PR4 | PR5 | PR6 | PR7 | PR8 |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
| Type | F | F | F | F | F | F | F | F |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGY1 | SIGY2 | SIGY3 | SIGY4 | SIGY5 | SIGY6 | SIGY7 | SIGY8 |
| Type | F | F | F | F | F | F | F | F |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ETAN1 | ETAN2 | ETAN3 | ETAN4 | ETAN5 | ETAN6 | ETAN7 | ETAN8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| T*i* | Temperatures. The minimum required is 2 while the maximum allowed is 8. |
| E*i* | Corresponding Young's modulus at temperature T*i* |
| PR*i* | Corresponding Poisson's ratio at temperature T*i* |
| ALPHA*i* | Corresponding coefficient of thermal expansion at temperature T*i* |
| SIGY*i* | Corresponding yield stress at temperature T*i* |
| ETAN*i* | Corresponding plastic hardening modulus at temperature T*i* |

**Remarks:**

1. **Material model.** The stress update for this material follows the standard approach for hypo-elastoplasticity, using the Jaumann rate for objectivity. The rate of Cauchy stress $\sigma$ can be expressed as

$$\dot{\sigma} = \mathbf{C}\left(\dot{\varepsilon} - \dot{\varepsilon}_T - \dot{\varepsilon}_p\right) + \dot{\mathbf{C}}\mathbf{C}^{-1}\sigma \ ,$$

where $\mathbf{C}$ is the temperature-dependent isotropic elasticity tensor, $\dot{\varepsilon}$ is the rate of deformation, $\dot{\varepsilon}_T$ is the thermal strain rate, and $\dot{\varepsilon}_p$ is the plastic strain rate. The coefficient of thermal expansion is defined as the instantaneous value. Thus, the thermal strain rate becomes

$$\dot{\varepsilon}_T = \alpha \dot{T}\mathbf{I}$$

where $\alpha$ is the temperature dependent thermal expansion coefficient, $\dot{T}$ is the rate of temperature and $\mathbf{I}$ is the identity tensor. Associated von Mises plasticity is adopted, resulting in

$$\dot{\varepsilon}_p = \dot{\varepsilon}_p \frac{3\mathbf{s}}{2\overline{\sigma}}$$

where $\dot{\varepsilon}_p$ is the effective plastic strain rate, $\mathbf{s}$ is the deviatoric stress tensor, and $\overline{\sigma}$ is the von Mises effective stress. The last term accounts for stress changes due to change in stiffness with respect to temperature, using the total elastic strain defined as $\varepsilon_e = \mathbf{C}^{-1}\sigma$. As a way to intuitively understand this contribution, the special case of small displacement elasticity, neglecting everything but the temperature-dependent elasticity parameters, gives

$$\dot{\sigma} = \frac{d}{dt}(\mathbf{C}\varepsilon) \ ,$$

showing that the stress may change without any change in strain.

2. **Model requirements and restrictions.** At least two temperatures and their corresponding material properties must be defined. The analysis will terminate if a material temperature falls outside the range specified in the input. If a thermo-elastic material is considered, do not define SIGY and ETAN.

3. **History variables.** This material has the following extra history variables.

| History Variable # | Description |
|---|---|
| 1 | Reference temperature |
| 3 | Current temperature |

## *MAT_SOIL_AND_FOAM

This is Material Type 5. It is a relatively simple material model for representing soil, concrete, or crushable foam. A table can be defined if thermal effects are considered in the pressure as a function of volumetric strain behavior.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | KUN | A0 | A1 | A2 | PC |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VCR | REF | LCID | | | | | |
| Type | F | F | F | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS9 | EPS10 | | | | | | |
| Type | F | F | | | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P9 | P10 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| KUN | Bulk modulus for unloading used for VCR = 0.0 |
| A0 | Yield function constant for plastic yield function below |
| A1 | Yield function constant for plastic yield function below |
| A2 | Yield function constant for plastic yield function below |
| PC | Pressure cutoff for tensile fracture (< 0) |
| VCR | Volumetric crushing option:<br>EQ.0.0: on<br>EQ.1.0: loading and unloading paths are the same |
| REF | Use reference geometry to initialize the pressure. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY. This option does not initialize the deviatoric stress state.<br>EQ.0.0: off<br>EQ.1.0: on |
| LCID | Load curve ID for compressive pressure (ordinate) as a function of volumetric strain (abscissa). If LCID is defined, then the curve is used instead of the input for EPS*i* and P*i*. It makes no difference whether the values of volumetric strain in the curve are input as positive or negative since internally, a negative sign is applied to the absolute value of each abscissa entry. If LCID refers to a table, the response is extended to being temperature dependent. |

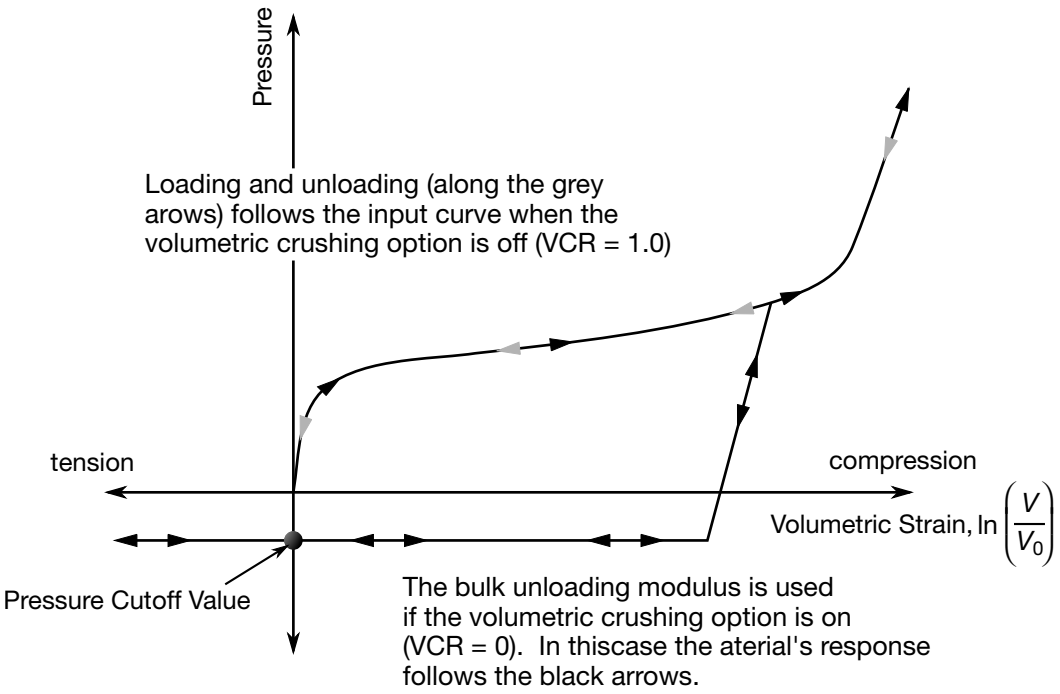



**Figure M5-1.** Pressure as a function of volumetric strain curve for soil and crushable foam model. The volumetric strain is given by the natural logarithm of the relative volume, $V$.

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1, … | Volumetric strain values in pressure as a function of volumetric strain curve (see Remarks below). A maximum of 10 values including 0.0 are allowed and a minimum of 2 values are necessary. If EPS1 is not 0.0, then a point $(0.0, 0.0)$ will be automatically generated and a maximum of nine values may be input. |
| P1, P2, …, PN | Pressures corresponding to volumetric strain values given on Cards 3 and 4. |

**Remarks:**

1. **Variable Definitions.** Pressure is positive in compression. Volumetric strain is given by the natural log of the relative volume and is negative in compression. Relative volume is a ratio of the current volume to the initial volume at the start of the calculation. The tabulated data should be given in order of increasing compression. If the pressure drops below the cutoff value specified, it is reset to that value. For a detailed description we refer to Krieg [1972].

2. **Yield Strength.** The deviatoric perfectly plastic yield function, $\phi$, is described in terms of the second invariant $J_2$,

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \ ,$$

pressure, $p$, and constants $a_0$, $a_1$, and $a_2$ as:

$$\phi = J_2 - \left[ a_0 + a_1 p + a_2 p^2 \right] \ .$$

On the yield surface $J_2 = \frac{1}{3}\sigma_y^2$ where $\sigma_y$ is the uniaxial yield stress, that is,

$$\sigma_y = \left[ 3\left( a_0 + a_1 p + a_2 p^2 \right) \right]^{1/2} \ .$$

There is no strain hardening on this surface.

To eliminate the pressure dependence of the yield strength, set:

$$a_1 = a_2 = 0 \quad \text{and} \quad a_0 = \frac{1}{3}\sigma_y^2 \ .$$

This approach is useful when a von Mises type elastic-plastic model is desired for use with the tabulated volumetric data.

3. **Equivalent Plastic Strain.** The history variable labeled as "plastic strain" by LS-PrePost is actually plastic volumetric strain. Note that when VCR = 1.0, plastic volumetric strain is zero.

## *MAT_VISCOELASTIC

This is Material Type 6. This material is for modeling viscoelastic behavior with beam (Hughes-Liu), shell, thick shell, and solid elements. Also see *MAT_GENERAL_VISCOELASTIC for a more general formulation.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | G0 | GI | BETA | | |
| Type | A | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Elastic bulk modulus.<br>LT.0.0: \|BULK\| is a load curve of bulk modulus as a function of temperature. |
| G0 | Short-time shear modulus, $G_0$; see the Remarks below.<br>LT.0.0: \|G0\| is a load curve of short-time shear modulus as a function of temperature. |
| GI | Long-time (infinite) shear modulus, $G_\infty$.<br>LT.0.0: \|GI\| is a load curve of long-time shear modulus as a function of temperature. |
| BETA | Decay constant.<br>LT.0.0: \|BETA\| is a load curve of decay constant as a function of temperature. |

### Remarks:

The shear relaxation behavior is described by [Hermann and Peterson, 1968]:

$$G(t) = G_\infty + (G_0 - G_\infty)\exp(-\beta t) .$$

A Jaumann rate formulation is used:

$$\overset{\nabla}{\sigma}'_{\mathrm{ij}} = 2\int_0^t G(t-\tau)D'_{ij}(\tau)d\tau \ ,$$

where the prime denotes the deviatoric part of the stress rate, $\overset{\nabla}{\sigma}_{ij}$, and the strain rate, $D_{ij}$.

## *MAT_BLATZ-KO_RUBBER

This is Material Type 7. This one-parameter material enables modeling nearly incompressible continuum rubber. The Poisson's ratio is fixed to 0.463.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | REF | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| REF | Use reference geometry to initialize the stress tensor. *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details) defines the reference geometry.<br>EQ.0.0: off<br>EQ.1.0: on |

**Remarks:**

1. **Stress-strain relationship.** The strain energy density potential for the Blatz-Ko rubber is

$$W(\mathbf{C}) = \frac{G}{2}\left[I_1 - 3 + \frac{1}{\beta}\left(I_3^{-\beta} - 1\right)\right]$$

where $G$ is the shear modulus, $I_1$ and $I_3$ are the first and third invariants of the right Cauchy-Green tensor $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$, and

$$\beta = \frac{v}{1-2v} \ .$$

The second Piola-Kirchhoff stress is computed as

$$\mathbf{S} = 2\frac{\partial W}{\partial \mathbf{C}} = G\left[\mathbf{I} - I_3^{-\beta}\mathbf{C}^{-1}\right]$$

from which the Cauchy stress is obtained by a push-forward from the reference to the current configuration divided by the relative volume $J = \det(\mathbf{F})$,

$$\sigma = \frac{1}{J}\mathbf{F}\mathbf{S}\mathbf{F}^{\mathrm{T}} = \frac{G}{J}\left[\mathbf{B} - I_3^{-\beta}\mathbf{I}\right].$$

Here we use $\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathrm{T}}$ to denote the left Cauchy-Green tensor, and Poisson's ratio, $v$, above is set internally to $v = 0.463$; also see Blatz and Ko [1962].

2. **History variables.** For solids, the nine history variables store the deformation gradient, whereas for shells, the gradient is stored in the slot for effective plastic strain along with the first eight history variables (the ninth stores an internal flag). If a dynain file is created using *INTERFACE_SPRINGBACK_LSDYNA, then NHSV should be set to 9 so that the *INITIAL_STRESS_SHELL cards have the correct deformation gradient from which the stresses are to be calculated.

## *MAT_HIGH_EXPLOSIVE_BURN

This is Material Type 8. It allows the modeling of the detonation of a high explosive. In addition, an equation of state must be defined. See Wilkins [1969] and Giroux [1973].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | D | PCJ | BETA | K | G | SIGY |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| D | Detonation velocity |
| PCJ | Chapman-Jouget pressure |
| BETA | Beta burn flag (see remarks below):<br>EQ.0.0: Beta and programmed burn<br>EQ.1.0: Beta burn only<br>EQ.2.0: Programmed burn only |
| K | Bulk modulus (BETA = 2.0 only) |
| G | Shear modulus (BETA = 2.0 only) |
| SIGY | $\sigma_y$, yield stress (BETA = 2.0 only) |

### Remarks:

Burn fractions, $F$, which multiply the equations of states for high explosives, control the release of chemical energy for simulating detonations. At any time, the pressure in a high explosive element is given by:

$$p = Fp_{\text{eos}}(V, E) ,$$

where $p_{\text{eos}}$ is the pressure from the equation of state (either types 2, 3, or 14), $V$ is the relative volume, and $E$ is the internal energy density per unit initial volume.

In the initialization phase, a lighting time, $t_l$, is computed for each element by dividing the distance from the detonation point to the center of the element by the detonation velocity, $D$. If multiple detonation points are defined, the closest detonation point determines $t_l$. The burn fraction $F$ is taken as the maximum,

$$F = \max(F_1, F_2) \ ,$$

where

$$F_1 = \begin{cases} \dfrac{2\ (t - t_l) D A_{e_{\max}}}{3 v_e} & \text{if } t > t_l \\ 0 & \text{if } t \le t_l \end{cases}$$

$$F_2 = \beta = \frac{1 - V}{1 - V_{CJ}}$$

where $V_{CJ}$ is the Chapman-Jouguet relative volume and $t$ is current time. If $F$ exceeds 1, it is reset to 1. This calculation of the burn fraction usually requires several time steps for $F$ to reach unity, thereby spreading the burn front over several elements. After reaching unity, $F$ is held constant. This burn fraction calculation is based on work by Wilkins [1964] and is also discussed by Giroux [1973].

If the beta burn option is used, BETA = 1.0, any volumetric compression will cause detonation and

$$F = F_2 \ .$$

$F_1$ is not computed. BETA = 1 does not allow for the initialization of the lighting time.

If the programmed burn is used, BETA = 2.0, the undetonated high explosive material will behave as an elastic perfectly plastic material if the bulk modulus, shear modulus, and yield stress are defined. Therefore, with this option the explosive material can compress without causing detonation. The location and time of detonation is controlled by *INITIAL_DETONATION.

As an option, the high explosive material can behave as an elastic perfectly-plastic solid prior to detonation. In this case we update the stress tensor, to an elastic trial stress, $*s_{ij}^{n+1}$,

$$*s_{ij}^{n+1} = s_{ij}^{n} + s_{ip}\Omega_{pj} + s_{jp}\Omega_{pi} + 2G\dot{\varepsilon}'_{ij}dt$$

where $G$ is the shear modulus, and $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. The von Mises yield condition is given by:

$$\phi = J_2 - \frac{\sigma_y^2}{3} \ ,$$

where the second stress invariant, $J_2$, is defined in terms of the deviatoric stress components as

$$J_2 = \frac{1}{2} s_{ij} s_{ij}$$

and the yield stress is $\sigma_y$. If yielding has occurred, namely, $\phi > 0$, the deviatoric trial stress is scaled to obtain the final deviatoric stress at time $n + 1$:

$$s_{ij}^{n+1} = \frac{\sigma_{\mathrm{y}}}{\sqrt{3J_2}} * s_{ij}^{n+1}$$

If $\phi \leq 0$, then

$$s_{ij}^{n+1} =* s_{ij}^{n+1}$$

Before detonation, pressure is given by the expression

$$p^{n+1} = \mathrm{K}\left(\frac{1}{V^{n+1}} - 1\right) ,$$

where $K$ is the bulk modulus. Once the explosive material detonates:

$$s_{ij}^{n+1} = 0 ,$$

and the material behaves like a gas.

### *MAT_NULL

This is Material Type 9.

In the case of solids and thick shells, this material allows equations of state to be considered without computing deviatoric stresses. Optionally, a viscosity can be defined. Also, erosion in tension and compression is possible.

Beams and shells that use this material type are completely bypassed in the element processing; however, the mass of the null beam or shell elements is computed and added to the nodal points which define the connectivity. The mass of null beams is ignored if the value of the density is less than $10^{-11}$. The Young's modulus and Poisson's ratio are used only for setting the contact stiffness, and it is recommended that reasonable values be input. The variables PC, MU, TEROD, and CEROD do not apply to beams and shells. Historically, null beams and/or null shells have been used as an aid in modeling of contact, but this practice is now seldom needed.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MU | TEROD | CEROD | YM | PR |
| Type | A | F | F | F | F | F | F | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0). See Remark 4. |
| MU | Dynamic viscosity, $\mu$ (optional). See Remark 1. |
| TEROD | Relative volume. $V/V_0$, for erosion in tension. Typically, use values greater than unity. If zero, erosion in tension is inactive. |
| CEROD | Relative volume, $V/V_0$, for erosion in compression. Typically, use values less than unity. If zero, erosion in compression is inactive. |
| YM | Young's modulus (used for null beams and shells only) |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio (used for null beams and shells only) |

**Remarks:**

These remarks apply to solids and thick shells only.

1. **Material Model.** When used with solids or thick shells, this material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form,

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij} \ ,$$

is computed for nonzero $\mu$, where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate and $\mu$ is the dynamic viscosity. Analyzing the dimensions of the above equation gives units of the components in SI of

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right] \ .$$

Therefore, $\mu$ may have units of [Pa × s].

2. **Hourglass Control.** Null materials have no shear stiffness (except from viscosity) and hourglass control must be used with great care. In some applications, the default hourglass coefficient may lead to significant energy losses. In general, for fluids the hourglass coefficient QM should be small (in the range $10^{-6}$ to $10^{-4}$), and the hourglass type IHQ should be set to 1 (default).

3. **Yield Strength.** The Null material has no yield strength and behaves in a fluid-like manner.

4. **Cut-off Pressure.** The cut-off pressure, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting the PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

## *MAT_ELASTIC_PLASTIC_HYDRO_{OPTION}

This is Material Type 10. This material is for modeling an elastic-plastic hydrodynamic material and requires an equation of state (*EOS).

Available options include:

<BLANK>

SPALL

STOCHASTIC

The keyword card can appear in three ways:

*MAT_ELASTIC_PLASTIC_HYDRO or MAT_010

*MAT_ELASTIC_PLASTIC_HYDRO_SPALL or MAT_010_SPALL

*MAT_ELASTIC_PLASTIC_HYDRO_STOCHASTIC or MAT_010_STOCHASTIC

### Card Summary:

**Card 1.** This card is required.

| MID | RO | G | SIGO | EH | PC | FS | CHARL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is included if and only if the SPALL keyword option is used.

| A1 | A2 | SPALL | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| EPS9 | EPS10 | EPS11 | EPS12 | EPS13 | EPS14 | EPS15 | EPS16 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| ES9 | ES10 | ES11 | ES12 | ES13 | ES14 | ES15 | ES16 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | SIG0 | EH | PC | FS | CHARL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | 0.0 | 0.0 | -∞ | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| SIG0 | Yield stress; see Remark 2. |
| EH | Plastic hardening modulus; see Remark 2. |
| PC | Pressure cutoff (≤ 0.0). If zero, a cutoff of -∞ is assumed. |
| FS | Effective plastic strain at which erosion occurs. |
| CHARL | Characteristic element thickness for deletion. This applies to 2D solid elements that lie on a boundary of a part. If the boundary element thins down due to stretching or compression, and if it thins to a value less than CHARL, the element will be deleted. The primary application of this option is to predict the break-up of axisymmetric shaped charge jets. |

**Spall Card.** Additional card for SPALL keyword option.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A1 | A2 | SPALL | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1 | Linear pressure hardening coefficient |
| A2 | Quadratic pressure hardening coefficient |
| SPALL | Spall type (see Remark 3):<br>EQ.0.0: Default set to "1.0"<br>EQ.1.0: Tensile pressure is limited by PC, that is, $p$ is always ≥ PC.<br>EQ.2.0: If $\sigma_{max} \geq -$PC element spalls and tension, $p < 0$, is never allowed.<br>EQ.3.0: $p <$ PC element spalls and tension, $p < 0$, is never allowed. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS9 | EPS10 | EPS11 | EPS12 | EPS13 | EPS14 | EPS15 | EPS16 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS*i* | Effective plastic strain (true). Define up to 16 values. Care must be taken that the full range of strains expected in the analysis is covered. Linear extrapolation is used if the strain values exceed the maximum input value. See Remark 2. |

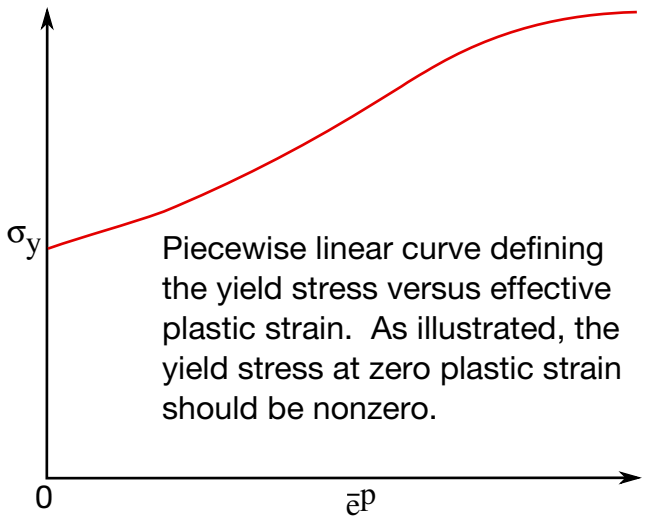



**Figure M10-1.** Effective stress as a function of effective plastic strain curve. See EPS and ES input.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES9 | ES10 | ES11 | ES12 | ES13 | ES14 | ES15 | ES16 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ES*i* | Effective stress. Define up to 16 values. See Remark 2. |

**Remarks:**

1. **Model Overview.** This model is applicable to a wide range of materials, including those with pressure-dependent yield behavior. The use of 16 points in the yield stress as a function of effective plastic strain curve allows complex post-yield hardening behavior to be accurately represented. In addition, the incorporation of an equation of state permits accurate modeling of a variety of different

materials. The spall model options permit incorporation of material failure, fracture, and disintegration effects under tensile loads.

The STOCHASTIC option allows spatially varying yield and failure behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

2. **Yield Stress and Plastic Hardening Modulus.** If ES and EPS values are undefined, the yield stress and plastic hardening modulus are taken from SIG0 and EH. In this case, the bilinear stress-strain curve shown in M10-1 is obtained with hardening parameter, $\beta = 1$. The yield strength is calculated as

$$\sigma_y = \sigma_0 + E_h\bar{\varepsilon}^p + (a_1 + pa_2)\max[p, 0] \ .$$

The quantity $E_h$ is the plastic hardening modulus defined in terms of Young's modulus, $E$, and the tangent modulus, $E_t$, as follows

$$E_h = \frac{E_t E}{E - E_t}.$$

The pressure, $p$, is taken as positive in compression.

If ES and EPS are specified, a curve like that shown in Figure M10-1 may be defined. Effective stress is defined in terms of the deviatoric stress tensor, $s_{ij}$, as:

$$\overline{\sigma} = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{1/2}$$

and effective plastic strain by:

$$\bar{\varepsilon}^p = \int_0^t \left(\frac{2}{3}D_{ij}^p D_{ij}^p\right)^{1/2} dt \ ,$$

where $t$ denotes time and $D_{ij}^p$ is the plastic component of the rate of deformation tensor. In this case the plastic hardening modulus on Card 1 is ignored and the yield stress is given as

$$\sigma_y = f(\bar{\varepsilon}^p) \ ,$$

where the value for $f(\bar{\varepsilon}^p)$ is found by interpolating the data curve.

3. **Spall Models.** A choice of three spall models is offered to represent material splitting, cracking, and failure under tensile loads.

   a) The pressure limit model, SPALL = 1, limits the hydrostatic tension to the specified value, $p_{\text{cut}}$. If pressures more tensile than this limit are calculated, the pressure is reset to $p_{\text{cut}}$. This option is not strictly a spall model, since the deviatoric stresses are unaffected by the pressure reaching the tensile cutoff, and the pressure cutoff value, $p_{\text{cut}}$, remains unchanged throughout the analysis.

b) The maximum principal stress spall model, SPALL = 2, detects spall if the maximum principal stress, $\sigma_{max}$, exceeds the limiting value -$p_{cut}$. Note that the negative sign is required because $p_{cut}$ is measured positive in compression, while $\sigma_{max}$ is positive in tension. Once spall is detected with this model, the deviatoric stresses are reset to zero, and no hydrostatic tension ($p < 0$) is permitted. If tensile pressures are calculated, they are reset to 0 in the spalled material. Thus, the spalled material behaves as a rubble or incohesive material.

c) The hydrostatic tension spall model, SPALL = 3, detects spall if the pressure becomes more tensile than the specified limit, $p_{cut}$. Once spall is detected the deviatoric stresses are reset to zero, and nonzero values of pressure are required to be compressive (positive). If hydrostatic tension ($p < 0$) is subsequently calculated, the pressure is reset to 0 for that element.

### *MAT_STEINBERG

This is Material Type 11. This material is available for modeling materials deforming at very high strain rates (> $10^5$ per second). The yield strength is a function of temperature and pressure. An equation of state determines the pressure.

This model applies to a wide range of materials, including those with pressure-dependent yield behavior. In addition, the incorporation of an equation of state permits accurate modeling of a variety of different materials. The spall model options permit the incorporation of material failure, fracture, and disintegration effects under tensile loads.

This material is available for solid, SPH, ALE, and thick shell (types 3, 5, and 7) elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | GO | SIGO | BETA | N | GAMA | SIGM |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B | BP | H | F | A | TMO | GAMO | SA |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PC | SPALL | RP | FLAG | MMN | MMX | ECO | EC1 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EC2 | EC3 | EC4 | EC5 | EC6 | EC7 | EC8 | EC9 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G0 | Basic shear modulus. See Remark 2. |
| SIGO | $\sigma_o$; see Remark 3 below. |
| BETA | $\beta$; see Remark 3 below. |
| N | $n$; see Remark 3 below. |
| GAMA | $\gamma_i$, initial plastic strain; see Remark 3 below. |
| SIGM | $\sigma_m$; see Remark 3. |
| B | $b$; see Remark 2. |
| BP | $b'$; see Remark 3. |
| H | $h$; see Remarks 2 and 3. |
| F | $f$; see Remark 3. |
| A | Atomic weight (if = 0.0, RP must be defined). See Remark 2. |
| TMO | $T_{mo}$; see Remark 2. |
| GAMO | $\gamma_o$; see Remark 2. |
| SA | $a$; see Remark 2. |
| PC | Pressure cutoff (default = $-10^{30}$). See Remark 5. |
| SPALL | Spall type (see Remark 5):<br>EQ.0.0: Default, set to "2.0"<br>EQ.1.0: $p \geq$ PC<br>EQ.2.0: If $\sigma_{max} \geq$ -PC, element spalls, and tension, $p < 0$, is never allowed.<br>EQ.3.0: If $p <$ PC element spalls, and tension, $p < 0$, is never allowed. |
| RP | $R'$. If $R' \neq 0.0$, $A$ is not defined. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| FLAG | Flag for cold compression energy fit (see Remarks 2 and 4):<br>EQ.0.0: $\eta$ coefficients (default)<br>EQ.1.0: $\mu$ coefficients |
| MMN | Optional $\mu$ or $\eta$ minimum value ($\mu_{\min}$ or $\eta_{\min}$), depending on FLAG. |
| MMX | Optional $\mu$ or $\eta$ maximum value ($\mu_{\max}$ or $\eta_{\max}$), depending on FLAG. |
| EC0, …, EC9 | Cold compression energy coefficients (optional). See Remark 2. |

**Remarks:**

1. **References.** Users who have an interest in this model are encouraged to study the paper by Steinberg and Guinan which provides the theoretical basis. Another useful reference is the KOVEC user's manual.

2. **Shear modulus.** In terms of the foregoing input parameters, we define the shear modulus, $G$, before the material melts as:

$$G = G_0\left[1 + bpV^{1/3} - h\left(\frac{E_i - E_c}{3R'} - 300\right)\right]e^{\frac{-fE_i}{E_m - E_i}} ,$$

where $p$ is the pressure, $V$ is the relative volume, $E_c$ (see Remark 4) is the cold compression energy, and $E_m$ is the melting energy. $E_c$ is given by:

$$E_c(x) = \int_0^x p(X)dX - \frac{900R'\exp(ax)}{(1-x)^{(\gamma_o - a)}}$$

with

$$x = 1 - V \ .$$

$E_m$ is defined as:

$$E_m(x) = E_c(x) + 3R'T_m(x) \ .$$

$E_m$ is in terms of the melting temperature $T_m(x)$:

$$T_m(x) = \frac{T_{\text{mo}}\exp(2ax)}{V^{2(\gamma_o - a - 1/3)}}$$

and the melting temperature at $\rho = \rho_o$, $T_{\text{mo}}$.

In the above equations $R'$ is defined by

$$R' = \frac{R\rho}{A},$$

where $R$ is the gas constant and $A$ is the atomic weight. If $R'$ is not defined, LS-DYNA computes it with $R$ in the cm-gram-microsecond system of units.

3. **Yield strength.** The yield strength, $\sigma_y$, is given by:

$$\sigma_y = \sigma_o' \left[1 + b' p V^{1/3} - h\left(\frac{E_i - E_c}{3R'} - 300\right)\right] e^{\frac{-fE_i}{E_m - E_i}}$$

if $E_m$ exceeds $E_i$ (see Remark 2). Here, $\sigma_o'$ is:

$$\sigma_o' = \sigma_o[1 + \beta(\gamma_i + \bar{\varepsilon}^p)]^n$$

where $\sigma_o$ is the initial yield stress and $\gamma_i$ is the initial plastic strain. If the work-hardened yield stress $\sigma_o'$ exceeds $\sigma_m$, $\sigma_o'$ is set equal to $\sigma_m$. After the materials melt, $\sigma_y$ and $G$ are set to one-half their initial value.

4. **Cold compression energy.** If the coefficients EC0, …, EC9 are not defined above, LS-DYNA will fit the cold compression energy to a ten-term polynomial expansion either as a function of $\mu$ or $\eta$ depending on field FLAG as:

$$E_c(\eta^i) = \sum_{i=0}^{9} \mathrm{EC}_i \eta^i$$

$$E_c(\mu^i) = \sum_{i=0}^{9} \mathrm{EC}_i \mu^i$$

where $\mathrm{EC}_i$ is the $i^{\text{th}}$ coefficient and:

$$\eta = \frac{\rho}{\rho_o}$$

$$\mu = \frac{\rho}{\rho_o} - 1$$

A linear least squares method is used to perform the fit.

5. **Spall models.** A choice of three spall models is offered to represent material splitting, cracking, and failure under tensile loads.

   a) *Pressure Limit Model.* The pressure limit model, SPALL = 1, limits the hydrostatic tension to the specified value, $p_{\text{cut}}$. If a pressure more tensile than this limit is calculated, the pressure is reset to $p_{\text{cut}}$. This option is not strictly a spall model, since the deviatoric stresses are unaffected by the pressure reaching the tensile cutoff, and the pressure cutoff value, $p_{\text{cut}}$, remains unchanged throughout the analysis.

   b) *Maximum Principal Stress Spall Model.* The maximum principal stress spall model, SPALL = 2, detects spall if the maximum principal stress, $\sigma_{\max}$, exceeds the limiting value $-p_{\text{cut}}$. Note that the negative sign is required

because $p_{cut}$ is measured positive in compression, while $\sigma_{max}$ is positive in tension. Once spall is detected with this model, the deviatoric stresses are reset to zero, and no hydrostatic tension ($p < 0$) is permitted. If tensile pressures are calculated, they are reset to 0 in the spalled material. Thus, the spalled material behaves as a rubble or incohesive material.

c) *Hydrostatic Tension Spall Model.* The hydrostatic tension spall model, SPALL = 3, detects spall if the pressure becomes more tensile than the specified limit, $p_{cut}$. Once spall is detected the deviatoric stresses are reset to zero, and nonzero values of pressure are required to be compressive (positive). If hydrostatic tension ($p < 0$) is subsequently calculated, the pressure is reset to 0 for that element.

## *MAT_STEINBERG_LUND

This is Material Type 11. This material is a modification of the Steinberg model above to include the rate model of Steinberg and Lund [1989]. An equation of state determines the pressure.

The keyword cards can appear in two ways:

*MAT_STEINBERG_LUND or MAT_011_LUND

### Card Summary:

**Card 1.** This card is required.

| MID | RO | G0 | SIGO | BETA | N | GAMA | SIGM |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| B | BP | H | F | A | TMO | GAMO | SA |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| PC | SPALL | RP | FLAG | MMN | MMX | EC0 | EC1 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| EC2 | EC3 | EC4 | EC5 | EC6 | EC7 | EC8 | EC9 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| UK | C1 | C2 | YP | YA | YM | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G0 | SIGO | BETA | N | GAMA | SIGM |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G0 | Basic shear modulus |
| SIGO | $\sigma_o$; see Remark 3 of *MAT_011. |
| BETA | $\beta$; see Remark 3 of *MAT_011. |
| N | $n$; see Remark 3 of *MAT_011. |
| GAMA | $\gamma_i$, initial plastic strain; see Remark 3 of *MAT_011. |
| SIGM | $\sigma_m$; see Remark 3 of *MAT_011. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B | BP | H | F | A | TMO | GAMO | SA |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| B | $b$; see Remark 2 of *MAT_011 |
| BP | $b'$; see Remark 3 of *MAT_011. |
| H | $h$; see Remarks 2 and 3 of *MAT_011. |
| F | $f$; see Remark 3 of *MAT_011. |
| A | Atomic weight (if = 0.0, RP must be defined). See Remark 2 of *MAT_011. |
| TMO | $T_{\mathrm{mo}}$; see Remark 2 of *MAT_011. |
| GAMO | $\gamma_o$; see Remark 2 of *MAT_011. |
| SA | $a$; see Remark 2 of *MAT_011. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PC | SPALL | RP | FLAG | MMN | MMX | EC0 | EC1 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EC2 | EC3 | EC4 | EC5 | EC6 | EC7 | EC8 | EC9 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| PC | Pressure cutoff ($p_{cut}$) or -$\sigma_f$ (default = -$10^{30}$) |
| SPALL | Spall type (see Remark 5 of *MAT_011):<br>EQ.0.0: Default, set to "2.0"<br>EQ.1.0: $p \geq$ PC<br>EQ.2.0: If $\sigma_{max} \geq$ -PC, element spalls, and tension, $p < 0$, is never allowed.<br>EQ.3.0: If $p <$ PC element spalls, and tension, $p < 0$, is never allowed. |
| RP | $R'$. If $R' \neq 0.0$, $A$ is not defined. See Remark 2 of *MAT_011. |
| FLAG | Flag for cold compression energy fit (see Remarks 2 and 4 of *MAT_011):<br>EQ.0.0: $\eta$ coefficients (default)<br>EQ.1.0: $\mu$ coefficients |
| MMN | Optional $\mu$ or $\eta$ minimum value ($\mu_{min}$ or $\eta_{min}$), depending on FLAG. |
| MMX | Optional $\mu$ or $\eta$ maximum value ($\mu_{max}$ or $\eta_{max}$), depending on FLAG. |
| EC0, …, EC9 | Cold compression energy coefficients (optional). See Remark 2 of *MAT_011. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UK | C1 | C2 | YP | YA | YM | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| UK | Activation energy for rate dependent model |
| C1 | Exponent pre-factor in rate dependent model |
| C2 | Coefficient of drag term in rate dependent model |
| YP | Peierls stress for rate dependent model |
| YA | A thermal yield stress for rate dependent model |
| YMAX | Work hardening maximum for rate model |

**Remarks:**

This model is similar in theory to the *MAT_STEINBERG above but with the addition of rate effects. When rate effects are included, the yield stress is given by:

$$\sigma_y = \{Y_T(\dot{\varepsilon}_p, T) + Y_A f(\varepsilon_p)\}\frac{G(p,T)}{G_0} \ .$$

There are two imposed limits on the yield stress. The first condition is on the nonthermal yield stress:

$$Y_A f(\varepsilon_p) = Y_A[1 + \beta(\gamma_i + \varepsilon^p)]^n \le Y_{\max}$$

and comes from the limit of the first term in $\sigma_y$ being small. In this case $Y_A f(\varepsilon_p)$ reduces to $\sigma_0'$ from the *MAT_011 material model (see Remark 3 of *MAT_011). The second limit is on the thermal part:

$$Y_T \le Y_P \ .$$

## *MAT_ISOTROPIC_ELASTIC_PLASTIC

This is Material Type 12. This is a very low cost isotropic plasticity model for three-dimensional solids. In the plane stress implementation for shell elements, a one-step radial return approach is used to scale the Cauchy stress tensor if the state of stress exceeds the yield surface. This approach to plasticity leads to inaccurate shell thickness updates and stresses after yielding. This is the only model in LS-DYNA for plane stress that does not default to an iterative approach.

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | SIGY | ETAN | BULK | | |
| Type | A | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus |
| BULK | Bulk modulus, $K$ |

**Remarks:**

The pressure is integrated in time from

$$\dot{p} = -K\dot{\varepsilon}_{ii} \ ,$$

where $\dot{\varepsilon}_{ii}$ is the volumetric strain rate.

## *MAT_ISOTROPIC_ELASTIC_FAILURE

This is Material Type 13. This is a non-iterative plasticity with simple plastic strain failure model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | SIGY | ETAN | BULK | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | 0.0 | none | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPF | PRF | REM | TREM | | | | |
| Type | F | F | F | F | | | | |
| Default | none | 0.0 | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus |
| BULK | Bulk modulus |
| EPF | Plastic failure strain |
| PRF | Failure pressure ($\leq 0.0$) |

| VARIABLE | DESCRIPTION |
|---|---|
| REM | Element erosion option:<br>EQ.0.0: failed element eroded after failure.<br>NE.0.0: element is kept, no removal except by $\Delta t$ below. |
| TREM | $\Delta t$ for element removal:<br>EQ.0.0: $\Delta t$ is not considered (default).<br>GT.0.0: element eroded if element time step size falls below $\Delta t$. |

**Remarks:**

When the effective plastic strain reaches the failure strain or when the pressure reaches the failure pressure, the element loses its ability to carry tension and the deviatoric stresses are set to zero, causing the material to behave like a fluid. If $\Delta t$ for element removal is defined, the element removal option is ignored.

The element erosion option based on $\Delta t$ must be used cautiously with the contact options. Nodes to surface contact is recommended with all nodes of the eroded brick elements included in the node list. As the elements are eroded the mass remains and continues to interact with the reference surface.

## *MAT_SOIL_AND_FOAM_FAILURE

This is Material Type 14. The input for this model is the same as for *MATERIAL_SOIL_-AND_FOAM (Type 5); however, when the pressure reaches the tensile failure pressure, the element loses its ability to carry tension. It should be used only in situations when soils and foams are confined within a structure or are otherwise confined by nodal boundary conditions.

## *MAT_JOHNSON_COOK_{OPTION}

This is Material Type 15. The Johnson/Cook strain and temperature sensitive plasticity is sometimes used for problems where the strain rates vary over a large range, and adiabatic temperature increases due to plastic heating cause material softening. When used with solid elements, this model requires an equation of state. If thermal effects and damage are unimportant, we recommend the much less expensive *MAT_SIMPLIFIED_JOHNSON_COOK model. The simplified model can be used with beam elements.

Material type 15 is applicable to the high-rate deformation of many materials, including most metals. Unlike the Steinberg-Guinan model, the Johnson-Cook model remains valid down to lower strain rates and even into the quasistatic regime. Typical applications include explosive metal forming, ballistic penetration, and impact.

Available options include:

<BLANK>

STOCHASTIC

The STOCHASTIC option enables spatially varying yield and failure behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | G | E | PR | DTF | VP | RATEOP |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| A | B | N | C | M | TM | TR | EPS0 |
|---|---|---|---|---|---|---|---|

**Card 3a.** Include this card if DMODEL = 0.

| CP | PC | SPALL | IT | D1 | D2 | D3 | D4 |
|---|---|---|---|---|---|---|---|

**Card 3b.** Include this card if DMODEL = 1.

| CP | PC | SPALL | IT | BETA | WC | DC | |
|---|---|---|---|---|---|---|---|

**Card 3c.** Include this card if DMODEL = 2.

| CP | PC | SPALL | IT | BETA | WC | PHI | GAMMA |
|---|---|---|---|---|---|---|---|

**Card 4a.** Include this card if RATEOP = 0.0 or 2.0 or if VP = 0.0.

| D5 | | EROD | EFMIN | NUMINT | | | DMODEL |
|---|---|---|---|---|---|---|---|

**Card 4b.** Include this card if RATEOP = 1.0, 3.0, or 4.0.

| D5 | C2/P/XNP | EROD | EFMIN | NUMINT | | | DMODEL |
|---|---|---|---|---|---|---|---|

**Card 4c.** Include this card if RATEOP = 5.0.

| D5 | D | EROD | EFMIN | NUMINT | K | EPS1 | DMODEL |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | E | PR | DTF | VP | RATEOP |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus. G and an equation-of-state are required for element types that use a 3D stress update, such as solids, 2D shell forms 13-15, and tshell forms 3, 5, and 7. For other element types, G is ignored, and E and PR must be provided. |
| E | Young's Modulus (see note above pertaining to G) |
| PR | Poisson's ratio (see note above pertaining to G) |
| DTF | Minimum time step size for automatic element deletion (shell elements). The element will be deleted when the solution time step size drops below DTF × TSSFAC where TSSFAC is the time step scale factor defined by *CONTROL_TIMESTEP. See Remark 4. |

| VARIABLE | DESCRIPTION |
|---|---|
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |
| RATEOP | Form of strain rate term. RATEOP is ignored if VP = 0. See Remark 5.<br>EQ.0.0: Log-linear Johnson-Cook (default)<br>EQ.1.0: Log-quadratic Huh-Kang (2 parameters)<br>EQ.2.0: Exponential Allen-Rule-Jones<br>EQ.3.0: Exponential Cowper-Symonds (2 parameters)<br>EQ.4.0: Nonlinear rate coefficient (2 parameters)<br>EQ.5.0: Log-exponential Couque (4 parameters) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | N | C | M | TM | TR | EPS0 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.0 | 0.0 | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| A | Constant $A$ in the flow stress formula or load curve giving the plastic strain term $\left(A + B\bar{\varepsilon}^{p^n}\right)$ in the flow stress formula: (see Remark 1):<br>GT.0.0: Constant $A$ term in the flow stress formula<br>LT.0.0: \|A\| refers to a load curve ID for the curve giving the plastic strain term $\left(A + B\bar{\varepsilon}^{p^n}\right)$ in the flow stress formula. The abscissa for the curve is the effective plastic strain, $\bar{\varepsilon}^p$. This curve is only available for solid elements. |
| B | Constant $B$ in the flow stress. See equations in Remark 1. This field is ignored if A < 0.0. |

| VARIABLE | DESCRIPTION |
|---|---|
| N | Constant $n$ in the flow stress. See equations in Remark 1. This field is ignored if A < 0.0. |
| C | Constant $C$ in the flow stress. See equations in Remarks 1 and 5. |
| M | Constant $m$ in the flow stress. See equations in Remark 1. |
| TM | Melt temperature |
| TR | Room temperature |
| EPS0 | Quasi-static threshold strain rate (see Remark 1). Ideally, this value represents the highest strain rate for which no rate adjustment to the flow stress is needed and is input in units of $[\text{time}]^{-1}$. For example, if strain rate effects on the flow stress first become apparent at strain rates greater than $10^{-2}$ $s^{-1}$, and the system of units for the model input is {kg, mm, ms}, then EPSO should be set to $10^{-5}$. |

**Johnson-Cook Failure and Damage Model Card.** This card is included for default DMODEL = 0 (see Cards 4a, 4b, and 4c).

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP | PC | SPALL | IT | D1 | D2 | D3 | D4 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0.0 | 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| CP | Specific heat (superseded by heat capacity in *MAT_THERMAL_*OPTION* if a coupled thermal/structural analysis) |
| PC | Tensile failure stress or tensile pressure cutoff (PC < 0.0) |
| SPALL | Spall type (see Remark 3):<br>EQ.0.0: Set to "2.0" (default).<br>EQ.1.0: Tensile pressure is limited by PC, that is, $p$ is always ≥ PC. |

| VARIABLE | DESCRIPTION |
|---|---|
| | *Shell Element* Specific Behavior:<br>EQ.2.0: Shell elements are deleted when $\sigma_{max} \geq -PC$.<br>EQ.3.0: Shell elements are deleted when $p < PC$.<br>*Solid Element* Specific Behavior<br>EQ.2.0: For solid elements $\sigma_{max} \geq -PC$ resets tensile stresses to zero. Compressive stresses are still allowed.<br>EQ.3.0: For solid elements $p < PC$ resets the pressure to zero, thereby disallowing tensile pressure. |
| IT | Plastic strain iteration option. This input applies to solid elements only because it is always necessary to iterate for the shell element plane stress condition.<br>EQ.0.0: No iterations (default)<br>EQ.1.0: Accurate iterative solution for plastic strain. This option is much more expensive than the default. |
| D1 – D4 | Failure parameters; see Remark 2. If D3 < 0.0, it will be converted to its absolute value. |

**Cockcroft-Latham Damage Model Card.** This card is included for DMODEL = 1 (see Cards 4a, 4b, and 4c).

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP | PC | SPALL | IT | BETA | WC | DC | |
| Type | F | F | F | F | F | F | F | |
| Default | none | 0.0 | 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| CP | Specific heat (superseded by heat capacity in *MAT_THERMAL_*OPTION* if a coupled thermal/structural analysis) |
| PC | Tensile failure stress or tensile pressure cutoff (PC < 0.0) |

| VARIABLE | DESCRIPTION |
|---|---|
| SPALL | Spall type (see Remark 3):<br>EQ.0.0: Set to "2.0" (default).<br>EQ.1.0: Tensile pressure is limited by PC, that is, $p$ is always $\geq$ PC.<br>*Shell Element* Specific Behavior:<br>EQ.2.0: Shell elements are deleted when $\sigma_{max} \geq -$PC.<br>EQ.3.0: Shell elements are deleted when $p <$ PC.<br>*Solid Element* Specific Behavior<br>EQ.2.0: For solid elements $\sigma_{max} \geq -$PC resets tensile stresses to zero. Compressive stresses are still allowed.<br>EQ.3.0: For solid elements $p <$ PC resets the pressure to zero, thereby disallowing tensile pressure. |
| IT | Plastic strain iteration option. This input applies to solid elements only because it is always necessary to iterate for the shell element plane stress condition.<br>EQ.0.0: No iterations (default)<br>EQ.1.0: Accurate iterative solution for plastic strain. This option is much more expensive than the default. |
| BETA | Damage coupling parameter; see Equation (107.3) in *MAT_107.<br>EQ.0.0: No coupling between ductile damage and the constitutive relation.<br>EQ.1.0: Full coupling between ductile damage and the constitutive relation. |
| WC | Critical Cockcroft-Latham parameter $W_c$; see Equation (107.19) in *MAT_107. When the plastic work per volume reaches this value, the element is eroded from the simulation. It defines the overall ductility of the material. |
| DC | Critical Cockcroft-Latham damage parameter $D_c$; see Equations (107.15) and (107.22) in *MAT_107. When the damage reaches this value, the element is eroded from the calculation. |

**Extended Cockcroft-Latham Damage Model Card.** This card is included for DMOD-EL = 2 (see Cards 4a, 4b, and 4c).

| Card 3c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP | PC | SPALL | IT | BETA | WC | PHI | GAMMA |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0.0 | 2.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| CP | Specific heat (superseded by heat capacity in *MAT_THERMAL_*OPTION* if a coupled thermal/structural analysis) |
| PC | Tensile failure stress or tensile pressure cutoff (PC < 0.0) |
| SPALL | Spall type (see Remark 3):<br>EQ.0.0: Set to "2.0" (default).<br>EQ.1.0: Tensile pressure is limited by PC, that is, $p$ is always $\geq$ PC.<br>*Shell Element* Specific Behavior:<br>EQ.2.0: Shell elements are deleted when $\sigma_{max} \geq -$PC.<br>EQ.3.0: Shell elements are deleted when $p <$ PC.<br>*Solid Element* Specific Behavior<br>EQ.2.0: For solid elements $\sigma_{max} \geq -$PC resets tensile stresses to zero. Compressive stresses are still allowed.<br>EQ.3.0: For solid elements $p <$ PC resets the pressure to zero, thereby disallowing tensile pressure. |
| IT | Plastic strain iteration option. This input applies to solid elements only because it is always necessary to iterate for the shell element plane stress condition.<br>EQ.0.0: No iterations (default)<br>EQ.1.0: Accurate iterative solution for plastic strain. This option is much more expensive than the default. |
| BETA | Damage coupling parameter; see Equation (107.3) in *MAT_107. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: No coupling between ductile damage and the constitutive relation.<br>EQ.1.0: Full coupling between ductile damage and the constitutive relation. |
| WC | Critical Cockcroft-Latham parameter $W_c$; see Equation (107.19) in *MAT_107. When the plastic work per volume reaches this value, the element is eroded from the simulation. It defines the overall ductility of the material. |
| PHI | Extended Cockcroft-Latham parameter $\phi$; see Equation (107.20) in *MAT_107. It controls the relative influence of the major principal and maximum shear stresses. $\phi = 1$ means the damage is only influenced by maximum principal stress, while $\phi = 0$ indicates that the damage is only influenced by the maximum shear stress. |
| GAMMA | Extended Cockcroft-Latham parameter $\gamma$; see Equation (107.20) in *MAT_107. It controls the intensity of the damage's dependence on the stress state. |

This card is included for RATEOP = 0.0 or 2.0 or for VP = 0.0.

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D5 | | EROD | EFMIN | NUMINT | | | DMODEL |
| Type | F | | F | F | I | | | I |
| Default | 0.0 | | 0.0 | $10^{-6}$ | 0 | | | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| D5 | Failure parameter; see Remark 2. This field is only used when DMODEL = 0. |
| EROD | Erosion flag:<br>EQ.0.0: Element erosion is allowed (default).<br>NE.0.0: Elements do not erode; the deviatoric stresses are set to zero when the element fails. |
| EFMIN | Lower bound for calculated strain at fracture (see Remark 2) |

| VARIABLE | DESCRIPTION |
|---|---|
| NUMINT | Number of through-thickness integration points that must fail before the shell element is deleted. If zero, all integration points must fail (the default). Since nodal fiber rotations limit strains at active integration points, we do not recommend the default because elements undergoing large strain are often not deleted using this criterion. Better results may be obtained when NUMINT is set to 1 or a number less than one-half of the number of through-thickness points. For example, if four through-thickness points are used, NUMINT should not exceed 2, even for fully integrated shells, which have 16 integration points. |
| DMODEL | Damage model:<br>EQ.0: Johnson-Cook damage and failure model (default).<br>EQ.1: Cockcroft-Latham damage model (same as in *MAT_107).<br>EQ.2: Extended Cockcroft-Latham damage model (same as in *MAT_107). |

This card is included for RATEOP = 1.0, 3.0, or 4.0.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D5 | C2/P/XNP | EROD | EFMIN | NUMINT | | | DMODEL |
| Type | F | F | F | F | I | | | I |
| Default | 0.0 | 0.0 | 0.0 | $10^{-6}$ | 0 | | | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| D5 | Failure parameter; see Remark 2. This field is only used when DMODEL = 0. |
| C2/P/XNP | Strain rate parameter |

| Field | Var | Model |
|---|---|---|
| C2 | $C_2$ | Huh-Kang |
| P | $P$ | Cowper-Symonds |
| XNP | $n'$ | Nonlinear Rate Coefficient |

See Remark 5 for a description of these models.

| VARIABLE | DESCRIPTION |
|---|---|
| EROD | Erosion flag:<br>EQ.0.0: Element erosion is allowed (default).<br>NE.0.0: Elements do not erode; the deviatoric stresses are set to zero when the element fails. |
| EFMIN | Lower bound for calculated strain at fracture (see Remark 2) |
| NUMINT | Number of through-thickness integration points that must fail before the shell element is deleted. If zero, all integration points must fail (the default). Since nodal fiber rotations limit strains at active integration points, we do not recommend the default because elements undergoing large strain are often not deleted using this criterion. Better results may be obtained when NUMINT is set to 1 or a number less than one-half of the number of through-thickness points. For example, if four through-thickness points are used, NUMINT should not exceed 2, even for fully integrated shells, which have 16 integration points. |
| DMODEL | Damage model:<br>EQ.0: Johnson-Cook damage and failure model (default).<br>EQ.1: Cockcroft-Latham damage model (same as in *MAT_107).<br>EQ.2: Extended Cockcroft-Latham damage model (same as in *MAT_107). |

This card is included for RATEOP = 5.0.

| Card 4c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D5 | D | EROD | EFMIN | NUMINT | K | EPS1 | DMODEL |
| Type | F | F | F | F | I | F | F | I |
| Default | 0.0 | 0.0 | 0.0 | $10^{-6}$ | 0 | 0.0 | none | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| D5 | Failure parameter; see Remark 2. This field is only used when DMODEL = 0. |

| VARIABLE | DESCRIPTION |
|---|---|
| D | Strain rate parameter $D$ for Couque term. See Remark 5. |
| EROD | Erosion flag:<br>EQ.0.0: Element erosion allowed (default).<br>NE.0.0: Element does not erode; deviatoric stresses set to zero when element fails. |
| EFMIN | Lower bound for calculated strain at fracture (see Remark 2) |
| NUMINT | Number of through-thickness integration points that must fail before the shell element is deleted. If zero, all integration points must fail (the default). Since nodal fiber rotations limit strains at active integration points, we do not recommend the default because elements undergoing large strain are often not deleted using this criterion. Better results may be obtained when NUMINT is set to 1 or a number less than one-half of the number of through-thickness points. For example, if four through-thickness points are used, NUMINT should not exceed 2, even for fully integrated shells, which have 16 integration points. |
| K | Strain rate parameter for Couque term. See Remark 5. |
| EPS1 | Reference strain rate for Couque term, characterizing the transition between the thermally activated regime and the viscous regime. Input in units of $[\text{time}]^{-1}$. See Remark 5. |
| DMODEL | Damage model:<br>EQ.0: Johnson-Cook damage and failure model (default).<br>EQ.1: Cockcroft-Latham damage model (same as in *MAT_107).<br>EQ.2: Extended Cockcroft-Latham damage model (same as in *MAT_107). |

**Remarks:**

1. **Flow stress.** Johnson and Cook express the flow stress as

$$\sigma_y = \left(A + B\bar{\varepsilon}^{p^n}\right)(1 + C\ln\dot{\varepsilon}^*)(1 - T^{*m}) \ ,$$

where

$A$, $B$, and $n$ = input constants or a load curve for the first term
$C$ and $m$ = input constants

$$\bar{\varepsilon}^p = \text{effective plastic strain}$$

$$\dot{\varepsilon}^* = \begin{cases} \dfrac{\dot{\bar{\varepsilon}}}{\text{EPS0}} & \text{for VP} = 0 \quad \text{(normalized effective total strain rate)} \\ \dfrac{\dot{\bar{\varepsilon}}^p}{\text{EPS0}} & \text{for VP} = 1 \quad \text{(normalized effective plastic strain rate)} \end{cases}$$

$$T^* = \text{homologous temperature} = \frac{T - T_{\text{room}}}{T_{\text{melt}} - T_{\text{room}}}$$

The quantity $T - T_{\text{room}}$ is stored as extra history variable 5. In the case of a mechanical-only analysis, this is the adiabatic temperature increase calculated as

$$T - T_{\text{room}} = \frac{\text{internal energy}}{(C_p \times \rho \times Vo)} \ ,$$

where

$$C_p \text{ and } \rho = \text{input constants}$$
$$Vo = \text{initial volume}$$

In a coupled thermal/mechanical analysis, $T - T_{\text{room}}$ includes heating/cooling from all sources, not just adiabatic heating from the internal energy.

Constants for a variety of materials are provided in Johnson and Cook [1983]. A fully viscoplastic formulation is optional (VP) which incorporates the rate equations within the yield surface. An additional cost is incurred, but the improvement in the results can be dramatic.

Due to nonlinearity in the dependence of flow stress on plastic strain, an accurate value of the flow stress requires iteration for the increment in plastic strain. However, by using a Taylor series expansion with linearization about the current time, we can solve for $\sigma_y$ with sufficient accuracy to avoid iteration.

2. **Strain at fracture for the Johnson-Cook damage and failure model.** The strain at fracture when DMODEL = 0 is given by

$$\varepsilon^f = \max([D_1 + D_2 \exp D_3 \sigma^*][1 + D_4 \ln \dot{\varepsilon}^*][1 + D_5 T^*], \text{EFMIN}) \ ,$$

where $\sigma^*$ is the ratio of pressure divided by effective stress

$$\sigma^* = \frac{p}{\sigma_{\text{eff}}} \ .$$

Fracture occurs when the damage parameter,

$$D = \sum \frac{\Delta \bar{\varepsilon}^p}{\varepsilon^f} \ ,$$

reaches the value of 1. $D$ is stored as extra history variable 4 in shell elements and extra history variable 6 in solid elements.

3. **Spall models.** A choice of three spall models is offered to represent material splitting, cracking, and failure under tensile loads:

   a) *Pressure Limit Model.* The pressure limit model limits the minimum hydrostatic pressure to the specified value, $p \geq p_{\min}$. If the calculated pressure is more tensile than this limit, the pressure is reset to $p_{\min}$. This option is not strictly a spall model since the deviatoric stresses are unaffected by the pressure reaching the tensile cutoff and the pressure cutoff value $p_{\min}$ remains unchanged throughout the analysis.

   b) *Maximum Principal Stress Model.* The maximum principal stress spall model detects spall if the maximum principal stress, $\sigma_{\max}$, exceeds the limiting value $\sigma_p$. Once spall in solids is detected with this model, the deviatoric stresses are reset to zero, and no hydrostatic tension is permitted. If tensile pressures are calculated, they are reset to 0 in the spalled material. Thus, the spalled material behaves as rubble.

   c) *Hydrostatic Tension Model.* The hydrostatic tension spall model detects spall if the pressure becomes more tensile than the specified limit, $p_{\min}$. Once spall in solids is detected with this model, the deviatoric stresses are set to zero, and the pressure is required to be compressive. If hydrostatic tension is calculated, then the pressure is reset to 0 for that element.

4. **Shell element deletion based on time step.** This material model also supports a shell element deletion criterion based on the maximum stable time step size for the element, $\Delta t_{\max}$ (see DTF on Card 1). Generally, $\Delta t_{\max}$ goes down as the element becomes more distorted. To assure stability of time integration, the global LS-DYNA time step is the minimum of the $\Delta t_{\max}$ values calculated for all elements in the model. Using this option allows the selective deletion of elements whose time step, $\Delta t_{\max}$, has fallen below the specified minimum time step, $\Delta t_{\text{crit}}$. Elements that are severely distorted often indicate that the material has failed and supports little load, but these same elements may have very small time steps and, therefore, control the cost of the analysis. This option allows these highly distorted elements to be deleted from the calculation, and, therefore, the analysis can proceed at a larger time step, and, thus, at a reduced cost. Deleted elements do not carry any load and are deleted from all applicable slide surface definitions. Clearly, this option must be judiciously used to obtain accurate results at a minimum cost.

5. **Optional strain rate forms.** The standard Johnson-Cook strain rate term is linear in the logarithm of the strain rate (see Remark 1):

$$1 + C \ln \dot{\varepsilon}^*$$

   You can replace this term by setting RATEOP > 0. These additional rate forms are currently available for solid and shell elements but only when the viscoplastic rate option is active (VP = 1). If VP is set to zero, RATEOP is ignored.

The first additional available rate form enables some additional data fitting by using the quadratic form proposed by Huh & Kang [2002]:

$$1 + C \ln \dot{\varepsilon}^* + C_2 (\ln \dot{\varepsilon}^*)^2$$

Four additional exponential forms are available, one due to Allen, Rule & Jones [1997]:

$$(\dot{\varepsilon}^*)^C \ ,$$

the Cowper-Symonds-like [1958] form:

$$1 + \left( \frac{\dot{\varepsilon}^p_{\text{eff}}}{C} \right)^{\frac{1}{P}} \ ,$$

the nonlinear rate coefficient:

$$1 + C \left( \varepsilon^p_{\text{eff}} \right)^{n'} \ln \dot{\varepsilon}^* .$$

and the Couque [2014] form,

$$1 + C \ln \dot{\varepsilon}^* + D \left( \frac{\dot{\varepsilon}^p_{\text{eff}}}{\text{EPS1}} \right)^k .$$

See Huh and Kang [2002], Allen, Rule, and Jones [1997], Cowper and Symonds [1958], and Couque [2014].

6. **History variables.** The following extra history variables may be output to the d3plot file (see *DATABASE_EXTENT_BINARY).

| | Description | |
|---|---|---|
| History Variable # | Shell Elements | Solid Elements |
| 1 | Failure value | |
| 3 | Current pressure cutoff | |
| 4 | Damage parameter, $D$ | |
| 5 | Temperature change, $T - T_{\text{room}}$ | Temperature change, $T - T_{\text{room}}$ |
| 6 | Failure strain | Damage parameter, $D$ |

## *MAT_PSEUDO_TENSOR

This is Material Type 16. This model has been used to analyze buried steel-reinforced concrete structures subjected to impulsive loadings.

This model can be used in two major modes - a simple tabular pressure-dependent yield surface and a potentially complex model featuring two yield-as-a-function-of-pressure functions with the means of migrating from one curve to the other. The Remarks section discusses these modes in detail. For both modes, load curve LCP is taken to be a strain rate multiplier for the yield strength.

Note that this model *must* be used with equation-of-state type 8, 9, or 11. If no EOS is set, the material model uses type 8 for a simple "generic" concrete model. See Remarks.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | G | PR | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SIGF | A0 | A1 | A2 | A0F | A1F | B1 | PER |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| ER | PRR | SIGY | ETAN | LCP | LCR | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| X9 | X10 | X11 | X12 | X13 | X14 | X15 | X16 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| YS1 | YS2 | YS3 | YS4 | YS5 | YS6 | YS7 | YS8 |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| YS9 | YS10 | YS11 | YS12 | YS13 | YS14 | YS15 | YS16 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | PR | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | optional | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus. If PR is set, this field is ignored. The shear modulus in this case is derived from PR and the bulk modulus of the EOS. |
| PR | Poisson's ratio |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGF | A0 | A1 | A2 | A0F | A1F | B1 | PER |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGF | Tensile cutoff (maximum principal stress for failure) |
| A0 | Cohesion |
| A1 | Pressure hardening coefficient |
| A2 | Pressure hardening coefficient |
| A0F | Cohesion for failed material |

| VARIABLE | DESCRIPTION |
|---|---|
| A1F | Pressure hardening coefficient for failed material |
| B1 | Damage scaling factor (or exponent in Mode II.C) |
| PER | Percent reinforcement |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ER | PRR | SIGY | ETAN | LCP | LCR | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | none | 0.0 | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ER | Elastic modulus for reinforcement |
| PRR | Poisson's ratio for reinforcement |
| SIGY | Initial yield stress |
| ETAN | Tangent modulus/plastic hardening modulus |
| LCP | Load curve ID giving rate sensitivity for principal material; see *DEFINE_CURVE. |
| LCR | Load curve ID giving rate sensitivity for reinforcement; see *DEFINE_CURVE. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | X9 | X10 | X11 | X12 | X13 | X14 | X15 | X16 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| X*n* | Effective plastic strain, damage, or pressure. See Remarks below. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | YS1 | YS2 | YS3 | YS4 | YS5 | YS6 | YS7 | YS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | YS9 | YS10 | YS11 | YS12 | YS13 | YS14 | YS15 | YS16 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| YS*n* | Yield stress (Mode I) or scale factor (Mode II.B or II.C) |

**Remarks:**

1. **Response Mode I (tabulated yield stress as a function of pressure).** This model is well suited for implementing standard geologic models like the Mohr-Coulomb yield surface with a Tresca limit, as shown in Figure M16-1. Examples

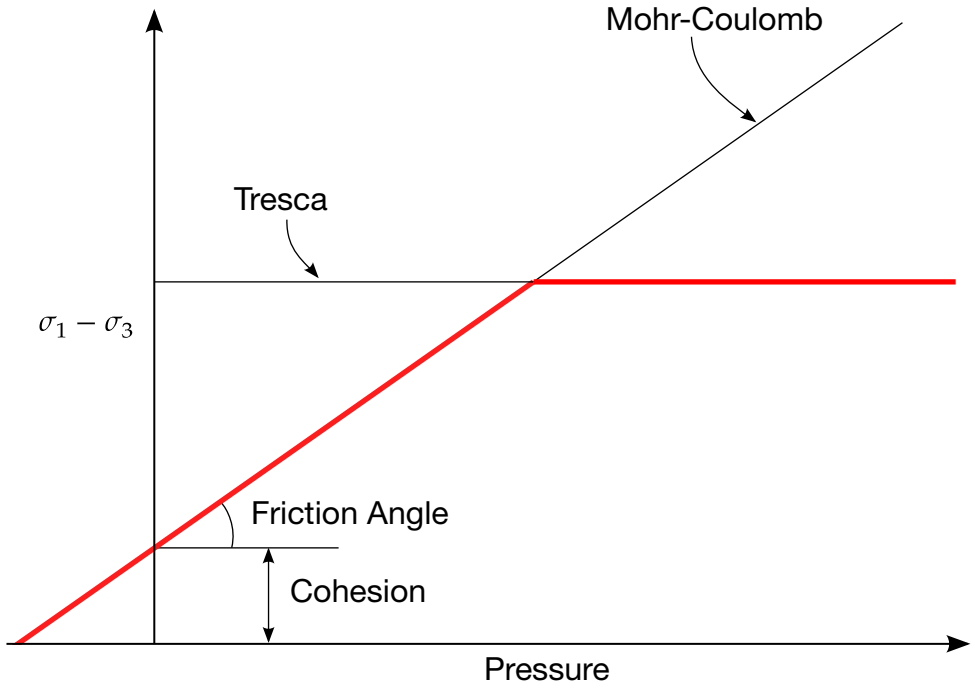



**Figure M16-1.** Mohr-Coulomb surface with a Tresca Limit.

of converting conventional triaxial compression data to this type of model are found in Desai and Siriwardane, 1984. Note that under conventional triaxial compression conditions, the LS-DYNA input corresponds to an ordinate of $\sigma_1 - \sigma_3$ rather than the more widely used $(\sigma_1 - \sigma_3)/2$, where $\sigma_1$ is the maximum principal stress and $\sigma_3$is the minimum principal stress.

This material combined with equation-of-state type 9 (saturated) has been used very successfully to model ground shocks and soil-structure interactions at pressures up to 100 kbars (approximately 1.5 × 106 psi).

To invoke Mode I of this model, set $a_0, a_1, a_2, b_1, a_{0f}$, and $a_{1f}$ to zero. The tabulated values of pressure should then be specified on Cards 4 and 5, and the corresponding values of yield stress should be specified on Cards 6 and 7. The parameters relating to reinforcement properties, initial yield stress, and tangent modulus are not used in this response mode and should be set to zero.

Note that $a_{1f}$ is reset internally to 1/3 even though it is input as zero; this defines a failed material curve of slope $3p$, where $p$ denotes pressure (positive in compression). In this case, the yield strength is taken from the tabulated yield as a function of pressure curve until the maximum principal stress ($\sigma_1$) in the element exceeds the tensile cutoff $\sigma_{cut}$ (input as variable SIGF). When $\sigma_1 > \sigma_{cut}$ is detected, the yield strength is scaled back by a fraction of the distance between the two curves in each of the next 20 time steps so that after those 20 time steps, the yield strength is defined by the failure curve. The only way to inhibit this feature is to set $\sigma_{cut}$ (SIGF) arbitrarily large.

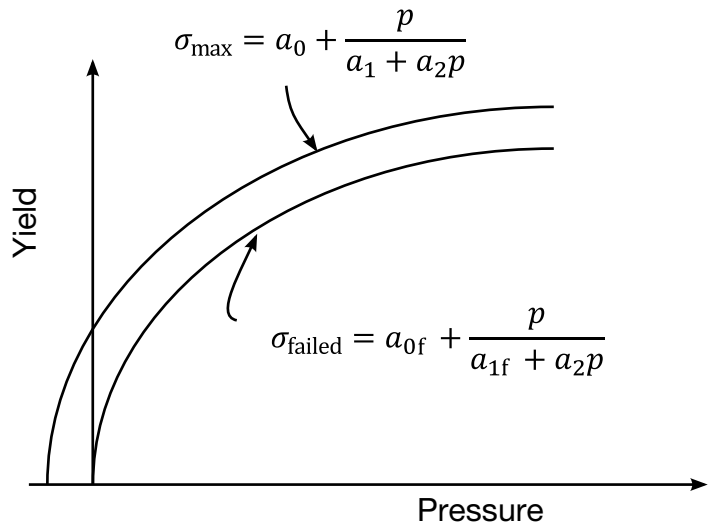



**Figure M16-2.** Two-curve concrete model with damage and failure

2. **Response Mode II (two curve model with damage and failure).** This approach uses two yield versus pressure curves of the form

$$\sigma_y = a_0 + \frac{p}{a_1 + a_2 p}$$

The upper curve is best described as the maximum yield strength curve and the lower curve is the failed material curve. There are a variety of ways of moving between the two curves and each is discussed below.

a) *Mode II.A (Simple Tensile Failure).* To use this mode, define $a_0$, $a_1$, $a_2$, $a_{0\text{f}}$, and $a_{1\text{f}}$, set $b_1$ to zero, and leave Cards 4 through 7 blank. In this case the yield strength is taken from the maximum yield curve until the maximum principal stress ($\sigma_1$) in the element exceeds the tensile cutoff ($\sigma_{\text{cut}}$). When $\sigma_1 > \sigma_{\text{cut}}$ is detected, the yield strength is scaled back by a fraction of the distance between the two curves in each of the next 20 time steps so that after those 20 time steps, the yield strength is defined by the failure curve.

b) *Mode II.B (Tensile Failure plus Plastic Strain Scaling).* Define $a_0$, $a_1$, $a_2$, $a_{0\text{f}}$, and $a_{1\text{f}}$, set $b_1$ to zero, and use Cards 4 through 7 to define a scale factor, $\eta$, (Cards 6 and 7) as a function of effective plastic strain (Cards 4 and 5). LS-DYNA evaluates $\eta$ at the current effective plastic strain and then calculates the yield stress as

$$\sigma_{\text{yield}} = \sigma_{\text{failed}} + \eta(\sigma_{\max} - \sigma_{\text{failed}}) ,$$

where $\sigma_{\max}$ and $\sigma_{\text{failed}}$ are found as shown in Figure M16-2. This yield strength is then subject to scaling for tensile failure as described above. This type of model describes a strain hardening or softening material, such as concrete.

c) *Model II.C (Tensile Failure plus Damage Scaling).* The change in yield stress as a function of plastic strain arises from the physical mechanisms such as internal cracking, and the extent of this cracking is affected by the hydrostatic pressure when the cracking occurs. This mechanism gives rise to the "confinement" effect on concrete behavior. To account for this phenomenon, a "damage" function was defined and incorporated. This damage function is given the form:

$$\lambda = \int_0^{\varepsilon^p} \left(1 + \frac{p}{\sigma_{\text{cut}}}\right)^{-b_1} d\varepsilon^p \quad .$$

To use this model, define $a_0$, $a_1$, $a_2$, $a_{0f}$, $a_{1f}$, and $b_1$. Cards 4 through 7 now give $\eta$ as a function of $\lambda$. $\eta$ scales the yield stress as

$$\sigma_{\text{yield}} = \sigma_{\text{failed}} + \eta(\sigma_{\text{max}} - \sigma_{\text{failed}})$$

before applying any tensile failure criteria.

3. **Mode II concrete model options.** Material Type 16 Mode II provides for the automatic internal generation of a simple "generic" model from concrete. If A0 is negative, then SIGF is assumed to be the unconfined concrete compressive strength, $f_c'$, and –A0 is assumed to be a conversion factor from LS-DYNA pressure units to psi. (For example, if the model stress units are MPa, A0 should be set to –145.) In this case, the parameter values generated internally are

$$f_c' = \text{SIGF} \qquad a_1 = \frac{1}{3} \qquad a_{0f} = 0$$

$$\sigma_{\text{cut}} = 1.7\left(\frac{f_c'^2}{-\text{A0}}\right)^{\frac{1}{3}} \qquad a_2 = \frac{1}{3f_c'} \qquad a_{1f} = 0.385$$

$$a_0 = \frac{f_c'}{4}$$

Note that these $a_{0f}$ and $a_{1f}$ defaults will be overridden by non-zero entries on Card 3. If plastic strain or damage scaling is desired, Cards 5 through 8 as well as $b_1$ should be specified in the input. When $a_0$ is input as a negative quantity, the equation of state can be given as 0 and a trilinear EOS Type 8 model will be automatically generated from the unconfined compressive strength and Poisson's ratio. The EOS 8 model is a simple pressure as a function of volumetric strain model with no internal energy terms, and should give reasonable results for pressures up to 5 kbar (approximately 75,000 psi).

4. **Mixture model.** A reinforcement fraction, $f_r$, can be defined (indirectly as PER/100) along with properties of the reinforcement material. The bulk

modulus, shear modulus, and yield strength are then calculated from a simple mixture rule. For example, for the bulk modulus the rule gives:

$$K = (1 - f_r)K_m + f_r K_r \ ,$$

where $K_m$ and $K_r$ are the bulk moduli for the geologic material and the reinforcement material, respectively. This feature should be used with caution. It gives an isotropic effect in the material instead of the true anisotropic material behavior. A reasonable approach would be to use the mixture elements only where the reinforcing exists and plain elements elsewhere. When the mixture model is being used, the strain rate multiplier for the principal material is taken from load curve N1 and the multiplier for the reinforcement is taken from load curve N2.

5. **Suggested parameters.** The LLNL DYNA3D manual from 1991 [Whirley and Hallquist] suggests using the damage function (Mode II.C) in Material Type 16 with the following set of parameters:

$$a_0 = \frac{f_c'}{4} \qquad a_2 = \frac{1}{3f_c'} \qquad a_{1f} = 1.5$$
$$a_1 = \frac{1}{3} \qquad a_{0f} = \frac{f_c'}{10} \qquad b_1 = 1.25$$

and a damage table of:

| | | | | | |
|---|---|---|---|---|---|
| Card 4: | 0.0 | 8.62E-06 | 2.15E-05 | 3.14E-05 | 3.95E-04 |
| | 5.17E-04 | 6.38E-04 | 7.98E-04 | | |
| Card 5: | 9.67E-04 | 1.41E-03 | 1.97E-03 | 2.59E-03 | 3.27E-03 |
| | 4.00E-03 | 4.79E-03 | 0.909 | | |
| Card 6: | 0.309 | 0.543 | 0.840 | 0.975 | 1.000 |
| | 0.790 | 0.630 | 0.469 | | |
| Card 7: | 0.383 | 0.247 | 0.173 | 0.136 | 0.114 |
| | 0.086 | 0.056 | 0.0 | | |

This set of parameters should give results consistent with Dilger, Koch, and Kowalczyk [1984] for plane concrete. It has been successfully used for reinforced structures where the reinforcing bars were modeled explicitly with embedded beam and shell elements. The model does not incorporate the major failure mechanism - separation of the concrete and reinforcement leading to catastrophic loss of confinement pressure. However, experience indicates that this physical behavior will occur when this model shows about 4% strain.

## *MAT_ORIENTED_CRACK

This is Material Type 17. This material may be used to model brittle materials which fail due to large tensile stresses.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | FS | PRF |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | none | 0.0 |

**Crack Propagation Card.** Optional card for crack propagation to adjacent elements (see remarks).

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SOFT | CVELO | | | | | | |
| Type | F | F | | | | | | |
| Default | 1.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus |
| FS | Fracture stress |
| PRF | Failure or cutoff pressure (≤ 0.0) |

| VARIABLE | DESCRIPTION |
|---|---|
| SOFT | Factor by which the fracture stress is reduced when a crack is coming from failed neighboring element. See remarks. |
| CVELO | Crack propagation velocity. See remarks. |

**Remarks:**

This is an isotropic elastic-plastic material which includes a failure model with an oriented crack. The von Mises yield condition is given by:

$$\phi = J_2 - \frac{\sigma_y^2}{3} \ ,$$

where the second stress invariant, $J_2$, is defined in terms of the deviatoric stress components as

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \ ,$$

and the yield stress, $\sigma_y$, is a function of the effective plastic strain, $\varepsilon_{\text{eff}}^p$, and the plastic hardening modulus, $E_p$:

$$\sigma_y = \sigma_0 + E_p \varepsilon_{\text{eff}}^p \ .$$

The effective plastic strain is defined as:

$$\varepsilon_{\text{eff}}^p = \int_0^t d\varepsilon_{\text{eff}}^p \ ,$$

where

$$d\varepsilon_{\text{eff}}^p = \sqrt{\frac{2}{3} d\varepsilon_{ij}^p d\varepsilon_{ij}^p}$$

and the plastic tangent modulus is defined in terms of the input tangent modulus, $E_t$, as

$$E_p = \frac{E E_t}{E - E_t} \ .$$

Pressure in this model is found from evaluating an equation of state. A pressure cutoff can be defined such that the pressure is not allowed to fall below the cutoff value.

The oriented crack fracture model is based on a maximum principal stress criterion. When the maximum principal stress exceeds the fracture stress, $\sigma_f$, the element fails on a plane perpendicular to the direction of the maximum principal stress. The normal stress and the two shear stresses on that plane are then reduced to zero. This stress reduction is done according to a delay function that reduces the stresses gradually to zero over a small number of time steps. This delay function procedure is used to reduce the ringing

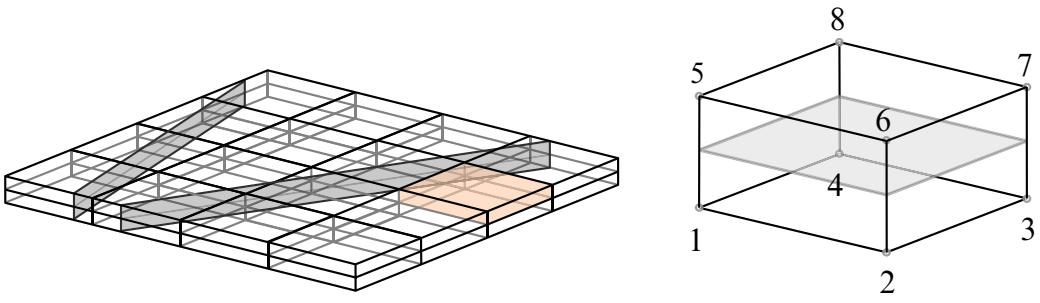



**Figure M17-1.** Thin structure (2 elements over thickness) with cracks and necessary element numbering.

that may otherwise be introduced into the system by the sudden fracture. The number of steps for stress reduction is 20 by default (CVELO = 0.0) or it is internally computed if CVELO > 0.0 is given, that is:

$$n_{\text{steps}} = \text{int}\left[\frac{L_e}{\text{CVELO} \times \Delta t}\right] ,$$

where $L_e$ is the characteristic element length and $\Delta t$ is the time step size.

After a tensile fracture, the element will not support tensile stress on the fracture plane, but in compression will support both normal and shear stresses. The orientation of this fracture surface is tracked throughout the deformation and is updated to properly model finite deformation effects. If the maximum principal stress subsequently exceeds the fracture stress in another direction, the element fails isotropically. In this case the element completely loses its ability to support any shear stress or hydrostatic tension, and only compressive hydrostatic stress states are possible. Thus, once isotropic failure has occurred, the material behaves like a fluid.

This model is applicable to elastic or elastoplastic materials under significant tensile or shear loading when fracture is expected. Potential applications include brittle materials such as ceramics as well as porous materials such as concrete in cases where pressure hardening effects are not significant.

Crack propagation behavior to adjacent elements can be controlled using parameter SOFT for thin, shell-like structures (for example, only 2 or 3 solids over thickness). Additionally, LS-DYNA must know where the plane or solid element midplane is at each integration point for projection of crack plane on this element midplane. Therefore, element numbering must be as shown in Figure M17-1. Currently, only solid element type 1 is supported with that option.

## *MAT_POWER_LAW_PLASTICITY

This is Material Type 18. This isotropic plasticity model with rate effects uses a power law hardening rule.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | K | N | SRC | SRP |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | 0.0 | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGY | VP | EPSF | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | 0.0 | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| K | Strength coefficient |
| N | Hardening exponent |
| SRC | Strain rate parameter, $C$. If zero, rate effects are ignored. |
| SRP | Strain rate parameter, $p$. If zero, rate effects are ignored. |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGY | Optional input parameter for defining the initial yield stress, $\sigma_{y,0}$. Generally, this parameter is not necessary and the elastic strain to initial yield, $\varepsilon_0$, is calculated as described in the remarks section below.<br>EQ.0.0: $\varepsilon_0$ is internally calculated. See Remarks.<br>GT.0.0.and.LT.0.02: $\varepsilon_0$ is SIGY.<br>GE.0.02: $\varepsilon_0$ is internally calculated with $\sigma_{y,0}$ = SIGY. See Remarks. |
| EPSF | Plastic failure strain for element deletion |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |

**Remarks:**

This model provides elastoplastic behavior with isotropic hardening. The yield stress, $\sigma_y$, is a function of plastic strain and obeys the equation:

$$\sigma_y = k\varepsilon^n = k(\varepsilon_0 + \bar{\varepsilon}^p)^n \ ,$$

where $\varepsilon_0$ is the elastic strain to initial yield and $\bar{\varepsilon}^p$is the effective plastic strain (logarithmic). If SIGY is set to zero, $\varepsilon_0$ is found by solving for the intersection of the linearly elastic loading equation with the strain hardening equation:

$$\sigma = E\varepsilon$$
$$\sigma = k\varepsilon^n$$

Thus:

$$\varepsilon_0 = \left(\frac{E}{k}\right)^{\left[\frac{1}{n-1}\right]} \ .$$

If SIGY is nonzero but less than 0.02, $\varepsilon_0$ = SIGY. If SIGY is nonzero and greater than 0.02, the following equation gives $\varepsilon_0$:

$$\varepsilon_0 = \left(\frac{\sigma_{y,0}}{k}\right)^{\left[\frac{1}{n}\right]} \ ,$$

where the initial yield stress, $\sigma_{y,0}$, is SIGY.

Strain rate is accounted for using the Cowper and Symonds model, which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p} ,$$

where $\dot{\varepsilon}$ is the strain rate. A fully viscoplastic formulation incorporating the Cowper and Symonds formulation within the yield surface is optional. An additional cost is incurred, but the improvement in results can be dramatic.

## *MAT_STRAIN_RATE_DEPENDENT_PLASTICITY

This is Material Type 19. A strain rate dependent material can be defined. For an alternative, see Material Type 24. A curve for the yield strength as a function of the effective strain rate must be defined. Optionally, Young's modulus and the tangent modulus can also be defined as a function of the effective strain rate. Also, optional failure of the material can be defined either by defining a von Mises stress at failure as a function of the effective strain rate (valid for solids/shells/thick shells) or by defining a minimum time step size (only for shells).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | VP | | | |
| Type | A | F | F | F | F | | | |
| Default | none | none | none | none | 0.0 | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC1 | ETAN | LC2 | LC3 | LC4 | TDEL | RDEF | |
| Type | F | F | F | F | F | F | F | |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| VP | Formulation for rate effects:<br>EQ.0.0: scale yield stress (default)<br>EQ.1.0: viscoplastic formulation |

| VARIABLE | DESCRIPTION |
|---|---|
| LC1 | Load curve ID defining the yield strength $\sigma_0$ as a function of the effective strain rate. |
| ETAN | Tangent modulus, $E_t$ |
| LC2 | Optional load curve ID defining Young's modulus as a function of the effective strain rate (available only when VP = 0; not recommended). |
| LC3 | Load curve ID defining tangent modulus as a function of the effective strain rate (optional) |
| LC4 | Load curve ID defining von Mises stress at failure as a function of the effective strain rate (optional) |
| TDEL | Minimum time step size for automatic element deletion. Use for shells only. |
| RDEF | Redefinition of failure curve:<br>EQ.1.0: effective plastic strain<br>EQ.2.0: maximum principal stress and absolute value of minimum principal stress<br>EQ.3.0: maximum principal stress (R5 of version 971) |

**Remarks:**

1. **Yield Stress.** In this model, a load curve is used to describe the yield strength $\sigma_0$ as a function of effective strain rate $\dot{\bar{\varepsilon}}$ where

$$\dot{\bar{\varepsilon}} = \left(\frac{2}{3}\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right)^{1/2}$$

and the prime denotes the deviatoric component. The strain rate is available for post-processing as the first stored history variable. If the viscoplastic option is active, the plastic strain rate is output; otherwise, the effective strain rate defined above is output.

The yield stress is defined as

$$\sigma_y = \sigma_0(\dot{\bar{\varepsilon}}) + E_p\bar{\varepsilon}^p \ ,$$

where $\bar{\varepsilon}^p$ is the effective plastic strain and $E_p$ is given in terms of Young's modulus and the tangent modulus by

$$E_p = \frac{EE_t}{E - E_t} \ .$$

Both the Young's modulus and the tangent modulus may optionally be made functions of strain rate by specifying a load curve ID giving their values as a function of strain rate. If these load curve IDs are input as 0, then the constant values specified in the input are used.

2. **Load Curves.** Note that all load curves used to define quantities as a function of strain rate must have the same number of points at the same strain rate values. This requirement is used to allow vectorized interpolation to enhance the execution speed of this constitutive model.

3. **Material Failure.** This model also contains a simple mechanism for modeling material failure. This option is activated by specifying a load curve ID defining the effective stress at failure as a function of strain rate. For solid elements, once the effective stress exceeds the failure stress the element is deemed to have failed and is removed from the solution. For shell elements the entire shell element is deemed to have failed if all integration points through the thickness have an effective stress that exceeds the failure stress. After failure the shell element is removed from the solution.

   In addition to the above failure criterion, this material model also supports a shell element deletion criterion based on the maximum stable time step size for the element, $\Delta t_{\max}$. Generally, $\Delta t_{\max}$ goes down as the element becomes more distorted. To assure stability of time integration, the global LS-DYNA time step is the minimum of the $\Delta t_{\max}$ values calculated for all elements in the model. Using this option allows the selective deletion of elements whose time step $\Delta t_{\max}$ has fallen below the specified minimum time step, $\Delta t_{\text{crit}}$. Elements which are severely distorted often indicate that material has failed and supports little load, but these same elements may have very small time steps and therefore control the cost of the analysis. This option allows these highly distorted elements to be deleted from the calculation, and, therefore, the analysis can proceed at a larger time step, and, thus, at a reduced cost. Deleted elements do not carry any load and are deleted from all applicable slide surface definitions. Clearly, this option must be judiciously used to obtain accurate results at a minimum cost.

4. **Viscoplastic Formulation.** A fully viscoplastic formulation is optional which incorporates the rate formulation within the yield surface. An additional cost is incurred but the improvement is results can be dramatic.

### *MAT_RIGID

This is Material Type 20. Parts made from this material are considered to belong to a rigid body (for each part ID). The coupling of a rigid body with MADYMO and CAL3D can also be defined using this material. Alternatively, a VDA surface can be attached as surface to model the geometry, such as for the tooling in metal forming applications. Optional global and local constraints on the mass center can be defined. A local consideration for output and user-defined airbag sensors may also optionally be chosen.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | N | COUPLE | M | ALIAS or RE |
|---|---|---|---|---|---|---|---|

**Card 2.** This card must be included but may be left blank.

| CMO | CON1 | CON2 | SPCNID | XSPC | YSPC | ZSPC | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card must be included but may be left blank.

| LCO or A1 | A2 | A3 | V1 | V2 | V3 | BNDLCO | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | N | COUPLE | M | ALIAS or RE |
| Type | A | F | F | F | F | F | F | A/F |
| Default | none | none | none | none | 0. | 0. | 0. | opt / none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus. Reasonable values must be chosen for contact analysis (choice of penalty); see Remarks below. |
| PR | Poisson's ratio. Reasonable values must be chosen for contact analysis (choice of penalty); see Remarks below. |
| N | MADYMO3D 5.4 coupling flag, *n*:<br>EQ.0.0: Use normal LS-DYNA rigid body updates.<br>GT.0.0: The rigid body is coupled to the MADYMO 5.4 ellipsoid number *n*.<br>LT.0.0: The rigid body is coupled to MADYMO 5.4 plane number, \|*n*\|. |
| COUPLE | Coupling option if applicable:<br>EQ.-1.0: Attach VDA surface in ALIAS (defined in the eighth field) and automatically generate a mesh for viewing the surface in LS-PREPOST.<br>MADYMO 5.4 / CAL3D coupling option:<br>EQ.0.0: The undeformed geometry input to LS-DYNA corresponds to the local system for MADYMO 5.4 / CAL3D. The finite element mesh is input.<br>EQ.1.0: The undeformed geometry input to LS-DYNA corresponds to the global system for MADYMO 5.4 / CAL3D.<br>EQ.2.0: Generate a mesh for the ellipsoids and planes internally in LS-DYNA. |
| M | MADYMO3D 5.4 coupling flag, *m*:<br>EQ.0.0: Use normal LS-DYNA rigid body updates,<br>EQ.m: This rigid body corresponds to the MADYMO rigid body number, *m*. Rigid body updates are performed by MADYMO. |
| ALIAS | VDA surface alias name; see Appendix L. |
| RE | MADYMO 6.0.1 External Reference Number |

**Constraint Card.** Must be included but may be left blank.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CMO | CON1 | CON2 | SPCNID | XSPC | YSPC | ZSPC | |
| Type | F | I | I | I | F | F | F | |
| Default | 0.0 | 0 | 0 | 0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| CMO | Constraint option, CMO (see Remark 5):<br>EQ.+2.0: Constraints applied in global directions at the coordinates given by XSPC, YSPC, and ZSPC or the initial coordinates of node SPCNID. Unless prescribed motion is applied to the rigid body, the constraint coordinates are fixed in time.<br>EQ.+1.0: Constraints applied in global directions,<br>EQ.0.0: No constraints,<br>EQ.-1.0: Constraints applied in local directions (SPC constraint).<br>EQ.-2.0: Constraints applied in local directions (SPC constraint) at coordinates given by XSPC, YSPC, and ZSPC or the initial coordinates of node SPCNID. Unless prescribed motion is applied to the rigid body, the constraint coordinates are fixed in time. |
| CON1 | First constraint parameter.<br>If CMO > 0.0, then specify the global translational constraint:<br>EQ.0: No constraints,<br>EQ.1: Constrained $x$ displacement,<br>EQ.2: Constrained $y$ displacement,<br>EQ.3: Constrained $z$ displacement,<br>EQ.4: Constrained $x$ and $y$ displacements,<br>EQ.5: Constrained $y$ and $z$ displacements,<br>EQ.6: Constrained $z$ and $x$ displacements, |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.7: Constrained $x$, $y$, and $z$ displacements.<br>If CMO < 0.0, then specify the local coordinate system ID. See *DEFINE_COORDINATE_OPTION. This coordinate system is fixed in time. |
| CON2 | Second constraint parameter:<br>If CMO > 0.0, then specify the global rotational constraint:<br>EQ.0: No constraints,<br>EQ.1: Constrained $x$ rotation,<br>EQ.2: Constrained $y$ rotation,<br>EQ.3: Constrained $z$ rotation,<br>EQ.4: Constrained $x$ and $y$ rotations,<br>EQ.5: Constrained $y$ and $z$ rotations,<br>EQ.6: Constrained $z$ and $x$ rotations,<br>EQ.7: Constrained $x$, $y$, and $z$ rotations.<br><br>If CMO < 0.0, then specify the local (SPC) constraint:<br>EQ.000000: No constraint,<br>EQ.100000: Constrained $x$ translation,<br>EQ.010000: Constrained $y$ translation,<br>EQ.001000: Constrained $z$ translation,<br>EQ.000100: Constrained $x$ rotation,<br>EQ.000010: Constrained $y$ rotation,<br>EQ.000001: Constrained $z$ rotation.<br>To specify a combination of local constraints, input the sum of the desired constraints. |
| SPCNID | For \|CMO\| = 2.0, the constraint coordinates (see below) are the (initial) coordinates of the node with this ID. |
| XSPC,YSPC,ZSPC | Coordinates where the constraints act. Superseded by SPC-NID. |

Optional for output (Must be included but may be left blank).

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCO or A1 | A2 | A3 | V1 | V2 | V3 | BNDLCO | |
| Type | F | F | F | F | F | F | I | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCO | Local coordinate system number for local output to rbdout. LCO also specifies the coordinate system used for *BOUNDARY_PRESCRIBED_MOTION_RIGID_LOCAL. Defaults to the principal coordinate system of the rigid body. |
| A1 - V3 | Alternative method for specifying the local system:<br>Define two vectors **a** and **v**, fixed to the rigid body which are used for output and the user defined airbag sensor subroutines. The output parameters are in the directions **a**, **b**, and **c** where the latter are given by the cross products $\mathbf{c} = \mathbf{a} \times \mathbf{v}$ and $\mathbf{b} = \mathbf{c} \times \mathbf{a}$. This input is optional. |
| BNDLCO | Flag specifying whether the forces and moments in the bndout file, generated by *BOUNDARY_PRESCRIBED_MOTION_RIGID, are expressed in the rigid body's local coordinate system.<br>EQ.0: Forces and moments are printed in the global coordinate system (default).<br>EQ.1: Forces and moments are printed in the local coordinate system. |

**Remarks:**

1. **Rigid material.** The rigid material type 20 provides a convenient way of turning one or more parts comprised of beams, shells, or solid elements into a rigid body. Approximating a deformable body as rigid is a preferred modeling technique in many real world applications. For example, in sheet metal forming problems the tooling can properly and accurately be treated as rigid. In the design of restraint systems the occupant can, for the purposes of early design studies, also be treated as rigid. Elements which are rigid are bypassed in the element

processing and no storage is allocated for storing history variables; consequently, the rigid material type is very cost efficient.

2. **Parts.** Two unique rigid part IDs may not share common nodes unless they are merged together using the rigid body merge option. A rigid body, however, may be made up of disjoint finite element meshes. LS-DYNA assumes this is the case since this is a common practice in setting up tooling meshes in forming problems.

   All elements which reference a given part ID corresponding to the rigid material should be contiguous, but this is not a requirement. If two disjoint groups of elements on opposite sides of a model are modeled as rigid, separate part IDs should be created for each of the contiguous element groups if each group is to move independently. This requirement arises from the fact that LS-DYNA internally computes the six rigid body degrees-of-freedom for each rigid body (rigid material or set of merged materials), and if disjoint groups of rigid elements use the same part ID, the disjoint groups will move together as one rigid body.

3. **Inertial properties.** Inertial properties for rigid materials may be defined in either of two ways. By default, the inertial properties are calculated from the geometry of the constituent elements of the rigid material and the density specified for the part ID. Alternatively, the inertial properties and initial velocities for a rigid body may be directly defined, and this overrides data calculated from the material property definition and nodal initial velocity definitions.

4. **Contact and material constants.** Young's modulus, E, and Poisson's ratio, $\upsilon$, are used for determining sliding interface parameters if the rigid body interacts in a contact definition. Realistic values for these constants should be defined since unrealistic values may contribute to numerical problems with contact.

5. **Constraints.** Constraint directions for rigid bodies (CMO $\neq$ 0) are fixed, that is, not updated, with time. To impose a constraint on a rigid body such that the constraint direction is updated as the rigid body rotates, use *BOUNDARY_PRESCRIBED_MOTION_RIGID_LOCAL. The constraint defined therein refers to the local system CID, which is updated with time.

   We strongly advise not applying nodal constraints, for instance, by *BOUNDARY_SPC_*OPTION*, to nodes of a rigid body as doing so may compromise the intended constraints in the case of an explicit simulation. In an implicit simulation, constraints can, however, be applied to the nodes of a rigid body.

   If the intended constraints are not with respect to the calculated center of mass of the rigid body in explicit simulations, the following alternative approaches can be used:

a) Set |CMO| = 2 and choose the point at which the constraints shall act. These coordinates are referenced both by the constraints given on this card and those on *BOUNDARY_PRESCRIBED_MOTION_*OPTION*. Note, |CMO| = 2 is not available for implicit simulations.

b) *CONSTRAINED_JOINT_*OPTION* may often be used to obtain the desired effect. This approach typically entails defining a second rigid body that is fully constrained and then defining a joint between the two rigid bodies.

c) Another alternative for defining rigid body constraints that are not with respect to the calculated center of mass of the rigid body is to manually specify the initial center of mass location using *PART_INERTIA. When using *PART_INERTIA, a full set of mass properties must be specified. Note that changing its mass properties will affect the rigid body's dynamic behavior.

Setting |CMO| = 2 in *BOUNDARY_PRESCRIBED_MOTION_*OPTION* not only allows for a constraint point other than the center of mass. The motion prescribed by *BOUNDARY_PRESCRIBED_MOTION_*OPTION* also acts on this point. In addition, setting |CMO| = 2 treats the constraints (including those from *BOUNDARY_PRESCRIBED_MOTION_*OPTION*) differently from |CMO| = 1. To allow for an arbitrary constraint point, the constraints are applied and solved during the kinematic update of the rigid body. Since the inertia does not need to be modified and no joints are involved, setting |CMO| = 2 is more accurate compared to options b and c above. No time penalty is to be expected.

To obtain reaction forces from constraints, see the SPC2BND flag of *CONTROL_OUTPUT.

6. **Coupling with MADYMO.** Only basic coupling is available for coupling with MADYMO 5.4.1. The coupling flags (N and M) must match with SYSTEM and ELLIPSOID/PLANE in the MADYMO input file and the coupling option (COUPLE) must be defined.

Both basic and extended coupling are available for coupling with MADYMO 6.0.1:

a) *Basic Coupling.* The external reference number (RE) must match the external reference number in the MADYMO XML input file. The coupling option (COUPLE) must be defined.

b) *Extended Coupling.* Under this option MADYMO will handle the contact between the MADYMO and LS-DYNA models. The external reference number (RE) and the coupling option (COUPLE) are not needed. All coupling surfaces that interface with the MADYMO models need to be defined in *CONTACT_COUPLING.

## *MAT_ORTHOTROPIC_THERMAL_{OPTION}

This is Material Type 21. It is a linearly elastic, orthotropic material with orthotropic thermal expansion. It is available for solids, shells, and thick shells.

Available options include:

<BLANK>

FAILURE

CURING

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | AA | AB | AC | AOPT | MACF |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
|---|---|---|---|---|---|---|---|

**Card 5a.** This card is included if and only if the keyword option FAILURE is used.

| A1 | A11 | A2 | A5 | A55 | A4 | NIP | |
|---|---|---|---|---|---|---|---|

**Card 5b.1.** This card is included if and only if the keyword option CURING is used.

| K1 | K2 | C1 | C2 | M | N | R | |
|---|---|---|---|---|---|---|---|

**Card 5b.2.** This card is included if and only if the keyword option CURING is used.

| LCCHA | LCCHB | LCCHC | LCAA | LCAB | LCAC | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in *a*-direction |
| EB | $E_b$, Young's modulus in *b*-direction |
| EC | $E_c$, Young's modulus in *c*-direction |
| PRBA | $\nu_{ba}$, Poisson's ratio, *ba* |
| PRCA | $\nu_{ca}$, Poisson's ratio, *ca* |
| PRCB | $\nu_{cb}$, Poisson's ratio, *cb* |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AA | AB | AC | AOPT | MACF |
| Type | F | F | F | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, Shear modulus, *ab* |
| GBC | $G_{bc}$, Shear modulus, *bc* |
| GCA | $G_{ca}$, Shear modulus, *ca* |
| AA | $\alpha_a$, coefficient of thermal expansion in the *a*-direction |

| VARIABLE | DESCRIPTION |
|---|---|
| AB | $\alpha_b$, coefficient of thermal expansion in the $b$-direction |
| AC | $\alpha_c$, coefficient of thermal expansion in the $c$-direction |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, $P$, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| VARIABLE | DESCRIPTION |
|---|---|
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3 (all element types). It may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, or *ELEMENT_SOLID_ORTHO. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details).<br>EQ.0.0: Off<br>EQ.1.0: On |

**Failure Card.** This card is only included if the FAILURE keyword option is used.

| Card 5a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A1 | A11 | A2 | A5 | A55 | A4 | NIP | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A11, A2 | Coefficients for the matrix dominated failure criterion |
| A5, A55, A4 | Coefficients for the fiber dominated failure criterion |
| NIP | Number of integration points that must fail in an element before an element fails and is deleted |

**Curing Card.** This card is included if and only if the CURING keyword option is used.

| Card 5b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | K2 | C1 | C2 | M | N | R | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| K1 | Parameter $k_1$ for Kamal model. For details see remarks below. |
| K2 | Parameter $k_2$ for Kamal model |
| C1 | Parameter $c_1$ for Kamal model |
| C2 | Parameter $c_2$ for Kamal model |
| M | Exponent $m$ for Kamal model |
| N | Exponent $n$ for Kamal model |
| R | Gas constant for Kamal model |

**Curing Card.** This card is included if and only if the CURING keyword option is used.

| Card 5b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCCHA | LCCHB | LCCHC | LCAA | LCAB | LCAC | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCCHA | Load curve for $\gamma_a$, coefficient of chemical shrinkage in the $a$-direction. Input $\gamma_a$ as function of state of cure $\beta$. |
| LCCHB | Load curve for $\gamma_b$, coefficient of chemical shrinkage in the $b$-direction. Input $\gamma_b$ as function of state of cure $\beta$. |
| LCCHC | Load curve for $\gamma_c$, coefficient of chemical shrinkage in the $c$-direction. Input $\gamma_c$ as function of state of cure $\beta$. |
| LCAA | Load curve or table ID for $\alpha_a$. If defined, parameter AA is ignored. If a load curve, then $\alpha_a$ is a function of temperature. If a table ID, the $\alpha_a$ is a function of the state of cure (table values) and temperature (see*DEFINE_TABLE). |
| LCAB | Load curve ID for $\alpha_b$. If defined parameter, AB is ignored. See LCAA for further details. |
| LCAC | Load curve ID for $\alpha_c$. If defined parameter, AC is ignored. See LCAA for further details. |

**Remarks:**

In the implementation for three-dimensional continua a total Lagrangian formulation is used. In this approach the material law that relates second Piola-Kirchhoff stress **S** to the Green-St. Venant strain **E** is

$$\mathbf{S} = \mathbf{C}\colon \mathbf{E} = \mathbf{T}^{\mathrm{T}}\mathbf{C}_l\mathbf{T}\colon \mathbf{E}$$

where **T** is the transformation matrix [Cook 1974].

$$\mathbf{T} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1m_1 & m_1n_1 & n_1l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2m_2 & m_2n_2 & n_2l_2 \\ l_3^2 & m_3^2 & m_3^2 & l_3m_3 & m_3n_3 & n_3l_3 \\ 2l_1l_2 & 2m_1m_2 & 2n_1n_2 & (l_1m_2+l_2m_1) & (m_1n_2+m_2n_1) & (n_1l_2+n_2l_1) \\ 2l_2l_3 & 2m_2m_3 & 2n_2n_3 & (l_2m_3+l_3m_2) & (m_2n_3+m_3n_2) & (n_2l_3+n_3l_2) \\ 2l_3l_1 & 2m_3m_1 & 2n_3n_1 & (l_3m_1+l_1m_3) & (m_3n_1+m_1n_3) & (n_3l_1+n_1l_3) \end{bmatrix}$$

$l_i$, $m_i$, $n_i$ are the direction cosines

$$x_i' = l_ix_1 + m_ix_2 + n_ix_3 \text{ for } i = 1,2,3$$

and $x_i'$ denotes the material axes. The constitutive matrix $\mathbf{C}_l$ is defined in terms of the material axes as

$$\mathbf{C}_l^{-1} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{v_{21}}{E_{22}} & -\frac{v_{31}}{E_{33}} & 0 & 0 & 0 \\ -\frac{v_{12}}{E_{11}} & \frac{1}{E_{22}} & -\frac{v_{32}}{E_{33}} & 0 & 0 & 0 \\ -\frac{v_{13}}{E_{11}} & -\frac{v_{23}}{E_{22}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}} \end{bmatrix}$$

where the subscripts denote the material axes, meaning

$$v_{ij} = v_{x_i'x_j'} \quad \text{and} \quad E_{ii} = E_{x_i'}$$

Since $\mathbf{C}_l$ is symmetric

$$\frac{v_{12}}{E_{11}} = \frac{v_{21}}{E_{22}}, \ldots$$

The vector of Green-St. Venant strain components is

$$\mathbf{E}^{\mathrm{T}} = [E_{11}, E_{22}, E_{33}, E_{12}, E_{23}, E_{31}]$$

which include the local thermal strains which are integrated in time:

$$\varepsilon_{aa}^{n+1} = \varepsilon_{aa}^{n} + \alpha_a\left(T^{n+1} - T^{n}\right)$$
$$\varepsilon_{bb}^{n+1} = \varepsilon_{bb}^{n} + \alpha_b\left(T^{n+1} - T^{n}\right)$$
$$\varepsilon_{cc}^{n+1} = \varepsilon_{cc}^{n} + \alpha_c\left(T^{n+1} - T^{n}\right)$$

where $T$ is temperature. After computing $S_{ij}$ we then obtain the Cauchy stress:

$$\sigma_{ij} = \frac{\rho}{\rho_0}\frac{\partial x_i}{\partial X_k}\frac{\partial x_j}{\partial X_l}S_{kl}$$

This model will predict realistic behavior for finite displacement and rotations as long as the strains are small.

In the implementation for shell elements, the stresses are integrated in time and are updated in the corotational coordinate system. In this procedure the local material axes are assumed to remain orthogonal in the deformed configuration. This assumption is valid if the strains remain small.

The failure models were derived by William Feng. The first one defines the matrix dominated failure mode,

$$F_m = A_1(I_1 - 3) + A_{11}(I_1 - 3)^2 + A_2(I_2 - 3) - 1 \ ,$$

and the second defines the fiber dominated failure mode,

$$F_f = A_5(I_5 - 1) + A_{55}(I_5 - 1)^2 + A_4(I_4 - 1) - 1 \ .$$

When either is greater than zero, the integration point fails, and the element is deleted after NIP integration points fail.

The coefficients $A_i$ are defined in the input and the invariants $I_i$ are the strain invariants

$$I_1 = \sum_{\alpha=1,3} C_{\alpha\alpha}$$
$$I_2 = \frac{1}{2}[I_1^2 - \sum_{\alpha,\beta=1,3} C_{\alpha\beta}^2]$$
$$I_3 = \det(\mathbf{C})$$
$$I_4 = \sum_{\alpha,\beta,\gamma=1,3} V_\alpha\, C_{\alpha\gamma} C_{\gamma\beta} V_\beta$$
$$I_5 = \sum_{\alpha,\beta=1,3} V_\alpha\, C_{\alpha\beta} V_\beta$$

and $\mathbf{C}$ is the Cauchy strain tensor and $\mathbf{V}$ is the fiber direction in the undeformed state. By convention in this material model, the fiber direction is aligned with the *a* direction of the local orthotropic coordinate system.

The curing option implies that orthotropic chemical shrinkage is to be considered, resulting from a curing process in the material. The state of cure $\beta$ is an internal material variable that follows the Kamal model

$$\frac{d\beta}{dt} = (K_1 + K_2\beta^m)(1-\beta)^n \quad \text{with} \quad K_1 = k_1 e^{-\frac{c_1}{RT}},\ K_2 = k_2 e^{-\frac{c_2}{RT}}$$

Chemical strains are introduced as:

$$\begin{aligned}
\varepsilon_{aa}^{n+1} &= \varepsilon_{aa}^{n} + \gamma_a\left(\beta^{n+1} - \beta^{n}\right) \\
\varepsilon_{bb}^{n+1} &= \varepsilon_{bb}^{n} + \gamma_b\left(\beta^{n+1} - \beta^{n}\right) \\
\varepsilon_{cc}^{n+1} &= \varepsilon_{cc}^{n} + \gamma_c\left(\beta^{n+1} - \beta^{n}\right)
\end{aligned}$$

The coefficients, $\gamma_a$, $\gamma_b$, and $\gamma_c$, can be defined as functions of the state of cure $\beta$. Furthermore, the coefficients of thermal expansion, $\alpha_a$, $\alpha_b$, and $\alpha_c$, can also be defined as functions of the state of cure, $\beta$, and the temperature, $T$, if the curing option is used.

The current degree of cure as well as the chemical shrinkage in the different directions is output in the history variables. For solid elements it can be found at positions 30 to 33 and for shell elements at positions 22 to 25.

## *MAT_COMPOSITE_DAMAGE

This is Material Type 22. With this model, an orthotropic material with optional brittle failure for composites can be defined following the suggestion of [Chang and Chang 1987a, 1987b]. Failure can be modeled with three criteria; see the LS-DYNA Theory Manual. By using the user defined integration rule (see *INTEGRATION_SHELL), the constitutive constants can vary through the shell thickness.

For all shells, except the DKT formulation, laminated shell theory can be activated to properly model the transverse shear deformation. Lamination theory is applied to correct for the assumption of a uniform constant shear strain through the thickness of the shell. For sandwich shells where the outer layers are much stiffer than the inner layers, the response will tend to be too stiff unless lamination theory is used. To turn on lamination theory, see *CONTROL_SHELL.

This material is available for shells, solids, thick shells, and SPH elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |
| Remarks | | | | | | 3 | 3 | 3 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | KFAIL | AOPT | MACF | ATRACK | |
| Type | F | F | F | F | F | I | I | |
| Default | none | none | none | 0.0 | 0.0 | 0 | 0 | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SC | XT | YT | YC | ALPH | SN | SYZ | SZX |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in $a$-direction |
| EB | $E_b$, Young's modulus in $b$-direction |
| EC | $E_c$, Young's modulus in $c$-direction |
| PRBA | $\nu_{ba}$, Poisson ratio, $ba$ |

| VARIABLE | DESCRIPTION |
|---|---|
| PRCA | $\nu_{ca}$, Poisson ratio, *ca* |
| PRCB | $\nu_{cb}$, Poisson ratio, *cb* |
| GAB | $G_{ab}$, Shear modulus, *ab* |
| GBC | $G_{bc}$, Shear modulus, *bc* |
| GCA | $G_{ca}$, Shear modulus, *ca* |
| KFAIL | Bulk modulus of failed material. Necessary for compressive failure. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying |

| VARIABLE | DESCRIPTION |
|---|---|
| | BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| ATRACK | Material *a*-axis tracking flag (shell elements only):<br>EQ.0: *a*-axis rotates with element (default).<br>EQ.1: *a*-axis also tracks deformation (see Remark 2). |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, or *ELEMENT_SOLID_ORTHO. |
| SC | Shear strength, *ab*-plane; see the LS-DYNA Theory Manual. |
| XT | Longitudinal tensile strength, *a*-axis; see the LS-DYNA Theory Manual. |
| YT | Transverse tensile strength, *b*-axis |
| YC | Transverse compressive strength, *b*-axis (positive value) |
| ALPH | Shear stress parameter for the nonlinear term in units of [stress$^{-3}$]; see the LS-DYNA Theory Manual. |
| SN | Normal tensile strength *(solid elements only)* |
| SYZ | Transverse shear strength *(solid elements only)* |
| SZX | Transverse shear strength *(solid elements only)* |

**Remarks:**

1. **History data.** The number of additional integration point variables for shells written to the d3plot database is specified using the *DATABASE_EXTENT_BINARY keyword on the NEIPS field. These additional history variables are enumerated below:

| History Variable[4] | Description | Value | LS-PrePost History Variable |
|---|---|---|---|
| ef($i$) | tensile fiber mode | 1 - elastic<br>0 - failed | See table below |
| cm($i$) | tensile matrix mode | | 1 |
| ed($i$) | compressive matrix mode | | 2 |

The following components are stored as element component 7 instead of the effective plastic strain. Note that ef($i$) for $i$ = 1,2,3 is not retrievable.

[4] $i$ ranges over the shell integration points.

| Description | Integration point |
|---|---|
| $\frac{1}{\text{nip}}\sum_{i=1}^{\text{nip}} \text{ef}(i)$ | 1 |
| $\frac{1}{\text{nip}}\sum_{i=1}^{\text{nip}} \text{cm}(i)$ | 2 |
| $\frac{1}{\text{nip}}\sum_{i=1}^{\text{nip}} \text{ed}(i)$ | 3 |
| $\text{ef}(i)$ for $i > 3$ | $i$ |

2. **The ATRACK field.** The initial material directions are set using AOPT and the related data. By default, the material directions in shell elements are updated each cycle based on the rotation of the 1-2 edge, or else the rotation of all edges if the invariant node numbering option is set on *CONTROL_ACCURACY. When ATRACK = 1, an optional scheme is used in which the *a*-direction of the material tracks element deformation as well as rotation.

   At the start of the calculation, a line is passed through each element center in the direction of the material *a*-axis. This line will intersect the edges of the element at two points. The referential coordinates of these two points are stored and then used throughout the calculation to locate these points in the deformed geometry. The material *a*-axis is assumed to be in the direction of the line that passes through both points. If ATRACK = 0, the layers of a layered composite will always rotate together. However, if ATRACK = 1, the layers can rotate independently which may be more accurate, particularly for shear deformation. This option is available only for shell elements.

3. **Poisson's ratio.** If EA > EB, PRBA is the minor Poisson's ratio if EA > EB, and the major Poisson's ratio will be equal to PRBA × (EA/EB). If EB > EA, then PRBA is the major Poisson's ratio. PRCA and PRCB are similarly defined. They are the minor Poisson's ratio if EA>EC or EB>EC, and the major Poisson's ratio if the EC > EA or EC > EB.

   Care should be taken when using material parameters from third party products regarding the directional indices *a*, *b* and *c*, as they may differ from the definition used in LS-DYNA. For the direction indices used in LS-DYNA, see the remarks section of *MAT_002 / *MAT_*OPTION*TROPIC_ELASTIC.

## *MAT_TEMPERATURE_DEPENDENT_ORTHOTROPIC

This is Material Type 23. It models an orthotropic elastic material with arbitrary temperature dependency. It is available for solids, shells, and thick shells.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | AOPT | REF | MACF | IHYPO | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 4.1.** Define one set of constants on two cards using Cards 4.1 and 4.2 for each temperature point. Up to 48 points (96 cards) can be defined. The next keyword ("*") card terminates the input.

| EAi | EBi | ECi | PRBAi | PRCAi | PRCBi | | |
|---|---|---|---|---|---|---|---|

**Card 4.2.** Define one set of constants on two cards using Cards 4.1 and 4.2 for each temperature point. Up to 48 points (96 cards) can be defined. The next keyword ("*") card terminates the input.

| AAi | ABi | ACi | GABi | GBCi | GCAi | Ti | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | AOPT | REF | MACF | IHYPO | | |
| Type | A | F | F | F | I | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): |

EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.

EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.

LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*).

| VARIABLE | DESCRIPTION |
|---|---|
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see description of this keyword for more details).<br>EQ.0.0: Off<br>EQ.1.0: On |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| IHYPO | Option to switch between two different elastic approaches (only available for solid elements):<br>EQ.0.0: Hyperelastic formulation, default<br>EQ.1.0: Hypoelastic formulation (allows stress initialization through *INITIAL_STRESS_SOLID) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, or *ELEMENT_SOLID_ORTHO. |

**First Temperature Card.** Define one set of constants on two cards using Cards 4.1 and 4.2 for each temperature point. Up to 48 points (96 cards) can be defined. The next keyword ("*") card terminates the input.

| Card 4.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EA*i* | EB*i* | EC*i* | PRBA*i* | PRCA*i* | PRCB*i* | | |
| Type | F | F | F | F | F | F | | |

**Second Temperature Card**

| Card 4.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AA*i* | AB*i* | AC*i* | GAB*i* | GBC*i* | GCA*i* | T*i* | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| EA*i* | $E_a$, Young's modulus in *a*-direction at temperature T*i* |
| EB*i* | $E_b$, Young's modulus in *b*-direction at temperature T*i* |
| EC*i* | $E_c$, Young's modulus in *c*-direction at temperature T*i* |
| PRBA*i* | $\nu_{ba}$, Poisson's ratio *ba* at temperature T*i* |
| PRCA*i* | $\nu_{ca}$, Poisson's ratio *ca* at temperature T*i* |
| PRCB*i* | $\nu_{cb}$, Poisson's ratio *cb* at temperature T*i* |
| AA*i* | $\alpha_a$, coefficient of thermal expansion in *a*-direction at temperature T*i* |
| AB*i* | $\alpha_B$ coefficient of thermal expansion in *b*-direction at temperature T*i*. |
| AC*i* | $\alpha_c$, coefficient of thermal expansion in *c*-direction at temperature T*i*. |
| GAB*i* | $G_{ab}$, Shear modulus *ab* at temperature T*i*. |
| GBC*i* | $G_{bc}$, Shear modulus *bc* at temperature T*i*. |
| GCA*i* | $G_{ca}$, Shear modulus *ca* at temperature T*i*. |
| T*i* | $i^{\text{th}}$ temperature |

**Remarks:**

In the implementation for three-dimensional continua a total Lagrangian formulation is used. In this approach the material law that relates second Piola-Kirchhoff stress **S** to the Green-St. Venant strain **E** is

$$\mathbf{S} = \mathbf{C}\colon\mathbf{E} = \mathbf{T}^{\mathrm{T}}\mathbf{C}_l\mathbf{T}\colon\mathbf{E}$$

where **T** is the transformation matrix [Cook 1974].

$$\mathbf{T} = \begin{bmatrix} l_1^2 & m_1^2 & n_1^2 & l_1m_1 & m_1n_1 & n_1l_1 \\ l_2^2 & m_2^2 & n_2^2 & l_2m_2 & m_2n_2 & n_2l_2 \\ l_3^2 & m_3^2 & m_3^2 & l_3m_3 & m_3n_3 & n_3l_3 \\ 2l_1l_2 & 2m_1m_2 & 2n_1n_2 & (l_1m_2+l_2m_1) & (m_1n_2+m_2n_1) & (n_1l_2+n_2l_1) \\ 2l_2l_3 & 2m_2m_3 & 2n_2n_3 & (l_2m_3+l_3m_2) & (m_2n_3+m_3n_2) & (n_2l_3+n_3l_2) \\ 2l_3l_1 & 2m_3m_1 & 2n_3n_1 & (l_3m_1+l_1m_3) & (m_3n_1+m_1n_3) & (n_3l_1+n_1l_3) \end{bmatrix}$$

$l_i$, $m_i$, $n_i$ are the direction cosines

$$x_i' = l_ix_1 + m_ix_2 + n_ix_3 \text{ for } i = 1,2,3$$

and $x_i'$ denotes the material axes. The temperature dependent constitutive matrix $\mathbf{C}_l$ is defined in terms of the material axes as

$$\mathbf{C}_l^{-1} = \begin{bmatrix} \frac{1}{E_{11}(T)} & -\frac{v_{21}(T)}{E_{22}(T)} & -\frac{v_{31}(T)}{E_{33}(T)} & 0 & 0 & 0 \\ -\frac{v_{12}(T)}{E_{11}(T)} & \frac{1}{E_{22}(T)} & -\frac{v_{32}(T)}{E_{33}(T)} & 0 & 0 & 0 \\ -\frac{v_{13}(T)}{E_{11}(T)} & -\frac{v_{23}(T)}{E_{22}(T)} & \frac{1}{E_{33}(T)} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}(T)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{23}(T)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{31}(T)} \end{bmatrix}$$

where the subscripts denote the material axes,

$$v_{ij} = v_{x_i'x_j'} \quad \text{and} \quad E_{ii} = E_{x_i'}$$

Since $\mathbf{C}_l$ is symmetric

$$\frac{v_{12}}{E_{11}} = \frac{v_{21}}{E_{22}}, \ldots$$

The vector of Green-St. Venant strain components is

$$\mathbf{E}^{\mathrm{T}} = \lfloor E_{11}, E_{22}, E_{33}, E_{12}, E_{23}, E_{31} \rfloor$$

which include the local thermal strains which are integrated in time:

$$\varepsilon_{aa}^{n+1} = \varepsilon_{aa}^{n} + \alpha_a \left( T^{n+\frac{1}{2}} \right) \left[ T^{n+1} - T^n \right]$$

$$\varepsilon_{bb}^{n+1} = \varepsilon_{bb}^{n} + \alpha_b \left( T^{n+\frac{1}{2}} \right) \left[ T^{n+1} - T^n \right]$$

$$\varepsilon_{cc}^{n+1} = \varepsilon_{cc}^{n} + \alpha_c \left( T^{n+\frac{1}{2}} \right) \left[ T^{n+1} - T^n \right]$$

where $T$ is temperature. After computing $S_{ij}$ we then obtain the Cauchy stress:

$$\sigma_{ij} = \frac{\rho}{\rho_0} \frac{\partial x_i}{\partial X_k} \frac{\partial x_j}{\partial X_l} S_{kl}$$

This model will predict realistic behavior for finite displacement and rotations as long as the strains are small.

For shell elements, the stresses are integrated in time and are updated in the corotational coordinate system. In this procedure the local material axes are assumed to remain orthogonal in the deformed configuration. This assumption is valid if the strains remain small.

***MAT_PIECEWISE_LINEAR_PLASTICITY**_*{OPTION}*

Available options include:

<BLANK>

LOG_INTERPOLATION

STOCHASTIC

MIDFAIL

2D

This is Material Type 24. It is an elasto-plastic material with an arbitrary stress as a function of strain curve that can also have an arbitrary strain rate dependency (see Remarks below). Failure based on a plastic strain or a minimum time step size can be defined. For another model with more comprehensive failure criteria, see *MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY. If considering laminated or sandwich shells with non-uniform material properties (this is defined through the user-specified integration rule), the model *MAT_LAYERED_LINEAR_PLASTICITY is recommended. If solid elements are used and if the elastic strains before yielding are finite, the model *MAT_FINITE_ELASTIC_STRAIN_PLASTICITY treats the elastic strains using a hyperelastic formulation.

The LOG_INTERPOLATION keyword option interpolates the strain rates in a table LCSS with logarithmic interpolation.

The STOCHASTIC keyword option allows spatially varying yield and failure behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

The MIDFAIL keyword option is available for thin shell elements and thick shell formulations which use thin shell material models. When included on the keyword line, this option causes failure to be checked only at the mid-plane of the element. If an element has an even number of layers, failure is checked in the two layers closest to the mid-plane.

The 2D keyword option is available only for shell elements. It invokes actual plane stress treatment, meaning transverse shear stresses are not part of the yield condition but are updated elastically.

All four keyword options can be combined with each other. The order of the options is arbitrary. Before R16, the combination of STOCHASTIC and 2D was not available. The shell-related keyword options, MIDFAIL and 2D, are ignored if used on solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | $10^{21}$ | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | VP | | RFILTF | |
| Type | F | F | I | I | F | | F | |
| Default | 0.0 | 0.0 | 0 | 0 | 0.0 | | 0.0 | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

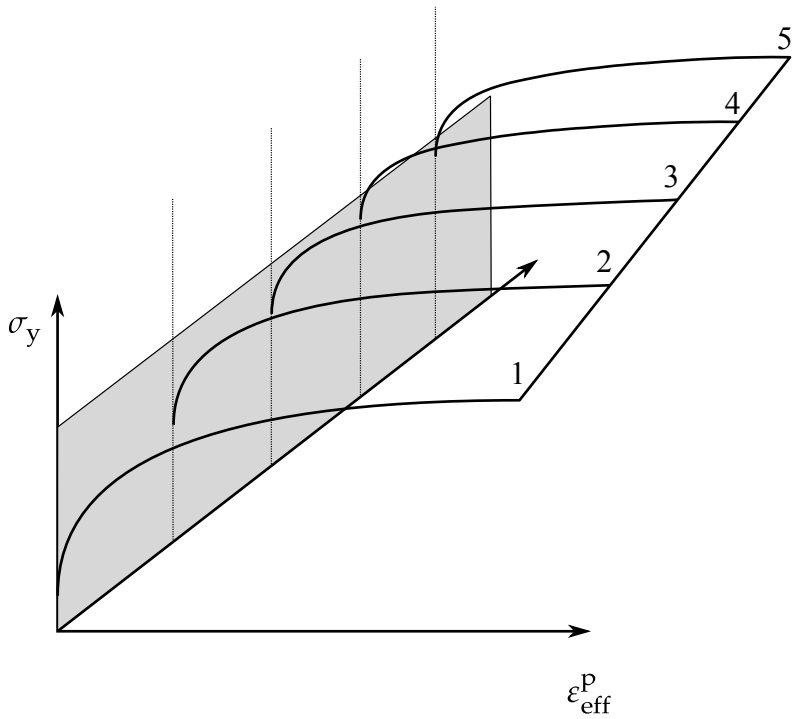



**Figure M24-1.** Rate effects may be accounted for by defining a table of curves. If a table ID is specified, a curve ID is given for each strain rate; see *DEFINE_TABLE. Intermediate values are found by interpolating between curves. Effective plastic strain as a function of yield stress is expected. If the strain rate values fall out of range, extrapolation is not used; rather, either the first or last curve determines the yield stress depending on whether the rate is low or high, respectively.

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress; ignored if LCSS > 0 except as described in Remark 1a. |
| ETAN | Tangent modulus; ignored if LCSS > 0 is defined. |
| FAIL | Failure flag:<br>LT.0.0: User-defined failure subroutine, `matusr_24` in dyn21.F, is called to determine failure. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Effective plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic element deletion |
| C | Strain rate parameter, $C$; see Remarks 1 and 3. |
| P | Strain rate parameter, $p$; see Remark 1. |
| LCSS | Load curve ID or Table ID<br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored. See Remark 7 for load curve rediscretization behavior.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see Figure M24-1. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the stress as a function of effective plastic strain curve for the highest value of strain rate is used. C, P, LCSR, EPS1 - EPS8, and ES1 - ES8 are ignored if a table ID is defined. Linear interpolation between the discrete strain rates is used by default; logarithmic interpolation is used when the LOG_INTERPOLATION option is invoked.<br>**Logarithmically Defined Tables.** Logarithmic interpolation between discrete strain rates is also assumed if the *first* value in the table is negative, in which case LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. Note that this option works only when the lowest strain rate has a value less than 1.0. For values greater than or equal to 1.0, use the LOG_INTERPOLATION option. There is some additional computational cost associated with invoking logarithmic interpolation. |

| VARIABLE | DESCRIPTION |
|---|---|
| | **Multi-Dimensional Tables.** With VP = 3.0 and 4.0, yield stress can be a function of plastic strain, strain rate, and up to seven history variables (see Remark 4). That means LCSS can refer to *DEFINE_TABLE_XD or *DEFINE_TABLE_COMPACT up to a level of 9. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress. If LCSR is negative, the load curve is evaluated using a binary search for the correct interval for the strain rate. The binary search is slower than the default incremental search, but in cases where large changes in the strain rate may occur over a single time step, it is more robust. This option is not necessary for the viscoplastic formulation. |
| VP | Formulation for rate effects:<br>EQ.-1.0: Cowper-Symonds with effective deviatoric strain rate rather than total<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation<br>EQ.3.0: Same as VP = 0, but with filtered effective *total* strain rates (see Remark 3)<br>EQ.4.0: Same as VP = 1, but with filtered effective *plastic* strain rates |
| RFILTF | Smoothing factor on the effective plastic strain rate (default is 0.0) used in the VP = 4 formulation. To include strain rate smoothing, set RFILTF to a value between 0.50 (low filtering) and 0.99 (high filtering).<br>$\dot{\varepsilon}^{p,\text{avg}}_{\text{eff},n} = \text{RFILTF} \times \dot{\varepsilon}^{p,\text{avg}}_{\text{eff},n\text{-}1} + (1 - \text{RFILTF}) \times \dot{\varepsilon}^{p}_{\text{eff},n}$ |
| EPS1 - EPS8 | Effective plastic strain values (optional). If used, at least 2 points should be defined. The first point must be zero, corresponding to the initial yield stress. WARNING: If the first point is nonzero, the yield stress is extrapolated to determine the initial yield. If this option is used, SIGY and ETAN are ignored and may be input as zero. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8 |

**Remarks:**

1. **Stress-strain behavior.** The stress-strain behavior may be treated by a bilinear stress-strain curve by defining the tangent modulus, ETAN. Alternately, a curve of effective stress as a function of effective plastic strain similar to that shown in Figure M10-1 may be defined by (EPS1, ES1) - (EPS8, ES8); however, a curve ID (LCSS) may be referenced instead if eight points are insufficient. The cost is roughly the same for either approach. Note that in the special case of uniaxial stress, true stress as a function of true plastic strain is equivalent to effective stress as a function of effective plastic strain. The most general approach is to use the table definition (LCSS) discussed below.

   Three options to account for strain rate effects are possible:

   a) Strain rate may be accounted for using the Cowper and Symonds model, which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p},$$

   where $\dot{\varepsilon}$ is the strain rate. $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$. If VP = -1.0, the deviatoric strain rates are used instead.

   If the viscoplastic option is active (VP = 1.0 or 4.0) and SIGY is > 0, the dynamic yield stress is computed from the sum of the static stress, $\sigma_y^s(\varepsilon_{\text{eff}}^p)$, which is typically given by a load curve ID, and the initial yield stress, SIGY, multiplied by the Cowper-Symonds rate term as follows:

$$\sigma_y(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p) = \sigma_y^s(\varepsilon_{\text{eff}}^p) + \text{SIGY} \times \left(\frac{\dot{\varepsilon}_{\text{eff}}^p}{C}\right)^{1/p}.$$

   Here, the plastic strain rate is used. With this latter approach, similar results to *MAT_ANISOTROPIC_VISCOPLASTIC can be obtained. If SIGY = 0, the following equation is used instead, where the static stress, $\sigma_y^s(\varepsilon_{\text{eff}}^p)$, must be defined by a load curve:

$$\sigma_y(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p) = \sigma_y^s(\varepsilon_{\text{eff}}^p)\left[1 + \left(\frac{\dot{\varepsilon}_{\text{eff}}^p}{C}\right)^{1/p}\right].$$

   This latter equation is always used if the viscoplastic option is off.

   b) For complete generality, a load curve (LCSR) to scale the yield stress may be input instead. In this curve, the scale factor as a function of strain rate is defined.

c) If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then, the table input in *DEFINE_TABLE must be used; see Figure M24-1.

2. **Viscoplastic formulation.** A fully viscoplastic formulation that incorporates the different options above within the yield surface is optional (VP = 1.0 or 4.0). An additional cost is incurred over the simple scaling, but the improvement in results can be dramatic.

3. **Filtered strain rates.** With the option VP = 3.0, it is possible to use filtered strain rates. This means that the total strain rate is used as with VP = 0.0, but this can now be filtered with the help of field C (not Cowper-Symonds in this case) and the following exponential moving average equation:

$$\dot{\varepsilon}_n^{\text{avg}} = \text{C} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{C}) \times \dot{\varepsilon}_n$$

This might be helpful if a table LCSS with crossing yield curves is used.

4. **Yield stress depending on history variables.** When VP = 3, the yield stress defined with LCSS can depend on up to seven history variables through a multi-dimensional table. These seven history variables are history variables 6 through 12, which you will have to set using *INITIAL_HISTORY_NODE or *INITIAL_-STRESS_SOLID/SHELL and whose meanings are, therefore, determined by you. For instance, you can set the values of history variables 6, 9, and 10 for certain nodes and have the value of yield stress depend upon history variables 6, 9, and 10. Note that these history variables are only initialized and do not evolve in time. See *DEFINE_TABLE_XD or *DEFINE_TABLE_COMPACT for more details.

5. **Implicit calculations.** For implicit calculations with this material involving severe nonlinear hardening, the radial return method may result in an inaccurate stress-strain response. Setting IACC = 1 on *CONTROL_ACCURACY activates a fully iterative plasticity algorithm, which will remedy this. This is not to be confused with the MITER flag on *CONTROL_SHELL, which governs the treatment of the plane stress assumption for shell elements. If failure is applied with this option, incident failure will initiate damage, and the stress will continuously degrade to zero before erosion for a deformation of 1% plastic strain. So, for instance, if the failure strain is FAIL = 0.05, then the element is eroded when $\bar{\varepsilon}^p = 0.06$ and the material goes from intact to completely damaged between $\bar{\varepsilon}^p = 0.05$ and $\bar{\varepsilon}^p = 0.06$. The reason is to enhance implicit performance by maintaining continuity in the internal forces.

6. **Failure output.** For a nonzero failure strain, *DEFINE_MATERIAL_HISTORIES can be used to output the failure indicator.

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Instability | - - - - | Failure indicator $\varepsilon^p_{\text{eff}}/\varepsilon^p_{\text{fail}}$, see FAIL |
| Plastic Strain Rate | - - - - | Effective plastic strain rate $\dot{\varepsilon}^p_{\text{eff}}$ |

7. **LCSS rediscretization.** In the special case where LCSS is a *DEFINE_CURVE, LCSS is not rediscretized (see LCINT in *DEFINE_CURVE).

## *MAT_GEOLOGIC_CAP_MODEL

This is Material Type 25. This is an inviscid two-invariant geologic cap model. This material model can be used for geomechanical problems or for materials such as concrete; see references cited below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | G | ALPHA | THETA | GAMMA | BETA |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R | D | W | XO | C | N | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PLOT | FTYPE | VEC | TOFF | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Initial bulk modulus, $K$ |
| G | Initial shear modulus |
| ALPHA | Failure envelope parameter, $\alpha$ |
| THETA | Failure envelope linear coefficient, $\theta$ |
| GAMMA | Failure envelope exponential coefficient, $\gamma$ |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Failure envelope exponent, $\beta$ |
| R | Cap, surface axis ratio |
| D | Hardening law exponent |
| W | Hardening law coefficient |
| X0 | Initial intersection of the cap surface with the $J_1$ axis, $X_0$ |
| C | Kinematic hardening coefficient, $\bar{c}$ |
| N | Kinematic hardening parameter |
| PLOT | Save the following variable for plotting in LS-PrePost, where it will be labeled as "effective plastic strain:"<br>EQ.1: hardening parameter, $\kappa$<br>EQ.2: cap -J1 axis intercept, $X(\kappa)$<br>EQ.3: volumetric plastic strain $\varepsilon_v^p$<br>EQ.4: first stress invariant, $J_1$<br>EQ.5: second stress invariant, $\sqrt{J_2}$<br>EQ.6: not used<br>EQ.7: not used<br>EQ.8: response mode number<br>EQ.9: number of iterations |
| FTYPE | Formulation flag:<br>EQ.1: soils (cap surface may contract)<br>EQ.2: concrete and rock (cap doesn't contract) |
| VEC | Vectorization flag:<br>EQ.0: vectorized (fixed number of iterations)<br>EQ.1: fully iterative<br>If the vectorized solution is chosen, the stresses might be slightly off the yield surface; however, on vector computers a much more efficient solution is achieved. |
| TOFF | Tension Cut Off, TOFF < 0 (positive in compression). |

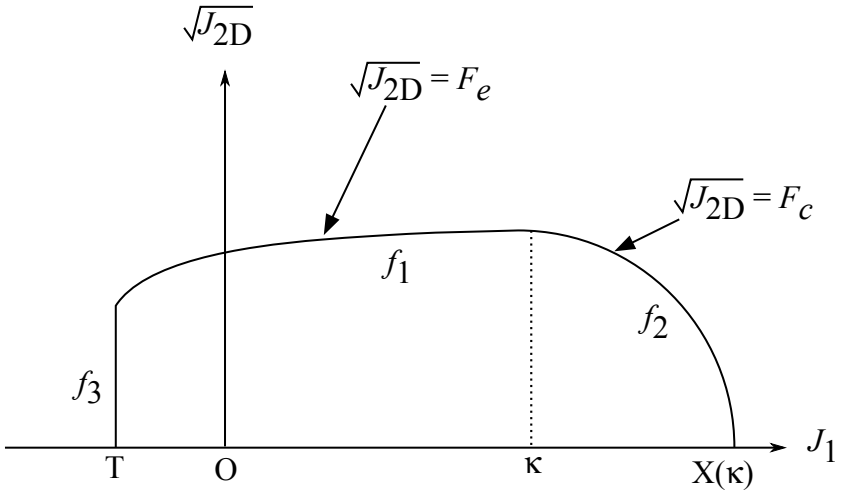



**Figure M25-1.** The yield surface of the two-invariant cap model in pressure $\sqrt{J_{2D}} - J_1$ space. Surface $f_1$ is the failure envelope, $f_2$ is the cap surface, and $f_3$ is the tension cutoff.

**Remarks:**

The implementation of an extended two-invariant cap model, suggested by Stojko [1990], is based on the formulations of Simo, et al. [1988, 1990] and Sandler and Rubin [1979]. In this model, the two-invariant cap theory is extended to include nonlinear kinematic hardening as suggested by Isenberg, Vaughan, and Sandler [1978]. A brief discussion of the extended cap model and its parameters is given below.

The cap model is formulated in terms of the invariants of the stress tensor. The square root of the second invariant of the deviatoric stress tensor, $\sqrt{J_{2D}}$ is found from the deviatoric stresses **s** as

$$\sqrt{J_{2D}} \equiv \sqrt{\frac{1}{2} S_{ij} S_{ij}}$$

and is the objective scalar measure of the distortional or shearing stress. The first invariant of the stress, $J_1$, is the trace of the stress tensor.

The cap model consists of three surfaces in $\sqrt{J_{2D}} - J_1$ space, as shown in Figure M25-1 First, there is a failure envelope surface, denoted $f_1$ in the figure. The functional form of $f_1$ is

$$f_1 = \sqrt{J_{2D}} - \min[F_e(J_1), T_{\text{mises}}],$$

where $F_e$ is given by

$$F_e(J_1) \equiv \alpha - \gamma \exp(-\beta J_1) + \theta J_1$$

and $T_{mises} \equiv |X(\kappa_n) - L(\kappa_n)|$. This failure envelop surface is fixed in $\sqrt{J_{2D}} - J_1$ space, and therefore, does not harden unless kinematic hardening is present. Next, there is a cap surface, denoted $f_2$ in the figure, with $f_2$ given by

$$f_2 = \sqrt{J_{2D}} - F_c(J_1, K)$$

where $F_c$ is defined by

$$F_c(J_1, \kappa) \equiv \frac{1}{R}\sqrt{[X(\kappa) - L(\kappa)]^2 - [J_1 - L(\kappa)]^2},$$

$X(\kappa)$ is the intersection of the cap surface with the $J_1$ axis

$$X(\kappa) = \kappa + RF_e(\kappa),$$

and $L(\kappa)$ is defined by

$$L(\kappa) \equiv \begin{cases} \kappa & \text{if} \quad \kappa > 0 \\ 0 & \text{if} \quad \kappa \leq 0 \end{cases}$$

The hardening parameter $\kappa$ is related to the plastic volume change $\varepsilon_v^p$ through the hardening law

$$\varepsilon_v^p = W\{1 - \exp[-D(X(\kappa) - X_0)]\}$$

Geometrically, $\kappa$ is seen in the figure as the $J_1$ coordinate of the intersection of the cap surface and the failure surface. Finally, there is the tension cutoff surface, denoted $f_3$ in the figure. The function $f_3$ is given by

$$f_3 \equiv T - J_1$$

where $T$ is the input material parameter which specifies the maximum hydrostatic tension sustainable by the material. The elastic domain in $\sqrt{J_{2D}} - J_1$ space is then bounded by the failure envelope surface above, the tension cutoff surface on the left, and the cap surface on the right.

An additive decomposition of the strain into elastic and plastic parts is assumed:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p \ ,$$

where $\boldsymbol{\varepsilon}^e$ is the elastic strain and $\boldsymbol{\varepsilon}^p$is the plastic strain. Stress is found from the elastic strain using Hooke's law,

$$\boldsymbol{\sigma} = \mathbf{C}(\boldsymbol{\varepsilon} \ - \boldsymbol{\varepsilon}^p),$$

where $\boldsymbol{\sigma}$ is the stress and $\mathbf{C}$ is the elastic constitutive tensor.

The yield condition may be written

$$f_1(s) \leq 0$$
$$f_2(s, \kappa) \leq 0$$
$$f_3(s) \leq 0$$

and the plastic consistency condition requires that

$$\dot{\lambda}_k f_k = 0$$
$$k = 1,2,3$$
$$\dot{\lambda}_k \geq 0$$

where $\lambda_k$ is the plastic consistency parameter for surface $k$. If $f_k < 0$ then, $\dot{\lambda}_k = 0$ and the response is elastic. If $f_k > 0$ then surface $k$ is active and $\dot{\lambda}_k$ is found from the requirement that $\dot{f}_k = 0$.

Associated plastic flow is assumed, so using Koiter's flow rule, the plastic strain rate is given as the sum of contribution from all of the active surfaces,

$$\dot{\varepsilon}^p = \sum_{k=1}^{3} \dot{\lambda}_k \frac{\partial f_k}{\partial s} \ .$$

One of the major advantages of the cap model over other classical pressure-dependent plasticity models is the ability to control the amount of dilatancy produced under shear loading. Dilatancy is produced under shear loading as a result of the yield surface having a positive slope in $\sqrt{J_{2D}} - J$ space, so the assumption of plastic flow in the direction normal to the yield surface produces a plastic strain rate vector that has a component in the volumetric (hydrostatic) direction (see Figure M25-1). In models such as the Drucker-Prager and Mohr-Coulomb, this dilatancy continues as long as shear loads are applied, and in many cases produces far more dilatancy than is experimentally observed in material tests. In the cap model, when the failure surface is active, dilatancy is produced just as with the Drucker-Prager and Mohr-Coulumb models. However, the hardening law permits the cap surface to contract until the cap intersects the failure envelope at the stress point, and the cap remains at that point. The local normal to the yield surface is now vertical, and therefore the normality rule assures that no further plastic volumetric strain (dilatancy) is created. Adjustment of the parameters that control the rate of cap contractions permits experimentally observed amounts of dilatancy to be incorporated into the cap model, thus producing a constitutive law which better represents the physics to be modeled.

Another advantage of the cap model over other models such as the Drucker-Prager and Mohr-Coulomb is the ability to model plastic compaction. In these models all purely volumetric response is elastic. In the cap model, volumetric response is elastic until the stress point hits the cap surface. Therefore, plastic volumetric strain (compaction) is generated at a rate controlled by the hardening law. Thus, in addition to controlling the amount of dilatancy, the introduction of the cap surface adds another experimentally observed response characteristic of geological material into the model.

The inclusion of kinematic hardening results in hysteretic energy dissipation under cyclic loading conditions. Following the approach of Isenberg, et al. [1978] a nonlinear kinematic hardening law is used for the failure envelope surface when nonzero values of $\bar{c}$ and $N$ are specified. In this case, the failure envelope surface is replaced by a family of yield surfaces bounded by an initial yield surface and a limiting failure envelope surface.

Thus, the shape of the yield surfaces described above remains unchanged, but they may translate in a plane orthogonal to the $J$ axis,

Translation of the yield surfaces is permitted through the introduction of a "back stress" tensor, $\boldsymbol{\alpha}$. The formulation including kinematic hardening is obtained by replacing the stress $\boldsymbol{\sigma}$ with the translated stress tensor $\eta \equiv \sigma - \alpha$ in all of the above equations. The history tensor $\boldsymbol{\alpha}$ is assumed deviatoric and therefore has only 5 unique components. The evolution of the back stress tensor is governed by the nonlinear hardening law

$$\alpha = \bar{c}\bar{F}(\sigma, \alpha)\dot{e}^p$$

where $\bar{c}$ is a constant, $\bar{F}$ is a scalar function of $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}$ and $\dot{e}^p$ is the rate of deviatoric plastic strain. The constant may be estimated from the slope of the shear stress - plastic shear strain curve at low levels of shear stress.

The function $\bar{F}$ is defined as

$$\bar{F} \equiv \max\left[0{,}1 - \frac{(\sigma - \alpha)\alpha}{2NF_e(J_1)}\right],$$

where $N$ is a constant defining the size of the yield surface. The value of $N$ may be interpreted as the radial distant between the outside of the initial yield surface and the inside of the limit surface. In order for the limit surface of the kinematic hardening cap model to correspond with the failure envelope surface of the standard cap model, the scalar parameter $\alpha$ must be replaced $\alpha - N$ in the definition $F_e$.

The cap model contains a number of parameters which must be chosen to represent a particular material and are generally based on experimental data. The parameters $\alpha$, $\beta$, $\theta$, and $\gamma$ are usually evaluated by fitting a curve through failure data taken from a set of triaxial compression tests. The parameters $W$, $D$, and $X_0$ define the cap hardening law. The value $W$ represents the void fraction of the uncompressed sample and $D$ governs the slope of the initial loading curve in hydrostatic compression. The value of $R$ is the ratio of major to minor axes of the quarter ellipse defining the cap surface. Additional details and guidelines for fitting the cap model to experimental data are found in Chen and Baladi [1985].

## *MAT_HONEYCOMB

This is Material Type 26. The major use of this material model is for honeycomb and foam materials with real anisotropic behavior. A nonlinear elastoplastic material behavior can be defined separately for all normal and shear stresses. These are considered to be fully uncoupled. See notes below. This material is available for solid elements and for thick shell formulations 3, 5, and 7.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | VF | MU | BULK |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | .05 | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCA | LCB | LCC | LCS | LCAB | LCBC | LCCA | LCSR |
| Type | F | F | F | F | F | F | F | F |
| Default | none | LCA | LCA | LCA | LCS | LCS | LCS | optional |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EAAU | EBBU | ECCU | GABU | GBCU | GCAU | AOPT | MACF |
| Type | F | F | F | F | F | F | | I |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | TSEF | SSEF | V1 | V2 | V3 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. |
| E | Young's modulus for compacted honeycomb material. |
| PR | Poisson's ratio for compacted honeycomb material. |
| SIGY | Yield stress for fully compacted honeycomb. |
| VF | Relative volume at which the honeycomb is fully compacted. |
| MU | $\mu$, material viscosity coefficient. The default, 0.05, is recommended. |
| BULK | Bulk viscosity flag:<br>EQ.0.0: Bulk viscosity is not used. This is recommended.<br>EQ.1.0: Bulk viscosity is active and $\mu = 0$. This will give results identical to previous versions of LS-DYNA. |
| LCA | Load curve ID (see *DEFINE_CURVE) for $\sigma_{aa}$ as a function of either relative volume or volumetric strain. See Remarks 1 and 3. |
| LCB | Load curve ID (see *DEFINE_CURVE) for $\sigma_{bb}$ as a function of either relative volume or volumetric strain. By default, LCB = LCA. See Remarks 1 and 3. |
| LCC | Load curve ID (see *DEFINE_CURVE) for $\sigma_{cc}$ as a function of either relative volume or volumetric strain. By default, LCC = LCA. See Remarks 1 and 3. |
| LCS | Load curve ID (see *DEFINE_CURVE) for shear stress as a function of either relative volume or volumetric strain. By default, LCS = LCA. Each component of shear stress may have its own load curve. See Remarks 1 and 3. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCAB | Load curve ID (see *DEFINE_CURVE) for $\sigma_{ab}$ as a function of either relative volume or volumetric strain. By default, LCAB = LCS. See Remarks 1 and 3. |
| LCBC | Load curve ID (see *DEFINE_CURVE) for $\sigma_{bc}$ as a function of either relative volume or volumetric strain. By default, LCBC = LCS. See Remarks 1 and 3. |
| LCCA | Load curve ID (see *DEFINE_CURVE) for $\sigma_{ca}$ as a function of either relative volume or volumetric strain. Default LCCA = LCS. See Remarks 1 and 3. |
| LCSR | Load curve ID (see *DEFINE_CURVE) for strain-rate effects defining the scale factor as a function of strain rate. This is optional. The curves defined above are scaled using this curve. See Remark 3. |
| EAAU | Elastic modulus $E_{aau}$ in uncompressed configuration. |
| EBBU | Elastic modulus $E_{bbu}$ in uncompressed configuration. |
| ECCU | Elastic modulus $E_{ccu}$ in uncompressed configuration. |
| GABU | Shear modulus $G_{abu}$ in uncompressed configuration. |
| GBCU | Shear modulus $G_{bcu}$ in uncompressed configuration. |
| GCAU | Shear modulus $G_{cau}$ in uncompressed configuration. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. BETA, if needed, is specified on *ELEMENT_SOLID_{*OPTION*}. |
| XP YP ZP | Coordinates of point *p* for AOPT = 1 and 4. |
| A1 A2 A3 | Components of vector **a** for AOPT = 2. |

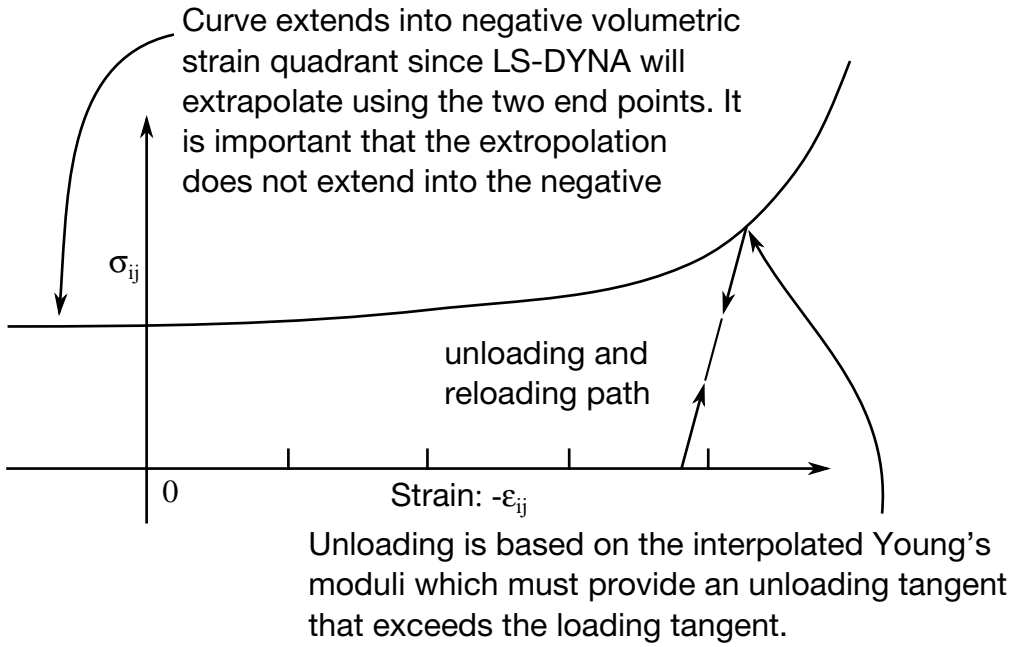



**Figure M26-1.** Stress quantity versus volumetric strain. Note that the "yield stress" at a volumetric strain of zero is non-zero. In the load curve definition, see *DEFINE_CURVE, the "time" value is the volumetric strain and the "function" value is the yield stress.

| VARIABLE | DESCRIPTION |
|---|---|
| D1 D2 D3 | Components of vector **d** for AOPT = 2. |
| V1 V2 V3 | Define components of vector **v** for AOPT = 3 and 4. |
| TSEF | Tensile strain at element failure (element will erode). |
| SSEF | Shear strain at element failure (element will erode). |

**Remarks:**

1. **Stress Load Curves.** For efficiency it is strongly recommended that the load curves with IDs LCA, LCB, LCC, LCS, LCAB, LCBC, and LCCA contain exactly the same number of points with corresponding strain values on the abscissa. If this recommendation is followed the cost of the table lookup is insignificant. Conversely, the cost increases significantly if the abscissa strain values are not consistent between load curves.

   The load curves define the magnitude of the average stress as the material changes density (relative volume); see Figure M26-1. There are two ways to define these curves, (1) as a function of relative volume, $V$, or (2) as a function of volumetric strain defined as:

$$\varepsilon_V = 1 - V \ .$$

In the former case, the first value in the curve should correspond to a value of relative volume slightly less than the fully compacted value. In the latter, the first value in the curve should be less than or equal to zero, corresponding to tension, and increase to full compaction. *Care should be taken when defining the curves so that extrapolated values do not lead to negative yield stresses.*

2. **Elastic/Shear Moduli during Compaction.** The behavior before compaction is orthotropic where the components of the stress tensor are uncoupled, meaning an $a$ component of strain will generate resistance in the local $a$-direction with no coupling to the local $b$ and $c$ directions. The elastic moduli vary, from their initial values to the fully compacted values at $V_f$, linearly with the relative volume $V$:

$$\begin{aligned}
E_{aa} &= E_{aau} + \beta(E - E_{aau}) \\
E_{bb} &= E_{bbu} + \beta(E - E_{bbu}) \\
E_{cc} &= E_{ccu} + \beta(E - E_{ccu}) \\
G_{ab} &= E_{abu} + \beta(G - G_{abu}) \\
G_{bc} &= E_{bcu} + \beta(G - G_{bcu}) \\
G_{ca} &= E_{cau} + \beta(G - G_{cau})
\end{aligned}$$

where

$$\beta = \max\left[\min\left(\frac{1-V}{1-V_f}, 1\right), 0\right]$$

and $G$ is the elastic shear modulus for the fully compacted honeycomb material

$$G = \frac{E}{2(1+v)}.$$

The relative volume, $V$, is defined as the ratio of the current volume to the initial volume. Typically, $V = 1$ at the beginning of a calculation. The viscosity coefficient $\mu$ (MU) should be set to a small number (usually .02 - .10 is okay). Alternatively, the two bulk viscosity coefficients on the control cards should be set to very small numbers to prevent the development of spurious pressures that may lead to undesirable and confusing results. The latter is not recommended since spurious numerical noise may develop.

3. **Stress Updates.** At the beginning of the stress update each element's stresses and strain rates are transformed into the local element coordinate system. For the uncompacted material, the trial stress components are updated using the elastic interpolated moduli (see Remark 2) according to:

$$\sigma_{aa}^{n+1^{\text{trial}}} = \sigma_{aa}^{n} + E_{aa}\Delta\varepsilon_{aa}$$
$$\sigma_{bb}^{n+1^{\text{trial}}} = \sigma_{bb}^{n} + E_{bb}\Delta\varepsilon_{bb}$$
$$\sigma_{cc}^{n+1^{\text{trial}}} = \sigma_{cc}^{n} + E_{cc}\Delta\varepsilon_{cc}$$
$$\sigma_{ab}^{n+1^{\text{trial}}} = \sigma_{ab}^{n} + 2G_{ab}\Delta\varepsilon_{ab}$$
$$\sigma_{bc}^{n+1^{\text{trial}}} = \sigma_{bc}^{n} + 2G_{bc}\Delta\varepsilon_{bc}$$
$$\sigma_{ca}^{n+1^{\text{trial}}} = \sigma_{ca}^{n} + 2G_{ca}\Delta\varepsilon_{ca}$$

Each component of the updated stresses is then independently checked to ensure that they do not exceed the permissible values determined from the load curves; for example, if

$$\left|\sigma_{ij}^{n+1^{\text{trial}}}\right| > \lambda\sigma_{ij}(V) \ ,$$

then

$$\sigma_{ij}^{n+1} = \sigma_{ij}(V)\frac{\lambda\sigma_{ij}^{n+1^{\text{trial}}}}{\left|\lambda\sigma_{ij}^{n+1^{\text{trial}}}\right|} \ .$$

The stress components are found using the curves defined on Card 2. The parameter $\lambda$ is either unity or a value taken from the load curve number, LCSR, that defines $\lambda$ as a function of strain-rate. Strain-rate is defined here as the Euclidean norm of the deviatoric strain-rate tensor.

For fully compacted material it is assumed that the material behavior is elastic-perfectly plastic and the stress components updated according to:

$$s_{ij}^{\text{trial}} = s_{ij}^{n} + 2G\Delta\varepsilon_{ij}^{dev^{n+1/2}} \ ,$$

where the deviatoric strain increment is defined as

$$\Delta\varepsilon_{ij}^{\text{dev}} = \Delta\varepsilon_{ij} - \frac{1}{3}\Delta\varepsilon_{kk}\delta_{ij} \ .$$

Now a check is made to see if the yield stress for the fully compacted material is exceeded by comparing the effective trial stress,

$$s_{\text{eff}}^{\text{trial}} = \left(\frac{3}{2}s_{ij}^{\text{trial}}s_{ij}^{\text{trial}}\right)^{1/2} \ ,$$

to the defined yield stress, SIGY. If the effective trial stress exceeds the yield stress the stress components are simply scaled back to the yield surface

$$s_{ij}^{n+1} = \frac{\sigma_y}{s_{\text{eff}}^{\text{trial}}}s_{ij}^{\text{trial}}.$$

Now the pressure is updated using the elastic bulk modulus, $K$,

$$p^{n+1} = p^{n} - K\Delta\varepsilon_{kk}^{n+1/2}$$

where

$$K = \frac{E}{3(1-2v)}$$

to obtain the final value for the Cauchy stress

$$\sigma_{ij}^{n+1} = s_{ij}^{n+1} - p^{n+1}\delta_{ij} \ .$$

After completing the stress the stresses are transformed back to the global configuration.

4. **Failure.** For *CONSTRAINED_TIED_NODES_WITH_FAILURE, the failure is based on the volume strain instead to the plastic strain.

## *MAT_MOONEY-RIVLIN_RUBBER

This is Material Type 27. A two-parametric material model for rubber can be defined.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | A | B | REF | | |
| Type | A | F | F | F | F | F | | |

**Least Squares Card.** If the values on Card 2 are nonzero, then a least squares fit is computed from the uniaxial data provided by the curve LCID. In this case A and B are ignored. If the A and B fields on Card 1 are left blank, then the fields on Card 2 *must* be nonzero.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LCID | | | | |
| Type | F | F | F | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PR | Poisson's ratio. A value between 0.49 and 0.5 is recommended; smaller values may not work. Setting to 0.5 for solid elements with implicit analysis activates a *U-P* formulation. See Remark 3 for details. |
| A | Constant; see literature and remarks below. This field is ignored if the fields on Card 2 are nonzero. |
| B | Constant; see literature and remarks below. This field is ignored if the fields on Card 2 are nonzero. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY. |

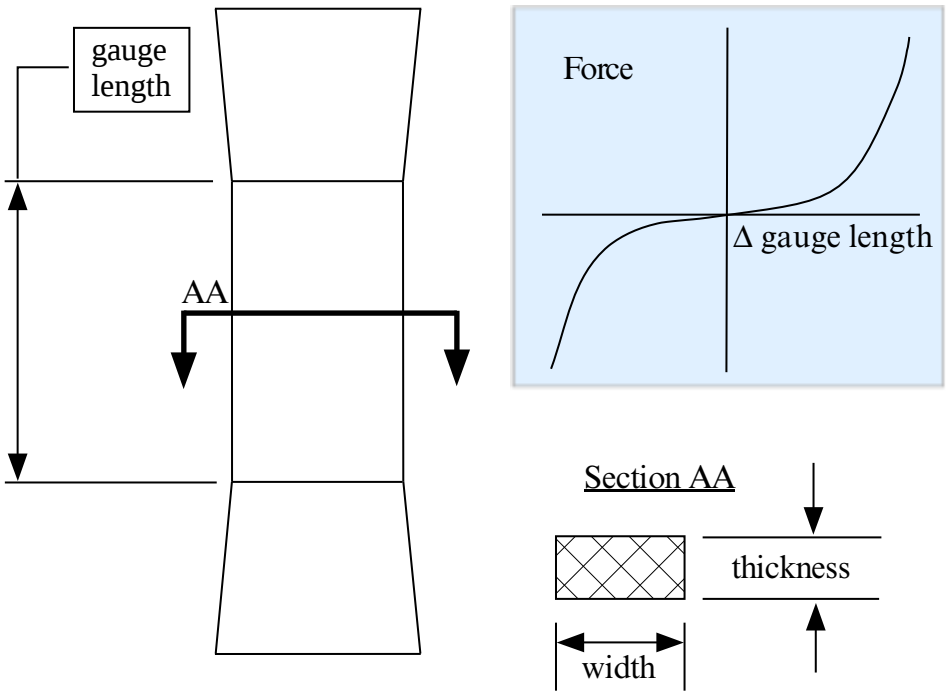



**Figure M27-1.** Uniaxial specimen for experimental data

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Off<br>EQ.1.0: On |
| SGL | Specimen gauge length $l_0$; see Figure M27-1. |
| SW | Specimen width; see Figure M27-1. |
| ST | Specimen thickness; see Figure M27-1. |
| LCID | Curve ID, see *DEFINE_CURVE, giving the force versus actual change $\Delta L$ in the gauge length. See Remark 2. See also Figure M27-2 for an alternative definition. LS-DYNA computes a least squares fit from this data. A and B are ignored if this field is defined. |

**Remarks:**

1. **Strain energy density function.** The strain energy density function is defined as:

$$W = A(I-3) + B(II-3) + C\left(III^{-2}-1\right) + D(III-1)^2$$

where

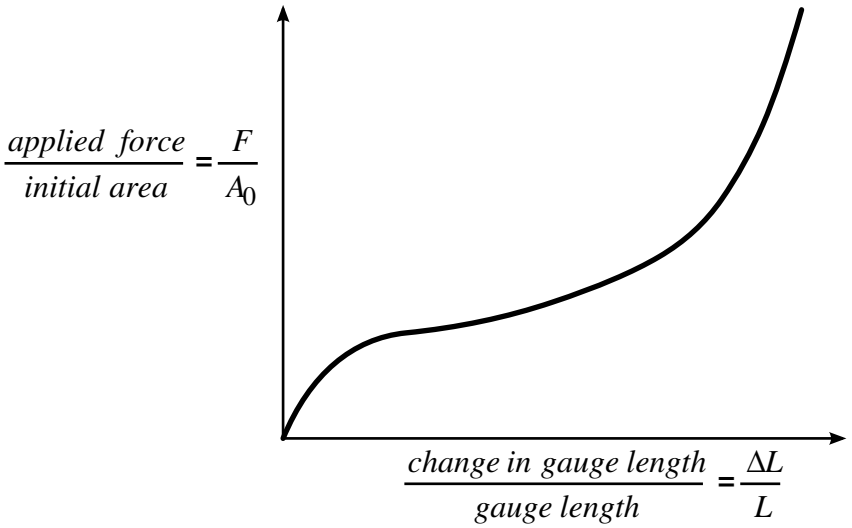



**Figure M27-2** The stress as a function of strain curve can be used instead of the force as a function of the change in the gauge length by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force. *MAT_077_O is a better alternative for fitting data resembling the curve above. *MAT_027 will provide a poor fit to a curve that exhibits a strong upturn in slope as strains become large.

$$C = 0.5A + B$$
$$D = \frac{A(5v-2) + B(11v-5)}{2(1-2v)}$$

Here, $A$ and $B$ are constants, $v$ is the Poisson's ratio, $2(A+B)$ is the shear modulus of linear elasticity, and $I$, $II$, and $III$ are the principal invariants of the right Cauchy-Green tensor, $\mathbf{C}$. Recall that $\mathbf{C} = \mathbf{F}^T\mathbf{F}$ where $\mathbf{F} = \nabla_{\mathbf{X}}\mathbf{x}$ is the deformation gradient, $\mathbf{x}$ is the current configuration, and $\mathbf{X}$ is the reference configuration.

2. **Experimental data for the material.** The load curve definition that provides the uniaxial data should give the change in gauge length, $\Delta L$, versus the corresponding force. In compression, both the force and the change in gauge length must be specified as negative values. In tension, the force and change in gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction, $\lambda_1$, is then given by

$$\lambda_1 = \frac{L_0 + \Delta L}{L_0}$$

with $L_0$ being the initial length and $L$ being the actual length.

Alternatively, the stress as a function of strain curve can also be input by setting the gauge length, thickness, and width to unity (1.0) and defining the

engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force; see Figure M27-1.

LS-DYNA performs a least square fit to the experimental data during the initialization phase. The d3hsp file provides a comparison between the fit and the actual. We recommend visually checking to make sure it is acceptable. d3hsp also contains the coefficients $A$ and $B$. We also advise examining the material model with the material driver (see Appendix K).

3. **Incompressible material.** If the material is incompressible with a Poisson ratio of exactly 0.5, LS-DYNA uses a mixed finite element method of displacement-pressure ($U$-$P$) type to avoid volumetric locking. Note that this formulation is only available for solid elements with implicit analysis. With this formulation, we enforce the incompressibility constraint, $J = 1$, with $J = \det(F)$, strongly using a Lagrange multiplier technique. In the absence of inertial and external forces, this amounts to seeking a stationary point to the Lagrangian

$$L(\mathbf{u}, \lambda) = \int W(\mathbf{C}) + \lambda(J - 1) d\mathbf{x} \ ,$$

where $\mathbf{u} = \mathbf{x} - \mathbf{X}$ is the displacement, and $\lambda$ is a pressure-like Lagrange multiplier for the constraint. The stiffness matrix resulting from the $U$-$P$ formulation is a saddle-point type (i.e., indefinite), and may therefore require special consideration regarding the choice of linear solver and stiffness reformation limit.

4. **Output to effective plastic strain location in d3plot.** The history variable labeled as "effective plastic strain" in LS-PrePost is internal energy density in the case of *MAT_MOONEY-RIVLIN_RUBBER.

## *MAT_RESULTANT_PLASTICITY

This is Material Type 28. It defines a resultant formulation for beam and shell elements including elasto-plastic behavior. This model is available for the Belytschko-Schwer beam, the $C^0$ triangular shell, the Belytschko-Tsay shell, and the fully integrated type 16 shell. For beams, the treatment is elastic-perfectly plastic, but for shell elements, isotropic hardening is approximately modeled. For a detailed description, we refer to the LS-DYNA Theory Manual. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | none | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus (for shells only) |

## *MAT_FORCE_LIMITED

This is Material Type 29. It is a force-limited resultant formulation. With this material model, for the Belytschko-Schwer beam only, plastic hinge formation at the ends of a beam can be modeled using curve definitions. Optionally, collapse can also be modeled. See also *MAT_139.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | DF | IAFLC | YTFLAG | ASOFT |
| Type | A | F | F | F | F | I | F | F |
| Default | none | none | none | none | 0.0 | 0 | 0.0 | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC1 | LC2 | LC3 | LC4 | LC5 | LC6 | LC7 | LC8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPS1 | SFS1 | LPS2 | SFS2 | YMS1 | YMS2 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0 | 1.0 | LPS1 | 1.0 | $10^{20}$ | YMS1 | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPT1 | SFT1 | LPT2 | SFT2 | YMT1 | YMT2 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0 | 1.0 | LPT1 | 1.0 | $10^{20}$ | YMT1 | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPR | SFR | YMR | | | | | |
| Type | F | F | F | | | | | |
| Default | 0 | 1.0 | $10^{20}$ | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| DF | Damping factor; see Remark 2. *A proper control for the timestep must be maintained by the user.* |

| VARIABLE | DESCRIPTION |
|---|---|
| IAFLC | Axial force load curve option:<br>EQ.0: axial load curves are force as a function of strain.<br>EQ.1: axial load curves are force as a function of change in length. |
| YTFLAG | Flag to allow beam to yield in tension:<br>EQ.0.0: beam does not yield in tension.<br>EQ.1.0: beam can yield in tension. |
| ASOFT | Axial elastic softening factor applied once hinge has formed. When a hinge has formed the stiffness is reduced by this factor. If zero, this factor is ignored. |
| M1, M2, …, M8 | Applied end moment for force as a function of (strain/change in length) curve. At least one must be defined. A maximum of 8 moments can be defined. The values should be in ascending order. |
| LC1, LC2, …, LC8 | Load curve ID (see *DEFINE_CURVE) defining axial force (collapse load) as a function of strain/change in length (see AOPT) for the corresponding applied end moment. Define the same number as end moments. Each curve must contain the same number of points. |
| LPS1 | Load curve ID for plastic moment as a function of rotation about the *s*-axis at node 1. If zero, this load curve is ignored. |
| SFS1 | Scale factor for plastic moment as a function of rotation curve about the *s*-axis at node 1. Default = 1.0. |
| LPS2 | Load curve ID for plastic moment as a function of rotation about the *s*-axis at node 2. Default: LPS1. |
| SFS2 | Scale factor for plastic moment as a function of rotation curve about the *s*-axis at node 2. Default: SFS1. |
| YMS1 | Yield moment about the *s*-axis at node 1 for interaction calculations (default set to $10^{20}$ to prevent interaction). |
| YMS2 | Yield moment about *s*-axis at node 2 for interaction calculations (default set to YMS1). |
| LPT1 | Load curve ID for plastic moment as a function of rotation about the *t*-axis at node 1. If zero, this load curve is ignored. |

| VARIABLE | DESCRIPTION |
|---|---|
| SFT1 | Scale factor for plastic moment as a function of rotation curve about the *t*-axis at node 1. Default = 1.0. |
| LPT2 | Load curve ID for plastic moment as a function of rotation about the *t*-axis at node 2. Default: LPT1. |
| SFT2 | Scale factor for plastic moment as a function of rotation curve about the *t*-axis at node 2. Default: SFT1. |
| YMT1 | Yield moment about the *t*-axis at node 1 for interaction calculations (default set to $10^{20}$ to prevent interactions) |
| YMT2 | Yield moment about the *t*-axis at node 2 for interaction calculations (default set to YMT1) |
| LPR | Load curve ID for plastic torsional moment as a function of rotation. If zero, this load curve is ignored. |
| SFR | Scale factor for plastic torsional moment as a function of rotation (default = 1.0). |
| YMR | Torsional yield moment for interaction calculations (default set to $10^{20}$ to prevent interaction) |

**Remarks:**

1. **Load Curves.** This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The moment as a function rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (0.0, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local *s* and *t* axes.

   Axial collapse occurs when the compressive axial load reaches the collapse load. Collapse load as a function of collapse deflection is specified in the form of a load curve. The points of the load curve are either (true strain, collapse force) or (change in length, collapse force). Both quantities should be entered as positive for all points and will be interpreted as compressive. The first point should be (0.0, initial collapse load).

The collapse load may vary with end moment as well as with deflections. In this case several load-deflection curves are defined, each corresponding to a different end moment. Each load curve should have the same number of points and the same deflection values. The end moment is defined as the average of the absolute moments at each end of the beam and is always positive.

2. **Damping.** Stiffness-proportional damping may be added using the damping factor $\lambda$. This is defined as follows:

$$\lambda = \frac{2 \times \xi}{\omega}$$

where $\xi$ is the damping factor at the reference frequency $\omega$ (in radians per second). For example if 1% damping at 2Hz is required

$$\lambda = \frac{2 \times 0.01}{2\pi \times 2} = 0.001592 \ .$$

If damping is used, a small timestep may be required. LS-DYNA does not check this, so to avoid instability it may be necessary to control the timestep using a load curve. As a guide, the timestep required for any given element is multiplied by $0.3L/c\lambda$ when damping is present ($L$ = element length, $c$ = sound speed).

3. **Moment Interaction.** Plastic hinges can form due to the combined action of moments about the three axes. This facility is activated only when yield moments are defined in the material input. A hinge forms when the following condition is first satisfied:

$$\left(\frac{M_r}{M_{\mathrm{r_{yield}}}}\right)^2 + \left(\frac{M_s}{M_{\mathrm{s_{yield}}}}\right)^2 + \left(\frac{M_t}{M_{\mathrm{t_{yield}}}}\right)^2 \geq 1 \ ,$$

where

$$M_r,\ M_s,\ M_t = \text{current moment}$$

$$M_{r_{\mathrm{yield}}}, M_{s_{\mathrm{yield}}}, M_{t_{\mathrm{yield}}} = \text{yield moment}$$

Note that scale factors for hinge behavior defined in the input will also be applied to the yield moments: for example, $M_{s_{\mathrm{yield}}}$ in the above formula is given by the input yield moment about the local axis times the input scale factor for the local $s$ axis. For strain-softening characteristics, the yield moment should generally be set equal to the initial peak of the moment-rotation load curve.

On forming a hinge, upper limit moments are set. These are given by

$$M_{r_{\mathrm{upper}}} = \max\left(M_r, \frac{M_{r_{\mathrm{yield}}}}{2}\right)$$

and similar conditions hold for $M_{s_{\mathrm{upper}}}$ and $M_{t_{\mathrm{upper}}}$.

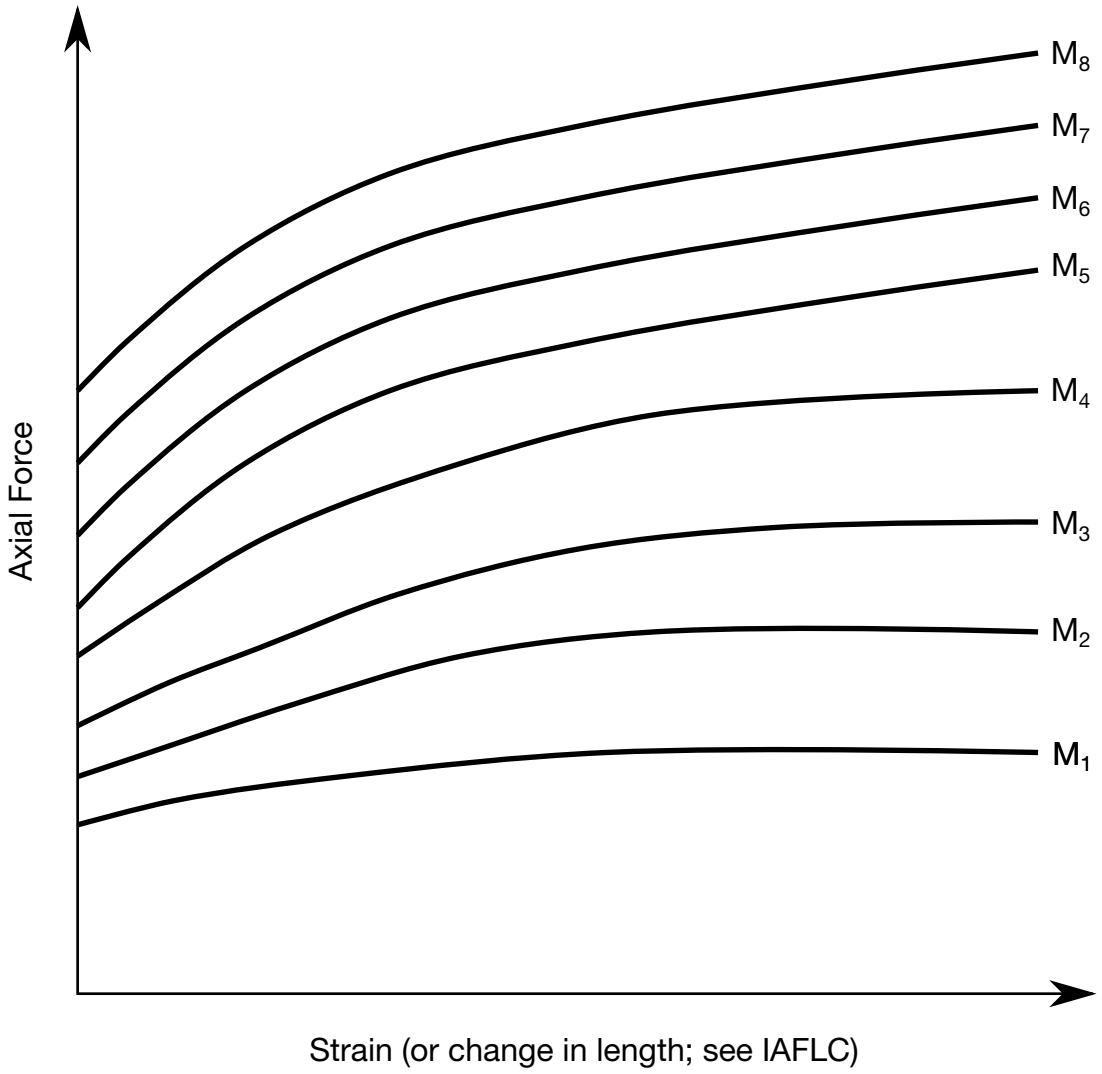



**Figure M29-1.** The force magnitude is limited by the applied end moment. For an intermediate value of the end moment LS-DYNA interpolates between the curves to determine the allowable force value.

Thereafter, the plastic moments will be given by

$$M_{r_p} = \min(M_{r_{\text{upper}}}, M_{r_{\text{curve}}})$$

where $M_{r_p}$ is the current plastic moment and $M_{r_{\text{curve}}}$ is the moment from the load curve at the current rotation scaled by the scale factor. $M_{s_p}$and $M_{t_p}$ satisfy similar conditions.

The effect of this is to provide an upper limit to the moment that can be generated; it represents the softening effect of local buckling at a hinge site. Thus if a member is bent about is local *s*-axis it will then be weaker in torsion and about its local *t*-axis. For moment-softening curves, the effect is to trim off the initial

peak (although if the curves subsequently harden, the final hardening will also be trimmed off).

It is not possible to make the plastic moment vary with axial load.

## *MAT_SHAPE_MEMORY

This is Material Type 30. This material model describes the superelastic response present in shape-memory alloys, that is, the peculiar material ability to undergo large deformations with a full recovery in loading-unloading cycles (see Figure M30-1). The material response is always characterized by a hysteresis loop. See the references by Auricchio, Taylor and Lubliner [1997] and Auricchio and Taylor [1997]. This model is available for shells, solids, and Hughes-Liu beam elements. The model supports von Mises or Ducker-Prager-based isotropic plasticity with an arbitrary effective stress as a function of effective plastic or total strain curve.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | LCSS | LCSSC | IDPP | |
| Type | A | F | F | F | I | I | I | |
| Default | none | none | none | none | 0 | 0 | 0 | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIG_ASS | SIG_ASF | SIG_SAS | SIG_SAF | EPSL | ALPHA | YMRT | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | 0.0 | 0.0 | |

**Optional Load Curve Card (starting with R7.1).** Load curves for mechanically induced phase transitions.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID_AS | LCID_SA | | | | | | |
| Type | I | I | | | | | | |
| Default | none | none | | | | | | |

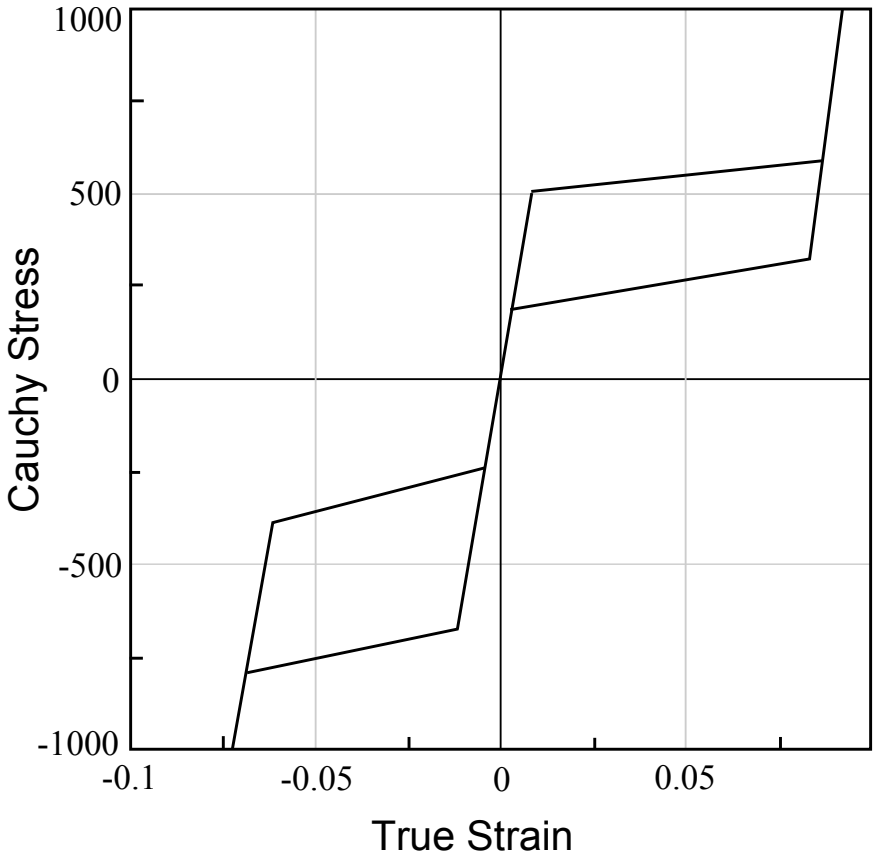



**Figure M30-1.** Superelastic Behavior for a Shape Memory Material

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| LCSS | The absolute value of LCSS is a load curve ID for effective stress as a function of effective plastic *or* effective total strain, depending on the first data point (this load curve is optional). If the first data point has a positive ordinate value, it indicates the initial yield stress at zero plastic strain. If it is zero, it is a curve with respect to effective total strain. The first abscissa value must always be zero.<br>For a negative value of LCSS, negative values of SIG_ASS, SIG_ASF, SIG_SAS, and SIG_SAF indicate dependence on plastic strain; see below. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSSC | The absolute value of LCSSC is a load curve ID for effective stress as a function of effective plastic *or* effective total strain (depending on the first data point) under *compressive* load (this load curve is optional). If the first data point has a positive ordinate value, it indicates the initial yield stress at zero plastic strain. If it is zero, it is a curve with respect to effective total strain. The first abscissa value must always be zero.<br>A negative value of LCSSC scales the load curve with the parameter ALPHA. For example, LCSSC = -LCSS will give the same relation between compressive and tensile hardening as between compressive and tensile phase transformation stresses.<br>If LCSSC is undefined or zero, LCSS specifies the behavior in compression. |
| IDPP | Plasticity model (see Remark 2):<br>EQ.0: von Mises plasticity (default)<br>EQ.1: Drucker-Prager-based plasticity |
| SIG_ASS | Starting value for the forward phase transformation (conversion of austenite into martensite) in the case of a uniaxial tensile state of stress.<br>LT.0.0: -SIG_ASS is a load curve ID defining the starting value as a function of temperature. If LCSS is also negative, then -SIG_ASS is either a load curve specifying the starting value as a function of effective plastic strain or a table of such load curves for different temperatures. |
| SIG_ASF | Final value for the forward phase transformation (conversion of austenite into martensite) in the case of a uniaxial tensile state of stress.<br>LT.0.0: -SIG_ASF is a load curve ID defining the final value as a function of temperature. If LCSS is also negative, -SIG_ASF is either a load curve specifying the final value as a function of effective plastic strain or a table of such load curves for different temperatures. |
| SIG_SAS | Starting value for the reverse phase transformation (conversion of martensite into austenite) in the case of a uniaxial tensile state of stress.<br>LT.0.0: -SIG_SAS is a load curve ID defining the starting value as a function of temperature. If LCSS is also negative, - |

| VARIABLE | DESCRIPTION |
|---|---|
| | SIG_SAS is either a load curve specifying the starting value as a function of effective plastic strain or a table of such load curves for different temperatures. |
| SIG_SAF | Final value for the reverse phase transformation (conversion of martensite into austenite) in the case of a uniaxial tensile state of stress.<br>LT.0.0: -SIG_SAF is a load curve ID specifying the reverse value as a function of temperature. If LCSS is also negative, -SIG_SAF is either a load curve specifying the final value as a function of effective plastic strain or a table of such load curves for different temperatures. |
| EPSL | Recoverable strain or maximum residual strain. It is a measure of the maximum deformation obtainable for all the martensite in one direction. |
| ALPHA | Parameter measuring the difference between material responses in tension and compression (set ALPHA = 0 for no difference). Also, see Remark 1. |
| YMRT | Young's modulus for the martensite if it is different from the modulus for the austenite. Defaults to the austenite modulus if it is set to zero. |
| LCID_AS | Load curve ID or table ID for the *forward* phase change (conversion of austenite into martensite).<br>1. *When LCID_AS is a load curve ID,* the curve is taken to be effective stress as a function of martensite fraction (ranging from 0 to 1).<br>2. *When LCID_AS is a table ID,* the table defines for each phase transition rate (derivative of martensite fraction) a load curve ID specifying the stress as a function of martensite fraction for that phase transition rate.<br>The stress as a function of martensite fraction curve for the lowest value of the phase transition rate is used if the phase transition rate falls below the minimum value. Likewise, the stress as a function of martensite fraction curve for the highest value of phase transition rate is used if the phase transition rate exceeds the maximum value. |

| VARIABLE | DESCRIPTION |
|---|---|

3. The values of SIG_ASS and SIG_ASF are overwritten when this option is used.

LCID_SA Load curve ID or table ID for *reverse* phase change (conversion of martensite into austenite).

1. *When LCID_SA is a load curve ID*, the curve is taken to be effective stress as a function of martensite fraction (ranging from 0 to 1).

2. *When LCID_SA is a table ID*, the table defines for each phase transition rate (derivative of martensite fraction) a load curve ID specifying the stress as a function of martensite fraction for that phase transition rate.

   The stress as a function of martensite fraction curve for the smallest value of the phase transition rate is used if the phase transition rate falls below the minimum value. Likewise, the stress as a function of martensite fraction curve for the largest value of the phase transition rate is used if the phase transition rate exceeds the maximum value.

3. The values of SIG_ASS and SIG_ASF are overwritten when this option is used.

**Remarks:**

1. **Difference in material responses in tension and compression.** The material parameter $\alpha$ ($-\sqrt{2/3} < \alpha < \sqrt{2/3}$) measures the difference between material responses in tension and compression. In particular, it is possible to relate the parameter $\alpha$ to the initial stress value of the austenite into martensite conversion from the expression

$$\alpha = \sqrt{\frac{2}{3}}\left(\frac{-\sigma_s^{AS,-} - \sigma_s^{AS,+}}{-\sigma_s^{AS,-} + \sigma_s^{AS,+}}\right),$$

   where $\sigma_s^{AS,+} > 0$ and $\sigma_s^{AS,-} < 0$ are the values in tension and compression, respectively. From the input parameters $\alpha$ and $\sigma_s^{AS,+}$, the stress in compression is then

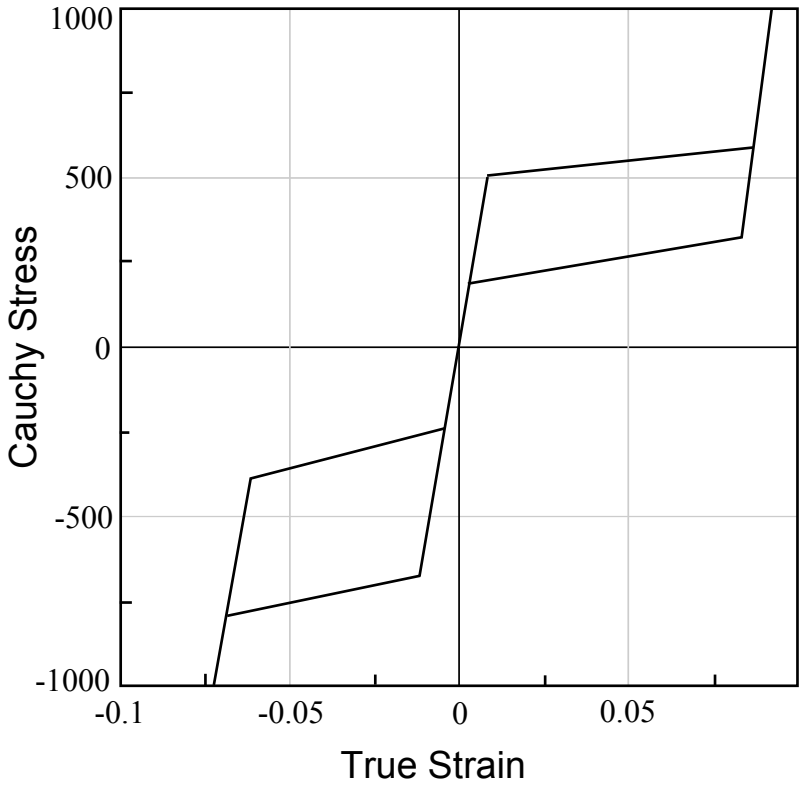



**Figure M30-2.** Complete loading-unloading test in tension and compression.

$$\sigma_s^{AS,-} = \frac{\alpha + \sqrt{\frac{2}{3}}}{\alpha - \sqrt{\frac{2}{3}}} \sigma_s^{AS,+}.$$

In Figure M30-2, we show the uniaxial Cauchy stress versus the logarithmic strain plot obtained from a simple test problem. The investigated problem is the complete loading-unloading test in tension and compression. We set the material properties to:

| Property | Value | Property | Value |
|---|---|---|---|
| E | 60000 MPa | SIG_SAF | 200 MPa |
| PR | 0.3 | EPSL | 0.07 |
| SIG_ASS | 520 MPa | ALPHA | 0.12 |
| SIG_ASF | 600 MPa | YMRT | 50000 MPa |
| SIG_SAS | 300 MPa | | |

2. **Plasticity.** A basic assumption for plasticity in this model is that plastic yielding occurs after the austenite-martensite transformation is complete. Otherwise, the material model breaks down, resulting in an error termination. For von Mises

plasticity, choosing a yield curve $\sigma_y$ larger than $\sigma_f^{AS}$ works well for low volumetric deformation, but for high volumetric deformation, yielding may still occur before a complete phase transformation. This premature yielding can occur because the material model uses a Drucker-Prager phase transition criterion:

$$F = \|\mathbf{t}\| + 3\alpha p \geq F_{tr} .$$

Here, $\mathbf{t} = \boldsymbol{\tau} - p\mathbf{I}$ is the deviatoric Kirchhoff stress, $p = \mathrm{tr}(\boldsymbol{\tau})/3$ is the pressure, and the transition criterion, $F_{tr}$, is derived from the uniaxial case

$$F_{tr} = \left(\alpha + \sqrt{\frac{2}{3}}\right)\sigma_{tr}, \qquad \sigma_{tr} = \{\sigma_s^{AS}, \sigma_f^{AS}, \sigma_s^{SA}, \sigma_f^{SA}\}.$$

Thus, the phase transitions are pressure dependent, but von Mises plasticity is not. To overcome this, the option IDPP enables a Drucker-Prager yield function:

$$F = \|\mathbf{t}\| + 3\alpha p \geq \left(\alpha + \sqrt{\frac{2}{3}}\right)\sigma_y(\varepsilon_p),$$

such that the yield stress corresponds to the transition stresses.

## *MAT_FRAZER_NASH_RUBBER_MODEL

This is Material Type 31. This model defines rubber from uniaxial test data. It is a modified form of the hyperelastic constitutive law first described in Kenchington [1988]. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | C100 | C200 | C300 | C400 | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C110 | C210 | C010 | C020 | EXIT | EMAX | EMIN | REF |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LCID | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PR | Poisson's ratio. Values between .49 and .50 are suggested. |
| C100 | $C_{100}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| C200 | $C_{200}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |

| VARIABLE | DESCRIPTION |
|---|---|
| C300 | $C_{300}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| C400 | $C_{400}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| C110 | $C_{110}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| C210 | $C_{210}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| C010 | $C_{010}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| C020 | $C_{020}$, constant in strain energy functional. If a least squares fit is being used, set this constant to 1.0 if the term it belongs to in the strain energy functional is to be included. See Remarks. |
| EXIT | Exit option (only in explicit analysis):<br>EQ.1.0: Stop if strain limits are exceeded (recommended)<br>NE.1.0: Continue if strain limits are exceeded. The curve is then extrapolated. |
| EMAX | Maximum strain limit, (Green-St, Venant Strain). |
| EMIN | Minimum strain limit, (Green-St, Venant Strain). |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details).<br>EQ.0.0: Off,<br>EQ.1.0: On. |
| SGL | Specimen gauge length; see Figure M27-1. |
| SW | Specimen width; see Figure M27-1. |

| VARIABLE | DESCRIPTION |
|---|---|
| ST | Specimen thickness; see Figure M27-1. |
| LCID | Load curve ID (see DEFINE_CURVE) giving the force as a function of actual change in gauge length. See also Figure M27-2 for an alternative definition. |

**Remarks:**

The constants can be defined directly, or a least squares fit can be performed if the uniaxial data (SGL, SW, ST and LCID) is available. *If a least squares fit is chosen, then the terms to be included in the energy functional are flagged by setting their corresponding coefficients to unity.* If all coefficients are zero, the default is to use only the terms involving $I_1$ and $I_2$. $C_{100}$ defaults to unity if the least square fit is used.

The strain energy functional $U$ is defined in terms of the input constants as

$$U = C_{100}I_1 + C_{200}I_1^2 + C_{300}I_1^3 + C_{400}I_1^4 + C_{110}I_1I_2 + C_{210}I_1^2I_2 + C_{010}I_2 + C_{020}I_2^2 + f(J)$$

where the invariants $I_1$, $I_2$ and $J$ can be expressed in terms of the deformation gradient matrix, $\mathbf{F}$, and the right stretch tensor, $\mathbf{C} = \mathbf{F}^T\mathbf{F}$:

$$J = \det\mathbf{F}$$
$$I_1 = \mathrm{tr}(\mathbf{C}) - 3$$
$$I_2 = \frac{1}{2}\left(\mathrm{tr}(\mathbf{C})^2 - \mathrm{tr}\left(\mathbf{C}^2\right)\right) - 3.$$

The dependence on the third invariant is given as

$$f(J) = \frac{2C_{100}(\nu - 4) + 4C_{010}(11\nu - 5)}{1 - 2\nu}\left(\frac{J^2}{2} - \ln J\right) + \frac{1}{2}(C_{100} + 2C_{010})\frac{1}{J^4}$$

where $\nu$ is the Poisson's ratio. The first term on the right-hand side of this expression should be interpreted as the constitutive law for the pressure while the second is necessary for providing zero stress at the reference configuration.

The derivative of $U$ with respect to $\mathbf{C}$ gives the 2nd Piola-Kirchhoff stress $\mathbf{S}$ as

$$\mathbf{S} = 2\frac{\partial U}{\partial \mathbf{C}}$$

and the Cauchy stress $\sigma$ is then given by

$$\sigma = \frac{1}{J}\mathbf{F}\mathbf{S}\mathbf{F}^T.$$

The load curve definition that provides the uniaxial data should give the change in gauge length, $\Delta L$, and the corresponding force. In compression both the force and the change in gauge length must be specified as negative values. In tension the force and change in

gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction, $\lambda_1$, is then given by

$$\lambda_1 = \frac{L_o + \Delta L}{L_o}$$

Alternatively, the stress as a function of strain curve can also be input by setting the gauge length, thickness, and width to unity and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force; see Figure M27-2 The least square fit to the experimental data is performed during the initialization phase, and a comparison between the fit and the actual input is provided in the printed file. It is a good idea to visually check the fit to make sure it is acceptable. The coefficients $C_{100}$ through $C_{020}$ are also printed in the output file.

## *MAT_LAMINATED_GLASS

This is Material Type 32. With this material model, layered glass including polymeric layers can be modeled. Failure of the glass part is possible. See notes below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EG | PRG | SYG | ETG | EFG | EP |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PRP | SYP | ETP | | | | | |
| Type | F | F | F | | | | | |

**Integration Point Cards.** Up to four of this card (specifying up to 32 values) may be input. This input is terminated by the next keyword ("*") card.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | F1 | F2 | F3 | F4 | F5 | F6 | F7 | F8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EG | Young's modulus for glass |
| PRG | Poisson's ratio for glass |
| SYG | Yield stress for glass |
| ETG | Plastic hardening modulus for glass |

| VARIABLE | DESCRIPTION |
|---|---|
| EFG | Plastic strain at failure for glass |
| EP | Young's modulus for polymer |
| PRP | Poisson's ratio for polymer |
| SYP | Yield stress for polymer |
| ETP | Plastic hardening modulus for polymer |
| F1, …, FN | Integration point material:<br>EQ.0.0: glass (default)<br>EQ.1.0: polymer<br>A user-defined integration rule must be specified; see *INTEGRATION_SHELL. See Remarks below. |

**Remarks:**

Isotropic hardening for both materials is assumed. The material to which the glass is bonded is assumed to stretch plastically without failure. A user defined integration rule specifies the thickness of the layers making up the glass. F*i* defines whether the integration point is glass (0.0) or polymer (1.0). The material definition, F*i*, must be given for the same number of integration points (NIPTS) as specified in the rule. A maximum of 32 layers is allowed.

If the recommended user defined rule is not defined, the default integration rules are used. The location of the integration points in the default rules are defined in the *SECTION_SHELL keyword description.

## *MAT_BARLAT_ANISOTROPIC_PLASTICITY

This is Material Type 33. This model was developed by Barlat, Lege, and Brem [1991] for modeling anisotropic material behavior in forming processes. The finite element implementation of this model is described in detail by Chung and Shah [1992] and is used here. It is based on a six parameter model, which is ideally suited for 3D continuum problems (see remarks below). For sheet forming problems, we recommend material 36 which is based on a 3-parameter model.

This material is available for shell, thick shell, and solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | K | EO | N | M |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | C | F | G | H | LCID | |
| Type | F | F | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | BETA | MACF | | | | | |
| Type | F | F | I | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| K | $k$, strength coefficient (see remarks below) |
| E0 | $\varepsilon_0$, strain corresponding to the initial yield (see remarks below) |
| N | $n$, hardening exponent for yield strength |
| M | $m$, flow potential exponent in Barlat's Model |
| A | $a$, anisotropy coefficient in Barlat's Model |
| B | $b$, anisotropy coefficient in Barlat's Model |
| C | $c$, anisotropy coefficient in Barlat's Model |
| F | $f$, anisotropy coefficient in Barlat's Model |
| G | $g$, anisotropy coefficient in Barlat's Model |
| H | $h$, anisotropy coefficient in Barlat's Model |
| LCID | Option load curve ID defining effective stress as a function of effective plastic strain. If nonzero, this curve will be used to define the yield stress. The load curve is implemented for solid elements only. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by |

| VARIABLE | DESCRIPTION |
|---|---|
| | element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. |
| | EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| BETA | Material angle in degrees for AOPT = 1 (shells only) and AOPT = 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, or *ELEMENT_SOLID_ORTHO. |
| MACF | Material axes change flag for solid elements: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-4: Switch material axes $b$ and $c$ before BETA rotation<br>EQ.-3: Switch material axes $a$ and $c$ before BETA rotation<br>EQ.-2: Switch material axes $a$ and $b$ before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes $a$ and $b$ after BETA rotation<br>EQ.3: Switch material axes $a$ and $c$ after BETA rotation<br>EQ.4: Switch material axes $b$ and $c$ after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

The yield function $\Phi$ is defined as:

$$\Phi = |S_1 - S_2|^m + |S_2 - S_3|^m + |S_3 - S_1|^m = 2\overline{\sigma}^m$$

where $\overline{\sigma}$ is the effective stress and $S_{i=1,2,3}$ are the principal values of the symmetric matrix $S_{\alpha\beta}$,

$$\begin{aligned} S_{xx} &= \left[c(\sigma_{xx} - \sigma_{yy}) - b(\sigma_{zz} - \sigma_{xx})\right]/3, & S_{yz} &= f\sigma_{yz} \\ S_{yy} &= \left[a(\sigma_{yy} - \sigma_{zz}) - c(\sigma_{xx} - \sigma_{yy})\right]/3, & S_{zx} &= g\sigma_{zx} \\ S_{zz} &= \left[b(\sigma_{zz} - \sigma_{xx}) - a(\sigma_{yy} - \sigma_{zz})\right]/3, & S_{xy} &= h\sigma_{xy} \end{aligned}$$

The material constants $a$, $b$, $c$, $f$, $g$ and $h$ represent anisotropic properties. When

$$a = b = c = f = g = h = 1,$$

the material is isotropic and the yield surface reduces to the Tresca yield surface for $m = 1$ and von Mises yield surface for $m = 2$ or 4.

For face centered cubic (FCC) materials $m = 8$ is recommended and for body centered cubic (BCC) materials $m = 6$ *is* used. The yield strength of the material is

$$\sigma_y = k(\varepsilon^p + \varepsilon_0)^n \ ,$$

where $\varepsilon_0$ is the strain corresponding to the initial yield stress and $\varepsilon^p$ is the plastic strain.

## *MAT_BARLAT_YLD96

This is Material Type 33. This model was developed by Barlat, Maeda, Chung, Yanagawa, Brem, Hayashida, Lege, Matsui, Murtha, Hattori, Becker, and Makosey [1997] for modeling anisotropic material behavior in forming processes, in particular for aluminum alloys. This model is available for shell elements and thick shell elements with element formulations 1, 2, and 6.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | K | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| E0 | N | ESR0 | M | HARD | A | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C1 | C2 | C3 | C4 | AX | AY | AZ0 | AZ1 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | BETA | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | K | | | |
| Type | A | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| K | $k$, strength coefficient or $a$ in Voce (see remarks below) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0 | N | ESR0 | M | HARD | A | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E0 | $\varepsilon_0$, strain corresponding to the initial yield or $b$ in Voce (see remarks below) |
| N | $n$, hardening exponent for yield strength or $c$ in Voce |
| ESR0 | $\varepsilon_{SR0}$, in power law rate sensitivity |
| M | $m$, exponent for strain rate effects |
| HARD | Hardening option:<br>LT.0.0: Absolute value defines the load curve ID<br>EQ.1.0: Power law<br>EQ.2.0: Voce |
| A | Flow potential exponent |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | AX | AY | AZ0 | AZ1 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C1 | $c_1$, see remarks below |
| C2 | $c_2$, see remarks below |
| C3 | $c_3$, see remarks below |
| C4 | $c_4$, see remarks below |
| AX | $a_x$, see remarks below |
| AY | $a_y$, see remarks below |
| AZ0 | $a_{z_0}$, see remarks below |
| AZ1 | $a_{z_1}$, see remarks below |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | BETA | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by |

| VARIABLE | DESCRIPTION |
|---|---|
| | offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector **v** with the normal to the plane of the element |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

The yield stress $\sigma_y$ is defined three ways. The first, the Swift equation, is given in terms of the input constants as:

$$\sigma_y = k(\varepsilon_0 + \varepsilon^p)^n \left(\frac{\dot{\varepsilon}}{\varepsilon_{SR0}}\right)^m \quad .$$

The second, the Voce equation, is defined as:

$$\sigma_y = a - be^{-c\varepsilon^p}.$$

The third option is to give a load curve ID that defines the yield stress as a function of effective plastic strain.

The yield function $\Phi$ is defined as:

$$\Phi = \alpha_1|s_1 - s_2|^a + \alpha_2|s_2 - s_3|^a + \alpha_3|s_3 - s_1|^a = 2\sigma_y^a \quad ,$$

Here $s_i$ is a principle component of the deviatoric stress tensor. In vector notation:

$$\mathbf{s} = \mathbf{L}\sigma \quad ,$$

where **L** is given as

$$\mathbf{L} = \begin{bmatrix} \frac{c_2 + c_3}{3} & \frac{-c_3}{3} & \frac{-c_2}{3} & 0 \\ \frac{-c_3}{3} & \frac{c_3 + c_1}{3} & \frac{-c_1}{3} & 0 \\ \frac{-c_2}{3} & \frac{-c_1}{3} & \frac{c_1 + c_2}{3} & 0 \\ 0 & 0 & 0 & c_4 \end{bmatrix}$$

A coordinate transformation relates the material frame to the principle directions of **s** is used to obtain the $\alpha_k$ coefficients consistent with the rotated principle axes:

$$\alpha_k = \alpha_x p_{1k}^2 + \alpha_y p_{2k}^2 + \alpha_z p_{3k}^2$$
$$\alpha_z = \alpha_{z0}\cos^2(2\beta) + \alpha_{z1}\sin^2(2\beta)$$

where $p_{ij}$ are components of the transformation matrix. The angle $\beta$ defines a measure of the rotation between the frame of the principal value of **s** and the principal anisotropy axes.

## *MAT_FABRIC

This is Material Type 34. This model is especially developed for airbag materials. The fabric model is a variation of the layered orthotropic composite model of material 22 and is valid for 3 and 4 node membrane elements only.

In addition to being a constitutive model, this model also invokes a special membrane element formulation, which is more suitable for the large deformations experienced by fabrics. For thin fabrics, buckling can result in an inability to support compressive stresses; thus, a flag is included for this option. A linearly elastic liner is also included, which can be used to reduce the tendency for these elements to be crushed when the no-compression option is invoked. An isotropic elastic option is also available.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | | PRBA | PRAB | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | | | CSE | EL | PRL | LRATIO | DAMP |
|---|---|---|---|---|---|---|---|

**Card 3a.** Include this card if 0 < X0 < 1 (see Card 5).

| AOPT | X2 | X3 | ELA | LNRC | FORM | FVOPT | TSRFAC |
|---|---|---|---|---|---|---|---|

**Card 3b.** Include this card if X0 = 0 or X0 = -1 (see Card 5) and FVOPT < 7.

| AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
|---|---|---|---|---|---|---|---|

**Card 3c.** Include this card if X0 = 0 or X0 = -1 (see Card 5) and FVOPT ≥ 7.

| AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
|---|---|---|---|---|---|---|---|

**Card 3d.** Include this card if X0 = 1 (see Card 5) and FVOPT < 7.

| AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
|---|---|---|---|---|---|---|---|

**Card 3e.** Include this card if X0 = 1 (see Card 5) and FVOPT ≥ 7.

| AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if FVOPT < 0.

| L | R | C1 | C2 | C3 | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| | RGBRTH | AOREF | A1 | A2 | A3 | X0 | X1 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | | | | BETA | ISREFG |
|---|---|---|---|---|---|---|---|

**Card 7.** Include this card if FORM = 4, 14, or -14.

| LCA | LCB | LCAB | LCUA | LCUB | LCUAB | RL | |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if FORM = -14.

| LCAA | LCBB | H | DT | | ECOAT | SCOAT | TCOAT |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | | PRBA | PRAB | |
| Type | A | F | F | F | | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | Young's modulus - longitudinal direction. For an isotropic elastic fabric material, *only* EA and PRBA are defined; they are used as the isotropic Young's modulus and Poisson's ratio, respectively. The input for the fiber directions and liner should be input as zero for the isotropic elastic fabric. |
| EB | Young's modulus - transverse direction, set to zero for isotropic elastic material. |
| PRBA | $\nu_{ba}$, Minor Poisson's ratio *ba* direction |
| PRAB | $\nu_{ab}$, Major Poisson's ratio *ab* direction (see Remark 15) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | | | CSE | EL | PRL | LRATIO | DAMP |
| Type | F | | | F | F | F | F | F |
| Remarks | | | | 1 | 4 | 4 | 4 | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus in the *ab* direction. Set to zero for an isotropic elastic material. |
| CSE | Compressive stress elimination option (see Remark 1):<br>EQ.0.0: Do not eliminate compressive stresses (*default*).<br>EQ.1.0: Eliminate compressive stresses. This option does not apply to the liner. |
| EL | Young's modulus for elastic liner (required if LRATIO > 0) |
| PRL | Poisson's ratio for elastic liner (required if LRATIO > 0) |
| LRATIO | A non-zero value activates the elastic liner and defines the ratio of liner thickness to total fabric thickness (optional). |
| DAMP | Rayleigh damping coefficient. A 0.05 coefficient is recommended, corresponding to 5% of critical damping. Sometimes larger values are necessary. |

Include this card if 0 < X0 < 1 (see Card 5).

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | X2 | X3 | ELA | LNRC | FORM | FVOPT | TSRFAC |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Also, please refer to Remark 5 for |

| VARIABLE | DESCRIPTION |
|---|---|
| | additional information specific to fiber directions for fabrics:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| X2 | Coefficient of the porosity from the equation in Anagonye and Wang [1999] |
| X3 | Coefficient of the porosity equation of Anagonye and Wang [1999] |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time.<br>The default value of zero means that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| LNRC | Flag to turn off compression in the liner until the reference geometry is reached, that is, the fabric element becomes tensile (see Remark 4):<br>EQ.0.0: Off<br>EQ.1.0: On<br>EQ.2.0: Liner's resistance force follows the strain restoration factor, TSRFAC. |
| FORM | Flag to modify membrane formulation for fabric material:<br>EQ.0.0: Default. Least costly and very reliable.<br>EQ.1.0: Invariant local membrane coordinate system |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2.0: Green-Lagrange strain formulation |
| | EQ.3.0: Large strain with nonorthogonal material angles. See Remark 5. |
| | EQ.4.0: Large strain with nonorthogonal material angles and nonlinear stress-strain behavior. Define optional load curve IDs on optional card. |
| | EQ.12.0: Enhanced version of formulation 2. See Remark 11. |
| | EQ.13.0: Enhanced version of formulation 3. See Remark 11. |
| | EQ.14.0: Enhanced version of formulation 4. See Remark 11. |
| | EQ.-14.0: Same as formulation 14 but invokes reading of Card 8. See Remark 14. |
| | EQ.24.0: Enhanced version of formulation 14. See Remark 11. |
| FVOPT | Fabric venting option (see Remarks 9, 17, and 18): |
| | EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered. |
| | EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered. |
| | EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered. |
| | EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered. |
| | EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered. |
| | EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered. |
| | EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card. |
| | EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as |

| VARIABLE | DESCRIPTION |
|---|---|
| | FAC in the *MAT_FABRIC card.<br>LT.0: \|FVOPT\| defines the same fabric venting options as above, but a new formula for the leakage area is used to replace the element area. See Remark 16. |
| TSRFAC | Strain restoration factor (see Remark 10):<br>LT.0: \|TSRFAC\| is the ID of a curve defining TSRFAC as a function of time.<br>GT.0 and LT.1: TSRFAC applied from time 0.<br>GT.1: TSRFAC is the ID of a curve that defines TSRFAC as a function of time using an alternate method (not available for FORM = 0 or 1). |

This card is included if X0 = 0 or X = -1 and FVOPT < 7.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
| Type | F | F | F | F | F | F | F | F |
| Remarks | | 2 | 2 | 3 | 4 | 11 | 9, 17, 18 | 10 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Also, please refer to Remark 5 for additional information specific to fiber directions for fabrics:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by |

| VARIABLE | DESCRIPTION |
|---|---|
| | the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0: Porous leakage flow coefficient.<br>LT.0: \|FLC\| is interpreted as a load curve ID defining FLC as a function of time. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0: Characteristic fabric parameter<br>LT.0: \|FAC\| is interpreted as a load curve ID defining FAC as a function of absolute pressure. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time.<br>The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| LNRC | Flag to turn off compression in the liner until the reference geometry is reached, that is, the fabric element becomes tensile:<br>EQ.0.0: Off<br>EQ.1.0: On |
| FORM | Flag to modify membrane formulation for fabric material:<br>EQ.0.0: Default. Least costly and very reliable.<br>EQ.1.0: Invariant local membrane coordinate system<br>EQ.2.0: Green-Lagrange strain formulation<br>EQ.3.0: Large strain with nonorthogonal material angles. See Remark 5.<br>EQ.4.0: Large strain with nonorthogonal material angles and nonlinear stress-strain behavior. Define optional load |

| VARIABLE | DESCRIPTION |
|---|---|
| | curve IDs on optional card. |
| | EQ.12.0: Enhanced version of formulation 2. See Remark 11. |
| | EQ.13.0: Enhanced version of formulation 3. See Remark 11. |
| | EQ.14.0: Enhanced version of formulation 4. See Remark 11. |
| | EQ.-14.0: Same as formulation 14 but invokes reading of Card 8. See Remark 14. |
| | EQ.24.0: Enhanced version of formulation 14. See Remark 11. |
| FVOPT | Fabric venting option (see Remarks 9, 17, and 18): |
| | EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered. |
| | EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered. |
| | EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered. |
| | EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered. |
| | EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered. |
| | EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered. |
| | LT.0: \|FVOPT\| defines the same fabric venting options as above, but a new formula for the leakage area is used to replace the element area. See Remark 16. |
| TSRFAC | Strain restoration factor: |
| | LT.0: \|TSRFAC\| is the ID of a curve defining TSRFAC as a function of time. |
| | GT.0 and LT.1: TSRFAC applied from time 0. |
| | GE.1: TSRFAC is the ID of a curve that defines TSRFAC as a function of time using an alternate method (not available for FORM = 0 or 1). |

This card is included if and only if X0 = 0 or X0 = -1 and FVOPT ≥ 7.

| Card 3c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
| Type | F | F | F | F | F | F | F | F |
| Remarks | | 2 | 2 | 3 | 4 | 11 | 9, 17, 18 | 10 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Also, please refer to Remark 5 for additional information specific to fiber directions for fabrics:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks for *AIRBAG_WANG_NEFSKE):<br>GE.0: Porous leakage flow coefficient<br>LT.0: \|FLC\| is interpreted as a load curve ID defining FLC as a function of time. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks for *AIRBAG_WANG_NEFSKE):<br>GE.0: Characteristic fabric parameter |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0: \|FAC\| is interpreted as a load curve ID giving *leakage volume flux rate* as a function of absolute pressure. The volume flux (per area) rate (per time) has the dimensions of $$d(\text{vol}_{\text{flux}})/\text{dt} \approx [\text{length}]^3/([\text{length}]^2[\text{time}])$$ $$\approx [\text{length}]/[\text{time}],$$ equivalent to relative porous gas speed. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3): |
| | LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| LNRC | Flag to turn off compression in liner until the reference geometry is reached, that is, the fabric element becomes tensile: |
| | EQ.0.0: Off |
| | EQ.1.0: On |
| FORM | Flag to modify membrane formulation for fabric material: |
| | EQ.0.0: Default. Least costly and very reliable. |
| | EQ.1.0: Invariant local membrane coordinate system |
| | EQ.2.0: Green-Lagrange strain formulation |
| | EQ.3.0: Large strain with nonorthogonal material angles. See Remark 5. |
| | EQ.4.0: Large strain with nonorthogonal material angles and nonlinear stress-strain behavior. Define optional load curve IDs on the optional card. |
| | EQ.12.0: Enhanced version of formulation 2. See Remark 11. |
| | EQ.13.0: Enhanced version of formulation 3. See Remark 11. |
| | EQ.14.0: Enhanced version of formulation 4. See Remark 11. |
| | EQ.-14.0: Same as formulation 14 but invokes reading of Card 8. See Remark 14. |
| | EQ.24.0: Enhanced version of formulation 14. See Remark 11. |
| FVOPT | Fabric venting option (see Remarks 9, 17, and 18): |
| | EQ.7: Leakage is based on gas volume outflow versus pressure |

| VARIABLE | DESCRIPTION |
|---|---|
| | load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card.<br>EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card. |
| TSRFAC | Strain restoration factor:<br>LT.0: \|TSRFAC\| is the ID of a curve defining TSRFAC as a function of time.<br>GT.0 and LT.1: TSRFAC applied from time 0.<br>GE.1: TSRFAC is the ID of a curve that defines TSRFAC as a function of time using an alternate method (not available for FORM = 0 or 1). |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

This card is included if X0 = 1 and FVOPT < 7.

| Card 3d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
| Type | F | F | F | F | F | F | F | F |
| Remarks | | 2 | 2 | 3 | 4 | 11 | 9, 17, 18 | 10 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Also, please refer to Remark 5 for additional information specific to fiber directions for fabrics:<br>EQ.0.0: Locally orthotropic with material axes determined by |

| VARIABLE | DESCRIPTION |
|---|---|
| | element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks for *AIRBAG_WANG_NEFSKE): |
| | GE.0: Porous leakage flow coefficient. |
| | LT.0: \|FLC\| is interpreted as a load curve ID defining FLC as a function of the stretching ratio defined as $r_s = A/A_0$. See notes below. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks for *AIRBAG_WANG_NEFSKE): |
| | GE.0: Characteristic fabric parameter |
| | LT.0: \|FAC\| is interpreted as a load curve defining FAC as a function of the pressure ratio $r_p = P_{\text{air}}/P_{\text{bag}}$. See Remark 2 below. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3): |
| | LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| LNRC | Flag to turn off compression in liner until the reference geometry is reached, that is, the fabric element becomes tensile: |
| | EQ.0.0: Off |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: On |
| FORM | Flag to modify membrane formulation for fabric material:<br>EQ.0.0: Default. Least costly and very reliable.<br>EQ.1.0: Invariant local membrane coordinate system<br>EQ.2.0: Green-Lagrange strain formulation<br>EQ.3.0: Large strain with nonorthogonal material angles. See Remark 5.<br>EQ.4.0: Large strain with nonorthogonal material angles and nonlinear stress strain behavior. Define optional load curve IDs on optional card.<br>EQ.12.0: Enhanced version of formulation 2. See Remark 11.<br>EQ.13.0: Enhanced version of formulation 3. See Remark 11.<br>EQ.14.0: Enhanced version of formulation 4. See Remark 11.<br>EQ.-14.0: Same as formulation 14 but invokes reading of Card 8. See Remark 14.<br>EQ.24.0: Enhanced version of formulation 14. See Remark 11. |
| FVOPT | Fabric venting option (see Remarks 9, 17, and 18):<br>EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.<br>EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.<br>EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.<br>EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.<br>EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.<br>EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered.<br>LT.0: \|FVOPT\| defines the same fabric venting options as above, but a new formula for the leakage area is used to replace |

| VARIABLE | DESCRIPTION |
|---|---|
| | the element area. See Remark 16. |
| TSRFAC | Strain restoration factor:<br>LT.0: \|TSRFAC\| is the ID of a curve defining TSRFAC as a function of time.<br>GT.0 and LT.1: TSRFAC applied from time 0.<br>GE.1: TSRFAC is the ID of a curve that defines TSRFAC as a function of time using an alternate method (not available for FORM = 0 or 1). |

This card is included if X0 = 1 and FVOPT ≥ 7.

| Card 3e | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | FLC | FAC | ELA | LNRC | FORM | FVOPT | TSRFAC |
| Type | F | F | F | F | F | F | F | F |
| Remarks | | 2 | 2 | 3 | 4 | 11 | 9, 17, 18 | 10 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Also, please refer to Remark 5 for additional information specific to fiber directions for fabrics:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_CO- |

| VARIABLE | DESCRIPTION |
|---|---|
| | ORDINATE_VECTOR). |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks for *AIRBAG_WANG_NEFSKE):<br>GE.0: Porous leakage flow coefficient.<br>LT.0: \|FLC\| is interpreted as a load curve ID defining FLC as a function of the stretching ratio defined as $r_s = A/A_0$. See notes below. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks for *AIRBAG_WANG_NEFSKE):<br>GE.0: Characteristic fabric parameter<br>LT.0: \|FAC\| is interpreted as a load curve defining leakage volume flux rate versus the pressure ratio defined as $r_p = P_{\text{air}}/P_{\text{bag}}$. See Remark 2 below. The volume flux (per area) rate (per time) has the unit of<br>$d(\text{vol}_{\text{flux}})/\text{dt} \approx [\text{length}]^3/([\text{length}]^2[\text{time}])$<br>$\approx [\text{length}]/[\text{time}]$,<br>equivalent to relative porous gas speed. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |
| LNRC | Flag to turn off compression in the liner until the reference geometry is reached, that is, the fabric element becomes tensile:<br>EQ.0.0: Off<br>EQ.1.0: On |
| FORM | Flag to modify membrane formulation for fabric material:<br>EQ.0.0: Default. Least costly and very reliable.<br>EQ.1.0: Invariant local membrane coordinate system<br>EQ.2.0: Green-Lagrange strain formulation<br>EQ.3.0: Large strain with nonorthogonal material angles. See |

| VARIABLE | DESCRIPTION |
|---|---|
| | Remark 5. |
| | EQ.4.0: Large strain with nonorthogonal material angles and nonlinear stress-strain behavior. Define optional load curve IDs on the optional card. |
| | EQ.12.0: Enhanced version of formulation 2. See Remark 11. |
| | EQ.13.0: Enhanced version of formulation 3. See Remark 11. |
| | EQ.14.0: Enhanced version of formulation 4. See Remark 11. |
| | EQ.-14.0: Same as formulation 14 but invokes reading of Card 8. See Remark 14. |
| | EQ.24.0: Enhanced version of formulation 14. See Remark 11. |
| FVOPT | Fabric venting option (see Remarks 9, 17, and 18): |
| | EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card. |
| | EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC in the *MAT_FABRIC card. |
| TSRFAC | Strain restoration factor: |
| | LT.0: \|TSRFAC\| is the ID of a curve defining TSRFAC as a function of time. |
| | GT.0 and LT.1: TSRFAC applied from time 0. |
| | GE.1: TSRFAC is the ID of a curve that defines TSRFAC as a function of time using an alternate method (not available for FORM = 0 or 1). |

Additional card for FVOPT < 0.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L | R | C1 | C2 | C3 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| L | Dimension of unit cell (length) |
| R | Radius of yarn (length) |
| C1 | Pressure coefficient (dependent on unit system) |
| C2 | Pressure exponent |
| C3 | Strain coefficient |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | RGBRTH | A0REF | A1 | A2 | A3 | X0 | X1 |
| Type | | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| RGBRTH | Material-dependent birth time of the airbag reference geometry. Non-zero RGBRTH overwrites the birth time defined in the *AIRBAG_REFERENCE_GEOMETRY_BIRTH keyword. RGBRTH also applies to reference geometry defined by *AIRBAG_SHELL_REFERENCE_GEOMETRY. |
| A0REF | Calculation option of initial area, $A_0$, used for airbag porosity leakage calculation.<br>EQ.0.: Default. Use the initial geometry defined in *NODE.<br>EQ.1.: Use the reference geometry defined in *AIRBAG_REFERENCE_GEOMETRY or *AIRBAG_SHELL_REFERENCE_GEOMETRY. |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$<br>X0.EQ.-1: Compressing seal vent option. The leakage area is evaluated as $A_{\text{leak}} = \max(A_{\text{current}} - A_0, 0)$. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | | | | BETA | ISREFG |
| Type | F | F | F | | | | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| BETA | Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA. |
| ISREFG | Initialize stress by *AIRBAG_REFERENCE_GEOMETRY. This option applies only to FORM = 12. Note that *MAT_FABRIC cannot be initialized using a dynain file because *INITIAL_STRESS_SHELL is not applicable to *MAT_FABRIC.<br>EQ.0.0: Default. Not active.<br>EQ.1.0: Active |

Additional card for FORM = 4, 14, -14, or 24.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCA | LCB | LCAB | LCUA | LCUB | LCUAB | RL | |
| Type | I | I | I | I | I | I | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCA | Load curve or table ID. Load curve ID defines the stress as a function of uniaxial strain along the *a*-axis fiber. Table ID defines for each strain rate a load curve representing stress as a function of uniaxial strain along the *a*-axis fiber. The load curve is available for |

| VARIABLE | DESCRIPTION |
|---|---|
| | FORM = 4, 14, –14, and 24, while the table is allowed only for FORM = -14. If zero, EA is used. For FORM = 14, -14, and 24, this curve can be defined in both tension and compression; see Remark 6 below. |
| LCB | Load curve or table ID. Load curve ID defines the stress as a function of uniaxial strain along the *b*-axis fiber. Table ID defines for each strain rate a load curve representing stress as a function of uniaxial strain along the *b*-axis fiber. The load curve is available for FORM = 4, 14, -14, and 24, while the table is allowed only for FORM = -14. If zero, EB is used. For FORM = 14, -14, and 24, this curve can be defined in both tension and compression; see Remark 6 below. |
| LCAB | Load curve ID for shear stress as a function of shear strain in the *ab*-plane. If zero, GAB is used. |
| LCUA | Unload/reload curve ID for stress as a function of strain along the *a*-axis fiber. If zero, LCA is used. |
| LCUB | Unload/reload curve ID for stress as a function of strain along the *b*-axis fiber. If zero, LCB is used. |
| LCUAB | Unload/reload curve ID for shear stress as a function of shear strain in the *ab*-plane. If zero, LCAB is used. |
| RL | Optional reloading parameter for FORM = 14 and 24. Values between 0.0 (reloading on unloading curve-default) and 1.0 (reloading on a minimum linear slope between unloading curve and loading curve) are possible. |

Additional card for FORM = -14.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCAA | LCBB | H | DT | | ECOAT | SCOAT | TCOAT |
| Type | I | I | F | F | | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCAA | Load curve or table ID. Load curve ID defines the stress along the *a*-axis fiber as a function of biaxial strain. Table ID defines for each |

| VARIABLE | DESCRIPTION |
|---|---|
| | directional strain rate a load curve representing stress along the *a*-axis fiber as a function of biaxial strain. If zero, LCA is used. |
| LCBB | Load curve or table ID. Load curve ID defines the stress along the *b*-axis fiber as a function of biaxial strain. Table ID defines for each directional strain rate a load curve representing stress along the *b*-axis fiber as a function of biaxial strain. If zero, LCB is used. |
| H | Normalized hysteresis parameter between 0 and 1 |
| DT | Strain rate averaging option:<br>EQ.0.0: Strain rate is evaluated using a running average<br>LT.0.0: Strain rate is evaluated using average of last 11 time steps<br>GT.0.0: Strain rate is averaged over the last DT time units |
| ECOAT | Young's modulus of coat material; see Remark 14. |
| SCOAT | Yield stress of coat material; see Remark 14. |
| TCOAT | Thickness of coat material, may be positive or negative; see Remark 14. |

**Remarks:**

1. **The compressive stress elimination option for airbag wrinkling.** Setting CSE = 1 switches off compressive stress in the fabric, thereby eliminating wrinkles. Without this "no compression" option, the geometry of the bag's wrinkles controls the amount of mesh refinement. In eliminating the wrinkles, this feature reduces the number of elements needed to attain an accurate solution.

   The no compression option can allow elements to collapse to a line which can lead to elements becoming tangled. The elastic liner option is one way to add some stiffness in compression to prevent this, see Remark 4. Alternatively, when using fabric formulations 14, -14, or 24 (see FORM) tangling can be reduced by defining stress/strain curves that include negative strain and stress values. See Remark 6.

2. **Porosity.** The parameters FLC and FAC are optional for the Wang-Nefske and Hybrid inflation models. It is possible for the airbag to be constructed of multiple fabrics having different values for porosity and permeability. Typically, FLC and FAC must be determined experimentally, and their variations in time or with pressure are optional to allow for maximum flexibility.

3. **Effects of airbag-structure interaction on porosity.** To calculate the leakage of gas through the fabric, it is necessary to accurately determine the leakage area. The dynamics of the airbag may cause the leakage area to change during the course of the simulation. In particular, the deformation may change the leakage area, but the leakage area may also decrease when the contact between the airbag and the structure blocks the flow. LS-DYNA can check the interaction between the bag and the structure and split the areas into blocked and unblocked regions, depending on whether the regions are in contact or not. Blockage effects may be controlled with the ELA field.

4. **Elastic liner.** An optional elastic liner can be defined using EL, PRL, and LRATIO. The liner is an isotropic layer that acts in both tension and compression. However, setting LNRC to 1.0 eliminates compressive stress in the liner until both principal stresses are tensile. The compressive stress elimination option, CSE = 1, has no influence on the liner behavior.

5. **Fiber axes.** For formulations 0, 1, and 2 (see FORM), the *a*-axis and *b*-axis fiber directions *are assumed to be orthogonal* and are completely defined by the material axes option, AOPT = 0, 2, or 3. For FORM = 3, 4, 13, or 14, the fiber directions *are not assumed to be orthogonal* and must be specified using the ICOMP = 1 option on *SECTION_SHELL. Offset angles should be input into the B1 and B2 fields used normally for integration points 1 and 2. The *a*-axis and *b*-axis directions will then be offset from the *a*-axis direction as determined by the material axis option, AOPT = 0, 2, or 3.

   When reference geometry is defined, the material axes are computed using coordinates from the reference geometry. The material axes are determined by computing the angle between the element system and the material direction.

6. **Stress as a function of strain curves.** For formulations (see FORM) 4, 14, -14, and 24, the second Piola-Kirchhoff stress as a function of Green's strain curves may be defined for *a*-axis, *b*-axis, and shear stresses for loading, unloading, and reloading. Alternatively, the *a*-axis and *b*-axis curves can be input using engineering stress as a function of strain by setting DATTYP = -2 on *DEFINE_CURVE. In that case, we recommend using enough points in the curve, such as 100, to enable a sufficiently accurate internal conversion to the second Piola-Kirchhoff stress as a function of Green's strain curve.

   Additionally, for formulations 14, -14, and 24, the uniaxial loading curves LCA and LCB may be defined for negative strain and stress, that is, a straightforward extension of the curves into the compressive region. This is available for modeling the compressive stresses resulting from the tight folding of airbags.

   The *a*-axis and *b*-axis stresses follow the curves for the entire defined strain region; if compressive behavior is desired, you should ensure the curve covers all

strains of interest. For strains below the first point on the curve, the curve is extrapolated using the stiffness from the constant values, EA or EB.

Shear stress/strain behavior is assumed symmetric, and curves should be defined for positive strain only. However, formulations 14, -14, and 24 allow the curves to be extended into the negative strain region to model asymmetric behavior. The asymmetric option cannot be used with a shear stress unload curve. If a load curve is omitted, the stress is calculated from the appropriate constant modulus, EA, EB, or GAB.

7. **Yield behavior.** When formulations 4, 14, -14, and 24 (see FORM) are used with loading and unloading curves, the initial yield strain is set equal to the strain of the first point in the load curve having a stress greater than zero. When the current strain exceeds the yield strain, the stress follows the load curve, and the yield strain is updated to the current strain. When unloading occurs, the unload/reload curve is shifted along the $x$-axis until it intersects the load curve at the current yield strain and then the stress is calculated from the shifted curve. When using unload curves, compressive stress elimination should be active to prevent the fibers from developing compressive stress during unloading when the strain remains tensile. *To use this option, the unload curve must have a non-negative second derivative so that the curve shifts right as the yield stress increases. In fact, if a shift to the left is needed, the unload curve is not used, and unloading follows the load curve instead.*

   If LCUA, LCUB, or LCUAB is input with a negative curve ID, then unloading is handled differently. Instead of shifting the unload curve along the $x$-axis, the curve is stretched in both the $x$-direction and $y$-direction such that the first point remains anchored at $(0,0)$ and the initial intersection point of the curves is moved to the current yield point. This option guarantees that the stress remains tensile while the strain is tensile, so compressive stress elimination is unnecessary. *To use this option, the unload curve should have an initial slope less steep than the load curve and should steepen such that it intersects the load curve at some positive strain value.*

8. **Shear unload-reload and fabric formulation.** Let $f$ be the shear stress unload-reload curve LCUAB. Then, the shear stress-strain relation is given by:

$$\sigma_{ab} = c_2 f(c_1 \varepsilon_{ab})$$

   where the scale factors $c_1$ and $c_2$ depend on the fabric formulation (see FORM).

| Fabric formulation | $c_1$ | $c_2$ |
|---|---|---|
| 4 | 2 | 1 |
| 14 and -14 | 1 | 2 |
| 24 | 1 | 1 |

When switching fabric formulations, the curve scale factors SFA and SFO on *DEFINE_CURVE can be used to offset this behavior.

9. **Per material venting option.** The FVOPT flag allows an airbag fabric venting equation to be assigned to a material. For control volume airbags, this feature is available for *AIRBAG_WANG_NEFSKE and *AIRBAG_HYBRID. For *AIRBAG_ALE, only FVOPT = 7 and 8 are available. See Remarks 17 and yy for a discussion of which options are available for CPM and CPG airbags.

   The anticipated use for this option is to allow a vent to be defined using FVOPT = 1 or 2 for one material and fabric leakage to be defined using FVOPT = 3, 4, 5, 6, 7, or 8 for other materials. To use FVOPT with *AIRBAG_WANG_NEFSKE or *AIRBAG_HYBRID, a venting option must first be defined for the airbag using the OPT parameter on the data cards for those airbags. If OPT = 0, then FVOPT is ignored. If OPT is defined and FVOPT is omitted, then FVOPT is set equal to OPT.

   For a description of the mass flow rate out of the airbag due to each of these models, see the Remarks section of the *AIRBAG_WANG_NEFSKE manual pages.

10. **TSRFAC option to restore element strains.** Airbags that use a reference geometry will typically have nonzero strains at the start of the calculation. To prevent such initial strains from prematurely opening an airbag, initial strains are stored and subtracted from the measured strain throughout the calculation.

$$\sigma = f(\varepsilon - \varepsilon_{\text{initial}})$$

   - Fabric formulations 2, 3, and 4 (see FORM) subtract off only the initial tensile strains so these forms are typically used with CSE = 1 and LNRC = 1.

   - Fabric formulations 12, 13, 14, -14, and 24 subtract off the total initial strains, so these forms may be used with CSE = 0 or 1 and LNRC = 0 or 1. A side effect of this strain modification is that airbags may not achieve the correct volume when they open. Therefore, the TSRFAC option is implemented to reduce the stored initial strain values over time thereby restoring the total stain which drives the airbag towards the correct volume.

   During each cycle, the stored initial strains are scaled by $(1.0 - \text{TSRFAC})$. A small value on the order of 0.0001 is typically sufficient to restore the strains in a few milliseconds of simulation time.

$$\sigma = f\left(\varepsilon - \varepsilon_{\text{adjustment}}\right)$$

   The adjustment to restore initial strain is then,

$$\varepsilon_{\text{adjustment}} = \varepsilon_{\text{initial}} \prod_i [1 - \text{TSFRAC}].$$

a) *Time-Dependent TSRFAC.* When TSRFAC < 0, |TSRFAC| becomes the ID of a curve that defines TSRFAC as a function of time. To delay the effect of TSRFAC, the curve ordinate value should be initially zero and should ramp up to a small number to restore the strain at an appropriate time during the simulation. The adjustment to restore initial strain is then,

$$\varepsilon_{\text{adjustment}}(t_i) = \varepsilon_{\text{initial}} \prod_i [1 - \text{TSFRAC}(t_i)].$$

To prevent airbags from opening prematurely, it is recommended to use the load curve option of TSRFAC to delay the strain restoration until the airbag is partially opened due to pressure loading.

b) *Alternate Time-Dependent TSRFAC.* For fabric formulations 2 and higher, a second curve option is invoked by setting TSRFAC ≥ 1 where TSRFAC is again the ID of a curve that defines TSRFAC as a function of time. Like the first curve option, the stored initial strain values are scaled by (1.0 − TSRFAC), but the modified initial strains are not saved, so the effect of TSRFAC does not accumulate. In this case the adjustment to eliminate initial strain

$$\varepsilon_{\text{adjustment}}(t_i) = [1 - \text{TSFRAC}(t_i)]\varepsilon_{\text{initial}}.$$

Therefore, the curve should ramp up from zero to one to fully restore the strain. This option gives the user better control of the rate of restoring the strain as it is a function of time rather than solution time step.

11. **Enhancements to the material formulations.** Material formulations (see FORM) 12, 13, and 14 are enhanced versions of formulations 2, 3, and 4, respectively. The most notable difference in their behavior is apparent when a reference geometry is used for the fabric. As discussed in Remark 10, the strain is modified to prevent initial strains from prematurely opening an airbag at the start of a calculation.

Formulations 2, 3, and 4 subtract the initial tensile strains, while the enhanced formulations subtract the total initial strains. Therefore, the enhanced formulations can be used without setting CSE = 1 and LNRC = 1 since compressive stress cutoff is not needed to prevent initial airbag movement. Formulations 2, 3, and 4 need a compressive stress cutoff when used with a reference geometry, or they can generate compressive stress at the start of a calculation. Available for formulation 12 only, the ISREFG parameter activates an option to calculate the initial stress by using a reference geometry.

Material formulation 24 is an enhanced version of formulation 14, implementing a correction for Poisson's effects when stress as a function of strain curves are input for the *a*-fiber or *b*-fiber. Also, for formulation 24, the outputted stress and strain in the elout or d3plot database files are engineering stress and strain rather

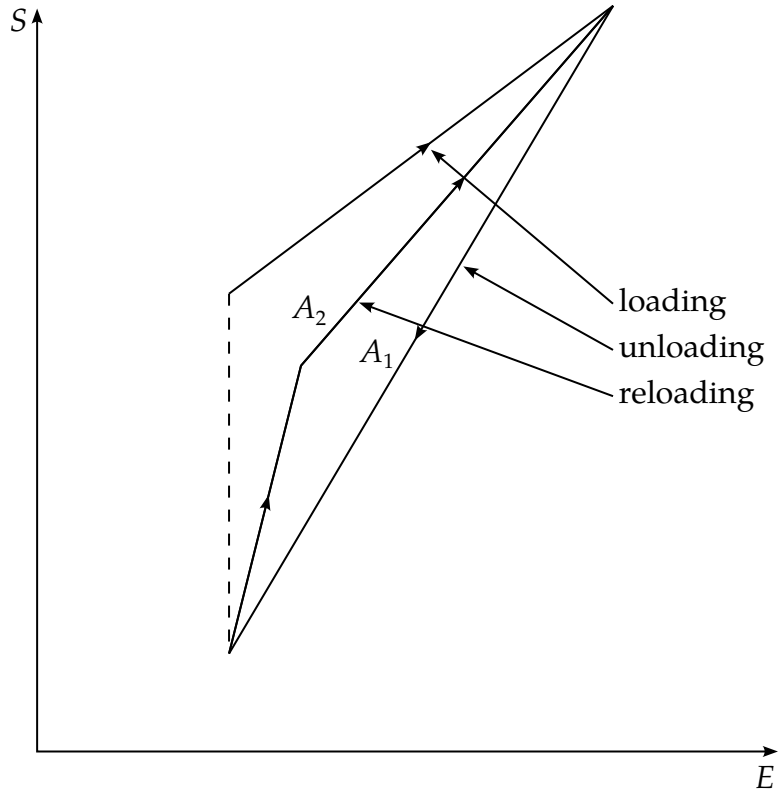



**Figure M34-1.** Hysteresis curve

than the 2nd Piola Kirchoff and Green's strain used by formulations other than 0 and 1.

12. **Noise reduction for the strain rate measure.** If tables are used, then the strain rate measure is the time derivative of the Green-Lagrange strain in the direction of interest. To suppress noise, the strain rate is averaged according to the value of DT. If DT > 0, it is recommended to use a large enough value to suppress the noise, while being small enough not to lose important information in the signal.

13. **Hysteresis.** The hysteresis parameter $H$ defines the fraction of dissipated energy during a load cycle in terms of the maximum possible dissipated energy. Referring to the Figure M34-1,

$$H \approx \frac{A_1}{A_1 + A_2} \ .$$

14. **Coating feature for additional rotational resistance.** It is possible to model the coating of the fabric using a sheet of elastic-ideal-plastic material where the Young's modulus, yield stress, and thickness are specified for the coating material. This coating feature adds rotational resistance to the fabric for more realistic behavior of coated fabrics. To read these properties, set FORM = -14, which adds an extra card containing the three fields ECOAT, SCOAT, and TCOAT, corresponding to the three abovementioned coat material properties. The

coating model includes transverse shear stiffness to avoid nonphysical zig-zagging. To adjust this stiffness, set SHRF on *SECTION_SHELL.

The thickness, TCOAT, applies to both sides of the fabric. The coat material for a certain fabric element deforms along with this and all elements connected to this element, which is how the rotations are "captured." Note that the coating will add to the membrane stiffness unless TCOAT is set to a negative value. For negative values of TCOAT, the thickness is set to |TCOAT| and the membrane contribution from the coating is suppressed. For this feature to work, the fabric parts must not include any T-intersections, and all of the surface normal vectors of connected fabric elements must point in the same direction. This option increases the computational complexity of this material.

15. **Poisson's ratios.** Fabric forms 12, 13, 14, -14, and 24 allow input of both the minor Poisson's ratio, $\nu_{ba}$, and the major Poisson's ratio, $\nu_{ab}$. This allows asymmetric Poisson's behavior to be modelled. If the major Poisson's ratio is left blank or input as zero, then it will be calculated using $\nu_{ab} = \nu_{ba}\, E_a/E_b$.

16. **St. Venant-Wantzel leakage.** If a negative value for the fabric venting option FVOPT is used (only -1 and -2 are supported), the mass flux through a fabric membrane is calculated according St.Venant-Wantzel by

$$\dot{m} = A_{\text{eff}}\, \Psi \sqrt{2 p_i \rho_i}$$

where $p_i$ describes the internal pressure, $\rho_i$ is the density of the outlet gas, and the effluence function $\psi$ depends on the character of the flow, the adiabatic exponent $\kappa$ and the pressure difference between the inside ($p_i$) and the outside ($p_a$) of the membrane. For subsonic flow, it is formulated as:

$$\Psi = \sqrt{\frac{\kappa}{\kappa - 1}\left[\left(\frac{p_a}{p_i}\right)^{\frac{2}{\kappa}} - \left(\frac{p_a}{p_i}\right)^{\frac{\kappa+1}{\kappa}}\right]}$$

and for sonic or critical flow as:

$$\Psi = \sqrt{\frac{\kappa}{2}\left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{\kappa-1}}}$$

The effective venting area of the membrane is determined according to M. Schlenger:

$$A_{\text{eff}} = \frac{A_0}{L^2}\left[\left(C_1 \Delta p^{C_2} - C_3\right)(L - 2r)^2 + C_3\left(L\lambda_1 - \frac{2r}{\sqrt{\lambda_2}}\right)\left(L\lambda_2 - \frac{2r}{\sqrt{\lambda_1}}\right)\right]\sin(\alpha_{12})$$

where $\lambda_i$ is the stretch in fiber direction $i$ and $\alpha_{12}$ is the angle between the fibers. The initial membrane area is equal to $A_0$. $r$ and $L$ represent the radius of the fabric fiber and the edge length of the fabric set, respectively. The coefficients, $L$, $r$, $C_1$, $C_2$, and $C_3$, must be defined on additional Card 4. This option is supported for *AIRBAG_WANG_NESFKE, *AIRBAG_HYBRID, and *AIRBAG_-

PARTICLE. No additional input in *AIRBAG cards is needed. All FORM options are supported, whereas $\alpha_{12}$ can only be different from 90 degrees for FORM = 3, 4, 13, or 14.

17. **CPM (*AIRBAG_PARTICLE) bags with FVOPT**. Only FVOPT = -2, -1, 1, 2, 7, and 8 are supported for CPM bags. If FVOPT = 0 is used, it defaults to FVOPT = 8. For FVOPT = -1 and -2, FLC is active and can be either a scalar or a curve defining the porous leakage flow coefficient as a function of time. The FAC coefficient, however, is *inactive* as the porous leakage velocity is computed using the formula specified in Remark 16 from the coefficient defined in Card 4. Note that both the FLC and FAC coefficients are active for uniform pressure airbags (*AIRBAG_HYBRID_...).

18. **CPG (*AIRBAG_CPG) bags with FVOPT.** Only FVOPT = 7 and 8 are supported for CPG bags. If FVOPT = 0, porous leakage is not considered.

### *MAT_FABRIC_MAP

This is Material Type 34 in which the stress response is given exclusively by tables, or maps, and where some obsolete features in *MAT_FABRIC have been deliberately excluded to allow for a clean input and better overview of the model. The response can be made temperature-dependent.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | PXX | PYY | SXY | DAMP | TH | T0 |
|---|---|---|---|---|---|---|---|

**Card 1.1.** Include this card if T0 > 0.

| T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
|---|---|---|---|---|---|---|---|

**Card 1.2.** Include this card if T0 > 0.

| PXX1 | PXX2 | PXX3 | PXX4 | PXX5 | PXX6 | PXX7 | PXX8 |
|---|---|---|---|---|---|---|---|

**Card 1.3.** Include this card if T0 > 0.

| PYY1 | PYY2 | PYY3 | PYY4 | PYY5 | PYY6 | PYY7 | PYY8 |
|---|---|---|---|---|---|---|---|

**Card 1.4.** Include this card if T0 > 0.

| SXY1 | SXY2 | SXY3 | SXY4 | SXY5 | SXY6 | SXY7 | SXY8 |
|---|---|---|---|---|---|---|---|

**Card 2a.** Include this card if 0 < X0 < 1.

| FVOPT | X0 | X1 | X2 | X3 | ELA | | |
|---|---|---|---|---|---|---|---|

**Card 2b.** Include this card if X0 = 0 or X0 = -1 and FVOPT < 7.

| FVOPT | X0 | X1 | FLC | FAC | ELA | | |
|---|---|---|---|---|---|---|---|

**Card 2c.** Include this card if X0 = 0 or X0 = -1 and FVOPT ≥ 7.

| FVOPT | X0 | X1 | FLC | FAC | ELA | | |
|---|---|---|---|---|---|---|---|

**Card 2d.** Include this card if X0 = 1 and FVOPT < 7.

| FVOPT | X0 | X1 | FLC | FAC | ELA | | |
|---|---|---|---|---|---|---|---|

**Card 2e.** Include this card if X0 = 1 and FVOPT ≥ 7.

| FVOPT | X0 | X1 | FLC | FAC | ELA | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| ISREFG | CSE | SRFAC | BULKC | JACC | FXX | FYY | DT |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | ECOAT | SCOAT | TCOAT | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PXX | PYY | SXY | DAMP | TH | T0 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PXX | Table giving engineering local $XX$-stress as a function of engineering local $XX$-strain and $YY$-strain |
| PYY | Table giving engineering local $YY$-stress as a function of engineering local $YY$-strain and $XX$-strain |
| SXY | Curve giving local 2nd Piola-Kirchhoff $XY$-stress as a function of local Green $XY$-strain. |
| DAMP | Damping coefficient for numerical stability |
| TH | Table giving hysteresis factor $0 \le H < 1$ as a function of engineering local $XX$-strain and $YY$-strain: |

| VARIABLE | DESCRIPTION |
|---|---|
| | GT.0.0: TH is table ID.<br>LE.0.0: -TH is used as constant value for hysteresis factor. |
| T0 | Flag to indicate temperature dependence and temperature corresponding to tables PXX, PYY, and SXY:<br>EQ.0.0: Do not consider temperature dependence for this model (default).<br>GT.0.0: Consider temperature dependence considered. T0 gives the temperature corresponding to tables PXX, PYY, and SXY. LS-DYNA expects Cards 1.1 through 1.4 to provide additional positive temperatures that correspond to similar tables. T0 represents a typical work temperature. It may be anywhere inside or outside the range between T1 - T8 defined in Card 1.1, but it cannot be equal to any of those individual values. Note that we are assuming that no matter the units the relevant temperatures are positive. |

Include this card if T0 > 0.

| Card 1.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| T*i* | Temperature values for which the tables and curves specified in Cards 1.2 - 1.4 apply. Temperature values must be increasing and positive, meaning T1 > 0, T2 > T1, T3 > T2, etc. If needing fewer than 8 temperature points, then set the first unused temperature value to zero. T0 may not take any of the positive T*i* values but will be properly inserted into the range so that all positive temperatures defined are in increasing order. |

Include this card if T0 > 0.

| Card 1.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PXX1 | PXX2 | PXX3 | PXX4 | PXX5 | PXX6 | PXX7 | PXX8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| PXX*i* | Table giving engineering local *XX*-stress as a function of engineering local *XX*-strain and *YY*-strain for temperature T*i*. |

Include this card if T0 > 0.

| Card 1.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PYY1 | PYY2 | PYY3 | PYY4 | PYY5 | PYY6 | PYY7 | PYY8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| PYY*i* | Table giving engineering local *YY*-stress as a function of engineering local *YY*-strain and *XX*-strain for temperature T*i*. |

Include this card if T0 > 0.

| Card 1.4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SXY1 | SXY2 | SXY3 | SXY4 | SXY5 | SXY6 | SXY7 | SXY8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SXY*i* | Curve giving local 2nd Piola-Kirchhoff *XY*-stress as function of local Green *XY*-strain for temperature T*i*. |

This card is included if 0 < X0 < 1.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVOPT | X0 | X1 | X2 | X3 | ELA | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVOPT | Fabric venting option (see Remarks 9, CPM (*AIRBAG_PARTICLE) bags with FVOPT. , and 18 of *MAT_FABRIC): |

EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.

EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.

EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.

EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.

EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.

EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered.

EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.

EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.

LT.0: |FVOPT| defines the same fabric venting options as above, but a new formula for the leakage area is used to replace the element area. See Remark 16 of *MAT_FABRIC.

| VARIABLE | DESCRIPTION |
|---|---|
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{leak} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$; see *MAT_FABRIC. |
| X2 | Coefficient of the porosity from the equation in Anagonye and Wang [1999] |
| X3 | Coefficient of the porosity equation of Anagonye and Wang [1999] |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3 of *MAT_FABRIC):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time.<br>The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

This card is included if and only if X0 = 0 or X0 = -1 and FVOPT < 7.

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVOPT | X0 | X1 | FLC | FAC | ELA | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVOPT | Fabric venting option (see Remarks 9, CPM (*AIRBAG_PARTICLE) bags with FVOPT. , and 18 of *MAT_FABRIC):<br>EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.<br>EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.<br>EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.<br>EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered.<br>EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered.<br>LT.0: \|FVOPT\| defines the same fabric venting options as above, but a new formula for the leakage area is used to replace the element area. See Remark 16 of *MAT_FABRIC. |
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$; see *MAT_FABRIC.<br>X0.EQ.-1: Compressing seal vent option. The leakage area is evaluated as $A_{\text{leak}} = \max(A_{\text{current}} - A_0, 0)$. |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0.0: Porous leakage flow coefficient.<br>LT.0.0: \|FLC\| is a load curve ID defining FLC as a function of time. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0.0: Characteristic fabric parameter<br>LT.0.0: \|FAC\| is a load curve ID defining FAC as a function of absolute pressure. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3 of *MAT_FABRIC):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time.<br>The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

This card is included if and only if X0 = 0 or X0 = -1 and FVOPT ≥ 7.

| Card 2c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVOPT | X0 | X1 | FLC | FAC | ELA | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVOPT | Fabric venting option (see Remarks 9, CPM (*AIRBAG_PARTICLE) bags with FVOPT. , and 18 of *MAT_FABRIC):<br>EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.<br>EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC. |
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$; see *MAT_FABRIC.<br>X0.EQ.-1: Compressing seal vent option. The leakage area is evaluated as $A_{\text{leak}} = \max(A_{\text{current}} - A_0, 0)$. |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0.0: Porous leakage flow coefficient<br>LT.0.0: \|FLC\| is a load curve ID defining FLC as a function of time. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0.0: Characteristic fabric parameter<br>LT.0.0: \|FAC\| is a load curve ID giving leakage volume flux rate as a function of absolute pressure. The volume flux (per area) rate (per time) has the dimensions of |

| VARIABLE | DESCRIPTION |
|---|---|
| | $d(\text{vol}_{\text{flux}})/\text{dt} \approx [\text{length}]^3/([\text{length}]^2[\text{time}])$<br>$\approx [\text{length}]/[\text{time}]$<br>equivalent to relative porous gas speed. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3 of *MAT_FABRIC):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time.<br>The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

This card is included if and only if X0 = 1 and FVOPT < 7.

| Card 2d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVOPT | X0 | X1 | FLC | FAC | ELA | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVOPT | Fabric venting option (see Remarks 9, CPM (*AIRBAG_PARTICLE) bags with FVOPT. , and 18 of *MAT_FABRIC):<br>EQ.1: Wang-Nefske formulas for venting through an orifice are used. Blockage is not considered.<br>EQ.2: Wang-Nefske formulas for venting through an orifice are used. Blockage of venting area due to contact is considered.<br>EQ.3: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage is not considered.<br>EQ.4: Leakage formulas of Graefe, Krummheuer, and Siejak [1990] are used. Blockage of venting area due to contact is considered.<br>EQ.5: Leakage formulas based on flow through a porous media are used. Blockage is not considered. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.6: Leakage formulas based on flow through a porous media are used. Blockage of venting area due to contact is considered.<br>LT.0: \|FVOPT\| defines the same fabric venting options as above, but a new formula for the leakage area is used to replace the element area. See Remark 16 of *MAT_FABRIC. |
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$; see *MAT_FABRIC. |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0.0: Porous leakage flow coefficient.<br>LT.0.0: \|FLC\| is interpreted as a load curve ID defining FLC as a function of the stretching ratio defined as $r_s = A/A_0$. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remakrs section of *AIRBAG_WANG_NEFSKE):<br>GE.0.0: Characteristic fabric parameter<br>LT.0.0: \|FAC\| is interpreted as a load curve defining FAC as a function of the pressure ratio $r_p = P_{\text{air}}/P_{\text{bag}}$. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3 of *MAT_FABRIC):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time.<br>The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

This card is included if and only if X0 = 1 and FVOPT ≥ 7.

| Card 2e | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVOPT | X0 | X1 | FLC | FAC | ELA | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVOPT | Fabric venting option (see Remarks 9, CPM (*AIRBAG_PARTICLE) bags with FVOPT. , and 18 of *MAT_FABRIC):<br>EQ.7: Leakage is based on gas volume outflow versus pressure load curve [Lian, 2000]. Blockage is not considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC.<br>EQ.8: Leakage is based on gas volume outflow versus pressure load curve [Lian 2000]. Blockage of venting or porous area due to contact is considered. Absolute pressure is used in the porous-velocity-versus-pressure load curve, given as FAC. |
| X0, X1 | Coefficients of Anagonye and Wang [1999] porosity equation for the leakage area: $A_{\text{leak}} = A_0(X_0 + X_1 r_s + X_2 r_p + X_3 r_s r_p)$; see *MAT_FABRIC. |
| FLC | Optional porous leakage flow coefficient (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0: Porous leakage flow coefficient.<br>LT.0: \|FLC\| is interpreted as a load curve ID defining FLC as a function of the stretching ratio defined as $r_s = A/A_0$. |
| FAC | Optional characteristic fabric parameter (see the theory manual and the remarks section of *AIRBAG_WANG_NEFSKE):<br>GE.0: Characteristic fabric parameter<br>LT.0: \|FAC\| is interpreted as a load curve defining leakage volume flux rate versus the pressure ratio defined as $r_p = P_{\text{air}}/P_{\text{bag}}$. The volume flux (per area) rate (per time) has the unit of<br>$d(\text{vol}_{\text{flux}})/\text{dt} \approx [\text{length}]^3/([\text{length}]^2[\text{time}])$<br>$\approx [\text{length}]/[\text{time}]$,<br>equivalent to relative porous gas speed. |
| ELA | Effective leakage area for blocked fabric, ELA (see Remark 3 of *MAT_FABRIC):<br>LT.0.0: \|ELA\| is the load curve ID of the curve defining ELA as a function of time. |

| VARIABLE | DESCRIPTION |
|---|---|
| | The default value of zero assumes that no leakage occurs. A value of .10 would assume that 10% of the blocked fabric is leaking gas. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ISREFG | CSE | SRFAC | BULKC | JACC | FXX | FYY | DT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ISREFG | Initial stress by reference geometry:<br>EQ.0.0: Not active<br>EQ.1.0: Active |
| CSE | Compressive stress elimination option:<br>EQ.0.0: Do not eliminate compressive stresses.<br>EQ.1.0: Eliminate compressive stresses. |
| SRFAC | Load curve ID for smooth stress initialization when using a reference geometry |
| BULKC | Bulk modulus for fabric compaction |
| JACC | Jacobian for the onset of fabric compaction |
| FXX | Load curve giving scale factor of uniaxial stress in first material direction as function of engineering strain rate |
| FYY | Load curve giving scale factor of uniaxial stress in second material direction as function of engineering strain rate |
| DT | Time window for smoothing strain rates used for FXX and FYY |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | ECOAT | SCOAT | TCOAT | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Also, please refer to Remark 5 of *MAT_FABRIC for additional information specific to fiber directions for fabrics:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| ECOAT | Young's modulus of coat material to include bending properties. This together with the following two parameters (SCOAT and TCOAT) encompass the same coating/bending feature as in *MAT_FABRIC. Please refer to these manual pages and associated remarks. |
| SCOAT | Yield stress of coat material, see *MAT_FABRIC. |
| TCOAT | Thickness of coat material, may be positive or negative, see *MAT_FABRIC. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

1. **Material model.** This material model invokes a special membrane element formulation regardless of the element choice. It is an anisotropic hyperelastic model, where the 2nd Piola-Kirchhoff stress **S** is a function of the Green-Lagrange strain **E** and possibly its history and temperature. Due to anisotropy, this strain is transformed to obtain the strains in each of the fiber directions, $E_{XX}$ and $E_{YY}$, together with the shear strain, $E_{XY}$. The associated stress components in the local system are given as functions of the strain components and temperature

$$S_{XX} = \gamma S_{XX}(E_{XX}, E_{YY}, T)\vartheta$$
$$S_{YY} = \gamma S_{YY}(E_{YY}, E_{XX}, T)\vartheta$$
$$S_{XY} = \gamma S_{XY}(E_{XY}, T)\vartheta$$

The factor $\gamma$ is used for dissipative effects and is described in more detail in Remark 5. For TH = 0, $\gamma = 1$. The function $\vartheta$ represents a strain rate scale factor (see Remark 7); for FXX = FYY = 0, this factor is 1. While the input curve SXY

directly gives the shear relation, the tabular input of the fiber stress components PXX and PYY is for the sake of convenience. PXX and PYY give the engineering stress as a function of engineering strain and optionally temperature, that is,

$$P_{XX} = P_{XX}(e_{XX}, e_{YY}, T)$$
$$P_{YY} = P_{YY}(e_{YY}, e_{XX}, T)$$

Because of this, the following conversion formulae are used between stresses and strains

$$e = \sqrt{1 + 2E} - 1$$
$$S = \frac{P}{1 + e}$$

which are applied in each of the two fiber directions.

2. **Temperature dependence.** We apply temperature dependence through input tables and curves for up to 9 different temperature values (see T0 and Cards 1.1 through 1.4). Whenever the temperature in an element is between two defined temperature values, interpolation of the values for the two temperature points gives the resulting value. If the temperature in an element falls below the smallest temperature defined or above the largest temperature defined, the resulting value is not extrapolated, but the first and last defined table/curve is used, respectively. Note that LS-DYNA inserts T0 and its associated data at the appropriate location so that all temperature values are internally in increasing order. For determining dissipation in the material, we use the properties at temperature T0.

3. **Compressive stress elimination.** Compressive stress elimination is optional through the CSE parameter, and when activated the principal components of the 2nd Piola-Kirchhoff stress is restricted to positive values.

4. **Reference geometry and smooth stress initialization.** If a reference geometry is used, then SRFAC is the curve ID for a function $\alpha(t)$ that should increase from zero to unity during a short time span. During this time span, the Green-Lagrange strain used in the formulae in Remark 1 above is substituted with

$$\tilde{\mathbf{E}} = \mathbf{E} - [1 - \alpha(t)]\mathbf{E}_0,$$

where $\mathbf{E}_0$ is the strain at time zero. This is done in order to smoothly initialize the stress resulting from using a reference geometry different from the geometry at time zero.

5. **Dissipative effects.** The factor $\gamma$ is a function of the strain history and is initially set to unity. It specifically depends on the internal work, $\epsilon$, given by the stress power

$$\dot{\epsilon} = \mathbf{S} : \dot{\mathbf{E}}.$$

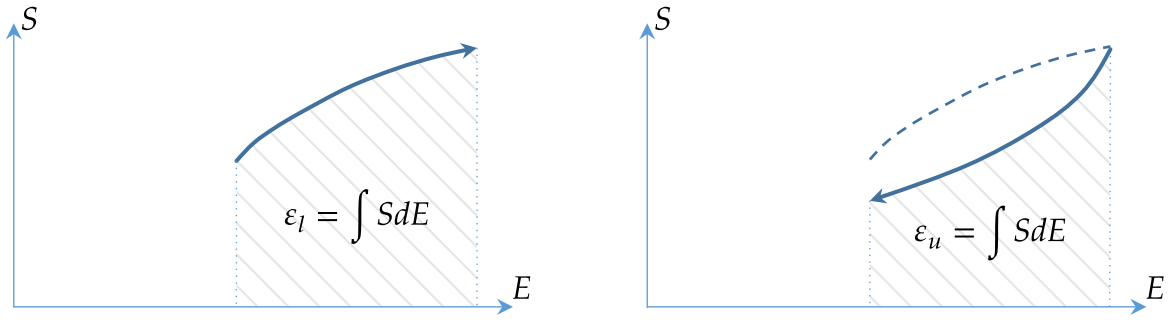



**Figure M34M-1.** Cyclic loading model for hysteresis model $H$

The evolution of $\gamma$ is related to the stress power since it increases on loading and decreases on unloading. As a result, it introduces dissipation. The exact mathematical formula is too complicated to reveal, but basically the function looks like

$$\gamma = \begin{cases} 1 - H(\bar{e}_{XX}, \bar{e}_{YY}) + H(\bar{e}_{XX}, \bar{e}_{YY})\exp[\beta(\epsilon - \overline{\epsilon})] & \dot{\epsilon} < 0 \\ 1 - H(\bar{e}_{XX}, \bar{e}_{YY})\exp[-\beta(\epsilon - \underline{\epsilon})] & \dot{\epsilon} \geq 0 \end{cases}$$

Here $\overline{\epsilon}$ is the maximum attained internal work up to this point in time, $\bar{e}_{XX}$ and $\bar{e}_{YY}$ are the engineering strain values associated with value. $H(\bar{e}_{XX}, \bar{e}_{YY})$ is the hysteresis factor specified through input parameter TH; it may or may not depend on the strains. $\beta$ is a decay constant that depends on $\bar{e}_{XX}$ and $\bar{e}_{YY}$, and $\underline{\epsilon}$ is the minimum attained internal work at any point in time after $\overline{\epsilon}$ was attained. In other words, on unloading, $\gamma$ will exponentially decay to $1 - H$, and on loading it will exponentially grow to 1 and always be restricted by the lower and upper bounds, $1 - H < \gamma \leq 1$. This formulation requires inputting a proper hysteresis factor $H$. With reference to a general loading/unloading cycle illustrated in Figure M34M-1, the relation $1 - H = \epsilon_u/\epsilon_l$ should hold with proper input. LS-DYNA uses the properties at the work temperature T0 for this dissipative treatment.

6. **Packing of yarn in compression.** To account for the packing of yarns in compression, a compaction effect is modeled by adding a term to the strain energy function of the form

$$W_c = K_c J \left\{ \ln\left(\frac{J}{J_c}\right) - 1 \right\}, \text{ for } J \leq J_c \ .$$

Here $K_c$ (BULKC) is a physical bulk modulus, $J = \det(\mathbf{F})$ is the Jacobian of the deformation and $J_c$ (JACC) is the critical Jacobian for when the effect commences. $\mathbf{F}$ is the deformation gradient. This contributes to the pressure with a term

$$p = K_c \ln\left(\frac{J_c}{J}\right), \text{ for } J \leq J_c$$

and thus prevents membrane elements from collapsing or inverting when subjected to compressive loads. The bulk modulus $K_c$ should be selected with the

slopes in the stress map tables in mind, presumably some order of magnitude(s) smaller.

7. **Strain rate scale factor.** As an option, the local membrane stress can be scaled based on the engineering strain rates via the function $\vartheta = \vartheta(\dot{e}, \mathbf{S})$. We set

$$\dot{e} = \max\left(\frac{\dot{\epsilon}}{\|\mathbf{FS}\|}, 0\right)$$

to be the equivalent engineering strain rate in the direction of loading and define

$$\vartheta(\dot{e}, \boldsymbol{S}) = \frac{F_{XX}(\dot{e})|S_{XX}| + F_{YY}(\dot{e})|S_{YY}| + 2|S_{XY}|}{|S_{XX}| + |S_{YY}| + 2|S_{XY}|},$$

meaning that the strain rate scale factor defaults to the user input data FXX and FYY for uniaxial loading in the two material directions, respectively. Note that we only consider strain rate scaling in loading and not in unloading, and furthermore that the strain rates used in evaluating the curves are pre-filtered using the time window DT to avoid excessive numerical noise. It is, therefore, recommended to set DT to a time corresponding to at least hundred time steps or so.

## *MAT_PLASTIC_GREEN-NAGHDI_RATE

This is Material Type 35. It is similar to model 3 but uses the Green-Naghdi Rate formulation rather than the Jaumann rate for the stress update. For some cases this might be helpful. This model also has a strain rate dependency following the Cowper-Symonds model. It is available for solid, thick shell (formulations 3, 5,and 7), and SPH elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGY | ETAN | SRC | SRP | BETA | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus |
| SRC | Strain rate parameter, $C$ |
| SRP | Strain rate parameter, $p$ |
| BETA | Hardening parameter, $0 < \beta' < 1$ |

## *MAT_3-PARAMETER_BARLAT_{OPTION}

This is Material Type 36. This model was developed by Barlat and Lian [1989] for modeling sheets with anisotropic materials under plane stress conditions. Lankford parameters may be used to define the anisotropy. This particular development is due to Barlat and Lian [1989]. *MAT_FLD_3-PARAMETER_BARLAT is a version of this material model that includes a flow limit diagram failure option.

Available options include:

<BLANK>

NLP

The NLP option estimates failure using the Formability Index (F.I.), which accounts for the non-linear strain paths seen in metal forming applications (see Remark 13). The NLP field in Card 4b *must* be defined when using this option. The NLP option is also available for *MAT_037, *MAT_125, and *MAT_226.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | HR | P1 | P2 | ITER |
|---|---|---|---|---|---|---|---|

**Card 2a.** Include this card if PB = 0 (see Card 4a/4b).

| M | R00 | R45 | R90 | LCID | E0 | SPI | P3 |
|---|---|---|---|---|---|---|---|

**Card 2b.** Include this card if PB > 0. (see Card 4a/4b).

| M | AB | CB | HB | LCID | E0 | SPI | P3 |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if M < 0.

| CRC1 | CRA1 | CRC2 | CRA2 | CRC3 | CRA3 | CRC4 | CRA4 |
|---|---|---|---|---|---|---|---|

**Card 4a.** Include this card if the keyword option is unset (<BLANK>).

| AOPT | C | P | VLCID | | PB | HTA | HTB |
|---|---|---|---|---|---|---|---|

**Card 4b.** This card is included if the keyword option is NLP.

| AOPT | C | P | VLCID | | PB | NLP | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| | | | A1 | A2 | A3 | HTC | HTD |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | HTFLAG |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is optional.

| USRFAIL | LCBI | LCSH | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | P1 | P2 | ITER |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = \|E\|, which defines Young's modulus as a function of plastic strain. See Remark 1. |
| PR | Poisson's ratio, $\nu$ |
| HR | Hardening rule:<br>EQ.1.0: Linear (default)<br>EQ.2.0: Exponential (Swift). See Remark 5.<br>EQ.3.0: Load curve or table with strain rate effects<br>EQ.4.0: Exponential (Voce). See Remark 5.<br>EQ.5.0: Exponential (Gosh). See Remark 5. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.6.0: Exponential (Hocket-Sherby). See Remark 5. |
| | EQ.7.0: Load curves in three directions |
| | EQ.8.0: Table with temperature dependence |
| | EQ.9.0: Three-dimensional table with temperature and strain rate dependence |
| | EQ.10.0: Table with prestrain dependence. See Remark 8. |
| P1 | Material parameter: |
| | HR.EQ.1.0: Tangent modulus |
| | HR.EQ.2.0: $k$, strength coefficient for Swift exponential hardening |
| | HR.EQ.4.0: $a$, coefficient for Voce exponential hardening |
| | HR.EQ.5.0: $k$, strength coefficient for Gosh exponential hardening |
| | HR.EQ.6.0: $a$, coefficient for Hocket-Sherby exponential hardening |
| | HR.EQ.7.0: Load curve ID for hardening in the 45° direction. See Remark 1. |
| P2 | Material parameter: |
| | HR.EQ.1.0: Initial yield stress |
| | HR.EQ.2.0: $n$, exponent for Swift exponential hardening |
| | HR.EQ.4.0: $c$, coefficient for Voce exponential hardening |
| | HR.EQ.5.0: $n$, exponent for Gosh exponential hardening |
| | HR.EQ.6.0: $c$, coefficient for Hocket-Sherby exponential hardening |
| | HR.EQ.7.0: Load curve ID for hardening in the 90° direction. See Remark 1. |
| ITER | Iteration flag for speed: |
| | EQ.0.0: Fully iterative |
| | EQ.1.0: Fixed at three iterations |
| | Generally, ITER = 0 is recommended. ITER = 1, however, is somewhat faster and may give acceptable results in most problems. |

**Lankford Parameters Card.** This card is included if PB = 0 (see Card 4a/4b).

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | R00 | R45 | R90 | LCID | E0 | SPI | P3 |
| Type | F | F | F | F | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| M | $m$, exponent in Barlat's yield surface. If negative, the absolute value is used. |
| R00 | $R_{00}$, Lankford parameter in the 0° direction:<br>GT.0.0: Constant value<br>LT.0.0: Load curve or table ID = \|R00\| which defines $R_{00}$ as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1, 2, and 3. |
| R45 | $R_{45}$, Lankford parameter in the 45° direction:<br>GT.0.0: Constant value<br>LT.0.0: Load curve or table ID = \|R45\| which defines $R_{45}$ as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1, 2, and 3. |
| R90 | $R_{90}$, Lankford parameter in the 90° direction:<br>GT.0.0: Constant value<br>LT.0.0: Load curve or table ID = \|R90\| which defines $R_{90}$ as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1, 2, and 3. |
| LCID | Load curve/table ID for hardening in the 0° direction. See Remark 1. |
| E0 | Material parameter:<br>HR.EQ.2.0: $\varepsilon_0$ for determining initial yield stress for Swift exponential hardening (default = 0.0) |

| VARIABLE | DESCRIPTION |
|---|---|
| | HR.EQ.4.0: $b$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $\varepsilon_0$ for determining initial yield stress for Gosh exponential hardening (default = 0.0)<br>HR.EQ.6.0: $b$, coefficient for Hocket-Sherby exponential hardening |
| SPI | Case I: If HR = 2.0 and E0 is zero, then $\varepsilon_0$ is determined by (see Remark 5):<br>EQ.0.0: $\varepsilon_0 = \left(\frac{E}{k}\right)^{\left[1/(n-1)\right]}$, default<br>LE.0.02: $\varepsilon_0$ = SPI<br>GT.0.02: $\varepsilon_0 = \left(\frac{\text{SPI}}{k}\right)^{\left[1/n\right]}$<br>Case II: If HR = 5.0 and E0 is zero, then the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR = 2.0. See Remark 5. |
| P3 | Material parameter:<br>HR.EQ.5.0: $p$, parameter for Gosh exponential hardening<br>HR.EQ.6.0: $n$, exponent for Hocket-Sherby exponential hardening |

**BARLAT89 Parameters Card.** This card is included if PB > 0 (see Card 4a/4b).

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | AB | CB | HB | LCID | E0 | SPI | P3 |
| Type | F | F | F | F | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| M | $m$, exponent in Barlat's yield surface. If negative, the absolute value is used. |
| AB | $a$, Barlat89 parameter |
| CB | $c$, Barlat89 parameter |
| HB | $h$, Barlat89 parameter |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID | Load curve/table ID for hardening in the 0° direction. See Remark 1. |
| E0 | Material parameter:<br>HR.EQ.2.0: $\varepsilon_0$ for determining initial yield stress for Swift exponential hardening (default = 0.0)<br>HR.EQ.4.0: $b$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $\varepsilon_0$ for determining initial yield stress for Gosh exponential hardening (default = 0.0)<br>HR.EQ.6.0: $b$, coefficient for Hocket-Sherby exponential hardening |
| SPI | Case I: If HR = 2.0 and E0 is zero, then $\varepsilon_0$ is determined by (see Remark 5):<br>EQ.0.0: $\varepsilon_0 = \left(\frac{E}{k}\right)^{\left[1/(n-1)\right]}$, default<br>LE.0.02: $\varepsilon_0 = \text{SPI}$<br>GT.0.02: $\varepsilon_0 = \left(\frac{\text{SPI}}{k}\right)^{\left[1/n\right]}$<br>Case II: If HR = 5.0 and E0 is zero, then the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR = 2.0. See Remark 5. |
| P3 | Material parameter:<br>HR.EQ.5.0: $p$, parameter for Gosh exponential hardening<br>HR.EQ.6.0: $n$, exponent for Hocket-Sherby exponential hardening |

Define the following card if M < 0.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CRC1 | CRA1 | CRC2 | CRA2 | CRC3 | CRA3 | CRC4 | CRA4 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CRC$n$ | Chaboche-Rousselier hardening parameters; see Remark 7. |

| VARIABLE | DESCRIPTION |
|---|---|
| CRA*n* | Chaboche-Rousselier hardening parameters; see Remark 7. |

This card is included if the keyword option is not used (<BLANK>)

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | C | P | VLCID | | PB | HTA | HTB |
| Type | F | F | F | I | | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by an angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector, **v**, with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR). |
| C | *C* in Cowper-Symonds strain rate model. See Remark 6. |
| P | *p* in Cowper-Symonds strain rate model. Set P to zero for no strain rate effects. See Remark 6. |
| VLCID | Volume correction curve ID defining the relative volume change (change in volume relative to the initial volume) as a function of the effective plastic strain. This is only used when nonzero. See Remark 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| PB | Barlat89 parameter, $p$. If PB > 0, parameters AB, CB, and HB are read instead of R00, R45, and R90. See Remark 3. |
| HTA | Load curve/table ID for postforming parameter $a$ in heat treatment |
| HTB | Load curve/table ID for postforming parameter $b$ in heat treatment |

This card is included if the keyword option is NLP.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | C | P | VLCID | | PB | NLP | |
| Type | F | F | F | I | | F | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by an angle BETA.<br>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector, **v**, with the element normal.<br>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR). |
| C | $C$ in Cowper-Symonds strain rate model. See Remark 6. |
| P | $p$ in Cowper-Symonds strain rate model. Set P to zero for no strain rate effects. See Remark 6. |

| VARIABLE | DESCRIPTION |
|---|---|
| VLCID | Volume correction curve ID defining the relative volume change (change in volume relative to the initial volume) as a function of the effective plastic strain. This is only used when nonzero. See Remark 1. |
| PB | Barlat89 parameter, $p$. If PB > 0, parameters AB, CB, and HB are read instead of R00, R45, and R90. See Remark 3. |
| NLP | ID of a load curve of the Forming Limit Diagram (FLD) under linear strain paths. In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when option NLP is used. See Remark 13. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | HTC | HTD |
| Type | | | | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| HTC | Load curve/table ID for postforming parameter $c$ in heat treatment |
| HTD | Load curve/table ID for postforming parameter $d$ in heat treatment |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | HTFLAG |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |
| HTFLAG | Heat treatment flag (see Remark 11):<br>EQ.0: Preforming stage<br>EQ.1: Heat treatment stage<br>EQ.2: Postforming stage |

Optional card.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | USRFAIL | LCBI | LCSH | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| USRFAIL | User-defined failure flag:<br>EQ.0: No user subroutine is called.<br>EQ.1: User subroutine matusr_24 in dyn21.f is called. |
| LCBI | HR.EQ.7: Load curve defining biaxial stress as a function of bi-axial strain for hardening rule; see discussion in the formulation section below for a definition.<br>HR.NE.7: Ignored |
| LCSH | HR.EQ.7: Load curve defining shear stress as a function of shear strain for hardening; see discussion in the formulation section below for a definition.<br>HR.NE.7: Ignored |

**Remarks:**

1. **Plastic strain in curve definitions.** The effective plastic strain used in this model is defined to be plastic work equivalent. A consequence of this is that for parameters defined as functions of effective plastic strain, the rolling (00) direction should be used as the reference direction. For instance, the hardening curve

for HR = 3 is the stress as a function of strain for uniaxial tension in the rolling direction, the VLCID curve should give the relative volume change as a function of strain for uniaxial tension in the rolling direction, and the load curve given by $-E$ should give the Young's modulus as a function of strain for uniaxial tension in the rolling direction. Optionally, the curve can be substituted for a table defining hardening as a function of plastic strain rate (HR = 3), temperature (HR = 8), or prestrain (HR = 10).

Exceptions to the rule above are curves defined as functions of plastic strain in the 45° and 90° directions, i.e., P1 and P2 for HR = 7 and negative R45 or R90; see Fleischer et.al. [2007]. The hardening curves are defined in these cases as measured stress as a function of measured plastic strain for uniaxial tension in the direction of interest, i.e., as determined from experimental testing using a standard procedure. The optional biaxial and shear hardening curves require some further elaboration, as we assume that a biaxial or shear test reveals that the true stress tensor in the material system, expressed as

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma & 0 \\ 0 & \pm\sigma \end{pmatrix}, \qquad \sigma \geq 0,$$

is a function of the (plastic) strain tensor

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \pm\varepsilon_2 \end{pmatrix}, \qquad \varepsilon_1 \geq 0, \qquad \varepsilon_2 \geq 0.$$

The input hardening curves are $\sigma$ as a function of $\varepsilon_1+\varepsilon_2$. The $\pm$ sign above distinguishes between the biaxial (+) and the shear (−) cases. Moreover, the curves defining the R-values are functions of the measured plastic strain for uniaxial tension in the direction of interest. These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in directions other than the rolling direction and may be somewhat confusing. Therefore, the von Mises work-equivalent plastic strain is output as history variable #2 if HR = 7 or any R-value is defined as a function of the plastic strain.

2. **Determining R-values from curves.** The R-values in curves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width $W$ and thickness $T$ are measured as functions of strain. Then the corresponding R-value is given by:

$$R = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T}$$

3. **Anisotropic yield criterion.** The anisotropic yield criterion, $\Phi$, for plane stress is defined as:

$$\Phi = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m = 2\sigma_Y^m$$

where $\sigma_Y$ is the yield stress and $K_{i=1,2}$ are given by:

$$K_1 = \frac{\sigma_x + h\sigma_y}{2}$$
$$K_2 = \sqrt{\left(\frac{\sigma_x - h\sigma_y}{2}\right)^2 + p^2\tau_{xy}^2}$$

If PB = 0, the anisotropic material constants $a$, $c$, $h$, and $p$ are obtained through $R_{00}$, $R_{45}$, and $R_{90}$:

$$a = 2 - 2\sqrt{\left(\frac{R_{00}}{1+R_{00}}\right)\left(\frac{R_{90}}{1+R_{90}}\right)}$$
$$c = 2 - a$$
$$h = \sqrt{\left(\frac{R_{00}}{1+R_{00}}\right)\left(\frac{1+R_{90}}{R_{90}}\right)}$$

The anisotropy parameter $p$ is calculated implicitly. According to Barlat and Lian, the R-value for any angle $\phi$ can be calculated from:

$$R_\phi = \frac{2m\sigma_Y^m}{\left(\frac{\partial\Phi}{\partial\sigma_x} + \frac{\partial\Phi}{\partial\sigma_y}\right)\sigma_\phi} - 1$$

where $\sigma_\phi$ is the uniaxial tension in the $\phi$ direction. This expression can be used to iteratively calculate the value of $p$. Let $\phi = 45$ and define a function $g$ as:

$$g(p) = \frac{2m\sigma_Y^m}{\left(\frac{\partial\Phi}{\partial\sigma_x} + \frac{\partial\Phi}{\partial\sigma_y}\right)\sigma_\phi} - 1 - R_{45}$$

An iterative search is used to find the value of $p$. If PB > 0, material parameters $a$(AB), $c$(CB), $h$(HB), and $p$(PB) are used directly.

4. **Effective stress.** The effective stress is given as

$$\sigma_{\text{eff}} = \left\{\frac{1}{2}\left(a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m\right)\right\}^{1/m}.$$

$K_1$ $K_2$, $a$, and $c$ are defined in Remark 3. For face-centered cubic (FCC) materials $m = 8$ is recommended and for body-centered cubic (BCC) materials $m = 6$ may be used.

It can be output to the d3plot database through *DEFINE_MATERIAL_HISTORIES.

| *DEFINE_MATERIAL_HISTORIES Properties | | |
|---|---|---|
| **Label** | **Attributes** | **Description** |
| Effective Stress | - - - - | Effective stress $\sigma_{\text{eff}}$, see above |

5. **Swift, Voce, Gosh, and Herbet-Sherby hardening laws.** For the Swift hardening law (HR = 2), the yield strength of the material is expressed in terms of $k$ and $n$:

$$\sigma_{\text{Y}} = k\varepsilon^n = k\left(\varepsilon_0 + \bar{\varepsilon}_{\text{p}}\right)^n$$

where $\varepsilon_0$ is the elastic strain to yield and $\bar{\varepsilon}_{\text{p}}$ is the effective plastic strain. If E0 is set to zero and SPI is set to zero, the strain to yield is found by solving for the intersection of the linearly elastic loading equation with the strain-hardening equation:

$$\sigma = E\varepsilon$$
$$\sigma = k\varepsilon^n$$

which gives the elastic strain at yield as:

$$\varepsilon_0 = \left(\frac{E}{k}\right)^{\frac{1}{n-1}}$$

If E0 is zero and SPI is nonzero and greater than 0.02, then:

$$\varepsilon_0 = \left(\frac{\sigma_{\text{Y}}}{k}\right)^{\frac{1}{n}}$$

Other available hardening models include the Voce equation (HR = 4) given by:

$$\sigma_{\text{Y}}\left(\bar{\varepsilon}_{\text{p}}\right) = a - be^{-c\bar{\varepsilon}_{\text{p}}},$$

the Gosh equation (HR = 5) given by:

$$\sigma_{\text{Y}}\left(\bar{\varepsilon}_{\text{p}}\right) = k\left(\varepsilon_0 + \bar{\varepsilon}_{\text{p}}\right)^n - p,$$

and finally the Hocket-Sherby equation given by:

$$\sigma_{\text{Y}}\left(\bar{\varepsilon}_{\text{p}}\right) = a - be^{-c\bar{\varepsilon}_{\text{p}}^n}.$$

For the Gosh hardening law, the interpretation of SPI is the same as for the Swift hardening law, i.e., if set to zero, the strain at yield is determined implicitly from the intersection of the strain-hardening equation with the linearly elastic loading equation.

6. **Cowper-Symonds' model.** To include strain-rate effects in the model, we multiply the yield stress by a factor that depends on the effective plastic strain rate. We use the Cowper-Symonds' model; hence, the yield stress can be written as:

$$\sigma_Y(\bar{\varepsilon}_p, \dot{\bar{\varepsilon}}_p) = \sigma_Y^s(\bar{\varepsilon}_p)\left\{1 + \left(\frac{\dot{\bar{\varepsilon}}_p}{C}\right)^{\frac{1}{p}}\right\}$$

where $\sigma_Y^s$ denotes the static yield stress, $C$ and $p$ are material parameters, and $\dot{\bar{\varepsilon}}_p$ is the effective plastic strain rate. With HR = 3, strain rate effects can be defined using a table, in which each load curve in the table defines the yield stress as a function of plastic strain for a given strain rate. In contrast to *MAT_024, when the strain rate is larger than that of any curve in the table, the table is extrapolated in the strain rate direction to find the appropriate yield stress.

7. **Kinematic hardening law.** A kinematic hardening model is implemented following the works of Chaboche and Roussilier. It is invoked by setting M to a negative number. A back stress $\boldsymbol{\alpha}$ is introduced such that the effective stress is computed as:

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12})$$

The back stress is the sum of up to four terms according to:

$$\alpha_{ij} = \sum_{k=1}^{4} \alpha_{ij}^k$$

and the evolution of each back stress component is as follows:

$$\delta\alpha_{ij}^k = C_k\left(a_k \frac{s_{ij} - \alpha_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k\right)\delta\bar{\varepsilon}_p$$

where $C_k$ and $a_k$ are material parameters, $s_{ij}$ is the deviatoric stress tensor, $\sigma_{\text{eff}}$ is the effective stress and $\bar{\varepsilon}_p$ is the effective plastic strain. The yield condition is, for this case, modified according to

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \bar{\varepsilon}_p) = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12}) - \left\{\sigma_Y(\bar{\varepsilon}_p, \dot{\bar{\varepsilon}}_p) - \sum_{k=1}^{4} a_k\left[1 - \exp(-C_k\bar{\varepsilon}_p)\right]\right\} \leq 0$$

in order to get the expected stress-strain response for uniaxial stress.

8. **Prestrain dependence.** With hardening rule HR = 10, the flow curves in a table definition can be based on different prestrain values. Hence, flow curves can have varying shapes as defined in the corresponding table. For example, the plastic strain distribution as obtained in a first step of a two-stage procedure is initialized in the next stage with *INITIAL_STRESS_SHELL and corresponding values for EPS. With HR = 10, this prestrain is initially transferred to history variable #29, and all stresses and other history variables are set to zero, assuming that the part was subjected to an annealing phase. With EPS stored on history variable #29, the table lookup for the actual yield value can be used to interpolate on differently shaped flow curves.

9. **Smoothing of the strain ratio.** *CONTROL_FORMING_STRAIN_RATIO_-SMOOTH applies a smoothing algorithm to reduce the output noise level of the strain ratio $\beta$ (in-plane minor strain increment/major strain increment) which is used to calculate the Formability Index.

10. **Non-integer flow potential exponent, $m$, for the NLP keyword option.** Starting in R12, a non-integer value of the exponent $m$ is supported for the option NLP. Note that a non-integer value was always supported for no keyword option.

11. **Heat treatment with variable HTFLAG.** Heat treatment for increasing the formability of prestrained aluminum sheets can be simulated through the use of HTFLAG, where the intention is to run a forming simulation in steps involving preforming, springback, heat treatment, and postforming. In each step, the history is transferred to the next via the use of dynain (see *INTERFACE_SPRINGBACK). The first two steps are performed with HTFLAG = 0 according to standard procedures, resulting in a plastic strain field $\varepsilon_p^0$ corresponding to the prestrain. The heat treatment step is performed using HTFLAG = 1 in a coupled thermomechanical simulation, where the blank is heated. The coupling between thermal and mechanical only involves storing the maximum temperature $T^0$ as a history variable in the material model, corresponding to the heat treatment temperature. Here, it is important to export all history variables to the dynain file for the postforming step. In the final postforming step, HTFLAG = 2, the yield stress is then augmented by the Hocket-Sherby-like term:

$$\Delta\sigma = b - (b-a)\exp\left[-c\left(\bar{\varepsilon}_\mathrm{p} - \bar{\varepsilon}_\mathrm{p}^0\right)^d\right]$$

Where $a$, $b$, $c$, and $d$ are given as tables as functions of the heat treatment temperature $T^0$ and prestrain $\bar{\varepsilon}_\mathrm{p}^0$. That is, in the table definitions, each load curve corresponds to a given prestrain, and the load curve value is with respect to the heat treatment temperature:

$$a = a\left(T^0, \bar{\varepsilon}_\mathrm{p}^0\right), \quad b = b\left(T^0, \bar{\varepsilon}_\mathrm{p}^0\right), \quad c = c\left(T^0, \bar{\varepsilon}_\mathrm{p}^0\right), \quad d = d\left(T^0, \bar{\varepsilon}_\mathrm{p}^0\right).$$

The effect of heat treatment is that the material strength decreases, but hardening increases, thus typically:

$$a \le 0, \quad b \ge a, \quad c > 0, \quad d > 0.$$

12. **History variables.** The following table lists the available history variables when the NLP keyword option is not used. For the NLP keyword option, see Remark 13. Set NEIPS to *DATABASE_EXTENT_BINARY to output history variables to d3plot.

| History Variable # | Description |
|---|---|
| 1 | Inverse of the previous time step's yield stress |

| History Variable # | Description |
|---|---|
| 2 | If HR = 7 or any R-value is defined as a function of the plastic strain: von Mises work-equivalent plastic strain (see Remark 1) |
| 3 | $\cos\alpha$, where $\alpha$ is the in-plane angle between the material coordinate system and the element coordinate system |
| 4 | $-\sin\alpha$, where $\alpha$ is defined in the description of history variable #3 |
| 6 | Effective strain rate |
| 7 | Current yield stress |
| 8 | Current hardening slope |
| 9 | Back stress component $\alpha_{11}$ |
| 10 | Back stress component $\alpha_{22}$ |
| 11 | Back stress component $\alpha_{12}$ |
| 24 | Temperature if HTFLAG = 1 |
| 29 | Prestrain (see Remark 8) |

13. **A failure criterion for nonlinear strain paths (NLP) in sheet metal forming.** When the NLP option is used, a necking failure criterion is activated to account for the non-linear strain-path effect in sheet metal forming. Based on the traditional Forming Limit Diagram (FLD) for the linear strain path, the Formability Index (F.I.) is calculated for every element in the model throughout the simulation. The entire F.I. time history for every element is stored in history variable #1 in d3plot files, accessible from *Post/History* menu in *LS-PrePost* v4.0. In addition to the F.I. output, other useful information stored in other history variables can be found as follows,

    a) Formability Index: #1

    b) Maximum failure: #2

    To enable the output of these history variables to the d3plot files, NEIPS on the *DATABASE_EXTENT_BINARY card must be set to at least 2. The history variables can also be plotted on the formed sheet blank as a color contour map, accessible from *Post/FriComp/Misc* menu. The index value starts from 0.0, with the onset of necking failure when it reaches 1.0. The F.I. is calculated based on the critical effect strain method, as explained in the manual pages for *MAT_037. The theoretical background based on two papers can also be found in the manual pages for *MAT_037.

    When d3plot files are used to plot the history variable #1 (the F.I.) in color contour, the value in the *Max* pull-down menu in *Post/FriComp* needs to be set to

*Min*, meaning that the necking failure occurs only when all integration points through the thickness have reached the critical value of 1.0 (refer to *Tharrett and Stoughton's paper in 2003 SAE 2003-01-1157*). We also recommend setting MAXINT in *DATABASE_EXTENT_BINARY to the same value as NIP in *SECTION_SHELL. In addition, the value in the *Avg* pull-down menu in *Post/FriRang* needs to be set to *None*. The strain path history (major vs. minor strain) of each element can be plotted with the radial dial button *Strain Path* in *Post/FLD*.

An example of a partial input for the material is provided below. The FLD for the linear strain path is defined by NLP with load curve ID 211, where the abscissae represent minor strains and the ordinates represent major strains.

```
*MAT_3-PARAMETER_BARLAT_NLP
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$      MID        RO         E        PR        HR        P1        P2      ITER
         1 2.890E-09  6.900E04     0.330     3.000
$        M       R00       R45       R90      LCID        E0       SPI        P3
     8.000     0.800     0.600     0.550        99
$     AOPT         C         P     VLCID                          NLP
     2.000                                                       211
$                                    A1        A2        A3
                                   0.000     1.000     0.000
$       V1        V2        V3        D1        D2        D3      BETA

$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$ Hardening Curve
*DEFINE_CURVE
        99
               0.000             130.000
               0.002             134.400
               0.006             143.000
               0.010             151.300
               0.014             159.300
                 ⋮                 ⋮
               0.900             365.000
               1.000             365.000
$ FLD Definition
*DEFINE_CURVE
211
                -0.2               0.325
              -0.1054             0.2955
              -0.0513             0.2585
               0.0000             0.2054
               0.0488             0.2240
               0.0953             0.2396
               0.1398             0.2523
               0.1823             0.2622
                 ⋮                 ⋮
```

As shown in Figures M36-1, M36-2 and M36-3, predictions and validations of forming limit curves (FLC) of various nonlinear strain paths on a single shell element were done using this new option, for an Aluminum alloy with $R_{00} = 0.8, R_{45} = 0.6$ *and* $R_{90} = 0.55$ and the yield at 130.0 MPa. In each case, the element is further strained in three different paths (uniaxial stress – U.A., plane strain – P.S., and equibiaxial strain – E.B.) separately, following a prestraining in uniaxial, plane strain and equibiaxial strain state, respectively. The forming limits are determined at the end of the secondary straining for each path, when the

F.I. has reached the value of 1.0. It is seen that the predicted FLCs (dashed curves) in case of the nonlinear strain paths are totally different from the FLCs under the linear strain paths. It is noted that the current predicted FLCs under nonlinear strain paths are obtained by connecting the ends of the three distinct strain paths. More detailed FLCs can be obtained by straining the elements in more paths between U.A. and P.S. and between P.S. and E.B. In Figure M36-4, time-history plots of F.I., strain ratio, and effective strain are shown for uniaxial prestrain followed by equibiaxial strain path on the same single element.

Typically, to assess sheet formability, F.I. contour of the entire part should be plotted. Based on the contour plot, the nonlinear strain path and the F.I. time history of a few elements in the area of concern can be plotted for further study. These plots are similar to those shown in the manual pages of *MAT_037.

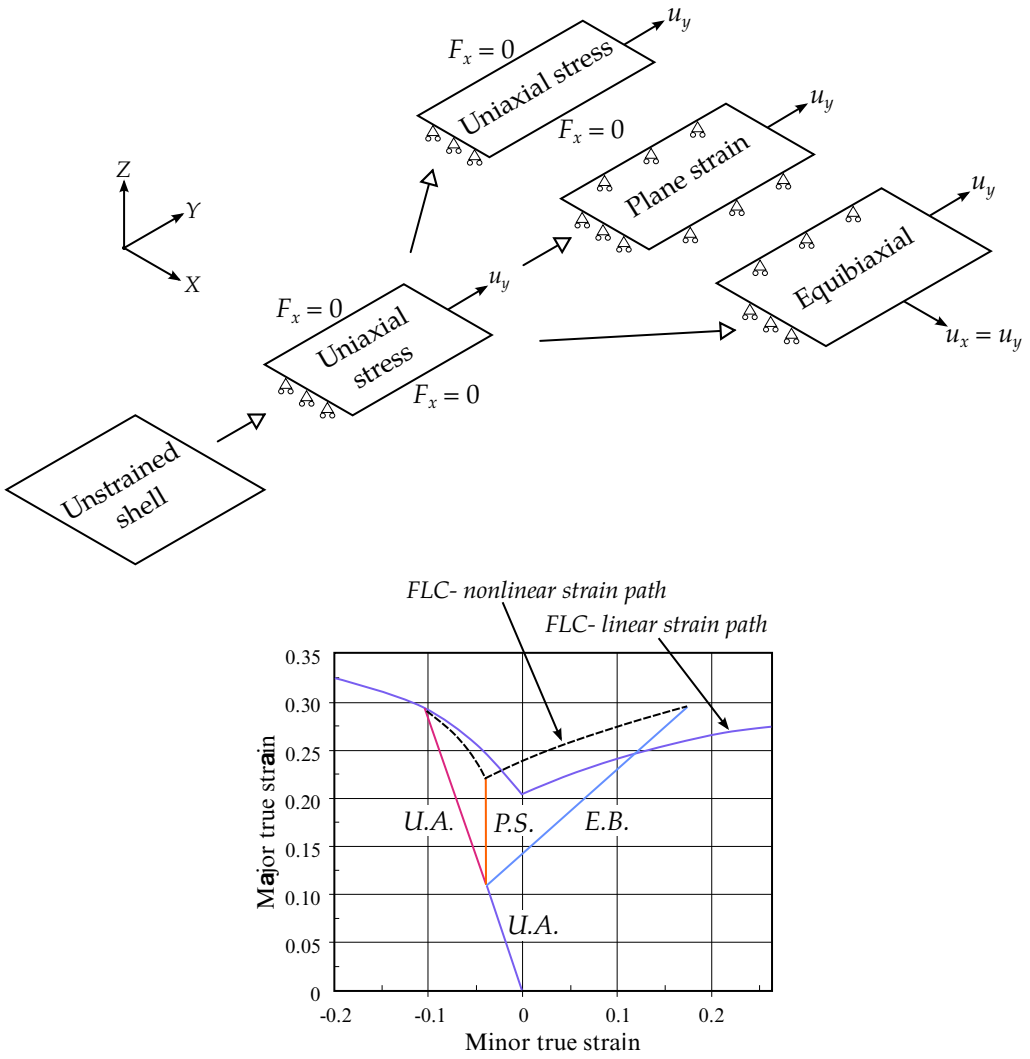



**Figure M36-1.** Nonlinear FLD prediction with uniaxial prestraining.

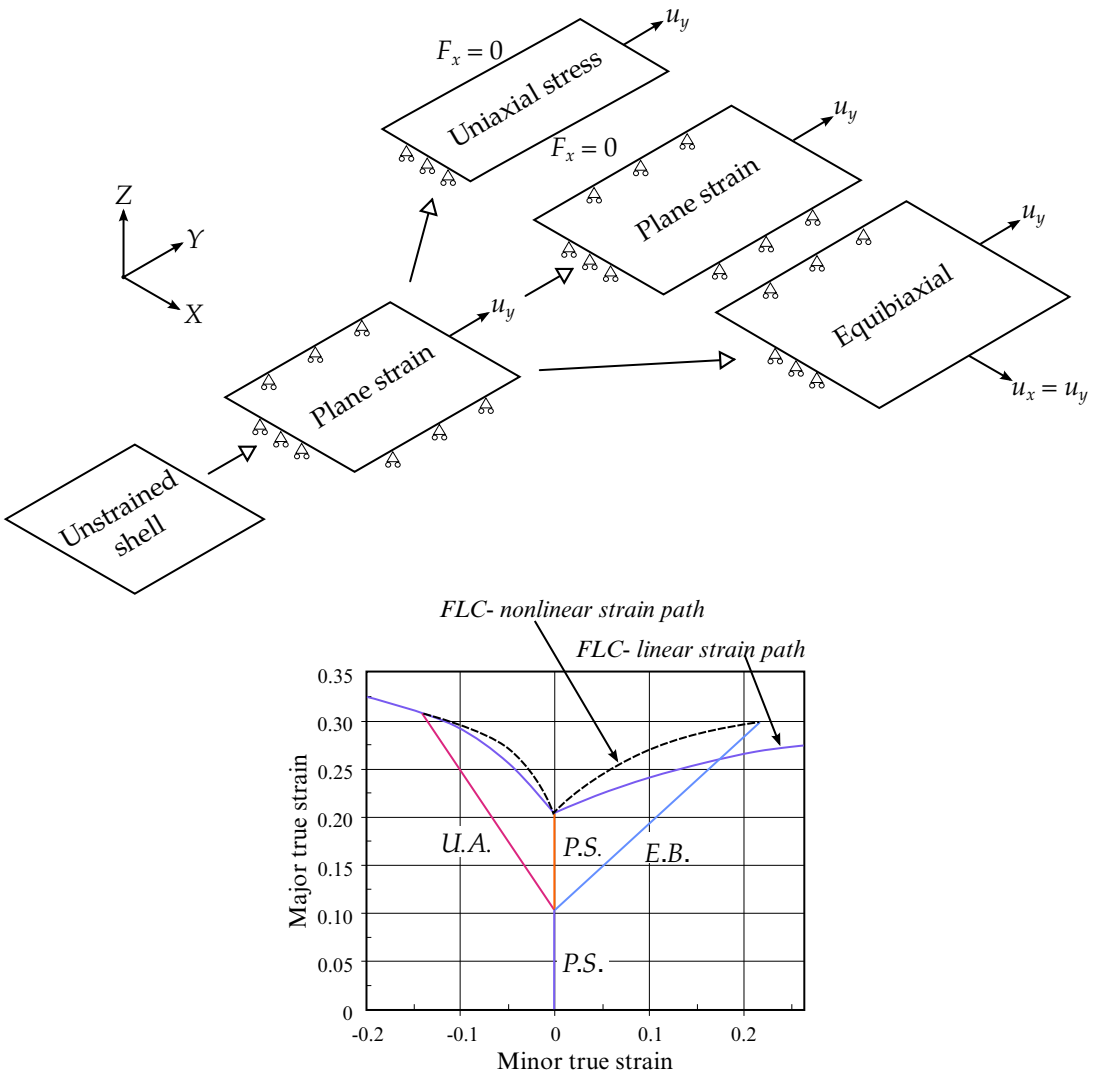



**Figure M36-2.** Nonlinear FLD prediction with plane strain prestraining.

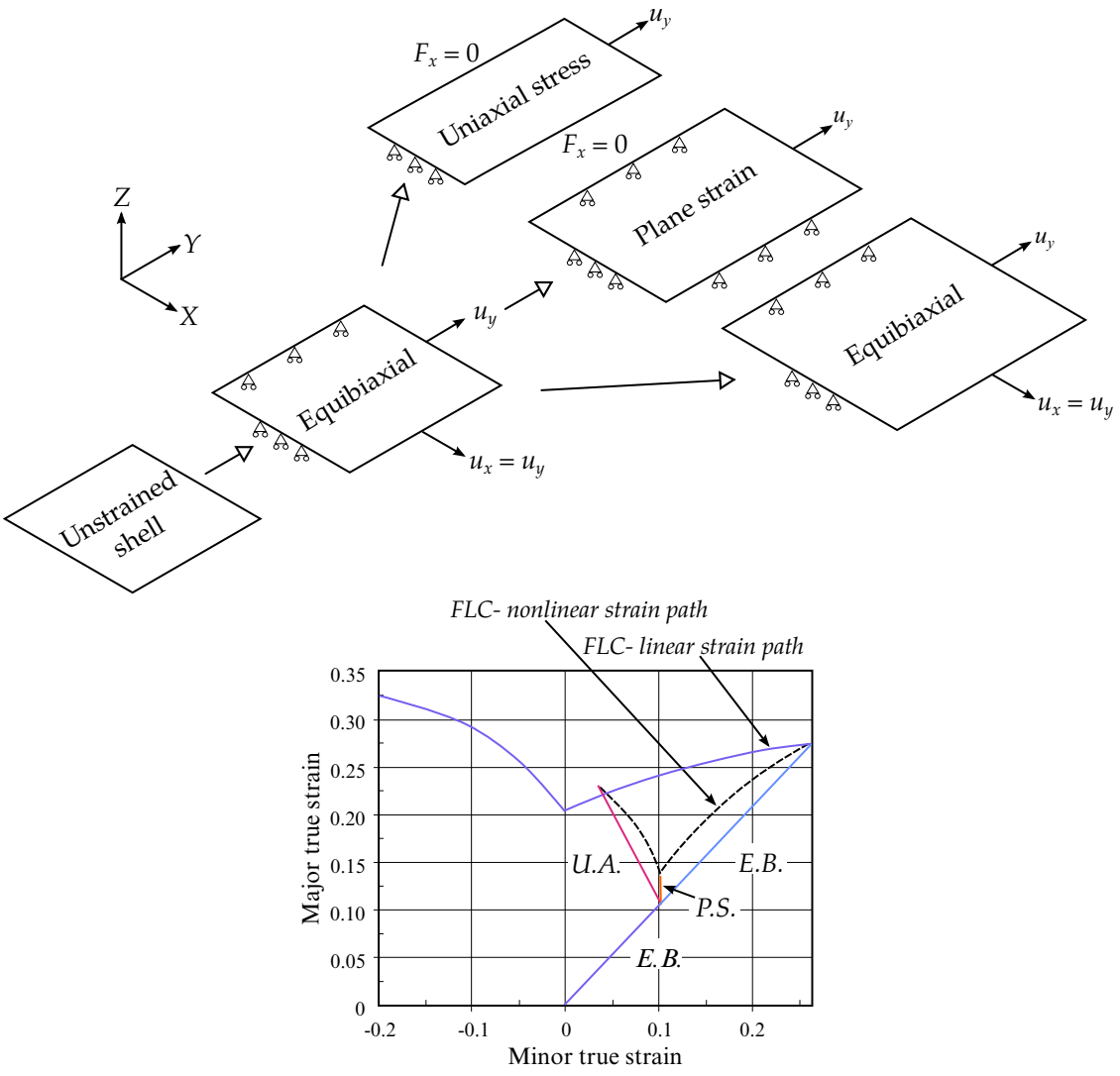



**Figure M36-3.** Nonlinear FLD prediction with equibiaxial prestraining.

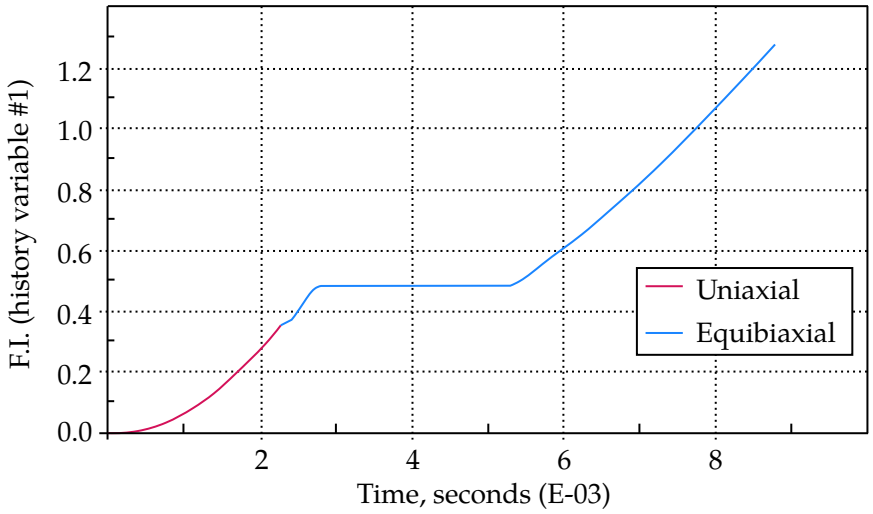



**Figure M36-4.** Time-history plots of the three history variables.

## *MAT_EXTENDED_3-PARAMETER_BARLAT

This is Material Type 36E. This model is an extension of the standard 3-parameter Barlat model, allowing for different hardening curves and R-values in different directions; see Fleischer et.al. [2007]. The directions in this context are the three uniaxial directions (0, 45 and 90 degrees) and optionally biaxial and shear.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCH00 | LCH45 | LCH90 | LCHBI | LCHSH | HOSF | TFLG | |
| Type | F | F | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCR00 | LCR45 | LCR90 | LCRBI | LCRSH | M | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. |
| E | Young's modulus, *E*. |
| PR | Poisson's ratio, ν. |
| LCH*XX* | Load curve/table defining uniaxial stress vs. uniaxial strain and strain rate in the given direction (*XX* is either 00, 45, or 90). The exact definition is discussed in the Remarks below. LCH00 must be defined. The others default to LCH00 if not defined. |
| LCHBI | Load curve/table defining biaxial stress vs. biaxial strain and strain rate, see discussion in the Remarks below for a definition. If not defined, this is determined from LCH00 and the initial *R*-values to yield a response close to the standard 3-parameter Barlat model. |
| LCHSH | Load curve/table defining shear stress vs. shear strain and strain rate, see discussion in the Remarks below for a definition. If not defined, this is ignored to yield a response close to the standard 3-parameter Barlat model. |
| HOSF | Hosford option for enhancing the convexity of the yield surface. Set to 1 to activate. |
| TFLG | Table evaluation flag:<br>EQ.0.0: Stresses in tables LCH*XX* depend on strain and strain rate (default). See description above.<br>EQ.1.0: Stresses in tables LCH*XX* depend on strain and history variable #12. Or they can be a TABLE_3D, in which case the stresses depend on strain, strain rate, and history variable #12. History variable #12 is an arbitrary user- |

| VARIABLE | DESCRIPTION |
|---|---|
| | defined variable that can be set with *INITIAL_-STRESS_SHELL or *INITIAL_HISTORY_NODE. |
| LCR*XX* | Load curve defining the standard *R*-value as a function of uniaxial strain in the given direction (*XX* is 00, 45, or 90). The exact definition is discussed in the Remarks below.The default is a constant *R*-value of 1.0. A negative input results in a constant *R*-value of –LCRXX. |
| LCRBI | Load curve defining the biaxial *R*-value as a function of biaxial strain. See the discussion in the Remarks below for a definition. The default is a constant *R*-value of 1.0. A negative input results in a constant *R*-value of –LCRBI. |
| LCRSH | Load curve defining the shear *R*-value as a function of shear strain. See the discussion in the Remarks below for a definition. The default is a constant *R*-value of 1.0. A negative input results in a constant *R*-value of –LCRSH. |
| M | Barlat flow exponent, *m*. It must be an integer value. |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Formulation:**

The standard 3-parameter Barlat model incorporates plastic anisotropy in a fairly moderate sense, allowing for the specification of *R*-values in three different directions, together with a stress level in the reference direction (termed *rolling* or *0 degree* direction), but not more than that. To allow for a more accurate representation of a more severe anisotropic material, like in rolled aluminum sheet components, you could switch to the Barlat YLD2000 model (material 133), which also allows for specifying stress levels in the two remaining directions as well as stress and strain data at an arbitrary point on the yield surface. The properties of extruded aluminum, however, are such that neither of these two material models is sufficient to describe its extreme anisotropy. One particular observation from experiments is that anisotropy evolves with deformation, a feature that is not captured in any of the material models discussed so far. We, therefore, developed the present extended version of material 36 in an attempt to fill this void in the material library. In short, this material allows for *R*-values and stress levels in the three directions, together with similar data in biaxial and shear directions. *And*, these properties are functions of the effective plastic strain, allowing for deformation-induced anisotropy. The following is an explanation of its parameters.

The hardening curves or tables LCH00, LCH45, and LCH90 are here defined as measured stress as a function of measured plastic strain (and potentially rate) for uniaxial tension in the direction of interest, meaning as determined from experimental testing using a standard procedure. The optional biaxial and shear hardening curves LCHBI and LCHSH require some further elaboration, as we assume that a biaxial or shear test reveals that the true stress tensor in the material system, expressed as

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma & 0 \\ 0 & \pm\sigma \end{pmatrix}, \qquad \sigma \geq 0,$$

is a function of the (plastic) strain tensor

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \pm\varepsilon_2 \end{pmatrix}, \qquad \varepsilon_1 \geq 0, \qquad \varepsilon_2 \geq 0.$$

The input hardening curves are $\sigma$ as a function of $\varepsilon_1+\varepsilon_2$. The $\pm$ sign above distinguishes between the biaxial (+) and the shear (−) cases.

Moreover, the curves LCR00, LCR45, and LCR90 defining the $R$-values are functions of the measured plastic strain for uniaxial tension in the direction of interest. The $R$-values in themselves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width, $W$, and thickness, $T$, are measured as functions of strain. Then, the corresponding $R$-value is given by:

$$R_\varphi = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T}.$$

These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in directions other than the rolling direction, which can be somewhat confusing. Therefore, the von Mises work equivalent plastic strain is output as history variable #2. As for hardening, the optional biaxial and shear $R$-value curves, LCRBI and LCRSH, are defined in a special way, for which we return to the local plastic strain tensor $\boldsymbol{\varepsilon}$ as defined above. The biaxial and shear $R$-values are defined as

$$R_{b/s} = \frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}$$

and again the curves are $R_{b/s}$ as a function of $\varepsilon_1+\varepsilon_2$. Note here that the suffix $b$ assumes loading biaxially and $s$ assumes loading in shear, so the $R$-values to be defined are always positive.

The option HOSF = 0 is equivalent to the standard Barlat model with HR = 7, whose yield function can be expressed by the potential $\Phi$ as given in the remarks for *MAT_3-PARAMETER_BARLAT. HOSF = 1 allows for a "Hosford-based" effective stress in the yield function instead of using the Barlat-based effective stress. If the material and principal axes are coincident, the plastic potential $\Phi$ for HOSF = 1 can be written as

$$\Phi(\sigma) = \frac{1}{2}\left(|\sigma_1|^m + |\sigma_2|^m + |\sigma_1 - \sigma_2|^m\right) - \sigma_y^m$$

The main difference is that the Barlat-based effective stress contains the orthotropic parameters $a$, $c$, $h$, and $p$ in the yield function. Meanwhile, the Hosford-based effective stress does not contain any information about the anisotropy. For HOSF = 1, the information about direction-dependent yielding is directly obtained from the hardening curves LCH00, LCH45, and LCH90. For materials exhibiting very dissimilar $R$-values in the different material directions (such as typical aluminum extrusion), HOSF = 0 might (but does not necessarily) lead to concave yield surfaces, which, in turn, might lead to numerical instabilities under certain circumstances. HOSF = 1 tends to reduce this effect.

More information on the theoretical and numerical foundations of HOSF = 1 can be found in the paper by Andrade, Borrvall, DuBois, and Feucht, *A Hosford-based orthotropic plasticity model in LS-DYNA* (2019).

## *MAT_TRANSVERSELY_ANISOTROPIC_ELASTIC_PLASTIC_{OPTION}

This is Material Type 37. This model is for simulating sheet forming processes with an anisotropic material. This model only considers transverse anisotropy. Optionally, a load curve can specify an arbitrary dependency of stress and effective plastic strain. This plasticity model is fully iterative. -It is available only for shell and thick shell (types 1, 2, and 6) elements.

Available options include:

<BLANK>

ECHANGE

NLP_FAILURE

NLP2

The ECHANGE option allows the Young's Modulus to change during the simulation. See Remark 4.

The NLP_FAILURE option estimates failure using the Formability Index (F.I.) which accounts for the nonlinear strain paths common in metal forming applications (see Remarks 5 and 7). The option NLP is also available for *MAT_036, *MAT_125, and *MAT_226. A related keyword is *CONTROL_FORMING_STRAIN_RATIO_SMOOTH, which applies a smoothing algorithm to reduce the noise level of the strain ratio $\beta$ (in-plane minor strain increment/major strain increment) when calculating the F.I.

The NLP_FAILURE option uses effective plastic strain to calculate the onset of necking, whichassumes the necking happens in an instant. However, necking may occur over a longer duration. We developed the keyword option NLP2 to address this issue. NLP2 calculates the damage during forming and accumulates it to predict the sheet metal failure. History variable #1 when output to d3plot gives this accumulated damage.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | ETAN | R | HLCID |
|---|---|---|---|---|---|---|---|

**Card 2a.** Include this card for the ECHANGE keyword option.

| IDSCALE | EA | COE | | | | | |
|---|---|---|---|---|---|---|---|

**Card 2b.** Include this card for the NLP_FAILURE keyword option.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | ICFLD | | STRAINLT | | |

**Card 2c.** Include this card for the NLP2 keyword option.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | ICFLD | | | | |

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | R | HLCID |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus. When this value is negative, normal stresses (either from contact or applied pressure) are considered and *LOAD_SURFACE_STRESS must be used to capture the stresses. This feature is applicable to both shell element types 2 and 16. It is found in some cases this inclusion can improve accuracy.<br><br>The negative local $z$-stresses caused by the contact pressure can be viewed from d3plot files. |
| R | Anisotropic parameter $\bar{r}$, also commonly called r-bar, in sheet metal forming literature. Its interpretation is given in Remark 1.<br><br>GT.0: Standard formulation<br><br>LT.0: The anisotropic parameter is set to $\|R\|$. When $R$ is set to a negative value the algorithm is modified for better stability in sheet thickness or thinning for sheet metal forming involving high strength steels or in cases when the |

| VARIABLE | DESCRIPTION |
|---|---|
| | simulation time is long. This feature is available to both element formulations 2 and 16. See Remark 2 and Figure M37-1. |
| HLCID | Load curve ID expressing effective yield stress as a function of effective plastic strain in uniaxial tension. |

**ECHANGE Card.** Additional card included if the using the ECHANGE keyword option.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDSCALE | EA | COE | | | | | |
| Type | I | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IDSCALE | Load curve ID expressing the scale factor for the Young's modulus as a function of effective plastic strain. If the EA and COE fields are specified, this curve is unnecessary. See Remark 4. |
| EA, COE | Coefficients defining the Young's modulus with respect to the effective plastic strain, EA is $E^A$ and COE is $\zeta$. If IDSCALE is defined, these two parameters are not necessary. See Remark 4. |

**NLP_FAILURE Card.** Additional card included if using the NLP_FAILURE keyword option.

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | ICFLD | | STRAINLT | | |
| Type | | | | F | | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ICFLD | ID of a load curve of the Forming Limit Diagram (FLD) under linear strain paths (see Remark 6). In the load curve, abscissas represent minor strains while ordinates represent major strains. |

| VARIABLE | DESCRIPTION |
|---|---|
| STRAINLT | Critical strain value at which strain averaging is activated. See Remark 8. |

**NLP2 Card.** Additional card included if using NLP2 keyword option.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | ICFLD | | | | |
| Type | | | | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ICFLD | ID of a load curve of the Forming Limit Diagram (FLD) under linear strain paths (see Remark 6). In the load curve, abscissas represent minor strains while ordinates represent major strains. |

**Remarks:**

1. **Formulation.** Consider Cartesian reference axes which are parallel to the three symmetry planes of anisotropic behavior. Then, the yield function suggested by Hill [1948] can be written as:

$$F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 - 1 = 0$$

where $\sigma_{y1}$, $\sigma_{y2}$, and $\sigma_{y3}$ are the tensile yield stresses and $\sigma_{y12}$, $\sigma_{y23}$, and $\sigma_{y31}$ are the shear yield stresses. The constants $F$, $G$, $H$, $L$, $M$, and $N$ are related to the yield stress by:

$$2F = \frac{1}{\sigma_{y2}^2} + \frac{1}{\sigma_{y3}^2} - \frac{1}{\sigma_{y1}^2} \qquad 2G = \frac{1}{\sigma_{y3}^2} + \frac{1}{\sigma_{y1}^2} - \frac{1}{\sigma_{y2}^2} \qquad 2H = \frac{1}{\sigma_{y1}^2} + \frac{1}{\sigma_{y2}^2} - \frac{1}{\sigma_{y3}^2}$$

$$2L = \frac{1}{\sigma_{y23}^2} \qquad 2M = \frac{1}{\sigma_{y31}^2} \qquad 2N = \frac{1}{\sigma_{y12}^2}$$

The isotropic case of von Mises plasticity can be recovered by setting:

$$F = G = H = \frac{1}{2\sigma_y^2}$$

and

$$L = M = N = \frac{3}{2\sigma_y^2}$$

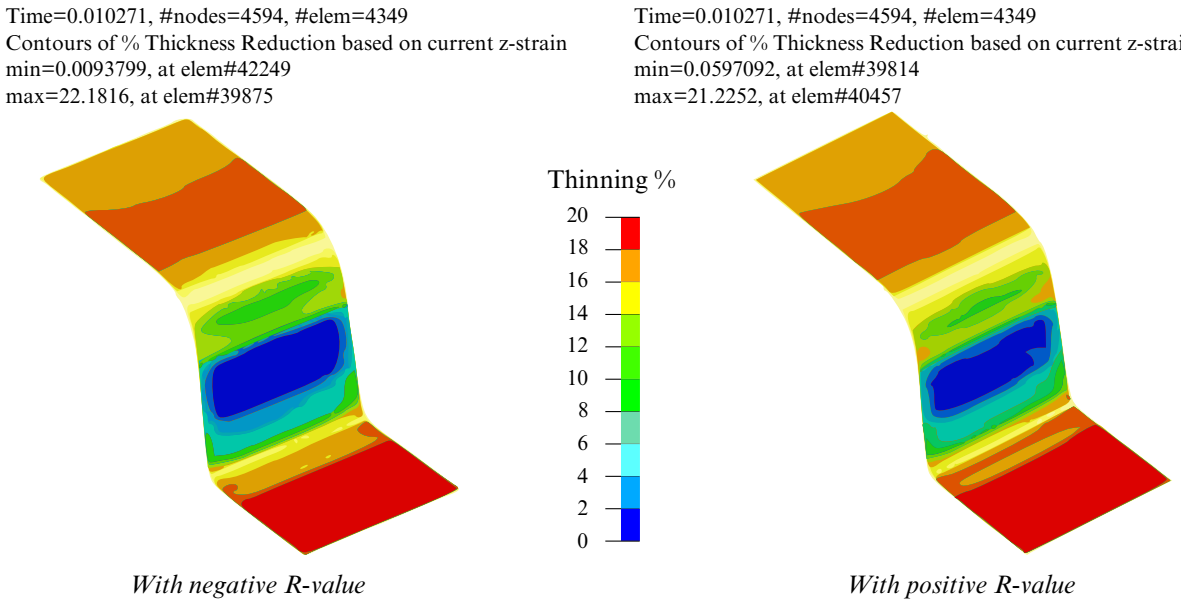



**Figure M37-1.** Thinning contour comparison.

For the particular case of transverse anisotropy, where properties do not vary in the $x_1 - x_2$ plane, the following relations hold:

$$2F = 2G = \frac{1}{\sigma_{y3}^2} \qquad 2H = \frac{2}{\sigma_y^2} - \frac{1}{\sigma_{y3}^2} \qquad N = \frac{2}{\sigma_y^2} - \frac{1}{2\sigma_{y3}^2}$$

where it has been assumed that $\sigma_{y1} = \sigma_{y2} = \sigma_y$.

Letting $K = \sigma_y / \sigma_{y3}$, the yield criteria can be written as:

$$F(\sigma) = \sigma_e = \sigma_y \ ,$$

where

$$F(\sigma) \equiv \Big[\sigma_{11}^2 + \sigma_{22}^2 + K^2\sigma_{33}^2 - K^2\sigma_{33}(\sigma_{11} + \sigma_{22}) - \left(2 - K^2\right)\sigma_{11}\sigma_{22} + 2L\sigma_y^2\left(\sigma_{23}^2 + \sigma_{31}^2\right) + 2\left(2 - \frac{1}{2}K^2\right)\sigma_{12}^2\Big]^{1/2}$$

The rate of plastic strain is assumed to be normal to the yield surface so $\dot{\varepsilon}_{ij}^p$ is found from:

$$\dot{\varepsilon}_{ij}^p = \lambda \frac{\partial F}{\partial \sigma_{ij}} \ .$$

Now consider the case of plane stress, where $\sigma_{33} = 0$. Also, define the anisotropy input parameter, $R$, as the ratio of the in-plane plastic strain rate to the out-of-plane plastic strain rate,

$$R = \frac{\dot{\varepsilon}_{22}^p}{\dot{\varepsilon}_{33}^p} \ .$$

It then follows that

$$R = \frac{2}{K^2} - 1 \ .$$

Using the plane stress assumption and the definition of $R$, the yield function may now be written as:

$$F(\sigma) = \left[\sigma_{11}^2 + \sigma_{22}^2 - \frac{2R}{R+1}\sigma_{11}\sigma_{22} + 2\frac{2R+1}{R+1}\sigma_{12}^2\right]^{1/2}.$$

2. **Anisotropic parameter R.** When the $R$ value is set to a negative value, it stabilizes the sheet thickness or thinning in sheet metal forming for some high strength types of steel or in cases where the simulation time is long. In Figure M37-1, a comparison of thinning contours is shown on a U-channel forming (one-half model) using negative and positive $R$ values. Maximum thinning on the draw wall is slight higher in the negative $R$ case than that in the positive $R$ case.

3. **Comparison to other material models.** This model and other plasticity models for shell elements, such as *MAT_PIECEWISE_LINEAR_PLASTICITY, differ in several ways. First, the yield function for plane stress does not include the transverse shear stress components which are updated elastically. Secondly, this model is always fully iterative. Consequently, when comparing results for the isotropic case where $R = 1.0$ with other isotropic model, differences in the results are expected, even though they are usually insignificant.

4. **ECHANGE.** In the original implementation, we assume that the Young's modulus is constant. However, some researchers have found that the Young's modulus decreases with respect to the increase of effective plastic strain. To accommodate this observation, we added the keyword option ECHANGE.

   We implemented two methods for defining the change of Young's modulus. For the first method, you specify a load curve to define the scale factor of the Young's modulus with respect to the effective plastic strain. The value of this scale factor should decrease from 1.0 to 0.0 with the increase of effective plastic strain. The second method uses a function as proposed by Yoshida [2003]:

   $$E = E^0 - \left(E^0 - E^A\right)[1 - \exp(-\zeta\overline{\varepsilon})].$$

5. **Nonlinear strain paths.** When the keyword option NLP_FAILURE is used, a necking failure criterion independent of strain path changes is activated. In sheet metal forming, as strain path history (plotted on in-plane major and minor strain space) of an element becomes non-linear, the position and shape of a traditional strain-based Forming Limit Diagram (FLD) changes. This option provides a simple formability index (F.I.) which remains invariant regardless of the presence of the non-linear strain paths in the model and can be used to identify if the element has reached its necking limit.

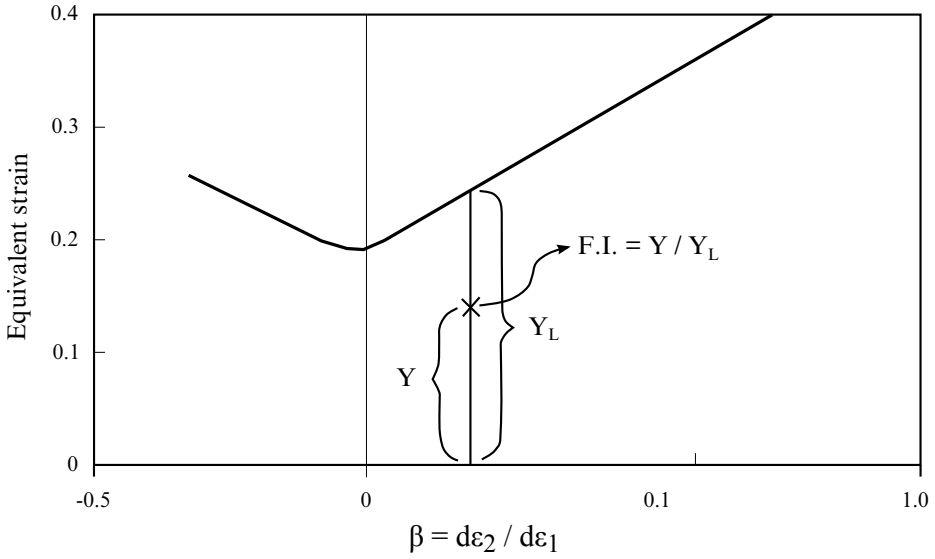



**Figure M37-2.** Calculation of F.I. based on critical equivalent strain method.

Formability index (F.I) is calculated, as illustrated in Figure M37-2, for every element in the sheet blank throughout the simulation duration. The value of F.I. is 0.0 for virgin material and reaches maximum of 1.0 when the material fails. The theoretical background can be found in two papers: *1) T.B. Stoughton, X. Zhu, "Review of Theoretical Models of the Strain-Based FLD and their Relevance to the Stress-Based FLD, International Journal of Plasticity", V. 20, Issues 8-9, P. 1463-1486, 2003; and 2) Danielle Zeng, Xinhai Zhu, Laurent B. Chappuis, Z. Cedric Xia, "A Path Independent Forming Limited Criterion for Sheet Metal Forming Simulations", 2008 SAE Proceedings, Detroit MI, April, 2008*.

6. **ICFLD.** The load curve input for ICFLD follows keyword format in *DEFINE_CURVE, with abscissas as minor strains and ordinates as major strains.

   ICFLD can also be specified using the *DEFINE_CURVE_FLC keyword where the sheet metal thickness and strain hardening value are used. Detailed usage information can be found in the manual entry for *DEFINE_CURVE_FLC.

7. **Formability index output.** The formability index is output as a history variable #1 in d3plot files. In addition to the F.I. values, starting in Revision 95599, the strain ratio $\beta$ and effective plastic strain $\bar{\varepsilon}$ are written to the d3plot database as history variables #2 and #3, respectively provided NEIPS on the second field of the first card of *DATABASE_EXTENT_BINARY is set to at least 3. The contour map of history variables can be plotted in LS-PrePost, accessible in *Post/FriComp*, under *Misc*, and by *Element*, under *Post/History*. It is suggested that variable MAXINT in *DATABASE_EXTENT_BINARY be set to the same value of as the NIP field for the *SECTION_SHELL keyword.

8. **STRAINLT.** By setting the STRAINLT field, strains (and strain ratios) can be averaged to reduce noise, which, in turn, affect the calculation of the formability index. The strain STRAINLT causes the formability index calculation to use only time averaged strains. Reasonable STRAINLT values range from $5 \times 10^{-3}$ to $10^{-2}$.

## *MAT_BLATZ-KO_FOAM

This is Material Type 38. This model is for the definition of rubber-like foams of polyurethane.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | REF | IMODEL | PR | FMOD | XLINV |
| Type | A | F | F | F | I | F | F | F |
| Default | none | none | none | 0 | 0 | 0.25 | 0.0 | 2.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| REF | Use reference geometry to initialize the stress tensor. *INITIAL_FOAM_REFERENCE_GEOMETRY defines the reference geometry.<br>EQ.0.0: Off<br>EQ.1.0: On |
| IMODEL | Model selection (optional):<br>EQ.0: Default foam model with a constant Poisson's ratio set to 0.25. See Remark 1.<br>EQ.1: General Blatz-Ko model with an interpolation factor set with FMOD and Poisson's ratio specified in field PR. See Remark 2.<br>EQ.2: Ciambella model (Ciambella and Saccomandi [2014]) with an interpolation factor set with FMOD and a Poisson's ratio specified with PR. See Remark 3. |
| PR | Poisson's ratio for IMODEL = 1 and 2. This field is optional. Only a physically valid value, meaning -1.0 < PR < 0.5, is accepted. If this value is not given or an invalid value is specified, the default |

| VARIABLE | DESCRIPTION |
|---|---|
| | value of 0.25 is used. |
| FMOD | Interpolation factor ($0.0 \le f \le 1.0$) to adjust the behavior in the General Blatz-Ko (IMODEL = 1) and Ciambella (IMODEL = 2) models. For the General Blatz-Ko model, $f = 0.0$ corresponds to a purely foam-like ($f = 0.0$) and $f = 1.0$ to a purely rubber-like response. See Remarks 2 and 3. |
| XLINV | Stretch value, $\lambda_I$, at which the material reverts from its auxetic behavior to the normal behavior for the Ciambella model. It is optional and only used by IMODEL = 2. See Remark 3. |

**Remarks:**

1. **Default compressible foam model.** The strain energy functional for the default compressible foam model (IMODEL = 0) is given by:

$$W = \frac{G}{2}\left(\frac{\text{II}}{\text{III}} + 2\sqrt{\text{III}} - 5\right) .$$

Blatz and Ko [1962] suggested this form for a 47 percent volume polyurethane foam rubber with a Poisson's ratio of 0.25. In terms of the strain invariants, I, II, and III, the second Piola-Kirchhoff stresses are given as

$$S^{ij} = G\left[\left(I\delta_{ij} - C_{ij}\right)\frac{1}{\text{III}} + \left(\sqrt{\text{III}} - \frac{\text{II}}{\text{III}}\right)C_{ij}^{-1}\right] ,$$

where $C_{ij}$ is the right Cauchy-Green strain tensor. This stress measure is transformed to the Cauchy stress, $\sigma_{ij}$, according to the relationship

$$\sigma^{ij} = \text{III}^{-1/2}F_{ik}F_{jl}S_{lk} ,$$

where $F_{ij}$ is the deformation gradient tensor.

2. **General Blatz-Ko model with an interpolation factor.** The general Blatz-Ko strain energy functional with an interpolation factor $f$ (IMODEL = 1) is given by:

$$W = \frac{Gf}{2}\left[(I-3) + \frac{1}{\beta}\left(\text{III}^{-\beta} - 1\right)\right] + \frac{G(1-f)}{2}\left[\frac{\text{II}}{\text{III}} - 3 + \frac{1}{\beta}\left(\text{III}^{\beta} - 1\right)\right]$$

where $f \in [0,1]$ is an interpolation parameter. To obtain compressible, foam-like behavior, set $f$ to 0. Set $f$ to 1 to model rubber-like behavior. The parameter $\beta$ is defined as:

$$\beta = \frac{\nu}{1-2\nu} .$$

The Cauchy stress tensor from this generalized Blatz-Ko energy functional is:

$$\sigma = G\left[\frac{f}{J} + \frac{(1-f)}{J^3}\mathrm{tr}(\mathbf{B})\right]\mathbf{B} - G\left[\frac{(1-f)}{J^3}\right]\mathbf{B}^2 + G\alpha\mathbf{I} \ ,$$

where $\alpha$ is:

$$\alpha = \frac{1}{J}\left[(1-f)\left(J^{2\beta} - \mathrm{tr}\left(\mathbf{B}^{-1}\right)\right) - fJ^{-2\beta}\right] \ ,$$

$J = \sqrt{\mathrm{III}} = \det(\mathbf{F})$, and $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ is the left Cauchy-Green tensor.

3. **Ciambella model.** The Ciambella model (IMODEL = 2) for auxetic materials uses Poisson's function (Ciambella and Saccomandi [2014]):

$$\nu = \frac{\nu_o}{[1 + \pi^2\nu_o^2(\lambda_u - 1)^2]^q} \ .$$

The following equation determines parameter $\lambda_u$:

$$J = \lambda_u\left[1 - \frac{\nu_o(\lambda_u - 1)}{[1 + \pi^2\nu_o^2(\lambda_u - 1)^2]^q}\right]^2 \ .$$

where $J$ is the $\det \mathbf{F}$. The exponent $q$ is defined as (Ciambella and Saccomandi [2014]):

$$q = \frac{1}{2}\left[1 + \frac{1}{\pi^2\nu_o^2(\lambda_I - 1)^2}\right] \ \text{with } q \geq \frac{1}{2} \ .$$

$\nu_o$ is the Poisson's ratio and $\lambda_I$ is the stretch value at which the material reverts from its auxetic behavior to the normal behavior. These material parameters are specified with XLINV.

Ciambella [2014] splits the strain energy function, so it depends on two terms as follows:

$$W(J_1, J_2, J) = W_3(J) + W_{12}(J_1, J_2) \ .$$

where $J_1$ and $J_2$ are invariants of the left Cauchy-Green tensor, $\mathbf{B}$. Here,

$$W_{12} = \frac{G}{2}f(J_1 - 3) + \frac{G}{2}(1-f)(J_2 - 2)$$

and

$$\frac{\partial W_3}{\partial J} = -\frac{G}{J}f\lambda_1^2 + \frac{G}{J}(1-f)\lambda_1^{-2}$$

where

$$\lambda_1 = 1 - \nu_o(\lambda_u - 1)\left[1 + \pi^2\nu_o^2(\lambda_u - 1)^2\right)^{-q}$$

and $f$ is an interpolation factor specified with FMOD.

## *MAT_FLD_TRANSVERSELY_ANISOTROPIC

This is Material Type 39. This model is for simulating sheet forming processes with anisotropic material. Only transverse anisotropy can be considered. Optionally, an arbitrary dependency of stress and effective plastic strain can be defined using a load curve. A Forming Limit Diagram (FLD) can be defined using a curve and is used to compute the maximum strain ratio, which can be plotted in LS-PrePost. This plasticity model is fully iterative. It is available only for shell elements and thick shell (types 1, 2, and 6) elements. Also see the Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | R | HLCID |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCFLD | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Plastic hardening modulus; see Remarks for MAT 37. |
| R | Anisotropic hardening parameter; see Remarks for MAT 37. |
| HLCID | Load curve ID defining effective stress as a function of effective plastic strain. The yield stress and hardening modulus are ignored with this option. |

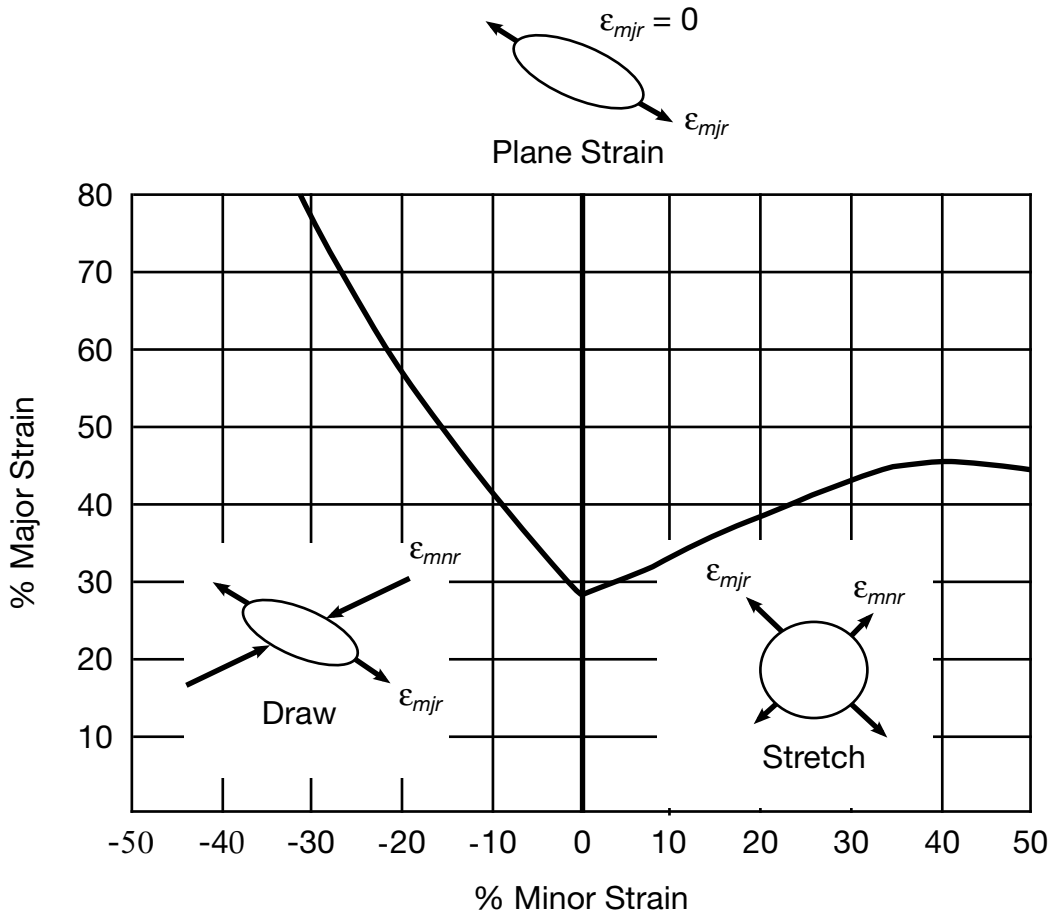



**Figure M39-1.** Forming limit diagram.

| VARIABLE | DESCRIPTION |
|---|---|
| LCFLD | Load curve ID defining the Forming Limit Diagram. Minor strains in percent are defined as abscissa values and major strains in percent are defined as ordinate values. The forming limit diagram is shown in Figure M39-1. In defining the curve list pairs of minor and major strains starting with the left most point and ending with the right most point; see *DEFINE_CURVE. |

**Remarks:**

See material model 37 for the theoretical basis. The first history variable is the maximum strain ratio:

$$\frac{\varepsilon_{\text{major}_{\text{workpiece}}}}{\varepsilon_{\text{major}_{\text{fld}}}},$$

corresponding to $\varepsilon_{\text{minor}_{\text{workpiece}}}$.

## *MAT_NONLINEAR_ORTHOTROPIC

This is Material Type 40. This model allows the definition of an orthotropic nonlinear elastic material based on a finite strain formulation with the initial geometry as the reference. Failure is optional with two failure criteria available. Optionally, stiffness proportional damping can be defined. In the stress initialization phase, temperatures can be varied to impose the initial stresses. This model is only available for shell elements, solid elements, and thick shell formulations 3, 5, and 7.

> **WARNING**: We do not recommend using this model at this time since it can be unstable especially if the stress-strain curves increase in stiffness with increasing strain.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | DT | TRAMP | ALPHA | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | 0.0 | 0.0 | 0.0 | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDA | LCIDB | EFAIL | DTFAIL | CDAMP | AOPT | MACF | ATRACK |
| Type | F | F | F | F | F | F | I | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 | 0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

Optional Card 6 (Applies to solid elements only)

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDC | LCIDAB | LCIDBC | LCIDCA | | | | |
| Type | F | F | F | F | | | | |
| Default | optional | optional | optional | optional | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. |

| VARIABLE | DESCRIPTION |
|---|---|
| EA | $E_a$, Young's modulus in $a$-direction. |
| EB | $E_b$, Young's modulus in $b$-direction. |
| EC | $E_c$, Young's modulus in $c$-direction. |
| PRBA | $\nu_{ba}$, Poisson's ratio $ba$. |
| PRCA | $\nu_{ba}$, Poisson's ratio $ca$. |
| PRCB | $\nu_{cb}$, Poisson's ratio $cb$. |
| GAB | $G_{ab}$, shear modulus $ab$. |
| GBC | $G_{bc}$, shear modulus $bc$. |
| GCA | $G_{ca}$, shear modulus $ca$. |
| DT | Temperature increment for isotropic stress initialization. This option can be used during dynamic relaxation. |
| TRAMP | Time to ramp up to the final temperature. |
| ALPHA | Thermal expansion coefficient. |
| LCIDA | Optional load curve ID defining the nominal stress versus strain along $a$-axis. Strain is defined as $\lambda_a - 1$ where $\lambda_a$ is the stretch ratio along the $a$-axis. |
| LCIDB | Optional load curve ID defining the nominal stress versus strain along $b$-axis. Strain is defined as $\lambda_b - 1$ where $\lambda_b$ is the stretch ratio along the $b$-axis. |
| EFAIL | Failure strain, $\lambda - 1$. |
| DTFAIL | Time step for automatic element erosion |
| CDAMP | Damping coefficient. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of |

| VARIABLE | DESCRIPTION |
|---|---|
| | the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4: Switch material axes $b$ and $c$ after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 5 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| ATRACK | Material $a$-axis tracking flag (shell elements only)<br>EQ.0: $a$-axis rotates with element (default)<br>EQ.1: $a$-axis also tracks deformation |
| XP, YP, ZP | Define coordinates of point $p$ for AOPT = 1 and 4. |
| A1, A2, A3 | $(a_1, a_2, a_3)$ define components of vector **a** for AOPT = 2. |
| D1, D2, D3 | $(d_1, d_2, d_3)$ define components of vector **d** for AOPT = 2. |
| V1, V2, V3 | $(v_1, v_2, v_3)$ define components of vector **v** for AOPT = 3 and 4. |
| BETA | Material angle in degrees for AOPT = 0 (shells and thick shells only) and AOPT = 3. BETA may be overridden on the element card, see *ELEMENT_SHELL_BETA. *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |
| LCIDC | Load curve ID defining the nominal stress versus strain along $c$-axis. Strain is defined as $\lambda_c - 1$ where $\lambda_c$ is the stretch ratio along the $c$-axis. |
| LCIDAB | Load curve ID defining the nominal $ab$ shear stress versus $ab$-strain in the $ab$-plane. Strain is defined as the $\sin(\gamma_{ab})$ where $\gamma_{ab}$ is the shear angle. |
| LCIDBC | Load curve ID defining the nominal $bc$ shear stress versus $bc$-strain in the $bc$-plane. Strain is defined as the $\sin(\gamma_{bc})$where $\gamma_{bc}$ is the shear angle. |
| LCIDCA | Load curve ID defining the nominal $ca$ shear stress versus $ca$-strain in the $ca$-plane. Strain is defined as the $\sin(\gamma_{ca})$ where $\gamma_{ca}$ is the shear angle. |

**Remarks:**

1. **The ATRACK field.** The initial material directions are set using AOPT and the related data. By default, the material directions in shell elements are updated each cycle based on the rotation of the 1-2 edge, or else the rotation of all edges if the invariant node numbering option is set on *CONTROL_ACCURACY. When ATRACK=1, an optional scheme is used in which the *a*-direction of the material tracks element deformation as well as rotation. For more information, see Remark 2 of *MAT_COMPOSITE_DAMAGE.

2. **Computing stresses.** The stress versus stretch curves LCIDA, LCIDB, LCIDC, LCIDAB, LCIDBC, and LCIDCA are only used to obtain the slope (stiffness) to fill up the |C| matrix and are not used directly to compute the stresses. The stresses are computed using the |C| matrix and the Green-St Venant strain tensor.

## *MAT_USER_DEFINED_MATERIAL_MODELS

These are Material Types 41 - 50. This keyword input defines material properties for a material subroutine that you provide. Isotropic, anisotropic, thermal, and hyperelastic material models with failure can be handled. See Appendix A for a discussion of user-defined features.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | MT | LMC | NHV | IORTHO | IBULK | IG |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| IVECT | IFAIL | ITHERM | IHYPER | IEOS | LMCA | EXT | EPSHV |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if IORTHO = 1 or 3.

| AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if IORTHO = 1 or 3.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | IEVTS |
|---|---|---|---|---|---|---|---|

**Card 5.** Include as many instantiations of this card as required to define LMC fields.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
|---|---|---|---|---|---|---|---|

**Card 6.** Include as many instantiations of this card as required to define LMCA fields.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | MT | LMC | NHV | IORTHO | IBULK | IG |
| Type | A | F | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| MT | User material type (41 - 50 inclusive). A number between 41 and 50 must be chosen. If MT < 0, subroutine `rwumat` in dyn21.f is called, where the material parameter reading can be modified. |
| | **WARNING**: If two or more materials in an input deck share the same MT value, those materials must have the same values of other variables on Cards 1 and 2 except for MID and RO. |
| LMC | Length of material constant array which is equal to the number of material constants to be input. See Remark 2. |
| NHV | Number of history variables to be stored; see Appendix A. When the model is to be used with an equation of state, NHV must be increased by 4 to allocate the storage required by the equation of state. |
| IORTHO/ ISPOT | Orthotropic/spot weld thinning flag:<br>EQ.0: If the material is not orthotropic and is not used with spot weld thinning<br>EQ.1: If the material is orthotropic<br>EQ.2: If material is used with spot weld thinning<br>EQ.3: If material is orthotropic and used with spot weld thinning |
| IBULK | Address of bulk modulus in material constants array; see Appendix A. |
| IG | Address of shear modulus in material constants array; see Appendix A. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IVECT | IFAIL | ITHERM | IHYPER | IEOS | LMCA | EXT | EPSHV |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| IVECT | Vectorization flag:<br>EQ.0: Off<br>EQ.1: On. A vectorized user subroutine must be supplied. |
| IFAIL | Failure flag.<br>EQ.0: No failure<br>EQ.1: Allows failure of shell and solid elements<br>LT.0: \|IFAIL\| is the address of NUMINT in the material constants array. NUMINT is defined as the number of failed integration points that will trigger element deletion. This option applies only to shell and solid elements (release 5 of version 971). |
| ITHERM | Temperature flag:<br>EQ.0: Off<br>EQ.1: Compute element center temperature.<br>EQ.2: Compute element integration point temperature. |
| IHYPER | Deformation gradient flag (see Appendix A):<br>EQ.0: Do not compute deformation gradient.<br>EQ.-1: Same as 1, except if IORTHO = 1 or 3, the deformation gradient is in the global coordinate system.<br>EQ.-10: Same as -1, except that this will enforce full integration for elements -1, -2 and 2.<br>EQ.1: Compute deformation gradient for bricks and shells. If IORTHO = 1 or 3, the deformation gradient is in the local coordinate system instead of the global coordinate system. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.10: Same as 1, except that this will enforce full integration for elements -1, -2 and 2.<br>EQ.3: Compute deformation gradient for shells from the nodal coordinates in the global coordinate system. |
| IEOS | Equation of state flag:<br>EQ.0: Off<br>EQ.1: On |
| LMCA | Length of additional material constant array |
| EXT | Flag to call external user material routines from other codes. See the file dyn21extumat.F for documentation. |
| EPSHV | Indicates which history variable is used to store effective plastic strain (if used). EPSHV is used in conjunction with EXT ≠ 0 to facilitate post-processing. |

**Orthotropic Card 1.** Additional card for IORTHO = 1 or 3.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
| Type | F | I | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see *MAT_002 for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle BETA<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDI- |

| VARIABLE | DESCRIPTION |
|---|---|
| | NATE_VECTOR. |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal. |
| | EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *p*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| MACF | Material axes change flag for brick elements for quick changes:<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b*<br>EQ.3: Switch material axes *a* and *c*<br>EQ.4: Switch material axes *b* and *c* |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

**Orthotropic Card 2.** Additional card for IORTHO = 1 or 3.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | IEVTS |
| Type | F | F | F | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. |
| IEVTS | Address of $E_a$ for orthotropic material with thick shell formulation 5 (see Remark 4) |

Define LMC material parameters using 8 parameters per card. See Remark 2.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |

Define LMCA material parameters using 8 parameters per card.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | First material parameter. |
| P2 | Second material parameter. |
| P3 | Third material parameter. |
| P4 | Fourth material parameter. |
| ⋮ | ⋮ |
| PLMC | LMC$^{th}$ material parameter. |

**Remarks:**

1. **Cohesive elements.** Material models for the cohesive element (solid element type 19) uses the first two *material parameters* to set flags in the *element formulation*.

a) *P1.* The P1 field controls how the density is used to calculate the mass when determining the tractions at mid-surface (tractions are calculated on a surface midway between the surfaces defined by nodes 1-2-3-4 and 5-6-7-8). If P1 is set to 1.0, then the density is per unit area of the mid-surface instead of per unit volume. Note that the cohesive element formulation permits the element to have zero or negative volume.

b) *P2.* The second parameter, P2, specifies the number of integration points (one to four) that are required to fail for the element to fail. If it is zero, the element will not fail regardless of IFAIL. The recommended value for P2 is 1.

c) *Other Parameters.* The cohesive element only uses MID, RO, MT, LMC, NHV, IFAIL and IVECT in addition to the material parameters.

d) *Appendix R.* See Appendix R for the specifics of the `umat` subroutine requirements for the cohesive element.

2. **Material constants.** If IORTHO = 0, LMC must be ≤ 48. If IORTHO = 1, LMC must be ≤ 40. If more material constants are needed, LMCA may be used to create an additional material constant array. There is no limit on the size of LMCA.

3. **Spot weld thinning.** If the user-defined material is used for beam or brick element spot welds that are tied to shell elements, and SPOTHIN > 0 on *CONTROL_CONTACT, then spot weld thinning will be done for those shells if ISPOT = 2. Otherwise, it will not be done.

4. **Thick shell formulation 5.** IEVTS is optional and is used only by thick shell formulation 5. It points to the position of $E_a$ in the material constants array. Following $E_a$, the next 5 material constants must be $E_b$, $E_c$, $\nu_{ba}$, $\nu_{ca}$, and $\nu_{cb}$. This data enables thick shell formulation 5 to calculate an accurate thickness strain, otherwise the thickness strain will be based on the elastic constants pointed to by IBULK and IG.

## *MAT_BAMMAN

This is Material Type 51. It allows the modeling of temperature and rate dependent plasticity with a fairly complex model that has many input parameters [Bammann 1989].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | T | HC | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C17 | C18 | A1 | A2 | A4 | A5 | A6 | KAPPA |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus (psi) |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio |
| T | Initial temperature (°R, degrees Rankine) |
| HC | Heat generation coefficient (°R/psi) |
| C1 | psi |
| C2 | °R |
| C3 | psi |
| C4 | °R |
| C5 | $s^{-1}$ |
| C6 | °R |
| C7 | 1/psi |
| C8 | °R |
| C9 | psi |
| C10 | °R |
| C11 | 1/psi-s |
| C12 | °R |
| C13 | 1/psi |
| C14 | °R |
| C15 | psi |
| C16 | °R |
| C17 | 1/psi-s |
| C18 | °R |
| A1 | $\alpha_1$, initial value of internal state variable 1 |
| A2 | $\alpha_2$, initial value of internal state variable 2. Note: $\alpha_3 = -(\alpha_1 + \alpha_2)$ |
| A4 | $\alpha_4$, initial value of internal state variable 3 |

| VARIABLE | DESCRIPTION |
|---|---|
| A5 | $\alpha_5$, initial value of internal state variable 4 |
| A6 | $\alpha_6$, initial value of internal state variable 5 |
| KAPPA | $\kappa$, initial value of internal state variable 6 |

*Unit Conversion Table*

| sec × psi × $^{o}$R | sec × MPa × $^{o}$R | sec × MPA × $^{o}$K |
|---|---|---|
| $C_1$ | × $1/145$ | × $1/145$ |
| $C_2$ | — | × $5/9$ |
| $C_3$ | × $1/145$ | × $1/145$ |
| $C_4$ | — | × $5/9$ |
| $C_5$ | — | — |
| $C_6$ | — | × $5/9$ |
| $C_7$ | × 145 | × 145 |
| $C_8$ | — | × $5/9$ |
| $C_9$ | × $1/145$ | × $1/145$ |
| $C_{10}$ | — | × $5/9$ |
| $C_{11}$ | × 145 | × 145 |
| $C_{12}$ | — | × $5/9$ |
| $C_{13}$ | × 145 | × 145 |
| $C_{14}$ | — | × $5/9$ |
| $C_{15}$ | × $1/145$ | × $1/145$ |
| $C_{16}$ | — | × $5/9$ |
| $C_{17}$ | × 145 | × 145 |
| $C_{18}$ | — | × $5/9$ |
| C0 = HC | × 145 | × $(145)(5/9)$ |
| E | × $1/145$ | × $1/145$ |
| $\upsilon$ | — | — |
| T | — | × $5/9$ |

**Remarks:**

The kinematics associated with the model are discussed in references [Hill 1948, Bammann and Aifantis 1987, Bammann 1989]. The description below is taken nearly verbatim from Bammann [1989].

With the assumption of linear elasticity, we can write:

$$\overset{o}{\boldsymbol{\sigma}} = \lambda \, \mathrm{tr}(\mathbf{D}^e)\mathbf{1} + 2\mu\mathbf{D}^e \ ,$$

where the Cauchy stress $\boldsymbol{\sigma}$ is convected with the elastic spin $\mathbf{W}^e$ as,

$$\overset{o}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} - \mathbf{W}^e\boldsymbol{\sigma} + \boldsymbol{\sigma}\mathbf{W}^e \ .$$

This is equivalent to writing the constitutive model with respect to a set of directors whose direction is defined by the plastic deformation [Bammann and Aifantis 1987, Bammann and Johnson 1987]. Decomposing both the skew symmetric and symmetric parts of the velocity gradient into elastic and plastic parts, we write for the elastic stretching $\mathbf{D}^e$ and the elastic spin $\mathbf{W}^e$,

$$\mathbf{D}^e = \mathbf{D} - \mathbf{D}^p - \mathbf{D}^{th}, \qquad \mathbf{W}^e = \mathbf{W} = \mathbf{W}^p \ .$$

Within this structure it is now necessary to prescribe an equation for the plastic spin $\mathbf{W}^p$ in addition to the normally prescribed flow rule for $\mathbf{D}^p$ and the stretching due to the thermal expansion $\mathrm{D}^{th}$. As proposed, we assume a flow rule of the form,

$$\mathbf{D}^p = f(T)\sinh\left[\frac{|\boldsymbol{\xi}| - \kappa - Y(T)}{V(T)}\right]\frac{\boldsymbol{\xi}'}{|\boldsymbol{\xi}'|} \ .$$

where $T$ is the temperature, $\kappa$ is the scalar hardening variable, and $\boldsymbol{\xi}'$ is the difference between the deviatoric Cauchy stress $\boldsymbol{\sigma}'$ and the tensor variable $\boldsymbol{\alpha}'$,

$$\boldsymbol{\xi}' = \boldsymbol{\sigma}' - \boldsymbol{\alpha}' \ ,$$

and $f(T)$, $Y(T)$, and $V(T)$ are scalar functions whose specific dependence upon the temperature is given below. Assuming isotropic thermal expansion and introducing the expansion coefficient $\dot{A}$, the thermal stretching can be written,

$$\mathbf{D}^{th} = \dot{A}\dot{T}\mathbf{1}$$

The evolution of the internal variables $\boldsymbol{\alpha}$ and $\boldsymbol{\kappa}$ are prescribed in a hardening minus recovery format as,

$$\overset{o}{\boldsymbol{\alpha}} = h(T)\mathbf{D}^p - [r_d(T)|\mathbf{D}^p| + r_s(T)]|\boldsymbol{\alpha}|\boldsymbol{\alpha}$$
$$\dot{\boldsymbol{\kappa}} = H(T)\mathbf{D}^p - [R_d(T)|\mathbf{D}^p| + R_s(T)]\boldsymbol{\kappa}^2$$

where $h$ and $H$ are the hardening moduli, $r_s(T)$ and $R_s(T)$ are scalar functions describing the diffusion controlled 'static' or 'thermal' recovery, and $r_d(T)$ and $R_d(T)$ are the functions describing dynamic recovery.

If we assume that $\mathbf{W}^p = 0$, we recover the Jaumann stress rate which results in the prediction of an oscillatory shear stress response in simple shear when coupled with a Prager kinematic hardening assumption [Johnson and Bammann 1984]. Alternatively, we can choose,

$$\mathbf{W}^p = \mathbf{R}^T\dot{\mathbf{U}}\mathbf{U}^{-1}\mathbf{R},$$

which recovers the Green-Naghdi rate of Cauchy stress and has been shown to be equivalent to Mandel's isoclinic state [Bammann and Aifantis 1987]. The model employing this

rate allows a reasonable prediction of directional softening for some materials, but in general under-predicts the softening and does not accurately predict the axial stresses which occur in the torsion of the thin walled tube.

The final equation necessary to complete our description of high strain rate deformation is one which allows us to compute the temperature change during the deformation. In the absence of a coupled thermo-mechanical finite element code we assume adiabatic temperature change and follow the empirical assumption that 90 - 95% of the plastic work is dissipated as heat. Hence,

$$\dot{T} = \frac{.9}{\rho C_v}(\boldsymbol{\sigma} \cdot \mathbf{D}^p),$$

where $\rho$ is the density of the material and $C_v$ is the specific heat.

In terms of the input parameters, the functions defined above become:

$$V(T) = \mathrm{C1}\exp(-\mathrm{C2}/T) \qquad r_s(T) = \mathrm{C11}\exp(-\mathrm{C12}/T)$$
$$Y(T) = \mathrm{C3}\exp(\mathrm{C4}/T) \qquad R_d(T) = \mathrm{C13}\exp(-\mathrm{C14}/T)$$
$$f(T) = \mathrm{C5}\exp(-\mathrm{C6}/T) \qquad H(T) = \mathrm{C15}\exp(\mathrm{C16}/T)$$
$$r_d(T) = \mathrm{C7}\exp(-\mathrm{C8}/T) \qquad R_s(T) = \mathrm{C17}\exp(-\mathrm{C18}/T)$$
$$h(T) = \mathrm{C9}\exp(\mathrm{C10}/T)$$

and the heat generation coefficient is

$$\mathrm{HC} = \frac{0.9}{\rho C_v}.$$

## *MAT_BAMMAN_DAMAGE

This is Material Type 52. This is an extension of model 51 which includes the modeling of damage. See Bamman et al. [1990].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | T | HC | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C9 | C10 | C11 | C12 | C13 | C14 | C15 | C16 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C17 | C18 | A1 | A2 | A3 | A4 | A5 | A6 |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | N | D0 | FS | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus (psi) |
| PR | Poisson's ratio |
| T | Initial temperature (°R, degrees Rankine) |
| HC | Heat generation coefficient ($^{\circ R}/_{psi}$) |
| C1 | Psi |
| C2 | °R |
| C3 | Psi |
| C4 | °R |
| C5 | $1/s$ |
| C6 | °R |
| C7 | $1/psi$ |
| C8 | °R |
| C9 | Psi |
| C10 | °R |
| C11 | $1/psi\text{-}s$ |
| C12 | °R |
| C13 | $1/psi$ |
| C14 | °R |
| C15 | psi |
| C16 | °R |

| VARIABLE | DESCRIPTION |
|---|---|
| C17 | $1/\text{psi-s}$ |
| C18 | °R |
| A1 | $\alpha_1$, initial value of internal state variable 1 |
| A2 | $\alpha_2$, initial value of internal state variable 2 |
| A3 | $\alpha_3$, initial value of internal state variable 3 |
| A4 | $\alpha_4$, initial value of internal state variable 4 |
| A5 | $\alpha_5$, initial value of internal state variable 5 |
| A6 | $\alpha_6$, initial value of internal state variable 6 |
| N | Exponent in damage evolution |
| D0 | Initial damage (porosity) |
| FS | Failure strain for erosion |

**Remarks:**

The evolution of the damage parameter, $\phi$ is defined by Bammann et al. [1990]

$$\dot{\phi} = \beta \left[ \frac{1}{(1-\phi)^N} - (1-\phi) \right]^{|D^p|}$$

in which

$$\beta = \sinh \left[ \frac{2(2N-1)p}{(2N-1)\overline{\sigma}} \right] ,$$

where $p$ is the pressure and $\overline{\sigma}$ is the effective stress.

## *MAT_CLOSED_CELL_FOAM

This is Material Type 53. This material models low density, closed cell polyurethane foam. It is for simulating impact limiters in automotive applications. The effect of the confined air pressure is included with the air being treated as an ideal gas. The general behavior is isotropic with uncoupled components of the stress tensor.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | A | B | C | P0 | PHI |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAMMA0 | LCID | | | | | | |
| Type | F | I | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| A | $a$, factor for yield stress definition; see Remarks below. |
| B | $b$, factor for yield stress definition; see Remarks below. |
| C | $c$, factor for yield stress definition; see Remarks below. |
| P0 | Initial foam pressure, $p_0$ |
| PHI | Ratio of foam to polymer density, $\phi$ |
| GAMMA0 | Initial volumetric strain,$\gamma_0$. The default is zero. |
| LCID | Optional load curve defining the von Mises yield stress as a function of $-\gamma$. If the load curve ID is given, the yield stress is taken from the curve and the constants $a$, $b$, and $c$ are not needed. The |

| VARIABLE | DESCRIPTION |
|---|---|
| | load curve is defined in the positive quadrant, that is, positive values of $\gamma$ are defined as negative values on the abscissa. |

**Remarks:**

A rigid, low density, closed cell, polyurethane foam model developed at Sandia Laboratories [Neilsen, Morgan and Krieg 1987] has been recently implemented for modeling impact limiters in automotive applications. A number of such foams were tested at Sandia and reasonable fits to the experimental data were obtained.

In some respects this model is similar to the crushable honeycomb model type 26 in that the components of the stress tensor are uncoupled until full volumetric compaction is achieved. However, unlike the honeycomb model this material possesses no directionality but includes the effects of confined air pressure in its overall response characteristics.

$$\sigma_{ij} = \sigma_{ij}^{\mathrm{sk}} - \delta_{ij}\sigma^{\mathrm{air}} \ ,$$

where $\sigma_{ij}^{\mathrm{sk}}$ is the skeletal stress and $\sigma^{\mathrm{air}}$ is the air pressure. $\sigma^{\mathrm{air}}$ is computed from the equation:

$$\sigma^{\mathrm{air}} = -\frac{p_0\gamma}{1+\gamma-\phi} \ ,$$

where $p_0$ is the initial foam pressure, usually taken as the atmospheric pressure, and $\gamma$ defines the volumetric strain

$$\gamma = V - 1 + \gamma_0 \ .$$

Here, $V$ is the relative volume, defined as the ratio of the current volume to the initial volume, and $\gamma_0$ is the initial volumetric strain, which is typically zero. The yield condition is applied to the principal skeletal stresses, which are updated independently of the air pressure. We first obtain the skeletal stresses:

$$\sigma_{ij}^{\mathrm{sk}} = \sigma_{ij} + \sigma_{ij}\sigma^{\mathrm{air}}$$

and compute the trial stress, $\sigma_{ij}^{\mathrm{skt}}$

$$\sigma_{ij}^{\mathrm{skt}} = \sigma_{ij}^{\mathrm{sk}} + E\,\dot{\varepsilon}_{ij}\,\Delta t \ ,$$

where $E$ is Young's modulus. Since Poisson's ratio is zero, the update of each stress component is uncoupled and $2G = E$ where $G$ is the shear modulus. The yield condition is applied to the principal skeletal stresses such that, if the magnitude of a principal trial stress component, $\sigma_i^{\mathrm{skt}}$, exceeds the yield stress, $\sigma_y$, then

$$\sigma_i^{\mathrm{sk}} = \min\left(\sigma_y, \left|\sigma_i^{\mathrm{skt}}\right|\right)\frac{\sigma_i^{\mathrm{skt}}}{\left|\sigma_i^{\mathrm{skt}}\right|} \ .$$

The yield stress is defined by

$$\sigma_y = a + b(1 + c\gamma) \ ,$$

where $a$, $b$, and $c$ are user defined input constants and $\gamma$ is the volumetric strain as defined above. After scaling the principal stresses they are transformed back into the global system and the final stress state is computed

$$\sigma_{ij} = \sigma_{ij}^{\mathrm{sk}} - \delta_{ij}\sigma^{\mathrm{air}}.$$

## *MAT_ENHANCED_COMPOSITE_DAMAGE

These are Material Types 54 - 55 which are enhanced versions of the composite model material type 22. Arbitrary orthotropic materials, such as unidirectional layers in composite shell structures, can be defined. Optionally, various types of failure can be specified following either the suggestions of [Chang and Chang 1987b] or [Tsai and Wu 1971]. In addition, special measures are taken for failure under compression. See [Matzenmiller and Schweizerhof 1991].

By using the user-defined integration rule, see *INTEGRATION_SHELL, the constitutive constants can vary through the shell thickness. For all shells, except the DKT formulation, laminated shell theory can be activated to properly model the transverse shear deformation. Lamination theory is applied to correct for the assumption of a uniform constant shear strain through the thickness of the shell.

For sandwich shells where the outer layers are much stiffer than the inner layers, the response will tend to be too stiff unless lamination theory is used. To turn on lamination theory, see *CONTROL_SHELL. A damage model for transverse shear strain to model interlaminar shear failure is available. The definition of minimum stress limits is available for thin/thick shells and solids.

**NOTE:** *MAT_054 is supported for shell, solid (see Remark 7), and thick shell elements. *MAT_055 is only supported for shell elements and thick shell formulations 1, 2, and 6. If *MAT_055 is used for solids, LS-DYNA automatically switches to *MAT_054.

**NOTE:** We recommend using *MAT_054 over *MAT_055.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | (KF) | AOPT | 2WAY | TI | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MANGLE | |
|---|---|---|---|---|---|---|---|

**Card 4a.** Include this card if the material is *MAT_054 and DFAILT ≠ 0.0.

| V1 | V2 | V3 | D1 | D2 | D3 | DFAILM | DFAILS |
|---|---|---|---|---|---|---|---|

**Card 4b.** Include this card if Card 4a is not included, meaning the material is *MAT_055 or the material is *MAT_054 with DFAILT = 0.0.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Card 5a.** Include this card if the material is *MAT_054.

| TFAIL | ALPH | SOFT | FBRT | YCFAC | DFAILT | DFAILC | EFS |
|---|---|---|---|---|---|---|---|

**Card 5b.** Include this card if the material is *MAT_055.

| TFAIL | ALPH | SOFT | FBRT | | | | |
|---|---|---|---|---|---|---|---|

**Card 6a.** Include this card if the 2WAY flag is 0.

| XC | XT | YC | YT | SC | CRIT | BETA | |
|---|---|---|---|---|---|---|---|

**Card 6b.** Include this card if the 2WAY flag is 1.

| XC | XT | YC | YT | SC | CRIT | BETA | |
|---|---|---|---|---|---|---|---|

**Card 7.** Only include this card for *MAT_054 (CRIT = 54).

| PFL | EPSF | EPSR | TSMD | SOFT2 | | | |
|---|---|---|---|---|---|---|---|

**Card 8a.** Only include this card for *MAT_054 (CRIT = 54) and 2WAY = 0.

| SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS | NCYRED | SOFTG | |
|---|---|---|---|---|---|---|---|

**Card 8b.** Only include this card for *MAT_054 (CRIT = 54) and 2WAY = 1.

| SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS | NCYRED | SOFTG | |
|---|---|---|---|---|---|---|---|

**Card 9.** Only include this card for *MAT_054 (CRIT = 54).

| LCXC | LCXT | LCYC | LCYT | LCSC | DT | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |
| Remarks | | | | | | 6 | 6 | 6 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus - longitudinal direction |
| EB | $E_b$, Young's modulus - transverse direction |
| EC | $E_c$, Young's modulus - normal direction |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | (KF) | AOPT | 2WAY | TI | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus *ab* |
| GBC | $G_{bc}$, shear modulus *bc* |
| GCA | $G_{ca}$, shear modulus *ca* |

| VARIABLE | DESCRIPTION |
|---|---|
| (KF) | Bulk modulus of failed material (not used) |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle MANGLE.<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (MANGLE) from a line in the plane of the element defined by the cross product of the vector, **v**, with the element normal.<br>EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *p*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| 2WAY | Flag to turn on 2-way fiber action:<br>EQ.0.0: Standard unidirectional behavior, meaning fibers run only in the *a*-direction<br>EQ.1.0: 2-way fiber behavior, meaning fibers run in both the *a*- and *b*-directions. The meaning of the fields DFAILT, DFAILC, YC, YT, SLIMT2 and SLIMC2 are altered if this flag is set. This option is only available for *MAT_054 using thin shells. |
| TI | Flag to turn on transversal isotropic behavior for *MAT_054 solid elements. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Standard unidirectional behavior |
| | EQ.1.0: Transversal isotropic behavior (see Remark 5) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MANGLE | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MANGLE | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. MANGLE may be overridden on the element card; see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. |

This card is included if the material is *MAT_054 and DFAILT (see Card 5a) is nonzero.

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | DFAILM | DFAILS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Define components of vector **v** for AOPT = 3. |
| D1, D2, D3 | Define components of vector **d** for AOPT = 2. |
| DFAILM | Maximum strain for matrix straining in tension or compression (active only for *MAT_054 and only if DFAILT > 0). The layer in the element is completely removed after the maximum strain in the matrix direction is reached. The input value is always positive. |
| DFAILS | Maximum tensorial shear strain (active only for *MAT_054 and only if DFAILT > 0). The layer in the element is completely removed after the maximum shear strain is reached. The input value |

| VARIABLE | DESCRIPTION |
|---|---|
| | is always positive. |

This card is included if Card 4a is not included.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Define components of vector **v** for AOPT = 3. |
| D1, D2, D3 | Define components of vector **d** for AOPT = 2. |

This card is included if the material is *MAT_054.

| Card 5a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TFAIL | ALPH | SOFT | FBRT | YCFAC | DFAILT | DFAILC | EFS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TFAIL | Time step size criteria for element deletion:<br>LE.0.0: No element deletion by time step size. The crashfront algorithm only works if TFAIL is set to a value greater than zero.<br>GT.0.0.and.LE.0.1: Element is deleted when its time step is smaller than the given value.<br>GT.0.1: Element is deleted when the quotient of the actual time step and the original time step drops below the given value. |
| ALPH | Shear stress parameter for the nonlinear term; see *MAT_022. |
| SOFT | Softening reduction factor for material strength in crashfront elements (default = 1.0). TFAIL must be greater than zero to activate |

| VARIABLE | DESCRIPTION |
|---|---|
| | this option. Crashfront elements are elements that are direct neighbors of failed (deleted) elements. See Remark 2. |
| FBRT | Softening for fiber tensile strength:<br>EQ.0.0: Tensile strength = XT<br>GT.0.0: Tensile strength = XT, reduced to XT × FBRT after failure has occurred in compressive matrix mode |
| YCFAC | Reduction factor for compressive fiber strength after matrix compressive failure. The compressive strength in the fiber direction after compressive matrix failure is reduced to:<br>$X_c$ = YCFAC × $Y_c$, (default: YCFAC = 2.0) |
| DFAILT | Maximum strain for fiber tension (*MAT_054 only). A value of 1 is 100% tensile strain. The layer in the element is completely removed after the maximum tensile strain in the fiber direction is reached. If a nonzero value is given for DFAILT (recommended), a nonzero, negative value must also be provided for DFAILC.<br><br>If the 2-way fiber flag is set, then DFAILT is the fiber tensile failure strain in the *a* and *b* directions. |
| DFAILC | Maximum strain for fiber compression. A value of -1 is 100% compression strain. The layer in the element is completely removed after the maximum compressive strain in the fiber direction is reached. The input value should be negative and is required if DFAILT > 0.<br><br>If the 2-way fiber flag is set, then DFAILC is the fiber compressive failure strain in the *a* and *b* directions. |
| EFS | Effective failure strain |

This card is included if the material is *MAT_055.

| Card 5b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TFAIL | ALPH | SOFT | FBRT | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TFAIL | Time step size criteria for element deletion:<br>LE.0.0: No element deletion by time step size. The crashfront algorithm only works if TFAIL is set to a value greater than zero.<br>GT.0.0.and.LE.0.1: Element is deleted when its time step is smaller than the given value.<br>GT.0.1: Element is deleted when the quotient of the actual time step and the original time step drops below the given value. |
| ALPH | Shear stress parameter for the nonlinear term; see *MAT_022. |
| SOFT | Softening reduction factor for material strength in crashfront elements (default = 1.0). TFAIL must be greater than zero to activate this option. Crashfront elements are elements that are direct neighbors of failed (deleted) elements. |
| FBRT | Softening for fiber tensile strength:<br>EQ.0.0: Tensile strength = XT<br>GT.0.0: Tensile strength = XT, reduced to XT × FBRT after failure has occurred in compressive matrix mode |

This card is included if 2WAY = 0.

| Card 6a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SC | CRIT | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength (absolute value is used):<br>GE.0.0: Poisson effect (PRBA) after failure is active.<br>LT.0.0: Poisson effect after failure is not active, meaning PRBA = 0. |
| XT | Longitudinal tensile strength; see Material Formulation below. |

| VARIABLE | DESCRIPTION |
|---|---|
| YC | Transverse compressive strength, *b*-axis (positive value). See Material Formulation below. |
| YT | Transverse tensile strength, *b*-axis. See Material Formulation below. |
| SC | Shear strength, *ab*-plane. See the Material Formulation below. |
| CRIT | Failure criterion (material number):<br>EQ.54.0: Chang-Chang criterion for matrix failure (as *MAT_022) (default),<br>EQ.55.0: Tsai-Wu criterion for matrix failure. |
| BETA | Weighting factor for shear term in tensile fiber mode. 0.0 ≤ BETA ≤ 1.0. |

This card is included if 2WAY = 1 (CRIT must be 54 in this case).

| Card 6b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SC | CRIT | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength (absolute value is used):<br>GE.0.0: Poisson effect (PRBA) after failure is active.<br>LT.0.0: Poisson effect after failure is not active, meaning PRBA = 0. |
| XT | Longitudinal tensile strength; see Material Formulation below. |
| YC | Fiber compressive failure stress in the *b*-direction. See Material Formulation below. |
| YT | Fiber tensile failure stress in the *b*-direction. See Material Formulation below. |
| SC | Shear strength, *ab*-plane. See the Material Formulation below. |

| VARIABLE | DESCRIPTION |
|---|---|
| CRIT | Failure criterion (material number):<br>EQ.54.0: Chang-Change criterion for matrix failure (as *MAT_-022) (default),<br>EQ.55.0: Tsai-Wu criterion for matrix failure. |
| BETA | Weighting factor for shear term in tensile fiber mode. 0.0 ≤ BETA ≤ 1.0. |

**Optional Card 7 (only for CRIT = 54).** This card is included for *MAT_054 only.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PFL | EPSF | EPSR | TSMD | SOFT2 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PFL | Percentage of layers which must fail before crashfront is initiated (thin and thick shells only). For example, if \|PFL\| = 80.0, then 80% of the layers must fail before strengths are reduced in neighboring elements. Default: all layers must fail. The sign of PFL determines how many in-plane integration points must fail for a single layer to fail:<br>GT.0.0: A single layer fails if 1 in-plane IP fail.<br>LT.0.0: A single layer fails if 4 in-plane IPs fail. |
| EPSF | Damage initiation transverse shear strain |
| EPSR | Final rupture transverse shear strain<br>LT.0.0: \|EPSR\| is final rupture transverse shear strain. In addition, the element erodes if transverse shear damage reaches TSMD. |
| TSMD | Transverse shear maximum damage (default = 0.90) |
| SOFT2 | Optional "orthogonal" softening reduction factor for material strength in crashfront elements (default = 1.0). See Remark 2 (thin and thick shells only). |

**Optional Card 8 (only for CRIT = 54).** This card is included for *MAT_054 only and 2WAY = 0.

| Card 8a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS | NCYRED | SOFTG | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| SLIMT1 | Factor to determine the minimum stress limit after stress maximum (fiber tension). Similar to *MAT_058. |
| SLIMC1 | Factor to determine the minimum stress limit after stress maximum (fiber compression). Similar to *MAT_058. |
| SLIMT2 | Factor to determine the minimum stress limit after stress maximum (matrix tension). Similar to *MAT_058. |
| SLIMC2 | Factor to determine the minimum stress limit after stress maximum (matrix compression). Similar to *MAT_058. |
| SLIMS | Factor to determine the minimum stress limit after stress maximum (shear). Similar to *MAT_058. |
| NCYRED | Number of cycles for stress reduction from maximum to minimum for DFAILT > 0. |
| SOFTG | Softening reduction factor for transverse shear moduli GBC and GCA in crashfront elements (thin and thick shells). Default = 1.0. |

**Optional Card 8 (only for CRIT = 54).** This card is included for *MAT_054 only and 2WAY = 1.

| Card 8b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS | NCYRED | SOFTG | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| SLIMT1 | Factor to determine the minimum stress limit after stress maximum (fiber tension) in the *a*-direction. Similar to *MAT_058. |
| SLIMC1 | Factor to determine the minimum stress limit after stress maximum (fiber compression) in the *a*-direction. Similar to *MAT_058. |
| SLIMT2 | Factor to determine the minimum stress limit after tensile failure stress is reached in the *b* fiber direction |
| SLIMC2 | Factor to determine the minimum stress limit after compressive failure stress is reached in the *b* fiber direction |
| SLIMS | Factor to determine the minimum stress limit after stress maximum (shear). Similar to *MAT_058. |
| NCYRED | Number of cycles for stress reduction from maximum to minimum for DFAILT > 0. |
| SOFTG | Softening reduction factor for transverse shear moduli GBC and GCA in crashfront elements (thin and thick shells). Default = 1.0. |

**Optional Card 9 (only for CRIT = 54).** This card is included for *MAT_054 only.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCXC | LCXT | LCYC | LCYT | LCSC | DT | | |
| Type | I | I | I | I | I | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCXC | Load curve ID for XC as a function of strain rate (XC is ignored with this option) |
| LCXT | Load curve ID for XT as a function strain rate (XT is ignored with this option) |
| LCYC | Load curve ID for YC as a function of strain rate (YC is ignored with this option) |
| LCYT | Load curve ID for YT as a function of strain rate (YT is ignored with this option) |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSC | Load curve ID for SC as a function of strain rate (SC is ignored with this option) |
| DT | Strain rate averaging option:<br>EQ.0.0: Strain rate is evaluated using a running average.<br>LT.0.0: Strain rate is evaluated using an average of the last 11 time steps.<br>GT.0.0: Strain rate is averaged over the last DT time units. |

**Material Formulation:**

***MAT_054 Failure Criteria**
The Chang-Chang (*MAT_054) criteria is given as follows:

1. For the tensile fiber mode,

$$\sigma_{aa} > 0 \Rightarrow e_f^2 = \left(\frac{\sigma_{aa}}{X_t}\right)^2 + \beta\left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \qquad \begin{matrix} e_f^2 \geq 0 \Rightarrow \text{failed} \\ e_f^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

$$E_a = E_b = G_{ab} = \nu_{ba} = \nu_{ab} = 0$$

2. For the compressive fiber mode,

$$\sigma_{aa} < 0 \Rightarrow e_c^2 = \left(\frac{\sigma_{aa}}{X_c}\right)^2 - 1, \qquad \begin{matrix} e_c^2 \geq 0 \Rightarrow \text{failed} \\ e_c^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

$$E_a = \nu_{ba} = \nu_{ab} = 0$$

3. For the tensile matrix mode,

$$\sigma_{bb} > 0 \Rightarrow e_m^2 = \left(\frac{\sigma_{bb}}{Y_t}\right)^2 + \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \qquad \begin{matrix} e_m^2 \geq 0 \Rightarrow \text{failed} \\ e_m^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

$$E_b = \nu_{ba} = 0 \Rightarrow G_{ab} = 0$$

4. For the compressive matrix mode,

$$\sigma_{bb} < 0 \Rightarrow e_d^2 = \left(\frac{\sigma_{bb}}{2S_c}\right)^2 + \left[\left(\frac{Y_c}{2S_c}\right)^2 - 1\right]\frac{\sigma_{bb}}{Y_c} + \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \qquad \begin{matrix} e_d^2 \geq 0 \Rightarrow \text{failed} \\ e_d^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

$$E_b = \nu_{ba} = \nu_{ab} = 0 \Rightarrow G_{ab} = 0$$

$$X_c = 2Y_c, \text{ for 50\% fiber volume}$$

For $\beta = 1$ we get the original criterion of Hashin [1980] in the tensile fiber mode. For $\beta = 0$ we get the maximum stress criterion which is found to compare better to experiments.

***MAT_054 with 2-Way Fiber Flag Failure Criteria**

If the 2-way fiber flag is set, then the failure criteria for tensile and compressive fiber failure in the local $x$-direction are unchanged. For the local $y$-direction, the same failure criteria as for the $x$-direction fibers are used.

1. For the tensile fiber mode in the local $y$-direction,

$$\sigma_{bb} > 0 \Rightarrow e_f^2 = \left(\frac{\sigma_{bb}}{Y_t}\right)^2 + \beta\left(\frac{\sigma_{ab}}{S_c}\right) - 1, \qquad \begin{matrix} e_f^2 \geq 0 \Rightarrow \text{failed} \\ e_f^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

2. For the compressive fiber mode in the local $y$-direction,

$$\sigma_{bb} < 0 \Rightarrow e_c^2 = \left(\frac{\sigma_{bb}}{Y_c}\right)^2 - 1, \qquad \begin{matrix} e_c^2 \geq 0 \Rightarrow \text{failed} \\ e_c^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

3. For 2WAY the matrix only fails in shear,

$$e_m^2 = \left(\frac{\sigma_{ab}}{S_c}\right)^2 - 1, \qquad \begin{matrix} e_m^2 \geq 0 \Rightarrow \text{failed} \\ e_m^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

***MAT_055 Failure Criteria**

For the Tsai-Wu (*MAT_055) criteria, the tensile and compressive fiber modes are treated the same as in the Chang-Chang criteria. The failure criterion for the tensile and compressive matrix mode is given as:

$$e_{m/d}^2 = \frac{\sigma_{bb}^2}{Y_c Y_t} + \left(\frac{\sigma_{ab}}{S_c}\right)^2 + \frac{(Y_c - Y_t)\,\sigma_{bb}}{Y_c Y_t} - 1, \qquad \begin{matrix} e_{m/d}^2 \geq 0 \Rightarrow \text{failed} \\ e_{m/d}^2 < 0 \Rightarrow \text{elastic} \end{matrix}$$

**Remarks:**

1. **Integration point failure.** In *MAT_054, failure can occur in any of four different ways:

   - If DFAILT is zero, failure occurs if the Chang-Chang failure criterion is satisfied in the tensile fiber mode.
   - If DFAILT is greater than zero, failure occurs if:
     - the fiber strain is greater than DFAILT or less than DFAILC
     - if absolute value of matrix strain is greater than DFAILM
     - if absolute value of tensorial shear strain is greater than DFAILS
   - If EFS is greater than zero, failure occurs if the effective strain is greater than EFS.
   - If TFAIL is greater than zero, failure occurs according to the element timestep as described in the definition of TFAIL above.

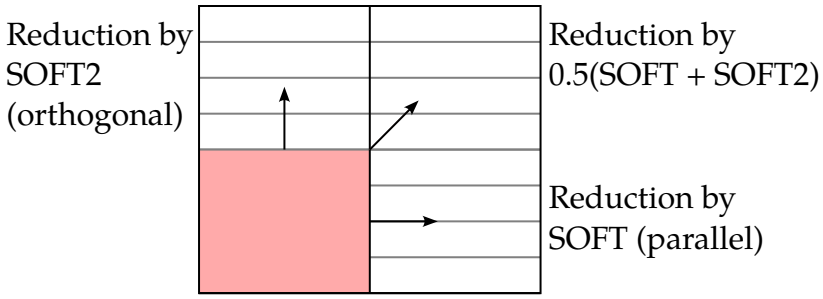



**Figure M54-1.** Direction dependent softening

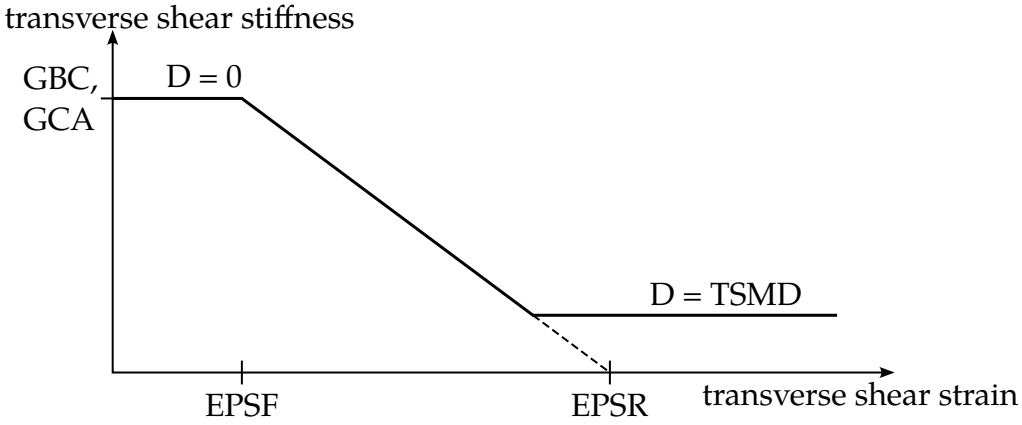



**Figure M54-2.** Linear Damage for transverse shear behavior

In *MAT_055, an integration point is deleted (all stresses go to zero) only if the tensile stress at that point reaches XT. Other strengths, XC, YT, YC, SC serve to cap stresses but do not delete the integration point.

When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. For bricks, the element is deleted after one integration point has met the failure criteria.

2. **Crashfront elements and strength reduction.** Elements that share nodes with a deleted element become "crashfront" elements and can have their strengths reduced by using the SOFT parameter with TFAIL greater than zero. An earlier initiation of crashfront elements is possible by using parameter PFL.

   An optional direction dependent strength reduction can be invoked by setting 0 < SOFT2 < 1. Then, SOFT equals a strength reduction factor for fiber parallel failure and SOFT2 equals a strength reduction factor for fiber orthogonal failure. Linear interpolation is used for angles in between. See Figure M54-1.

3. **Transverse shear strain damage model.** In an optional damage model for transverse shear strain, out-of-plane stiffness (GBC and GCA) can linearly

decrease to model interlaminar shear failure. Damage starts when effective transverse shear strain

$$\varepsilon_{56}^{\text{eff}} = \sqrt{\varepsilon_{yz}^2 + \varepsilon_{zx}^2}$$

reaches EPSF. Final rupture occurs when effective transverse shear strain reaches EPSR. A maximum damage of TSMD (0.0 < TSMD < 0.99) cannot be exceeded. See Figure M54-2.

4. **Failure/damage status.** The status in each layer (integration point) and element can be plotted using additional integration point history variables. NEIPH and NEIPS on *DATABASE_EXTENT_BINARY sets the number of additional integration point history variables output for solids and shells, respectively. The number of additional integration point history variables for shells and solids written to the database is input by the *DATABASE_EXTENT_BINARY definition as variable . For Models 54 and 55 these additional history variables are tabulated below ($i$ = integration point):

**Table M54-1.** Additional history variables for *MAT_054

| History Variable # | Description for shells and thick shell types 1, 2, and 6 | Description for solids and thick shell types 3, 5, and 7 | Value |
|---|---|---|---|
| 1 | Tensile fiber failure mode, $\text{ef}(i)$ | Tensile fiber failure mode, $\text{ef}(i)$ | 1: elastic<br>0: failed |
| 2 | Compressive fiber failure mode, $\text{ec}(i)$ | Compressive fiber failure mode, $\text{ec}(i)$ | 1: elastic<br>0: failed |
| 3 | Tensile (shear for 2WAY) matrix mode, $em(i)$ | Tensile (shear for 2WAY) matrix mode, $em(i)$ | 1: elastic<br>0: failed |
| 4 | Compressive matrix mode, $ed(i)$ | Compressive matrix mode, $ed(i)$ | 1: elastic<br>0: failed |
| 5 | Total failure | Total failure | 1: elastic<br>0: failed |
| 6 | Damage parameter (SOFT) | Damage parameter (SOFT) | -1: element intact<br>$10^{-8}$: element in crashfront<br>1: element failed |
| 8 | $\cos(\alpha)$, where $\alpha$ is the in-plane angle between the material coordinate system and | | |

| History Variable # | Description for shells and thick shell types 1, 2, and 6 | Description for solids and thick shell types 3, 5, and 7 | Value |
|---|---|---|---|
| | the element coordinate system | | |
| 9 | $\sin\alpha$ | | |
| 10 | Local strain in the *a*-direction | | |
| 11 | Local strain in the *b*-direction | | |
| 12 | Local shear strain (*ab*-plane) | | |
| 15 | | Local strain in the *a*-direction | |
| 16 | Transverse shear damage | Local strain in the *b*-direction | |
| 17 | | Local shear strain (*ab*-plane) | |

**Table M54-2.** Additional history variables for *MAT_055

| History Variable # | Description for shells and thick shell types 1, 2, and 6 | Value |
|---|---|---|
| 1 | Tensile fiber failure mode, $ef(i)$ | 1: elastic<br>0: failed |
| 2 | Compressive fiber failure mode, $ec(i)$ | 1: elastic<br>0: failed |
| 3 | Tensile matrix mode, $em(i)$ | 1: elastic<br>0: failed |
| 4 | Compressive matrix mode, $ed(i)$ | 1: elastic<br>0: failed |
| 5 | Total failure | 1: elastic<br>0: failed |
| 6 | Damage parameter (SOFT) | -1: element intact<br>$10^{-8}$: element in crashfront<br>1: element failed |
| 8 | $\cos(\alpha)$, where $\alpha$ is the in-plane angle between the material coordinate system and the element coordinate system | |
| 9 | $\sin(\alpha)$ | |

The three element history variables in the table below represent the fraction of elastic (non-failed) integration points in tensile fiber, compressive fiber, and

tensile matrix failure modes. They are labeled as "effective plastic strain" by LS-PrePost for integration points 1, 2, and 3. In the table $i$ indexes the integration points in the element and nip is the number of integration points in the element.

| Description | Integration Point |
|---|---|
| $\frac{1}{\text{nip}}\sum_{i=1}^{\text{nip}} \text{ef}(i)$ | 1 |
| $\frac{1}{\text{nip}}\sum_{i=1}^{\text{nip}} \text{ec}(i)$ | 2 |
| $\frac{1}{\text{nip}}\sum_{i=1}^{\text{nip}} \text{em}(i)$ | 3 |

5. **TI flag**. This applies only to transversal isotropic behavior for *MAT_054 solid elements. The behavior in the $bc$-plane is assumed to be isotropic, thus the elastic constants EC, PRCA and GCA are ignored and set according to the given values EA, EB, PRAB, and GAB. Damage in transverse shear (EPSF, EPSR, TSMD, SOFTG) is ignored. The failure criterion is evaluated by replacing $\sigma_{bb}$ and $\sigma_{ab}$ with the corresponding stresses $\sigma_{11}$ and $\sigma_{a1}$ in a principal stress frame rotated around the local $a$-axis. The principal axes 1 and 2 in the $bc$-plane are chosen such that $|\sigma_{11}| \geq |\sigma_{22}|$ is fulfilled.

6. **Material parameters.** PRBA is the minor Poisson's ratio if EA>EB, and the major Poisson's ratio will be equal to PRBA*(EA/EB). If EB > EA, then PRBA is the major Poisson's ratio. PRCA and PRCB are similarly defined. They are the minor Poisson's ratio if EA > EC or EB > EC, and the major Poisson's ratio if the EC > EA or EC > EB.

   Care should be taken when using material parameters from third party products regarding the directional indices $a$, $b$ and $c$, as they may differ from the definition used in LS-DYNA. For the direction indices used in LS-DYNA see Material Directions in *MAT_002/*MAT_*OPTION*TROPIC_ELASTIC.

7. **Limitations of *MAT_054 with solid elements.** The out-of-plane damage treatment for solid elements is limited. As described in Remark 3, EPSF, EPSR, and TSMD provide a simplified damage model for the transverse shear response by reducing $G_{bc}$ and $G_{ca}$. However, there is no corresponding damage treatment for the normal $c$-direction response, which is treated elastically. *MAT_054 is intended for continuous-fiber, unidirectional laminated structures. Because of these limitations, it is not well suited for general three-dimensional composite components where out-of-plane damage is important. For such applications, consider *MAT_058 with the SOLID keyword option, which provides a more comprehensive treatment of out-of-plane damage.

## *MAT_LOW_DENSITY_FOAM

This is Material Type 57 for modeling highly compressible low density foams. Its main applications are for seat cushions and padding on the Side Impact Dummies (SID). Optionally, a tension cut-off failure can be defined. A table can be defined if thermal effects are considered in the nominal stress as a function of strain behavior. Also, see the remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | LCID | TC | HU | BETA | DAMP |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | $10^{20}$ | 1. | none | 0.05 |
| Remarks | | | | | | 3 | 1 | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SHAPE | FAIL | BVFLAG | ED | BETA1 | KCON | REF | |
| Type | F | F | F | F | F | F | F | |
| Default | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| Remarks | 3 | | 2 | 5 | 5 | 6 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus used in tension. For implicit problems E is set to the initial slope of load curve LCID. |
| LCID | Load curve ID (see *DEFINE_CURVE) or table ID defining the nominal stress as a function of nominal strain. If a table is used, a |

| VARIABLE | DESCRIPTION |
|---|---|
| | family of curves is defined each corresponding to a discrete temperature; see *DEFINE_TABLE. |
| TC | Cut-off for the nominal tensile stress, $\tau_i$. |
| HU | Hysteretic unloading factor between 0.0 and 1.0 (default = 1.0, that is, no energy dissipation); see also Figure M57-1. |
| BETA | Decay constant to model creep in unloading, $\beta$. |
| DAMP | Viscous coefficient (.05 < recommended value < .50) to model damping effects.<br>LT.0.0: \|DAMP\| is the load curve ID, which defines the damping constant as a function of the maximum strain in compression defined as:<br>$\varepsilon_{\max} = \max(1-\lambda_1, 1-\lambda_2, 1.-\lambda_3)$ .<br>In tension, the damping constant is set to the value corresponding to the strain at 0.0. The abscissa should be defined from 0.0 to 1.0. |
| SHAPE | Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor. Values less than one reduces the energy dissipation and greater than one increases dissipation; see also Figure M57-1. |
| FAIL | Failure option after cutoff stress is reached:<br>EQ.0.0: tensile stress remains at cut-off value.<br>EQ.1.0: tensile stress is reset to zero. |
| BVFLAG | Bulk viscosity activation flag:<br>EQ.0.0: no bulk viscosity (recommended)<br>EQ.1.0: bulk viscosity active |
| ED | Optional Young's relaxation modulus, $E_d$, for rate effects. |
| BETA1 | Optional decay constant, $\beta_1$. |
| KCON | Stiffness coefficient for contact interface stiffness. If undefined the maximum slope in the stress as a function of strain curve is used. When the maximum slope is taken for the contact, the time step size for this material is reduced for stability. In some cases, $\Delta t$ may |

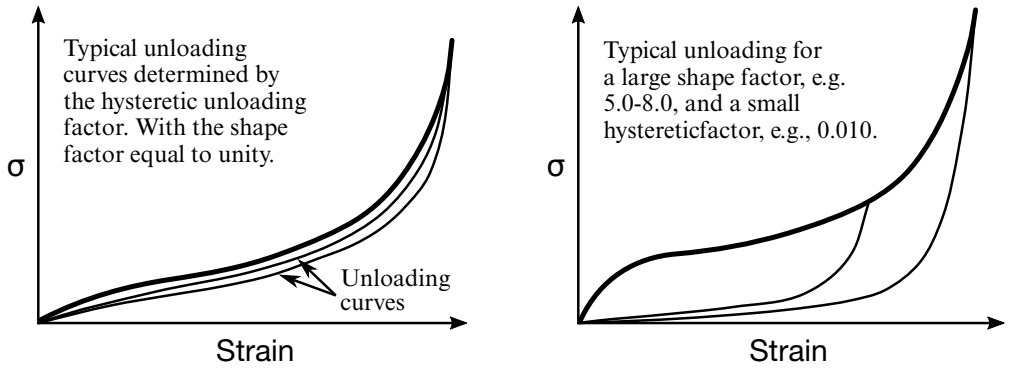



**Figure M57-1.** Behavior of the low density urethane foam model

| VARIABLE | DESCRIPTION |
|---|---|
| | be significantly smaller, so defining a reasonable stiffness is recommended. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Material Formulation:**

The compressive behavior is illustrated in Figure M57-1 where hysteresis upon unloading is shown. This behavior under uniaxial loading is assumed not to significantly couple in the transverse directions. In tension the material behaves in a linear fashion until tearing occurs. Although our implementation may be somewhat unusual, it was motivated by Storakers [1986].

The model uses tabulated input data for the loading curve where the nominal stresses are defined as a function of the elongations, $\varepsilon_i$, which are defined in terms of the principal stretches, $\lambda_i$, as:

$$\varepsilon_i = \lambda_i - 1$$

The negative of the principal elongations (negative of principal engineering strains) are stored in an arbitrary order as extra history variables 16, 17, and 18 if ED = 0 and as extra history variables 28, 29, and 30 if ED > 0. (See NEIPH in *DATABASE_EXTENT_BINARY for output of extra history variables.) The stretch ratios are found by solving for the eigenvalues of the left stretch tensor, $V_{ij}$, which is obtained using a polar decomposition of the deformation gradient matrix, $F_{ij}$. Recall that,

$$F_{ij} = R_{ik}U_{kj} = V_{ik}R_{kj}$$

The update of $V_{ij}$ follows the numerically stable approach of Taylor and Flanagan [1989]. After solving for the principal stretches, we compute the elongations and, if the elongations are compressive, the corresponding values of the nominal stresses, $\tau_i$ are interpolated. If the elongations are tensile, the nominal stresses are given by

$$\tau_i = E\varepsilon_i$$

and the Cauchy stresses in the principal system become

$$\sigma_i = \frac{\tau_i}{\lambda_j \lambda_k} \ .$$

The stresses can now be transformed back into the global system for the nodal force calculations.

**Remarks:**

1. **Decay constant and hysteretic unloading.** When hysteretic unloading is used the reloading will follow the unloading curve if the decay constant, $\beta$, is set to zero. If $\beta$ is nonzero the decay to the original loading curve is governed by the expression:

$$1 - e^{-\beta t} \ .$$

2. **Bulk viscosity.** The bulk viscosity, which generates a rate dependent pressure, may cause an unexpected volumetric response and, consequently, it is optional with this model.

3. **Hysteretic unloading factor.** The hysteretic unloading factor results in the unloading curve to lie beneath the loading curve as shown in Figure M57-1 This unloading provides energy dissipation which is reasonable in certain kinds of foam.

4. **Output.** Note that since this material has no effective plastic strain, the internal energy per initial volume is written into the output databases.

5. **Rate effects.** Rate effects are accounted for through linear viscoelasticity by a convolution integral of the form

$$\sigma_{ij}^r = \int_0^t g_{ijkl}\,(t-\tau)\,\frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau \ ,$$

where $g_{ijkl}(t-\tau)$ is the relaxation function. The stress tensor, $\sigma_{ij}^r$, augments the stresses determined from the foam, $\sigma_{ij}^f$; consequently, the final stress, $\sigma_{ij}$, is taken as the summation of the two contributions:

$$\sigma_{ij} = \sigma_{ij}^f + \sigma_{ij}^r \ .$$

Since we wish to include only simple rate effects, the relaxation function is represented by one term from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = E_d e^{-\beta_1 t} .$$

This model is effectively a Maxwell fluid which consists of a damper and spring in series. We characterize this in the input by a Young's modulus, $E_d$, and decay constant, $\beta_1$. The formulation is performed in the local system of principal stretches where only the principal values of stress are computed and triaxial coupling is avoided. Consequently, the one-dimensional nature of this foam material is unaffected by this addition of rate effects. The addition of rate effects necessitates twelve additional history variables per integration point. The cost and memory overhead of this model comes primarily from the need to "remember" the local system of principal stretches.

6. **Time step size.** The time step size is based on the current density and the maximum of the instantaneous loading slope, $E$, and KCON. If KCON is undefined, the maximum slope in the loading curve is used instead.

## *MAT_LAMINATED_COMPOSITE_FABRIC_{OPTION}

Available options include:

SOLID

This is Material Type 58. Depending on the type of failure surface, this material can model composite materials with unidirectional layers, complete laminates, and woven fabrics. We implemented this model for shell, thick shell, and solid elements. Shell elements (and thick shell types 1, 2, and 6) require no keyword option, while solid elements (and thick shell element types 3, 5, and 7) require the SOLID keyword option.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | TAU1 | GAMMA1 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | TSIZE | ERODS | SOFT | FS | EPSF | EPSR | TSMD |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | LCDFAIL |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| E11C | E11T | E22C | E22T | GMS | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| XC | XT | YC | YT | SC | | | |
|---|---|---|---|---|---|---|---|

**Card 8.1.** This card is required for the SOLID keyword option.

| E33C | E33T | GMS23 | GMS31 | | | | |
|---|---|---|---|---|---|---|---|

**Card 8.2.** This card is required for the SOLID keyword option.

| ZC | ZT | SC23 | SC31 | | | | |
|---|---|---|---|---|---|---|---|

**Card 8.3.** This card is required for the SOLID keyword option.

| SLIMT3 | SLIMC3 | SLIMS23 | SLIMS31 | TAU2 | GAMMA2 | TAU3 | GAMMA3 |
|---|---|---|---|---|---|---|---|

***Card 9.*** This card is optional. (shells and solids)

| LCXC | LCXT | LCYC | LCYT | LCSC | LCTAU | LCGAM | DT |
|---|---|---|---|---|---|---|---|

***Card 10.*** This card is optional. (shells and solids)

| LCE11C | LCE11T | LCE22C | LCE22T | LCGMS | LCEFS | | |
|---|---|---|---|---|---|---|---|

***Card 11.*** This card is optional. (solids only!)

| LCZC | LCZT | LCSC23 | LCSC31 | LCTAU2 | LCGAM2 | LCTAU3 | LCGAM3 |
|---|---|---|---|---|---|---|---|

***Card 12.*** This card is optional. (solids only!)

| LCE33C | LCE33T | LCGMS23 | LCGMS31 | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | TAU1 | GAMMA1 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | GT.0.0: $E_a$, Young's modulus - longitudinal direction<br>LT.0.0: Load Curve ID or Table ID = (-EA). See Remark 8.<br>**Load Curve.** When -EA is equal to a load curve ID, it is taken as defining the uniaxial elastic stress as a function |

| VARIABLE | DESCRIPTION |
|---|---|
| | of strain behavior in the longitudinal direction. Negative data points correspond to compression, and positive values to tension.<br><br>**Tabular Data.** When -EA is equal to a table ID, it defines a load curve ID for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the longitudinal direction.<br><br>**Logarithmically Defined Tables.** Suppose the first uniaxial elastic stress as a function of strain curve in the table corresponds to a negative strain rate. In that case, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves. |
| EB | GT.0.0: $E_b$, Young's modulus - transverse direction<br><br>LT.0.0: Load Curve ID or Table ID = (-EB). See Remark 8.<br><br>**Load Curve.** When -EB is equal to a load curve ID, it is taken as defining the uniaxial elastic stress as a function of strain behavior in the transverse direction. Negative data points correspond to compression, and positive values to tension.<br><br>**Tabular Data.** When -EB corresponds to a table ID, it specifies a load curve ID for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the transverse direction.<br><br>**Logarithmically Defined Tables.** Suppose the first uniaxial elastic stress as a function of strain curve in the table corresponds to a negative strain rate. In that case, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves. |
| EC | $E_c$, Young's modulus - normal direction (used only by thick shells and solids). See Remark 6.<br><br>GT.0.0: $E_c$, Young's modulus - normal direction<br><br>LT.0.0: Load Curve ID or Table ID = (-EC) (solids only). See Remark 8.<br><br>**Load Curve.** When -EC is equal to a load curve ID, it is taken as defining the uniaxial elastic stress as a function of strain behavior in the transverse direction. Negative data points correspond to compression, and positive |

| VARIABLE | DESCRIPTION |
|---|---|
| | values to tension.<br><br>**Tabular Data.** When -EC corresponds to a table ID, it specifies a load curve ID for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the transverse direction.<br><br>**Logarithmically Defined Tables.** Suppose the first uniaxial elastic stress as a function of strain curve in the table corresponds to a negative strain rate. In that case, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves. |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| TAU1 | $\tau_1$, stress limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve. The values $\tau_1$ and $\gamma_1$ help define a shear stress as a function of shear strain curve. Input these values if you set FS to -1 (see Card 3). |
| GAMMA1 | $\gamma_1$, strain limit of the first slightly nonlinear part of the shear stress as a function of engineering shear strain curve |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | GT.0.0: $G_{ab}$, shear modulus in the *ab*-direction<br><br>LT.0.0: Load Curve ID or Table ID = (-GAB)<br><br>**Load Curve.** When -GAB is equal to a load curve ID, it is taken as defining the elastic shear stress as a function of shear strain behavior in the *ab*-direction.<br><br>**Tabular Data.** When -GAB corresponds to a table ID, it defines a load curve ID for each strain rate value. The load curves give the elastic shear stress as a function of shear strain behavior in the *ab*-direction.<br><br>**Logarithmically Defined Tables.** If the *first* elastic |

| VARIABLE | DESCRIPTION |
|---|---|
| | shear stress as a function of shear strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* shear stress-shear strain curves. |
| GBC | GT.0.0: $G_{bc}$, shear modulus in the *cb*-direction<br>LT.0.0: Load Curve ID or Table ID = (-GBC) (solids only)<br>**Load Curve.** When -GBC is equal to a load curve ID, it is taken as defining the elastic shear stress as a function of shear strain behavior in the *bc*-direction.<br>**Tabular Data.** When -GBC corresponds to a table ID, it defines a load curve ID for each strain rate value. The load curves give the elastic shear stress as a function of shear strain behavior in the *bc*-direction.<br>**Logarithmically Defined Tables.** If the *first* elastic shear stress as a function of shear strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* shear stress-shear strain curves. |
| GCA | GT.0.0: $G_{ca}$, shear modulus in the *ca*-direction<br>LT.0.0: Load Curve ID or Table ID = (-GCA) (solids only)<br>**Load Curve.** When -GCA is equal to a load curve ID, it is taken as defining the elastic shear stress as a function of shear strain behavior in the *ca*-direction.<br>**Tabular Data.** When -GCA refers to a table ID, it defines a load curve ID for each strain rate value. The load curves give the elastic shear stress as a function of shear strain behavior in the *ca*-direction.<br>**Logarithmically Defined Tables.** If the *first* elastic shear stress as a function of shear strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* shear stress-shear strain curves. |
| SLIMT1 | Factor to determine the minimum stress limit after stress maximum (fiber tension) |

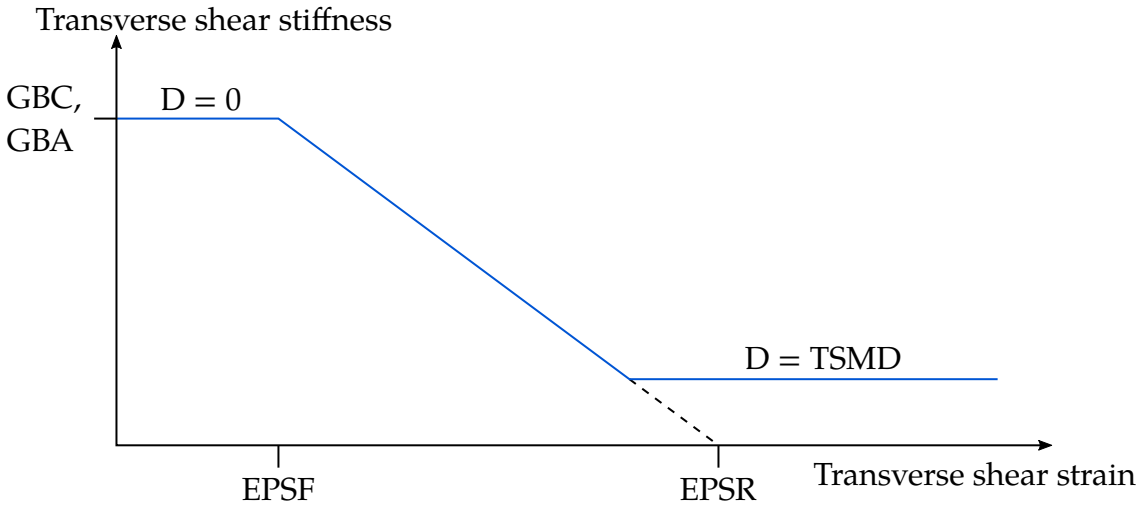



**Figure M58-1.** Linear Damage for Transverse Shear Behavior

| VARIABLE | DESCRIPTION |
|---|---|
| SLIMC1 | Factor to determine the minimum stress limit after stress maximum (fiber compression) |
| SLIMT2 | Factor to determine the minimum stress limit after stress maximum (matrix tension) |
| SLIMC2 | Factor to determine the minimum stress limit after stress maximum (matrix compression) |
| SLIMS | Factor to determine the minimum stress limit after stress maximum (shear) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | TSIZE | ERODS | SOFT | FS | EPSF | EPSR | TSMD |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see *MAT_002 for a complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes, as shown in Figure M2-1. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a |

| VARIABLE | DESCRIPTION |
|---|---|
| | point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element (see Figure M2-1). The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then an angle BETA, which you set in the element's keyword input or the input for this keyword, rotates **a** and **b** about **c**. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: \|AOPT\| is a coordinate system ID (see *DEFINE_COORDINATE_*OPTION*). |
| TSIZE | Time step for automatic element deletion |
| ERODS | Maximum effective strain for element layer failure. A value of unity would equal 100% strain (see Remark 1). |
| | GT.0.0: Fails when effective strain calculated assuming the material is volume preserving exceeds ERODS (old way) |
| | LT.0.0: Fails when effective strain calculated from the full strain tensor exceeds \|ERODS\| |
| SOFT | Softening reduction factor for strength in the crashfront (see Remark 3) |
| FS | Failure surface type (see Remarks 4 and 5): |
| | EQ.1.0: Smooth failure surface with a quadratic criterion for both the fiber (a) and transverse (b) directions. This |

| VARIABLE | DESCRIPTION |
|---|---|
| | option can be used with complete laminates and fabrics. |
| | EQ.0.0: Smooth failure surface in the transverse (*b*) direction with a limiting value in the fiber (*a*) direction. This model is appropriate for unidirectional (UD) layered composites only. |
| | EQ.-1.0: Faceted failure surface. When the strength values are reached, damage evolves in tension and compression for the fiber and transverse direction. Shear behavior is also considered. This option can be used with complete laminates and fabrics. |
| EPSF | Damage initiation transverse shear strain |
| EPSR | Final rupture transverse shear strain |
| TSMD | Transverse shear maximum damage; default = 0.90. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | PRCA | PRCB |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* (default = PRBA) |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* (default = PRBA) |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | LCDFAIL |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2 V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Angle in degrees of a material rotation about the c-axis, available for AOPT = 0 (shells and tshells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |
| LCDFAIL | Load curve ID, which defines orientation-dependent failure strains. The ordinate values in the load curve define the various failure strains in the following order:<br>1. EF_11T: tensile failure strain in longitudinal *a*-direction<br>2. EF_11C: compressive failure strain in longitudinal *a*-direction<br>3. EF_22T: tensile failure strain in transverse *b*-direction<br>4. EF_22C: compressive failure strain in transverse *b*-direction<br>5. EF_12: in-plane shear failure strain in *ab*-plane<br>6. EF_33T: tensile failure strain in transverse *c*-direction<br>7. EF_33C: compressive failure strain in transverse *c*-direction<br>8. EF_23: out-of-plane shear failure strain in *bc*-plane<br>9. EF_31: out-of-plane shear failure strain in *ca*-plane<br>Thus, the load curve to define these values must have either five (shells) or nine (solids) entries in its definition. You may input a load curve with nine entries for shells, but LS-DYNA ignores the last four entries. The ignored abscissa values need to be ascending, such as 1.0, 2.0, …, 9.0. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E11C | E11T | E22C | E22T | GMS | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E11C | Strain at longitudinal compressive strength, *a*-axis (positive) |
| E11T | Strain at longitudinal tensile strength, *a*-axis |
| E22C | Strain at transverse compressive strength, *b*-axis |
| E22T | Strain at transverse tensile strength, *b*-axis |
| GMS | Engineering shear strain at shear strength, *ab*-plane |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SC | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength (positive value); see Remark 2. |
| XT | Longitudinal tensile strength; see Remark 2. |
| YC | Transverse compressive strength, *b*-axis (positive value); see Remark 2. |
| YT | Transverse tensile strength, *b*-axis; see Remark 2. |
| SC | Shear strength, *ab*-plane; see below Remark 2. |

**Card 8.1 for SOLID Keyword Option.**

| Card 8.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E33C | E33T | GMS23 | GMS31 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E33C | Strain at transverse compressive strength, *c*-axis. |
| E33T | Strain at transverse tensile strength, *c*-axis. |
| GMS23 | Engineering shear strain at shear strength, *bc*-plane. |
| GMS31 | Engineering shear strain at shear strength, *ca*-plane. |

**Card 8.2 for SOLID Keyword Option.**

| Card 8.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ZC | ZT | SC23 | SC31 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ZC | Transverse compressive strength, *c*-axis (positive value). |
| ZT | Transverse tensile strength, *c*-axis. |
| SC23 | Shear strength, *bc*-plane. |
| SC31 | Shear strength, *ca*-plane. |

**Card 8.3 for SOLID Keyword Option.**

| Card 8.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SLIMT3 | SLIMC3 | SLIMS23 | SLIMS31 | TAU2 | GAMMA2 | TAU3 | GAMMA3 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SLIMT3 | Factor to determine the minimum stress limit after stress maximum (matrix tension, *c*-axis). |
| SLIMC3 | Factor to determine the minimum stress limit after stress maximum (matrix compression, *c*-axis). |
| SLIMS23 | Factor to determine the minimum stress limit after stress maximum (shear, *bc*-plane). |
| SLIMS31 | Factor to determine the minimum stress limit after stress maximum (shear, *ca*-plane). |
| TAU2 | $\tau_2$, stress limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve. The values $\tau_2$ and $\gamma_2$ are used to define a shear stress as a function of shear strain curve. Input these values if FS = -1 (see Card 3). These values are for the *bc*-plane. |
| GAMMA2 | $\gamma_2$, strain limit of the first slightly nonlinear part of the shear stress as a function of engineering shear strain curve (*bc*-plane). |
| TAU3 | $\tau_3$, stress limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve. The values $\tau_3$ and $\gamma_3$ help define a shear stress as a function of shear strain curve. Input these values if FS = -1 on Card 3 (*ca*-plane). |
| GAMMA3 | $\gamma_3$, strain limit of the first slightly nonlinear part of the shear stress as a function of engineering shear strain curve (*bc*-plane). |

**First Optional Strain Rate Dependence Card. (shells and solids)**

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCXC | LCXT | LCYC | LCYT | LCSC | LCTAU | LCGAM | DT |
| Type | I | I | I | I | I | I | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCXC | Load curve ID defining longitudinal compressive strength XC as a function of strain rate (XC is ignored with that option). If the first strain rate value in the curve is negative, LS-DYNA assumes that |

| VARIABLE | DESCRIPTION |
|---|---|
| | you input all the strain rate values as the natural logarithm of the strain rate. |
| LCXT | Load curve ID defining longitudinal tensile strength XT as a function of strain rate (XT is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCYC | Load curve ID defining transverse compressive strength YC as a function of strain rate (YC is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCYT | Load curve ID defining transverse tensile strength YT as a function of strain rate (YT is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCSC | Load curve ID defining shear strength SC as a function of strain rate (SC is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCTAU | Load curve ID defining TAU1 as a function of strain rate (TAU1 is ignored with this option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCGAM | Load curve ID defining GAMMA1 as a function of strain rate (GAMMA1 is ignored with this option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| DT | Strain rate averaging option:<br>EQ.0.0: Strain rate is evaluated using a running average.<br>LT.0.0: Strain rate is evaluated using the average over the last 11 time steps.<br>GT.0.0: Strain rate is averaged over the last DT time units. |

**Second Optional Strain Rate Dependence Card. (shells and solids)**

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCE11C | LCE11T | LCE22C | LCE22T | LCGMS | LCEFS | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCE11C | Load curve ID defining E11C as a function of strain rate (E11C is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCE11T | Load curve ID defining E11T as a function of strain rate (E11T is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCE22C | Load curve ID defining E22C as a function of strain rate (E22C is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCE22T | Load curve ID defining E22T as a function of strain rate (E22T is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCGMS | Load curve ID defining GMS as a function of strain rate (GMS is ignored with this option). If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |
| LCEFS | Load curve ID defining ERODS as a function of strain rate (ERODS is ignored with this option). LS-DYNA uses the full strain tensor to compute the equivalent strain. If the first strain rate value in the curve is negative, LS-DYNA assumes that you input all the strain rate values as the natural logarithm of the strain rate. |

**Third Optional Strain Rate Dependence Card. (solid only!)**

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCZC | LCZT | LCSC23 | LCSC31 | LCTAU2 | LCGAM2 | LCTAU3 | LCGAM3 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| LCZC | Load curve ID defining transverse compressive strength ZC as a function of strain rate (ZC is ignored with that option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCZT | Load curve ID defining transverse tensile strength ZT as a function of strain rate (ZT is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCSC23 | Load curve ID defining shear strength SC23 as a function of strain rate (SC23 is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCSC31 | Load curve ID defining shear strength SC31 as a function of strain rate (SC31 is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCTAU2 | Load curve ID defining TAU2 as a function of strain rate (TAU2 is ignored with this option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCGAM2 | Load curve ID defining GAMMA2 as a function of strain rate (GAMMA2 is ignored with this option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCTAU3 | Load curve ID defining TAU3 as a function of strain rate (TAU3 is ignored with this option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, all the strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| | values are assumed to be given as the natural logarithm of the strain rate. |
| LCGAM3 | Load curve ID defining GAMMA3 as a function of strain rate (GAMMA3 is ignored with this option). This value is only used for FS = -1. If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |

**Fourth Optional Strain Rate Dependence Card. (solids only!)**

| Card 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCE33C | LCE33T | LCGMS23 | LCGMS31 | | | | |
| Type | I | I | I | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCE33C | Load curve ID defining E33C as a function of strain rate (E33C is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCE33T | Load curve ID defining E33T as a function of strain rate (E33T is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCGMS23 | Load curve ID defining GMS23 as a function of strain rate (GMS23 is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |
| LCGMS31 | Load curve ID defining GMS31 as a function of strain rate (GMS31 is ignored with this option). If the first strain rate value in the curve is negative, all the strain rate values are assumed to be given as the natural logarithm of the strain rate. |

**Remarks:**

1. **Failure of an element layer.** ERODS, the maximum effective strain, controls the failure of an element layer. The maximum value of ERODS, 1, is 100% straining. The layer in the element is completely removed after the maximum effective strain (compression/tension including shear) is reached.

2. **Stress limits.** The stress limits are factors used to limit the stress in the softening part to a given value,

$$\sigma_{\min} = \text{SLIM}xx \times \text{strength}.$$

Thus, the damage value is slightly modified to achieve elastoplastic-like behavior with the threshold stress. The SLIM*xx* fields may range between 0.0 and 1.0. With a factor of 1.0, the stress remains at a maximum value identical to the strength, similar to ideal elastoplastic behavior. A small value for SLIMT*x* is often reasonable for tensile failure; however, SLIMCx = 1.0 is preferred for compression. This is also valid for the corresponding shear value.

If SLIM*xx* is smaller than 1.0, then localization can be observed depending on the total behavior of the layer. If intentionally using SLIM*xx* < 1.0, we generally recommend avoiding a drop to zero and setting the value to something between 0.05 and 0.10. Then elastoplastic behavior is achieved in the limit, which often leads to fewer numerical problems. The defaults for SLIM*xx* are $10^{-8}$.

3. **Crashfront.** To start the crashfront algorithm, input a value for TSIZE. Note that the time step size, with element elimination after the actual time step, becomes smaller than TSIZE.

4. **Damage.** The damage parameters can be written to the post-processing database for each integration point as the first three additional element variables and can be visualized.

Material models with FS = 1 or FS = -1 are better for complete laminates and fabrics, as all directions are treated similarly.

For FS = 1, the model assumes an interaction between the normal and shear stresses for damage evolution in the *a* and *b*-directions. The shear damage is always the maximum damage value from the criterion in the *a* or *b*-direction.

For FS = -1, we assume that the damage evolution is independent of any of the other stresses. The elastic material parameters and the complete structure provide the only coupling. In the tensile and compression directions, as well as in the *b*-direction, the material can have different failure surfaces. The damage values monotonically increase. Thus, a load reversal from tension to compression, or compression to tension, does not reduce damage.

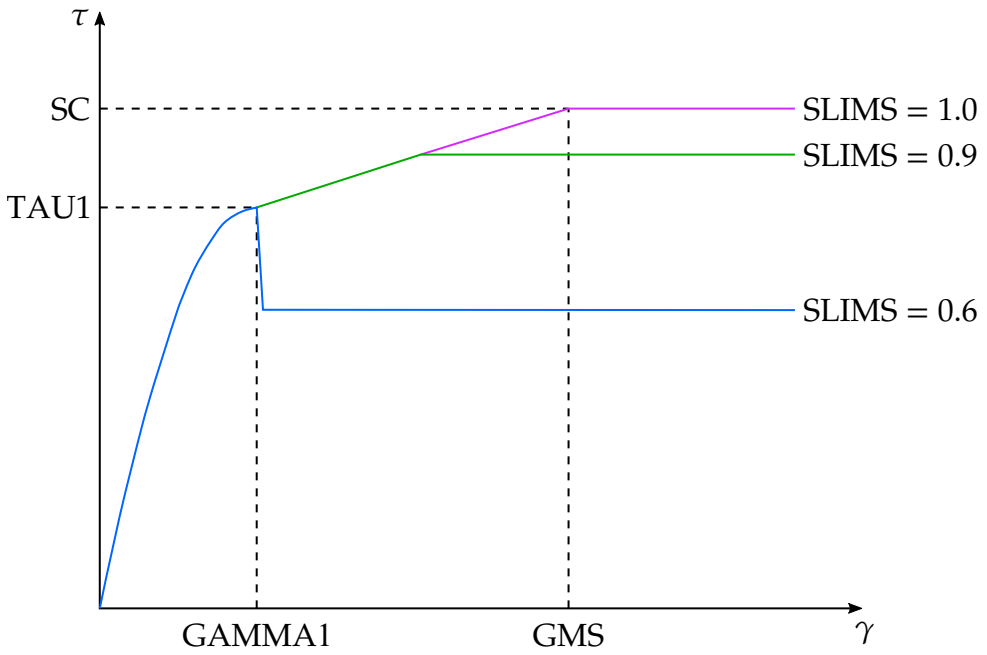



**Figure M58-2.** Stress-strain diagram for shear

5. **Shear failure of fabrics.** For fabric materials, we can assume a nonlinear stress-strain curve for the shear part for failure surface FS = -1, as given below. This is not possible for other values of FS.

   Three points define the curve as shown in Figure M58-2:

   a) the origin (0,0) is assumed,

   b) the limit of the first slightly nonlinear part (must be input), stress (TAU1) and strain (GAMMA1), and

   c) the shear strength at failure and shear strain at failure.

   In addition, a stress limiter can be used to keep the stress constant using the SLIMS field. This value must be positive and less than or equal to 1.0. It leads to elastoplastic behavior for the shear part. The default is $10^{-8}$, assuming almost brittle failure once the strength limit SC is reached.

6. **EC.** The EC field is ignored when thin shells use this material model. When used with thick shell elements of form 1, 2, or 6, a positive EC value will be used to evaluate thickness stress. If EC is set to zero or a negative number, then the minimum of EA and EB is used for the thickness stress calculation.

7. **Strain rate.** LS-DYNA uses the smoothed, direction-appropriate strain rate for any property specified to be strain-rate-dependent. For example, LS-DYNA uses strain rate in the $a$-direction when assessing properties in the $a$-direction. LS-

DYNA, however, uses the effective strain rate when determining the rate-dependence of ERODS for load curve LCEFS.

8. **EA / EB / EC < 0.0.** If a load curve specifies the uniaxial elastic stress as a function of strain behavior, the range of the strain space (abscissa values) must span from at least 5% negative (compressive) to 5% positive (tensile) strain.

## *MAT_COMPOSITE_FAILURE_*OPTION*_MODEL

This is Material Type 59.

Available options include:

SHELL

SOLID

SPH

depending on the element type the material is to be used with; see *PART. An equation of state (*EOS) is optional for SPH elements and is invoked by setting EOSID to a nonzero value in *PART. If an equation of state is used, only the deviatoric stresses are calculated by the material model and the pressure is calculated by the equation of state.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | KF | AOPT | MACF | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 5a.1.** This card is included if the SHELL keyword option is used.

| TSIZE | ALP | SOFT | FBRT | SR | SF | | |
|---|---|---|---|---|---|---|---|

**Card 5a.2.** This card is included if the SHELL keyword option is used.

| XC | XT | YC | YT | SC | | | |
|---|---|---|---|---|---|---|---|

**Card 5b.1.** This card is included if either the SOLID or SPH keyword option is used.

| SBA | SCA | SCB | XXC | YYC | ZZC | | |
|---|---|---|---|---|---|---|---|

**Card 5b.2.** This card is included if either the SOLID or SPH keyword option is used.

| XXT | YYT | ZZT | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definition:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Density |
| EA | $E_a$, Young's modulus - longitudinal direction |
| EB | $E_b$, Young's modulus - transverse direction |
| EC | $E_c$, Young's modulus - normal direction |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | KF | AOPT | MACF | | |
| Type | F | F | F | F | F | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus |
| GBC | $G_{bc}$, shear modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| GCA | $G_{ca}$, shear modulus |
| KF | Bulk modulus of failed material |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, $P$, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| VARIABLE | DESCRIPTION |
|---|---|
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP YP ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1 V2 V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO. |

**Card 5 for SHELL Keyword Option.**

| Card 5a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TSIZE | ALP | SOFT | FBRT | SR | SF | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TSIZE | Time step for automatic element deletion |
| ALP | Nonlinear shear stress parameter |
| SOFT | Softening reduction factor for material strength in crashfront elements |
| FBRT | Softening of fiber tensile strength |
| SR | $s_r$, reduction factor (default = 0.447) |
| SF | $s_f$, softening factor (default = 0.0) |

**Card 6 for SHELL Keyword Option.**

| Card 5a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SC | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength, *a*-axis (positive value) |

| VARIABLE | DESCRIPTION |
|---|---|
| XT | Longitudinal tensile strength, *a*-axis |
| YC | Transverse compressive strength, *b*-axis (positive value) |
| YT | Transverse tensile strength, *b*-axis |
| SC | Shear strength, *ab*-plane:<br>GT.0.0: Faceted failure surface theory<br>LT.0.0: Ellipsoidal failure surface theory |

**Card 5 for SPH and SOLID Keyword Options.**

| Card 5b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SBA | SCA | SCB | XXC | YYC | ZZC | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SBA | In plane shear strength |
| SCA | Transverse shear strength |
| SCB | Transverse shear strength |
| XXC | Longitudinal compressive strength *a*-axis (positive value) |
| YYC | Transverse compressive strength *b*-axis (positive value) |
| ZZC | Normal compressive strength *c*-axis (positive value) |

**Card 6 for SPH and SOLID Keyword Options.**

| Card 5b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XXT | YYT | ZZT | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XXT | Longitudinal tensile strength *a*-axis |
| YYT | Transverse tensile strength *b*-axis |
| ZZT | Normal tensile strength *c*-axis |

## *MAT_ELASTIC_WITH_VISCOSITY

This is Material Type 60, which was developed to simulate the forming of glass products (such as car windshields) at high temperatures. Deformation is by viscous flow, but elastic deformations can also be large. The material model, which allows for temperature-dependent viscosity, is suitable for treating a wide range of viscous flow problems. It is implemented for solid, shell, thick shell, and SPH elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | VO | A | B | C | LCID | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PR1 | PR2 | PR3 | PR4 | PR5 | PR6 | PR7 | PR8 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
| Type | F | F | F | F | F | F | F | F |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| V0 | Temperature independent dynamic viscosity coefficient, $V_0$. If defined, the temperature dependent viscosity defined below is skipped; see type i and ii definitions for viscosity below. |
| A | Dynamic viscosity coefficient; see type i and ii definitions below. |
| B | Dynamic viscosity coefficient; see type i and ii definitions below. |
| C | Dynamic viscosity coefficient; see type i and ii definitions below. |
| LCID | Load curve (see *DEFINE_CURVE) defining viscosity as a function of temperature; see type iii. (Optional.) |
| T1, T2, …, TN | Temperatures, $T_i$, define up to 8 values |
| PR1, PR2, …, PRN | Poisson's ratios for the temperatures $T_i$ |
| V1, V2, …, VN | Corresponding dynamic viscosity coefficients (define only one if not varying with temperature) |
| E1, E2, …, EN | Corresponding Young's moduli coefficients (define only one if not varying with temperature) |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA1, …, ALPHAN. | Corresponding thermal expansion coefficients |

**Remarks:**

Volumetric behavior is treated as linear elastic. The deviatoric strain rate is considered to be the sum of elastic and viscous strain rates:

$$\dot{\boldsymbol{\varepsilon}}'_{\text{total}} = \dot{\boldsymbol{\varepsilon}}'_{\text{elastic}} + \dot{\boldsymbol{\varepsilon}}'_{\text{viscous}} = \frac{\dot{\boldsymbol{\sigma}}'}{2G} + \frac{\boldsymbol{\sigma}'}{2v} \ ,$$

where $G$ is the elastic shear modulus, $v$ is the viscosity coefficient, and bold indicates a tensor. The stress increment over one timestep $dt$ is

$$d\sigma' \ = 2G\dot{\boldsymbol{\varepsilon}}_{\text{total}}dt - \frac{G}{v}dt\ \boldsymbol{\sigma}' \ .$$

The stress before the update is used for $\boldsymbol{\sigma}'$. For shell elements the through-thickness strain rate is calculated as follows:

$$d\sigma_{33} = 0 = K(\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33})dt + 2G\dot{\varepsilon}'_{33}dt - \frac{G}{v}dt\sigma'_{33} \ ,$$

where the subscript 33 denotes the through-thickness direction and $K$ is the elastic bulk modulus. This leads to:

$$\dot{\varepsilon}_{33} = -a(\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22}) + bp$$

$$a = \frac{\left(K - \frac{2}{3}G\right)}{\left(K + \frac{4}{3}G\right)}$$

$$b = \frac{Gdt}{v\left(K + \frac{4}{3}G\right)}$$

in which $p$ is the pressure defined as the negative of the hydrostatic stress.

The variation of viscosity with temperature can be defined in any one of the 3 ways.

i) Constant, $V = V_0$. Do not define constants, A, B, and C, or the piecewise curve (leave Card 4 blank).

ii) $V = V_0 \times 10^{\left(\frac{A}{T-B}+C\right)}$

iii) Piecewise curve: define the variation of viscosity with temperature.

> **NOTE:** Viscosity is inactive during dynamic relaxation.

## *MAT_ELASTIC_WITH_VISCOSITY_CURVE

This is Material Type 60, which was developed to simulate the forming of glass products (such as car windshields) at high temperatures. Deformation is by viscous flow, but elastic deformations can also be large. The material model, which allows for temperature-dependent viscosity, is suitable for treating a wide range of viscous flow problems. It is implemented for solid, shell, thick shell, and SPH elements. Load curves are used to represent the temperature dependence of Poisson's ratio, Young's modulus, the coefficient of expansion, and the viscosity.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | V0 | A | B | C | LCID | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PR_LC | YM_LC | A_LC | V_LC | V_LOG | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| V0 | Temperature independent dynamic viscosity coefficient, $V_0$. If defined, the temperature dependent viscosity curve, V_LC, is skipped; see type i and ii definitions for viscosity below. |
| A | Dynamic viscosity coefficient; see type i and ii definitions below. |
| B | Dynamic viscosity coefficient; see type i and ii definitions below. |
| C | Dynamic viscosity coefficient; see type i and ii definitions below. |
| LCID | Load curve (see *DEFINE_CURVE) defining factor on dynamic viscosity as a function of temperature; see type iii. (Optional). |

| VARIABLE | DESCRIPTION |
|---|---|
| PR_LC | Load curve (see *DEFINE_CURVE) defining Poisson's ratio as a function of temperature. |
| YM_LC | Load curve (see *DEFINE_CURVE) defining Young's modulus as a function of temperature. |
| A_LC | Load curve (see *DEFINE_CURVE) defining the coefficient of thermal expansion as a function of temperature. |
| V_LC | Load curve (see *DEFINE_CURVE) or table for defining the dynamic viscosity<br>GT.0: Load curve ID for defining dynamic viscosity as a function of temperature<br>LT.0: \|V_LC\| is table ID giving dynamic viscosity as a function of shear strain rate and temperature. The dynamic viscosity as a function of temperature curves are indexed by the shear strain rate. |
| V_LOG | Flag for the form of V_LC:<br>EQ.1.0: The value specified in V_LC is the natural logarithm of the viscosity, $\ln V$. The value interpolated from the curve is then exponentiated to obtain the viscosity. The logarithmic form is useful if the value of the viscosity changes by orders of magnitude over the temperature range of the data.<br>EQ.0.0: The value specified in V_LC is the viscosity. |

**Remarks:**

Volumetric behavior is treated as linear elastic. The deviatoric strain rate is considered to be the sum of elastic and viscous strain rates:

$$\dot{\boldsymbol{\varepsilon}}'_{\text{total}} = \dot{\boldsymbol{\varepsilon}}'_{\text{elastic}} + \dot{\boldsymbol{\varepsilon}}'_{\text{viscous}} = \frac{\dot{\boldsymbol{\sigma}}'}{2G} + \frac{\boldsymbol{\sigma}'}{2v}$$

where $G$ is the elastic shear modulus, $v$ is the viscosity coefficient, and bold indicates a tensor. The stress increment over one timestep $dt$ is

$$d\boldsymbol{\sigma}' = 2G\dot{\boldsymbol{\varepsilon}}'_{\text{total}}\, dt - \frac{G}{v}\, dt\, \boldsymbol{\sigma}'$$

The stress before the update is used for $\boldsymbol{\sigma}'$. For shell elements the through-thickness strain rate is calculated as follows.

$$d\sigma_{33} = 0 = K(\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33})dt + 2G\dot{\varepsilon}'_{33}dt - \frac{G}{v}dt\sigma'_{33}$$

where the subscript 33 denotes the through-thickness direction and $K$ is the elastic bulk modulus. This leads to:

$$\dot{\varepsilon}_{33} = -a(\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22}) + bp$$

$$a = \frac{\left(K - \frac{2}{3}G\right)}{\left(K + \frac{4}{3}G\right)}$$

$$b = \frac{Gdt}{v\left(K + \frac{4}{3}G\right)}$$

in which $p$ is the pressure defined as the negative of the hydrostatic stress.

The variation of viscosity with temperature can be defined in any one of the 3 ways:

i) Constant, $V = V_0$. Do not define constants, A, B, and C, or the curve, V_LC.

ii) $V = V_0 \times 10^{(\frac{A}{T-B}+C)}$

iii) Piecewise curve: define the variation of viscosity with temperature.

**NOTE:** Viscosity is inactive during dynamic relaxation.

## *MAT_KELVIN-MAXWELL_VISCOELASTIC

This is Material Type 61. This material is a classical Kelvin-Maxwell model for modeling viscoelastic bodies, such as foams. This model is valid for solid elements, SPH elements, and thick shell formulations 3, 5, and 7. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | G0 | GI | DC | FO | SO |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Bulk modulus (elastic) |
| G0 | Short-time shear modulus, $G_0$ |
| GI | Long-time (infinite) shear modulus, $G_\infty$ |
| DC | Constant depending on formulation:<br>FO.EQ.0.0: Maxwell decay constant<br>FO.EQ.1.0: Kelvin relaxation constant |
| FO | Formulation option:<br>EQ.0.0: Maxwell<br>EQ.1.0: Kelvin |
| SO | Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step:<br>EQ.0.0: Maximum principal strain that occurs during the calculation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Maximum magnitude of the principal strain values that occurs during the calculation |
| | EQ.2.0: Maximum effective strain that occurs during the calculation |

**Remarks:**

The shear relaxation behavior is described for the Maxwell model by:

$$G(t) = G + (G_0 - G_\infty)e^{-\beta t}$$

A Jaumann rate formulation is used

$$\overset{\nabla}{\sigma}'_{\mathrm{ij}} = 2\int_0^t G(t-\tau)\ D'_{ij}(\tau)dt\ ,$$

where the prime denotes the deviatoric part of the stress rate, $\overset{\nabla}{\sigma}_{ij}$, and the strain rate $D_{ij}$. For the Kelvin model, the stress evolution equation is defined as:

$$\dot{s}_{ij} + \frac{1}{\tau}s_{ij} = \left(1+\delta_{ij}\right)G_0\dot{e}_{ij} + \left(1+\delta_{ij}\right)\frac{G_\infty}{\tau}\dot{e}_{ij}$$

The strain data as written to the database may be used to predict damage; see [Bandak 1991].

## *MAT_VISCOUS_FOAM

This is Material Type 62. It was developed to represent the Confor Foam on the ribs of EuroSID side impact dummy. It is only valid for solid elements and thick shell formulations 3, 5, and 7, mainly under compressive loading.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | N1 | V2 | E2 | N2 | PR |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E1 | Initial Young's modulus, $E_1$ |
| N1 | Exponent in power law for Young's modulus, $n_1$ |
| V2 | Viscous coefficient, $V_2$ |
| E2 | Elastic modulus for viscosity, $E_2$; see Remarks below. |
| N2 | Exponent in power law for viscosity, $n_2$ |
| PR | Poisson's ratio, $\nu$ |

### Remarks:

The model consists of a nonlinear elastic stiffness in parallel with a viscous damper. The elastic stiffness is intended to limit total crush while the viscosity absorbs energy. The stiffness $E_2$ exists to prevent timestep problems. It is used for time step calculations as long as $E_1^t$ is smaller than $E_2$. It has to be carefully chosen to take into account the stiffening effects of the viscosity. Both $E_1$ and $V_2$ are nonlinear with crush as follows:

$$E_1^t = E_1(V^{-n_1})$$
$$V_2^t = V_2|1 - V|^{n_2}$$

Here, $V$ is the relative volume defined by the ratio of the current to initial volume. Viscosity generates a shear stress given by

$$\tau = V_2\dot{\gamma} \ .$$

$\dot{\gamma}$ is the engineering shear strain rate.

Table showing typical values (units of N, mm, s):

| Variable | Value |
|---|---|
| E1 | 0.0036 |
| N1 | 4.0 |
| V2 | 0.0015 |
| E2 | 100.0 |
| N2 | 0.2 |
| PR | 0.05 |

## *MAT_CRUSHABLE_FOAM

This is Material Type 63. This material type models crushable foam. It includes three different formulations. The original formulation (MODEL = 0) has optional damping and tension cutoff. It also features fully elastic unloading and treats tension as elastic-perfectly-plastic at the tension cut-off value. *MAT_MODIFIED_CRUSHABLE_FOAM is a modified version of this original formulation that includes strain rate effects. The two alternative formulations (invoked with MODEL = 1 or 2) incorporate an elliptical yield surface in $p$-$q$ space and include independent definitions of elastic and plastic Poisson's ratio. They also both support rate dependence. See Remarks 2 and 3 for further details.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | LCID | TSC | DAMP | MODEL |
| Type | A | F | F | F | F | F | F | I |
| Default | none | none | none | none | none | 0.0 | 0.10 | 0 |

Optional card.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PRP | K | RFILTF | BVFLAG | SRCRT | ESCAL | KT | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus. For MODEL ≤ 0, E may affect contact stiffness but otherwise is not used. The final slope of the curve LCID determines the elastic stiffness for loading and unloading. The time step calculation also uses this slope. For MODEL = 1 or 2, the material law uses E as the Young's modulus. |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | (Elastic) Poisson's ratio |
| LCID | Load curve (or table) ID with a meaning that depends on MODEL:<br>MODEL.LE.0: Load curve ID defining yield stress as a function of volumetric strain, $\gamma$ (see Figure M63-1).<br>MODEL.GE.1: Load curve, table ID, or 3D table ID. If specifying a load curve ID, the load curve defines uniaxial yield stress under compression, $\sigma_c$, as a function of equivalent plastic strain. If specifying a table ID, each strain rate references a load curve ID that gives uniaxial yield stress as a function of equivalent plastic strain. If specifying a 3D table ID, uniaxial yield stress is given as a function of history variable #8 (3D table), strain rate (table), and equivalent plastic strain (curve). |
| TSC | Tensile stress cutoff (only for MODEL ≤ 0). A nonzero, positive value is strongly recommended for realistic behavior. |
| DAMP | Rate sensitivity via damping coefficient (.05 < recommended value < .50). Only available for MODEL ≤ 0. |
| MODEL | Choice of material model formulation:<br>EQ.-1: Same as MODEL = 0, but with a potentially wrong stress response in unloading. A bug fix for MODEL = 0 in R17 may change results, so this can be seen as a fallback option for obtaining the old behavior.<br>EQ.0: Original approach (default)<br>EQ.1: Elliptical yield surface in $p$-$q$ space with symmetric tension-compression behavior (isotropic hardening)<br>EQ.2: Elliptical yield surface in $p$-$q$ space with asymmetric tension-compression behavior (volumetric hardening) |
| PRP | Plastic Poisson's ratio (only for MODEL = 1 or 2). PRP determines the yield potential, that is, the plastic flow direction. It ranges from -1 to 0.5. |
| K | Ratio of $\sigma_c^0$, initial uniaxial yield stress, to $p_c^0$, initial hydrostatic yield stress under compression (only for MODEL = 1 or 2). K determines the shape of the yield ellipse. |

| VARIABLE | DESCRIPTION |
|---|---|
| RFILTF | Rate filtering parameter for MODEL = 1 or 2 (0.0 ≤ RFILTF < 1.0):<br>EQ.0.0: Plastic strain rates are used if LCID is a table (default).<br>GT.0.0: Filtered total strain rates are used if LCID is a table. See Remark 2. |
| BVFLAG | Bulk viscosity deactivation flag (for MODEL = 1 or 2):<br>EQ.0.0: Bulk viscosity active (default).<br>EQ.1.0: No bulk viscosity |
| SRCRT | Critical stretch ratio for high compression regime (MODEL = 1 or 2). For instance, a value of 0.1 for SRCRT means adding an elastic stiffness at 10% residual length to avoid excessive compression. |
| ESCAL | Scale factor for high compression stiffness as a multiple of the Young's modulus (MODEL = 1 or 2). |
| KT | Ratio of $p_t$, absolute yield stress in hydrostatic tension, to $p_c^0$, initial yield stress in hydrostatic compression (only for MODEL = 2). KT defines the shift of the yield ellipse in the direction of the $p$-axis. With KT = 1, the initial yield ellipse is symmetric (with respect to the $q$-axis), but it always becomes unsymmetric through hardening. |

**Remarks:**

1. **Volumetric strain.** The volumetric strain is defined in terms of the relative volume, $V$, as:

$$\gamma = 1 - V$$

The relative volume is the ratio of the current volume to the initial volume. In place of the effective plastic strain in the d3plot database, the integrated volumetric strain (natural logarithm of the relative volume) is output.

2. **Symmetric elliptical yield surface formulation.** Setting MODEL = 1 invokes an alternative formulation for crushable with the following yield condition:

$$F = \sqrt{q^2 + \alpha^2 p^2} - Y_s = 0 \ ,$$

This yield condition corresponds to an elliptical yield surface in the pressure ($p$) – deviator Mises stress ($q$) space; see Figure M63-2. In the above yield condition,

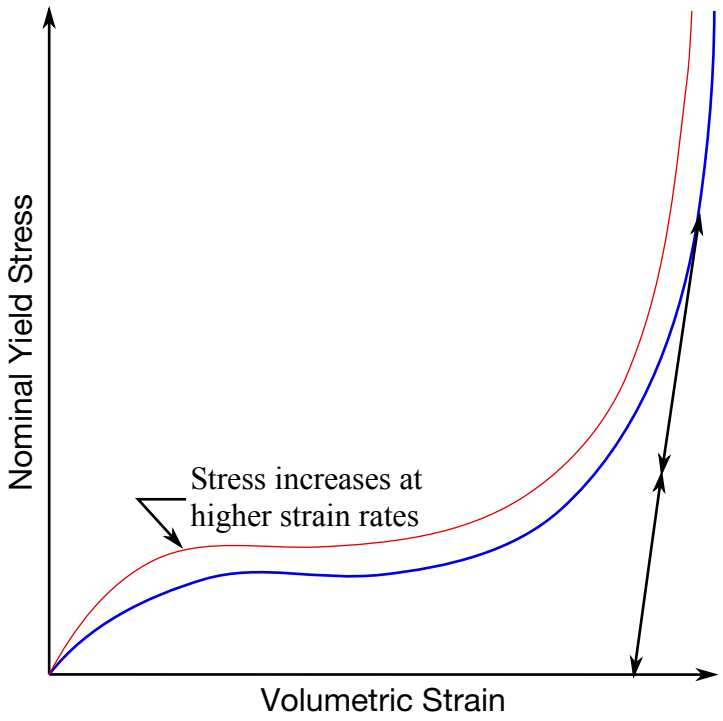



**Figure M63-1.** Behavior of strain rate sensitive crushable foam. Unloading is elastic to the tension cutoff. Subsequent reloading follows the unloading curve.

$$p = -\frac{1}{3} I_1$$

$$q = \sqrt{3 J_2}$$

$$\alpha = \frac{3k}{\sqrt{9 - k^2}}$$

$$Y_s = \sigma_c \sqrt{1 + \left(\frac{\alpha}{3}\right)^2}$$

$Y_s$, the yield stress, gives the size of the elliptical yield surface. $k$, the stress ratio, is given by

$$k = \frac{\sigma_c^0}{p_c^0} \ .$$

$k$ describes the shape of the yield surface and is input as field K. It ranges from 0 (von Mises) to less than 3.

For lateral straining, define individual Poisson's ratios for the elastic (PR) and the plastic (PRP) regimes. The associated flow potential is given by

$$G = \sqrt{q^2 + \beta^2 p^2} \ ,$$

where

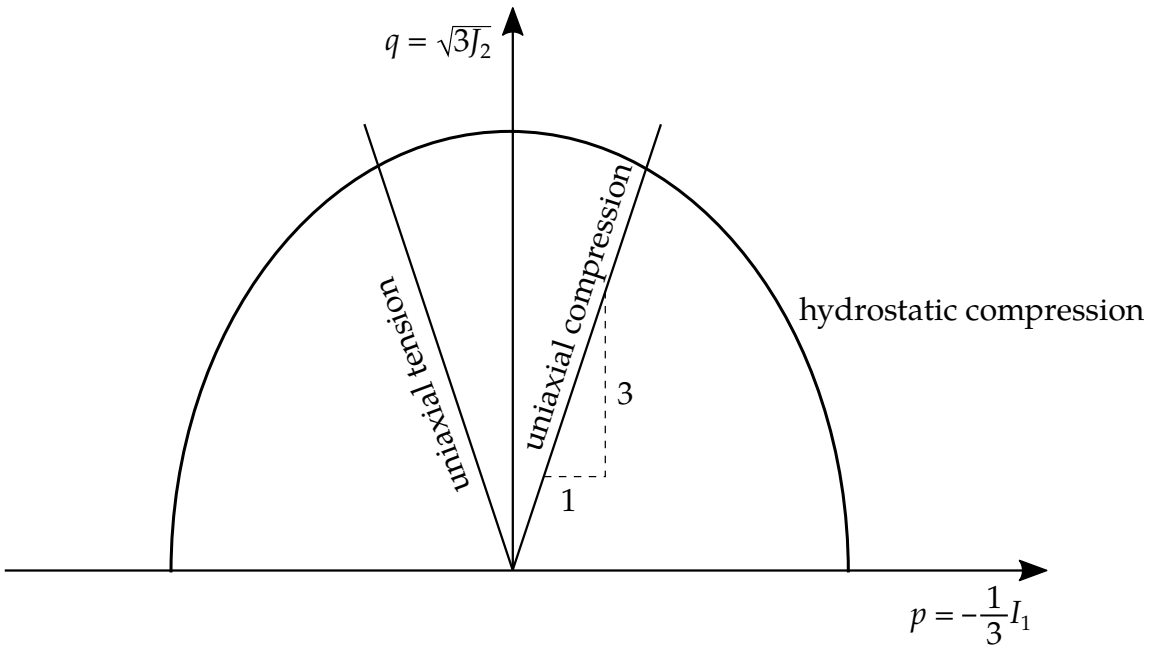



**Figure M63-2.** Yield surface for MODEL = 1 in $p$-$q$ space

$$\beta = \frac{3}{\sqrt{2}}\sqrt{\frac{1-2\nu^{\mathrm{pl}}}{1+\nu^{\mathrm{pl}}}}$$

with plastic Poisson's ratio $\nu^{\mathrm{pl}}$ (PRP). A yield curve or table specified with LCID defines the hardening. If LCID is a yield curve, it relates uniaxial yield stress, $\sigma_c$, as a function of equivalent plastic strain. To consider rate dependence, make LCID a table. RFILTF determines if the algorithm uses plastic strain rates (RFILT = 0.0) or filtered total strain rates (0.0 < RFILTF < 1.0). In the latter case, we use exponential smoothing:

$$\dot{\varepsilon}^{\mathrm{avg}} = \mathrm{RFILTF} \times \dot{\varepsilon}_{n-1}^{\mathrm{avg}} + (1 - \mathrm{RFILTF}) \times \dot{\varepsilon}_{n}^{\mathrm{cur}} \ ,$$

Thus, as RFILTF increases, more filtering occurs.

3. **Asymmetric elliptical yield surface formulation.** Setting MODEL = 2 invokes another formulation for crushable foam with different plastic deformation behavior under tension and compression. It has the following yield condition:

$$F = \sqrt{q^2 + \alpha^2(p - \bar{p})^2} - Y_s = 0 \ ,$$

corresponding to an elliptical yield surface in the pressure ($p$) – deviator Mises stress ($q$) space with its center at $\bar{p} = (p_c - p_t)/2$; see Figure M63-3. In the above yield condition,

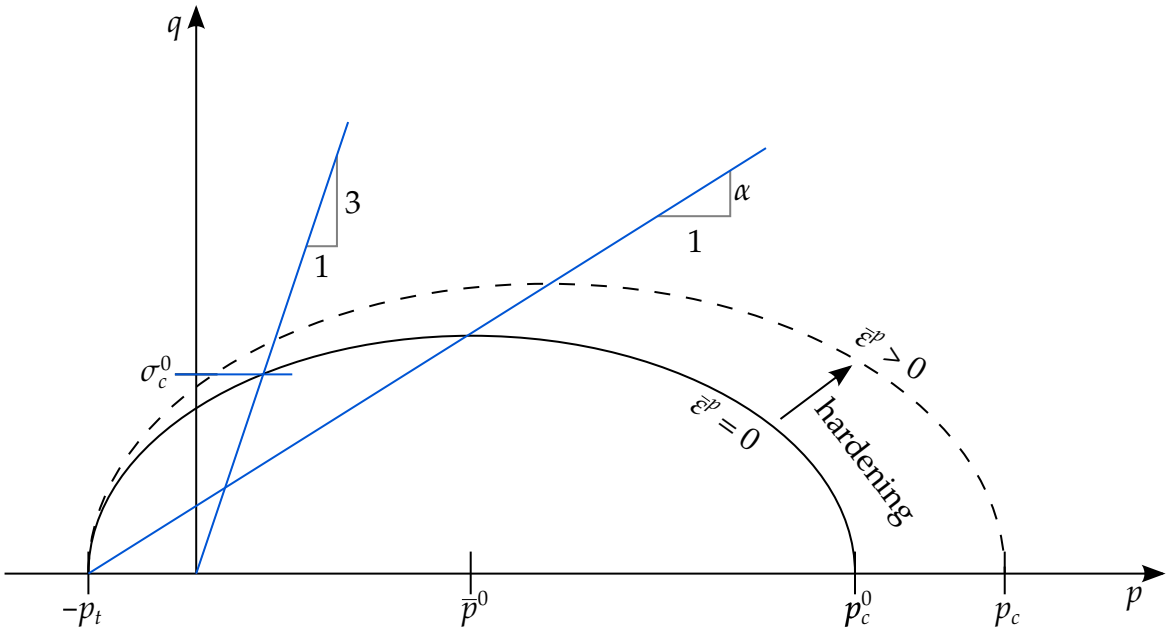



**Figure M63-3.** Yield surface for MODEL = 2 in $p$-$q$ space

$$p = -\frac{1}{3}I_1$$
$$q = \sqrt{3J_2}$$
$$\alpha = \frac{3k}{\sqrt{(3k_t + k)(3 - k)}}$$
$$Y_s = \alpha\frac{p_c + p_t}{2}$$

The yield stress, $Y_s$, gives the size of the elliptical yield surface. The yield stress in hydrostatic compression, $p_c$, is a function of the uniaxial yield stress, $\sigma_c$, through this equation:

$$p_c = \frac{\sigma_c\left(\sigma_c\left(\frac{1}{\alpha^2} + \frac{1}{9}\right) + \frac{1}{3}p_t\right)}{p_t + \frac{1}{3}\sigma_c}.$$

The following equations give the stress ratios $k$ and $k_t$:

$$k = \frac{\sigma_c^0}{p_c^0}, \quad k_t = \frac{p_t}{p_c^0}.$$

$k$ describes the shape of the yield surface. Input field K sets $k$. It ranges from 0 (von Mises) to less than 3. $k_t$ describes the shift of the yield ellipse on the $p$-axis. Input field KT sets $k_t$.

The flow potential $G$ is the same as for MODEL = 1. The other statements from Remark 2 on plastic Poisson's ratio PRP, yield curve input with LCID, and strain rate filtering with RFILTF also hold for MODEL = 2.

## *MAT_RATE_SENSITIVE_POWERLAW_PLASTICITY

This is Material Type 64 which will model strain rate sensitive elasto-plastic material with a power law hardening. Optionally, the coefficients can be defined as functions of the effective plastic strain.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | K | M | N | E0 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0001 | none | 0.0002 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VP | EPS0 | RFILTF | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | 1.0 | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus of elasticity |
| PR | Poisson's ratio |
| K | Material constant, $k$. If K < 0, the absolute value of K is taken as the load curve number that defines $k$ as a function of effective plastic strain. |
| M | Strain hardening coefficient, $m$. If M < 0, the absolute value of M is taken as the load curve number that defines $m$ as a function of effective plastic strain. |

| VARIABLE | DESCRIPTION |
|---|---|
| N | Strain rate sensitivity coefficient, $n$. If N < 0, the absolute value of N is taken as the load curve number that defines $n$ as a function of effective plastic strain. |
| E0 | Initial strain rate (default = 0.0002) |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |
| EPS0 | Quasi-static threshold strain rate. See description under *MAT_-015. |
| RFILTF | Smoothing factor on the effective strain rate for solid elements when VP = 0: $\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$ |

**Remarks:**

1. **Constitutive Relationship.** This material model follows a constitutive relationship of the form:

$$\sigma = k\varepsilon^m \dot{\varepsilon}^n \ ,$$

where $\sigma$ is the yield stress, $\varepsilon$ is the effective plastic strain, and $\dot{\varepsilon}$ is the effective total strain rate (VP = 0), or the effective plastic strain rate (VP = 1). The constants $k$, $m$, and $n$ can be expressed as functions of effective plastic strain or can be constant with respect to the plastic strain. The case of no strain hardening can be obtained by setting the exponent of the plastic strain equal to a very small positive value, such as 0.0001.

The initial yield stress is obtained through:

$$\sigma_0 = k\varepsilon_0^m \dot{\varepsilon}^n \ ,$$

with an initial effective strain of

$$\varepsilon_0 = \max\left(0.001, \left(\frac{E}{k\dot{\varepsilon}^n}\right)^{1/(m-1)}\right) \ .$$

2. **Superplastic Forming.** This model can be combined with the superplastic forming input (see *LOAD_SUPERPLASTIC_FORMING) to control the magnitude of the pressure in the pressure boundary conditions. Controlling the pressure limits the effective plastic strain rate so that it does not exceed a maximum value at any integration point within the model.

3. **Viscoplastic Formulation.** A fully viscoplastic formulation is optional. An additional cost is incurred, but the improvement in results can be dramatic.

## *MAT_MODIFIED_ZERILLI_ARMSTRONG

This is Material Type 65 which is a rate and temperature sensitive plasticity model that is sometimes preferred in ordnance design calculations.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | E0 | N | TROOM | PC | SPALL |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | EFAIL | VP |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B1 | B2 | B3 | G1 | G2 | G3 | G4 | BULK |
| Type | F | F | F | F | F | F | F | F |

**Optional FCC Metal Card.** This card is optional.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| E0 | $\dot{\varepsilon}_0$, factor to normalize strain rate |
| N | $n$, exponent for bcc metal |
| TROOM | $T_r$, room temperature |
| PC | $p_0$, Pressure cutoff |
| SPALL | Spall Type:<br>EQ.1.0: minimum pressure limit<br>EQ.2.0: maximum principal stress<br>EQ.3.0: minimum pressure cutoff |
| C*i* | $C_i$, coefficients for flow stress; see Remark 1 below. |
| EFAIL | Failure strain for erosion |
| VP | Formulation for rate effects:<br>EQ.0.0: scale yield stress (default)<br>EQ.1.0: viscoplastic formulation |
| B*i* | $B_i$, coefficients for polynomial to represent temperature dependency of flow stress yield |
| G*i* | $G_i$, coefficients for defining heat capacity and temperature dependency of heat capacity |
| BULK | Bulk modulus defined for shell elements only. Do not input for solid elements. |
| M | $m$, exponent for FCC metal (default = 0.5). This field is only used when N = 0.0 on Card 1. |

**Remarks:**

1. **Flow Stress.** The Zerilli-Armstrong Material Model expresses the flow stress as follows.

   a) For FCC metals ($n = 0$),

$$\sigma = C_1 + \left\{C_2(\varepsilon^p)^{\mathrm{m}}\left[e^{[-C_3+C_4\ln(\dot{\varepsilon}^*)]T}\right] + C_5\right\}\left[\frac{\mu(T)}{\mu(293)}\right],$$

where $\varepsilon^p$ is the effective plastic strain and $\dot{\varepsilon}^*$ is the effective plastic strain rate defined as

$$\dot{\varepsilon}^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \ .$$

$\dot{\varepsilon}_0 = 1, 10^{-3}, 10^{-6}$ for time units of seconds, milliseconds, and microseconds, respectively.

b) For BCC metals ($n > 0$),

$$\sigma = C_1 + C_2 e^{[-C_3 + C_4 \ln(\dot{\varepsilon}^*)]T} + [C_5(\varepsilon^p)^n + C_6]\left[\frac{\mu(T)}{\mu(293)}\right] \ ,$$

where

$$\frac{\mu(T)}{\mu(293)} = B_1 + B_2 T + B_3 T^2 \ .$$

2. **Heat Capacity.** The relationship between heat capacity (specific heat) and temperature may be characterized by a cubic polynomial equation as follows:

$$C_p = G_1 + G_2 T + G_3 T^2 + G_4 T^3$$

3. **Viscoplastic Formulation.** A fully viscoplastic formulation is optional. An additional cost is incurred but the improvement is results can be dramatic.

## *MAT_LINEAR_ELASTIC_DISCRETE_BEAM

This is Material Type 66. This material model is for simulating the effects of a linear elastic beam by using six springs, each acting about one of the six local degrees of freedom. The two nodes defining a beam may be coincident to give a zero-length beam or offset to give a finite-length beam. For finite-length discrete beams, the absolute value of the variable SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam, to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this model. A triad is used to orient the beam for the directional springs. Translational/rotational stiffness and viscous damping effects are considered for a local Cartesian system; see Remark 1. Applications for this material include the modeling of joint stiffnesses.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TKR | TKS | TKT | RKR | RKS | RKT |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TDR | TDS | TDT | RDR | RDS | RDT | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FOR | FOS | FOT | MOR | MOS | MOT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also “volume” in the *SECTION_BEAM definition. |

| VARIABLE | DESCRIPTION |
|---|---|
| TKR | Translational stiffness along local *r*-axis; see Remarks 1 and 2 below. |
| TKS | Translational stiffness along local *s*-axis |
| TKT | Translational stiffness along local *t*-axis |
| RKR | Rotational stiffness about the local *r*-axis |
| RKS | Rotational stiffness about the local *s*-axis |
| RKT | Rotational stiffness about the local *t*-axis |
| TDR | Translational viscous damper along local *r*-axis (optional) |
| TDS | Translational viscous damper along local *s*-axis (optional) |
| TDT | Translational viscous damper along local *t*-axis (optional) |
| RDR | Rotational viscous damper about the local *r*-axis (optional) |
| RDS | Rotational viscous damper about the local *s*-axis (optional) |
| RDT | Rotational viscous damper about the local *t*-axis (optional) |
| FOR | Preload force in *r*-direction (optional) |
| FOS | Preload force in *s*-direction (optional) |
| FOT | Preload force in *t*-direction (optional) |
| MOR | Preload moment about *r*-axis (optional) |
| MOS | Preload moment about *s*-axis (optional) |
| MOT | Preload moment about *t*-axis (optional) |

**Remarks:**

1. **Coordinate System and Orientation.** The formulation of the discrete beam (type 6) assumes that the beam is of zero length and requires no orientation node. A small distance between the nodes joined by the beam is permitted. The local coordinate system which determines $(r, s, t)$ is given by the coordinate ID (see *DEFINE_COORDINATE_*OPTION*) in the cross-sectional input (see *SECTION_BEAM), where the global system is the default. The local coordinate

system axes can rotate with either node of the beam or an average rotation of both nodes (see SCOOR variable in *SECTION_BEAM).

2. **Null Stiffness.** For null stiffness coefficients, no forces corresponding to these null values will develop. The viscous damping coefficients are optional.

3. **Rotational Displacement.** Rotational displacement is measured in radians.

## *MAT_NONLINEAR_ELASTIC_DISCRETE_BEAM

This is Material Type 67. This material model is defined for simulating the effects of nonlinear elastic and nonlinear viscous beams by using six springs, each acting about one of the six local degrees of freedom. The two nodes defining a beam may be coincident to give a zero-length beam or offset to give a finite-length beam. For finite-length discrete beams, the absolute value of the variable SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which aligns the local *r*-axis along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs. Arbitrary curves can be used to model transitional/rotational stiffness and damping effects. See remarks below.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | LCIDTR | LCIDTS | LCIDTT | LCIDRR | LCIDRS | LCIDRT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCIDTDR | LCIDTDS | LCIDTDT | LCIDRDR | LCIDRDS | LCIDRDT | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| FOR | FOS | FOT | MOR | MOS | MOT | | |
|---|---|---|---|---|---|---|---|

**Card 4.** To consider failure, this card must be defined. Otherwise it is optional.

| FFAILR | FFAILS | FFAILT | MFAILR | MFAILS | MFAILT | | |
|---|---|---|---|---|---|---|---|

**Card 5.** To consider failure, this card must be defined. Otherwise it is optional.

| UFAILR | UFAILS | UFAILT | TFAILR | TFAILS | TFAILT | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | LCIDTR | LCIDTS | LCIDTT | LCIDRR | LCIDRS | LCIDRT |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM. |
| LCIDTR | Load curve ID defining translational force resultant along local *r*-axis as a function of relative translational displacement; see Remarks 1 and 3 and Figure M67-1. |
| LCIDTS | Load curve ID defining translational force resultant along local *s*-axis as a function of relative translational displacement. |
| LCIDTT | Load curve ID defining translational force resultant along local *t*-axis as a function of relative translational displacement. |
| LCIDRR | Load curve ID defining rotational moment resultant about local *r*-axis as a function of relative rotational displacement. |
| LCIDRS | Load curve ID defining rotational moment resultant about local *s*-axis as a function of relative rotational displacement. |
| LCIDRT | Load curve ID defining rotational moment resultant about local *t*-axis as a function of relative rotational displacement. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDTDR | LCIDTDS | LCIDTDT | LCIDRDR | LCIDRDS | LCIDRDT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDTDR | Load curve ID defining translational damping force resultant along local *r*-axis as a function of relative translational velocity. |
| LCIDTDS | Load curve ID defining translational damping force resultant along local *s*-axis as a function of relative translational velocity. |
| LCIDTDT | Load curve ID defining translational damping force resultant along local *t*-axis as a function of relative translational velocity. |
| LCIDRDR | Load curve ID defining rotational damping moment resultant about local *r*-axis as a function of relative rotational velocity. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDRDS | Load curve ID defining rotational damping moment resultant about local *s*-axis as a function of relative rotational velocity. |
| LCIDRDT | Load curve ID defining rotational damping moment resultant about local *t*-axis as a function of relative rotational velocity. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FOR | FOS | FOT | MOR | MOS | MOT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FOR | Preload force in *r*-direction (optional). |
| FOS | Preload force in *s*-direction (optional). |
| FOT | Preload force in *t*-direction (optional). |
| MOR | Preload moment about *r*-axis (optional). |
| MOS | Preload moment about *s*-axis (optional). |
| MOT | Preload moment about *t*-axis (optional). |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FFAILR | FFAILS | FFAILT | MFAILR | MFAILS | MFAILT | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FFAILR | Optional failure parameter. If zero, the corresponding force, $F_r$, is not considered in the failure calculation. See Remark 4. |

| VARIABLE | DESCRIPTION |
|---|---|
| FFAILS | Optional failure parameter. If zero, the corresponding force, $F_s$, is not considered in the failure calculation. |
| FFAILT | Optional failure parameter. If zero, the corresponding force, $F_t$, is not considered in the failure calculation. |
| MFAILR | Optional failure parameter. If zero, the corresponding moment, $M_r$, is not considered in the failure calculation. |
| MFAILS | Optional failure parameter. If zero, the corresponding moment, $M_s$, is not considered in the failure calculation. |
| MFAILT | Optional failure parameter. If zero, the corresponding moment, $M_t$, is not considered in the failure calculation. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UFAILR | UFAILS | UFAILT | TFAILR | TFAILS | TFAILT | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| UFAILR | Optional failure parameter. If zero, the corresponding displacement, $u_r$, is not considered in the failure calculation. See Remark 4. |
| UFAILS | Optional failure parameter. If zero, the corresponding displacement, $u_s$, is not considered in the failure calculation. |
| UFAILT | Optional failure parameter. If zero, the corresponding displacement, $u_t$, is not considered in the failure calculation. |
| TFAILR | Optional failure parameter. If zero, the corresponding rotation, $\theta_r$, is not considered in the failure calculation. |
| TFAILS | Optional failure parameter. If zero, the corresponding rotation, $\theta_s$, is not considered in the failure calculation. |

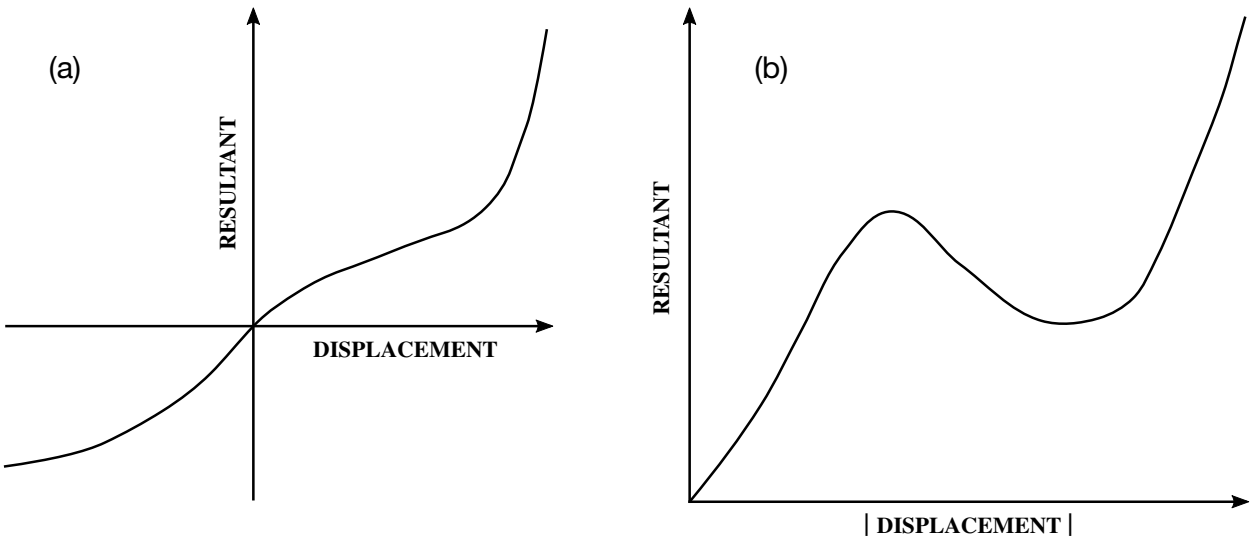



**Figure M67-1.** The resultant forces and moments are determined by a table lookup. If the origin of the load curve is at [0,0] as in (b) and tension and compression responses are symmetric.

| VARIABLE | DESCRIPTION |
|---|---|
| TFAILT | Optional failure parameter. If zero, the corresponding rotation, $\theta_t$, is not considered in the failure calculation. |

**Remarks:**

1. **Null Load Curve IDs.** For null load curve IDs, no forces are computed.

2. **Discrete Beam Formulation.** The formulation of the discrete beam (type 6) assumes that the beam is of zero length and requires no orientation node. A small distance between the nodes joined by the beam is permitted. The local coordinate system which determines $(r,s,t)$ is given by the coordinate ID (see *DEFINE_COORDINATE_*OPTION*) in the cross-sectional input (see *SECTION_BEAM) where the global system is the default. The local coordinate system axes can rotate with either node of the beam or an average rotation of both nodes (see SCOOR variable in *SECTION_BEAM).

3. **Tension and Compression.** If different behavior in tension and compression is desired in the calculation of the force resultants, the load curve(s) must be defined in the negative quadrant starting with the most negative displacement then increasing monotonically to the most positive. If the load curve behaves similarly in tension and compression, define only the positive quadrant. Whenever displacement values fall outside of the defined range, the resultant forces will be extrapolated. Figure M67-1 depicts a typical load curve for a force resultant. Load curves used for determining the damping forces and moment resultants always act identically in tension and compression, since only the

positive quadrant values are considered, that is, start the load curve at the origin [0,0].

4. **Failure.** Catastrophic failure based on force resultants occurs if the following inequality is satisfied.

$$\left(\frac{F_r}{F_r^{\text{fail}}}\right)^2+\left(\frac{F_s}{F_s^{\text{fail}}}\right)^2+\left(\frac{F_t}{F_t^{\text{fail}}}\right)^2+\left(\frac{M_r}{M_r^{\text{fail}}}\right)^2+\left(\frac{M_s}{M_s^{\text{fail}}}\right)^2+\left(\frac{M_t}{M_t^{\text{fail}}}\right)^2-1.\geq 0\ .$$

After failure the discrete element is deleted. Likewise, catastrophic failure based on displacement resultants occurs if the following inequality is satisfied:

$$\left(\frac{u_r}{u_r^{\text{fail}}}\right)^2+\left(\frac{u_s}{u_s^{\text{fail}}}\right)^2+\left(\frac{u_t}{u_t^{\text{fail}}}\right)^2+\left(\frac{\theta_r}{\theta_r^{\text{fail}}}\right)^2+\left(\frac{\theta_s}{\theta_s^{\text{fail}}}\right)^2+\left(\frac{\theta_t}{\theta_t^{\text{fail}}}\right)^2-1.\geq 0\ .$$

After failure, the discrete element is deleted. If failure is included, either or both of the criteria may be used.

5. **Rotational Displacement.** Rotational displacement is measured in radians.

## *MAT_NONLINEAR_PLASTIC_DISCRETE_BEAM

This is Material Type 68. This material model is for simulating the effects of nonlinear elastoplastic, linear viscous beams by using six springs, each acting about one of the six local degrees of freedom. The two nodes defining a beam may be coincident to give a zero-length beam or offset to give a finite-length beam. For finite-length discrete beams, the absolute value of the variable SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which aligns the local *r*-axis along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad orients the beam for the directional springs. Translational/rotational stiffness and damping effects can be considered. The plastic behavior is modeled using force/moment curves as a function of displacements/rotation. Optionally, failure can be specified based on a force/moment criterion and a displacement rotation criterion. See also the remarks below.

### Card Summary:

**Card 1.** This card is required

| MID | RO | TKR | TKS | TKT | RKR | RKS | RKT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| TDR | TDS | TDT | RDR | RDS | RDT | RYLD | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| LCPDR | LCPDS | LCPDT | LCPMR | LCPMS | LCPMT | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| FFAILR | FFAILS | FFAILT | MFAILR | MFAILS | MFAILT | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| UFAILR | UFAILS | UFAILT | TFAILR | TFAILS | TFAILT | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| FOR | FOS | FOT | MOR | MOS | MOT | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TKR | TKS | TKT | RKR | RKS | RKT |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume on *SECTION_BEAM definition. |
| TKR | Translational stiffness along local *r*-axis<br>LT.0.0: \|TKR\| is the load curve ID defining the elastic translational force along the local *r*-axis as a function of relative translational displacement. Useful for nonlinear elastic behavior. |
| TKS | Translational stiffness along local *s*-axis<br>LT.0.0: \|TKS\| is the load curve ID for defining the elastic translational force along the local *s*-axis as a function of relative translational displacement. Useful for nonlinear elastic behavior. |
| TKT | Translational stiffness along local *t*-axis<br>LT.0.0: \|TKT\| is the load curve ID defining the elastic translational force along the local *t*-axis as a function of relative translational displacement. Useful for nonlinear elastic behavior. |
| RKR | Rotational stiffness about the local *r*-axis<br>LT.0.0: \|RKR\| is the load curve ID defining the elastic rotational moment along the local *r*-axis as a function of relative rotational displacement. Useful for nonlinear elastic behavior. |

| VARIABLE | DESCRIPTION |
|---|---|
| RKS | Rotational stiffness about the local *s*-axis<br>LT.0.0: \|RKS\| is the load curve ID defining the elastic rotational moment along the local *s*-axis as a function of relative rotational displacement. Useful for nonlinear elastic behavior. |
| RKT | Rotational stiffness about the local *t*-axis<br>LT.0.0: \|RKT\| is the load curve ID defining the elastic rotational moment along the local *t*-axis as a function of relative rotational displacement. Useful for nonlinear elastic behavior. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TDR | TDS | TDT | RDR | RDS | RDT | RYLD | |
| Type | F | F | F | F | F | F | I | |
| Default | none | none | none | none | none | none | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| TDR | Translational viscous damper along local *r*-axis |
| TDS | Translational viscous damper along local *s*-axis |
| TDT | Translational viscous damper along local *t*-axis |
| RDR | Rotational viscous damper about the local *r*-axis |
| RDS | Rotational viscous damper about the local *s*-axis |
| RDT | Rotational viscous damper about the local *t*-axis |
| RYLD | Flag for method of computing plastic yielding:<br>EQ.0: Original method of determining plastic yielding (default)<br>EQ.1: Compute yield displacement/rotation by taking the first point of the relevant curve as the yield force/moment and dividing it by the relevant stiffness |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCPDR | LCPDS | LCPDT | LCPMR | LCPMS | LCPMT | | |
| Type | I | I | I | I | I | I | | |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCPDR | Load curve (or table) ID for yield force as a function of plastic displacement along the local *r*-axis (and translational velocity in the *r*-direction, if table). If the curve/table ID is zero, and TKR is nonzero, then elastic behavior is obtained for this component. |
| LCPDS | Load curve (or table) ID-yield force as a function of plastic displacement along the *s*-axis (and translational velocity in the *s*-direction, if table). If the curve/table ID is zero, and TKS is nonzero, then elastic behavior is obtained for this component. |
| LCPDT | Load curve (or table) ID-yield force as a function of plastic displacement along the *t*-axis (and translational velocity in the *t*-direction, if table). If the curve/table ID is zero, and TKT is nonzero, then elastic behavior is obtained for this component. |
| LCPMR | Load curve (or table) ID-yield moment as a function of plastic rotation about the *r*-axis (and rotational velocity about the *r*-axis, if table). If the curve/table ID is zero, and RKR is nonzero, then elastic behavior is obtained for this component. |
| LCPMS | Load curve (or table) ID-yield moment as a function of plastic rotation about the *s*-axis (and rotational velocity about the *s*-axis, if table). If the curve/table ID is zero, and RKS is nonzero, then elastic behavior is obtained for this component. |
| LCPMT | Load curve (or table) ID-yield moment as a function of plastic rotation about the *t*-axis (and rotational velocity about the *t*-axis, if table). If the curve/table ID is zero, and RKT is nonzero, then elastic behavior is obtained for this component. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FFAILR | FFAILS | FFAILT | MFAILR | MFAILS | MFAILT | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FFAILR | Optional failure parameter. If zero, the corresponding force, $F_r$, is not considered in the failure calculation.<br>LT.0.0: \|FFAILR\| is the load curve ID defining $F_r$ as a function of translational velocity along the local $r$-axis. |
| FFAILS | Optional failure parameter. If zero, the corresponding force, $F_s$, is not considered in the failure calculation.<br>LT.0.0: \|FFAILS\| is the load curve ID defining $F_s$ as a function of translational velocity along the local $s$-axis. |
| FFAILT | Optional failure parameter. If zero, the corresponding force, $F_t$, is not considered in the failure calculation.<br>LT.0.0: \|FFAILT\| is the load curve ID defining $F_t$ as a function of translational velocity along the local $t$-axis. |
| MFAILR | Optional failure parameter. If zero, the corresponding moment, $M_r$, is not considered in the failure calculation.<br>LT.0.0: \|MFAILR\| is the load curve ID defining $M_r$ as a function of rotational velocity about the local $r$-axis. |
| MFAILS | Optional failure parameter. If zero, the corresponding moment, $M_s$, is not considered in the failure calculation.<br>LT.0.0: \|MFAILS\| is the load curve ID defining $M_s$ as a function of rotational velocity about the local $s$-axis. |
| MFAILT | Optional failure parameter. If zero, the corresponding moment, $M_t$, is not considered in the failure calculation.<br>LT.0.0: \|MFAILT\| is the load curve ID defining $M_t$ as a function of rotational velocity about the local $t$-axis. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UFAILR | UFAILS | UFAILT | TFAILR | TFAILS | TFAILT | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| UFAILR | Optional failure parameter. If zero, the corresponding displacement, $u_r$, is not considered in the failure calculation.<br>LT.0.0: \|UFAILR\| is the load curve ID defining $u_r$ as a function of translational velocity along the local $r$-axis. |
| UFAILS | Optional failure parameter. If zero, the corresponding displacement, $u_s$, is not considered in the failure calculation.<br>LT.0.0: \|UFAILS\| is the load curve ID defining $u_s$ as a function of translational velocity along the local $s$-axis. |
| UFAILT | Optional failure parameter. If zero, the corresponding displacement, $u_t$, is not considered in the failure calculation.<br>LT.0.0: \|UFAILT\| is the load curve ID defining $u_t$ as a function of translational velocity along the local $t$-axis. |
| TFAILR | Optional failure parameter. If zero, the corresponding rotation, $\theta_r$, is not considered in the failure calculation.<br>LT.0.0: \|TFAILR\| is the load curve ID defining $\theta_r$ as a function of rotational velocity about the local $r$-axis. |
| TFAILS | Optional failure parameter. If zero, the corresponding rotation, $\theta_s$, is not considered in the failure calculation.<br>LT.0.0: \|TFAILS\| is the load curve ID defining $\theta_s$ as a function of rotational velocity about the local $s$-axis. |
| TFAILT | Optional failure parameter. If zero, the corresponding rotation, $\theta_t$, is not considered in the failure calculation.<br>LT.0.0: \|TFAILT\| is the load curve ID defining $\theta_t$ as a function of rotational velocity about the local $t$-axis. |

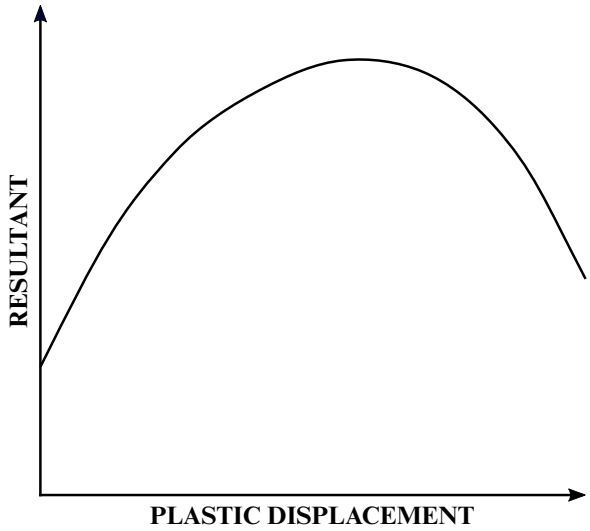



**Figure M68-1.** The resultant forces and moments are limited by the yield definition. The initial yield point corresponds to a plastic displacement of zero.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FOR | FOS | FOT | MOR | MOS | MOT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FOR | Preload force in *r*-direction (optional) |
| FOS | Preload force in *s*-direction (optional) |
| FOT | Preload force in *t*-direction (optional) |
| MOR | Preload moment about *r*-axis (optional) |
| MOS | Preload moment about *s*-axis (optional) |
| MOT | Preload moment about *t*-axis (optional) |

**Remarks:**

1. **Elastic behavior.** For the translational and rotational degrees of freedom where elastic behavior is desired, set the load curve ID to zero.

2. **Plastic displacement.** The plastic displacement for the load curves is defined as:

$$\text{plastic displacement} = \text{total displacement} - \text{yield force/elastic stiffness} \ .$$

3. **Discrete beam formulation.** The formulation of the discrete beam (type 6) assumes that the beam is of zero length and requires no orientation node. A small distance between the nodes joined by the beam is permitted. The local coordinate system which determines $(r,s,t)$ is given by the coordinate ID (see *DEFINE_COORDINATE_*OPTION*) in the cross-sectional input (see *SECTION_BEAM) where the global system is the default. The local coordinate system axes can rotate with either node of the beam or an average rotation of both nodes (see SCOOR variable in *SECTION_BEAM).

4. **Failure.** Catastrophic failure based on force resultants occurs if the following inequality is satisfied.

$$\left(\frac{F_r}{F_r^{\text{fail}}}\right)^2 + \left(\frac{F_s}{F_s^{\text{fail}}}\right)^2 + \left(\frac{F_t}{F_t^{\text{fail}}}\right)^2 + \left(\frac{M_r}{M_r^{\text{fail}}}\right)^2 + \left(\frac{M_s}{M_s^{\text{fail}}}\right)^2 + \left(\frac{M_t}{M_t^{\text{fail}}}\right)^2 - 1. \geq 0.$$

After failure, the discrete element is deleted. Likewise, catastrophic failure based on displacement resultants occurs if the following inequality is satisfied:

$$\left(\frac{u_r}{u_r^{\text{fail}}}\right)^2 + \left(\frac{u_s}{u_s^{\text{fail}}}\right)^2 + \left(\frac{u_t}{u_t^{\text{fail}}}\right)^2 + \left(\frac{\theta_r}{\theta_r^{\text{fail}}}\right)^2 + \left(\frac{\theta_s}{\theta_s^{\text{fail}}}\right)^2 + \left(\frac{\theta_t}{\theta_t^{\text{fail}}}\right)^2 - 1. \geq 0.$$

After failure the discrete element is deleted. If failure is included, either or both of the criteria may be used.

5. **Rotational displacement.** Rotational displacement is measured in radians.

6. **History variables.** The following additional history variables are available for this material by setting NEIPB on *DATABASE_EXTENT_BINARY:

| History Variable # | Description |
|---|---|
| 12 | Flag for failure from resultant forces:<br>EQ.0: Intact<br>EQ.1: Failed |
| 13 | Flag for failure from displacement resultants:<br>EQ.0: Intact<br>EQ.1: Failed |

## *MAT_SID_DAMPER_DISCRETE_BEAM

This is Material Type 69. The side impact dummy uses a damper that is not adequately treated by the nonlinear force as a function of relative velocity curves since the force characteristics are dependent on the displacement of the piston. See Remarks below.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | ST | D | R | H | K | C |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| C3 | STF | RHOF | C1 | C2 | LCIDF | LCIDD | S0 |
|---|---|---|---|---|---|---|---|

**Card 3.** Include one card per orifice. Read in up to 15 orifice locations. The next keyword ("*") card terminates this input.

| ORFLOC | ORFRAD | SF | DC | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ST | D | R | H | K | C |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume on *SECTION_BEAM definition. |
| ST | $S_t$, piston stroke. $S_t$ must equal or exceed the length of the beam element; see Figure M69-1 below. |
| D | $d$, piston diameter |
| R | $R$, default orifice radius |
| H | $h$, orifice controller position |

| VARIABLE | DESCRIPTION |
|---|---|
| K | $K$, damping constant<br>LT.0.0: \|K\| is the load curve number ID (see *DEFINE_CURVE) defining the damping coefficient as a function of the absolute value of the relative velocity. |
| C | $C$, discharge coefficient |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C3 | STF | RHOF | C1 | C2 | LCIDF | LCIDD | S0 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C3 | Coefficient for fluid inertia term |
| STF | $k$, stiffness coefficient if piston bottoms out |
| RHOF | $\rho_{\text{fluid}}$, fluid density |
| C1 | $C_1$, coefficient for linear velocity term |
| C2 | $C_2$, coefficient for quadratic velocity term |
| LCIDF | Load curve number ID defining force as a function of piston displacement, $s$, that is, term $f(s + s_0)$. Compressive behavior is defined in the positive quadrant of the force displacement curve. Displacements falling outside of the defined force displacement curve are extrapolated. Care must be taken to ensure that extrapolated values are reasonable. |
| LCIDD | Load curve number ID defining damping coefficient as a function of piston displacement, $s$, that is, $g(s + s_0)$. Displacements falling outside the defined curve are extrapolated. Care must be taken to ensure that extrapolated values are reasonable. |
| S0 | Initial displacement, $s_0$; typically set to zero. A positive displacement corresponds to compressive behavior. |

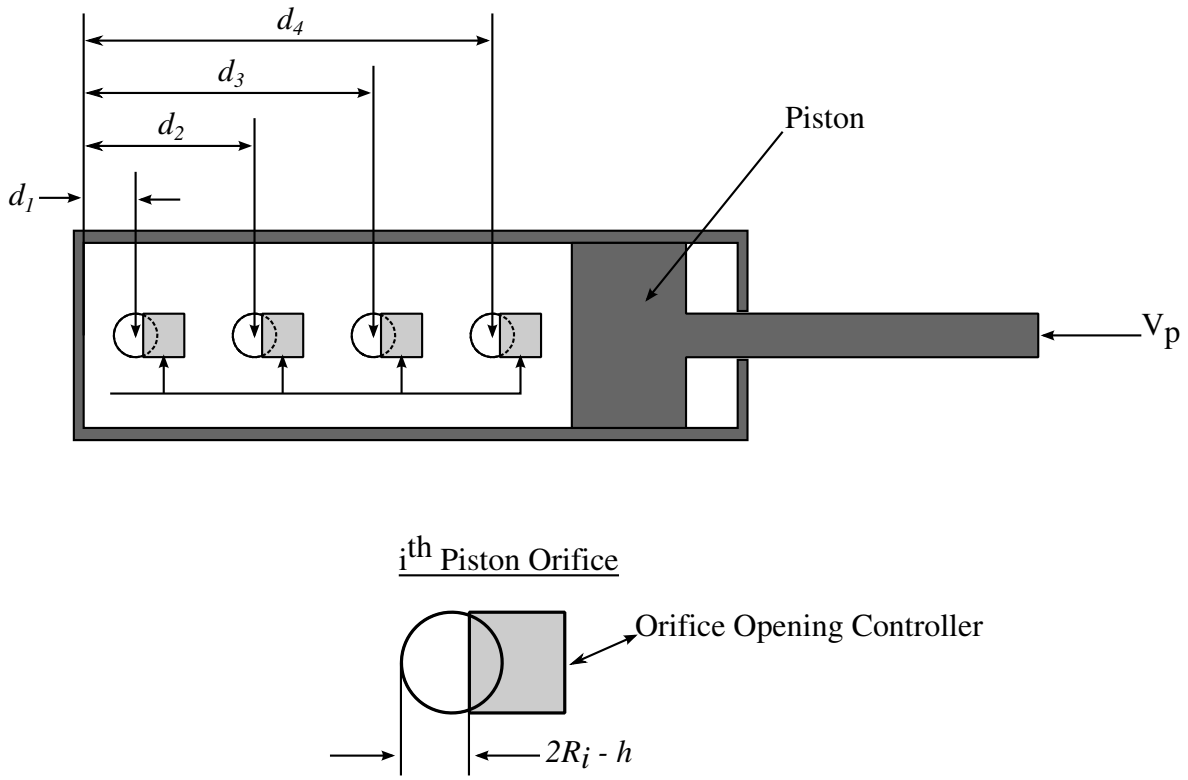



**Figure M69-1.** Mathematical model for the Side Impact Dummy damper.

**Orifice Cards.** Include one card per orifice. Read in up to 15 orifice locations. The next keyword ("*") card terminates this input. On the first card below the optional input parameters SF and DF may be specified.

| Cards 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ORFLOC | ORFRAD | SF | DC | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ORFLOC | $d_i$, orifice location of $i^{th}$ orifice relative to the fixed end |
| ORFRAD | $r_i$, orifice radius of $i^{th}$ orifice<br>EQ.0.0: R on Card 1 is used. |
| SF | Scale factor on calculated force. The default is set to 1.0. |
| DC | $c$, linear viscous damping coefficient used after damper bottoms out either in tension or compression |

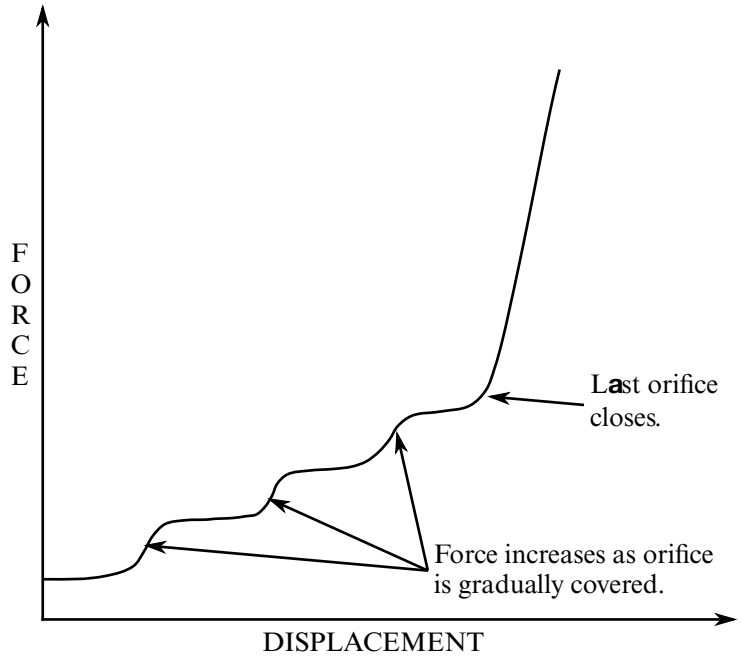



**Figure M69-2.** Force as a function of displacement as orifices are covered at a constant relative velocity. Only the linear velocity term is active.

**Remarks:**

As the damper moves, the fluid flows through the open orifices to provide the necessary damping resistance. While moving as shown in Figure M69-1, the piston gradually blocks off and effectively closes the orifices. The number of orifices and the size of their opening control the damper resistance and performance. The damping force is computed from,

$$F = \mathrm{SF} \times \left\{ KA_pV_p \left\{ \frac{C_1}{A_0^t} + C_2|V_p|\rho_{\mathrm{fluid}} \left[ \left( \frac{A_p}{CA_0^t} \right)^2 - 1 \right] \right\} - f(s+s_0) + V_p g(s+s_0) \right\} ,$$

where $K$ is a user defined constant or a tabulated function of the absolute value of the relative velocity, $V_p$ is the piston velocity, $C$ is the discharge coefficient, $A_p$ is the piston area, $A_0^t$ is the total open areas of orifices at time $t$, $\rho_{\mathrm{fluid}}$ is the fluid density, $C_1$ is the coefficient for the linear term, and $C_2$ is the coefficient for the quadratic term.

In the implementation, the orifices are assumed to be circular with partial covering by the orifice controller. As the piston closes, the closure of the orifice is gradual. This gradual closure is properly taken into account to insure a smooth response. If the piston stroke is exceeded, the stiffness value, $k$, limits further movement, meaning if the damper bottoms out in tension or compression the damper forces are calculated by replacing the damper by a bottoming out spring and damper, $k$ and $c$, respectively. The piston stroke must exceed the initial length of the beam element. The time step calculation is based in part

on the stiffness value of the bottoming out spring. A typical force as a function of displacement curve at constant relative velocity is shown in Figure M69-2.

The factor, SF, which scales the force defaults to 1.0 and is analogous to the adjusting ring on the damper.

## *MAT_HYDRAULIC_GAS_DAMPER_DISCRETE_BEAM

This is Material Type 70. This special purpose element represents a combined hydraulic and gas-filled damper which has a variable orifice coefficient. A schematic of the damper is shown in Figure M70-1. Dampers of this type are sometimes used on buffers at the end of railroad tracks and as aircraft undercarriage shock absorbers. This material can be used only as a discrete beam element. See also the remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | CO | N | P0 | PA | AP | KH |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID | FR | SCLF | CLEAR | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label not must be specified (see *PART). |
| RO | Mass density, see also volume in *SECTION_BEAM definition. |
| CO | Length of gas column, $C_0$ |
| N | Adiabatic constant, $n$ |
| P0 | Initial gas pressure, $P_0$ |
| PA | Atmospheric pressure, $P_a$ |
| AP | Piston cross sectional area, $A_p$ |
| KH | Hydraulic constant, $K$ |
| LCID | Load curve ID (see *DEFINE_CURVE) defining the orifice area, $a_0$, as a function of element deflection $S$ |

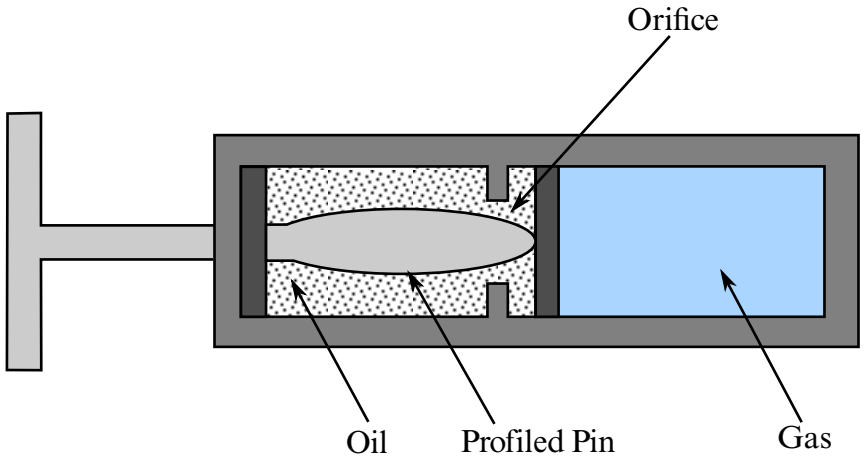



**Figure M70-1.** Schematic of Hydraulic/Gas damper.

| VARIABLE | DESCRIPTION |
|---|---|
| FR | Return factor on orifice force. This acts as a factor on the hydraulic force only and is applied when unloading. It is intended to represent a valve that opens when the piston unloads to relieve hydraulic pressure. Set it to 1.0 for no such relief. |
| SCLF | Scale factor on force (default = 1.0). |
| CLEAR | Clearance. If nonzero, no tensile force develops for positive displacements and negative forces develop only after the clearance is closed. |

**Remarks:**

As the damper is compressed two actions contribute to the force which develops. First, the gas is adiabatically compressed into a smaller volume. Secondly, oil is forced through an orifice. A profiled pin may occupy some of the cross-sectional area of the orifice; thus, the orifice area available for the oil varies with the stroke. The force is assumed proportional to the square of the velocity and inversely proportional to the available area.

The equation for this element is:

$$F = \text{SCLF} \times \left\{ K_h \left( \frac{V}{a_0} \right)^2 + \left[ P_0 \left( \frac{C_0}{C_0 + S} \right)^n - P_a \right] A_p \right\}$$

where $S$ is the element deflection (positive in tension) and $V$ is the relative velocity across the element.

## *MAT_CABLE_DISCRETE_BEAM

This is Material Type 71. This model permits elastic cables to be realistically modeled; thus, no force will develop in compression.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | LCID | F0 | TMAXF0 | TRAMP | IREAD |
| Type | A | F | F | F | F | F | F | I |
| Default | none | none | none | none | 0 | 0 | 0 | 0 |

Additional card for IREAD > 1.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OUTPUT | TSTART | FRACL0 | MXEPS | MXFRC | | | |
| Type | I | F | F | F | F | | | |
| Default | 0 | 0 | 0 | 1.0E+20 | 1.0E+20 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, see also volume in *SECTION_BEAM definition. |
| E | GT.0.0: Young's modulus<br>LT.0.0: Stiffness |
| LCID | Load curve ID, see *DEFINE_CURVE, defining the stress versus engineering strain. (Optional). |
| F0 | Initial tensile force. If F0 is defined, an offset is not needed for an initial tensile force. |
| TMAXF0 | Time for which pre-tension force will be held |

| VARIABLE | DESCRIPTION |
|---|---|
| TRAMP | Ramp-up time for pre-tension force |
| IREAD | Set to 1 to read second line of input |
| OUTPUT | Flag = 1 to output axial strain (see note below concerning OUTPUT) |
| TSTART | Time at which the ramp-up of pre-tension begins |
| FRACL0 | Fraction of initial length that should be reached over time period of TRAMP. Corresponding tensile force builds up as necessary to reach cable length = FRACL0 × L0 at time t = TRAMP. |
| MXEPS | Maximum strain at failure |
| MXFRC | Maximum force at failure |

**Remarks:**

The force, $F$, generated by the cable is nonzero if and only if the cable is tension. The force is given by:

$$F = \max(F_0 + K\Delta L, 0.)$$

where ΔL is the change in length

$$\Delta L = \text{current length} - (\text{initial length} - \text{offset})$$

and the stiffness (E > 0.0 only ) is defined as:

$$K = \frac{E \times \text{area}}{(\text{initial length} - \text{offset})}$$

Note that a constant force element can be obtained by setting:

$$F_0 > 0 \text{ and } K = 0$$

although the application of such an element is unknown.

The area and offset are defined on either the cross section or element cards. For a slack cable the offset should be input as a negative length. For an initial tensile force the offset should be positive.

If a load curve is specified the Young's modulus will be ignored and the load curve will be used instead. The points on the load curve are defined as engineering stress versus engineering strain, i.e., the change in length over the initial length. The unloading behavior follows the loading.

By default, cable pretension is applied only at the start of the analysis. If the cable is attached to flexible structure, deformation of the structure will result in relaxation of the cables, which will therefore lose some or all of the intended preload.

This can be overcome by using TMAXF0. In this case, it is expected that the structure will deform under the loading from the cables and that this deformation will take time to occur during the analysis. The unstressed length of the cable will be continuously adjusted until time TMAXF0 such that the force is maintained at the user-defined pre-tension force – this is analogous to operation of the pre-tensioning screws in real cables. After time TMAXF0, the unstressed length is fixed and the force in the cable is determined in the normal way using the stiffness and change of length.

Sudden application of the cable forces at time zero may result in an excessively dynamic response during pre-tensioning. A ramp-up time TRAMP may optionally be defined. The cable force ramps up from zero at time TSTART to the full pre-tension F0 at time TSTART + TRAMP. TMAXF0, if set less than TSTART + TRAMP by the user, will be internally reset to TSTART + TRAMP.

If the model does not use dynamic relaxation, it is recommended that damping be applied during pre-tensioning so that the structure reaches a steady state by time TMAXF0.

If the model uses dynamic relaxation, TSTART, TRAMP, and TMAXF0 apply only during dynamic relaxation. The cable preload at the end of dynamic relaxation carries over to the start of the subsequent transient analysis.

The cable mass will be calculated from length × area × density if VOL is set to zero on *SECTION_BEAM. Otherwise, VOL × density will be used.

If OUTPUT is set to 1, one additional history variable representing axial strain is output to d3plot for the cable elements. This axial strain can be plotted by LS-PrePost by selecting the beam component labeled as “axial stress”. Though the label says “axial stress”, it is actually axial strain.

If the stress-strain load curve option, LCID, is combined with preload, two types of behavior are available:

1. If the preload is applied using the TMAXF0/TRAMP method, the initial strain is calculated from the stress-strain curve to achieve the desired preload.

2. If TMAXF0/TRAMP are not used, the preload force is taken as additional to the force calculated from the stress/strain curve. Thus, the total stress in the cable will be higher than indicated by the stress/strain curve.

## *MAT_CONCRETE_DAMAGE

This is Material Type 72. This model has been used to analyze buried steel reinforced concrete structures subjected to impulsive loadings. A newer version of this model is available as *MAT_CONCRETE_DAMAGE_REL3.

### Card Summary:

**Card 1.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| MID | RO | PR | | | | | |

**Card 2.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| SIGF | A0 | A1 | A2 | | | | |

**Card 3.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A0Y | A1Y | A2Y | A1F | A2F | B1 | B2 | B3 |

**Card 4.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| PER | ER | PRR | SIGY | ETAN | LCP | LCR | |

**Card 5.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| λ1 | λ2 | λ3 | λ4 | λ5 | λ6 | λ7 | λ8 |

**Card 6.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| λ9 | λ10 | λ11 | λ12 | λ13 | | | |

**Card 7.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| η1 | η2 | η3 | η4 | η5 | η6 | η7 | η8 |

**Card 8.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| η9 | η10 | η11 | η12 | η13 | | | |

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | | | | | |
| Type | A | F | F | | | | | |
| Default | none | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PR | Poisson's ratio |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGF | A0 | A1 | A2 | | | | |
| Type | F | F | F | F | | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGF | Maximum principal stress for failure |
| A0 | Cohesion |
| A1 | Pressure hardening coefficient |
| A2 | Pressure hardening coefficient |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A0Y | A1Y | A2Y | A1F | A2F | B1 | B2 | B3 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| A0Y | Cohesion for yield |
| A1Y | Pressure hardening coefficient for yield limit |
| A2Y | Pressure hardening coefficient for yield limit |
| A1F | Pressure hardening coefficient for failed material |
| A2F | Pressure hardening coefficient for failed material |
| B1 | Damage scaling factor |
| B2 | Damage scaling factor for uniaxial tensile path |
| B3 | Damage scaling factor for triaxial tensile path |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PER | ER | PRR | SIGY | ETAN | LCP | LCR | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | none | 0.0 | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| PER | Percent reinforcement |
| ER | Elastic modulus for reinforcement |
| PRR | Poisson's ratio for reinforcement |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGY | Initial yield stress |
| ETAN | Tangent modulus/plastic hardening modulus |
| LCP | Load curve ID giving rate sensitivity for principal material; see *DEFINE_CURVE. |
| LCR | Load curve ID giving rate sensitivity for reinforcement; see *DEFINE_CURVE. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | λ1 | λ2 | λ3 | λ4 | λ5 | λ6 | λ7 | λ8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | λ9 | λ10 | λ11 | λ12 | λ13 | | | |
| Type | F | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| λ1 - λ13 | Tabulated damage function |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | η1 | η2 | η3 | η4 | η5 | η6 | η7 | η8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | η9 | η10 | η11 | η12 | η13 | | | |
| Type | F | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| η1 - η13 | Tabulated scale factor. |

**Remarks:**

1. **Cohesion.** Cohesion for failed material $a_{0f} = 0$.
2. **B3.** B3 must be positive or zero.
3. **Damage Function.** $\lambda_n \leq \lambda_{n+1}$. The first point must be zero.

## *MAT_CONCRETE_DAMAGE_REL3

This is Material Type 72R3. The Karagozian & Case (K&C) Concrete Model - Release III is a three-invariant model. It uses three shear failure surfaces, includes damage and strain-rate effects, and has origins based on the Pseudo-TENSOR Model (Material Type 16). The most significant user improvement provided by Release III is a model parameter generation capability, based solely on the unconfined compression strength of the concrete.

An open-source reference that precedes the parameter generation capability is provided in Malvar et al. [1997]. A workshop proceedings reference, Malvar et al. [1996], is useful, but may be difficult to obtain. More recent, but *limited-distribution reference materials, such as those by* Malvar et al. [2000], may be obtained by contacting Karagozian & Case.

Seven card images are required to define the *complete* set of model parameters for the K&C Concrete Model. An equation of state is also required for the pressure-volume strain response. Brief descriptions of all the input parameters are provided below; however, it is expected that this model will be used primarily with the option to automatically generate the model parameters based on the unconfined compression strength of the concrete. These generated material parameters, along with the generated parameters for *EOS_TABULATED_COMPACTION, are written to the d3hsp file.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | | | | | |
| Type | A | F | F | | | | | |
| Default | none | none | none | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FT | A0 | A1 | A2 | B1 | OMEGA | A1F | |
| Type | F | F | F | F | F | F | F | |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | none | 0.0 | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Sλ | NOUT | EDROP | RSIZE | UCF | LCRATE | LOCWID | NPTS |
| Type | F | F | F | F | F | I | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | λ01 | λ02 | λ03 | λ04 | λ05 | λ06 | λ07 | λ08 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | λ09 | λ10 | λ11 | λ12 | λ13 | B3 | A0Y | A1Y |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | 0.0 | 0.0 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | η01 | η02 | η03 | η04 | η05 | η06 | η07 | η08 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | η09 | η10 | η11 | η12 | η13 | B2 | A2F | A2Y |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PR | Poisson's ratio, $\nu$ |
| FT | Uniaxial tensile strength, $f_t$ |
| A0 | Maximum shear failure surface parameter, $a_0$, **or** $-f_c'$ for **parameter generation** (recommended) |
| A1 | Maximum shear failure surface parameter, $a_1$ |
| A2 | Maximum shear failure surface parameter, $a_2$ |
| B1 | Compressive damage scaling parameter, $b_1$ |
| OMEGA | Fractional dilatancy, $\omega$ |
| A1F | Residual failure surface coefficient, $a_{1f}$ |
| Sλ | λ stretch factor, $s$ |
| NOUT | Output selector for effective plastic strain (see table) |
| EDROP | Post peak dilatancy decay, $N^{\alpha}$ |
| RSIZE | Unit conversion factor for length (inches/user-unit). For example, set to 39.37 if user length unit in meters. |
| UCF | Unit conversion factor for stress (psi/user-unit). For instance set to 145 if $f_c'$ in MPa. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCRATE | Define (load) curve number for strain-rate effects; effective strain rate on abscissa (negative = tension) and strength enhancement on ordinate. If LCRATE is set to -1, strain rate effects are automatically included, based on equations provided in Wu, Crawford, Lan, and Magallanes [2014]. LCRATE = -1 is applicable to models which use time units of seconds, for other time units, the strain-rate effects should be input by means of a curve. |
| LOCWID | Three times the maximum aggregate diameter (input in user length units). |
| NPTS | Number of points in $\lambda$ versus $\eta$ damage relation; must be 13 points. |
| λ01 | 1st value of damage function, (a.k.a., 1st value of "modified" effective plastic strain; see references for details). |
| λ02 | 2nd value of damage function, |
| λ03 | 3rd value of damage function, |
| λ04 | 4th value of damage function, |
| λ05 | 5th value of damage function, |
| λ06 | 6th value of damage function, |
| λ07 | 7th value of damage function, |
| λ08 | 8th value of damage function, |
| λ09 | 9th value of damage function, |
| λ10 | 10th value of damage function, |
| λ11 | 11th value of damage function, |
| λ12 | 12th value of damage function, |
| λ13 | 13th value of damage function. |
| B3 | Damage scaling coefficient for triaxial tension, $b_3$. |
| A0Y | Initial yield surface cohesion, $a_{0y}$. |
| A1Y | Initial yield surface coefficient, $a_{1y}$. |
| η01 | 1st value of scale factor, |

| VARIABLE | DESCRIPTION |
|---|---|
| η02 | 2nd value of scale factor, |
| η03 | 3rd value of scale factor, |
| η04 | 4th value of scale factor, |
| η05 | 5th value of scale factor, |
| η06 | 6th value of scale factor, |
| η07 | 7th value of scale factor, |
| η08 | 8th value of scale factor, |
| η09 | 9th value of scale factor, |
| η10 | 10th value of scale factor, |
| η11 | 11th value of scale factor, |
| η12 | 12th value of scale factor, |
| η13 | 13th value of scale factor. |
| B2 | Tensile damage scaling exponent, $b_2$. |
| A2F | Residual failure surface coefficient, $a_{2f}$. |
| A2Y | Initial yield surface coefficient, $a_{2y}$. |

λ, sometimes referred to as "modified" effective plastic strain, is computed internally as a function of effective plastic strain, strain rate enhancement factor, and pressure. η is a function of λ as specified by the η as a function of λ curve. The η value, which is always between 0 and 1, is used to interpolate between the yield failure surface and the maximum failure surface or between the maximum failure surface and the residual failure surface, depending on whether λ is to the left or right of the first peak in the η as a function of λ curve. The "scaled damage measure" ranges from 0 to 1 as the material transitions from the yield failure surface to the maximum failure surface, and thereafter ranges from 1 to 2 as the material ranges from the maximum failure surface to the residual failure surface. See the references for details.

### Output of Selected Variables:

The quantity labeled as "plastic strain" by LS-PrePost is actually the quantity described in Table M72-1, in accordance with the input value of NOUT (see Card 3 above).

| NOUT | Function | Description |
|---|---|---|
| 1 | | Current shear failure surface radius |
| 2 | $\delta = 2\lambda/(\lambda + \lambda_m)$ | Scaled damage measure |
| 3 | $\dot{\sigma}_{ij}\dot{\varepsilon}_{ij}$ | Strain energy (rate) |
| 4 | $\dot{\sigma}_{ij}\dot{\varepsilon}^{p}_{ij}$ | Plastic strain energy (rate) |

**Table M72-1.** Description of quantity labeled "plastic strain" by LS-PrePost.

An additional six extra history variables as shown in Table M72-2 may be written by setting NEIPH = 6 on the keyword *DATABASE_EXTENT_BINARY. The extra history variables are labeled as "history var#1" through "history var#6" in LS-PrePost.

| Label | Description |
|---|---|
| history var#1 | Internal energy |
| history var#2 | Pressure from bulk viscosity |
| history var#3 | Volume in previous time step |
| history var#4 | Plastic volumetric strain |
| history var#5 | Slope of damage evolution (η vs. λ) curve |
| history var#6 | "Modified" effective plastic strain (λ) |

**Table M72-2.** Extra History Variables for *MAT_072R3

### Sample Input for Concrete:

As an example of the K&C Concrete Model material parameter generation, the following sample input for a 45.4 MPa (6,580 psi) unconfined compression strength concrete is provided. The basic units for the provided parameters are length in millimeters (mm), time in milliseconds (msec), and mass in grams (g). This base unit set yields units of force in Newtons (N) and pressure in Mega-Pascals (MPa).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | | | | | |
| Value | 72 | 2.3E-3 | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FT | A0 | A1 | A2 | B1 | OMEGA | A1F | |
| Value | | -45.4 | | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Sλ | NOUT | EDROP | RSIZE | UCF | LCRATE | LOCWID | NPTS |
| Value | | | | 3.94E-2 | 145.0 | 723 | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | λ01 | λ02 | λ03 | λ04 | λ05 | λ06 | λ07 | λ08 |
| Value | | | | | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | λ09 | λ10 | λ11 | λ12 | λ13 | B3 | A0Y | A1Y |
| Value | | | | | | | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | η01 | η02 | η03 | η04 | η05 | η06 | η07 | η08 |
| Value | | | | | | | | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | η09 | η10 | η11 | η12 | η13 | B2 | A2F | A2Y |
| Value | | | | | | | | |

Shear strength enhancement factor as a function of effective strain rate is given by a curve (*DEFINE_CURVE) with LCID 723. The sample input values, see Malvar & Ross [1998], are given in Table M72-3.

| Strain-Rate (1/ms) | Enhancement |
|---|---|
| -3.0E+01 | 9.70 |
| -3.0E-01 | 9.70 |
| -1.0E-01 | 6.72 |
| -3.0E-02 | 4.50 |
| -1.0E-02 | 3.12 |
| -3.0E-03 | 2.09 |
| -1.0E-03 | 1.45 |
| -1.0E-04 | 1.36 |
| -1.0E-05 | 1.28 |
| -1.0E-06 | 1.20 |
| -1.0E-07 | 1.13 |
| -1.0E-08 | 1.06 |
| 0.0E+00 | 1.00 |
| 3.0E-08 | 1.00 |
| 1.0E-07 | 1.03 |
| 1.0E-06 | 1.08 |
| 1.0E-05 | 1.14 |
| 1.0E-04 | 1.20 |
| 1.0E-03 | 1.26 |
| 3.0E-03 | 1.29 |
| 1.0E-02 | 1.33 |

| Strain-Rate (1/ms) | Enhancement |
|---|---|
| 3.0E-02 | 1.36 |
| 1.0E-01 | 2.04 |
| 3.0E-01 | 2.94 |
| 3.0E+01 | 2.94 |

**Table M72-3.** Enhancement as a function of effective strain rate for 45.4 MPa concrete (sample). When defining curve LCRATE, input negative (tensile) values of effective strain rate first. The enhancement should be positive and should be 1.0 at a strain rate of zero.

## *MAT_LOW_DENSITY_VISCOUS_FOAM

This is Material Type 73. This material is for modeling low-density urethane foam with high compressibility and rate sensitivity, which can be characterized by a relaxation curve. Its main applications are for seat cushions, padding on the Side Impact Dummies (SID), bumpers, and interior foams. Optionally, a tension cut-off failure can be defined. See the remarks below and the description of material 57.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | LCID | TC | HU | BETA | DAMP |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SHAPE | FAIL | BVFLAG | KCON | LCID2 | BSTART | TRAMP | NV |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included if and only if LCID2 = 0. Include up to 6 of this card. The next keyword ("*") card terminates this input.

| G*i* | BETA*i* | REF | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3b.** This card is included if and only if LCID2 = -1.

| LCID3 | LCID4 | SCALEW | SCALEA | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | LCID | TC | HU | BETA | DAMP |
| Type | A | F | F | I | F | F | F | F |
| Default | none | none | none | none | $10^{20}$ | 1. | none | 0.05 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus used in tension. For implicit problems E is set to the initial slope of load curve LCID. |
| LCID | Load curve ID (see *DEFINE_CURVE) for nominal stress as a function of strain |
| TC | Tension cut-off stress |
| HU | Hysteretic unloading factor between 0 and 1 (default = 1, that is, no energy dissipation); see Figure M57-1 and Remark 3. |
| BETA | $\beta$, decay constant to model creep in unloading (see Remark 3)<br>EQ.0.0: No relaxation |
| DAMP | Viscous coefficient (.05 < recommended value <.50) to model damping effects.<br>LT.0.0: \|DAMP\| is the load curve ID which defines the damping constant as a function of the maximum strain in compression defined as:<br>$\varepsilon_{max} = \max(1-\lambda_1, 1-\lambda_2, 1.-\lambda_3)$ .<br>In tension, the damping constant is set to the value corresponding to the strain at 0. The abscissa should be defined from 0 to 1. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SHAPE | FAIL | BVFLAG | KCON | LCID2 | BSTART | TRAMP | NV |
| Type | F | F | F | F | I | F | F | I |
| Default | 1.0 | 0.0 | 0.0 | 0.0 | 0 | 0.0 | 0.0 | 6 |

| VARIABLE | DESCRIPTION |
|---|---|
| SHAPE | Shape factor for unloading which is active for nonzero values of HU. SHAPE less than one reduces the energy dissipation and greater than one increases dissipation; see Figure M57-1. |
| FAIL | Failure option after cutoff stress is reached: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: tensile stress remains at cut-off value.<br>EQ.1.0: tensile stress is reset to zero. |
| BVFLAG | Bulk viscosity activation flag (see remarks below):<br>EQ.0.0: No bulk viscosity (recommended)<br>EQ.1.0: Bulk viscosity active |
| KCON | Stiffness coefficient for contact interface stiffness. If undefined, the maximum slope in the stress-strain curve is used. When the maximum slope is taken for the contact, the time step size for this material is reduced for stability. In some cases, $\Delta t$ may be significantly smaller, so defining a reasonable stiffness is recommended. |
| LCID2 | Load curve ID of relaxation curve. If LCID2 > 0, constants $G_i$ and $\beta_i$ are determined using a least squares fit. An example is shown in Figure M76-1. This model ignores the constant stress. |
| BSTART | Fit parameter. In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times greater than $\beta_3$ , and so on. If zero, BSTART = .01. |
| TRAMP | Optional ramp time for loading |
| NV | Number of terms in fit. Currently, the maximum number is 6. Since each term used adds significantly to the cost, 2 or 3 terms are recommended. Caution should be exercised when taking the results from the fit. Preferably, all generated coefficients should be positive because negative values may lead to unstable results. Once a satisfactory fit has been achieved, we recommend using the output coefficients in future runs. |

**Relaxation Constant Cards.** If LCID2 = 0, then include this card. Up to 6 cards may be input. The next keyword ("*") card terminates this input.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | REF | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional decay constant if $i^{th}$ term |
| REF | Use reference geometry to initialize the stress tensor. *INITIAL_-FOAM_REFERENCE_GEOMETRY defines the reference geometry.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Frequency Dependence Card.** If LCID2 = -1 then include this card.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID3 | LCID4 | SCALEW | SCALEA | | | | |
| Type | I | I | I | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID3 | Load curve ID giving the magnitude of the shear modulus as a function of the frequency. LCID3 must use the same frequencies as LCID4. |
| LCID4 | Load curve ID giving the phase angle of the shear modulus as a function of the frequency. LCID4 must use the same frequencies as LCID3. |
| SCALEW | Flag for the form of the frequency data:<br>EQ.0: Frequency is in cycles per unit time.<br>EQ.1: Circular frequency |
| SCALEA | Flag for the units of the phase angle:<br>EQ.0: Degrees<br>EQ.1: Radians |

**Remarks:**

1. **Material formulation.** This viscoelastic foam material formulation models highly compressible viscous foams. The hyperelastic formulation of this model follows that of Material 57.

2. **Rate effects.** Rate effects are accounted for through linear viscoelasticity by a convolution integral of the form

$$\sigma_{ij}^r = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \ ,$$

where $g_{ijkl}(t-\tau)$ is the relaxation function. The stress tensor, $\sigma_{ij}^r$, augments the stresses determined from the foam, $\sigma_{ij}^f$; consequently, the final stress, $\sigma_{ij}$, is taken as the summation of the two contributions:

$$\sigma_{ij} = \sigma_{ij}^f + \sigma_{ij}^r \ .$$

Since we wish to include only simple rate effects, the relaxation function is represented by up to six terms of the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta_m t} \ .$$

This model is effectively a Maxwell fluid, which consists of dampers and springs in series. The formulation is performed in the local system of principal stretches, where only the principal values of stress are computed and triaxial coupling is avoided. Consequently, the one-dimensional nature of this foam material is unaffected by this addition of rate effects. The addition of rate effects necessitates 42 additional history variables per integration point. The cost and memory overhead of this model come primarily from the need to "remember" the local system of principal stretches and the evaluation of the viscous stress components.

Frequency data can be fit to the Prony series. Using Fourier transforms, the relationship between the relaxation function and the frequency-dependent data is

$$G_s(\omega) = \alpha_0 + \sum_{m=1}^{N} \frac{\alpha_m(\omega/\beta_m)^2}{1+(\omega/\beta_m)^2}$$
$$G_\ell(\omega) = \sum_{m=1}^{N} \frac{\alpha_m \omega/\beta_m}{1+\omega/\beta_m}$$

where the storage modulus and loss modulus are defined in terms of the frequency-dependent magnitude $G$ and phase angle $\phi$ given by load curves LCID3 and LCID4 respectively,

$$G_s(\omega) = G(\omega)\cos[\phi(\omega)]$$
$$G_l(\omega) = G(\omega)\sin[\phi(\omega)]$$

3. **Hysteretic unloading.** When hysteretic unloading is used, the reloading will follow the unloading curve if the decay constant, $\beta$, is set to zero. If $\beta$ is nonzero, the decay to the original loading curve is governed by the expression:

$$1 - e^{-\beta t}$$

The bulk viscosity, which generates a rate-dependent pressure, may cause an unexpected volumetric response and, consequently, it is optional with this model.

The hysteretic unloading factor results in the unloading curve lying beneath the loading curve, as shown in Figure M57-1. This unloading provides energy dissipation, which is reasonable in certain kinds of foam.

## *MAT_ELASTIC_SPRING_DISCRETE_BEAM

This is Material Type 74. This model permits elastic springs with damping to be combined and represented with discrete beam element type 6. Linear stiffness and damping coefficients can be defined, and for nonlinear behavior, force-deflection and force-rate curves can be used. Displacement-based failure and an initial force are optional.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | F0 | D | CDF | TDF | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FLCID | HLCID | C1 | C2 | DLE | GLCID | | |
| Type | F | F | F | F | F | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| K | Stiffness coefficient. |
| F0 | Optional initial force. This option is inactive if this material is referenced in a part referenced by material type *MAT_ELASTIC_6DOF_SPRING. |
| D | Viscous damping coefficient. |
| CDF | Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs.<br>EQ.0.0: inactive. |
| TDF | Tensile displacement at failure. After failure, no forces are carried. |

| VARIABLE | DESCRIPTION |
|---|---|
| FLCID | Load curve ID (see *DEFINE_CURVE) defining force as a function of deflection for nonlinear behavior. |
| HLCID | Load curve ID (see *DEFINE_CURVE) defining force as a function of relative velocity for nonlinear behavior (optional). If the origin of the curve is at (0,0), the force magnitude is identical for a given magnitude of the relative velocity, that is, only the sign changes. |
| C1 | Damping coefficient for nonlinear behavior (optional). |
| C2 | Damping coefficient for nonlinear behavior (optional). |
| DLE | Factor to scale time units. The default is unity. |
| GLCID | Optional load curve ID (see *DEFINE_CURVE) defining a scale factor as a function of deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed. |

**Remarks:**

If the linear spring stiffness is used, the force, $F$, is given by:

$$F = F_0 + \mathrm{K}\Delta L + \mathrm{D}\Delta\dot{L} \ .$$

But if the load curve ID is specified, the force is then given by:

$$F = F_0 + \mathrm{K}f(\Delta L)\left\{1 + \mathrm{C1}\times\Delta\dot{L} + \mathrm{C2}\times\mathrm{sgn}(\Delta\dot{L})\ln\left[\max\left(1.,\frac{\Delta\dot{L}}{\mathrm{DLE}}\right)\right]\right\} + \mathrm{D}\Delta\dot{L} + g(\Delta L)h(\Delta\dot{L}) \ .$$

In these equations, $\Delta L$ is the change in length, that is,

$$\Delta L = \text{current length} - \text{initial length} \ .$$

The cross-sectional area is defined on the section card for the discrete beam elements; see *SECTION_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

## *MAT_BILKHU/DUBOIS_FOAM

This is Material Type 75. This model is for simulating isotropic crushable foams. Uniaxial and triaxial test data are used to describe the behavior.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | LCPY | LCUYS | VC | PC | VPC |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TSC | VTSC | LCRATE | PR | KCON | ISFLG | NCYCLE | |
| Type | I | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| LCPY | Load curve ID giving pressure for plastic yielding as a function of volumetric strain; see Figure M75-1. |
| LCUYS | Load curve ID giving uniaxial yield stress as a function of volumetric strain (see Figure M75-1). All abscissa values should be positive if only the results of a compression test are included. Optionally the results of a tensile test can be added (corresponding to negative values of the volumetric strain); in this case PC, VPC, TC and VTC will be ignored. |
| VC | Viscous damping coefficient (0.05 < recommended value < 0.50; default is 0.05) |
| PC | Pressure cutoff for hydrostatic tension. If zero, the default is set to one-tenth of $p_0$, the yield pressure corresponding to a volumetric strain of zero. PC will be ignored if TC is nonzero. |

| VARIABLE | DESCRIPTION |
|---|---|
| VPC | Variable pressure cutoff for hydrostatic tension as a fraction of pressure yield value. If nonzero this will override the pressure cutoff value PC. |
| TC | Tension cutoff for uniaxial tensile stress. Default is zero. A nonzero value is recommended for better stability. |
| VTC | Variable tension cutoff for uniaxial tensile stress as a fraction of the uniaxial compressive yield strength. If nonzero; this will override the tension cutoff value TC. |
| LCRATE | Load curve ID giving a scale factor for the previous yield curves, dependent upon the volumetric strain rate |
| PR | Poisson's ratio, which applies to both elastic and plastic deformations. It must be smaller then 0.5. |
| KCON | Stiffness coefficient for contact interface stiffness. If undefined one-third of Young's modulus, E, is used. KCON is also considered in the element time step calculation; therefore, large values may reduce the element time step size. |
| ISFLG | Flag for tensile response (active only if negative abscissa are present in load curve LCUYS):<br>EQ.0: load curve abscissa in tensile region correspond to volumetric strain<br>EQ.1: load curve abscissa in tensile region correspond to effective strain (for large PR, when volumetric strain vanishes) |
| NCYCLE | Number of cycles to determine the average volumetric strain rate. NCYCLE is 1 by default (no smoothing) and cannot exceed 100. |

**Remarks:**

1. **Volumetric Strain.** The logarithmic volumetric strain is defined in terms of the relative volume, $V$, as:

$$\gamma = -\ln(V)$$

If option ISFLG = 1 is used, the effective strain is defined in the usual way:

$$\varepsilon_{\text{eff}} = \sqrt{\frac{2}{3}\text{tr}(\boldsymbol{\varepsilon}^{\text{t}}\boldsymbol{\varepsilon})}$$

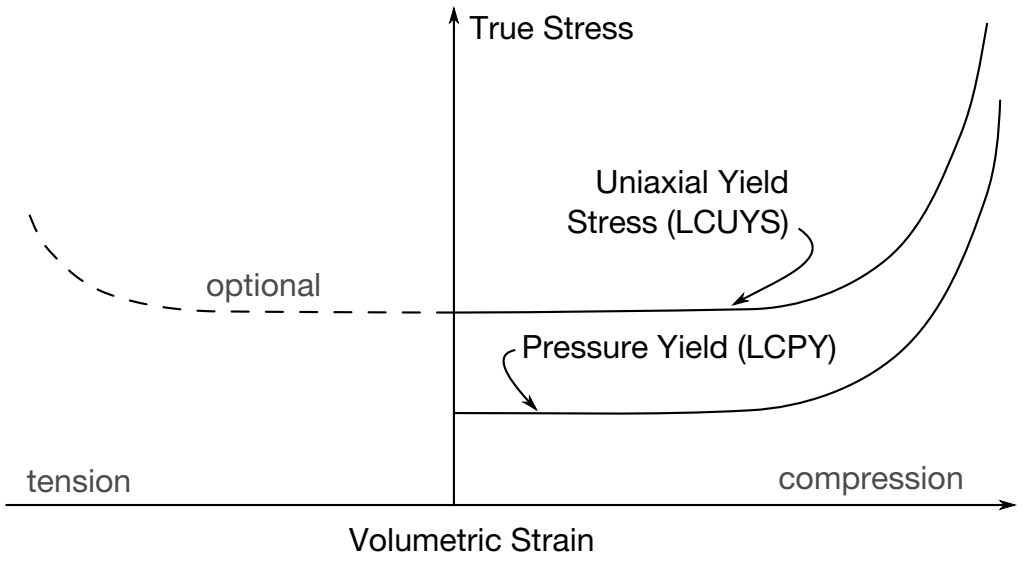



**Figure M75-1.** Behavior of crushable foam. Unloading is elastic.

The stress and strain pairs in load curve LCPY should be positive values starting with a volumetric strain value of zero.

2. **LCUYS.** The load curve LCUYS can optionally contain the results of the tensile test (corresponding to negative values of the volumetric strain); if it does, then the load curve information will override PC, VPC, TC and VTC. This is the recommended approach, because the necessary continuity between tensile and compressive regime becomes obvious (see Figure M75-1).

3. **Yield Surface.** The yield surface is defined as an ellipse in the equivalent pressure and von Mises stress plane. This ellipse is characterized by three points:

   a) the hydrostatic compression limit (LCPY),

   b) the uniaxial compression limit (LCUYS), and

   c) either the pressure cutoff for hydrostatic stress (PC,VPC), the tension cutoff for uniaxial tension (TC,VTC), or the optional tensile part of LCUYS.

4. **High Frequency Oscillations.** To prevent high frequency oscillations in the strain rate from causing similar high frequency oscillations in the yield stress, a modified volumetric strain rate is used to obtain the scaled yield stress. The modified strain rate is obtained as follows. If NYCLE is > 1, then the modified strain rate is obtained by a time average of the actual strain rate over NCYCLE solution cycles. The averaged strain rate is stored in history variable #3.

## *MAT_GENERAL_VISCOELASTIC_{OPTION}

The available options include:

<BLANK>

MOISTURE

This is Material Type 76. This material model provides a general viscoelastic Maxwell model having up to 18 terms in the Prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. Either the coefficients of the Prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior.

The material model can also be used with laminated shells. Either an elastic or viscoelastic layer can be defined with the laminated formulation. To activate laminated shells, you must set the laminated formulation flag on *CONTROL_SHELL. With the laminated option you must also define an integration rule. The addition of an elastic or viscoelastic layer was implemented by Professor Ala Tabiei, and the laminated shells feature was developed and implemented by Professor Ala Tabiei.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | BULK | PCF | EF | TREF | A | B |
|---|---|---|---|---|---|---|---|

**Card 2.** Leave blank if the Prony Series Cards (Card 4) are used below. Also, leave blank if an elastic layer is defined in a laminated shell.

| LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if the MOISTURE keyword option is used.

| MREF | ALPHA | BETA | GAMMA | MST | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Up to 18 cards may be input. This input is terminated at the next keyword ("*") card.

| G*i* | BETA*i* | K*i* | BETAK*i* | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | PCF | EF | TREF | A | B |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Elastic bulk modulus |
| PCF | Tensile pressure elimination flag for solid elements only. If set to unity, tensile pressures are set to zero. |
| EF | Elastic flag:<br>EQ.0: The layer is viscoelastic.<br>EQ.1: The layer is elastic. |
| TREF | Reference temperature for shift function (must be greater than zero) |
| A | Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions |
| B | Coefficient for the Williams-Landel-Ferry shift function |

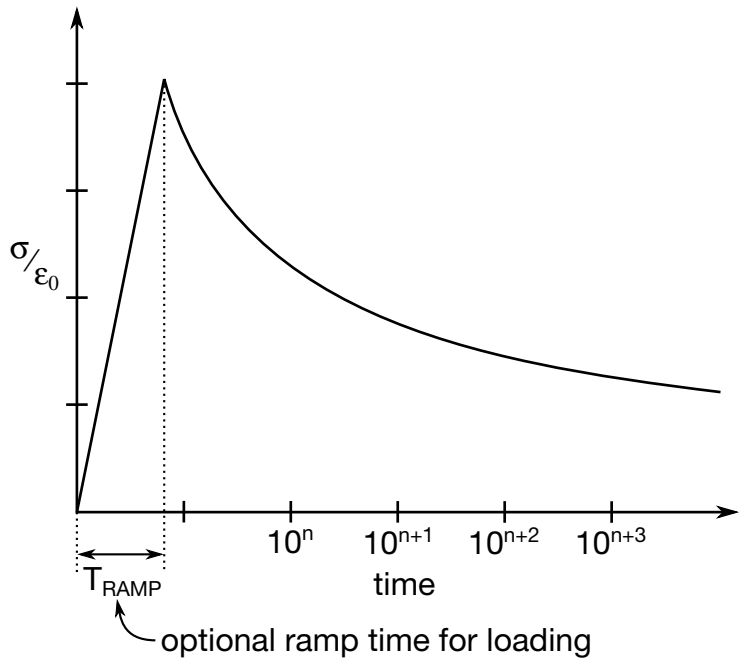



**Figure M76-1.** Relaxation curves for deviatoric behavior and bulk behavior. The ordinate of LCID is the deviatoric stress divided by 2 times the constant value of deviatoric strain, where the stress and strain are in the direction of the prescribed strain, or in non-directional terms, the effective stress divided by 3 times the effective strain. LCIDK defines the mean stress divided by the constant value of volumetric strain imposed in a hydrostatic stress relaxation experiment as a function of time. For best results, the points defined in the curve should be equally spaced on the logarithmic scale. *Note that the abscissa values are input as time, not log(time).* Furthermore, the curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by a least squares fit, an error termination will occur after the completion of the initialization phase. If the ramp time for loading is included, the relaxation that occurs during the loading phase is accounted for. This effect may or may not be important.

**Relaxation Curve Card.** Leave blank if the *Prony Series Cards* are used below. Also, leave blank if an elastic layer is defined in a laminated shell.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
| Type | F | I | F | F | F | I | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID | Load curve ID for deviatoric relaxation behavior. If LCID is given, constants $G_i$, and $\beta_i$ are determined using a least squares fit. See Figure M76-1 for an example relaxation curve. |
| NT | Number of terms in shear fit. If zero, the default is 6. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is set to 18. |
| BSTART | In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times $\beta_3$, and so on. If zero, BSTART is determined by an iterative trial and error scheme. |
| TRAMP | Optional ramp time for loading. |
| LCIDK | Load curve ID for bulk relaxation behavior. If LCIDK is given, constants $K_i$, and $\beta_{ki}$ are determined via a least squares fit. See Figure M76-1 for an example relaxation curve. |
| NTK | Number of terms desired in bulk fit. If zero, the default is 6. Currently, the maximum number is set to 18. |
| BSTARTK | In the fit, $\beta_{k1}$ is set to zero, $\beta_{k2}$ is set to BSTARTK, $\beta_{k3}$ is 10 times $\beta_{k2}$, $\beta_{k4}$ is 100 times $\beta_{k3}$, and so on. If zero, BSTARTK is determined by an iterative trial and error scheme. |
| TRAMPK | Optional ramp time for bulk loading. |

**Moisture Card.** Additional card for the MOISTURE keyword option.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MREF | ALPHA | BETA | GAMMA | MST | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MREF | Reference moisture, $M_{\text{ref}}$, for moisture-induced strains. The default value is zero. |
| ALPHA | Specifies $\alpha$ as a function of moisture.<br>GT.0.0: Specifies a curve ID. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: Specifies the negative of a constant value. |
| BETA | Specifies $\beta$ as a function of moisture.<br>GT.0.0: Specifies a curve ID.<br>LT.0.0: Specifies the negative of a constant value. |
| GAMMA | Specifies $\gamma$ as a function of moisture.<br>GT.0.0: Specifies a curve ID.<br>LT.0.0: Specifies the negative of a constant value. |
| MST | Moisture, *M*. If the moisture is 0.0, the moisture option is disabled.<br>GT.0.0: Specifies a curve ID giving moisture as a function of time.<br>LT.0.0: Specifies the negative of a constant value of moisture. |

**Prony Series cards.** Card Format for viscoelastic constants. Up to 18 cards may be input. If fewer than 18 cards are used, the next keyword ("*") card terminates this input. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: insert zero if a term is not included. If an elastic layer is defined you only need to define G*i* and K*i* (note in an elastic layer only one card is needed)

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional shear decay constant for the $i^{th}$ term |
| K*i* | Optional bulk relaxation modulus for the $i^{th}$ term |
| BETAK*i* | Optional bulk decay constant for the $i^{th}$ term |

**Remarks:**

Rate effects are taken into accounted through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau$$

where $g_{ijkl(t-\tau)}$ is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by 18 terms from the Prony series:

$$g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t}$$

We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. An arbitrary number of terms, up to 18, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^{N} K_m e^{-\beta_{k_m} t}$$

The Arrhenius and Williams-Landel-Ferry (WLF) shift functions account for the effects of the temperature on the stress relaxation. A scaled time, $t'$,

$$t' = \int_0^t \Phi(T)dt$$

is used in the relaxation function instead of the physical time. The Arrhenius shift function is

$$\Phi(T) = \exp\left[-A\left(\frac{1}{T} - \frac{1}{T_{\text{REF}}}\right)\right]$$

and the Williams-Landel-Ferry shift function is

$$\Phi(T) = \exp\left(-A\frac{T - T_{\text{REF}}}{B + T - T_{\text{REF}}}\right)$$

If all three values (TREF, A, and B) are nonzero, the WLF function is used; the Arrhenius function is used if B is zero; and no scaling is applied if all three values are zero.

The moisture model allows the scaling of the material properties as a function of the moisture content, $M$, of the material. The shear and bulk moduli are scaled by $\alpha$, the decay constants are scaled by $\beta$. A strain induced by moisture changes is introduced, analogous to the thermal strain:

$$\varepsilon^{\text{M}} = \gamma(M)[M - M_{\text{ref}}] - \gamma(M_{t=0})[M_{t=0} - M_{\text{ref}}].$$

Note that strain can only be induced by non-constant moisture values.

## *MAT_HYPERELASTIC_RUBBER

This is Material Type 77. This material model provides a general hyperelastic rubber model combined optionally with linear viscoelasticity, as outlined by Christensen [1980].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | PR | N | NV | G | SIGF | REF |
|---|---|---|---|---|---|---|---|

**Card 2.** Include this card if PR < 0.

| TBHYS | LCBI | LCPL | WBI | WPL | D1 | D2 | D3 |
|---|---|---|---|---|---|---|---|

**Card 3a.** Include this card if N > 0.

| SGL | SW | ST | LCID1 | DATA | LCID2 | BSTART | TRAMP |
|---|---|---|---|---|---|---|---|

**Card 3b.** Include this card if N = 0.

| C10 | C01 | C11 | C20 | C02 | C30 | THERML | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include up to 12 of this card. The next keyword ("*") card terminates this input. Note that VFLAG is only included in the first card of this set.

| G*i* | BETA*i* | G*j* | SIGF*j* | VFLAG | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | N | NV | G | SIGF | REF |
| Type | A | F | F | I | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio. If set to a negative number, the Poisson's ratio is the absolute value, and Card 2 is included for extra parameters. Setting to 0.5 activates a *U-P* formulation for implicit analysis; see Remark 3 of *MAT_027 (the Mooney-Rivlin rubber model). |
| N | Number of hyperelastic constants to solve for from LCID1 or combinations of LCID1, LCBI, and LCPL:<br>EQ.0: Set hyperelastic constants directly.<br>EQ.1: Solve for C10 and C01.<br>EQ.2: Solve for C10, C01, C11, C20, and C02.<br>EQ.3: Solve for C10, C01, C11, C20, C02, and C30. |
| NV | Number of Prony series terms used in fitting curve LCID2. This field is ignored if LCID2 is not defined. If NV is set to zero, it defaults to a value of 6. Currently, 12 is the maximum number. We recommend values smaller than 12, possibly 3 – 5, since each term used adds significantly to the cost. Exercise caution when taking the results from the fit. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once you have achieved a satisfactory fit, we recommend inputting the coefficients written into the output file for future runs. |
| G | Shear modulus for frequency-independent damping. Frequency-independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF, defined below. For the best results, the value of G should be 250 - 1000 times greater than SIGF. |
| SIGF | Limit stress for frequency-independent frictional damping |
| REF | Use reference geometry to initialize the stress tensor. *INITIAL_FOAM_REFERENCE_GEOMETRY defines the reference geometry.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Hysteresis Card.** Additional card included when PR < 0.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TBHYS | LCBI | LCPL | WBI | WPL | D1 | D2 | D3 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TBHYS | Table ID for hysteresis, which can be positive or negative; see Remarks 1 and 2. This field only applies to solid elements. |
| LCBI | Load curve ID giving force as a function of displacement for the biaxial test used for parameter fitting. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3. |
| LCPL | Load curve ID giving force as a function of displacement for the planar test used for parameter fitting. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3. |
| WBI | Weight factor giving the relative influence of the biaxial test data in the fitting of material parameters. A value of 1.0 means that the biaxial test data is of equal importance as the uniaxial test data. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3. |
| WPL | Weight factor giving the relative influence of planar test data in the fitting of material parameters. A value of 1.0 means that the planar test data is of equal importance as the uniaxial test data. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3. |
| D1 | Compression compliance constant. If this parameter is greater than zero, then LS-DYNA does not use the value of PR set on Card 1 for Poisson's ratio. |
| D2 | Compression compliance constant |
| D3 | Compression compliance constant |

**Card 3 for N > 0.** For N > 0, LS-DYNA computes a least squares fit from the uniaxial or combined data.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LCID1 | DATA | LCID2 | BSTART | TRAMP |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SGL | Specimen gauge length |
| SW | Specimen width |
| ST | Specimen thickness |
| LCID1 | Load curve ID giving the force as a function of actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), LCID1 is engineering stress as a function of engineering strain. The curve should have both negative (compressive) and positive (tensile) values. |
| DATA | Type of experimental data (only active if LCBI, LCPL, WBI, and WPL are all zero on Card 2 or Card 2 is not activated):<br>EQ.0.0: Uniaxial data (only option for this model) |
| LCID2 | Load curve ID of the deviatoric stress relaxation curve, neglecting the long term deviatoric stress. If LCID2 is specified, constants $G_i$ and $\beta_i$ are determined internally using a least squares fit. See Figure M76-1 for an example relaxation curve. The ordinate of the curve is the viscoelastic deviatoric stress divided by 2 times the constant value of deviatoric strain where the stress and strain are in the direction of the prescribed strain, or in non-directional terms, the effective stress divided by 3 times the effective strain. |
| BSTART | In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times $\beta_3$, and so on. If zero, BSTART is determined by an iterative trial and error scheme. |
| TRAMP | Optional ramp time for loading |

**Card 3 for N = 0.** Set the hyperelastic material parameters directly.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C10 | C01 | C11 | C20 | C02 | C30 | THERML | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| C10 | $C_{10}$ |
| C01 | $C_{01}$ |
| C11 | $C_{11}$ |
| C20 | $C_{20}$ |
| C02 | $C_{02}$ |
| C30 | $C_{30}$ |
| THERML | Flag for the thermal option. If THERML > 0.0, then G, SIGF, C10 and C01 must *all* specify curve IDs (zero is not permitted) that define the values as functions of temperature. If THERML < 0.0, then G, SIGF, C10 and C01, C11, C20, C02, and C30 must *all* specify curve IDs (zero is not permitted) that define the values as functions of temperature. A 'flat' curve may be used to define a constant value that does not change with temperature. This thermal option is available only for solid elements. |

**Optional Viscoelastic Constants & Frictional Damping Constant Cards.** Up to 12 cards may be input. The next keyword ("*") card terminates this input.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | G*j* | SIGF*j* | VFLAG | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term. Not used if LCID2 is given. |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA*i* | Optional decay constant of the $i^{th}$ term. Not used if LCID2 is given. |
| G*j* | Optional shear modulus for frequency independent damping represented as the $j^{th}$ spring and slider in series in parallel to the rest of the stress contributions. |
| SIGF*j* | Limit stress for frequency independent, frictional, damping represented as the $j^{th}$ spring and slider in series in parallel to the rest of the stress contributions. |
| VFLAG | Flag for the viscoelasticity formulation. This field appears only in the first Card 4 line.<br>EQ.0: Standard viscoelasticity formulation (default)<br>EQ.1: Viscoelasticity formulation using the instantaneous elastic stress (only applicable to solid elements). |

**Background:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term, $W_H(J)$, is included in the strain energy functional which is function of the relative volume, $J$, [Ogden 1984]:

$$W(J_1, J_2, J) = \sum_{p,q=0}^{n} C_{pq}(J_1 - 3)^p (J_2 - 3)^q + W_H(J)$$

$$J_1 = I_1 I_3^{-1/3}$$

$$J_2 = I_2 I_3^{-2/3}$$

To prevent volumetric work from contributing to the hydrostatic work, the first and second invariants are modified as shown. If D1 is positive, then

$$W_H(J) = \sum_{i=1}^{3} \frac{(J-1)^{2i}}{D_i} \ .$$

Otherwise, it is

$$W_H(J) = \frac{K}{2}(J-1)^2$$

with $K$ being the linear bulk modulus determined from the corresponding linear shear modulus $G = 2(C_{10} + C_{01})$ and Poisson's ratio. Historically this model has been used for incompressible behavior, but it is also valid for compressible data. This procedure is described in more detail by Sussman and Bathe [1987]. The second Piola-Kirchhoff and Cauchy stress tensors are obtained from the strain energy functional as

$$\boldsymbol{S} = \frac{\partial W}{\partial \boldsymbol{E}}, \qquad \boldsymbol{\sigma}_W = \frac{1}{J}\boldsymbol{F}\boldsymbol{S}\boldsymbol{F}^T,$$

where $\boldsymbol{E}$ is the Green strain tensor and $\boldsymbol{F}$ is the deformation gradient. We use the subscript $W$ here to denote the contribution from the strain energy potential, and with no other contributions the resulting Cauchy stress is simply

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_W.$$

Rate effects are taken into account through linear viscoelasticity by adding a sequence of stress contributions

$$\boldsymbol{\sigma}_V = \sum_{i=1}^{n} \boldsymbol{\sigma}_V^i$$

where each term is known as a *Prony* term. Each such stress component $\boldsymbol{\sigma}_V^i$ evolves with deformation and time as

$$\left(\boldsymbol{\sigma}_V^i\right)^{\nabla} = 2G_i\left(\boldsymbol{D} - e^{-\beta_i(t-t_0)}\beta_i\boldsymbol{\varepsilon}_V^i\right), \qquad \left(\boldsymbol{\varepsilon}_V^i\right)^{\nabla} = e^{\beta_i(t-t_0)}\boldsymbol{D}.$$

Here $\nabla$ denotes the Jaumann rate. $\boldsymbol{D}$ is the rate-of-deformation tensor, $t$ is time and $t_0$ is an arbitrary time point. Each term has an internal strain $\boldsymbol{\varepsilon}_V^i$ associated with itself, which incorporates the memory properties a viscoelastic material typically possesses. This stress is added to the stress tensor determined from the strain energy functional, so that

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_W + \boldsymbol{\sigma}_V.$$

This model is effectively a Maxwell fluid which consists of dampers and springs in series. An arbitrary number of such Prony terms can be input, each characterized by the shear modulus, $G_i$, and relaxation coefficient, $\beta_i$. To avoid a constant shear modulus from this viscoelastic formulation, a term in the series is included only when $\beta_i > 0$.

For the sake of understanding the influence these terms have on the rate effects of viscoelasticity, let's investigate the model in a situation with no spin and constant rate-of-deformation with $\boldsymbol{D} \neq \boldsymbol{0}$. This means that the Jaumann rate is simply differentiation with time, and we can look at the implications a specific term has. To make some physical sense of things, we deal with both the no hyperelastic material present and the hyperelastic material present cases. For the latter we assume an elastic shear modulus, $G$, for the hyperelastic material.

1. **Constant strain rate, $\boldsymbol{D}$.** For the special case of constant strain rate, $\boldsymbol{D}$, we have the following expression for the stress rate

   $$\dot{\boldsymbol{\sigma}}_V^i = 2G_i e^{-\beta_i t}\boldsymbol{D},$$

   so each term contributes with an instantaneous shear stiffness of $G_i$ that decays with time at a rate determined by $\beta_i$. If we define the relaxation time as

   $$\tau_i = 1/\beta_i,$$

we see that the term will not contribute much to the response when $t > 5\tau_i$. So with several Prony terms with different relaxation properties, the overall viscoelastic stiffness decays roughly with steps of $G_i$ in time spans of $\tau_i$. This information can be used for determining the material data by making clever use of tensile tests at different strain rates. Looking at the corresponding stress contribution from each term

$$\sigma_V^i = 2\frac{G_i}{\beta_i}(1 - e^{-\beta_i t})\boldsymbol{D}$$

we see that the stress stabilizes at a nonzero level $2\frac{G_i}{\beta_i}\boldsymbol{D}$ as time goes to infinity. See Figure M77-1.

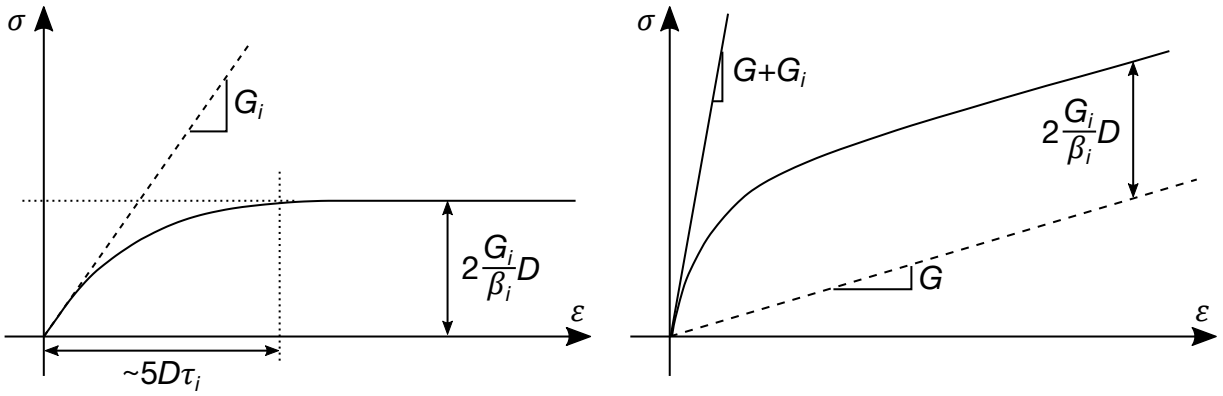



**Figure M77-1.** Material response with a constant strain rate

2. **Relaxation.** To see its effect on stress relaxation, we assume the material has deformed with a constant rate-of-deformation $\boldsymbol{D}_0 \neq \boldsymbol{0}$ between time 0 and $t_0$, and then continues with another constant rate-of-deformation $\boldsymbol{D}$ (which we allow to be zero) after time $t_0$ (see Figure M77-2). The expression for the stress is

$$\sigma_V^i = e^{-\beta_i(t-t_0)}\sigma_0^i + 2\frac{G_i}{\beta_i}(1 - e^{-\beta_i(t-t_0)})\boldsymbol{D},$$

where $\sigma_0^i$ is the stress level that was reached at time $t_0$. Stress relaxation occurs when $\boldsymbol{D} = \boldsymbol{0}$ for which we see that the stress decays (or relaxes) to zero at a rate determined by $\beta_i$. When a hyperelastic material is included, the stress is relaxed to the hyperelastic stress, illustrated by a dashed line in the figure. As before, when combining many terms with different relaxation properties, the stress relaxes in steps of $\sigma_0^i$ in time spans of $\tau_i$ and essentially determines the shape of the relaxation curve. This can also be used as a basis for estimating material parameters.

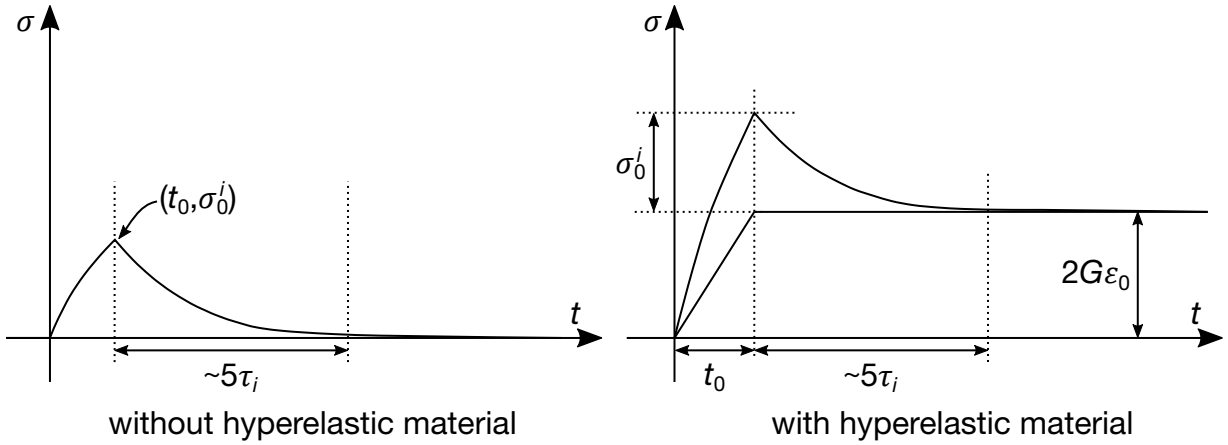



**Figure M77-2.** Stress relaxation curves

3. **Creep.** For creep, we assume the same situation but instead of prescribing the strain rate, $\boldsymbol{D}$, we enforce the stress, $\sigma$, to be constant after time $t_0$. The expression for the creep strain, $\varepsilon_c$, in the non-presence of a hyperelastic material becomes

$$\varepsilon_c = \frac{\beta_i}{2G_i}(t - t_0)\sigma_0^i,$$

which indicates that the creep strain evolves linearly with time (see Figure M77-3). This is a rather non-physical behavior, but in the presence of a hyperelastic material the creep evolves as

$$\varepsilon_c = \frac{1}{2G_i}\ln\left\{\frac{G + G_i}{G + G_i e^{-\beta_i(t-t_0)}}\right\}\sigma_0^i$$

and saturates as one would expect to a constant value. With many such terms, the creep evolves in a quantitatively different manner, but the qualitative behavior is to be understood as described.

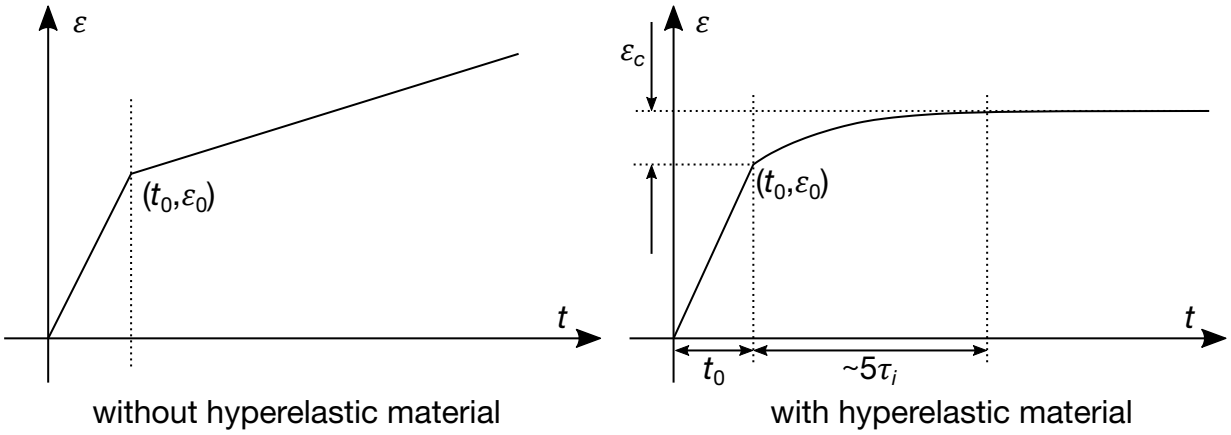
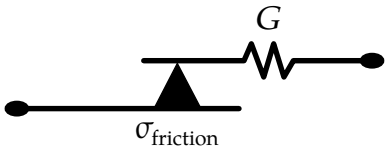



**Figure M77-3.** Creep curves

The Mooney-Rivlin rubber model (model 27) is obtained by specifying $N = 1$. Despite the differences in formulations, we find that the results obtained with this model are nearly identical with those of material 27 as long as large values of Poisson's ratio are used.

Frequency independent damping is obtained by having a spring and slider in series as shown in the following sketch:




Several springs and sliders in series can be defined that are put in parallel to the rest of the stress contributions of this material model.

**Remarks:**

1. **Hysteresis (TBHYS > 0).** *If a positive table ID for hysteresis is defined*, then TBHYS is a table having curves that are functions of strain-energy density. Let $W_{\text{dev}}$ be the current value of the deviatoric strain energy density as calculated above. Furthermore, let $\overline{W}_{\text{dev}}$ be the peak strain energy density reached up to this point in time. It is then assumed that the resulting stress is reduced by a damage factor according to

$$\mathbf{S} = D(W_{\text{dev}}, \overline{W}_{\text{dev}}) \frac{\partial W_{\text{dev}}}{\partial \mathbf{E}} + \frac{\partial W_{\text{vol}}}{\partial \mathbf{E}} \ ,$$

where $D(W_{\text{dev}}, \overline{W}_{\text{dev}})$ is the damage factor which is input as the table, TBHYS. This table consists of curves giving stress reduction (between 0 and 1) as a function of $W_{\text{dev}}$ indexed by $\overline{W}_{\text{dev}}$.

Each $\overline{W}_{\text{dev}}$ curve must be valid for strain energy densities between 0 and $\overline{W}_{\text{dev}}$. It is *recommended* that each curve be monotonically increasing, and it is *required* that each curve equals 1 when $W_{\text{dev}} > \overline{W}_{\text{dev}}$. Additionally, *DEFINE_TABLE *requires* that each curve have the same beginning and end point and, furthermore, that they not cross except at the boundaries, although they are not required to cross.

This table can be roughly estimated from a uniaxial quasistatic compression test as follows (see Figure  for an illustration of the different curves):

a) Load the specimen to a maximum displacement $\bar{d}$ and measure the force as function of displacement, $f_{\text{load}}(d)$.

b) Unload the specimen again measuring the force as a function of displacement, $f_{\text{unload}}(d)$.

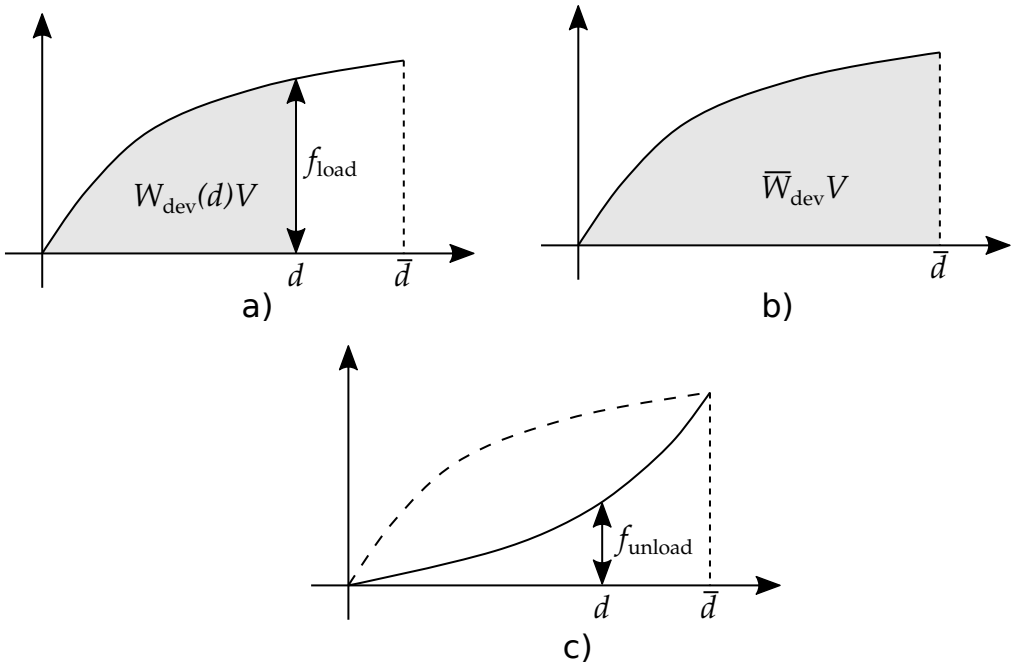



**Figure M77-4.** Illustration of curves needed from experiments to obtain $D(W_{dev}, \overline{W}_{dev})$. a) indicates the response during a uniaxial quasistatic compression test from which you can find $W_{dev}(d)$ (area under the curve). Each test is associated with a maximum displacement and thus a peak strain energy, $\overline{W}_{dev}$ (area under the curve in b)). c) indicates the unloading curve during the test. Inverting $W_{dev}(d)$ allows you to find $D(W_{dev}, \overline{W}_{dev})$ from the loading and unloading curves for a value of $\overline{W}_{dev}$.

c) The strain energy density is, then, given as a function of the loaded displacement as

$$W_{dev}(d) = \frac{1}{V}\int_0^d f_{load}(s)ds \ .$$

i) The peak energy, which is used to index the data set, is given by

$$\overline{W}_{dev} = W_{dev}(\bar{d}) \ .$$

ii) From this energy curve we can also determine the inverse, $d(W_{dev})$. Using this inverse the load curve for LS-DYNA is then given by:

$$D(W_{dev}, \overline{W}_{dev}) = \frac{f_{unload}[d(W_{dev})]}{f_{load}[d(W_{dev})]} \ .$$

d) This procedure is repeated for different values of $\bar{d}$ (or equivalently $\overline{W}_{dev}$).

2. **Hysteresis (TBHYS < 0).** If a negative table ID for hysteresis is defined, then all of the above holds with the difference being that the load curves comprising table, |TBHYS|, must give the strain-energy density, $W_{dev}$, as a function of the

stress reduction factor. *This scheme guarantees that all curves have the same beginning point, 0, and the same end point, 1.* For negative TBHYS the user provides $W_{\text{dev}}(D, \overline{W}_{\text{dev}})$ instead of $D(W_{\text{dev}}, \overline{W}_{\text{dev}})$. In practice, this case corresponds to swapping the load curve axes.

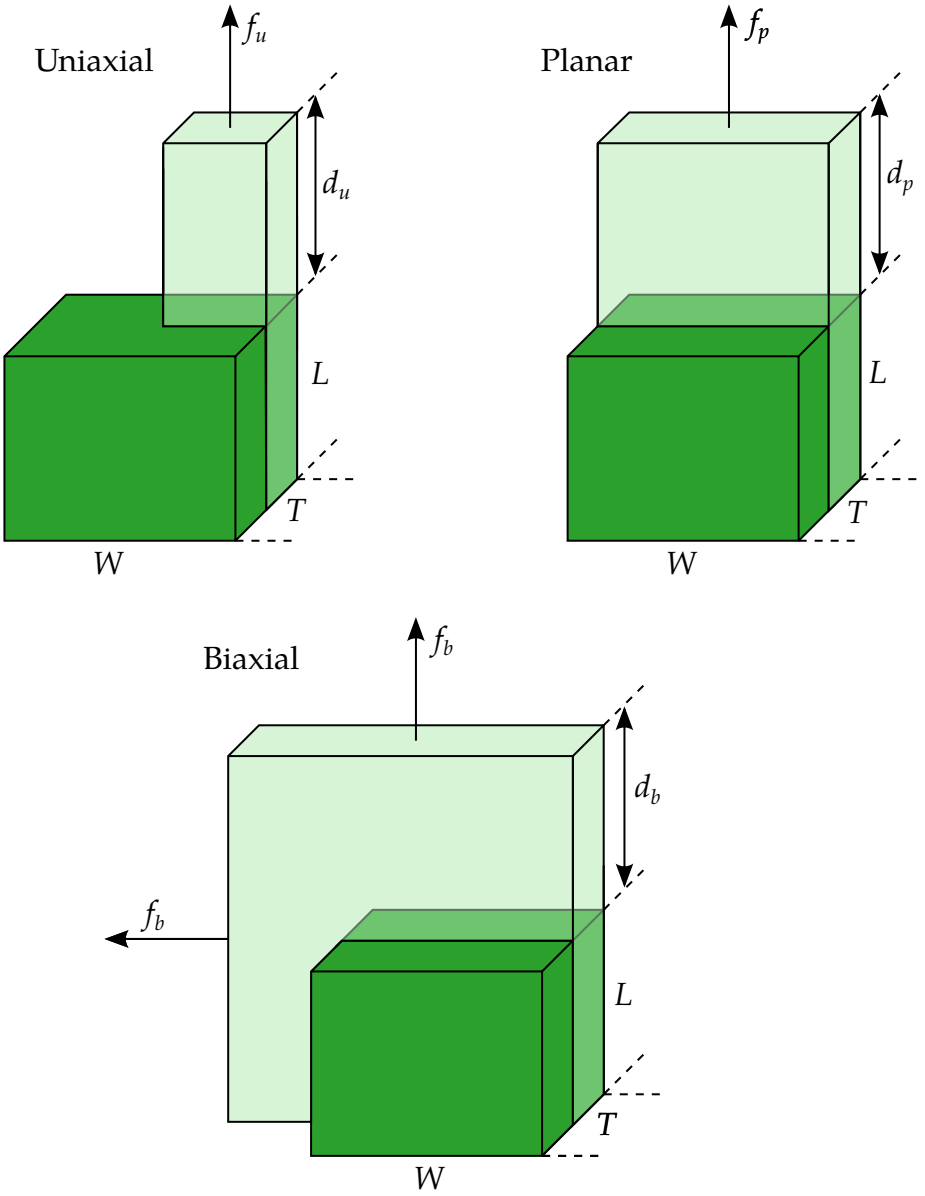



**Figure M77-5.** Tests for parameter fitting

3. **Parameter fitting.** For general fitting of material parameters we refer to Figure M77-5. If at least one of LCBI with a positive WBI ($w_b$) or LCPL with a positive WPL ($w_p$) is set, parameters determined by N on Card 1 are fitted using a nonlinear least square optimization problem. We assume that LCID1 corresponds to a load curve giving $f_u$ as a function of $d_u$, while LCBI and LCPL are load curves giving $f_b$ as a function of $d_b$ and $f_p$ as a function of $d_p$, respectively.

To obtain the test data, load a specimen of dimensions $L \times W \times T$ as shown in the figure. The displacements must increase in the curves, and both compressive and tensile data is allowed. Let $g_u$, $g_b$ and $g_p$ be the simulated forces for the displacement data given, then the material parameters are determined to minimize the potential

$$h = \sum_{d_u} \left(1 - \frac{g_u}{f_u}\right)^2 + w_b \sum_{d_b} \left(1 - \frac{g_b}{f_b}\right)^2 + w_p \sum_{d_p} \left(1 - \frac{g_p}{f_p}\right)^2 .$$

The sums are supposed to be over the data points provided for each test. Note that the weight factors can be used to determine the relative influence of each test. Each term in the sums corresponds to the relative force error for the corresponding data point, this to obtain a better fit for smaller strains.

## *MAT_OGDEN_RUBBER

This is also Material Type 77. This material model provides the Ogden [1984] rubber model combined optionally with linear viscoelasticity as outlined by Christensen [1980].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | PR | N | NV | G | SIGF | REF |
|---|---|---|---|---|---|---|---|

**Card 2.** Include this card when PR < 0.

| TBHYS | LCBI | LCPL | WBI | WPL | D1 | D2 | D3 |
|---|---|---|---|---|---|---|---|

**Card 3a.** Include this card if N > 0.

| SGL | SW | ST | LCID1 | DATA | LCID2 | BSTART | TRAMP |
|---|---|---|---|---|---|---|---|

**Card 3b.1.** Include this card if N = 0 or -1.

| MU1 | MU2 | MU3 | MU4 | MU5 | MU6 | MU7 | MU8 |
|---|---|---|---|---|---|---|---|

**Card 3b.2.** Include this card if N = 0 or -1.

| ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
|---|---|---|---|---|---|---|---|

**Card 4.** Include up to 12 of this card. This input ends with the next keyword ("*") card.

| G*i* | BETA*i* | VFLAG | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | N | NV | G | SIGF | REF |
| Type | A | F | F | I | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| PR | Poisson's ratio. If set to a negative number, the Poisson's ratio is the absolute value, and Card 2 is included for extra parameters. |
| N | Order of fit to curve LCID1 or combinations of LCID1, LCBI, and LCPL for the Ogden model (currently < 9, 2 generally works okay). LS-DYNA prints the constants generated during the fit to d3hsp. To save the cost of performing the nonlinear fit in future runs, directly input the constants from this fit. You can visually evaluate the goodness of the fit by plotting data in the output file curveplot*. To do this with LS-PrePost, click *XYplot* → *Add* to read the curveplot* file.<br>EQ.0: Allows you to specify the material parameters directly with Cards 3b.1 and 3b.2<br>EQ.-1: Same as N = 0 but invokes a thermal option: parameters MU*i* and ALPHA*i* are read as load curves IDs and thereby define these parameters as functions of temperature. It is available only for solid elements. VFLAG must be 0. |
| NV | Number of Prony series terms for fitting curve LCID2. This field is ignored if LCID2 is not defined. If NV is set to zero, it defaults to a value of 6. Currently, 12 is the maximum number. We recommend values smaller than 12, possibly 3 – 5, since each term used adds significantly to the cost. Exercise caution when taking the results from the fit. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once you have achieved a satisfactory fit, we recommend inputting the coefficients written into the output file for future runs. |
| G | Shear modulus for frequency independent damping. Frequency independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250 - 1000 times greater than SIGF. |
| SIGF | Limit stress for frequency independent frictional damping |
| REF | Use reference geometry to initialize the stress tensor. *INITIAL_FOAM_REFERENCE_GEOMETRY defines the reference geometry. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Off |
| | EQ.1.0: On |

**Hysteresis Card.** Additional card included when PR < 0.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TBHYS | LCBI | LCPL | WBI | WPL | D1 | D2 | D3 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TBHYS | Table ID for hysteresis, could be positive or negative; see Remarks 1 and 2 in the manual page for *MAT_HYPERELASTIC_RUBBER. This field only applies to solid elements. |
| LCBI | Load curve ID giving force as a function of displacement for the biaxial test used in parameter fitting. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3 in the manual page for *MAT_HYPERELASTIC_RUBBER. |
| LCPL | Load curve ID giving force as a function of displacement for the planar test used in parameter fitting. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3 in the manual page for *MAT_HYPERELASTIC_RUBBER. |
| WBI | Weight factor giving the relative influence of the biaxial test data in the fitting of material parameters, a value of 1.0 means that it is of equal importance as the uniaxial test data. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3 in the manual page for *MAT_HYPERELASTIC_RUBBER. |
| WPL | Weight factor giving the relative influence of the planar test data in the fitting of material parameters, a value of 1.0 means that it is of equal importance as the uniaxial test data. Make sure N > 0 on Card 1 if setting this parameter. See Remark 3 in the manual page for *MAT_HYPERELASTIC_RUBBER. |
| D1 | Compression compliance constant. If this parameter is greater than zero, then LS-DYNA does not use the value of PR set on Card 1 for Poisson's ratio. |

| VARIABLE | DESCRIPTION |
|---|---|
| D2 | Compression compliance constant |
| D3 | Compression compliance constant |

**Least Squares Card.** For N > 0, a least squares fit to curve LCID1 or LCID1/LCBI/LCPL is computed.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LCID1 | DATA | LCID2 | BSTART | TRAMP |
| Type | F | F | F | F | F | F | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SGL | Specimen gauge length |
| SW | Specimen width |
| ST | Specimen thickness |
| LCID1 | Load curve ID giving the force as a function of actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), LCID1 is engineering stress as a function of engineering strain. The curve should have both negative (compressive) and positive (tensile) values. |
| DATA | Type of experimental data (only active if LCBI, LCPL, WBI, and WPL are all zero on Card 2 or Card 2 is not activated):<br>EQ.1.0: Uniaxial data (default)<br>EQ.2.0: Biaxial data<br>EQ.3.0: Pure shear data |
| LCID2 | Load curve ID of the deviatoric stress relaxation curve, neglecting the long term deviatoric stress. If LCID2 is given, constants $G_i$ and $\beta_i$ are determined using a least squares fit. See M76-1 for an example relaxation curve. The ordinate of the curve is the viscoelastic deviatoric stress divided by the quantity 2 times the constant value of deviatoric strain where the stress and strain are in the direction of the prescribed strain. If in non-directional terms, it is the effective stress divided by the quantity 3 times the effective strain. |

| VARIABLE | DESCRIPTION |
|---|---|
| BSTART | In the fit, $\beta_i$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times $\beta_3$, and so on. If zero, BSTART is determined by an iterative trial and error scheme. |
| TRAMP | Optional ramp time for loading |

**Material Parameters Card.** Include for N = 0 or N = -1 to set the material parameters directly.

| Card 3b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MU1 | MU2 | MU3 | MU4 | MU5 | MU6 | MU7 | MU8 |
| Type | F | F | F | F | F | F | F | F |

**Material Parameters Card.** Include for N = 0 or N = -1 to set the material parameters directly.

| Card 3b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MU*i* | $\mu_i$, the $i^{th}$ shear modulus (N = 0). $i$ varies up to 8. For N = -1, each MU*i* is a load curve ID for specifying the $i^{th}$ shear modulus as a function of temperature, that is, $\mu_i(T)$. If a curve ID is zero, then the corresponding shear modulus is a constant with value zero. |
| ALPHA*i* | $\alpha_i$, the $i^{th}$ exponent (N = 0). $i$ varies up to 8. For N = -1, each ALPHA*i* is a load curve ID for specifying the $i^{th}$ exponent as a function of temperature, that is, $\alpha_i(T)$. If a curve IDs is zero, then the corresponding exponent is a constant with value zero. |

**Optional Viscoelastic Constants Cards.** Up to 12 cards may be input. The next keyword ("*") card terminates this input if fewer than 12 cards are used.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | VFLAG | | | | | |
| Type | F | F | I | | | | | |
| Default | none | none | 0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term. Not used if LCID2 is given. |
| BETA*i* | Optional decay constant if $i^{th}$ term. Not used if LCID2 is given. |
| VFLAG | Flag for the viscoelasticity formulation. This appears only on the first line defining G*i*, BETA*i*, and VFLAG. If VFLAG = 0, the standard viscoelasticity formulation is used (the default), and if VFLAG = 1 (only applicable to solid elements), the viscoelasticity formulation using the instantaneous elastic stress is used. |

**Remarks:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material, a hydrostatic work term is included in the strain energy functional that is a function of the relative volume, $J$, [Ogden 1984]:

$$W^* = \sum_{i=1}^{3}\sum_{j=1}^{n}\frac{\mu_j}{\alpha_j}\left(\lambda_i^{*\alpha_j} - 1\right) + W_H(J)$$

The asterisk (*) indicates that the volumetric effects have been eliminated from the principal stretches, $\lambda_j^*$. The number of terms, $n$, may vary from 1 to 8 inclusive. If D1 is positive, then

$$W_H(J) = \sum\nolimits_{i=1}^{3}\frac{(J-1)^{2i}}{D_i}$$

whereas otherwise it is

$$W_H(J) = K(J - 1 - \ln J)$$

with $K$ being the linear bulk modulus determined from the corresponding linear shear modulus $G = \frac{1}{2}\sum_{j=1}^{n} \mu_j \alpha_j$ and Poisson's ratio. Although this material is commonly used for incompressible rubber behavior, the theory is valid for compressible data as well.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau$$

or in terms of the second Piola-Kirchhoff stress, $S_{ij}$ , and Green's strain tensor, $E_{ij}$,

$$S_{ij} = \int_0^t G_{ijkl}(t-\tau)\frac{\partial E_{kl}}{\partial \tau} d\tau$$

where $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$ are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta t}$$

given by,

$$g(t) = \sum_{i=1}^{n} G_i e^{-\beta_i t} \quad .$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. The viscoelastic behavior is optional, and an arbitrary number of terms may be used. In order to avoid a constant shear modulus from this viscoelastic formulation, a term in the series is included only when $\beta_i > 0$.

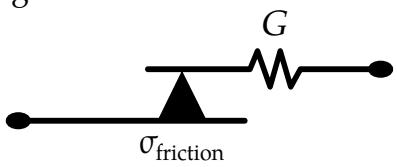
For VFLAG = 1, the viscoelastic term is

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \sigma^E_{kl}}{\partial \tau} d\tau$$

where $\sigma^E_{kl}$ is the instantaneous stress evaluated from the internal energy functional. The coefficients in the Prony series therefore correspond to normalized relaxation moduli instead of elastic moduli.

The Mooney-Rivlin rubber model (model 27) is obtained by specifying $n = 1$. In spite of the differences in formulations with Model 27, we find that the results obtained with this model are nearly identical with those of Material 27 as long as large values of Poisson's ratio are used.

The frequency independent damping is obtained by the having a spring and slider in series as shown in the following sketch:



## *MAT_SOIL_CONCRETE

This is Material Type 78. This model permits concrete and soil to be efficiently modeled. See the remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | K | LCPV | LCYP | LCFP | LCRP |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PC | OUT | B | FAIL | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| K | Bulk modulus. This value is only used for determining the contact stiffness and the time step. Note that for the time step, the solver compares K to the maximum slope in LCPV and takes the maximum value. |
| LCPV | Load curve ID for pressure as a function of volumetric strain. The pressure as a function of volumetric strain curve is defined in compression only. The sign convention requires that both pressure and compressive strain be defined as positive values, where the compressive strain is taken as the negative value of the natural logarithm of the relative volume. |
| LCYP | Load curve ID for yield as a function of pressure:<br>GT.0: von Mises stress as a function of pressure,<br>LT.0: Second stress invariant, $J_2$, as a function of pressure. This |

| VARIABLE | DESCRIPTION |
|---|---|
| | curve must be defined. |
| LCFP | Load curve ID for plastic strain at which fracture begins as a function of pressure. LCFP must be defined if B > 0.0. |
| LCRP | Load curve ID for plastic strain at which residual strength is reached as a function of pressure. LCRP must be defined if B > 0.0. |
| PC | Pressure cutoff for tensile fracture |
| OUT | Output option for plastic strain in database:<br>EQ.0: Volumetric plastic strain<br>EQ.1: Deviatoric plastic strain |
| B | Residual strength factor after cracking; see Figure M78-1. |
| FAIL | Flag for failure:<br>EQ.0: No failure<br>EQ.1: When pressure reaches failure, pressure element is eroded.<br>EQ.2: When pressure reaches failure, pressure element loses its ability to carry tension. |

**Remarks:**

Pressure is positive in compression. Volumetric strain is defined as the natural log of the relative volume and is *positive* in compression where the relative volume, $V$, is the ratio of the current volume to the initial volume. The tabulated data should be given in order of increasing compression. If the pressure drops below the cutoff value specified, it is reset to that value, and the deviatoric stress state is eliminated.

If the load curve ID (LCYP) is provided as a positive number, the deviatoric, perfectly plastic, pressure-dependent, yield function, $\phi$, is given as

$$\phi = \sqrt{3J_2} - F(p) = \sigma_y - F(p)$$

where, $F(p)$ is a tabulated function of yield stress as a function of pressure, and the second invariant, $J_2$, is defined in terms of the deviatoric stress tensor as:

$$J_2 = \frac{1}{2} S_{ij} S_{ij} \ .$$

If LCYP is negative, then the yield function becomes:

$$\phi = J_2 - F(p) \quad .$$

If cracking is invoked by setting the residual strength factor, B, on Card 2 to a value between 0.0 and 1.0, the yield stress is multiplied by a factor $f$ which reduces with plastic strain according to a trilinear law as shown in Figure M78-1.

$b$ = residual strength factor
$\varepsilon_1$ = plastic stain at which cracking begins
$\varepsilon_2$ = plastic stain at which residual strength is reached

$\varepsilon_1$ and $\varepsilon_2$ are tabulated functions of pressure that are defined by load curves, see Figure M78-2. The values on the curves are strain as a function of pressure and should be entered in order of increasing pressure. The strain values should always increase monotonically with pressure.

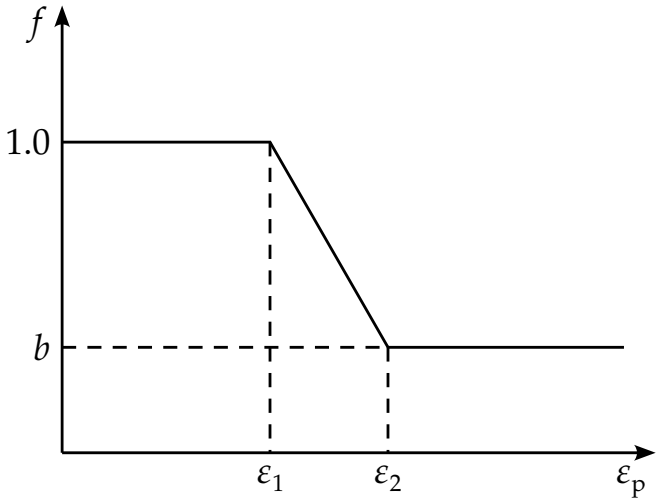



**Figure M78-1.** Strength reduction factor.

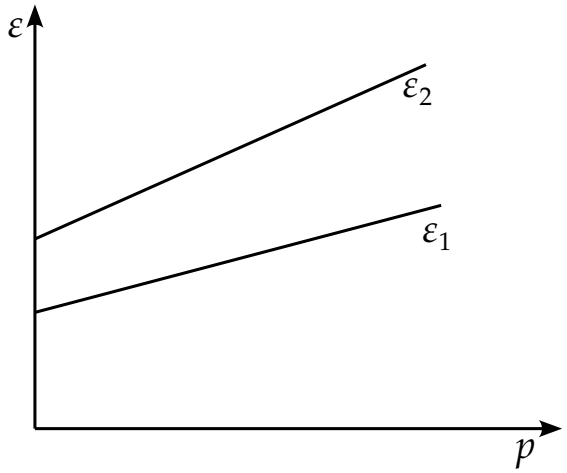



**Figure M78-2.** Cracking strain versus pressure.

By properly defining the load curves, it is possible to obtain the desired strength and ductility over a range of pressures; see Figure M78-3.

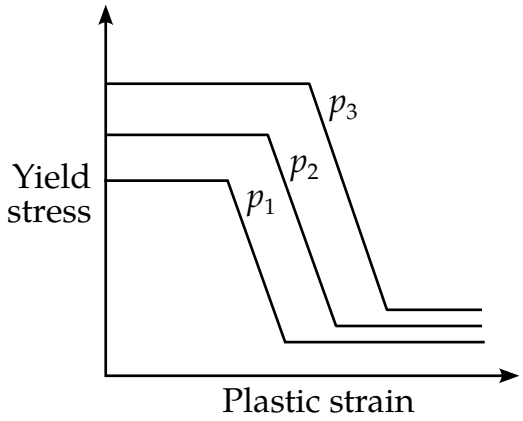



**Figure M78-3.** Yield stress as a function of plastic strain.

## *MAT_HYSTERETIC_SOIL

This is Material Type 79. For this material, you supply a shear stress-strain curve. LS-DYNA converts this curve into a nested surface model with up to twenty superposed "layers" of elastic-perfectly-plastic material, each with its own elastic modulus and yield stress. The hysteretic behavior follows from the yielding of the different "layers" at different stresses and follows the so-called "Masing" rules. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | KO | PO | B | A0 | A1 | A2 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DF | RP | LCID | SFLC | DIL_A | DIL_B | DIL_C | DIL_D |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAM1 | GAM2 | GAM3 | GAM4 | GAM5 | LCD | LCSR | PINIT |
| Type | F | F | F | F | F | I | I | I |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TAU1 | TAU2 | TAU3 | TAU4 | TAU5 | FLAG5 | | |
| Type | F | F | F | F | F | | | |

Include this card if FLAG5 = 1.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGTH | SIGR | CHI | TPINIT | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K0 | Bulk modulus at the reference pressure. See Remark 1. |
| P0 | Cut-off/datum pressure. P0 is irrelevant if $B = A1 = A2 = 0$. Otherwise, P0 must be < 0 (meaning tensile); a very small negative value is acceptable. Below this pressure, stiffness and strength go to zero. This is also the "zero" pressure for pressure-varying properties. See Remark 3. |
| B | Exponent for the pressure-sensitive elastic moduli, $b$. B must be in the range $0 \le B < 1$. We do not recommend values too close to 1 because the pressure becomes indeterminate. See Remark 1. |
| A0 | Yield function constant $a_0$ (default = 1.0); see Remark 5. |
| A1 | Yield function constant $a_1$ (default = 0.0); see Remark 5. |
| A2 | Yield function constant $a_2$ (default = 0.0); see Remark 5. |
| DF | Damping factor (must be in the range $0 \le$ DF $\le 1$):<br>EQ.0: No damping<br>EQ.1: Maximum damping |
| RP | Reference pressure for following input data; see Remarks 1, 2, and 5. |
| LCID | Load curve ID defining shear stress as a function of shear strain. Up to 20 points may be specified in the load curve. See *DEFINE_CURVE and Remarks 4 and 7. |
| SFLC | Scale factor to apply to shear stress in LCID |

| VARIABLE | DESCRIPTION |
|---|---|
| DIL_A | Dilation parameter A, see Remark 11. |
| DIL_B | Dilation parameter B, see Remark 11. |
| DIL_C | Dilation parameter C, see Remark 11. |
| DIL_D | Dilation parameter D, see Remark 11. |
| GAM1 | $\gamma_1$, shear strain (ignored if LCID is nonzero) |
| GAM2 | $\gamma_2$, shear strain (ignored if LCID is nonzero) |
| GAM3 | $\gamma_3$, shear strain (ignored if LCID is nonzero) |
| GAM4 | $\gamma_4$, shear strain (ignored if LCID is nonzero) |
| GAM5 | $\gamma_5$, shear strain (ignored if LCID is nonzero) |
| LCD | Optional load curve ID defining the damping ratio of hysteresis at different strain amplitudes (overrides Masing rules for unload/reload). The $x$-axis is the shear strain, and the $y$-axis is the damping ratio (such as 0.05 for 5% damping). The strains ($x$-axis values) of curve LCD must be identical to those of curve LCID. See Remark 15. |
| LCSR | Load curve ID defining plastic strain rate scaling effect on yield stress. See *DEFINE_CURVE. The $x$-axis is the plastic strain rate; the $y$-axis is the yield enhancement factor. See Remark 12. |
| PINIT | Flag for pressure sensitivity. Positive values apply to both B (elastic stiffness scaling) and the A0, A1, and A2 (strength scaling) equations. Negative values apply only to B, while the A0, A1, and A2 equations use the current pressure like PINIT = 0. See TPINIT below for changing the time PINIT applies, and see Remarks 9 and 10.<br>\|PINIT\|.EQ.0: Use current pressure (will vary during the analysis).<br>\|PINIT\|.EQ.1: Use pressure from the initial stress state.<br>\|PINIT\|.EQ.2: Use initial "plane stress" pressure $(\sigma_v + \sigma_h)/2$.<br>\|PINIT\|.EQ.3: User (compressive) initial vertical stress. |
| TAU1 | $\tau_1$, shear stress at $\gamma_1$(ignored if LCID is nonzero) |

| VARIABLE | DESCRIPTION |
|---|---|
| TAU2 | $\tau_2$, shear stress at $\gamma_2$ (ignored if LCID is nonzero) |
| TAU3 | $\tau_3$, shear stress at $\gamma_3$ (ignored if LCID is nonzero) |
| TAU4 | $\tau_4$, shear stress at $\gamma_4$ (ignored if LCID is nonzero) |
| TAU5 | $\tau_5$, shear stress at $\gamma_5$ (ignored if LCID is nonzero) |
| FLAG5 | If FLAG5 = 1, optional Card 5 will be read. |
| SIGTH | Threshold shear stress ratio for cyclic degradation, see Remark 13. |
| SIGR | Residual shear stress ratio for cyclic degradation, see Remark 13. |
| CHI | Cyclic degradation parameter, see Remark 13. |
| TPINIT | Time at which PINIT applies. See Remark 10. |

**Remarks:**

1. **Elastic moduli.** The elastic moduli G and K are pressure sensitive:

$$G(p) = \frac{G_0(p - p_0)^b}{(p_{\text{ref}} - p_0)^b}$$
$$K(p) = \frac{K_0(p - p_0)^b}{(p_{\text{ref}} - p_0)^b}$$

In the above $K_0$ is the input value K0, $p$ is the current pressure, $p_0$ is the cut-off or datum pressure given by input value P0 (must be zero or negative), $p_{\text{ref}}$ is the reference pressure given by the input value RP, $b$ is the input value B, and $G_0$ is the initial shear modulus at small shear strain:

$$G_0 = \text{SFLC} \times \frac{\tau_1}{\gamma_1} \ .$$

In the above $(\gamma_1, \tau_1)$, is the first (nonzero) point in LCID. $G_0$ is also the total of the shear moduli of all the nested layers; see Remark 6. For limitations on the value of B, see Remark 9.

2. **Volumetric response.** The following equation gives the pressure in compression:

$$p = p_{\text{ref}} \left[ -\frac{K_0}{p_{\text{ref}}} \ln(V) \right]^{1/(1-b)} \ .$$

Here $V$ is the relative volume, the ratio between the original and current volume. $p_{\text{ref}}$ and $b$ are the input values RP and B, respectively. This formula results in an

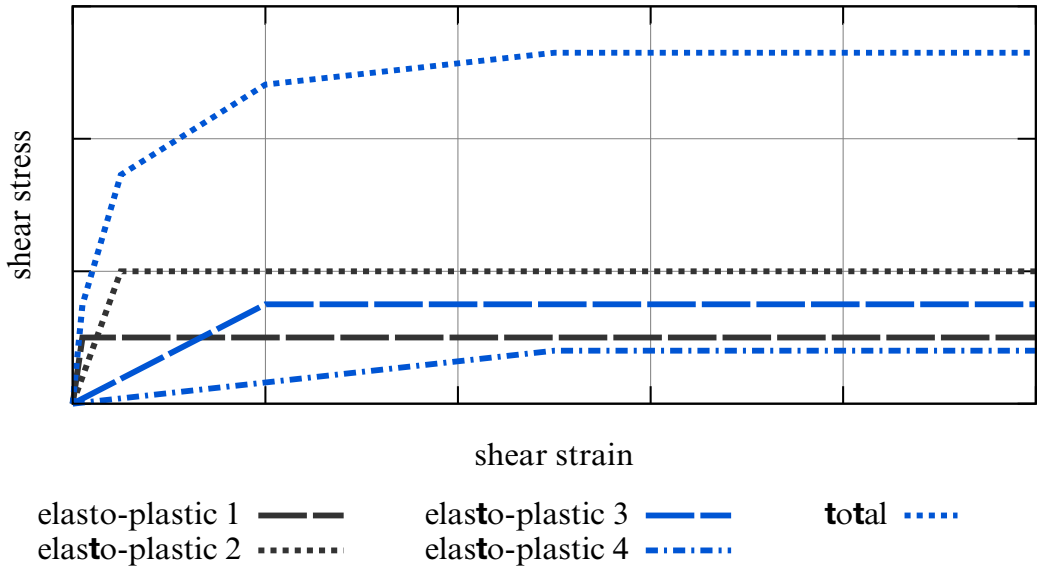



**Figure M79-1.** Family of stress-strain curves. The curve labeled total represents LCID × SFLC. The other curves represent one "layer" in the material model.

instantaneous bulk modulus that is proportional to $p^b$ and whose value at the reference pressure equals $K_0/(1-b)$.

3. **Tensile cut-off.** If $p$ falls below $p_0$ (i.e., becomes more tensile than input value P0), the shear stresses are set to zero, and the pressure is set to $p_0$. Thus, the material has no stiffness or strength when the pressure is more tensile than $p_0$.

4. **Shear stress-strain curve.** LCID and SFLC define a curve giving shear stress ($\tau$) as a function of shear strain ($\gamma$). The shear strains are the $x$-axis values in LCID. The shear stresses are the $y$-axis values in LCID multiplied by SFLC. Starting from version R14, LCID may contain up to 20 points (in versions up to R13, the limit was 10 points). The first point on the curve is assumed by default to be (0,0) and does not need to be entered. The slope of the curve must decrease with increasing $\gamma$.

5. **Pressure-sensitivity of the shear response.** The curve LCID applies at the reference pressure (input value RP); at other pressures, the curve is scaled by

$$\frac{\tau(p,\gamma)}{\tau(p_{\text{ref}},\gamma)} = \sqrt{\frac{[a_0 + a_1(p-p_0) + a_2(p-p_0)^2]}{[a_0 + a_1(p_{\text{ref}}-p_0) + a_2(p_{\text{ref}}-p_0)^2]}}$$

The constants $a_0$, $a_1$, and $a_2$ govern the pressure sensitivity of the yield stress. Only the ratios between these values are important - the absolute stress values are taken from the stress-strain curve scaled, as shown above.

6. **Nested yield surface approach.** LS-DYNA automatically converts the shear stress-strain curve (with points $(\gamma_1,\tau_1)$, $(\gamma_2,\tau_2)$,…, $(\gamma_N,\tau_N)$) into a series of $N$ elastic-perfectly-plastic curves such that $\sum(\tau_i,(\gamma)) = \tau(\gamma)$, as shown in

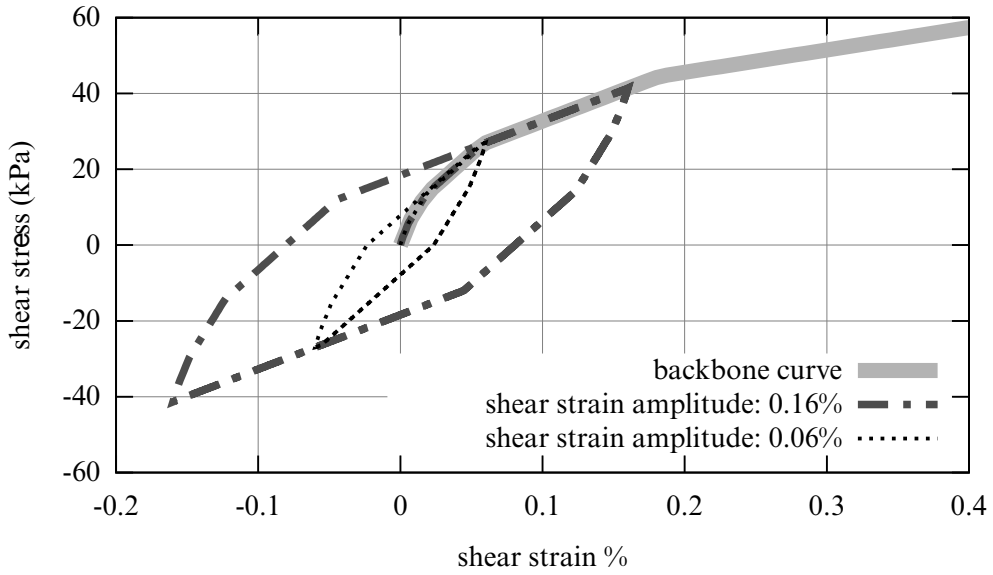



**Figure M79-2.** Small and large strain cycles superposed on the input curve

Figure M79-1. Each elastic-perfectly-plastic curve represents one "layer" in the material model. Deviatoric stresses are stored and calculated separately for each layer. The total deviatoric stress is the sum of the deviatoric stresses in each layer. This method generates hysteretic (energy-absorbing) stress-strain curves in response to any strain cycle of amplitude greater than the lowest yield strain of any layer. The example in Figure M79-2 shows the response to small and large strain cycles superposed on the input curve (thick line labeled backbone curve).

7. **Definition of shear strain and shear stress.** Different definitions of "shear strain" and "shear stress" are possible when applied to three-dimensional stress states. *MAT_079 uses the following definitions. Input shear stress $\tau$ (from LCID multiplied by SFLC) and shear strain $\gamma$ (from LCID) are treated by the material model as:

$$\tau = 0.5 \times \text{Von Mises Stress} = \sqrt{(3\sigma' : \sigma'/8)}$$
$$\gamma = 1.5 \times \text{Von Mises Strain} = \sqrt{(3\varepsilon' : \varepsilon'/2)}$$

where $\sigma'$ and $\varepsilon'$ are the deviatoric stress and strain tensors respectively. For a particular stress or strain state (defined by the relationship among the three principal stresses or strains), a scaling factor may be needed to convert between the definitions given above and the shear stress or strain that an engineer would expect. The *MAT_079 definitions of shear stress and shear strain are derived from triaxial testing in which one principal stress is applied while the other two principal stresses are equal to a confining stress which is held constant. In other words, the principal stresses have the form $(a + q, a, a)$, and the shear stress, as defined above, is $0.5q$. If instead you wish the input curve to represent a test in which a pure shear strain is applied over a hydrostatic pressure, such as a shear-

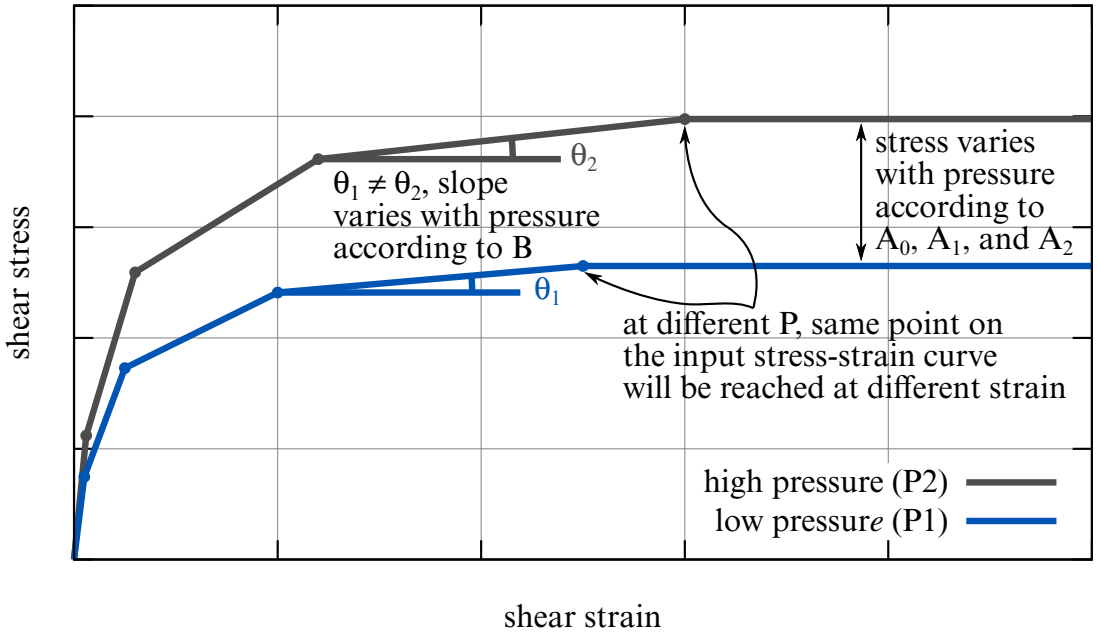



**Figure M79-3**. Sensitivity of curves to pressure

box test, then we recommend scaling both the $x$-axis and the $y$-axis of the curve LCID by 0.866. This factor assumes principal stresses of the form $(p + t, p - t, p)$ where $t$ is the applied shear stress, and similarly for the principal strains.

8. **More about pressure sensitivity.** The yield stresses of the layers, and hence the stress at each point on the shear stress-strain input curve, vary with pressure according to constants A0, A1, and A2. The elastic moduli, and hence also the slope of each section of the shear stress-strain curve, vary with pressure according to constant B. These effects combine to modify the shear stress-strain curve according to pressure, as shown in Figure M79-3.

9. **PINIT.** Pressure sensitivity can make the solution sensitive to numerical noise. In cases where the expected pressure changes are small compared to the initial stress state, using pressure from the initial stress state instead of current pressure as the basis for the pressure sensitivity (option PINIT) may be preferable. This causes the bulk modulus and shear stress-strain curve to be calculated once for each element at the analysis's start and remain fixed thereafter. Positive settings of PINIT affect both stiffness scaling (calculated using B) and strength scaling (calculated using A0, A1, and A2). If using PINIT options 2 ("plane stress" pressure) or 3 (vertical stress), these quantities substitute for pressure $p$ in the equations above. Input values of $p_{\text{ref}}$ and $p_0$ should then also be "plane stress" pressure or vertical stress, respectively. Negative settings of PINIT have these effects only on stiffness scaling (B), while the strength scaling is re-calculated every time step from the current pressure as for PINIT = 0. If PINIT is nonzero, B is allowed to be as high as 1.0 (stiffness proportional to initial pressure); otherwise, we do not recommend values of B higher than about 0.5.

10. **TPINIT.** TPINIT is relevant only when PINIT is nonzero. When TPINIT = 0.0 (the default), PINIT acts at the start of the analysis. The pressures in each element on the first cycle (for instance, due to *INITIAL_STRESS_... cards) determine the pressure-sensitive stiffness and strength parameters which remain constant for the duration of the analysis. If TPINIT is nonzero, the stiffness and strength properties vary dynamically with pressure (same as PINIT = 0) until time TPINIT when they become frozen based on the pressure that exists at time TPINIT. For example, this feature can be used to build up stresses under gravity loading before applying PINIT. If using dynamic relaxation and TPINIT > 0.0, then PINIT acts at time TPINIT in the transient phase.

11. **Dilatancy.** Parameters DIL_A, DIL_B, DIL_C, and DIL_D control the compaction and dilatancy in sandy soils due to shearing motion. Using this feature with pore water pressure (see *CONTROL_PORE_FLUID) can model liquefaction. However, note that the compaction/dilatancy algorithm used in this material model is very unsophisticated compared to recently published research findings.

The dilatancy is expressed as a volume strain, $\varepsilon_v$:

$$\begin{aligned}
\varepsilon_v &= \varepsilon_r + \varepsilon_g \\
\varepsilon_r &= \mathrm{DIL_A}(\Gamma)^{\mathrm{DIL_B}} \\
\varepsilon_g &= \frac{\int (d\gamma_{xz}^2 + d\gamma_{yz}^2)^{1/2}}{\mathrm{DIL_C} + \mathrm{DIL_D} \times \int (d\gamma_{xz}^2 + d\gamma_{yz}^2)^{1/2}} \\
\Gamma &= (\gamma_{xz}^2 + \gamma_{yz}^2)^{1/2} \\
\gamma_{xz} &= 2\varepsilon_{xz} \\
\gamma_{yz} &= 2\varepsilon_{yz}
\end{aligned}$$

$\varepsilon_r$ describes soil's dilation due to the magnitude of the shear strains; this is caused by the soil particles having to climb over each other to develop shear strain. $\varepsilon_g$ describes the compaction of the soil due to the collapse of weak areas and voids caused by continuous shear straining.

Recommended inputs for sandy soil when modeling dilatancy are:

| DIL_A | DIL_B | DIL_C | DIL_D |
|---|---|---|---|
| 10 | 1.6 | 100 | 10 |

DIL_A and DIL_B may cause instabilities in some models.

12. **Strain rate sensitivity.** Scaling the yield stress of each layer by a "rate enhancement factor" accounts for strain rate effects (see optional input field LCSR). This factor is a function of the plastic strain rate in that layer. The stress-strain curve defined by LCID and SFLC is for quasi-static loading. The rate enhancement factor (on the $y$-axis) is input as a function of plastic strain rate (on the $x$-axis) in

curve LCSR. All rate enhancement factors must be equal to or larger than 1.0. Because the rate enhancement factor applies to the strength but not the stiffness and is calculated separately for each layer, situations in which not all the layers are yielding cause an overall enhancement factor between 1.0 and the value in LCSR.

13. **Cyclic degradation.** See optional input fields SIGR, CHI, and SIGTH. Reducing the size of all yield surfaces proportionally based on the accumulation of the damage strain accounts for cyclic degradation. The following equation determines the strength reduction factor, $f$:

$$f = 1 - (1 - \Sigma_R)\left(1 - e^{\frac{-\chi\gamma_d}{1-\Sigma_R}}\right)$$

where $\Sigma$ is the shear stress ratio (defined as current shear stress divided by shear strength at the current pressure); $\Sigma_R$ is the residual shear strength ratio SIGR; $\chi$ is the input parameter CHI; and $\gamma_d$ is the damage strain, defined as the summation of absolute incremental changes in Von Mises strain that accumulate whenever $\Sigma$ exceeds the threshold shear stress ratio SIGTH.

14. **Saturated soil.** When modeling saturated soil, we do not recommend attempting to represent the additional stiffness of the pore water by increasing the bulk modulus on the *MAT card (this method is sometimes termed a "total stress" model). Using that method causes the pressure calculated by the material model to represent the total pressure (the pore water pressure plus the "effective pressure" which is the component of pressure associated with contact between the soil grains). Pressure-sensitive properties, such as shear strength, then depend unrealistically on total pressure, whereas in real-life soils, they depend on effective pressure. To obtain the latter behavior, model the pore pressure effects with *CONTROL_PORE_FLUID and *BOUNDARY_PORE_FLUID and set the properties on the *MAT card to represent the effective stress properties.

15. **Non-Masing damping.** See optional input field LCD. Hysteresis damping arises from the energy absorbed during each stress-strain cycle, meaning the area enclosed by the hysteresis loops, such as those shown in Figure M79-2. The nested yield surface approach of this material model governs the shape of the hysteresis loops, providing a level of damping known as "Masing damping" that depends on the shape of the input shear stress-strain curve and the cyclic strain amplitude. Masing damping often overestimates the actual hysteresis damping shown by soils in cyclic tests, particularly at high cyclic shear strains. To counteract this, specify input curve LCD to define "non-Masing damping." In this material model, non-Masing damping acts like a damage model, progressively reducing the properties of the separate nested layers as the strain increases, making the hysteresis loops thinner. Thus, this feature only reduces the damping compared to the default Masing damping. It cannot increase the damping. Furthermore, the gap between Masing and non-Masing damping

must increase monotonically with increasing shear strain. The amount of hysteresis damping will not follow an input LCD if it does not obey these rules. In this case, LS-DYNA writes a warning and a table that includes the Masing damping for each point of the input stress-strain curve to the message file. LS-DYNA outputs the table to provide information for adjusting the damping ratios in LCD to meet the above requirements.

## *MAT_RAMBERG-OSGOOD

This is Material Type 80. This model is intended as a simple model of shear behavior and can be used in seismic analysis.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | GAMY | TAUY | ALPHA | R | BULK | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| GAMY | Reference shear strain, $\gamma_y$ |
| TAUY | Reference shear stress, $\tau_y$ |
| ALPHA | Stress coefficient, $\alpha$ |
| R | Stress exponent, $r$ |
| BULK | Elastic bulk modulus |

### Remarks:

The Ramberg-Osgood equation is an empirical constitutive relation to represent the one-dimensional elastic-plastic behavior of many materials, including soils. This model allows a simple rate independent representation of the hysteretic energy dissipation observed in soils subjected to cyclic shear deformation. For monotonic loading, the stress-strain relationship is given by:

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y} + \alpha \left| \frac{\tau}{\tau_y} \right|^r \qquad \text{for } \gamma \geq 0$$

$$\frac{\gamma}{\gamma_y} = \frac{\tau}{\tau_y} - \alpha \left| \frac{\tau}{\tau_y} \right|^r \qquad \text{for } \gamma < 0$$

where $\gamma$ is the shear and $\tau$ is the stress. The model approaches perfect plasticity as the stress exponent $r \to \infty$. These equations must be augmented to correctly model

unloading and reloading material behavior. The first load reversal is detected by $\gamma\dot{\gamma} < 0$. After the first reversal, the stress-strain relationship is modified to

$$\frac{(\gamma - \gamma_0)}{2\gamma_y} = \frac{(\tau - \tau_0)}{2\tau_y} + \alpha \left|\frac{(\tau - \tau_0)}{2\tau_y}\right|' \qquad \text{for } \gamma \geq 0$$

$$\frac{(\gamma - \gamma_0)}{2\gamma_y} = \frac{(\tau - \tau_0)}{2\tau_y} - \alpha \left|\frac{(\tau - \tau_0)}{2\tau_y}\right|' \qquad \text{for } \gamma < 0$$

where $\gamma_0$ and $\tau_0$ represent the values of strain and stress at the point of load reversal. Subsequent load reversals are detected by $(\gamma - \gamma_0)\dot{\gamma} < 0$.

The Ramberg-Osgood equations are inherently one-dimensional and are assumed to apply to shear components. To generalize this theory to the multidimensional case, it is assumed that each component of the deviatoric stress and deviatoric tensorial strain is independently related by the one-dimensional stress-strain equations. A projection is used to map the result back into deviatoric stress space if required. The volumetric behavior is elastic, and, therefore, the pressure p is found by

$$p = -K\varepsilon_v$$

where $\varepsilon_v$ is the volumetric strain.

## *MAT_PLASTICITY_WITH_DAMAGE_*{OPTION}*

This manual entry applies to *both* types 81 and 82. Materials 81 and 82 model an elasto-visco-plastic material with user-defined *isotropic* stress versus strain curves, which, themselves, may be strain-rate dependent. This model accounts for the effects of damage prior to rupture based on an effective plastic-strain measure. Additionally, failure can be triggered when the time step drops below some specified value. Adding an orthotropic damage option will invoke material type 82. Since type 82 must track directional strains it is, computationally, more expensive.

Available options include:

<BLANK>

ORTHO

ORTHO_RCDC

ORTHO_RCDC1980

STOCHASTIC

The keyword card can appear in the following ways:

*MAT_PLASTICITY_WITH_DAMAGE or *MAT_081

*MAT_PLASTICITY_WITH_DAMAGE_ORTHO or *MAT_082

*MAT_PLASTICITY_WITH_DAMAGE_ORTHO_RCDC or *MAT_082_RCDC

*MAT_PLASTICITY_WITH_DAMAGE_ORTHO_RCDC1980 or *MAT_082_RCDC1980

*MAT_PLASTICITY_WITH_DAMAGE_STOCHASTIC or *MAT_081_STOCHASTIC

The ORTHO option invokes an orthotropic damage model, an extension that was first added as for modelling failure in aluminum panels. Directional damage begins after a defined failure strain is reached in tension and continues to evolve until a tensile rupture strain is reached in either one of the two orthogonal directions. After rupture is detected at *all* integration points, the element is deleted.

The ORTHO_RCDC option invokes the damage model developed by Wilkins [Wilkins, et al. 1977]. The ORTHO_RCDC1980 option invokes a damage model based on strain invariants as developed by Wilkins [Wilkins, et al. 1980]. A nonlocal formulation, which requires additional storage, is used if a characteristic length is defined. The RCDC option, which was added at the request of Toyota, works well in predicting failure in cast aluminum; see Yamasaki, et al., [2006].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | EPPF | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | $10^{12}$ | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | EPPFR | VP | LCDM | NUMINT |
| Type | F | F | F | F | F | F | F | I |
| Default | 0 | 0 | 0 | 0 | $10^{14}$ | 0 | 0 | 0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

**Ortho RCDC Card.** Additional card for keyword options ORTHO_RCDC and ORTHO_-RCDC1980.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | BETA | GAMMA | D0 | B | LAMBDA | DS | L |
| Type | F | F | F | F | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus, ignored if (LCSS > 0) is defined. |
| EPPF | $\varepsilon^p_{\text{failure}}$, effective plastic strain at which material softening begins |
| TDEL | Minimum time step size for automatic element deletion |
| C | Strain rate parameter, $C$; see formula below. |
| P | Strain rate parameter, $P$; see formula below. |
| LCSS | Load curve ID or Table ID<br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that rate; see Figure M24-1. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress as a function of effective plastic strain curve for the highest value of strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| | is used. C, P, LCSR, EPS1 - EPS8, and ES1 - ES8 are ignored if a table ID is defined.<br><br>**Logarithmically Defined Tables.** If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress |
| EPPFR | $\varepsilon^p_{\text{rupture}}$, effective plastic strain at which material ruptures |
| VP | Formulation for rate effects:<br><br>EQ.0.0: Scale yield stress (default)<br><br>EQ.1.0: Viscoplastic formulation |
| LCDM | Optional curve ID defining nonlinear damage curve. If this curve is specified, either EPPF or EPPFR must also be input. If LCDM, EPPF, and EPPFR are all nonzero, then EPPFR is ignored. |
| NUMINT | Number of through thickness integration points which must fail before a shell element is deleted. (If zero, all points must fail.) The default of all integration points is not recommended since shells undergoing large strain are often not deleted due to nodal fiber rotations which limit strains at active integration points after most points have failed. Better results are obtained if NUMINT is set to 1 or a number less than one half of the number of through thickness points. For example, if four through thickness points are used, NUMINT should not exceed 2, even for fully integrated shells which have 16 integration points. |
| EPS1 - EPS8 | Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8 |
| ALPHA | Parameter $\alpha$ for the Rc-Dc model |
| BETA | Parameter $\beta$ for the Rc-Dc model |
| GAMMA | Parameter $\gamma$ for the Rc-Dc model |

| VARIABLE | DESCRIPTION |
|---|---|
| D0 | Parameter $D_0$ for the Rc-Dc model |
| B | Parameter $b$ for the Rc-Dc model |
| LAMBDA | Parameter $\lambda$ for the Rc-Dc model |
| DS | Parameter $D_s$ for the Rc-Dc model |
| L | Optional characteristic element length for this material. If zero, nodal values of the damage function are used to compute the damage gradient. See discussion below. |

**Remarks:**

1. The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in Figure M24-1 is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress versus effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

   Three options to account for strain rate effects are possible:

   a) Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

   $$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/6}$$

   where $\dot{\varepsilon}$ is the strain rate, $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$.

   If the viscoplastic option is active, VP = 1.0, and if SIGY is > 0, then the dynamic yield stress is computed from the sum of the static stress, $\sigma_y^s(\varepsilon_{\text{eff}}^p)$, which is typically given by a load curve ID, and the initial yield stress, SIGY, multiplied by the Cowper-Symonds rate term as follows:

   $$\sigma_y(\varepsilon_{\text{eff}}^p, \dot{\varepsilon}_{\text{eff}}^p) = \sigma_y^s(\varepsilon_{\text{eff}}^p) + \text{SIGY} \times \left(\frac{\dot{\varepsilon}_{\text{eff}}^p}{C}\right)^{1/p},$$

   where the plastic strain rate is used. With this latter approach similar results can be obtained between this model and material model: *MAT_ANISOTROPIC_VISCOPLASTIC. If SIGY = 0, the following equation is used instead where the static stress, $\sigma_y^s(\varepsilon_{\text{eff}}^p)$, must be defined by a load curve:

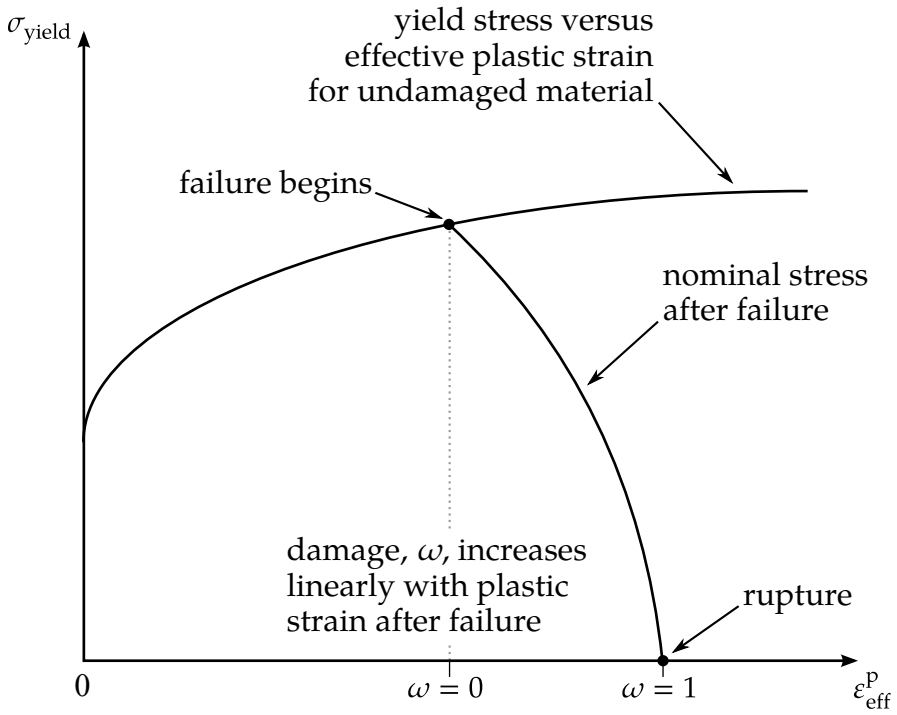



**Figure M81-1.** Stress strain behavior when damage is included

$$\sigma_y\left(\varepsilon^p_{\text{eff}}, \dot{\varepsilon}^p_{\text{eff}}\right) = \sigma^s_y\left(\varepsilon^p_{\text{eff}}\right)\left[1 + \left(\frac{\dot{\varepsilon}^p_{\text{eff}}}{C}\right)^{1/p}\right]$$

This latter equation is always used if the viscoplastic option is off.

b) For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.

c) If different stress as a function of strain curves can be provided for various strain rates, a table (LCSS) can be used. Then the table input in *DEFINE_TABLE is expected; see Figure M24-1.

2. **Damage.** The constitutive properties for the damaged material are obtained from the undamaged material properties. The amount of damage is represented $\omega$ which varies from zero if no damage has occurred to unity for complete rupture. For uniaxial loading, the nominal stress in the damaged material is given by

$$\sigma_{\text{nominal}} = \frac{P}{A}$$

where $P$ is the applied load and $A$ is the surface area. The true stress is given by:

$$\sigma_{\text{true}} = \frac{P}{A - A_{\text{loss}}}$$

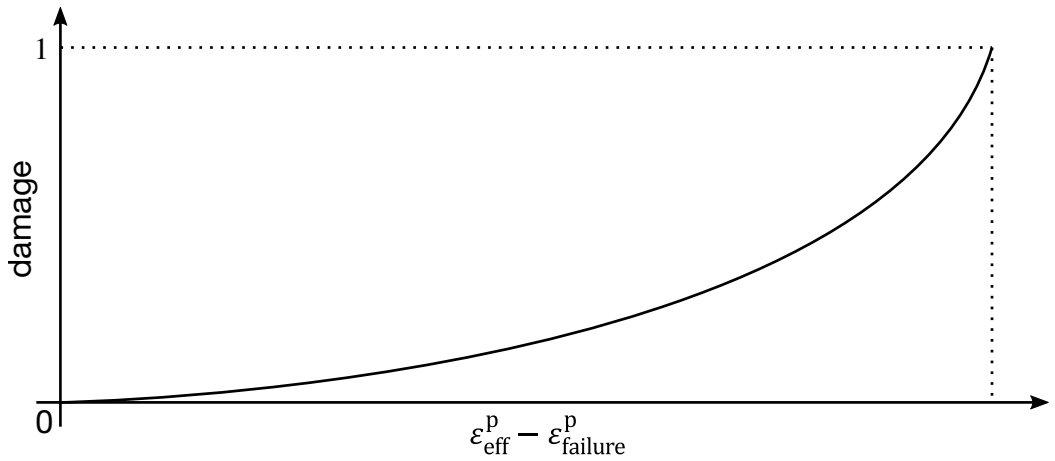



**Figure M81-2.** A nonlinear damage curve is optional. Note that the origin of the curve is at (0,0). The nonlinear damage curve is useful for controlling the softening behavior after the failure strain EPPF is reached.

where $A_{\mathrm{loss}}$ is the void area. The damage variable can then be defined:

$$\omega = \frac{A_{\mathrm{loss}}}{A}$$

such that

$$0 \le \omega \le 1.$$

In this model, unless LCDM is defined, damage is defined in terms of effective plastic strain after the failure strain is exceeded as follows:

$$\omega = \frac{\varepsilon^p_{\mathrm{eff}} - \varepsilon^p_{\mathrm{failure}}}{\varepsilon^p_{\mathrm{rupture}} - \varepsilon^p_{\mathrm{failure}}}, \qquad \varepsilon^p_{\mathrm{failure}} \le \varepsilon^p_{\mathrm{eff}} \le \varepsilon^p_{\mathrm{rupture}}$$

After exceeding the failure strain, softening begins and continues until the rupture strain is reached.

3. **Rc-Dc Model.** The damage, $D$, for the Rc-Dc model is given by:

$$D = \int \omega_1 \omega_2 d\varepsilon^p$$

where $\varepsilon^p$ is the effective plastic strain,

$$\omega_1 = \left(\frac{1}{1 - \gamma\sigma_m}\right)^{\alpha}$$

is a triaxial stress weighting term and

$$\omega_2 = (2 - A_D)^{\beta}$$

is an asymmetric strain weighting term. In the above $\sigma_m$ is the mean stress. For $A_D$ we use

$$A_D = \min\left(\left|\frac{\sigma_2}{\sigma_3}\right|, \left|\frac{\sigma_3}{\sigma_2}\right|\right),$$

where $\sigma_i$ are the principal stresses and $\sigma_1 > \sigma_2 > \sigma_3$. Fracture is initiated when the accumulation of damage is

$$\frac{D}{D_c} > 1 \ ,$$

where $D_c$ is the critical damage given by

$$D_c = D_0\left(1 + b|\nabla D|^{\lambda}\right) \ .$$

A fracture fraction,

$$F = \frac{D - D_c}{D_s} \ ,$$

defines the degradations of the material by the Rc-Dc model.

For the Rc-Dc model the gradient of damage needs to be estimated. The damage is connected to the integration points, and, thus, the computation of the gradient requires some manipulation of the LS-DYNA source code. Provided that the damage is connected to nodes, it can be seen as a standard bilinear field and the gradient is easily obtained. To enable this, the damage at the integration points are transferred to the nodes as follows. Let $E_n$ be the set of elements sharing node $n$, $|E_n|$ be the number of elements in that set, $P_e$ be the set of integration points in element $e$ and $|P_e|$ be the number of points in that set. The average damage $\overline{D}_e$ in element $e$ is computed as

$$\overline{D}_e = \frac{\sum_{p \in P_e} D_p}{|P_e|}$$

where $D_p$ is the damage in integration point $p$. Finally, the damage value in node $n$ is estimated as

$$D_n = \frac{\sum_{e \in E_n} \overline{D}_e}{|E_n|}.$$

This computation is performed in each time step and requires additional storage. Currently we use three times the total number of nodes in the model for this calculation, but this could be reduced by a considerable factor if necessary.

There is an Rc-Dc option for the Gurson dilatational-plastic model. In the implementation of this model, the norm of the gradient is computed differently. Let $E_f^l$ be the set of elements from within a distance $l$ of element, $f$, not including the element itself, and let $\left|E_f^l\right|$ be the number of elements in that set. The norm of the gradient of damage is estimated roughly as

$$\|\nabla D\|_f \approx \frac{1}{\left|E_f^l\right|} \sum_{e \in E_f^l} \frac{|D_e - D_f|}{d_{ef}}$$

where $d_{ef}$ is the distance between element $f$ and $e$.

The reason for taking the first approach is that it should be a better approximation of the gradient; it can for one integration point in each element be seen as a weak gradient of an elementwise constant field. The memory consumption as well as computational work should not be much higher than for the other approach.

The RCDC1980 model is identical to the RCDC model except the expression for $A_D$is in terms of the principal stress deviators and takes the form

$$A_D = \max\left(\left|\frac{S_2}{S_3}\right|, \left|\frac{S_2}{S_1}\right|\right)$$

4. **STOCHASTIC Option.** The STOCHASTIC option allows spatially varying yield and failure behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

5. **Material Histories.** *DEFINE_MATERIAL_HISTORIES can be used to output the instability, plastic strain rate, and damage, following

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Instability | - - - - | Failure indicator $\varepsilon^p_{\text{eff}}/\varepsilon^p_{\text{fail}}$, see EPPF |
| Plastic Strain Rate | - - - - | Effective plastic strain rate $\dot{\varepsilon}^p_{\text{eff}}$ |
| Damage | - - - - | Damage $\omega$ |

***MAT_FU_CHANG_FOAM**_{OPTION}

This is Material Type 83.

Available options include:

DAMAGE_DECAY

LOG_LOG_INTERPOLATION

PATH_DEPENDENT

Rate effects can be modeled in low and medium density foams; see Figure M83-1. Hysteretic unloading behavior in this model is a function of the rate sensitivity, with the most rate-sensitive foams providing the largest hysteresis and vice versa. The unified constitutive equations for foam materials by Chang [1995] provide the basis for this model. The mathematical description given below is excerpted from the reference. Further improvements have been incorporated based on work by Hirth, Du Bois, and Weimar [1998]. Their improvements permit: load curves generated from a drop tower test to be directly input, a choice of principal or volumetric strain rates, load curves to be defined in tension, and the volumetric behavior to be specified by a load curve.

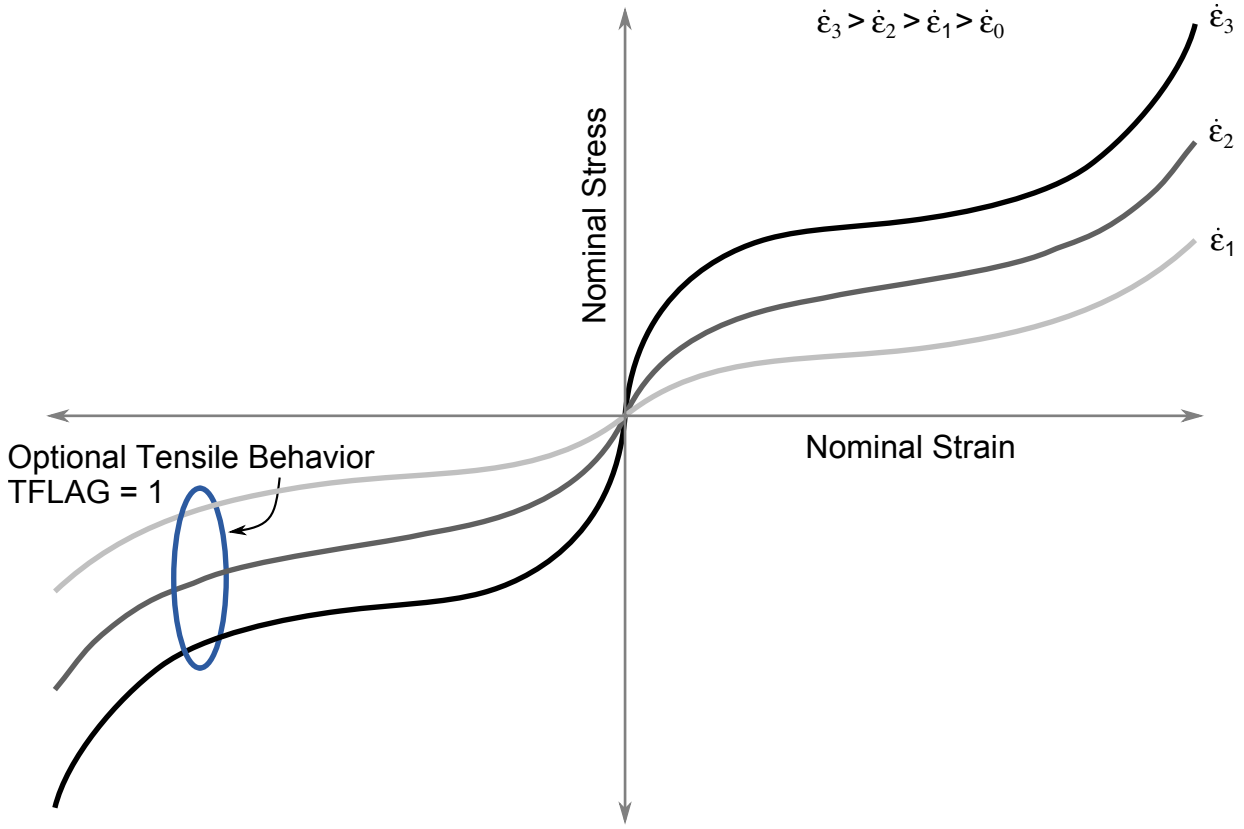



**Figure M83-1.** Rate effects in the nominal stress versus engineering strain curves, which are used to model rate effects in Fu Chang's foam model.

The unloading response was generalized by Kolling, Hirth, Erhart, and Du Bois [2006] to allow the Mullin's effect to be modeled, meaning that after the first loading and unloading, further reloading occurs on the unloading curve. If it is desired to reload on the loading curves with the new generalized unloading, the DAMAGE_DECAY option is available, which allows the reloading to quickly return to the loading curve as the damage parameter decays back to zero in tension and compression.

Keyword option PATH_DEPENDENT invokes an alternative formulation of the Fu Chang model. In contrast to the original approach (total update of Cauchy stresses), it uses an incremental update of the second Piola-Kirchhoff stresses with respect to the Green-Lagrange strains. If this keyword option is used, TBID should be defined as a 3D table for nominal stress, giving nominal stress as a function of volumetric change or maximum volumetric change (TABLE_3D), strain rate (TABLE), and nominal strain (CURVE). This formulation enables modeling a path-dependent response as sometimes observed in compressive loading followed by shear deformation or vice versa, for instance. Also, with this keyword option, FMATRX = 2 should be set on *CONTROL_SOLID.

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | E | KCON | TC | FAIL | DAMP | TBID |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| BVFLAG | SFLAG | RFLAG | TFLAG | PVID | SRAF | REF | HU |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included if the DAMAGE_DECAY keyword option is used.

| MINR | MAXR | SHAPE | BETAT | BETAC | | | |
|---|---|---|---|---|---|---|---|

**Card 3b.** This card is included if the DAMAGE_DECAY keyword option is *not* used.

| D0 | N0 | N1 | N2 | N3 | C0 | C1 | C2 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is included if the DAMAGE_DECAY keyword option is *not* used.

| C3 | C4 | C5 | AIJ | SIJ | MINR | MAXR | SHAPE |
|---|---|---|---|---|---|---|---|

***Card 5.*** This card is optional.

| EXPON | RIULD | PDOPT | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | KCON | TC | FAIL | DAMP | TBID |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | $10^{20}$ | none | 0.05 | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| KCON | Optional Young's modulus used to compute the sound speed. This will influence the time step, contact forces, hourglass stabilization forces, and the numerical damping (DAMP).<br><br>If TBID ≠ 0, the time step is based on a stiffness of<br>max (KCON, E, max. current slope of the stress-strain curve)<br>The "max .current slope" is taken from the three principal directions. The tensile (negative) portion of the stress-strain curves is included in this evaluation if TFLAG = 1. |
| TC | Tension cut-off stress |
| FAIL | Failure option after cutoff stress is reached:<br>EQ.0.0: Tensile stress remains at cut-off value.<br>EQ.1.0: Tensile stress is reset to zero.<br>EQ.2.0: The element is eroded. |
| DAMP | Viscous coefficient to model damping effects (0.05 < recommended value < 0.50; default is 0.05) |
| TBID | Table ID (see *DEFINE_TABLE) for nominal stress as a function of strain data at a given strain rate. If the table ID is provided, Cards 3 and 4 may be left blank and the input curves will be used |

| VARIABLE | DESCRIPTION |
|---|---|
| | directly in the model. The Table ID can be positive or negative (see Remark 6 below). If TBID < 0, enter \|TBID\| on the *DEFINE_-TABLE keyword.<br>For the PATH_DEPENDENT keyword option, TBID defines a 3D table for nominal stress, giving it as a function of volumetric change or maximum volumetric change depending on PDOPT (TABLE_3D), strain rate (TABLE), and nominal strain (CURVE). |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | BVFLAG | SFLAG | RFLAG | TFLAG | PVID | SRAF | REF | HU |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| BVFLAG | Toggle to turn bulk viscosity off or on (see Remark 1):<br>LT.1.0: No bulk viscosity (recommended)<br>GE.1.0: Bulk viscosity active. |
| SFLAG | Strain rate flag (see Remark 2 below):<br>EQ.0.0: True constant strain rate<br>EQ.1.0: Engineering strain rate |
| RFLAG | Strain rate evaluation flag (see Remark 3):<br>EQ.0.0: First principal direction<br>EQ.1.0: Principal strain rates for each principal direction<br>EQ.2.0: Volumetric strain rate |
| TFLAG | Tensile stress evaluation:<br>EQ.0.0: Linear (follows E) in tension<br>EQ.1.0: Input via load curves with the tensile response corresponds to negative values of stress and strain. |

| VARIABLE | DESCRIPTION |
|---|---|
| PVID | Optional load curve ID defining pressure as a function of volumetric strain. See Remark 4. |
| SRAF | Strain rate averaging flag (see Remark 5):<br>LT.0.0: Use exponential moving average.<br>EQ.0.0: Use weighted running average.<br>GT.0.0.AND.LE.0.9999: Filter window for averaging strain rates, suppressing the time step dependence of the operation.<br>EQ.1.0: Average the last twelve values.<br>GE.1.0001: SRAF – 1.0 is a filter window for averaging strain rates, suppressing the time step dependence of the operation. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |
| HU | Hysteretic unloading factor between 0.0 and 1.0. See Remark 6 and Figure M83-4. |

**DAMAGE_DECAY Card.** Card 3 for DAMAGE_DECAY keyword option.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MINR | MAXR | SHAPE | BETAT | BETAC | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MINR | Minimum strain rate of interest |
| MAXR | Maximum strain rate of interest |

| VARIABLE | DESCRIPTION |
|---|---|
| SHAPE | Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor HU. Values less than one reduce energy dissipation and greater than one increase dissipation; see Figure M83-4. |
| BETAT | Decay constant for damage in tension. The damage decays after loading ceases according to $e^{-\text{BETAT}\times t}$. |
| BETAC | Decay constant for damage in compression. The damage decays after loading ceases according to $e^{-\text{BETAC}\times t}$. |

**Material Constants Card.** Card 3 for keyword option *NOT* set to DAMAGE_DECAY.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D0 | N0 | N1 | N2 | N3 | C0 | C1 | C2 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| D0 | Material constant; see Material Formulation. |
| N0 | Material constant; see Material Formulation. |
| N1 | Material constant; see Material Formulation. |
| N2 | Material constant; see Material Formulation. |
| N3 | Material constant; see Material Formulation. |
| C0 | Material constant; see Material Formulation. |
| C1 | Material constant; see Material Formulation. |
| C2 | Material constant; see Material Formulation. |
| C3 | Material constant; see Material Formulation. |
| C4 | Material constant; see Material Formulation. |

| VARIABLE | DESCRIPTION |
|---|---|
| C5 | Material constant; see Material Formulation. |
| AIJ | Material constant; see Material Formulation. |
| SIJ | Material constant; see Material Formulation. |

**Material Constants Card.** Card 4 for keyword option *NOT* set to DAMAGE_DECAY.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C3 | C4 | C5 | AIJ | SIJ | MINR | MAXR | SHAPE |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| C3 | Material constant; see Material Formulation. |
| C4 | Material constant; see Material Formulation. |
| C5 | Material constant; see Material Formulation. |
| AIJ | Material constant; see Material Formulation. |
| SIJ | Material constant; see Material Formulation. |
| MINR | Minimum strain rate of interest |
| MAXR | Maximum strain rate of interest |
| SHAPE | Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor HU. Values less than one reduces the energy dissipation and greater than one increases dissipation; see Figure M83-2. |

**Unloading Card.** This card is optional.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EXPON | RIULD | PDOPT | | | | | |
| Type | F | F | F | | | | | |
| Default | 1.0 | 0.0 | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EXPON | Exponent for unloading. Active for nonzero values of the hysteretic unloading factor HU. Default is 1.0 |
| RIULD | Flag for rate-independent unloading, see Remark 6.<br>EQ.0.0: Off<br>EQ.1.0: On |
| PDOPT | Flag for choice of third argument in TBID when PATH_DEPENDENT is used:<br>EQ.0.0: Volumetric change, $1 - J$<br>EQ.1.0: Maximum volumetric change. In other words, at each time step for each integration point, the stored largest volumetric change is compared to the current volumetric change, and the larger volumetric change is used and stored. |

**Material Formulation:**

The strain is divided into two parts: a linear part and a non-linear part of the strain

$$\mathbf{E}(t) = \mathbf{E}^L(t) + \mathbf{E}^N(t) \ ,$$

and the strain rate becomes

$$\dot{\mathbf{E}}(t) = \dot{\mathbf{E}}^L(t) + \dot{\mathbf{E}}^N(t) \ ,$$

where $\dot{\mathbf{E}}^N$ is an expression for the past history of $\mathbf{E}^N$. A postulated constitutive equation may be written as:

$$\sigma(t) = \int_{\tau=0}^{\infty} \left[\mathbf{E}_t^N(\tau), \mathbf{S}(t)\right] d\tau \ ,$$

where $\mathbf{S}(t)$ is the state variable and $\int_{.\tau=0}^{\infty}$ is a functional of all values of $\tau$ in $T_\tau: 0 \le \tau \le \infty$ and

$$\mathbf{E}_t^N(\tau) = \mathbf{E}^N(t-\tau) \ ,$$

where $\tau$ is the history parameter:

$$\mathbf{E}_t^N(\tau = \infty) \Leftrightarrow \text{the virgin material} \ .$$

It is assumed that the material remembers only its immediate past, that is, a neighborhood about $\tau = 0$. Therefore, an expansion of $\mathbf{E}_t^N(\tau)$ in a Taylor series about $\tau = 0$ yields:

$$\mathbf{E}_t^N(\tau) = \mathbf{E}^N(0) + \frac{\partial \mathbf{E}_t^N}{\partial t}(0)dt \ .$$

Hence, the postulated constitutive equation becomes:

$$\sigma(t) = \sigma^*\left[\mathbf{E}^N(t), \dot{\mathbf{E}}^N(t), \mathbf{S}(t)\right] \ ,$$

where we have replaced $\frac{\partial \mathbf{E}_t^N}{\partial t}$ by $\dot{\mathbf{E}}^N$, and $\sigma^*$ is a function of its arguments.

For a special case,

$$\sigma(t) = \sigma^*\left(\mathbf{E}^N(t), \mathbf{S}(t)\right) \ ,$$

we may write

$$\dot{\mathbf{E}}_t^N = f(\mathbf{S}(t), \mathbf{s}(t))$$

which states that the nonlinear strain rate is the function of stress and a state variable which represents the history of loading. Therefore, the proposed kinetic equation for foam materials is:

$$\dot{\mathbf{E}}_t^N = \frac{\sigma}{\|\sigma\|} D_0 \exp\left\{-c_0\left[\frac{\sigma : \mathbf{S}}{(\|\sigma\|)^2}\right]^{2n_0}\right\} \ ,$$

where $D_0$, $c_0$, and $n_0$ are material constants, and $\mathbf{S}$ is the overall state variable. If either $D_0 = 0$ or $c_0 \to \infty$ then the nonlinear strain rate vanishes.

$$\begin{aligned}
\dot{S}_{ij} &= \left[c_1\left(a_{ij}R - c_2 S_{ij}\right)P + c_3 W^{n_1}\left(\|\dot{\mathbf{E}}^N\|\right)^{n_2} I_{ij}\right]R \\
R &= 1 + c_4\left[\frac{\|\dot{\mathbf{E}}^N\|}{c_5} - 1\right]^{n_3} \\
P &= \sigma : \dot{\mathbf{E}}^N \\
W &= \int \sigma : (d\mathbf{E})
\end{aligned}$$

where $c_1$, $c_2$, $c_3$, $c_4$, $c_5$, $n_1$, $n_2$, $n_3$, and $a_{ij}$ are material constants and:

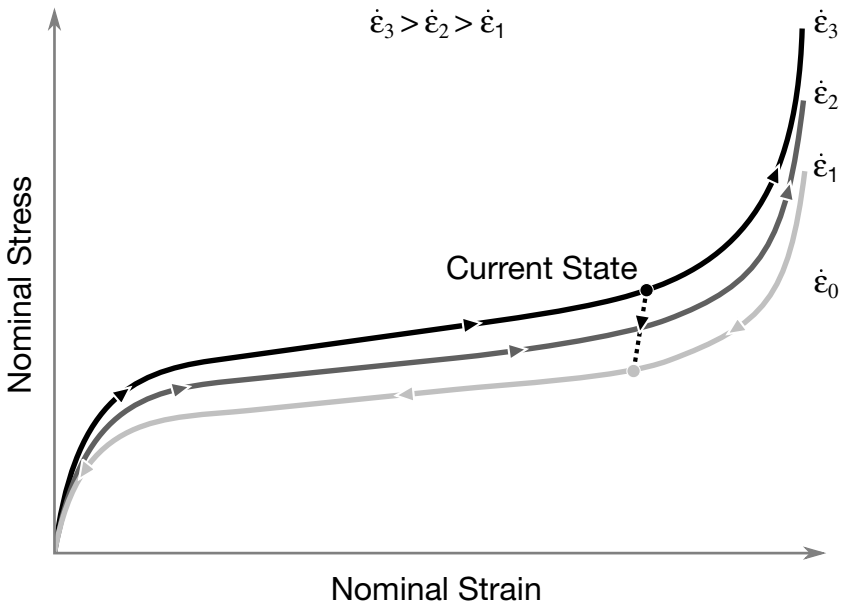



**Figure M83-2.** HU = 0, TBID > 0

$$\|\sigma\| = \left(\sigma_{ij}\sigma_{ij}\right)^{\frac{1}{2}}$$
$$\|\dot{\mathbf{E}}\| = \left(\dot{E}_{ij}\dot{E}_{ij}\right)^{\frac{1}{2}}$$
$$\|\dot{\mathbf{E}}^N\| = \left(\dot{E}^N{}_{ij}\dot{E}^N{}_{ij}\right)^{\frac{1}{2}}$$

In the implementation by Fu Chang the model was simplified such that the input constants $a_{ij}$ and the state variables $S_{ij}$ are scalars.

**Additional Remarks:**

1. **Bulk viscosity.** The bulk viscosity, which generates a rate-dependent pressure, may cause an unexpected volumetric response. Consequently, it is optional with this model.

2. **Constant velocity loading.** Dynamic compression tests at the strain rates of interest in vehicle crash are usually performed with a drop tower. In this test, the loading velocity is nearly constant, but the true strain rate, which depends on the instantaneous specimen thickness, is not. Therefore, the engineering strain rate input is optional so that the stress-strain curves obtained at constant velocity loading can be used directly. See the SFLAG field.

3. **Strain rates with multiaxial loading**. To further improve the response under multiaxial loading, the strain rate parameter can either be based on the principal strain rates or the volumetric strain rate. See the RFLAG field.

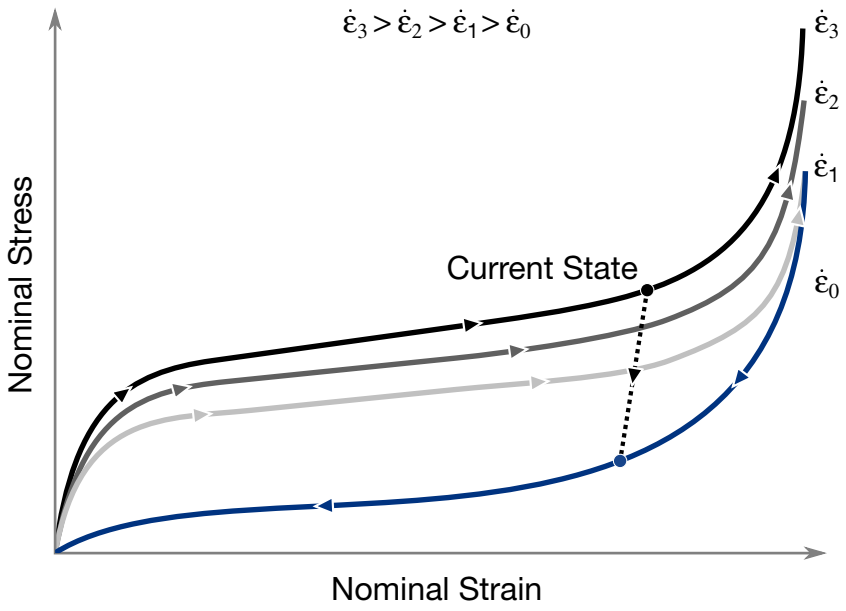



**Figure M83-3.** HU = 0, TBID < 0

4. **Triaxial loading.** Correlation under triaxial loading is achieved by directly inputting the results of hydrostatic testing in addition to the uniaxial data. Without this additional information which is fully optional, triaxial response tends to be underestimated. See the PVID field.

5. **Strain rate averaging.** Four different options are available. The default, SRAF = 0.0, uses a weighted running average with a weight of 1/12 on the current strain rate. With the second option, SRAF = 1.0, the last twelve strain rates are averaged. The third option, SRAF < 0, uses an exponential moving average with factor |SRAF| representing the degree of weighting decrease ($-1 \leq$ SRAF $<$ 0). The averaged strain rate at time $t_n$ is obtained by:

$$\dot{\varepsilon}_n^{\text{averaged}} = |\text{SRAF}|\dot{\varepsilon}_n + (1 - |\text{SRAF}|)\dot{\varepsilon}_{n-1}^{\text{averaged}}$$

To suppress time step dependence, you can select a filter window for averaging strain rates. Depending on units and the wanted filter size, you can input either $0.0 <$ SRAF $\leq 0.999$ for which SRAF becomes the filter size or SRAF $\geq 1.0001$ for which SRAF $-$ 1.0 becomes the filter size. This rather awkward way of inputting the filter size is for allowing *any* filter size to accurate precision, including a size of 1.0.

6. **Unloading response options.** Several options are available to control the unloading response for MAT_083:

   a) *HU = 0 and TBID > 0.* See Figure M83-2.

      This is the old way. In this case, the unloading response will follow the curve with the lowest strain rate and is rate-independent. The curve with

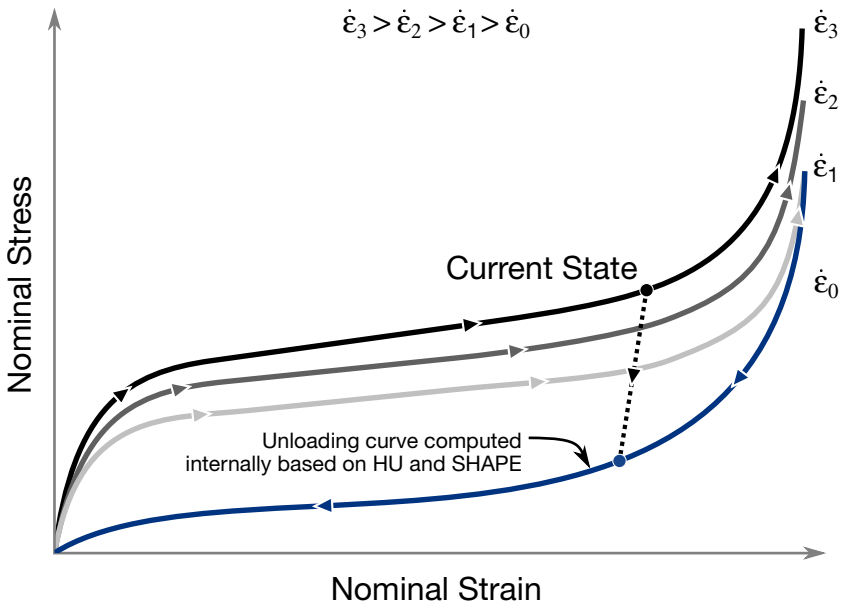



**Figure M83-4.** HU > 0, TBID > 0

the lowest strain rate value (typically zero) in TBID should correspond to the unloading path of the material as measured in a quasistatic test. The quasistatic loading path then corresponds to a realistic (small) value of the strain rate.

b) *HU = 0 and TBID < 0.* See Figure M83-3.

In this case, the curve with the lowest strain rate value (typically zero) in TBID must correspond to the unloading path of the material as measured in a quasistatic test. The quasistatic loading path then corresponds to a realistic (small) value of the strain rate. At least three curves should be used in the table: one for unloading, one for quasistatic, and one or more for dynamic response. The quasistatic loading and unloading path (thus the first two curves of the table) should form a closed loop. The unloading response is given by a damage formulation for the principal stresses as follows:

$$\sigma_i = (1 - d)\sigma_i$$

The damage parameter, $d$, is computed internally in such a way that the unloading path under uniaxial tension and compression is fitted exactly in the simulation. The unloading response is rate dependent in this case. In some cases, this rate dependence for loading *and* unloading can lead to noisy results. To reduce that noise, it is possible to switch to rate independent unloading with RIULD = 1.

The internal computation of $d$ using the first two curves of the table only works well if they are both nicely shaped and smooth, and no extreme final slope is present under compression, which is often hard to fulfill. Therefore, it is preferable to use the next option, HU > 0 with TBID > 0, instead.

c) *HU > 0 and TBID > 0.* See Figure M83-4.

No unloading curve should be provided in the table and the curve with the lowest strain rate value in TBID should correspond to the loading path of the material as measured in a quasistatic test. At least two curves should be used in the table: one for quasistatic and one or more for dynamic response. In this case the unloading response is given by a damage formulation for the principal stresses as follows:

$$\sigma_i = (1 - d)\sigma_i$$

$$d = (1 - HU)\left[1 - \left(\frac{W_{\text{cur}}}{W_{\text{max}}}\right)^{\text{SHAPE}}\right]^{\text{EXPON}}$$

where $W_{cur}$ corresponds to the current value of the hyperelastic energy per unit undeformed volume. The unloading response is rate dependent in this case. In some cases, this rate dependence for loading *and* unloading can lead to noisy results. To reduce that noise, it is possible to switch to rate-independent unloading with RIULD = 1.

The LOG_LOG_INTERPOLATION option uses log-log interpolation for table TBID in the strain rate direction.

## *MAT_WINFRITH_CONCRETE

This is Material Type 84 with optional rate effects. The Winfrith concrete model is a smeared crack (sometimes known as pseudo crack), smeared rebar model. We implemented this model for the 8-node single integration point solid element (ELFORM = 1 on *SECTION_SOLID) and the 4-node single integration point tetrahedral element (ELFORM = 10). We recommend using a double precision executable for simulations that include this material model. Single precision may produce unstable results.

Broadhouse and Neilson [1987] and Broadhouse [1995] developed this model over many years, and experiments have validated it. Much of the input documentation given here comes directly from the report by Broadhouse. In releases R15 onwards, further developments by Arup are available by setting the input parameter RATE to 8; see Cards 5 through 7.

Rebar may be defined using the keyword *MAT_WINFRITH_CONCRETE_REINFORCEMENT, which appears in the following section, or may be modeled with beam elements fixed to the concrete using *CONSTRAINED_BEAM_IN_SOLID.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | TM | PR | UCS | UTS | FE | ASIZE |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| E | YS | EH | UELONG | RATE | CONM | CONL | CONT |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required but may be left blank.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required but may be left blank.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card when RATE = 8.

| MAXSHR | LCYMT | LCFCT | LCFTT | | | LCTST | LCCMP |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card when RATE = 8. It may be left blank.

| CRFAC | COD1 | TENPWR | TENRSD | LCFIB | RO_G | ZSURF | LCFTIM |
|---|---|---|---|---|---|---|---|

**Card 7.** Include this card when RATE = 8. It may be left blank.

| OTTO | DILATD | DILRAT | DEGRAD | TFAC8 | TLOSSC | CDSF | LCRATE |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TM | PR | UCS | UTS | FE | ASIZE |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| TM | Initial tangent (Young's) modulus of concrete, $E_s$. See Remarks 2 and 3. |
| PR | Poisson's ratio, $\nu$. See Remarks 2 and 3. |
| UCS | Uniaxial compressive strength (see Remarks 2 and 3) |
| UTS | Uniaxial tensile strength (see Remarks 2, 6 and 11) |
| FE | The meaning of FE depends on the value of RATE (see Remark 8):<br>RATE.EQ.0: Fracture energy (energy per unit area dissipated in opening the crack).<br>RATE.GT.0: Crack width at which the crack-normal tensile stress goes to zero. |
| ASIZE | Aggregate size, depending on the value of RATE.<br>RATE.LE.1: Aggregate radius in model length units.<br>RATE.GE.2: Aggregate diameter in meters. The formula for shear stress carried across cracks with aggregate interlock uses this field; see Remark 11. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E | YS | EH | UELONG | RATE | CONM | CONL | CONT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus of rebar |
| YS | Yield stress of rebar |
| EH | Hardening modulus of rebar |
| UELONG | Ultimate elongation before rebar fails. |
| RATE | Material model option (see Remarks 8 and 13):<br>EQ.0.0: Original Broadhouse implementation with strain rate effects included. WARNING: This option does not guarantee energy conservation.<br>EQ.1.0: Original Broadhouse implementation with strain rate effects turned off.<br>EQ.2.0: Like RATE = 1 but includes an improved crack algorithm. It is superseded by RATE = 8.<br>EQ.8.0: Improved crack algorithm plus additional inputs on Cards 5 through 7 (recommended). |
| CONM | Units (conversion) flag:<br>GT.0.0: Factor to convert model mass units to kg<br>EQ.-1.0: Mass, length, and time units in the model are lbf × sec2/in, inch, and sec.<br>EQ.-2.0: Mass, length, and time units in the model are g, cm, and microsec.<br>EQ.-3.0: Mass, length, and time units in the model are g, mm, and msec.<br>EQ.-4.0: Mass, length, and time units in the model are metric ton, mm, and sec.<br>EQ.-5.0: Mass, length, and time units in the model are kg, mm, and msec. |

| VARIABLE | DESCRIPTION |
|---|---|
| CONL | Length units conversion factor:<br>CONM.GT.0: CONL is the conversion factor from model length units to meters (for instance, CONL = 0.001 for millimeters).<br>CONM.LE.0: CONL is ignored. |
| CONT | Time units conversion factor:<br>CONM.GT.0: CONT is the conversion factor from time units to seconds (for example, CONT = 0.001 for milliseconds).<br>CONM.LE.0: CONT is ignored. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1, EPS2, … | Volumetric strain values (natural logarithmic values); see Remark 3. If this card is not left blank, a minimum of 2 values must be defined and a maximum of 8 values are allowed. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| P1, P2, … | Pressures corresponding to the volumetric strain values given on Card 3. See Remark 3. |

Additional card for RATE = 8.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MAXSHR | LCYMT | LCFCT | LCFTT | | | LCTST | LCCMP |
| Type | F | I | I | I | | | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MAXSHR | Maximum shear stress that can be carried across a crack under conditions of zero normal stress on the crack and zero crack opening displacement. The default value is 1.161 times UTS; see Remark 11. |
| LCYMT | Optional load curve ID governing the variation of elastic stiffness with temperature. The $x$-axis is temperature, and the $y$-axis is a nondimensional factor on elastic modulus TM. |
| LCFCT | Optional load curve ID governing the variation of compressive strength with temperature. The $x$-axis is temperature, and the $y$-axis is a nondimensional factor on compressive strength UCS. |
| LCFTT | Optional load curve ID governing the variation of tensile compressive strength with temperature. The $x$-axis is temperature, and the $y$-axis is a nondimensional factor on tensile strength UTS. See Remark 9. |
| LCTST | Optional load curve ID governing the post-cracking tensile response. See Remark 8. The $x$-axis is crack-opening displacement (length units), and the $y$-axis is a nondimensional factor on tensile strength UTS. If LCTST is defined, it overrides FE on Card 1. The first point should be (0, 1). |
| LCCMP | Optional load curve ID governing post-yield compression/shear response. The $x$-axis is plastic strain,and the $y$-axis is a nondimensional factor that scales UCS. See Remark 14. |

Additional card for RATE = 8.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CRFAC | COD1 | TENPWR | TENRSD | LCFIB | RO_G | ZSURF | LCFTIM |
| Type | F | F | F | F | I | F | F | I |
| Default | 0.0 | 0.0 | 1.0 | 0.01 | none | 0.0 | 0.0 | optional |

| VARIABLE | DESCRIPTION |
|---|---|
| CRFAC | Scale the tensile strength of uncracked elements by (1 – CRFAC) if an adjacent element has cracked. See Remark 9. |
| COD1 | Crack opening displacement (length units) of the adjacent element at which the full value of CRFAC applies. For crack opening displacements smaller than COD1, linear interpolation is applied. See Remark 9. |
| TENPWR | Power law term governing tensile strength when at least one principal stress is compressive. See Remark 9. |
| TENRSD | Residual factor term governing tensile strength when at least one principal stress is compressive. See Remark 9. |
| LCFIB | Optional load curve ID for fiber-reinforced concrete. The $x$-axis of the curve is crack-opening displacement (length units), and the $y$-axis is additional tensile stress due to the presence of the fibers. See Remark 10. |
| RO_G | A nonzero RO_G invokes the option for water pressure to be applied within cracks. The value of RO_G is water density times acceleration due to gravity. See Remark 22. |
| ZSURF | Global $z$-coordinate of the water surface, used for calculating water pressure in cracks. See Remark 22. |
| LCFTIM | Optional load curve ID giving scaling factor on tensile strength (UTS) as a function of time. See Remarks 9 and 21. |

Additional card for RATE = 8.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OTTO | DILATD | DILRAT | DEGRAD | TFAC8 | TLOSSC | CDSF | LCRATE |
| Type | I | F | F | F | F | F | F | I |
| Default | 1 | 0.0 | 0.0 | 0.0 | 0.9 | 1.0 | 8.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| OTTO | Option for automatic calculation of the Ottosen yield surface constants (see Remark 13):<br>EQ.1: fib Model Code 2010, normal weight concrete<br>EQ.2: fib Model Code 2010, lightweight concrete<br>EQ.3: Same as RATE = 0, 1 or 2 (Broadhouse model) |
| DILATD | Maximum dilation displacement (in model length units) due to crack sliding or yielding |
| DILRAT | Initial dilation ratio |
| DEGRAD | Lower limit on the factor by which the compressive strength of cracked elements is scaled (see Remark 15):<br>EQ.0: No reduction of compressive strength<br>GT.0: Equation from Eurocode 2 with lower limit = DEGRAD |
| TFAC8 | Nondimensional modification factor applied to any tensile principal stresses when calculating the Ottosen yield function; see Remark 16. |
| TLOSSC | Nondimensional parameter controlling loss of tensile strength in crushed elements; see Remark 9. |
| CDSF | Nondimensional ductility factor for confined concrete. CDSF controls scaling of the *x*-axis of LCCMP; see Remark 14. |
| LCRATE | Optional load curve ID, \|LCRATE\|, giving the strength enhancement factor as a function of strain rate (see Remark 23):<br>GT.0: Rate effects treated by a simple scaling of the strength |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0: Rate effects treated by a fully viscoplastic formulation |

**Remarks:**

1. **Minimum input recommendations.** All of the input parameters on Card 1 should be defined, together with RATE and the unit conversion factors CONM, CONL, and CONT on Card 2 (CONL and CONT may be left blank if CONM is negative). If yielding or failure in compression or shear is anticipated, we recommend RATE = 8. For RATE = 8, we recommend defining LCCMP. All the other input parameters on Cards 5, 6, and 7 may be left blank because the defaults are intended to provide a reasonably realistic response.

2. **Basic properties.** The elastic properties are defined by Young's modulus TM and Poisson's Ratio PR. UCS is the compressive strength under uniaxial stress conditions as measured by a standard cylinder compression test. Note that the strength obtained from standard cube tests is typically 15-25% greater than the uniaxial compressive strength. UTS, the tensile strength, may be estimated from tables or equations in codes and standards such as Eurocode 2 or ACI-318. It is important that a realistic tensile strength (as opposed to an artificially low "design" value) is input, for reasons explained in Remark 13.

3. **Volumetric response.** Cards 3 and 4 enable providing the volumetric response curve. In this curve, pressure is positive in compression. Volumetric strain is defined as the natural log of the relative volume and is negative in compression. The tabulated data must be provided in order of increasing compression, with no initial zero point.

   If omitting the volume compaction curve, i.e., if Cards 3 and 4 are left blank, LS-DYNA uses the scaled curve in Table M84-1. $p_1$ in the curve is the pressure at uniaxial compressive failure:

   $$p_1 = \frac{\text{UCS}}{3} \ ,$$

   $K$ (referenced in the Table below) is the bulk unloading modulus computed from:

   $$K = \frac{E_s}{3(1-2v)} \ .$$

   Here $E_s$ is the input tangent modulus for concrete (input parameter TM), and $v$ is Poisson's ratio.

**Table M84-1.** Default pressure as a function of volumetric strain curve for concrete if the curve is not defined.

| Volumetric Strain | Pressure |
|---|---|
| $-p_1/K$ | $1.00p_1$ |
| $-0.002$ | $1.50p_1$ |
| $-0.004$ | $3.00p_1$ |
| $-0.010$ | $4.80p_1$ |
| $-0.020$ | $6.00p_1$ |
| $-0.030$ | $7.50p_1$ |
| $-0.041$ | $9.45p_1$ |
| $-0.051$ | $11.55p_1$ |
| $-0.062$ | $14.25p_1$ |
| $-0.094$ | $25.05p_1$ |

4. **Binary crack output database.** The Winfrith concrete model can generate an additional binary output database containing information on crack locations, directions, and widths. Generating the crack database requires modifyinh the LS-DYNA execution line by adding the following:

q=**crf**

where **crf** is the desired name of the crack database, such as q = **d3crack**. LS-DYNA writes the output at the same times as the d3plot database.

LS-PrePost can display the cracks on the deformed mesh plots. To do so, read the d3plot database into LS-PrePost and select *File* → *Open* → *Crack* from the top menu bar. Or, open the crack database by adding the following to the LS-PrePost execution line:

q=**crf**

where **crf** is the name of the crack database.

By default, LS-PrePost shows all the cracks in visible elements. Setting a minimum crack width for displayed cracks eliminates narrow cracks from the display. To do this, choose *Settings* → *Post Settings* → *Concrete Crack Width*. From the top menu bar of LS-PrePost, choosing *Misc* → *Model Info* reveals the number of cracked elements and the maximum crack width in a given plot state.

5. **Crack summary output file.** Including *DATABASE_BINARY_D3CRACK in the input deck causes LS-DYNA to write an ASCII output file named aea_crack. This command does not have any bearing on the aforementioned binary crack database.

6. **Crack plane directions.** The crack algorithm uses a non-rotating approach. Once cracks have initiated, their directions remain fixed relative to the element's local axis system. Up to three mutually perpendicular cracks can form in each element. The first crack is initiated on a plane normal to the maximum tensile principal stress when that principal stress reaches the tensile strength (input parameter UTS; see Remark 9 for RATE = 8). A second crack can initiate on any plane normal to the first crack and does so if the tensile stress acting perpendicular to that plane reaches the tensile strength. After creating two cracks, the only possible plane for the third crack is the one mutually perpendicular to the first two cracks. The third crack initiates if the tensile stress acting perpendicular to that plane reaches the tensile strength.

7. **Limitation of non-rotating cracks.** The algorithm prevents the tensile stress from exceeding UTS only in directions normal to actual or potential crack planes. It is possible to observe tensile principal stresses greater than UTS in the results if the loading directions rotate after a crack has formed. This result is a limitation of the non-rotating crack approach.

8. **Crack tensile response.** We model cracks with the "smeared crack" approach, meaning that the stress-strain relationships model the presence of a crack instead of the mesh breaking apart. To reduce the sensitivity of results to mesh size, these relationships are formulated using displacement instead of strain as the abscissa. Displacement, $\delta$, is calculated from strain, $\varepsilon$, and the initial element volume, $V_0$, as follows:

$$\delta = L_0 \varepsilon$$
$$L_0 = V_0^{1/3}$$

After the initiation of a crack, the tensile stress decays with increasing crack opening displacement. For RATE = 1 and 2, the decay follows a linear relationship with the tensile strength reaching zero at a crack opening displacement equal to the input parameter FE. For RATE = 0, the decay follows a bilinear relationship. LS-DYNA automatically scales the displacement axis of this bilinear relationship based on the input parameter FE which represents the fracture energy.

For RATE = 8 the default is to use FE in the same way as RATE = 1 and 2, but this may optionally be overridden using the load curve LCTST. If defined, the first point of LCTST should be (0, 1), i.e., at zero crack opening displacement, the uniaxial tensile strength is equal to unity times UTS. It is expected, but not essential, that the $y$-axis values drop to zero at some finite $x$-axis value, meaning the tensile strength drops to zero at a finite crack opening displacement. See also Remark 10 regarding fiber-reinforced concrete.

9. **Modifications to tensile strength (RATE = 8).** When RATE = 8, the tensile strength of a given element may be different from UTS for the following reasons:

a) If compressive principal stresses are also present, the tensile strength is scaled by a factor $k_{TC}$ defined as follows:

$$k_{TC} = \text{TENRSD} + (1 - \text{TENRSD})\left(1 - f^{\text{TENPWR}}\right)$$
$$f = {}^{-\sigma_{\text{comp}}}/_{\text{UCS}} \text{ subject to } 0 \le f \le 1$$

Here, $\sigma_{\text{comp}}$ is the most compressive principal stress, and TENRSD and TENPWR are defined on Card 6. The default values provide a linear reduction of tensile strength from UTS to 0.01 × UTS as the most compressive principal stress increases from zero to UCS.

b) If any neighboring elements are cracked (where "neighboring" means elements that share at least one node with the uncracked element being considered), the tensile strength is scaled by a crack propagation factor $k_{CP}$ intended to represent the effect of stress concentrations near a crack tip:

$$k_{CP} = 1 - \text{CRFAC}\left(\min(1.0, \frac{\delta_{\text{crack,max}}}{\text{COD1}})\right)$$

Here, $\delta_{\text{crack,max}}$ is the greatest crack opening displacement in any neighboring element, and CRFAC and COD1 are input parameters on Card 6.

c) If the element has yielded in compression (see Remark 13), it is assumed that the damage to bonds within the material caused by crushing rapidly eliminates the tensile strength. To represent this effect, the tensile strength is scaled by a factor calculated as follows:

$$k_Y = \max\left[0.0, {}^{L_0\varepsilon_p}/_{(\text{TLOSSC} \times \delta_{\sigma=0})}\right]$$

In the above equation, $L_0$ is the initial element size as defined in Remark 8, $\varepsilon_p$ is the plastic strain associated with yielding on the Ottosen yield surface, TLOSSC is a nondimensional input parameter defined on Card 7, and $\delta_{\sigma=0}$ is the crack opening displacement at which the tensile stress reduces to zero. By default, $\delta_{\sigma=0}$ is equal to FE, but if LCTST is defined, it is the $x$-axis value at which the $y$-axis value falls to zero.

d) If LCFTIM is defined (see Remark 21), the current $y$-axis value of LCFTIM is a scaling factor $k_t$. Otherwise, $k_t = 1$.

e) If LCFTT is defined (see Card 5), a temperature-dependent scaling factor $k_T$ is applied. Otherwise, $k_T = 1$.

f) The initial tensile strength of an element, $f_t$, is calculated from UTS and the above factors as follows:

$$f_t = k_{TC}k_{CP}k_Y k_t k_T \times \text{UTS}$$

10. **Fiber-reinforced concrete.** Steel fibers increase the ductility of concrete because they resist the opening of cracks. In order for cracks to widen, the fibers that span across the crack must be pulled out or stretched. This effect may be modeled using the load curve LCFIB on Card 6. The $x$-axis is crack opening displacement in length units. The $y$-axis is additional tensile stress resisting further opening of the crack. This additional tensile strength is not subject to the reduction factors described in Remark 9, which are appropriate only for the concrete itself and not for the effect of the fibers. For this reason, we recommend using LCFIB instead of combining the influence of the fibers with the tensile response of the concrete into the curve LCTST. We recommend that the first point of LCFIB should be (0, 0) so that the fibers do not influence the overall initial tensile strength.

11. **Shear transfer across cracks (RATE = 2 and 8).** When RATE = 2 and 8, the following equations model the aggregate-interlock that allows cracked concrete to carry shear loading. The equations were proposed by Vecchio & Collins (1986) and subsequently adopted into Norwegian standard NS3473. The maximum shear stress, $\tau_{\max}$, that the crack plane can carry depends on the compressive stress on the crack, $\sigma_c$, for a closed crack or on the crack opening width, $w$, for an open crack:

$$\tau_{\max} = 0.18\tau_{\mathrm{rm}} + 1.64\sigma_c - 0.82\frac{\sigma_c^2}{\tau_{\mathrm{rm}}}$$

$$\tau_{\mathrm{rm}} = \frac{2f_{t,sh}}{0.31 + \dfrac{0.024w}{(\mathrm{ASIZE} + 0.016)}}$$

ASIZE is the aggregate diameter in meters defined on Card 1. For this purpose, CONL is ignored, and the input value of ASIZE should be in meters even if the model units are not meters.

In the above equations, $f_{t,sh}$ is a tensile strength that depends on RATE and, if RATE = 8, on MAXSHR on Card 5. If RATE = 8 and MAXSHR = 0.0 (recommended), $f_{t,sh}$ is equal to the tensile strength (i.e., UTS defined on Card 1) in accordance with Vecchio & Collins. The above equations then give $\tau_{\max} = 1.16\mathrm{UTS}$ when $\sigma_c$ and $w$ are both zero. For RATE = 8, if MAXSHR is nonzero, $f_{t,sh}$ is automatically set such that $\tau_{\max} = \mathrm{MAXSHR}$ when $\sigma_c$ and $w$ are both zero.

If RATE = 2, MAXSHR is unavailable. In this case, $f_{t,sh} = \max(\mathrm{UTS}, 0.086\mathrm{UCS})$, giving $\tau_{\max} = \max(1.16\mathrm{UTS}, 0.1\mathrm{UCS})$ when $\sigma_c$ and $w$ are both zero. The 0.1UCS does not comply with the recommendations of Vecchio & Collins; this is one of the reasons why RATE = 2 is not recommended.

12. **Compression response: general comments.** The compression response of concrete varies according to the stress state. Under uniaxial conditions, such as occur in a cylinder test, concrete exhibits a brittle response, failing rapidly after

reaching its compressive strength. Under confined conditions, which occur in reinforced concrete structures when the reinforcement cage resists expansion of the concrete in the directions perpendicular to the main compressive load, the stress state in the concrete consists of one large compressive stress in the loading direction and, typically, two smaller compressive stresses ("confining stresses") in the perpendicular directions. The influence of the confining stresses is two-fold: firstly, the compressive strength in the loading direction is increased from the uniaxial value to an enhanced value (denoted here as $\sigma_c$ and $\sigma_{cc}$, respectively); and secondly, the compressive response becomes more ductile, meaning that the rate of softening with strain is reduced and the strain to failure is increased.

13. **Compression response: yield surface.** The Ottosen yield surface governs yielding under compressive stress states. This surface captures the influence of confining stresses on the compressive strength for all settings of RATE. The following equation defines the Ottosen yield surface:

$$\alpha \frac{J_2}{\sigma_c^2} + \lambda \frac{\sqrt{J_2}}{\sigma_c} + \beta \frac{I_1}{\sigma_c} - 1 = 0 \ ,$$

where

$$\lambda = c_1 \cos\left[\frac{1}{3}\arccos(c_2 \cos(3\theta))\right] \ .$$

In the above $I_1$, $J_2$ and $J_3$ are the first, second and third stress invariants, $\sigma_c$ is the uniaxial compressive strength (input parameter UCS for RATE = 0, 1 and 2, and see Remark 14 for RATE = 8), and $\theta$ is the Lode Angle. $\alpha$, $\beta$, $c_1$, and $c_2$ are calibration constants that LS-DYNA internally calculates to fit the yield surface through four reference stress states which are chosen automatically.

Two of the reference stress states are uniaxial compression and uniaxial tension, defined by input parameters UCS and UTS. This calibration has a counterintuitive side effect whereby the input value of tensile strength (UTS) affects the whole yield surface, including stress states in which no tensile stresses are present. For this reason, it is important to use values for UTS and UCS that are in similar proportions to each other as for real concrete. This applies to all settings of RATE.

Two further reference stress states are needed for calibration. These differ according to the setting of RATE. RATE = 0 and 1 are as described by Broadhouse. RATE = 8 offers a choice via the input parameter OTTO, with the default being to adopt the recommendations of fib Model Code for Concrete Structures 2010 ("MC2010") Section 5.1.6. The choice of reference stress state influences the degree to which small confining stresses increase the compressive strength, with the MC2010 method giving a smaller increase than the Broadhouse method.

For RATE = 2, the yield surface is calibrated using the same method as RATE = 0 and 1 except that an enhanced tensile strength, namely max(1.25UTS, 0.1UCS), is used instead of the actual tensile strength UTS for purpose of calibrating the yield surface. Using an enhannced tensile strength was done in order to reduce the counter-intuitive influence of tensile strength on all-compressive stress states, but it does not accord with recommendations in the literature. For this reason, RATE = 8 is preferred over RATE = 2.

14. **Compression Response Post-Yield.** For RATE = 0, 1 and 2, the post-yield response is perfectly-plastic, which fails to capture the brittle response under uniaxial stress conditions and is not representative of real concrete. These settings of RATE are unsuitable for assessing failure under compressive stress states.

    For RATE = 8, the compressive stress-strain relation under uniaxial stress conditions is controlled by the loadcurve LCCMP, which should be calibrated by the user to obtain the desired brittle response:

$$\sigma_c = \text{LCCMP}\left(\varepsilon_{p,\text{uniaxial}}\right) \times \text{UCS}$$

    The uniaxial-equivalent plastic strain, $\varepsilon_{p,\text{uniaxial}}$, is calculated from the equation below so as to provide increased ductility under confined conditions compared to uniaxial conditions by stretching the load curve LCCMP along the $x$-axis:

$$\varepsilon_{p,\text{uniaxial}} = \sum \left[ \frac{d\varepsilon_p}{1 + \text{CDSF}(\sigma_{cc}/\sigma_c - 1)} \right]$$

    Here, the actual plastic strain increments are denoted by $d\varepsilon_p$, CDSF is an input parameter on Card 7, and $\sigma_{cc}$ is the confined compressive strength defined by most compressive principal stress at the point on the Ottosen yield surface corresponding to the current stress state.

15. **Influence of cracking on compressive strength.** By default, cracking of an element has no influence on its compressive strength. In practice, the compressive strength parallel to an open crack is reduced to some degree. This may be modelled using the optional input parameter DEGRAD which invokes the following equations based on Eurocode 2:

$$\sigma_c = k_{cr}\sigma_{c,\text{uncracked}}$$
$$k_{cr} = {}^{1}\!/_{(0.8 + 170\varepsilon_{cr})}$$

    where $\varepsilon_{cr} = \max(\delta_{cr1}, \delta_{cr1}, \delta_{cr1})\ /\text{Vol}^{1/3}$ subject to $\text{DEGRAD} \le k_{cr} \le 1.0$. Here, $\delta_{cr1,2,3}$ are the high-tide crack opening displacements.

16. **Input parameter TFAC8.** For RATE = 8, yielding (governed by the Ottosen yield surface) is treated as a separate deformation mechanism from cracking. Since the Ottosen surface covers the whole stress space including where one or

more principal stress is tensile, and since the yield surface is calibrated to pass through the point of uniaxial tensile failure at a stress equal to UTS, either or both deformation mechanisms could potentially occur under conditions where only cracking is expected. The parameter TFAC8 modifies the Ottosen yield surface such that the activation of cracking rather than yielding becomes unambiguous under these circumstances. It does this by calculating the yield surface from principal stresses where any tensile values are artificially scaled down by TFAC8. Thus, for compressive-only stress states, the yield surface calibration is unaffected by TFAC8, while the expected cracking occurs under tensile stress states. The default value of 0.9 is recommended.

17. **Output history variables.** The meaning of "plastic strain" in output files differs depending on the setting of RATE. Use NEIPH on *DATABASE_EXTENT_BINARY to request extra history variables with NEIPH on *DATABASE_EXTENT_BINARY. The meanings of these also depend on RATE. The following tables give the meanings of a selection of these variables. In the tables, Crack 1, Crack 2, and Crack 3 refer to the first, second, and third cracks, respectively, to form in the element.

| Plastic Strain or Extra History Variable | Description for RATE = 0 |
|---|---|
| Plastic strain | Volumetric plastic strain, see compaction curve defined on Cards 3 and 4. |
| 1 | Maximum current value of the crack status flag across all three cracks; see Remark 18. |
| 2 | Energy absorbed by crack formation |
| 3-5 | High-tide value of the non-dimensional crack opening parameter for Cracks 1, 2, and 3. This value is capped at 5.16. See Remark 19. |
| 36-38 | Crack status flags for Cracks 1, 2, and 3. See Remark 18. |
| 45-47 | Current crack opening displacement for Cracks 1, 2, and 3. See Remark 19. |
| 48-50 | Initiation time for Cracks 1, 2, and 3 |

| Plastic Strain or Extra History Variable | Description for RATE = 1 |
|---|---|
| Plastic strain | Volumetric plastic strain, see compaction curve defined on Cards 3 and 4. |
| 1 | Maximum current value of the crack status flag across all three cracks. See Remark 18. |

| Plastic Strain or Extra History Variable | Description for RATE = 1 |
|---|---|
| 30-32 | High-tide opening displacement for Cracks 1, 2, and 3, capped at a displacement equal to input parameter FE. See Remark 19. |
| 36-38 | Crack status flags for Cracks 1, 2, and 3. See Remark 18. |
| 45-47 | Current crack opening displacement for Cracks 1, 2, and 3. See Remark 19. |
| 48-50 | Initiation time for Cracks 1, 2, and 3 |

| Plastic Strain or Extra History Variable | Description for RATE = 2 and 8 |
|---|---|
| Plastic strain | (Starting from R15): "Damage deformation" in length units, see Remark 20. |
| 1 | Number of cracks that have formed (0, 1, 2, or 3). |
| 2 | Plastic strain due to yielding on Ottosen surface (RATE = 8 only) |
| 3-5 | High-tide opening displacement for Cracks 1, 2, and 3 (not capped). See Remark 19. |
| 27 | (Starting from R15): Volumetric plastic strain, see compaction curve defined on Cards 3 and 4. |
| 36-38 | Crack status flags for Cracks 1, 2, and 3. See Remark 18. |
| 45-47 | Current crack opening displacement for Cracks 1, 2, and 3. See Remark 19. |
| 71 | Effective total strain rate in compression (if LCRATE is positive), or plastic strain rate in compression (if LCRATE is negative) |
| 72-74 | Effective total strain rate in tension (if LCRATE is positive), or crack-opening strain rate for Cracks 1,2 and 3 (if LCRATE is negative) |
| 75 | Compressive strength enhancement factor due to rate effects |
| 76-78 | Tensile strength enhancement factor for Cracks 1, 2, and 3 due to rate effects |

18. **Crack status flags.** The crack status flags referred to in the Extra History Variables have the following meanings:

    EQ.0: No crack has formed.

EQ.1: The crack is opening and on the descending branch of the stress-displacement relationship, such that the tensile strength has not yet reached zero.

EQ.2: The crack has partially closed (unloading/reloading branch).

EQ.3: The crack has fully opened such that the tensile strength has reached zero.

19. **Extra history variables related to crack opening displacement.** The *current* crack opening displacement may be output for all settings of RATE in Extra History Variables 45 through 47. It can rise and fall during an analysis as cracks open and close due to changes in loading. These values are not capped, meaning that they reflect the total width of cracks within the element even if the crack opens further after the tensile strength has reached zero.

The *high-tide* crack opening displacement is the maximum width that has occurred up to that point in the analysis. It differs from the current value in cases where the cracks open and then fully or partially close. This output parameter is available for RATE = 2 and 8. For RATE = 0 and RATE = 1, high-tide output parameters are available, but we cap them at the value where the tensile strength reaches zero. Although cracks can continue to open further with zero resistance, the capped output parameters do not reflect the additional opening. The capped output parameters are useful for assessing how much of the tensile capacity has been lost but not for assessing crack widths. For RATE = 1, the high-tide output is the crack displacement capped at a value equal to the input parameter FE. For RATE = 0, the high-tide output is in a non-dimensional form and is capped at a value of 5.16 which is the point at which the tensile strength reaches zero.

20. **Damage deformation.** Starting from R15, the parameter output in place of plastic strain for RATE = 2 and 8, $\delta_{\text{dam}}$, is defined as follows:

$$\delta_{\text{dam}} = \varepsilon_p \text{Vol}_0^{1/3} + \delta_{\text{crack1}} + \delta_{\text{crack2}} + \delta_{\text{crack3}}$$

In this equation, $\delta_{\text{crack1,2,3}}$ are the high-tide crack opening displacements, $\varepsilon_p$ is the plastic strain associated with yielding on the Ottosen surface and $\text{Vol}_0$ is the initial element volume.

21. **Low "design" values of tensile strength.** Users may wish to check that the performance of a structure is not reliant on the tensile strength of concrete, but this should *not* be done by setting an artificially low value for UTS. Doing so would distort the yield surface as explained in Remark 13 and may also cause cracks to form at random angles due to small tensile stresses occurring dynamically during application of the load. Instead, we recommend scaling down the tensile strength as a function of time, starting from a realistic value given by UTS which will be used to calibrate the yield surface, and then reducing to the desired

low "design" value after loads have been applied. This may be achieved using the load curve LCFTIM. The ordinate of LCFTIM is a scaling factor applied to UTS. The abscissa is time. The first point of LCFTIM should be (0, 1).

22. **Water pressure in cracks.** When water seeps into cracks in underwater structures, the water pressure acts to push the crack surfaces apart, balancing the effect of pressure on the structure's outer surfaces which would tend to push the crack together. The input parameters RO_G and ZSURF model the effect of water in any cracks that form by applying an additional compressive stress normal to the crack, irrespective of whether or not there is a path for the water to reach the cracked element from the outer surface of the structure. To use this feature, the model should be oriented such that the global $z$-coordinate is vertically upwards. The additional compressive stress, $\sigma_{\text{water}}$, ramps up from zero to its full value as the crack opening displacement increases from zero to FE, as follows:

$$\sigma_{\text{water}} = \rho g(z_0 - z_{\text{el}}) \times \min(1.0, \delta_{\max}/\text{FE})$$

Here, $\rho g$ is the input parameter RO_G, $z_0$ is the input parameter ZSURF, $z_{\text{el}}$ is the $z$-coordinate of the element center, $\delta_{\max}$ is the maximum opening displacement of the crack so far during the analysis, and FE is the input parameter on Card 1.

23. **Strain rate effects.** The definition of load curve LCRATE follows the convention adopted in *MAT_072R3, where negative strain rate values (on the $x$-axis) denote the tensile strength enhancement factor (on the $y$-axis), and positive strain rate values denote the compressive strength enhancement factor. The curve must include a point at zero strain rate with an enhancement factor of 1.0. If the load curve ID is given as a positive value, the $x$-axis of the curve represents *effective deviatoric* strain rate, and rate effects are treated by scaling the tensile and compressive strength of concrete by the enhancement factors interpolated from the curve. If the load curve ID is given as a negative value, the $x$-axis represents *plastic* strain rate in compression and *crack-opening* strain rate in tension. In this case, a viscoplastic formulation is used to incorporate strain rate effects within the Ottosen yield function and the cracking algorithm. This approach is computationally more expensive than the simple strength scaling, but it significantly reduces numerical noise and can greatly improve solution quality. When LCRATE is defined, Extra History Variables 71 through 78 may be output to track the relevant strain rates and enhancement factors.

**References:**

[1] Ottosen N.S., "A failure criterion for concrete". Journal of the Engineering Mechanics Division 103(4):527-35, 1977.

[2] Sturt, R., Montalbini, G. & Jung, H-I., "Developments in *MAT_WINFRITH_CONCRETE and Application to Modelling Tunnel Linings", 14th European LS-DYNA Conference, 2023.

[3] Taerwe L, Matthys S., "Fib model code for concrete structures", CEB-FIP, 2010.
[4] Vecchio, F.J. & Collins, M.P., "The Modified Compression-Field Theory for Reinforced Concrete Elements Subjected to Shear", ACI J 83(2) (1986) pp219-231, 1986.

## *MAT_WINFRITH_CONCRETE_REINFORCEMENT

This is *MAT_084_REINF for rebar reinforcement supplemental to concrete defined using Material type 84. Reinforcement may be defined in specific groups of elements, but it is usually more convenient to define a two-dimensional material in a specified layer of a specified part. Reinforcement quantity is defined as the ratio of the cross-sectional area of steel relative to the cross-sectional area of concrete in the element (or layer). These cards may follow either one of two formats below and may also be defined in any order.

### Card Summary:

**Card 1a.** Reinforcement is defined in specific groups of elements.

| EID1 | EID2 | INC | XR | YR | ZR | | |
|---|---|---|---|---|---|---|---|

**Card 1b.** Reinforcement is defined in two-dimensional layers by part ID. This option is active when the first entry is left blank.

| | PID | AXIS | COOR | RQA | RQB | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

**Option 1.** Reinforcement quantities in element groups

| Card 1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EID1 | EID2 | INC | XR | YR | ZR | | |
| Type | I | I | I | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EID1 | First element ID in group |
| EID2 | Last element ID in group |
| INC | Element increment for generation |
| XR | $x$-reinforcement quantity (for bars running parallel to global $x$-axis) |
| YR | $y$-reinforcement quantity (for bars running parallel to global $y$-axis) |
| ZR | $z$-reinforcement quantity (for bars running parallel to global $z$-axis) |

**Option 2.** Two dimensional layers by part ID. Option 2 is active when first entry is left blank.

| Card 1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | PID | AXIS | COOR | RQA | RQB | | |
| Type | blank | I | I | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PID | Part ID of reinforced elements. If PID = 0, the reinforcement is applied to all parts which use the Winfrith concrete model. |
| AXIS | Axis normal to layer:<br>EQ.1: A and B are parallel to global $y$ and $z$, respectively.<br>EQ.2: A and B are parallel to global $x$ and $z$, respectively.<br>EQ.3: A and B are parallel to global $x$ and $y$, respectively. |
| COOR | Coordinate location of layer:<br>AXIS.EQ.1: $x$-coordinate<br>AXIS.EQ.2: $y$-coordinate<br>AXIS.EQ.3: $z$-coordinate |
| RQA | Reinforcement quantity (A). |
| RQB | Reinforcement quantity (B). |

**Remarks:**

1. **Reinforcement Quantity.** Reinforcement quantity is the ratio of area of reinforcement in an element to the element's total cross-sectional area in a given direction. This definition is true for both Options 1 and 2. Where the options differ is in the manner in which it is decided which elements are reinforced. In Option 1, the reinforced element IDs are spelled out. In Option 2, elements of part ID PID which are cut by a plane (layer) defined by AXIS and COOR are reinforced.

## *MAT_ORTHOTROPIC_VISCOELASTIC

This is Material Type 86. It allows for the definition of an orthotropic material with a viscoelastic part. This model applies to shell elements and thick shell formulations 1, 2, and 6.

> **NOTE:** This material does not support specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | VF | K | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G0 | GINF | BETA | PRBA | PRCA | PRCB | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | MANGLE | | | |
| Type | F | F | F | F | F | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | Young's Modulus $E_a$ |
| EB | Young's Modulus $E_b$ |
| EC | Young's Modulus $E_c$ |
| VF | Volume fraction of viscoelastic material |
| K | Elastic bulk modulus |
| G0 | $G_0$, short-time shear modulus |
| GINF | $G_\infty$, long-time shear modulus |
| BETA | $\beta$, decay constant |
| PRBA | Poisson's ratio, $\nu_{ba}$ |
| PRCA | Poisson's ratio, $\nu_{ca}$ |
| PRCB | Poisson's ratio, $\nu_{cb}$ |
| GAB | Shear modulus, $G_{ab}$ |
| GBC | Shear modulus, $G_{bc}$ |
| GCA | Shear modulus, $G_{ca}$ |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDI- |

| VARIABLE | DESCRIPTION |
|---|---|
| | NATE_NODES, and then rotated about the shell element normal by an angle MANGLE<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_ECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, MANGLE, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| MANGLE | Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card; see *ELEMENT_SHELL_BETA. |
| A1 A2 A3 | Define components of vector **a** for AOPT = 2 |
| V1 V2 V3 | Define components of vector **v** for AOPT = 3 |
| D1 D2 D3 | Define components of vector **d** for AOPT = 2 |

**Remarks:**

See material types 2 and 24 for the orthotropic definition.

## *MAT_CELLULAR_RUBBER

This is Material Type 87. This material model provides a cellular rubber model with confined air pressure combined with linear viscoelasticity as outlined by Christensen [1980]. See Figure M87-1.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | PR | N | | | | |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if and only if N > 0.

| SGL | SW | ST | LCID | | | | |
|---|---|---|---|---|---|---|---|

**Card 2b.** This card is included if and only if N = 0.

| C10 | C01 | C11 | C20 | C02 | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| P0 | PHI | IVS | G | BETA | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PR | N | | | | |
| Type | A | F | F | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PR | Poisson's ratio; typical values are between 0.0 to 0.2. Due to the large compressibility of air, large values of Poisson's ratio generate physically meaningless results. |

| VARIABLE | DESCRIPTION |
|---|---|
| N | Order of fit for material model (currently < 3). If N > 0, then a least square fit is computed with uniaxial data. The parameters given on Card 2a should be specified. Also see *MAT_MOONEY_RIVLIN_RUBBER (material model 27). A Poisson's ratio of .5 is assumed for the void free rubber during the fit. The Poisson's ratio defined on Card 1 is for the cellular rubber. A void fraction formulation is used. |

**Material Least Squares Fit Card.** Card 2 if N > 0, a least squares fit is computed from uniaxial data

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LCID | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SGL | Specimen gauge length, $l_0$ |
| SW | Specimen width |
| ST | Specimen thickness |
| LCID | Load curve ID giving the force as a function of actual change in the gauge length, $\Delta L$. If SGL, SW, and ST are set to unity (1.0), then curve LCID is also engineering stress as a function of engineering strain. |

**Material Constants Card.** Card 2 if N = 0, define the following constants

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C10 | C01 | C11 | C20 | C02 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| C10 | Coefficient, $C_{10}$ |

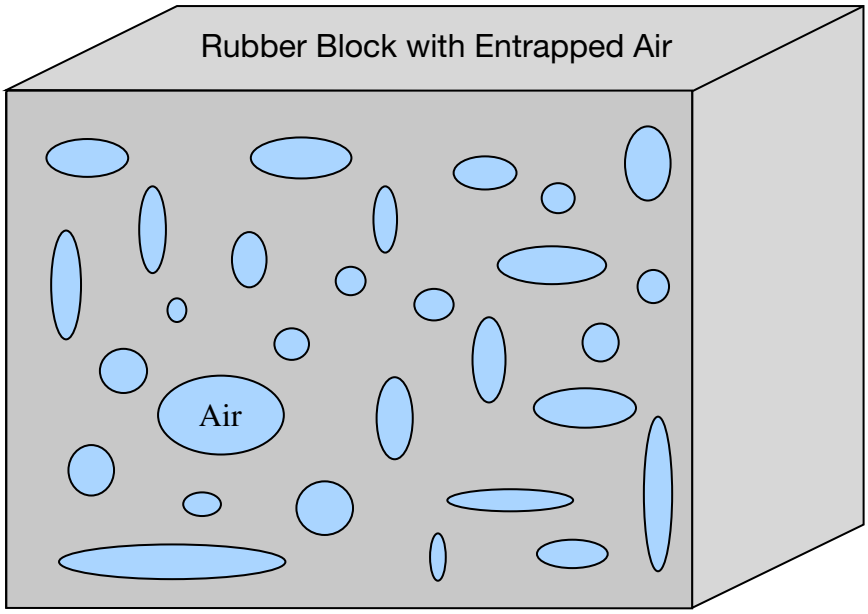



**Figure M87-1.** Cellular rubber with entrapped air. By setting the initial air pressure to zero, an open cell, cellular rubber can be simulated.

| VARIABLE | DESCRIPTION |
|---|---|
| C01 | Coefficient, $C_{01}$ |
| C11 | Coefficient, $C_{11}$ |
| C20 | Coefficient, $C_{20}$ |
| C02 | Coefficient, $C_{02}$ |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P0 | PHI | IVS | G | BETA | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P0 | Initial air pressure, $p_0$ |
| PHI | Ratio of cellular rubber to rubber density, $\phi$ |
| IVS | Initial volumetric strain, $\gamma_0$ |
| G | Optional shear relaxation modulus, $G$, for rate effects (viscosity) |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Optional decay constant, $\beta_1$ |

**Remarks:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material a hydrostatic work term, $W_H(J)$, is included in the strain energy functional which is function of the relative volume, $J$, [Ogden 1984]:

$$W(J_1, J_2, J) = \sum_{p,q=0}^{n} C_{pq}(J_1 - 3)^p (J_2 - 3)^q + W_H(J)$$

$$J_1 = I_1 I_3^{-1/3}$$

$$J_2 = I_2 I_3^{-2/3}$$

In order to prevent volumetric work from contributing to the hydrostatic work the first and second invariants are modified as shown. This procedure is described in more detail by Sussman and Bathe [1987].

The effects of confined air pressure in its overall response characteristics is included by augmenting the stress state within the element by the air pressure, that is,

$$\sigma_{ij} = \sigma_{ij}^{sk} - \delta_{ij}\sigma^{\text{air}} \ ,$$

where $\sigma_{ij}^{sk}$ is the bulk skeletal stress and $\sigma^{air}$ is the air pressure. $\sigma^{\text{air}}$ is computed from:

$$\sigma^{\text{air}} = -\frac{p_0\gamma}{1+\gamma-\phi} \ ,$$

where $p_0$ is the initial foam pressure usually taken as the atmospheric pressure and $\gamma$ defines the volumetric strain. The volumetric is found with

$$\gamma = V - 1 + \gamma_0 \ ,$$

where $V$ is the relative volume of the voids and $\gamma_0$ is the initial volumetric strain which is typically zero. The rubber skeletal material is assumed to be incompressible.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau \ ,$$

or in terms of the second Piola-Kirchhoff stress, $S_{ij}$, and Green's strain tensor, $E_{ij}$,

$$S_{ij} = \int_0^t G_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau} d\tau \ ,$$

where $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

Since we wish to include only simple rate effects, the relaxation function is represented by one term from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = E_d e^{-\beta_1 t}.$$

This model is effectively a Maxwell fluid which consists of a damper and spring in series. We characterize this in the input by a shear modulus, $G$, and decay constant, $\beta_1$.

The Mooney-Rivlin rubber model (model 27) is obtained by specifying N = 1 without air pressure and viscosity. In spite of the differences in formulations with Model 27, we find that the results obtained with this model are nearly identical with those of material type 27 as long as large values of Poisson's ratio are used.

### *MAT_MTS

This is Material Type 88. The MTS model is due to Mauldin, Davidson, and Henninger [1990] and is available for applications involving large strains, high pressures, and strain rates. As described in the foregoing reference, this model is based on dislocation mechanics and provides a better understanding of the plastic deformation process for ductile materials by using an internal state variable called the mechanical threshold stress. This kinematic quantity tracks the evolution of the material's microstructure along some arbitrary strain, strain rate, and temperature-dependent path using a differential form that balances dislocation generation and recovery processes. Given a value for the mechanical threshold stress, the flow stress is determined using either a thermal-activation-controlled or a drag-controlled kinetics relationship. An equation of state is required for solid elements (as well as SPH and thick shell formulations 3, 5, and 7), and a bulk modulus must be defined below for shell elements (as well as thick shell formulations 1, 3, and 6).

#### Card Summary:

**Card 1.** This card is required.

| MID | RO | SIGA | SIGI | SIGS | SIG0 | BULK | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| HF0 | HF1 | HF2 | SIGS0 | EDOTS0 | BURG | CAPA | BOLTZ |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| SM0 | SM1 | SM2 | EDOT0 | G0 | PINV | QINV | EDOTI |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| G0I | PINVI | QINVI | EDOTS | G0S | PINVS | QINVS | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| RHOCPR | TEMPRF | ALPHA | EPS0 | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | SIGA | SIGI | SIGS | SIG0 | BULK | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| SIGA | $\hat{\sigma}_a$, dislocation interactions with long-range barriers (force/area) |
| SIGI | $\hat{\sigma}_i$, dislocation interactions with interstitial atoms (force/area) |
| SIGS | $\hat{\sigma}_s$, dislocation interactions with solute atoms (force/area) |
| SIG0 | $\hat{\sigma}_0$, initial value of $\hat{\sigma}$ at zero plastic strain (force/area) NOT USED. |
| BULK | Bulk modulus defined for shell elements only. Do not input for solid elements. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HF0 | HF1 | HF2 | SIGS0 | EDOTS0 | BURG | CAPA | BOLTZ |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| HF0 | $a_0$, dislocation generation material constant (force/area) |
| HF1 | $a_1$, dislocation generation material constant (force/area) |
| HF2 | $a_2$, dislocation generation material constant (force/area) |
| SIGS0 | $\hat{\sigma}_{\varepsilon so}$, saturation threshold stress at 0$^o$ K (force/area) |
| EDOTS0 | $\dot{\varepsilon}_{\varepsilon so}$, reference strain-rate (time$^{-1}$). |

| VARIABLE | DESCRIPTION |
|---|---|
| BURG | Magnitude of Burgers vector (interatomic slip distance) |
| CAPA | Material constant, $A$ |
| BOLTZ | Boltzmann's constant, $k$ (energy/degree). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SM0 | SM1 | SM2 | EDOT0 | G0 | PINV | QINV | EDOTI |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SM0 | $G_0$, shear modulus at zero degrees Kelvin (force/area) |
| SM1 | $b_1$, shear modulus constant (force/area) |
| SM2 | $b_2$, shear modulus constant (degree) |
| EDOT0 | $\dot{\varepsilon}_o$, reference strain-rate (time$^{-1}$) |
| G0 | $g_0$, normalized activation energy for a dislocation/dislocation interaction |
| PINV | $1/p$, material constant |
| QINV | $1/q$, material constant |
| EDOTI | $\dot{\varepsilon}_{o,i}$, reference strain-rate (time$^{-1}$) |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G0I | PINVI | QINVI | EDOTS | G0S | PINVS | QINVS | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| G0I | $g_{0,i}$, normalized activation energy for a dislocation/interstitial interaction |

| VARIABLE | DESCRIPTION |
|---|---|
| PINVI | $1/p_i$, material constant |
| QINVI | $1/q_i$, material constant |
| EDOTS | $\dot{\varepsilon}_{o,s}$, reference strain-rate (time$^{-1}$) |
| G0S | $g_{0,s}$, normalized activation energy for a dislocation/solute interaction |
| PINVS | $1/p_s$, material constant |
| QINVS | $1/q_s$, material constant |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RHOCPR | TEMPRF | ALPHA | EPS0 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| RHOCPR | $\rho c_p$, product of density and specific heat |
| TEMPRF | $T_{\text{ref}}$, initial element temperature in degrees K |
| ALPHA | $\alpha$, material constant (typical value is between 0 and 2) |
| EPS0 | $\varepsilon_o$, factor to normalize strain rate in the calculation of $\Theta_o$ (time$^{-1}$) |

**Remarks:**

The flow stress $\sigma$ is given by:

$$\sigma = \hat{\sigma}_a + \frac{G}{G_0}\left[s_{\text{th}}\hat{\sigma} + s_{\text{th},i}\hat{\sigma}_i + s_{\text{th},s}\hat{\sigma}_s\right] \ .$$

The first product in the equation for $\sigma$ contains a micro-structure evolution variable, $\hat{\sigma}$. which is multiplied by a constant-structure deformation variable $s_{\text{th}}$: $s_{\text{th}}$. $\hat{\sigma}$ is the *Mechanical Threshold Stress* (MTS) and is a function of absolute temperature, $T$, and the plastic strain-rates, $\dot{\varepsilon}^{\text{P}}$. The evolution equation for $\hat{\sigma}$ is a differential hardening law representing dislocation-dislocation interactions:

$$\frac{\partial}{\partial \varepsilon^p} \equiv \Theta_o \left[ 1 - \frac{\tanh\left(\alpha \frac{\hat{\sigma}}{\hat{\sigma}_{\varepsilon s}}\right)}{\tanh(\alpha)} \right] .$$

The term $\frac{\partial \hat{\sigma}}{\partial \varepsilon^p}$ represents the hardening due to dislocation generation while the stress ratio, $\frac{\hat{\sigma}}{\hat{\sigma}_{\varepsilon s}}$, represents softening due to dislocation recovery. The threshold stress at zero strain-hardening, $\hat{\sigma}_{\varepsilon s}$, is called the saturation threshold stress. $\Theta_o$ is given as:

$$\Theta_o = a_o + a_1 \ln\left(\frac{\dot{\varepsilon}^p}{\varepsilon_0}\right) + a_2 \sqrt{\frac{\dot{\varepsilon}^p}{\varepsilon_0}}$$

which contains the material constants, $a_o$, $a_1$, and $a_2$. The constant, $\hat{\sigma}_{\varepsilon s}$, is given as:

$$\hat{\sigma}_{\varepsilon s} = \hat{\sigma}_{\varepsilon so} \left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_{\varepsilon so}}\right)^{kT/Gb^3 A}$$

which contains the input constants: $\hat{\sigma}_{\varepsilon so}$, $\dot{\varepsilon}_{\varepsilon so}$, $b$, $A$, and $k$. The shear modulus, $G$, appearing in these equations is assumed to be a function of temperature and is given by the correlation.

$$G = G_0 - b_1 / \left(e^{b_2/T} - 1\right)$$

which contains the constants: $G_0$, $b_1$, and $b_2$. For thermal-activation controlled deformation $s_{\text{th}}$ is evaluated using an Arrhenius rate equation of the form:

$$s_{\text{th}} = \left\{ 1 - \left[ \frac{kT \ln\left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}^p}\right)}{Gb^3 g_0} \right]^{\frac{1}{q}} \right\}^{\frac{1}{p}} .$$

The absolute temperature is given as:

$$T = T_{\text{ref}} + \frac{E}{\rho c_p} ,$$

where $E$ is the internal energy density per unit initial volume.

## *MAT_PLASTICITY_POLYMER

This is Material Type 89. An elasto-plastic material with an arbitrary stress as a function of strain curve and arbitrary strain rate dependency can be defined. It is intended for applications where the elastic and plastic sections of the response are not as clearly distinguishable as they are for metals. Rate dependency of failure strain is included. Many polymers show a more brittle response at high rates of strain. This material is supported for the commonly used solid, shell, and thick shell elements. 2D plane strain stress, plane strain, and axisymmetric elements are *not* supported.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | | | | |
| Type | F | F | I | I | | | | |
| Default | 0 | 0 | 0 | 0 | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFTX | DAMP | RFAC | LCFAIL | NUMINT | | | |
| Type | F | F | F | I | F | | | |
| Default | 0 | 0 | 0 | 0 | 0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| C | Strain rate parameter, $C$, (Cowper Symonds) |
| P | Strain rate parameter, $P$, (Cowper Symonds) |
| LCSS | Load curve ID or Table ID<br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining effective stress as a function of total effective strain.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective strain for that rate.<br>**Logarithmically Defined Tables.** If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress. If LCSR is negative, the load curve is evaluated using a binary search for the correct interval for the strain rate. The binary search is slower than the default incremental search, but in cases where large changes in the strain rate may occur over a single time step, it is more robust. |
| EFTX | Failure flag:<br>EQ.0.0: Failure determined by maximum tensile strain (default).<br>EQ.1.0: Failure determined only by tensile strain in local $x$ direction.<br>EQ.2.0: Failure determined only by tensile strain in local $y$ direction. |
| DAMP | Viscous damping factor in the units of [stress × time]. Typical values are $10^{-3}$ Ns/mm$^2$ or $10^{-4}$ Ns/mm$^2$. If set too high, instabilities can result. |

| VARIABLE | DESCRIPTION |
|---|---|
| RFAC | Filtering factor for strain rate effects. Must be between 0 (no filtering) and 1 (infinite filtering). The filter is a simple low pass filter to remove high frequency oscillation from the strain rates before they are used in rate effect calculations. The cut off frequency of the filter is [(1 - RFAC) / timestep] rad/sec. |
| LCFAIL | Load curve ID giving variation of failure strain with strain rate. The points on the $x$-axis should be natural log of strain rate, while the $y$-axis should be the true strain to failure. Typically, this is measured by a uniaxial tensile test, and the strain values are converted to true strain. |
| NUMINT | Number of integration points which must fail before the element is deleted. This option is available for shells only.<br>LT.0.0: \|NUMINT\| is percentage of integration points/layers which must fail before shell element fails. |

**Remarks:**

1. ***MAT_089 compared to *MAT_024.** *MAT_089 is the same as *MAT_024 except for the following points:
   - Load curve lookup for yield stress is based on equivalent uniaxial strain, not plastic strain (see Remarks 2 and 3).
   - Elastic stiffness is initially equal to $E$ but will be increased according to the slope of the stress-strain curve (see Remark 7).
   - Special strain calculation is used for failure and damage (see Remark 2).
   - Failure strain depends on strain rate (see Remark 4).

2. **Strain calculation for failure and damage.** The strain used for failure and damage calculation, $\varepsilon_{\text{pm}}$, is based on an approximation of the greatest value of maximum principal strain encountered during the analysis:

$$\varepsilon_{\text{pm}} = \max_{\text{i}\le\text{n}}(\varepsilon_H^i + \varepsilon_{\text{vm}}^i) \ ,$$

where

$$\begin{aligned} n &= \text{current time step index} \\ \max_{i\le n}(\dots) &= \text{maximum value attained by the argument during the calculation} \\ \varepsilon_H &= \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3} \\ \varepsilon_x, \varepsilon_y, \varepsilon_z &= \text{cumulative strain in the local x, y, or z direction} \end{aligned}$$

$\varepsilon_{\text{vm}} = \sqrt{\frac{2}{3}\text{tr}(\boldsymbol{\varepsilon}'^{\text{T}}\boldsymbol{\varepsilon}')}$, the usual definition of equivalent uniaxial strain
$\boldsymbol{\varepsilon}'$ = deviatoric strain tensor, where each $\varepsilon_x$, $\varepsilon_y$, and $\varepsilon_z$ is cumulative

3. **Yield stress load curves.** When looking up yield stress from the load curve LCSS, the $x$-axis value is $\varepsilon_{\text{vm}}$.

4. **Failure strain load curves.**

$\varepsilon_{\text{sr}} = \frac{\text{d}\varepsilon_{\text{pm}}}{\text{d}t}$ = strain rate for failure and damage calculation
$\varepsilon_F$ = LCFAIL($\varepsilon_{sr}$)
= Instantanous true strain to failure from look-up on the curve LCFAIL

5. **Damage.** A damage approach is used to avoid sudden shocks when the failure strain is reached. Damage begins when the "strain ratio," $R$, reaches 1.0, where

$$R = \int \frac{d\varepsilon_{\text{pm}}}{\varepsilon_F}.$$

Damage is complete, and the element fails and is deleted, when $R = 1.1$. The damage,

$$D = \begin{cases} 1.0 & R < 1.0 \\ 10(1.1 - R) & 1.0 < R < 1.1 \end{cases},$$

is a reduction factor applied to all stresses. For example, when $R = 1.05$, then $D = 0.5$.

6. **Strain definitions.** Unlike other LS-DYNA material models, both the input stress-strain curve and the strain to failure are defined as total true strain, not plastic strain. The input can be defined from uniaxial tensile tests; nominal stress and nominal strain from the tests must be converted to true stress and true strain. The elastic component of strain must not be subtracted out.

7. **Elastic stiffness scaling.** The stress-strain curve is permitted to have sections steeper (i.e. stiffer) than the elastic modulus. When these are encountered the elastic modulus is increased to prevent spurious energy generation. The elastic stiffness is scaled by a factor $f_{\text{e}}$, which is calculated as follows:

$$f_e = \max\left(1.0, \frac{S_{\text{max}}}{3G}\right)$$

where

$G$ = initial shear modulus
$S_{\text{max}}$ = maximum slope of stress-strain curve encountered during the analysis

8. **Precision.** Double precision is recommended when using this material model, especially if the strains become high.

9. **Shell numbering.** Invariant shell numbering is recommended when using this material model. See *CONTROL_ACCURACY.

## *MAT_ACOUSTIC

This is Material Type 90. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This material is intended for general acoustic applications in either the time domain or frequency domain. See Appendix W for a description of applications. Depending on the application, it can be used with the implicit or explicit solvers.

This model is appropriate for tracking low pressure stress waves in an acoustic media, such as air or water, and can be used only with the acoustic pressure element formulation. The acoustic pressure element requires only one unknown per node. This element is very cost effective. Optionally, cavitation can be allowed.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | C | BETA | CF | ATMOS | GRAV | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | XN | YN | ZN | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| C | Sound speed |
| BETA | Damping factor. Recommended values are between 0.1 and 1.0. |
| CF | Cavitation flag:<br>EQ.0.0: Off<br>EQ.1.0: On |
| ATMOS | Atmospheric pressure (optional) |

| VARIABLE | DESCRIPTION |
|---|---|
| GRAV | Gravitational acceleration constant (optional) |
| XP | *x*-coordinate of free surface point |
| YP | *y*-coordinate of free surface point |
| ZP | *z*-coordinate of free surface point |
| XN | *x*-direction cosine of free surface normal vector |
| YN | *y*-direction cosine of free surface normal vector |
| ZN | *z*-direction cosine of free surface normal vector |

## *MAT_ACOUSTIC_COMPLEX

This is Material Type 90_COMPLEX. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This material only works with acoustic elements It is intended for direct, steady state vibration simulations with real and imaginary material properties. The model should be used with *CONTROL_IMPLICIT_SSD_DIRECT and thus only works with the implicit solver. See Appendix W for a description of this application.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHOR | BULKR | RHOI | BULKI | | | |
| Type | A | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDRR | LCIDKR | LCIDRI | LCIDKI | | | | |
| Type | I | I | I | I | | | | |
| Default | 0 | 0 | 0 | 0 | | | | |
| Remarks | 2 | 2 | 2 | 2 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RHOR | Real part of the density, $\rho_r$ |
| BULKR | Real part of the bulk modulus, $K_r$ |
| RHOI | Imaginary part of the density, $\rho_i$ |
| BULKI | Imaginary part of the bulk modulus, $K_i$ |
| LCIDRR | Load curve ID for specifying frequency variation of $\rho_r$. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDKR | Load curve ID for specifying frequency variation of $K_r$. |
| LCIDRI | Load curve ID for specifying frequency variation of $\rho_i$. |
| LCIDKI | Load curve ID for specifying frequency variation of $K_i$. |

**Remarks:**

1. **Mass and Stiffness.** The contributions of elements using this material model are

$$\left[\overline{M}_f\right] = \frac{-K_r}{(K_r^2 + K_i^2)} \int_V N_f^T N_f \, dV + \frac{iK_i}{(K_r^2 + K_i^2)} \int_V N_f^T N_f \, dV$$

$$\left[\overline{K}_f\right] = \frac{-\rho_r}{(\rho_r^2 + \rho_i^2)} \int_V \nabla N_f^T \nabla N_f \, dV + \frac{i\rho_i}{(\rho_r^2 + \rho_i^2)} \int_V \nabla N_f^T \nabla N_f \, dV$$

2. **Frequency Dependence.** If the load curve specifying the frequency variation is undefined, then the property is constant with frequency.

## *MAT_ACOUSTIC_DAMP

This is Material Type 90_DAMP. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This model can only be used with acoustic elements. This material works for explicit transient and direct, steady state vibration applications. See Appendix W for a description of applications. Depending on the application, it can be used with the implicit or explicit solvers.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | CEE | BETA | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | 0.0 | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | VDC | BETA2 |
| Type | | | | | | | F | F |
| Default | | | | | | | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| CEE | Sound speed, $c$ |
| BETA | Linear bulk viscosity coefficient, $\beta$ |
| VDC | Volumetric drag coefficient, $r$ |
| BETA2 | Quadratic bulk viscosity coefficient, $\beta_2$ |

**Remarks:**

1. **Usage in Direct Steady State Vibration.** The bulk viscosity parameters, BETA and BETA2, are ignored in steady, state vibration simulations invoked with *CONTROL_IMPLICIT_SSD_DIRECT. The volumetric drag coefficient, $r$, contributes to the fluid damping matrix:

$$[W_f] = \frac{-r}{\rho^2 c^2} \int_V N_f^T N_f \, dV \quad .$$

   $r$ has dimensions of force / volume / velocity.

2. **Usage in Explicit Transient Analysis.** For spectral analyses (see *CONTROL_ACOUSTIC_SPECTRAL), the bulk viscosity parameters, BETA and BETA2, contribute an artificial pressure:

$$\Delta p = \beta \Delta t \dot{p} + \beta_2 \frac{\Delta t^2}{\rho c^2} \dot{p} \max(\dot{p}, 0) \quad .$$

   Nonzero values of BETA and BETA2 will adversely affect the time step.

## *MAT_ACOUSTIC_POROUS_DB

This is Material Type 90_POROUS_DB. It specifies the material properties of a linear acoustic fluid. This fluid is assumed to be compressible, irrotational, inviscid, and subject to small displacements. This material works with acoustic elements. It is intended for direct, steady state forced vibration of porous materials having a rigid frame, such as glass wool. It should be used with *CONTROL_IMPLICIT_SSD_DIRECT and thus can only be used with the implicit solver. See Appendix W for a description of applications.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHO0 | CEE0 | SIGMA | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RHO0 | Mass density in air, $\rho_0$ |
| CEE0 | Sound speed in air, $c_0$ |
| SIGMA | Flow resistivity, $\sigma$ |
| C*i* | Constants of the material model. See Remark 2. |

**Remarks:**

1. **Characteristic Impedance and Propagation Constant.** The characteristic impedance is

$$Z = \rho_o c_o \left(1 + c_1 X^{c_2} - i c_3 X^{c_4}\right) ,$$

and the propagation constant is

$$\Gamma = \frac{2\pi f}{c_o}\left(c_5 X^{c_6} + i(1 + c_7 X^{c_8})\right) ,$$

where

$$f = \frac{\omega}{2\pi} , \qquad X = \frac{\rho_o f}{\sigma} .$$

2. **Delany-Bazley, Miki and Allard-Champoux Models.** C1 to C8 of various regression models for the impedance and propagation constant, including those of Delany-Bazley, Miki, and Allard-Champoux, are listed in the journal Applied Acoustics, Sound absorption of porous materials – Accuracy of prediction methods, Oliva and Hongisto, 74 (2013) 1473-1479.

## *MAT_SOFT_TISSUE_{OPTION}

Available options include:

<BLANK>

VISCO

This is Material Type 91 (*OPTION* = <BLANK>) or Material Type 92 (*OPTION* = VISCO). This material is a transversely isotropic hyperelastic model for representing biological soft tissues, such as ligaments, tendons, and fascia. The representation provides an isotropic Mooney-Rivlin matrix reinforced by fibers having a strain energy contribution with the qualitative material behavior of collagen. The model has a viscoelasticity option, which activates a six-term Prony series kernel for the relaxation function. In this case, the hyperelastic strain energy represents the elastic (long-time) response. See Weiss et al. [1996] and Puso and Weiss [1998] for additional details.

> **NOTE:** This material does not support the specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | C1 | C2 | C3 | C4 | C5 | REF |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| XK | XLAM | FANG | XLAM0 | FAILSF | FAILSM | FAILSHR | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | AX | AY | AZ | BX | BY | BZ | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required. This input does not apply to shells, so it may be included as a blank line.

| LA1 | LA2 | LA3 | MACF | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card for the VISCO keyword option.

| S1 | S2 | S3 | S4 | S5 | S6 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card for the VISCO keyword option.

| T1 | T2 | T3 | T4 | T5 | T6 | | |
|---|---|---|---|---|---|---|---|

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | C1 | C2 | C3 | C4 | C5 | REF |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| C1 - C5 | Hyperelastic coefficients (see the equations in the Material Formulation section below) |
| REF | Use reference geometry to initialize the stress tensor. *INITIAL_FOAM_REFERENCE_GEOMETRY defines the reference geometry.<br>EQ.0.0: Off<br>EQ.1.0: On |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XK | XLAM | FANG | XLAM0 | FAILSF | FAILSM | FAILSHR | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XK | Bulk modulus |
| XLAM | Stretch ratio at which fibers are straightened |
| FANG | Angle in degrees of a material rotation about the *c*-axis, available for AOPT = 0 (shells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELE- |

| VARIABLE | DESCRIPTION |
|---|---|
| | MENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. See Remark 1. |
| XLAM0 | Initial fiber stretch (optional). See Remark 2. |
| FAILSF | Stretch ratio for ligament fibers at failure (applies to shell elements only). If zero, failure is not considered. |
| FAILSM | Stretch ratio for surrounding matrix material at failure (applies to shell elements only). If zero, failure is not considered. |
| FAILSHR | Shear strain at failure at a material point (applies to shell elements only). If zero, failure is not considered. This failure value is independent of FAILSF and FAILSM. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | AX | AY | AZ | BX | BY | BZ | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details). The fiber direction depends on this coordinate system (see Remark 1).<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle FANG on this keyword or BETA on the *ELEMENT_SHELL_*{OPTION}* input.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a |

| VARIABLE | DESCRIPTION |
|---|---|
| | vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedra. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or with FANG on this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying the angle rotation, depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| AX, AY, AZ | Vector components that depend on the value of AOPT:<br>AOPT.LT.0.0: Ignored<br>AOPT.EQ.1.0: Components of point *p* (XP, YP, ZP)<br>AOPT.EQ.2.0: Components of vector **a** (A1, A2, A3)<br>AOPT.GT.2.0: Components of vector **v** (V1, V2, V3) |
| BX, BY, BZ | Vector components that depend on the value of AOPT:<br>AOPT.LE.1.0: Ignored<br>AOPT.EQ.2.0: Components of vector **d** (D1, D2, D3)<br>AOPT.EQ.3.0: Ignored<br>AOPT.EQ.4.0: Components of point *p* (XP, YP, ZP) |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LA1 | LA2 | LA3 | MACF | | | | |
| Type | F | F | F | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LAX, LAY, LAZ | Local fiber orientation vector (solids only) |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA or FANG rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA or FANG rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA or FANG rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA or FANG rotation<br>EQ.3: Switch material axes *a* and *c* after BETA or FANG rotation<br>EQ.4: Switch material axes *b* and *c* after BETA or FANG rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If defined, the BETA on *ELEMENT_SOLID_{*OPTION*} is used for the rotation for all AOPT options. If BETA is not used for the element, then a rotation only occurs for AOPT = 3, where FANG is applied. |

**Prony Series Card 1.** Additional card for the VISCO keyword option.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S1 | S2 | S3 | S4 | S5 | S6 | | |
| Type | F | F | F | F | F | F | | |

**Prony Series Card 2.** Additional card for the VISCO keyword option.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| S1 – S6 | Factors in the Prony series (see Material Formulation and Remark 3) |
| T1 - T6 | Characteristic times for Prony series relaxation kernel (see Material Formulation and Remark 3) |

**Material Formulation:**

The overall strain energy, $W$, is "uncoupled" and includes two isotropic deviatoric matrix terms, a fiber term, $F$, and a bulk term:

$$W = C_1(\tilde{I}_1 - 3) + C_2(\tilde{I}_2 - 3) + F(\lambda) + \frac{1}{2}K[\ln(J)]^2$$

Here, $\tilde{I}_1$ and $\tilde{I}_2$ are the deviatoric invariants of the right Cauchy deformation tensor, $\lambda$ is the deviatoric part of the stretch along the current fiber direction, and $J = \det\mathbf{F}$ is the volume ratio. The material coefficients $C_1$ and $C_2$ are the Mooney-Rivlin coefficients, while $K$ is the effective bulk modulus of the material (input parameter XK).

The derivatives of the fiber term $F$ are defined to capture the behavior of crimped collagen. The fibers are assumed to be unable to resist compressive loading; thus, the model is isotropic when $\lambda < 1$. An exponential function describes the straightening of the fibers, while a linear function describes the behavior of the fibers once they are straightened past a critical fiber stretch level $\lambda \geq \lambda^*$ (input parameter XLAM):

$$\frac{\partial F}{\partial \lambda} = \begin{cases} 0 & \lambda < 1 \\ \dfrac{C_3}{\lambda}[\exp(C_4(\lambda - 1)) - 1] & \lambda < \lambda^* \\ \dfrac{1}{\lambda}(C_5\lambda + C_6) & \lambda \geq \lambda^* \end{cases}$$

Coefficients $C_3$, $C_4$, and $C_5$ must be defined by the user. $C_6$ is determined by LS-DYNA to ensure stress continuity at $\lambda = \lambda^*$. Sample values for the material coefficients $C_1 - C_5$ and $\lambda^*$ for ligament tissue can be found in Quapp and Weiss [1998]. The bulk modulus

$K$ should be at least three orders of magnitude larger than $C_1$ to ensure near-incompressible material behavior.

Viscoelasticity is included through a convolution integral representation for the time-dependent second Piola-Kirchhoff stress $\mathbf{S}(\mathbf{C},t)$:

$$\mathbf{S}(\mathbf{C},t) = \mathbf{S}^e(\mathbf{C}) + \int_0^t 2G(t-s)\frac{\partial W}{\partial \mathbf{C}(s)}ds$$

Here, $\mathbf{S}^e$ is the elastic part of the second Piola-Kirchhoff stress as derived from the strain energy, and $G(t-s)$ is the reduced relaxation function, represented by a Prony series:

$$G(t) = \sum_{i=1}^{6} S_i \exp\left(\frac{t}{T_i}\right) .$$

Puso and Weiss [1998] describe a graphical method to fit the Prony series coefficients to relaxation data that approximates the behavior of the continuous relaxation function proposed by Y-C. Fung, as quasilinear viscoelasticity.

**Remarks:**

1. **Fiber direction.** For shell elements, the fiber direction lies in the plane of the element. The fiber direction is along the $a$-axis material direction. This direction depends on the value of AOPT.

   For solid elements, the local coordinate system depends on the value of AOPT. The fiber direction is oriented in the local system using input parameters LAX, LAY, and LAZ. By default, (LAX,LAY,LAZ) = (1,0,0), and the fiber is aligned with the local $a$-direction.

2. **Initial fiber stretch.** An optional initial fiber stretch can be specified using XLAM0. The initial stretch is applied during the first time step. This creates preload in the model as soft tissue contacts, and equilibrium is established. For example, a ligament tissue "uncrimping strain" of 3% can be represented with an initial stretch value of 1.03.

3. **Prony series input.** If the VISCO keyword option is included, at least one Prony series term (S1, T1) must be defined.

## *MAT_ELASTIC_6DOF_SPRING_DISCRETE_BEAM

This is Material Type 93. This material model is defined for simulating the effects of nonlinear elastic and nonlinear viscous beams by using six springs each acting about one of the six local degrees-of-freedom. The input consists of part IDs that reference material type, *MAT_ELASTIC_SPRING_DISCRETE_BEAM (type 74 above). Generally, these referenced parts are used only for the definition of this material model and are not referenced by any elements. The two nodes defining a beam may be coincident to give a zero length beam or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TPIDR | TPIDS | TPIDT | RPIDR | RPIDS | RPIDT |
| Type | A | F | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| TPIDR | Translational motion in the local *r*-direction is governed by part ID TPIDR. If zero, no force is computed in this direction. |
| TPIDS | Translational motion in the local *s*-direction is governed by part ID TPIDS. If zero, no force is computed in this direction. |
| TPIDT | Translational motion in the local *t*-direction is governed by part ID TPIDT. If zero, no force is computed in this direction. |
| RPIDR | Rotational motion about the local *r*-axis is governed by part ID RPIDR. If zero, no moment is computed about this axis. |
| RPIDS | Rotational motion about the local *s*-axis is governed by part ID RPIDS. If zero, no moment is computed about this axis. |

| VARIABLE | DESCRIPTION |
|---|---|
| RPIDT | Rotational motion about the local *t*-axis is governed by part ID RPIDT. If zero, no moment is computed about this axis. |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_INELASTIC_SPRING_DISCRETE_BEAM

This is Material Type 94. This model permits elastoplastic springs with damping to be represented with a discrete beam element type 6. A yield force as a function deflection curve is used which can vary in tension and compression.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | F0 | D | CDF | TDF | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FLCID | HLCID | C1 | C2 | DLE | GLCID | | |
| Type | F | F | F | F | F | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| K | Elastic loading/unloading stiffness. This is required input. |
| F0 | Optional initial force. This option is inactive if this material is referenced by a part referenced by material type *MAT_INELASTIC_6DOF_SPRING. |
| D | Optional viscous damping coefficient. |
| CDF | Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs.<br>EQ.0.0: inactive. |
| TDF | Tensile displacement at failure. After failure, no forces are carried.<br>EQ.0.0: inactive. |

| VARIABLE | DESCRIPTION |
|---|---|
| FLCID | Load curve ID (see *DEFINE_CURVE) defining the yield force as a function of plastic deflection. If the origin of the curve is at (0,0), the force magnitude is identical in tension and compression, that is, only the sign changes. If not, the yield stress in the compression is used when the spring force is negative. The plastic displacement increases monotonically in this implementation. The load curve is required input. |
| HLCID | Load curve ID (see *DEFINE_CURVE) defining force as a function of relative velocity (optional). If the origin of the curve is at (0,0), the force magnitude is identical for a given magnitude of the relative velocity, that is, only the sign changes. |
| C1 | Damping coefficient |
| C2 | Damping coefficient |
| DLE | Factor to scale time units |
| GLCID | Optional load curve ID (see *DEFINE_CURVE) defining a scale factor as a function of deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed. |

**Remarks:**

1. **Force.** To determine the force, a trial force is first computed as:

$$F^T = F^n + \mathrm{K} \times \Delta\dot{L}(\Delta t)$$

The yield force is taken from the load curve:

$$F^Y = F_y\left(\Delta L^{\text{plastic}}\right) \ ,$$

where $L^{\text{plastic}}$ is the plastic deflection, given by

$$\Delta L^{\text{plastic}} = \frac{F^T - F^Y}{S + K^{\max}} \ .$$

Here $S$ is the slope of FLCID and $K^{\max}$ is the maximum elastic stiffness:

$$K^{\max} = \max(K, 2 \times S^{\max}) \ .$$

The trial force is, then, checked against the yield force to determine $F$:

$$F = \begin{cases} F^Y & \text{if } F^T > F^Y \\ F^T & \text{if } F^T \le F^Y \end{cases}$$

The final force, which includes rate effects and damping, is given by:

$$F^{n+1} = F \times \left[ 1 + \text{C1} \times \Delta\dot{L} + \text{C2} \times \text{sgn}(\Delta\dot{L}) \ln \left( \max \left\{ 1., \frac{|\Delta\dot{L}|}{\text{DLE}} \right\} \right) \right] + \text{D} \times \Delta\dot{L} + g(\Delta L) h(\Delta\dot{L}) \ .$$

2. **Yield Force Curve.** Unless the origin of the curve starts at $(0,0)$, the negative part of the curve is used when the spring force is negative where the negative of the plastic displacement is used to interpolate, $F_y$. The positive part of the curve is used whenever the force is positive. In these equations, $\Delta L$ is the change in length

$$\Delta L = \text{current length - initial length} \ .$$

3. **Cross-Sectional Area.** The cross-sectional area is defined on the section card for the discrete beam elements, See *SECTION_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

## *MAT_INELASTIC_6DOF_SPRING_DISCRETE_BEAM

This is Material Type 95. This material model is defined for simulating the effects of nonlinear inelastic and nonlinear viscous beams by using six springs each acting about one of the six local degrees-of-freedom. The input consists of part IDs that reference material type *MAT_INELASTIC_SPRING_DISCRETE_BEAM above (type 94). Generally, these referenced parts are used only for the definition of this material model and are not referenced by any elements. The two nodes defining a beam may be coincident to give a zero length beam, or offset to give a finite length beam. For finite length discrete beams, the absolute value of the variable SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam, to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad must be used to orient the beam for zero length beams.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TPIDR | TPIDS | TPIDT | RPIDR | RPIDS | RPIDT |
| Type | A | F | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, see also volume in *SECTION_BEAM definition. |
| TPIDR | Translational motion in the local *r*-direction is governed by part ID TPIDR. If zero, no force is computed in this direction. |
| TPIDS | Translational motion in the local *s*-direction is governed by part ID TPIDS. If zero, no force is computed in this direction. |
| TPIDT | Translational motion in the local *t*-direction is governed by part ID TPIDT. If zero, no force is computed in this direction. |
| RPIDR | Rotational motion about the local *r*-axis is governed by part ID RPIDR. If zero, no moment is computed about this axis. |
| RPIDS | Rotational motion about the local *s*-axis is governed by part ID RPIDS. If zero, no moment is computed about this axis. |

| VARIABLE | DESCRIPTION |
|---|---|
| RPIDT | Rotational motion about the local *t*-axis is governed by part ID RPIDT. If zero, no moment is computed about this axis. |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_BRITTLE_DAMAGE

This is Material Type 96. It is an anisotropic brittle damage model designed primarily for concrete though it can be applied to a wide variety of brittle materials.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | TLIMIT | SLIMIT | FTOUGH | SRETEN |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VISC | FRA_RF | E_RF | YS_RF | EH_RF | FS_RF | SIGY | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| TLIMIT | Tensile limit, $f_n$ |
| SLIMIT | Shear limit, $f_s$ |
| FTOUGH | Fracture toughness, $g_c$ |
| SRETEN | Shear retention, $\beta$ |
| VISC | Viscosity, $\eta$ |
| FRA_RF | Fraction of reinforcement in section |
| E_RF | Young's modulus of reinforcement |
| YS_RF | Yield stress of reinforcement |

| VARIABLE | DESCRIPTION |
|---|---|
| EH_RF | Hardening modulus of reinforcement |
| FS_RF | Failure strain (true) of reinforcement |
| SIGY | Compressive yield stress, $\sigma_y$<br>EQ.0: No compressive yield |

**Remarks:**

A full description of the tensile and shear damage parts of this material model is given in Govindjee, Kay and Simo [1994,1995]. This model admits progressive degradation of tensile and shear strengths across smeared cracks that are initiated under tensile loadings. Compressive failure is governed by a simplistic J2 flow correction that can be disabled if not desired. Damage is handled by treating the rank 4 elastic stiffness tensor as an evolving internal variable for the material. Softening induced mesh dependencies are handled by a characteristic length method [Oliver 1989].

Description of properties:

1. $E$ is the Young's modulus of the undamaged material also known as the virgin modulus.

2. $\nu$ is the Poisson's ratio of the undamaged material also known as the virgin Poisson's ratio.

3. $f_n$ is the initial principal tensile strength (stress) of the material. Once this stress has been reached at a point in the body a smeared crack is initiated there with a normal that is co-linear with the 1st principal direction. Once initiated, the crack is fixed at that location, though it will convect with the motion of the body. As the loading progresses the allowed tensile traction normal to the crack plane is progressively degraded to a small machine dependent constant.

   The degradation is implemented by reducing the material's modulus normal to the smeared crack plane according to a maximum dissipation law that incorporates exponential softening. The restriction on the normal tractions is given by

   $$\phi_t = (\mathbf{n} \otimes \mathbf{n}) : \sigma - f_n + (1 - \varepsilon) f_n (1 - \exp[-H\alpha]) \leq 0$$

   where $\mathbf{n}$ is the smeared crack normal, $\varepsilon$ is the small constant, $H$ is the softening modulus, and $\alpha$ is an internal variable. $H$ is set automatically by the program; see $g_c$ below. $\alpha$ measures the crack field intensity and is output in the equivalent plastic strain field, $\bar{\varepsilon}^p$, in a normalized fashion.

The evolution of $\alpha$ is governed by a maximum dissipation argument. When the normalized value reaches unity, the material's strength has been reduced to 2% of its original value in the normal and parallel directions to the smeared crack. Note that for plotting purposes it is never output greater than 5.

4. $f_s$ is the initial shear traction that may be transmitted across a smeared crack plane. The shear traction is limited to be less than or equal to $f_s(1-\beta)(1-\exp[-H\alpha])$ through the use of two orthogonal shear damage surfaces. Note that the shear degradation is coupled to the tensile degradation through the internal variable $\alpha$ which measures the intensity of the crack field. $\beta$ is the shear retention factor defined below. The shear degradation is taken care of by reducing the material's shear stiffness parallel to the smeared crack plane.

5. $g_c$ is the fracture toughness of the material. It should be entered as fracture energy per unit area crack advance. Once entered the softening modulus is automatically calculated based on element and crack geometries.

6. $\beta$ is the shear retention factor. As the damage progresses the shear tractions allowed across the smeared crack plane asymptote to the product $\beta f_s$.

7. $\eta$ represents the viscosity of the material. Viscous behavior is implemented as a simple Perzyna regularization method which allows for the inclusion of first order rate effects. The use of some viscosity is recommend as it serves as regularizing parameter that increases the stability of calculations.

8. $\sigma_y$ is a uniaxial compressive yield stress. A check on compressive stresses is made using the J2 yield function

$$\mathbf{s}:\mathbf{s} - \sqrt{\frac{2}{3}}\sigma_y \le 0 \ ,$$

where **s** is the stress deviator. If violated, a J2 return mapping correction is executed. This check is executed (1) when no damage has taken place at an integration point yet; (2) when damage has taken place at a point, but the crack is currently closed; and (3) during active damage after the damage integration (i.e. as an operator split). Note that if the crack is open the plasticity correction is done in the plane-stress subspace of the crack plane.

A variety of experimental data has been replicated using this model from quasi-static to explosive situations. Reasonable properties for a standard grade concrete would be:

| Property | Value |
|---|---|
| $E$ | $3.15 \times 10^6$ psi |

| Property | Value |
|---|---|
| $f_n$ | 450 psi |
| $f_s$ | 2100 psi |
| $\nu$ | 0.2 |
| $g_c$ | 0.8 lbs/in |
| $\beta$ | 0.03 |
| $\eta$ | 0.0 psi-sec |
| $\sigma_y$ | 4200 psi |

For stability, values of $\eta$ between 104 to 106 psi/sec are recommended. Our limited experience thus far has shown that many problems require nonzero values of $\eta$ to run to avoid error terminations.

Various other internal variables such as crack orientations and degraded stiffness tensors are internally calculated but currently not available for output.

## *MAT_GENERAL_JOINT_DISCRETE_BEAM

This is Material Type 97. This model is used to define a general joint constraining any combination of degrees of freedom between two nodes. The nodes may belong to rigid or deformable bodies. In most applications the end nodes of the beam are coincident and the local coordinate system (*r*, *s*, *t* axes) is defined by CID (see *SECTION_BEAM).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TR | TS | TT | RR | RS | RT |
| Type | A | F | I | I | I | I | I | I |
| Remarks | 1 | | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RPST | RPSR | | | | | | |
| Type | F | F | | | | | | |
| Remarks | 2 | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| TR | Translational constraint code along the *r*-axis:<br>EQ.0: Free<br>EQ.1: Constrained |
| TS | Translational constraint code along the *s*-axis:<br>EQ.0: Free<br>EQ.1: Constrained |

| VARIABLE | DESCRIPTION |
|---|---|
| TT | Translational constraint code along the *t*-axis:<br>EQ.0: Free<br>EQ.1: Constrained |
| RR | Rotational constraint code about the *r*-axis:<br>EQ.0: Free<br>EQ.1: Constrained |
| RS | Rotational constraint code about the *s*-axis:<br>EQ.0: Free<br>EQ.1: Constrained |
| RT | Rotational constraint code about the *t*-axis:<br>EQ.0: Free<br>EQ.1: Constrained |
| RPST | Penalty stiffness scale factor for translational constraints |
| RPSR | Penalty stiffness scale factor for rotational constraints |

**Remarks:**

1. **Inertia and Stability.** For explicit calculations, the additional stiffness due to this joint may require addition mass and inertia for stability. Mass and rotary inertia for this beam element is based on the defined mass density, the volume, and the mass moment of inertia defined in the *SECTION_BEAM input.

2. **Penalty Stiffness.** The penalty stiffness applies to explicit calculations. For implicit calculations, constraint equations are generated and imposed on the system equations; therefore, these constants, RPST and RPSR, are not used.

## *MAT_SIMPLIFIED_JOHNSON_COOK_{OPTION}

Available options include:

<BLANK>

STOCHASTIC

This is Material Type 98 implementing Johnson/Cook strain sensitive plasticity. It is used for problems where the strain rates vary over a large range. In contrast to the full Johnson/Cook model (material type 15) this model introduces the following simplifications:

1. thermal effects and damage are ignored,

2. and the maximum stress is directly limited since thermal softening which is very significant in reducing the yield stress under adiabatic loading is not available.

An iterative plane stress update is used for the shell elements, but due to the simplifications related to thermal softening and damage, this model is 50% faster than the full Johnson/Cook implementation. To compensate for the lack of thermal softening, limiting stress values are introduced to keep the stresses within reasonable limits.

A resultant formulation for the Belytschko-Tsay, the C0 Triangle, and the fully integrated type 16 shell elements is available and can be activated by specifying either zero or one through thickness integration point on the *SECTION_SHELL card. While less accurate than through thickness integration, this formulation runs somewhat faster. Since the stresses are not computed in the resultant formulation, the stresses written to the databases for the resultant elements are set to zero.

This model is also available for the Hughes-Liu beam, the Belytschko-Schwer beam, and for the truss element. For the resultant beam formulation, the rate effects are approximated by the axial rate, since the thickness of the beam about it bending axes is unknown. Because this model is primarily used for structural analysis, the pressure is determined using the linear bulk modulus.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | VP | | | |
| Type | A | F | F | F | F | | | |
| Default | none | none | none | none | 0.0 | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | N | C | PSFAIL | SIGMAX | SIGSAT | EPSO |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.0 | 0.0 | $10^{17}$ | SIGSAT | $10^{28}$ | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| VP | Formulation for rate effects:<br>EQ.0.0: scale yield stress (default)<br>EQ.1.0: viscoplastic formulation<br>This option applies only to the 4-node shell and 8-node thick shell if and only if through thickness integration is used. |
| A | See Remark 1. |
| B | See Remark 1. |
| N | See Remark 1. |
| C | See Remark 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| PSFAIL | Effective plastic strain at failure. If zero, failure is not considered. |
| SIGMAX | Maximum stress obtainable from work hardening before rate effects are added (optional). This option is ignored if VP = 1.0 |
| SIGSAT | Saturation stress which limits the maximum value of effective stress which can develop after rate effects are added (optional). |
| EPS0 | Quasi-static threshold strain rate. See description under *MAT_-015. |

**Remarks:**

1. **Flow Stress.** Johnson and Cook express the flow stress as

$$\sigma_y = \left(A + B\bar{\varepsilon}^{p^n}\right)(1 + C\ln\dot{\varepsilon}^*)$$

where $A$, $B$, and $C$ are input constants and $\bar{\varepsilon}^p$ is the effective plastic strain. $\dot{\varepsilon}^*$ is the normalized effective strain rate:

$$\dot{\varepsilon}^* = \frac{\dot{\bar{\varepsilon}}}{\text{EPS0}} \quad .$$

The maximum stress is limited by SIGMAX and SIGSAT by:

$$\sigma_y = \min\left\{\min\left[A + B\bar{\varepsilon}^{p^n}, \text{SIGMAX}\right](1 + c\ln\dot{\varepsilon}^*), \text{SIGSAT}\right\} \quad .$$

Failure occurs when the effective plastic strain exceeds PSFAIL.

2. **Viscoplastic.** If the viscoplastic option is active (VP = 1.0), the parameters SIGMAX and SIGSAT are ignored since these parameters make convergence of the plastic strain iteration loop difficult to achieve. The viscoplastic option replaces the effective strain rate in the forgoing equations by the effective plastic strain rate. Numerical noise is substantially reduced by the viscoplastic formulation.

3. **STOCHASTIC.** The STOCHASTIC option allows spatially varying yield and failure behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

## *MAT_SIMPLIFIED_JOHNSON_COOK_ORTHOTROPIC_DAMAGE

This is Material Type 99. This model, which is implemented with multiple through thickness integration points, is an extension of model 98 to include orthotropic damage as a means of treating failure in aluminum panels. Directional damage begins after a defined failure strain is reached in tension and continues to evolve until a tensile rupture strain is reached in either one of the two orthogonal directions. After rupture is detected at NUMINT integration points, the element is deleted.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | VP | EPPFR | LCDM | NUMINT |
| Type | A | F | F | F | F | F | I | I |
| Default | none | none | none | none | 0.0 | $10^{16}$ | optional | {all} |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | N | C | PSFAIL | SIGMAX | SIGSAT | EPSO |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.0 | 0.0 | $10^{17}$ | SIGSAT | $10^{28}$ | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |

| VARIABLE | DESCRIPTION |
|---|---|
| | This option applies only to the 4-node shell and 8-node thick shell if and only if through thickness integration is used. |
| EPPFR | Plastic strain at which material ruptures (logarithmic) |
| LCDM | Load curve ID defining nonlinear damage curve. See Figure M81-2. |
| NUMINT | Number of through thickness integration points which must fail before the element is deleted. If zero, all points must fail. The default of all integration points is not recommended since elements undergoing large strain are often not deleted due to nodal fiber rotations which limit 0 strains at active integration points after most points have failed. Better results are obtained if NUMINT is set to 1 or a number less than one half of the number of through thickness points. For example, if four through thickness points are used, NUMINT should not exceed 2, even for fully integrated shells which have 16 integration points. |
| A | See Remark 1 in *MAT_098. |
| B | See Remark 1 in *MAT_098. |
| N | See Remark 1 in *MAT_098. |
| C | See Remark 1 in *MAT_098. |
| PSFAIL | Principal plastic strain at failure. If zero, failure is not considered. |
| SIGMAX | Maximum stress obtainable from work hardening before rate effects are added (optional). This option is ignored if VP = 1.0. |
| SIGSAT | Saturation stress which limits the maximum value of effective stress which can develop after rate effects are added (optional). |
| EPS0 | Quasi-static threshold strain rate. See description under *MAT_015. |

**Remarks:**

See the description for the SIMPLIFIED_JOHNSON_COOK model above.

## *MAT_SPOTWELD_{OPTION1}_{OPTION2}

This is Material Type 100. The material model applies to beam element type 9 and to solid element type 1. The failure models apply to both beam and solid elements.

For solid elements, if hourglass type 4 is specified, it will be used; otherwise, hourglass type 6 will be automatically assigned. Hourglass type 6 is preferred.

The beam elements, based on the Hughes-Liu beam formulation, can be placed between any two deformable shell surfaces and tied with constraint contact, *CONTACT_SPOTWELD, which eliminates the need to have adjacent nodes at spot weld locations. Beam spot welds may be placed between rigid bodies and rigid/deformable bodies by making the node on one end of the spot weld a rigid body node, which can be an extra node for the rigid body; see *CONSTRAINED_EXTRA_NODES_*OPTION*. In the same way, rigid bodies may also be tied together with this spot weld option. This weld option should not be used with rigid body switching. The foregoing advice is valid if solid element spot welds are used; however, since the solid elements have just three degrees of freedom at each node, *CONTACT_TIED_SURFACE_TO_SURFACE must be used instead of *CONTACT_SPOTWELD.

In flat topologies, the shell elements have an unconstrained drilling degree of freedom, which prevents torsional forces from being transmitted. If the torsional forces are deemed to be important, brick elements should be used to model the spot welds.

Beam and solid element force resultants for MAT_SPOTWELD are written to the spot weld force file, swforc, and the file for element stresses and resultants for designated elements, elout.

*It is advisable to include all spot welds, which provide the tracked nodes, and spot welded materials, which define the reference segments, within a single *CONTACT_SPOTWELD interface for beam element spot welds or a *CONTACT_TIED_SURFACE_TO_SURFACE interface for solid element spot welds.* As a constraint method, these interfaces are treated independently, which can lead to significant problems if such interfaces share common nodal points. An added benefit is that memory usage can be substantially less with a single interface.

Available options for *OPTION1* include:

<BLANK>

DAMAGE-FAILURE

The DAMAGE-FAILURE option causes one additional line to be read with the damage parameter and a flag that determines how failure is computed from the resultants. On this line the parameter, RS, if nonzero, invokes damage mechanics combined with the plasticity model to achieve a smooth drop off of the resultant forces prior to the removal

of the spot weld. The parameter OPT determines the method used in computing resultant based failure, which is unrelated to damage.

Available options for *OPTION2* include:

<BLANK>

UNIAXIAL

The UNIAXIAL keyword option causes the transverse stresses and transverse strains to be zero for solid spot welds. The older uniaxial method, invoked with E < 0.0 on Card 1, assumed only the transverse stresses are zero. Compared to the older method, the UNIAXIAL keyword option increases the stability of the solver. See Remark 2 for more details.

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | EH | DT | TFAIL |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if no keyword option is used (<BLANK>) for *OPTION1*.

| EFAIL | NRR | NRS | NRT | MRR | MSS | MTT | NF |
|---|---|---|---|---|---|---|---|

**Card 2b.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = -2, -1 or 0 on Card 3.

| EFAIL | NRR | NRS | NRT | MRR | MSS | MTT | NF |
|---|---|---|---|---|---|---|---|

**Card 2c.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 1 on Card 3.

| EFAIL | SIGAX | SIGTAU | | | | | NF |
|---|---|---|---|---|---|---|---|

**Card 2d.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 2, 12, or 22 on Card 3.

| EFAIL | USRV1 | USRV2 | USRV3 | USRV4 | USRV5 | USRV6 | NF |
|---|---|---|---|---|---|---|---|

**Card 2e.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 3 or 4 on Card 3.

| EFAIL | ZD | ZT | ZALP1 | ZALP2 | ZALP3 | ZRRAD | NF |
|---|---|---|---|---|---|---|---|

**Card 2f.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 5 on Card 3.

| EFAIL | ZD | ZT | ZT2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 2g.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 6, 7, 9, -9 or 10 on Card 3.

| EFAIL | | | | | | | NF |
|---|---|---|---|---|---|---|---|

**Card 2h.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 11 on Card 3.

| EFAIL | LCT | LCC | | | | | NF |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if the DAMAGE-FAILURE keyword option is used.

| RS | OPT | FVAL | TRUE_T | ASFF | BETA | | DMGOPT |
|---|---|---|---|---|---|---|---|

**Card 3.1.** Include this card if the DAMAGE-FAILURE keyword option is used and DMGOPT = -1 on Card 3.

| DMGOPT | FMODE | FFCAP | TTOPT | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 12 or 22 on Card 3.

| USRV7 | USRV8 | USRV9 | USRV10 | USRV11 | USRV12 | USRV13 | USRV14 |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card if the DAMAGE-FAILURE keyword option is used and OPT = 12 or 22 on Card 3.

| USRV15 | USRV16 | USRV17 | USRV18 | USRV19 | USRV20 | USRV21 | USRV22 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | EH | DT | TFAIL |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus.<br>LT.0.0: \|E\| is the Young's modulus. E < 0 invokes uniaxial stress for solid spot welds with the transverse stresses assumed to be zero. See Remark 2. This is for when OPTION2 is unset (<BLANK>) only. |
| PR | Poisson's ratio |
| SIGY | Yield Stress:<br>GT.0: Initial yield stress<br>EQ.0: Default to 1% of E<br>LT.0: A yield curve or table is assigned by \|SIGY\|; see Remark 5. |
| EH | Plastic hardening modulus, $E_h$ |
| DT | Time step size for mass scaling, $\Delta t$ |
| TFAIL | Failure time if nonzero. If zero, this option is ignored. |

**Card 2 for No Failure.** Additional card when no keyword option is used (<BLANK>).

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | NRR | NRS | NRT | MRR | MSS | MTT | NF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in weld material at failure. The spot weld element is deleted when the plastic strain at each integration point exceeds EFAIL. If zero, failure due to effective plastic strain is not considered. |
| NRR | Axial force resultant $N_{rr_F}$ at failure. If zero, failure due to this component is not considered. |

| VARIABLE | DESCRIPTION |
|---|---|
| | GT.0: Constant value<br>LT.0: \|NRR\| is a load curve ID, defining the axial force resultant $N_{rr_F}$ at failure as a function of the effective strain rate. |
| NRS | Force resultant $N_{rs_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|NRS\| is a load curve ID, defining the force resultant $N_{rs_F}$ at failure as a function of the effective strain rate. |
| NRT | Force resultant $N_{rt_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|NRT\| is a load curve ID, defining the force resultant $N_{rt_F}$ at failure as a function of the effective strain rate. |
| MRR | Torsional moment resultant $M_{rr_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|MRR\| is a load curve ID, defining the torsional moment resultant $M_{rr_F}$ at failure as a function of the effective strain rate. |
| MSS | Moment resultant $M_{ss_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|MSS\| is a load curve ID, defining the moment resultant $M_{ss_F}$ at failure as a function of the effective strain rate. |
| MTT | Moment resultant $M_{tt_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|MTT\| is a load curve ID, defining the moment resultant $M_{tt_F}$ at failure as a function of the effective strain rate. |
| NF | Number of force vectors stored for filtering |

**Card 2 for Resultant-Based Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = -2, -1 or 0.

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | NRR | NRS | NRT | MRR | MSS | MTT | NF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| NRR | Axial force resultant $N_{rr_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|NRR\| is a load curve ID, defining the axial force resultant $N_{rr_F}$ at failure as a function of the effective strain rate. |
| NRS | Force resultant $N_{rs_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|NRS\| is a load curve ID, defining the force resultant $N_{rs_F}$ at failure as a function of the effective strain rate. |
| NRT | Force resultant $N_{rt_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|NRT\| is a load curve ID, defining the force resultant $N_{rt_F}$ at failure as a function of the effective strain rate. |
| MRR | Torsional moment resultant $M_{rr_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|MRR\| is a load curve ID, defining the torsional moment resultant $M_{rr_F}$ at failure as a function of the effective strain rate. |

| VARIABLE | DESCRIPTION |
|---|---|
| MSS | Moment resultant $M_{ss_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|MSS\| is a load curve ID, defining the moment resultant $M_{ss_F}$ at failure as a function of the effective strain rate. |
| MTT | Moment resultant $M_{tt_F}$ at failure. If zero, failure due to this component is not considered.<br>GT.0: Constant value<br>LT.0: \|MTT\| is a load curve ID, defining the moment resultant $M_{tt_F}$ at failure as a function of the effective strain rate. |
| NF | Number of force vectors stored for filtering |

**Card 2 for Stress-Based Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 1.

| Card 2c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | SIGAX | SIGTAU | | | | | NF |
| Type | F | F | F | | | | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| SIGAX | Maximum axial stress $\sigma_{rr}^F$ at failure.<br>GT.0.0: Constant maximum axial stress at failure<br>EQ.0.0: Failure due to this component is not considered.<br>LT.0.0: \|SIGAX\| is a load curve ID defining the maximum axial stress at failure as a function of the effective strain rate. |
| SIGTAU | Maximum shear stress $\tau^F$ at failure.<br>GT.0.0: Constant maximum shear stress at failure |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Failure due to this component is not considered. |
| | LT.0.0: \|SIGTAU\| is a load curve ID defining the maximum shear stress at failure as a function of the effective strain rate. |
| NF | Number of force vectors stored for filtering |

**Card 2 for User Subroutine Based Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 2, 12, or 22.

| Card 2d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | USRV1 | USRV2 | USRV3 | USRV4 | USRV5 | USRV6 | NF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| USRV$n$ | Failure constants for user failure subroutine, $n = 1, 2, \ldots, 6$ |
| NF | Number of force vectors stored for filtering |

**Card 2 for Notch Stress Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 3 or 4.

| Card 2e | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | ZD | ZT | ZALP1 | ZALP2 | ZALP3 | ZRRAD | NF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in the weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| ZD | Notch diameter |

| VARIABLE | DESCRIPTION |
|---|---|
| ZT | Sheet thickness |
| ZALP1 | Correction factor $\alpha_1$ |
| ZALP2 | Correction factor $\alpha_2$ |
| ZALP3 | Correction factor $\alpha_3$ |
| ZRRAD | Notch root radius (OPT = 3 only) |
| NF | Number of force vectors stored for filtering |

**Card 2 for Structural Stress Failure.** Additional card for DAMAGE-FAILURE keyword option with OPT = 5.

| Card 2f | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | ZD | ZT | ZT2 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in the weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| ZD | Notch diameter |
| ZT | Sheet thickness |
| ZT2 | Second sheet thickness |

**Card 2 for Stress-Based Failure from Resultants/Rate Effects.** Additional card for DAMAGE-FAILURE keyword option with OPT = 6, 7, 9, -9 or 10.

| Card 2g | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | | | | | | | NF |
| Type | F | | | | | | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| NF | Number of force vectors stored for filtering |

**Card 2 for Resultant-Based Failure for Beams depending on Loading Direction.** Additional card for DAMAGE-FAILURE keyword option with OPT =11.

| Card 2h | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | LCT | LCC | | | | | NF |
| Type | F | F | F | | | | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EFAIL | Effective plastic strain in the weld material at failure. The plastic strain must exceed the rupture strain (RS) at each integration point before deletion occurs. See Card 3. |
| LCT | Load curve or Table ID. The load curve defines resultant failure force under tension as a function of loading direction (in degree range 0 to 90). The table defines these curves as functions of strain rates. See remarks. |
| LCC | Load curve or Table ID. The load curve defines resultant failure force under compression as a function of loading direction (in degree range 0 to 90). The table defines these curves as functions of strain rates. See remarks. |
| NF | Number of force vectors stored for filtering |

Additional card for the DAMAGE-FAILURE option.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RS | OPT | FVAL | TRUE_T | ASFF | BETA | | DMGOPT |
| Type | F | F | F | F | I | F | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| RS | Rupture strain (or rupture time if DMGOPT = 2 or 12). Define only if damage is active. |
| OPT | Failure option: |

EQ.-9: OPT = 9 failure is evaluated and written to the swforc file, but element failure is suppressed.

EQ.-2: Same as option –1 but in addition, the peak value of the failure criteria and the time it occurs is stored and is written into the swforc database. This information may be necessary since the instantaneous values written at specified time intervals may miss the peaks. Additional storage is allocated to store this information.

EQ.-1: OPT = 0 failure is evaluated and written into the swforc file, but element failure is suppressed.

EQ.0: Resultant-based failure

EQ.1: Stress-based failure computed from resultants (Toyota)

EQ.2: User subroutine `uweldfail` to determine failure

EQ.3: Notch stress-based failure (beam and hex assembly welds only)

EQ.4: Stress intensity factor at failure (beam and hex assembly welds only)

EQ.5: Structural stress at failure (beam and hex assembly welds only)

EQ.6: Stress based failure computed from resultants (Toyota). In this option a shell strain rate dependent failure model is used (beam and hex assembly welds only). The static failure stresses are defined by part ID using the keyword *DEFINE_SPOTWELD_RUPTURE_STRESS.

EQ.7: Stress based failure for solid elements (Toyota) with peak stresses computed from resultants, and strength values input for pairs of parts; see *DEFINE_SPOTWELD_FAILURE_RESULTANTS. Strain rate effects are optional.

EQ.8: Not used

EQ.9: Stress based failure from resultants (Toyota). In this option a shell strain rate dependent failure model is used (beam welds only). The static failure stresses are defined

| VARIABLE | DESCRIPTION |
|---|---|
| | by part ID using the keyword *DEFINE_SPOTWELD_-RUPTURE_PARAMETER. |
| | EQ.10: Stress based failure with rate effects. Failure data is defined by material using the keyword *DEFINE_SPOWELD_FAILURE. |
| | EQ.11: Resultant based failure (beams only). In this option load curves or tables LCT (tension) and LCC (compression) can be defined as resultant failure force as a function loading direction (curve) or resultant failure force as a function of loading direction for each strain rate (table). |
| | EQ.12: User subroutine `uweldfail12` with 22 material constants to determine damage and failure |
| | EQ.22: user subroutine `uweldfail22` with 22 material constants to determine failure |
| FVAL | Failure parameter. |
| | OPT.EQ.-2: Not used |
| | OPT.EQ.-1: Not used |
| | OPT.EQ.0: Function ID (*DEFINE_FUNCTION) to define alternative Weld Failure. If this is set, the values given for NRR, NRS, NRT, MRR, MSS and MTT in Card 2 are ignored. See description of Weld Failure for OPT = 0. |
| | OPT.EQ.1: Not used |
| | OPT.EQ.2: Not used |
| | OPT.EQ.3: Notch stress value at failure ($\sigma_{KF}$) |
| | OPT.EQ.4: Stress intensity factor value at failure ($K_{eqF}$) |
| | OPT.EQ.5: Structural stress value at failure ($\sigma_{sF}$) |
| | OPT.EQ.6: Number of cycles that failure condition must be met to trigger beam deletion. |
| | OPT.EQ.7: Not used |
| | OPT.EQ.9: Number of cycles that failure condition must be met to trigger beam deletion. |
| | OPT.EQ.10: ID of data defined by *DEFINE_SPOTWELD_FAILURE. |

| VARIABLE | DESCRIPTION |
|---|---|
| | OPT.EQ.12: Number of history variables available in user defined failure subroutine, `uweldfaill2`. |
| TRUE_T | True weld thickness. This optional value is available for solid element failure and is used to reduce the moment contribution to the failure calculation from artificially thick weld elements under shear loading, so shear failure can be modeled more accurately. Note that the behavior of TRUE_T depends on TTOPT. See Remark 8. |
| | **NOTE:** We do not recommend using TRUE_T. Instead, we recommend using TTOPT = 2 and leaving TRUE_T = 0.0. In many cases, TTOPT = 2 does a better job of removing the spurious moments. See Remark 9. |
| ASFF | Weld assembly simultaneous failure flag:<br>EQ.0: Damaged elements fail individually.<br>EQ.1: All elements in the weld assembly fail when one element in the assembly is completely damaged. |
| BETA | Damage model decay rate |
| DMGOPT | Damage option flag:<br>EQ.-1: Flag to include Card 3.1 for additional damage fields. DMGOPT will be set on Card 3.1.<br>EQ.0: Plastic strain based damage<br>EQ.1: Plastic strain based damage with post damage stress limit<br>EQ.2: Time based damage with post damage stress limit<br>EQ.10: Like DMGOPT = 0, but failure option will initiate damage<br>EQ.11: Like DMGOPT = 1, but failure option will initiate damage<br>EQ.12: Like DMGOPT = 2, but failure option will initiate damage |

**Damage Option Card.** Optional additional card for the DAMAGE-FAILURE option; read only if DMGOPT = -1 on Card 3.

| Card 3.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DMGOPT | FMODE | FFCAP | TTOPT | | | | |
| Type | F | F | F | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| DMGOPT | Damage option flag:<br>EQ.0: Plastic strain-based damage<br>EQ.1: Plastic strain-based damage with post-damage stress limit<br>EQ.2: Time-based damage with post-damage stress limit<br>EQ.10: Like DMGOPT = 0, but the failure option will initiate damage<br>EQ.11: Like DMGOPT = 1, but the failure option will initiate damage<br>EQ.12: Like DMGOPT = 2, but the failure option will initiate damage |
| FMODE | Failure surface ratio for damage or failure, for DMGOPT = 10, 11, or 12<br>EQ.0: Damage initiates<br>GT.0: Damage or failure (see Remark 6) |
| FFCAP | Failure function limit for OPT = 0 or -1 and DMGOPT = 10, 11, or 12<br>EQ.0: Damage initiates<br>GT.0: Damage or failure (see Remark 6) |
| TTOPT | Options for TRUE_T / weld failure behavior:<br>EQ.0: TRUE_T behavior of version R9 and later (see Remark 8)<br>EQ.1: TRUE_T behavior of version R8 and earlier (see Remark 8)<br>EQ.2: Weld failure is invariant with respect to the node numbering of weld elements. For this case, there is no need for the TRUE_T correction. We recommend using this option with TRUE_T set to 0.0. See Remark 9. |

**Failure Constants Card.** Additional card for OPT = 12 or 22.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | USRV7 | USRV8 | USRV9 | USRV10 | USRV11 | USRV12 | USRV13 | USRV14 |
| Type | F | F | F | F | F | F | F | F |

**Failure Constants Card.** Additional card for OPT = 12 or 22.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | USRV15 | USRV16 | USRV17 | USRV18 | USRV19 | USRV20 | USRV21 | USRV22 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| USRV$n$ | Failure constants for OPT = 12 or 22 user-defined failure, $n = 7, 8, \ldots, 22$ |

**Remarks:**

1. **Failure model overview.** Spot weld material is modeled with isotropic hardening plasticity coupled to failure models. EFAIL specifies a failure strain that fails each integration point in the spot weld independently. The OPT parameter is used to specify a failure criterion that fails the entire weld element when the criterion is met. Alternatively, EFAIL and OPT may be used to initiate damage when the DAMAGE-FAILURE option is active using RS, BETA, and DMGOPT as described below.

   Beam spot weld elements can use any OPT value except 7. Brick spot weld elements can use any OPT value except 3, 4, 5, 6, 9, and -9. Hex assembly spot welds can use any OPT value except 9 and -9.

   For all OPT failure criteria, if a zero is input for a failure parameter on Card 2, the corresponding term will be omitted from the equation. For example, if for OPT = 0, only $N_{rr_F}$ is nonzero, the failure surface is reduced to $|N_{rr}| = N_{rr_F}$ (see below).

   Similarly, if the failure strain EFAIL is set to zero, the failure strain model is not used. Both EFAIL and OPT may be active at the same time.

2. **Loading solid welds uniaxially.** We have implemented two methods of loading solid and solid weld assemblies uniaxially. The older method is invoked by defining the elastic modulus, $E$, as a negative number where the absolute value of $E$ is the desired value for $E$. This uniaxial option causes the two transverse stress terms to be assumed to be zero throughout the calculation. This assumption eliminates parasitic transverse stress that causes slow growth of plastic strain-based damage.

   The other method is invoked by setting *OPTION2* to UNIAXIAL. This method is preferred. It causes the two transverse stress terms and the two transverse strains terms to be set to zero. It was added because we found that the older method sometimes induced spurious oscillations in the axial force, leading to premature failure.

   The motivation for the uniaxial options can be explained with a weld loaded in tension. Due to Poisson's effect, an element in tension would be expected to contract in the transverse directions. However, because the weld nodes are constrained to the mid-plane of shell elements, such contraction is only possible to the degree that the shell element contracts. In other words, the uniaxial stress state cannot be represented by the weld. For plastic strain-based damage, this effect can be particularly apparent as it causes tensile stress to continue to grow very large as the stress state becomes very nearly triaxial tension. In this stress state, plastic strain grows very slowly so it appears that damage calculation is failing to knock down the stress. By simply assuming that the transverse stresses are zero, the plastic strain grows as expected and damage is much more effective.

3. **NF.** NF specifies the number of terms used to filter the stresses or resultants used in the OPT failure criterion. NF cannot exceed 30. The default value is set to zero which is generally recommended unless oscillatory resultant forces are observed in the time history databases. Although welds should not oscillate significantly, this option was added for consistency with the other spot weld options. NF affects the storage since it is necessary to store the resultant forces as history variables.

4. **Time scaling.** The inertias of the spot welds are scaled during the first time step so that their stable time step size is $\Delta t$. A strong compressive load on the spot weld at a later time may reduce the length of the spot weld so that the stable time step size drops below $\Delta t$. If the value of $\Delta t$ is zero, mass scaling is not performed, and the spot welds will probably limit the time step size. Under most circumstances, the inertias of the spot welds are small enough that scaling them will have a negligible effect on the structural response, and the use of this option is encouraged.

5. **Yield curve or table for SIGY.** When using a yield curve or table for SIGY, a simplified plasticity algorithm is used, assuming a linear behavior within one time increment. Thus, no iterative return mapping has to be performed.

6. **Damage.** When the DAMAGE-FAILURE option is invoked, the constitutive properties for the damaged material are obtained from the undamaged material properties. The amount of damage evolved is represented by the constant, $\omega$, which varies from zero if no damage has occurred to unity for complete rupture. For uniaxial loading, the nominal stress in the damaged material is given by

$$\sigma_{\text{nominal}} = \frac{P}{A}$$

where $P$ is the applied load and $A$ is the surface area. The true stress is given by:

$$\sigma_{\text{true}} = \frac{P}{A - A_{\text{loss}}}$$

where $A_{\text{loss}}$ is the void area. The damage variable can then be defined:

$$\omega = \frac{A_{\text{loss}}}{A} \ ,$$

where

$$0 \le \omega \le 1 \ .$$

In this model, damage is initiated when the effective plastic strain in the weld exceeds the failure strain, EFAIL. If DMGOPT = 10, 11, or 12, damage will initiate when the effective plastic strain exceeds EFAIL, or when the failure criterion is met, whichever occurs first. The failure criterion is specified by the OPT parameter. If the inputted value of EFAIL = 0 and DMGOPT = 10, 11, or 12, then damage will only be initiated if the failure criterion is met. After damage initiates, the damage variable is evaluated by one of two ways:

a) For DMGOPT = 0, 1, 10, or 11, the damage variable is a function of effective plastic strain in the weld:

$$\varepsilon^p_{\text{failure}} \le \varepsilon^p_{\text{eff}} \le \varepsilon^p_{\text{rupture}} \Rightarrow \omega = \frac{\varepsilon^p_{\text{eff}} - \varepsilon^p_{\text{failure}}}{\varepsilon^p_{\text{rupture}} - \varepsilon^p_{\text{failure}}} \ ,$$

where $\varepsilon^p_{\text{failure}}$ = EFAIL and $\varepsilon^p_{\text{rupture}}$ = RS. If DMGOPT = 10 or 11, and damage initiates by the failure criterion, then $\varepsilon^p_{\text{failure}}$is set equal to the effective plastic strain in the weld at the time of damage initiation.

b) For DMGOPT = 2 or 12, the damage variable is a function of time:

$$t_{\text{failure}} \le t \le t_{\text{rupture}} \Rightarrow \omega = \frac{t - t_{\text{failure}}}{t_{\text{rupture}}}$$

where $t_{\text{failure}}$ is the time at which damage initiates, and $t_{\text{rupture}}$ = RS. For DMGOPT = 2, $t_{\text{failure}}$ is set equal to the time at which $\varepsilon_{\text{eff}}^{p}$ exceeds EFAIL. For DMGOPT = 12, $t_{\text{failure}}$ is set equal to either the time when $\varepsilon_{\text{eff}}^{p}$ exceeds EFAIL or the time when the failure criterion is met, whichever occurs first.

If DMGOPT = 0, 1, or 2, inputting EFAIL = 0 will cause damage to initiate as soon as the weld stress reaches the yield surface. Prior to version 9.1, inputting EFAIL = 0 for DMGOPT = 10, 11, 12 would similarly cause damage to initiate when the stress state reaches the yield surface, but version 9.1 and later will ignore EFAIL = 0 and only initiate damage when the failure criterion is met. If the effective plastic strain is zero when damage initiates by the failure criterion, then the yield stress of the weld is reduced to the current effective stress so that the stress state is on the yield surface and plastic strain can start to grow.

For DMGOPT = 1, the damage behavior is the same as for DMGOPT = 0, but an additional damage variable is calculated to prevent stress growth during softening. Similarly, DMGOPT = 11 behaves like DMGOPT = 10 except for the additional damage variable. This additional function is also used with DMGOPT = 2 and 12. The effect of this additional damage function is noticed only in brick and brick assembly welds in tension loading where it prevents growth of the tensile force in the weld after damage initiates.

For DMGOPT = 10, 11, or 12 an optional FMODE parameter determines whether a weld that reaches the failure surface will fail immediately or initiate damage. The failure surface calculation has shear terms, which may include the torsional moment as well as normal and bending terms. If FMODE is input with a value between 0 and 1, then when the failure surface is reached, the sum of the square of the shear terms is divided by the sum of the square of all terms. If this ratio exceeds FMODE, then the weld will fail immediately. If the ratio is less than or equal to FMODE, then damage will initiate. The FMODE option is available only for brick and brick assembly welds.

For resultant based failure (OPT = -1 or 0) and DMGOPT = 10, 11, or 12 an optional FFCAP parameter determines whether a weld that reaches the failure surface will fail immediately. After damage initiation, the failure function can reach values above 1.0. This can now be limited by the FFCAP value (should be larger than 1.0):

$$\left(\left[\frac{\max(N_{rr},0)}{N_{rr_F}}\right]^2+\left[\frac{N_{rs}}{N_{rs_F}}\right]^2+\left[\frac{N_{rt}}{N_{rt_F}}\right]^2+\left[\frac{M_{rr}}{M_{rr_F}}\right]^2+\left[\frac{M_{ss}}{M_{ss_F}}\right]^2+\left[\frac{M_{tt}}{M_{tt_F}}\right]^2\right)^{\frac{1}{2}} < \text{FFCAP}$$

7. **BETA.** If *BETA* is specified, the stress is multiplied by an exponential using $\omega$ defined in the equations define in Remark 6,

$$\sigma_d = \sigma\exp(-\beta\omega)\ .$$

For weld elements in an assembly (see RPBHX on *CONTROL_SPOTWELD_-BEAM or *DEFINE_HEX_SPOTWELD_ASSEMBLY), the failure criterion is evaluated using the assembly cross section. If damage is not active, all elements will be deleted when the failure criterion is met. If damage is active, then damage is calculated independently in each element of the assembly. By default, elements of the assembly are deleted as damage in each element is complete. If ASFF = 1, then failure and deletion of all elements in the assembly will occur simultaneously when damage is complete in any one of the elements.

8. **TRUE_T and TTOPT.** Solid weld elements and weld assemblies are tied to the mid-plane of shell materials and so typically have a thickness that is half the sum of the thicknesses of the welded shell sections. As a result, a weld under shear loading can be subject to an artificially large moment which will be balanced by normal forces transferred through the tied contact. These normal forces will cause the out-of-plane bending moment used in the failure calculation to be artificially high.

   TRUE_T was our original implementation to fix this issue. Inputting a TRUE_T value that is smaller than the modeled thickness, for example, 10%-30% of true thickness will scale down the moment or stress that results from the balancing moment and provide more realistic failure calculations for solid elements and weld assemblies. TRUE_T effects only the failure calculation, not the weld element behavior. If TRUE_T = 0.0 or data is omitted, the modeled weld element thickness is used. Our preferred solution to this problem is keeping TRUE_-T = 0.0 and setting TTOPT = 2 which is discussed in Remark 9.

   The behavior of TRUE_T depends on TTOPT. In LS-DYNA version R9, a modification to the TRUE_T behavior was made to address a condition of weld assemblies that are tied to shell elements of significantly different stiffness. This change had unintended effects on the behavior of weld failure, so TTOPT was added to revert the behavior of TRUE_T to that of R8 and earlier versions. The default behavior of TTOPT is to perform the R9 method but setting TTOPT = 1 will cause the earlier method to be used. TTOPT = 1 also invokes a second correction. With TTOPT = 0, weld assemblies use TRUE_T as if it was a scale factor, but single element welds use it as a thickness value. Setting TTOPT = 1 corrects this so that both weld assemblies and single welds use TRUE_T as a thickness value.

   For OPT = 0 (see below), the two out-of-plane moments, $M_{ss}$ and $M_{tt}$ are replaced by modified terms $\widehat{M}_{ss}$ and $\widehat{M}_{tt}$:

$$\left[\frac{\max(N_{rr},0)}{N_{rr_F}}\right]^2+\left[\frac{N_{rs}}{N_{rs_F}}\right]^2+\left[\frac{N_{rt}}{N_{rt_F}}\right]^2+\left[\frac{M_{rr}}{M_{rr_F}}\right]^2+\left[\frac{\widehat{M}_{ss}}{M_{ss_F}}\right]^2+\left[\frac{\widehat{M}_{tt}}{M_{tt_F}}\right]^2-1=0$$

$$\widehat{M}_{ss} = \begin{cases} M_{ss} - N_{rt}t(1 - t_{\text{true}}) & \text{for solid weld assemblies with TTOPT} = 0 \\ M_{ss} - N_{rt}(t - t_{\text{true}}) & \text{otherwise} \end{cases}$$

$$\widehat{M}_{tt} = \begin{cases} M_{tt} - N_{rs}t(1 - t_{\text{true}}) & \text{for solid weld assemblies with TTOPT} = 0 \\ M_{tt} - N_{rs}(t - t_{\text{true}}) & \text{otherwise} \end{cases}$$

In the above, $t$ is the element thickness and $t_{\text{true}}$ is the TRUE_T parameter. For OPT = 1 (see below), the same modification is done to the moments that contribute to the normal stress, as shown below:

$$\sigma_{rr} = \frac{N_{rr}}{A} + \frac{\sqrt{\widehat{M}_{ss}^2 + \widehat{M}_{tt}^2}}{Z} \ .$$

9. **TTOPT = 2.** By default, failure is calculated using forces on the bottom surface of the weld as defined by nodes 1 to 4 of each element. Setting TTOPT = 2 causes the average of the forces on the bottom and top to be used so that failure is invariant. When TTOPT = 2 is used, the averaging causes the spurious moments or peak normal stress to cancel, so there is no need for a TRUE_T correction. Therefore, the best practice is to use TTOPT = 2 and TRUE_T = 0.0.

10. **History data output files.** Spot weld force history data is written into the swforc ASCII file. In this database, the resultant moments are not available, but they are in the binary time history database and in the ASCII elout file.

11. **Material histories.** The probability of failure in solid or beam spotwelds can be estimated by retrieving the corresponding material histories for output to the d3plot database.

**DEFINE_MATERIAL_HISTORIES Properties*

| Label | Attributes | Description |
|---|---|---|
| Instability | - - - - | A measure between 0 and 1 related to how close the spotweld element is to fail |
| Damage | - - - - | Damage in the spotweld element between 0 and 1 |

These two labels are supported for all options (OPT and DMGOPT, including assemblies and beams), except for user defined failure. The instability measure is the maximum over time; namely, it gives the maximum value for a given element throughout the simulation. If a damage option is invoked, then damage will initiate and increment when the instability reaches unity, and elements are not deleted until the damage value reaches unity. If no damage option is invoked, then the damage output is always zero and elements will be deleted at the point when the instability measure reaches unity

**OPT = -1 or 0**

OPT = 0 and OPT = -1 invoke a resultant-based failure criterion that fails the weld if the resultants are outside of the failure surface defined by:

$$\left[\frac{\max(N_{rr},0)}{N_{rr_F}}\right]^2+\left[\frac{N_{rs}}{N_{rs_F}}\right]^2+\left[\frac{N_{rt}}{N_{rt_F}}\right]^2+\left[\frac{M_{rr}}{M_{rr_F}}\right]^2+\left[\frac{M_{ss}}{M_{ss_F}}\right]^2+\left[\frac{M_{tt}}{M_{tt_F}}\right]^2-1=0$$

where the *numerators* in the equation are the resultants calculated in the local coordinates of the cross section, and the *denominators* are the values specified in the input. If OPT = -1, the failure surface equation is evaluated, but element failure is suppressed. This allows easy identification of vulnerable spot welds when post-processing. Failure is likely to occur if FC > 1.0.

Alternatively, a *DEFINE_FUNCTION could be used to define the Weld Failure for OPT = 0. Then set FVAL = function ID. Such a function could look like this:

```
*DEFINE_FUNCTION
       100
func(nrr,nrs,nrt,mrr,mss,mtt)= (nrr/5.0)*(nrr/5.0)
```

The six arguments for this function (nrr, …, mtt) are the force and moment resultants during the computation.

**OPT = 1:**

OPT = 1 invokes a stress-based failure model, which was developed by *Toyota Motor Corporation* and is based on the peak axial and transverse shear stresses. The weld fails if the stresses are outside of the failure surface defined by

$$\left(\frac{\sigma_{rr}}{\sigma_{rr}^F}\right)^2+\left(\frac{\tau}{\tau^F}\right)^2-1=0$$

If strain rates are considered, then the failure criteria becomes:

$$\left[\frac{\sigma_{rr}}{\sigma_{rr}^F(\dot{\varepsilon}_{\text{eff}})}\right]^2+\left[\frac{\tau}{\tau^F(\dot{\varepsilon}_{\text{eff}})}\right]^2-1=0$$

where $\sigma_{rr}^F(\dot{\varepsilon}_{\text{eff}})$ and $\tau^F(\dot{\varepsilon}_{\text{eff}})$ are defined by load curves (SIGAX and SIGTAU are less than zero). The peak stresses are calculated from the resultants using simple beam theory:

$$\sigma_{rr}=\frac{N_{rr}}{A}+\frac{\sqrt{M_{ss}^2+M_{tt}^2}}{Z}$$

$$\tau=\frac{M_{rr}}{2Z}+\frac{\sqrt{N_{rs}^2+N_{rt}^2}}{A}$$

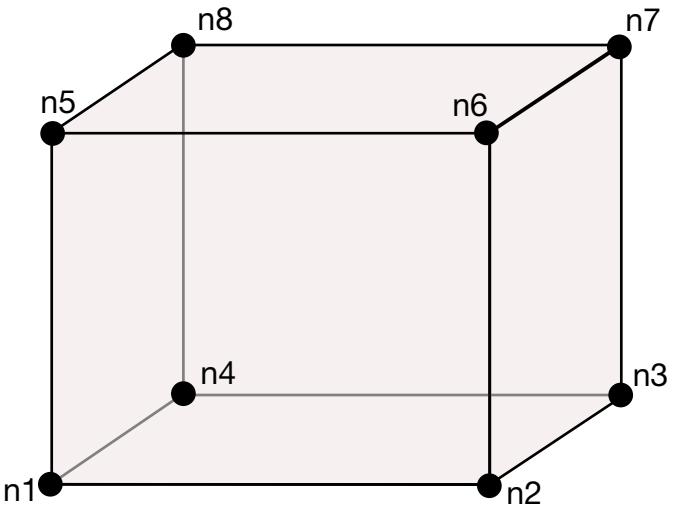



**Figure M100-1.** A solid element used as spot weld is shown. When resultant based failure is used orientation is very important. Nodes n1 - n4 attach to the lower shell mid-surface and nodes n5 - n8 attach to the upper shell mid-surface. The resultant forces and moments are computed based on the assumption that the brick element is properly oriented.

where the area and section modulus are given by:

$$A = \pi \frac{d^2}{4}$$

$$Z = \pi \frac{d^3}{32}$$

In the above equations, $d$ is the equivalent diameter of the beam element or solid element used as a spot weld.

### OPT = 2

OPT = 2 invokes a user-written subroutine `uweldfail`, documented in Appendix Q.

### OPT = 12 or 22

OPT = 12 and OPT = 22 invoke similar user-written subroutines, `uweldfail12` and `uweldfail22`, respectively. Both allow up to 22 failure parameters to be used rather than the 6 allowed with OPT = 2. OPT = 12 also allows user control of weld damage.

### OPT = 3

OPT = 3 invokes a failure based on notch stress; see Zhang [1999]. Failure occurs when the failure criterion:

$$\sigma_k - \sigma_{kF} \geq 0$$

is satisfied. The notch stress is given by the equation:

$$\sigma_k = \alpha_1 \frac{4F}{\pi dt}\left(1 + \frac{\sqrt{3}+\sqrt{19}}{8\sqrt{\pi}}\sqrt{\frac{t}{\rho}}\right) + \alpha_2 \frac{6M}{\pi dt^2}\left(1 + \frac{2}{\sqrt{3\pi}}\sqrt{\frac{t}{\rho}}\right) + \alpha_3 \frac{4F_{rr}}{\pi d^2}\left(1 + \frac{5}{3\sqrt{2\pi}}\frac{d}{t}\sqrt{\frac{t}{\rho}}\right)$$

Here,

$$F = \sqrt{F_{rs}^2 + F_{rt}^2}$$
$$M = \sqrt{M_{ss}^2 + M_{tt}^2}$$

and the $\alpha_i$ $(i = 1, 2, 3)$ are input correction factors with default values of unity. If spot welds are between sheets of unequal thickness, the minimum thickness of the spot-welded sheets may be introduced as a crude approximation.

### OPT = 4

OPT = 4 invokes failure based on structural stress intensity, see Zhang [1999]. Failure occurs when the failure criterion:

$$K_{\text{eq}} - K_{\text{eqF}} \geq 0$$

is satisfied where

$$K_{\text{eq}} = \sqrt{K_I^2 + K_{II}^2}$$

and

$$K_I = \alpha_1 \frac{\sqrt{3}F}{2\pi d\sqrt{t}} + \alpha_2 \frac{2\sqrt{3}M}{\pi dt\sqrt{t}} + \alpha_3 \frac{5\sqrt{2}F_{rr}}{3\pi d\sqrt{t}}$$
$$K_{II} = \alpha_1 \frac{2F}{\pi d\sqrt{t}}$$

Here, $F$ and $M$ are as defined above for the notch stress formulas and again, $\alpha_i$ $(i = 1, 2, 3)$ are input corrections factors with default values of unity. If spot welds are between sheets of unequal thickness, the minimum thickness of the spot welded sheets may be used as a crude approximation.

The maximum structural stress at the spot weld was utilized successfully for predicting the fatigue failure of spot welds, see Rupp, et. al. [1994] and Sheppard [1993]. The corresponding results from] Rupp, et. al. are listed below where it is assumed that they may be suitable for crash conditions.

### OPT = 5

OPT = 5 invokes failure by

$$\max(\sigma_{v1}, \sigma_{v2}, \sigma_{v3}) - \sigma_{sF} = 0 \ ,$$

where $\sigma_{sF}$ is the critical value of structural stress at failure. It is noted that the forces and moments in the equations below refer to the beam node 1, beam node 2, and the midpoint, respectively. The three stress values, $\sigma_{v1}, \sigma_{v2}, \sigma_{v3}$, are defined by:

$$\sigma_{v1}(\zeta) = \frac{F_{rs1}}{\pi d t_1}\cos\zeta + \frac{F_{rt1}}{\pi d t_1}\sin\zeta - \frac{1.046\beta_1 F_{rr1}}{t_1\sqrt{t_1}} - \frac{1.123 M_{ss1}}{d t_1\sqrt{t_1}}\sin\zeta + \frac{1.123 M_{tt1}}{d t_1\sqrt{t_1}}\cos\zeta$$

with

$$\beta_1 = \begin{cases} 0 & F_{rr1} \le 0 \\ 1 & F_{rr1} > 0 \end{cases}$$

$$\sigma_{v2}(\zeta) = \frac{F_{rs2}}{\pi d t_2}\cos\zeta + \frac{F_{rt2}}{\pi d t_2}\sin\zeta - \frac{1.046\beta_1 F_{rr2}}{t_2\sqrt{t_2}} + \frac{1.123 M_{ss2}}{d t_2\sqrt{t_2}}\sin\zeta - \frac{1.123 M_{tt2}}{d t_2\sqrt{t_2}}\cos\zeta$$

with

$$\beta_2 = \begin{cases} 0 & F_{rr2} \le 0 \\ 1 & F_{rr2} > 0 \end{cases}$$

$$\sigma_{v3}(\zeta) = 0.5\sigma(\zeta) + 0.5\sigma(\zeta)\cos(2\alpha) + 0.5\tau(\zeta)\sin(2\alpha)$$

where

$$\sigma(\zeta) = \frac{4\beta_3 F_{rr}}{\pi d^2} + \frac{32 M_{ss}}{\pi d^3}\sin\zeta - \frac{32 M_{tt}}{\pi d^3}\cos\zeta$$

$$\tau(\zeta) = \frac{16 F_{rs}}{3\pi d^2}\sin^2\zeta + \frac{16 F_{rt}}{3\pi d^2}\cos^2\zeta$$

$$\alpha = \frac{1}{2}\tan^{-1}\frac{2\tau(\zeta)}{\sigma(\zeta)}$$

$$\beta_3 = \begin{cases} 0 & F_{rr} \le 0 \\ 1 & F_{rr} > 0 \end{cases}$$

The stresses are calculated for all directions, $0° \le \zeta \le 90°$, in order to find the maximum.

### OPT = 10

OPT = 10 invokes the failure criterion developed by Lee and Balur (2011). It is available for welds modeled by beam elements, solid elements, or solid assemblies. A detailed discussion of the criterion is given in the user's manual section for *DEFINE_SPOTWELD_FAILURE.

### OPT = 11

OPT = 11 invokes a resultant force-based failure criterion for beams. With corresponding load curves or tables, LCT and LCC, the resultant force at failure $F_{fail}$ can be defined as a function of loading direction $\gamma$ (curve) or loading direction $\gamma$ and effective strain rate $\dot{\varepsilon}$ (table):

$$F_{\text{fail}} = f(\gamma) \quad \text{or} \quad F_{\text{fail}} = f(\gamma, \dot{\varepsilon})$$

with the following definitions for loading direction (in degree) and effective strain rate:

$$\gamma = \tan^{-1}\left(\left|\frac{F_{\text{shear}}}{F_{\text{axial}}}\right|\right), \qquad \dot{\varepsilon} = \left[\frac{2}{3}\left(\dot{\varepsilon}_{\text{axial}}{}^2 + \dot{\varepsilon}^2_{\text{shear}}\right)\right]^{1/2}$$

It depends on the sign of the axial beam force, if LCT or LCC are used for failure condition:

$$F_{\text{axial}} > 0: \qquad \left[F^2_{\text{axial}} + F^2_{\text{shear}}\right]^{1/2} > \text{F}_{\text{fail,LCT}} \quad \rightarrow \quad \text{failure}$$

$$F_{\text{axial}} < 0: \qquad \left[F^2_{\text{axial}} + F^2_{\text{shear}}\right]^{1/2} > \text{F}_{\text{fail,LCC}} \quad \rightarrow \quad \text{failure}$$

### *MAT_SPOTWELD_DAIMLERCHRYSLER_{OPTION}

This is Material Type 100. The material model applies only to solid element type l. If hourglass type 4 is specified, then hourglass type 4 will be used; otherwise, hourglass type 6 will be automatically assigned. Hourglass type 6 is preferred.

Spot weld elements may be placed between any two deformable shell surfaces and tied with constraint contact, *CONTACT_TIED_SURFACE_TO_SURFACE, which eliminates the need to have adjacent nodes at spot weld locations. Spot weld failure is modeled using this card and *DEFINE_CONNECTION_PROPERTIES data. Details of the failure model can be found in Seeger, Feucht, Frank, Haufe, and Keding [2005].

> **NOTE:** It is advisable to include all spot welds, which provide the tracked nodes, and spot welded materials, which define the reference segments, within a single *CONTACT_TIED_SURFACE_TO_SURFACE interface. This contact type uses constraint equations. If multiple interfaces are treated independently, significant problems can occur if such interfaces share common nodes. An added benefit is that memory usage can be substantially less with a single interface.

Available options include:

<BLANK>

UNIAXIAL

The UNIAXIAL keyword option causes the transverse stresses and transverse strains to be zero for solid spot welds. The older uniaxial method, invoked with E < 0.0 on Card 1, assumed only the transverse stresses are zero. Compared to the older method, the UNIAXIAL keyword option increases the stability of the solver. See Remark 1 for more details.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | DT | TFAIL |
| Type | A | F | F | F | | | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EFAIL | | | | | | | NF |
| Type | F | | | | | | | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RS | ASFF | | TRUE_T | CON_ID | RFILTF | JTOL | DMGOPT |
| Type | F | I | | F | F | F | F | F |

**Damage Option Card.** LS-DYNA reads this optional card only if DMGOPT = -1 on Card 3.

| Card 3.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | TTOPT | | | | |
| Type | | | | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus.<br>LT.0.0: \|E\| is the Young's modulus. E < 0 invokes uniaxial stress for solid spot welds with the transverse stresses assumed to be zero. See Remark 1. This is for when the keyword option is unset (<BLANK>) only. |
| PR | Poisson's ratio |
| DT | Time step size for mass scaling, $\Delta t$ |
| TFAIL | Failure time if nonzero. If zero, this option is ignored. |
| EFAIL | Effective plastic strain in weld material at failure. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| NF | Number of failure function evaluations stored for filtering by time averaging. The default value is set to zero which is generally recommended unless oscillatory resultant forces are observed in the time history databases. Even though these welds should not oscillate significantly, this option was added for consistency with the other spot weld options. NF affects the storage since it is necessary to store the failure terms. When NF is nonzero, the resultants in the output databases are filtered. NF cannot exceed 30. |
| RS | Rupture strain. See Remark 2. |
| ASFF | Weld assembly simultaneous failure flag (see Remark 3):<br>EQ.0: Damaged elements fail individually.<br>EQ.1: Damaged elements fail when first reaches failure criterion. |
| TRUE_T | True weld thickness for single hexahedron solid weld elements. Note that the behavior of TRUE_T depends on TTOPT. See Remark 8 on *MAT_SPOTWELD.<br>**NOTE:** We do not recommend using TRUE_T. Instead, we recommend using TTOPT = 2 and leaving TRUE_T = 0.0. In many cases, TTOPT = 2 does a better job of removing the spurious moments. See Remark 9 on *MAT_SPOTWELD. |
| CON_ID | Connection ID of *DEFINE_CONNECTION card. A negative CON_ID deactivates failure; see Remark 5. |
| RFILTF | Smoothing factor on the effective strain rate (default is 0.0), potentially used in table DSIGY < 0 and in functions for PRUL.ge.2 (see *DEFINE_CONNECTION_PROPERTIES).<br>$\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$ |
| JTOL | Tolerance value for relative volume change (default: JTOL = 0.01). Solid element spot welds with a Jacobian less than JTOL will be eroded. |
| DMGOPT | Damage option flag:<br>EQ.-1: Flag to include Card 3.1 for additional damage fields. |

| VARIABLE | DESCRIPTION |
|---|---|
| TTOPT | Options for TRUE_T / weld failure behavior:<br>EQ.0: TRUE_T behavior of version R9 and later (see Remark 8 on *MAT_SPOTWELD)<br>EQ.1: TRUE_T behavior of version R8 and earlier (see Remark 8 on *MAT_SPOTWELD)<br>EQ.2: Weld failure is invariant with respect to the node numbering of weld elements. For this case, there is no need for the TRUE_T correction. We recommend using this option with TRUE_T set to 0.0. See Remark 9 on *MAT_SPOTWELD. |

**Remarks:**

1. **Loading solid welds uniaxially.** We have implemented two methods of loading solid and solid weld assemblies uniaxially. The older method is invoked by defining the elastic modulus, *E*, as a negative number where the absolute value of *E* is the desired value for *E*. This uniaxial option causes the two transverse stress terms to be assumed to be zero throughout the calculation. This assumption eliminates parasitic transverse stress that causes slow growth of plastic strain-based damage.

   The other method is invoked by setting *OPTION* to UNIAXIAL. This method is preferred. It causes the two transverse stress terms and the two transverse strains terms to be set to zero. It was added because we found that the older method sometimes induced spurious oscillations in the axial force, leading to premature failure.

   The motivation for the uniaxial options can be explained with a weld loaded in tension. Due to Poisson's effect, an element in tension would be expected to contract in the transverse directions. However, because the weld nodes are constrained to the mid-plane of shell elements, such contraction is only possible to the degree that the shell element contracts. In other words, the uniaxial stress state cannot be represented by the weld. For plastic strain-based damage, this effect can be particularly apparent as it causes tensile stress to continue to grow very large as the stress state becomes very nearly triaxial tension. In this stress state, plastic strain grows very slowly so it appears that damage calculation is failing to knock down the stress. By simply assuming that the transverse stresses are zero, the plastic strain grows as expected and damage is much more effective

2. **Connection properties.** This weld material is modeled with isotropic hardening plasticity. The yield stress and constant hardening modulus are assumed to

be those of the welded shell elements as defined in a *DEFINE_CONNECTION_PROPERTIES table. *DEFINE_CONNECTION_PROPERTIES data also define a failure function and the damage type. The interpretation of EFAIL and RS is determined by the choice of damage type. This is discussed in Remark 4 on *DEFINE_CONNECTION_PROPERTIES.

3. **Weld assembly failure.** For weld elements in an assembly (see RPBHX on *CONTROL_SPOTWELD_BEAM or *DEFINE_HEX_SPOTWELD_ASSEMBLY), the failure criterion is evaluated using the assembly cross section. If damage is not active, all elements will be deleted when the failure criterion is met. If damage is active, then damage is calculated independently in each element of the assembly. By default, elements of the assembly are deleted as damage in each element is complete. If ASFF = 1, then failure and deletion of all elements in the assembly will occur simultaneously when damage is complete in any one of the elements.

4. **Output.** Solid element force resultants for *MAT_SPOTWEL_DAIMLERCHRYSLER are written to the spot weld force file, swforc, and the file for element stresses and resultants for designated elements, ELOUT. Also, spot weld failure data is written to the file, dcfail.

5. **Deactivating weld failure.** An option to deactivate weld failure is switched on by setting CON_ID to a negative value where the absolute value of CON_ID becomes the connection ID. When weld failure is deactivated, the failure function is evaluated and output to swforc and dcfail, but the weld retains its full strength.

## *MAT_GEPLASTIC_SRATE_2000a

This is Material Type 101. The GEPLASTIC_SRATE_2000a material model characterizes General Electric's commercially available engineering thermoplastics subjected to high strain rate events. This material model features the variation of yield stress as a function of strain rate, cavitation effects of rubber modified materials, and automatic element deletion of either ductile or brittle materials.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | RATESF | EDOT0 | ALPHA | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSS | LCFEPS | LCFSIG | LCE | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's Modulus |
| PR | Poisson's ratio |
| RATESF | Constant in plastic strain rate equation |
| EDOT0 | Reference strain rate |
| ALPHA | Pressure sensitivity factor |
| LCSS | Load curve ID or table ID that defines the post yield material behavior. The values of this stress-strain curve are the difference of the yield stress and strain, respectively. This means the first values for both stress and strain should be zero. All subsequent values will define softening or hardening. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCFEPS | Load curve ID that defines the plastic failure strain as a function of strain rate |
| LCFSIG | Load curve ID that defines the maximum principal failure stress as a function of strain rate |
| LCE | Load curve ID that defines the unloading moduli as a function of plastic strain |

**Remarks:**

The constitutive model for this approach is:

$$\dot{\varepsilon}_p = \dot{\varepsilon}_0 \exp\{A[\sigma - S(\varepsilon_p)]\} \times \exp(-p\alpha A)$$

where $\dot{\varepsilon}_0$ and $A$ are rate dependent yield stress parameters, $S(\varepsilon_p)$ is the internal resistance (strain hardening), and $\alpha$ is a pressure dependence parameter.

In this material the yield stress may vary throughout the finite element model as a function of strain rate and hydrostatic stress. Post yield stress behavior is captured in material softening and hardening values. Finally, ductile or brittle failure measured by plastic strain or maximum principal stress, respectively, is accounted for by automatic element deletion.

Although this may be applied to a variety of engineering thermoplastics, GE Plastics have constants available for use in a wide range of commercially available grades of their engineering thermoplastics.

## *MAT_INV_HYPERBOLIC_SIN_{OPTION}

This is Material Type 102. It allows the modeling of temperature and rate-dependent plasticity, Sheppard and Wright [1979].

Available options include:

<BLANK>

THERMAL

such that the keyword card can appear as:

*MAT_INV_HYPERBOLIC_SIN or *MAT_102

*MAT_INV_HYPERBOLIC_SIN_THERMAL or *MAT_102_T

### Card Summary:

**Card 1a.** This card is included if the keyword option is unset (<BLANK>).

| MID | RO | E | PR | T | HC | VP | |
|---|---|---|---|---|---|---|---|

**Card 1b.** This card is included if the THERMAL keyword option is used.

| MID | RO | ALPHA | N | A | Q | G | EPSO |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if the keyword option is unset (<BLANK>).

| ALPHA | N | A | Q | G | EPSO | LCQ | |
|---|---|---|---|---|---|---|---|

**Card 2b.** This card is included if the THERMAL keyword option is used.

| LCE | LCPR | LCCTE | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

Card 1 for no keyword option (<BLANK>)

| Card 1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | T | HC | VP | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's Modulus |
| PR | Poisson's ratio |
| T | Initial temperature |
| HC | Heat generation coefficient |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |

Card 1 for the THERMAL keyword option

| Card 1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ALPHA | N | A | Q | G | EPS0 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| ALPHA | $\alpha$. See Remark 1. This $\alpha$ is not the coefficient of thermal expansion. |
| N | See Remark 1. |
| A | See Remark 1. |
| Q | See Remark 1. |
| G | See Remark 1. |
| EPS0 | Minimum strain rate considered in calculating $Z$ |

Card 2 for no keyword option (<BLANK>)

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | N | A | Q | G | EPS0 | LCQ | |
| Type | F | F | F | F | F | F | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | $\alpha$. See Remark 1. This $\alpha$ is not the coefficient of thermal expansion. |
| N | See Remark 1. |
| A | See Remark 1. |
| Q | See Remark 1. |
| G | See Remark 1. |
| EPS0 | Minimum strain rate considered in calculating $Z$. |
| LCQ | ID of curve specifying parameter $Q$:<br>GT.0: $Q$ as function of plastic strain.<br>LT.0: $Q$ as function of temperature. |

Card 2 for the THERMAL keyword option

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCE | LCPR | LCCTE | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCE | ID of curve defining the Young's modulus as a function of temperature |
| LCPR | ID of curve defining Poisson's ratio as a function of temperature |

| VARIABLE | DESCRIPTION |
|---|---|
| LCCTE | ID of curve defining the coefficient of thermal expansion as a function of temperature |

**Remarks:**

1. **Material description.** Resistance to deformation is both temperature and strain rate dependent. The flow stress equation is:

$$\sigma = \frac{1}{\alpha}\sinh^{-1}\left[\left(\frac{Z}{A}\right)^{\frac{1}{N}}\right]$$

where $Z$, the Zener-Holloman temperature compensated strain rate, is:

$$Z = \max(\dot{\varepsilon},\text{EPS0}) \times \exp\left(\frac{Q}{\text{GT}}\right) .$$

The units of the material constitutive constants are as follows: $A$ (1/sec), $N$ (dimensionless), $\alpha$ (1/MPa), the activation energy for flow, $Q$(J/mol), and the universal gas constant, $G$ (J/mol K). The value of $G$ only varies with the unit system chosen. Typically, it is either 8.3144 J/(mol K), or 40.8825 lb in/(mol R).

The final equation necessary to complete our description of high strain rate deformation is one that enables computing the temperature change during the deformation. In the absence of a coupled thermo-mechanical finite element code, we assume adiabatic temperature change and follow the empirical assumption that 90-95% of the plastic work is dissipated as heat. Thus, the heat generation coefficient is

$$\text{HC} \approx \frac{0.9}{\rho C_v}$$

where $\rho$ is the density of the material and $C_v$ is the specific heat.

2. **History variables.** $Z$ is output as history variable #11 when using the THERMAL keyword option and as history variable #8 when not using the THERMAL keyword option. See NEIPH and NEIPS on *DATABASE_EXTENT_BINARY to set the number of extra history variables output to d3plot.

## *MAT_ANISOTROPIC_VISCOPLASTIC

This is Material Type 103. This anisotropic-viscoplastic material model applies to shell, thick shell, solid, and SPH elements. The material constants may be fit directly or, if desired, stress as a function of strain data may be input and a least squares fit will be performed by LS-DYNA to determine the constants. Kinematic, isotropic, or a combination of kinematic and isotropic hardening may be used. A detailed description of this model can be found in the following references: Berstad, Langseth, and Hopperstad [1994]; Hopperstad and Remseth [1995]; and Berstad [1996]. Failure is based on effective plastic strain or by a user defined subroutine.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | FLAG | LCSS | ALPHA |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required

| QR1 | CR1 | QR2 | CR2 | QX1 | CX1 | QX2 | CX2 |
|---|---|---|---|---|---|---|---|

**Card 3a.** Include this card for shell elements and thick shell formulations 1, 2, and 6.

| VK | VM | R00 | R45 | R90 | | | |
|---|---|---|---|---|---|---|---|

**Card 3b.** Include this card for solid elements, SPH elements, and thick shell formulations 3, 5, and 7.

| VK | VM | F | G | H | L | M | N |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | FAIL | NUMINT | MACF | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | FLAG | LCSS | ALPHA |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress |
| FLAG | Flag:<br>EQ.0: Give all material parameters<br>EQ.1: Material parameters $Q_{r1}$, $C_{r1}$, $Q_{r2}$, and $C_{r2}$ for pure isotropic hardening ($\alpha = 1$) are determined by a least squares fit to the curve or table specified by the variable LCSS. If $\alpha$ is input as less than 1, $Q_{r1}$ and $Q_{r2}$ are then modified by multiplying them by the factor $\alpha$, while the factors $Q_{x1}$ and $Q_{x2}$ are taken as the product of the factor $(1-\alpha)$ and the original parameters $Q_{r1}$and $Q_{r2}$, respectively, for pure isotropic hardening. $C_{x1}$ is set equal to $C_{r1}$ and $C_{x2}$ is set equal to $C_{r2}$. $\alpha$ is input as variable ALPHA on Card 1.<br>EQ.2: Use load curve directly, that is, no fitting is required for the parameters $Q_{r1}$, $C_{r1}$, $Q_{r2}$, and $C_{r2}$. A table is not allowed and only isotropic hardening is implemented.<br>EQ.4: Use table definition directly. No fitting is required and the values for $Q_{r1}$, $C_{r1}$, $Q_{r2}$, $C_{r2}$, $V_k$, and $V_m$ are ignored. Only isotropic hardening is implemented, and this option is only available for solids. |
| LCSS | Load curve ID or Table ID. Card 2 is ignored with this option.<br>**Load Curve ID.** The load curve ID defines effective stress as a |

| VARIABLE | DESCRIPTION |
|---|---|
| | function of effective plastic strain. For this load curve case, viscoplasticity is modeled when the coefficients $V_k$ and $V_m$ are provided.<br>**Table ID.** The table consists of stress as a function of effective plastic strain curves indexed by strain rate. See Figure M24-1.<br>FLAG.EQ.1: Table is used to calculate the coefficients $V_k$ and $V_m$.<br>FLAG.EQ.4: Table is interpolated and used directly. This option is available only for solid elements. |
| ALPHA | $\alpha$ distribution of hardening used in the curve-fitting. $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QR1 | CR1 | QR2 | CR2 | QX1 | CX1 | QX2 | CX2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| QR1 | Isotropic hardening parameter $Q_{r1}$ |
| CR1 | Isotropic hardening parameter $C_{r1}$ |
| QR2 | Isotropic hardening parameter $Q_{r2}$ |
| CR2 | Isotropic hardening parameter $C_{r2}$ |
| QX1 | Kinematic hardening parameter $Q_{\chi 1}$ |
| CX1 | Kinematic hardening parameter $C_{\chi 1}$ |
| QX2 | Kinematic hardening parameter $Q_{\chi 2}$ |
| CX2 | Kinematic hardening parameter $C_{\chi 2}$ |

**Shell Elements Card.** This card is included for shell elements and thick shell formulations 1, 2, and 6.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VK | VM | R00 | R45 | R90 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| VK | Viscous material parameter $V_k$ |
| VM | Viscous material parameter $V_m$ |
| R00 | $R_{00}$ for shell (default = 1.0) |
| R45 | $R_{45}$ for shell (default = 1.0) |
| R90 | $R_{90}$ for shell (default = 1.0) |

**Solid Elements Card.** This card is included for solid elements, SPH elements, and thick shell formulations 3, 5, and 7.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VK | VM | F | G | H | L | M | N |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| VK | Viscous material parameter $V_k$ |
| VM | Viscous material parameter $V_m$ |
| F | $F$ in yield criteria (default = 1/2); see Remarks |
| G | $G$ in yield criteria (default = 1/2); see Remarks |
| H | $H$ in yield criteria (default = 1/2); see Remarks |
| L | $L$ in yield criteria (default = 3/2); see Remarks |
| M | $M$ in yield criteria (default = 3/2); see Remarks |

| VARIABLE | DESCRIPTION |
|---|---|
| N | $N$ in yield criteria (default = 3/2); see Remarks |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | FAIL | NUMINT | MACF | | | | |
| Type | F | F | F | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): |

EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying

| VARIABLE | DESCRIPTION |
|---|---|
| | BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| FAIL | Failure flag:<br>LT.0.0: User defined failure subroutine is called to determine failure. This is subroutine named, `MATUSR_103`, in dyn21.f.<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| NUMINT | Number of integration points which must fail before element deletion. If zero, all points must fail. This option applies to shell elements only. For the case of one point shells, NUMINT should be set to a value that is less than the number of through thickness integration points. Nonphysical stretching can sometimes appear in the results if all integration points have failed except for one point away from the midsurface because unconstrained nodal rotations will prevent strains from developing at the remaining integration point. In fully integrated shells, similar problems can occur. |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_- |

| VARIABLE | DESCRIPTION |
|---|---|
| | SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 6 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells and thick shells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |

**Remarks:**

The uniaxial stress-strain curve is given on the following form

$$\sigma(\varepsilon^p_{\text{eff}}, \dot{\varepsilon}^p_{\text{eff}}) = \sigma_0 + Q_{r1}\left[(1 - \exp(-C_{r1}\varepsilon^p_{\text{eff}}))\right] + Q_{r2}\left[1 - \exp\left(-C_{r2}\varepsilon^p_{\text{eff}}\right)\right] + Q_{\chi 1}\left[(1 - \exp(-C_{\chi 1}\varepsilon^p_{\text{eff}}))\right] + Q_{\chi 2}\left[(1 - \exp(-C_{\chi 2}\varepsilon^p_{\text{eff}}))\right] + V_k \dot{\varepsilon}^{p\ V_m}_{\text{eff}}$$

For solids the following yield criteria is used

$$F(\sigma_{22} - \sigma_{33})^2 + G(\sigma_{33} - \sigma_{11})^2 + H(\sigma_{11} - \sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2 = \left[\sigma(\varepsilon^p_{\text{eff}}, \dot{\varepsilon}^p_{\text{eff}})\right]^2$$

where $\varepsilon^p_{\text{eff}}$ is the effective plastic strain and $\dot{\varepsilon}^p_{\text{eff}}$ is the effective plastic strain rate. For shells the anisotropic behavior is given by $R_{00}$, $R_{45}$ and $R_{90}$. The model will work when the three first parameters in Card 3 are given values. When $V_k = 0$, the material will behave elasto-plastically. Default values are given by:

$$F = G = H = \frac{1}{2}$$

$$L = M = N = \frac{3}{2}$$

$$R_{00} = R_{45} = R_{90} = 1$$

Strain rates are accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}^p_{\text{eff}}}{C}\right)^{1/p}$$

To convert these constants set the viscoelastic constants, $V_k$ and $V_m$, to the following values:

$$V_k = \text{SIGY}\left(\frac{1}{C}\right)^{\frac{1}{p}}$$

$$V_m = \frac{1}{p}$$

If LCSS is nonzero, substitute the initial, quasi-static yield stress for SIGY in the equation for $V_k$ above.

This model properly treats rate effects. The viscoplastic rate formulation is an option in other plasticity models in LS-DYNA, such as *MAT_003 and *MAT_024, invoked by setting the parameter VP to 1.

## *MAT_ANISOTROPIC_PLASTIC

This is Material Type 103_P. This anisotropic-plastic material model is a simplified version of the MAT_ANISOTROPIC_VISCOPLASTIC above. This material model applies only to shell elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | LCSS | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QR1 | CR1 | QR2 | CR2 | | | | |
| Type | F | F | F | F | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R00 | R45 | R90 | S11 | S22 | S33 | S12 | |
| Type | F | F | F | F | F | F | F | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress |
| LCSS | Load curve ID. The load curve ID defines effective stress as a function of effective plastic strain. Card 2 is ignored with this option. |
| QR1 | Isotropic hardening parameter $Q_{r1}$ |
| CR1 | Isotropic hardening parameter $C_{r1}$ |
| QR2 | Isotropic hardening parameter $Q_{r2}$ |
| CR2 | Isotropic hardening parameter $C_{r2}$ |
| R00 | $R_{00}$ for anisotropic hardening |
| R45 | $R_{45}$ for anisotropic hardening |
| R90 | $R_{90}$ for anisotropic hardening |
| S11 | Yield stress in local $x$-direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$. |
| S22 | Yield stress in local $y$-direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$. |
| S33 | Yield stress in local $z$-direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$. |

| VARIABLE | DESCRIPTION |
|---|---|
| S12 | Yield stress in local $xy$-direction. This input is ignored if $(R_{00}, R_{45}, R_{90}) > 0$. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later. |
| XP, YP, ZP | $x_p, y_p, z_p$ define coordinates of point **p** for AOPT = 1 and 4. |
| A1, A2, A3 | $a_1, a_2, a_3$ define components of vector **a** for AOPT = 2. |
| D1, D2, D3 | $d_1, d_2, d_3$ define components of vector **d** for AOPT = 2. |
| V1, V2, V3 | $v_1, v_2, v_3$ define components of vector **v** for AOPT = 3 and 4. |
| BETA | Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card, see *ELEMENT_SHELL_BETA. |

**Remarks:**

If no load curve is defined for the effective stress versus effective plastic strain, the uniaxial stress-strain curve is given on the following form

$$\sigma(\varepsilon^p_{\text{eff}}) = \sigma_0 + Q_{r1}\left[1 - \exp\left(-C_{r1}\varepsilon^p_{\text{eff}}\right)\right] + Q_{r2}\left[1 - \exp\left(-C_{r2}\varepsilon^p_{\text{eff}}\right)\right]$$

where $\varepsilon_{\text{eff}}^{p}$ is the effective plastic strain. For shells the anisotropic behavior is given by $R_{00}$, $R_{45}$ and $R_{90}$, or the yield stress in the different direction. Default values are given by:

$$R_{00} = R_{45} = R_{90} = 1$$

if the variables R00, R45, R90, S11, S22, S33 and S12 are set to zero.

## *MAT_DAMAGE_1

This is Material Type 104. This is a continuum damage mechanics (CDM) model which includes anisotropy and viscoplasticity. The CDM model applies to shell, thick shell, and solid elements. A more detailed description of this model can be found in the paper by Berstad, Hopperstad, Lademo, and Malo [1999]. This material model can also model anisotropic damage behavior by setting FLAG to -1 in Card 2.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | LCSS | LCDS | |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if FLAG = -1.

| Q1 | C1 | Q2 | C2 | EPSD | EPSR | | FLAG |
|---|---|---|---|---|---|---|---|

**Card 2b.** This card is included if FLAG ≥ 0.

| Q1 | C1 | Q2 | C2 | EPSD | S | DC | FLAG |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included if the element type is shells or thick shells.

| VK | VM | R00 | R45 | R90 | | | |
|---|---|---|---|---|---|---|---|

**Card 3b.** The is card is included if the element type is solids.

| VK | VM | F | G | H | L | M | N |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | | CPH | MACF | YO | ALPHA | THETA | ETA |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | LCSS | LCDS | |
| Type | A | F | F | F | F | I | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress, $\sigma_0$ |
| LCSS | Load curve ID defining effective stress as a function of effective plastic strain. Isotropic hardening parameters on Card 2 are ignored with this option. |
| LCDS | Load curve ID defining nonlinear damage curve for FLAG = -1. |

**Anisotropic Damage Card.** This card is included if FLAG = -1.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | C1 | Q2 | C2 | EPSD | EPSR | | FLAG |
| Type | F | F | F | F | F | F | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| Q1 | Isotropic hardening parameter $Q_1$ |
| C1 | Isotropic hardening parameter $C_1$ |
| Q2 | Isotropic hardening parameter $Q_2$ |
| C2 | Isotropic hardening parameter $C_2$ |

| VARIABLE | DESCRIPTION |
|---|---|
| EPSD | Damage threshold $\varepsilon^p_{\text{eff,d}}$. Damage effective plastic strain when material softening begins (default = 0.0). |
| EPSR | Effective plastic strain at which material ruptures (logarithmic). |
| FLAG | Damage type flag:<br>EQ.-1: Anisotropic damage check (*only for shell elements*)<br>EQ.0: Standard isotropic damage (default)<br>EQ.1: Standard isotropic damage plus strain localization check (*only for shell elements*)<br>EQ.10: Enhanced isotropic damage<br>EQ.11: Enhanced isotropic damage plus strain localization check (*only for shell elements*) |

**Isotropic Damage Only Card.** This card is included if FLAG ≥ 0.

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | C1 | Q2 | C2 | EPSD | S | DC | FLAG |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| Q1 | Isotropic hardening parameter $Q_1$ |
| C1 | Isotropic hardening parameter $C_1$ |
| Q2 | Isotropic hardening parameter $Q_2$ |
| C2 | Isotropic hardening parameter $C_2$ |
| EPSD | Damage threshold $\varepsilon^p_{\text{eff,d}}$. Damage effective plastic strain when material softening begins (default = 0.0). |
| S | Damage material constant $S$ (default = $\frac{\sigma_0}{200}$) |
| DC | Critical damage value $D_C$ (default = 0.5). When the damage value, $D$, reaches this value, the element is deleted from the calculation. |

| VARIABLE | DESCRIPTION |
|---|---|
| FLAG | Damage type flag:<br>EQ.-1: Anisotropic damage check (*only for shell elements*)<br>EQ.0: Standard isotropic damage (default)<br>EQ.1: Standard isotropic damage plus strain localization check (*only for shell elements*)<br>EQ.10: Enhanced isotropic damage<br>EQ.11: Enhanced isotropic damage plus strain localization check (*only for shell elements*) |

**Shell Element Material Parameters Card.** This card is included for shell or thick shell elements.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VK | VM | R00 | R45 | R90 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| VK | Viscous material parameter, $V_k$ |
| VM | Viscous material parameter, $V_m$ |
| R00 | $R_{00}$ for shell (default = 1.0) |
| R45 | $R_{45}$ for shell (default = 1.0) |
| R90 | $R_{90}$ for shell (default = 1.0) |

**Brick Element Material Parameters Card.** This card is included for solid elements.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VK | VM | F | G | H | L | M | N |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| VK | Viscous material parameter, $V_k$ |
| VM | Viscous material parameter, $V_m$ |
| F | $F$ for solid (default = 1/2) |
| G | $G$ for solid (default = 1/2) |
| H | $H$ for solid (default = 1/2) |
| L | $L$ for solid (default = 3/2) |
| M | $M$ for solid (default = 3/2) |
| N | $N$ for solid (default = 3/2) |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | CPH | MACF | YO | ALPHA | THETA | ETA |
| Type | F | | F | I | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between |

| VARIABLE | DESCRIPTION |
|---|---|
| | the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| CPH | Microdefects closure parameter *h* for enhanced damage (FLAG ≥ 10). |
| MACF | Material axes change flag for solid elements: |
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation |
| | EQ.-3: Switch material axes *a* and *c* before BETA rotation |
| | EQ.-2: Switch material axes *a* and *b* before BETA rotation |
| | EQ.1: No change, default |
| | EQ.2: Switch material axes *a* and *b* after BETA rotation |
| | EQ.3: Switch material axes *a* and *c* after BETA rotation |
| | EQ.4: Switch material axes *b* and *c* after BETA rotation |
| | Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 6 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| VARIABLE | DESCRIPTION |
|---|---|
| Y0 | Initial damage energy release rate, $Y_0$, for enhanced damage (FLAG ≥ 10). |
| ALPHA | Exponent $\alpha$ for enhanced damage (FLAG ≥ 10) |
| THETA | Exponent $\theta$ for enhanced damage (FLAG ≥ 10) |
| ETA | Exponent $\eta$ for enhanced damage (FLAG ≥ 10) |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| BETA | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |

**Remarks:**

1. **Standard isotropic damage model (FLAG = 0 or 1).** The Continuum Damage Mechanics (CDM) model is based on an approach proposed by Lemaitre [1992]. The effective stress $\tilde{\sigma}$, which is the stress calculated over the section that effectively resist the forces, reads

$$\tilde{\sigma} = \frac{\sigma}{1-D}$$

where $D$ is the damage variable. The evolution equation for the damage variable is defined as

$$\dot{D} = \begin{cases} 0 & \text{for} \quad \varepsilon^p_{\text{eff}} \le \varepsilon^p_{\text{eff,d}} \\ \dfrac{Y}{S}\dot{\varepsilon}^p_{\text{eff}} & \text{for} \quad \varepsilon^p_{\text{eff}} > \varepsilon^p_{\text{eff,d}} \quad \text{and} \quad \sigma_1 > 0 \end{cases}$$

where $\varepsilon^p_{\text{eff,d}}$ is the damage threshold, $S$ is the so-called damage energy release rate, and $\sigma_1$ is the maximum principal stress. The damage energy density release rate is

$$Y = \frac{1}{2}\mathbf{e_e}\colon\mathbf{C}\colon\mathbf{e_e} = \frac{\sigma^2_{vm}R_v}{2E(1-D)^2}$$

where $E$ is Young's modulus and $\sigma_{vm}$ is the equivalent von Mises stress. The triaxiality function $R_v$ is defined as

$$R_v = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{vm}}\right)^2$$

with Poisson's ratio $\nu$ and hydrostatic stress $\sigma_H$.

2. **Enhanced isotropic damage model (FLAG = 10 or 11).** A more sophisticated damage model that includes crack closure effects (reduced damage under compression) and more flexibility in stress state dependence and functional expressions is invoked by setting FLAG = 10 or 11. The corresponding evolution equation for the damage variable is defined as

$$\dot{D} = \left(\frac{2\tau_{\max}}{\sigma_{vm}}\right)^{\eta}\left\langle\frac{Y-Y_0}{S}\right\rangle^{\alpha}(1-D)^{1-\theta}\,\dot{\varepsilon}^p_{\text{eff}}$$

where $\tau_{\max}$ is the maximum shear stress, $Y_0$ is the initial damage energy release rate, and $\eta$, $\alpha$, and $\theta$ are additional material constants. $\langle\ \rangle$ are the *Macauley brackets*. The damage energy density release rate is

$$Y = \frac{1+\nu}{2E}\left(\sum_{i=1}^{3}\left(\langle\tilde{\sigma}_i\rangle^2 + h\langle-\tilde{\sigma}_i\rangle^2\right)\right) - \frac{\nu}{2E}\left(\langle\tilde{\sigma}_H\rangle^2 + h\langle-\tilde{\sigma}_H\rangle^2\right)$$

where $\tilde{\sigma}_i$ are the principal effective stresses and $h$ is the microdefects closure parameter that accounts for different damage behavior in tension and compression. A value of $h \approx 0.2$ is typically observed in many experiments as stated in

Lemaitre [2000]. A parameter set of $h = 1$, $Y_0 = 0$, $\alpha = 1$, $\theta = 1$, and $\eta = 0$ should give the same results as the standard isotropic damage model (FLAG = 0 or 1) with $\varepsilon^p_{\text{eff,d}} = 0$ as long as $\sigma_1 > 0$.

3. **Strain localization check (FLAG = 1 or 11).** In order to add strain localization computation to the damage models above, parameter FLAG should be set to 1 (standard damage) or 11 (enhanced damage). An acoustic tensor-based bifurcation criterion is checked and history variable no. 4 is set to 1.0 if strain localization is indicated. This is only available for shell elements.

4. **Anisotropic damage model (FLAG = -1).** At each thickness integration points, an anisotropic damage law acts on the plane stress tensor in the directions of the principal total shell strains, $\varepsilon_1$ and $\varepsilon_2$, as follows:

$$\sigma_{11} = [1 - D_1(\varepsilon_1)]\sigma_{110}$$

$$\sigma_{22} = [1 - D_2(\varepsilon_2)]\sigma_{220}$$

$$\sigma_{12} = \left[1 - \frac{D_1 + D_2}{2}\right]\sigma_{120}$$

The transverse plate shear stresses in the principal strain directions are assumed to be damaged as follows:

$$\sigma_{13} = (1 - D_1/2)\sigma_{130}$$
$$\sigma_{23} = (1 - D_2/2)\sigma_{230}$$

In the anisotropic damage formulation, $D_1(\varepsilon_1)$ and $D_2(\varepsilon_2)$ are anisotropic damage functions for the loading directions 1 and 2, respectively. Stresses $\sigma_{110}$, $\sigma_{220}$, $\sigma_{120}$, $\sigma_{130}$ and $\sigma_{230}$ are stresses in the principal shell strain directions as calculated from the undamaged elastic-plastic material behavior. The strains $\varepsilon_1$ and $\varepsilon_2$ are the magnitude of the principal strains calculated upon reaching the damage thresholds. Damage can only develop for tensile stresses, and the damage functions $D_1(\varepsilon_1)$ and $D_2(\varepsilon_2)$are identical to zero for negative strains $\varepsilon_1$ and $\varepsilon_2$. The principal strain directions are fixed within an integration point as soon as either principal strain exceeds the initial threshold strain in tension. A more detailed description of the damage evolution for this material model is given in the description of Material 81.

5. **Anisotropic viscoplasticity.** The uniaxial stress-strain curve is given in the following form

$$\sigma\left(r, \dot{\varepsilon}^p_{\text{eff}}\right) = \sigma_0 + Q_1[1 - \exp(-C_1 r)] + Q_2[1 - \exp(-C_2 r)] + V_k \dot{\varepsilon}^{p\ V_m}_{\text{eff}} ,$$

where $r$ is the damage accumulated plastic strain, which can be calculated by

$$\dot{r} = \dot{\varepsilon}^p_{\text{eff}}(1 - D) .$$

For bricks the following yield criterion associated with the Hill criterion is used

$$F(\tilde{\sigma}_{22}-\tilde{\sigma}_{33})^2+G(\tilde{\sigma}_{33}-\tilde{\sigma}_{11})^2+H(\tilde{\sigma}_{11}-\tilde{\sigma}_{22})^2 \\ +2L\tilde{\sigma}_{23}^2+2M\tilde{\sigma}_{31}^2+2N\tilde{\sigma}_{12}^2=\sigma(r,\dot{\varepsilon}_{\text{eff}}^p)$$

where $r$ is the damage effective viscoplastic strain and $\dot{\varepsilon}_{\text{eff}}^p$ is the effective viscoplastic strain rate. For shells the anisotropic behavior is given by the R-values: $R_{00}$, $R_{45}$, and $R_{90}$. When $V_k = 0$, the material will behave as an elastoplastic material without rate effects. Default values for the anisotropic constants are given by:

$$F=G=H=\frac{1}{2}$$

$$L=M=N=\frac{3}{2}$$

$$R_{00}=R_{45}=R_{90}=1$$

so that isotropic behavior is obtained.

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1+\left(\frac{\dot{\varepsilon}}{C}\right)^{1/p}$$

To convert these constants, set the viscoelastic constants, $V_k$ and $V_m$, to the following values:

$$V_k=\sigma\left(\frac{1}{C}\right)^{\frac{1}{p}}$$

$$V_m=\frac{1}{p}$$

## *MAT_DAMAGE_2

This is Material Type 105. This is an elastic viscoplastic material model combined with continuum damage mechanics (CDM). This material model applies to shell, thick shell, and brick elements. The elastoplastic behavior is described in the description of material model 24. A more detailed description of the CDM model is given in the description of material model 104 above.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| C | P | LCSS | LCSR | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EPSD | S | DC | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | $10^{20}$ | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus; ignored if LCSS > 0 |
| FAIL | Failure flag:<br>EQ.0.0: Failure due to plastic strain is not considered.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic element deletion |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | | | | |
| Type | F | F | I | I | | | | |
| Default | 0.0 | 0.0 | 0 | 0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Strain rate parameter, *C*; see Remarks below. |
| P | Strain rate parameter, *p*; see Remarks below. |
| LCSS | Load curve ID or Table ID. Load curve ID defining effective stress as a function of effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored. The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate, See Figure M24-1. The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. |

| VARIABLE | DESCRIPTION |
|---|---|
| | Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters, C and P; the curve ID, LCSR; EPS1 - EPS8; and ES1 - ES8 are ignored if a Table ID is defined. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSD | S | DC | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | ↓ | 0.5 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EPSD | Damage threshold, $r_d$. Damage effective plastic strain when material softening begins. |
| S | Damage material constant $S$. Default = $\sigma_0/200$. |
| DC | Critical damage value $D_C$. When the damage value $D$ reaches this value, the element is deleted from the calculation. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1 - EPS8 | Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8. |

**Remarks:**

By defining the tangent modulus (ETAN), the stress-strain behavior becomes a bilinear curve. Alternately, a curve similar to that shown in Figure M10-1 is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress as a function of effective plastic strain curve ID (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition with table ID, LCSS, discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p},$$

where $\dot{\varepsilon}$ is the strain rate, $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.

3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in *DEFINE_TABLE must be used; see Figure M24-1

A fully viscoplastic formulation is used in this model.

## *MAT_ELASTIC_VISCOPLASTIC_THERMAL

This is Material Type 106. This is an elastic viscoplastic material with thermal effects or effects from an external variable (see *LOAD_EXTERNAL_VARIABLE and Remark 4).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | ALPHA | LCSS | FAIL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| QR1 | CR1 | QR2 | CR2 | QX1 | CX1 | QX2 | CX2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C | P | LCE | LCPR | LCSIGY | LCR | LCX | LCALPH |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| LCC | LCP | TREF | LCFAIL | NUSHIS | T1PHAS | T2PHAS | TOL |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card when NUSHIS > 0.

| FUSHI1 | FUSHI2 | FUSHI3 | FUSHI4 | FUSHI5 | FUSHI6 | FUSHI7 | FUSHI8 |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ALPHA | LCSS | FAIL |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio |
| SIGY | Initial yield stress |
| ALPHA | Coefficient of thermal expansion |
| LCSS | Load curve ID or Table ID. The load curve ID defines effective stress as a function of effective plastic strain. The table ID defines for each temperature value a load curve ID giving the stress as a function of effective plastic strain for that temperature (*DEFINE_-TABLE) or it defines for each temperature value a table ID which defines for each strain rate a load curve ID giving the stress as a function of effective plastic strain (*DEFINE_TABLE_3D). The stress as a function of effective plastic strain curve for the lowest value of temperature or strain rate is used if the temperature or strain rate falls below the minimum value. Likewise, maximum values cannot be exceeded. See Remark 1. |
| FAIL | Effective plastic failure strain for erosion |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QR1 | CR1 | QR2 | CR2 | QX1 | CX1 | QX2 | CX2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| QR1 | Isotropic hardening parameter, $Q_{r1}$ |
| CR1 | Isotropic hardening parameter, $C_{r1}$ |
| QR2 | Isotropic hardening parameter, $Q_{r2}$ |
| CR2 | Isotropic hardening parameter, $C_{r2}$ |
| QX1 | Kinematic hardening parameter, $Q_{\chi 1}$ |
| CX1 | Kinematic hardening parameter, $C_{\chi 1}$ |
| QX2 | Kinematic hardening parameter, $Q_{\chi 2}$ |
| CX2 | Kinematic hardening parameter, $C_{\chi 2}$ |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCE | LCPR | LCSIGY | LCR | LCX | LCALPH |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Viscous material parameter, $C$ |
| P | Viscous material parameter, $p$ |
| LCE | Load curve defining Young's modulus as a function of temperature or external variable (see Remark 4). E on Card 1 is ignored with this option. |
| LCPR | Load curve defining Poisson's ratio as a function of temperature or external variable (see Remark 4). PR on Card 1 is ignored with this option. |
| LCSIGY | Load curve defining the initial yield stress as a function of temperature or external variable (see Remark 4). SIGY on Card 1 is ignored with this option. |
| LCR | Load curve for scaling the isotropic hardening parameters QR1 and QR2 or the stress given by the load curve LCSS as a function of temperature or external variable (see Remark 4) |
| LCX | Load curve for scaling the kinematic hardening parameters QX1 and QX2 as a function of temperature or external variable (see Remark 4) |
| LCALPH | Load curve ID defining the instantaneous coefficient of thermal expansion as a function of temperature (or external variable; see Remark 4): $$d\varepsilon_{ij}^{\text{thermal}} = \alpha(T)dT\delta_{ij}\ .$$ ALPHA on Card 1 is ignored with this option. If LCALPH is defined as the negative of the load curve ID, the curve is assumed to define the coefficient relative to a reference temperature, TREF below, such that the total thermal strain is given by $$\varepsilon_{ij}^{\text{thermal}} = [\alpha(T)(T - T_{\text{ref}}) - \alpha(T_0)(T_0 - T_{\text{ref}})]\delta_{ij}\ .$$ |

| VARIABLE | DESCRIPTION |
|---|---|
| | Here, temperature $T_0$ is the initial temperature. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCC | LCP | TREF | LCFAIL | NUSHIS | T1PHAS | T2PHAS | TOL |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCC | Load curve for scaling the viscous material parameter $C$ as a function of temperature or external variable (see Remark 4). See Remark 1. |
| LCP | Load curve for scaling the viscous material parameter $P$ as a function of temperature or external variable (see Remark 4) |
| TREF | Reference temperature required if LCALPH is given with a negative curve ID |
| LCFAIL | Load curve defining the plastic failure strain as a function of temperature or external variable (see Remark 4). FAIL on Card 1 is ignored with this option. |
| NUHIS | Number of additional user-defined history variables, not used for EXTVAR keyword option. See Remarks 2 and 3. |
| T1PHAS | Lower temperature limit for cooling rate evaluation. Cooling rate can be used as input for user defined variables. |
| T2PHAS | Upper temperature limit for cooling rate evaluation. Cooling rate can be used as input for user-defined variables. |
| TOL | Optional tolerance for plasticity update. The default is $10^{-6}$ for solid elements and $10^{-3}$ for shells. This parameter overrides the default tolerance for all element types. |

**User History Card.** Additional card only for NUSHIS > 0.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FUSHI1 | FUSHI2 | FUSHI3 | FUSHI4 | FUSHI5 | FUSHI6 | FUSHI7 | FUSHI8 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| FUSH*i* | Function ID for user-defined history variables. See Remarks 2 and 3. |

**Remarks:**

1. **Viscous effects.** If LCSS is not given any value, the uniaxial stress-strain curve has the form:

$$\sigma(\varepsilon_{\text{eff}}^p) = \sigma_0 + Q_{r1}\left[1 - \exp(-C_{r1}\varepsilon_{\text{eff}}^p)\right] + Q_{r2}\left[1 - \exp(-C_{r2}\varepsilon_{\text{eff}}^p)\right] + Q_{\chi 1}\left[1 - \exp(-C_{\chi 1}\varepsilon_{\text{eff}}^p)\right] + Q_{\chi 2}\left[1 - \exp(-C_{\chi 2}\varepsilon_{\text{eff}}^p)\right] \ .$$

Viscous effects are accounted for using the Cowper and Symonds model, which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}_{\text{eff}}^p}{C}\right)^{1/p} .$$

2. **User-defined history data.** The user can define up to eight additional history variables that are added to the list of history variables (see table in Remark 3). These values can, for example, be used to evaluate the hardness of the material.

   The additional variables are to be given by respective *DEFINE_FUNCTION keywords as functions of the cooling rate between T2PHASE and T1PHASE, temperature, time, user-defined histories themselves, equivalent plastic strain, rate of the equivalent plastic strain, and the first six history variables (see table in Remark 3). A function declaration should, thus, look as follows:

```
*DEFINE_FUNCTION
1,user-defined history 1
uhist(trate,temp,time,usrhst1,usrhst2,…,usrhstn,epspl,
epsplrate,hist2,hist3,…,hist6)= ...
```

3. **History values.** The most important history variables of this material model are listed in the following table. To be able to post-process that data, parameters NEIPS (shells) or NEIPH (solids) must be defined on *DATABASE_EXTENT_-BINARY.

| History Variable # | Description |
|---|---|
| 1 | Temperature |
| 2 | Young's modulus |
| 3 | Poisson's ratio |
| 4 | Yield stress |
| 5 | Isotropic scaling factor |
| 6 | Kinematic scaling factor |
| 9 | Effective plastic strain rate |
| 10→ 9+NUSHIS | User-defined history variables |

4. **Effect of external variables.** By default, many of the material properties can be defined as a function of the temperature field, but this material supports material definitions based on a given distribution of an external variable defined by the keyword *LOAD_EXTERNAL_VARIABLE instead. Depending on the input in that loading card, one or more of the load curves LCE, LCPR, LCSIGY, LCR; LCX, LCALPH, LCC, LCP, and LCFAIL are evaluated not based on the temperature, but on the external variable. To do this, set IMP on *LOAD_EXTERNAL_-VARIABLE to the material property index for the desired material property. The following table lists the material property indices:

| Property index | Property name | Load curve |
|---|---|---|
| 1 | Young's modulus, $E$ | LCE |
| 2 | Poisson's ratio | LCPR |
| 3 | Initial yield stress | LCSIGY |
| 4 | Scale factor on the isotropic hardening parameters | LCR |
| 5 | Scale factor on the kinematic hardening parameters | LCX |
| 6 | Instantaneous coefficient of thermal expansion | LCALPH |
| 7 | Scale factor on the viscous material parameter $C$ | LCC |
| 8 | Scale factor on the viscous material parameter $P$ | LCP |
| 9 | Plastic failure strain | LCFAIL |

## *MAT_MODIFIED_JOHNSON_COOK

This is Material Type 107. Adiabatic heating is included in the material formulation. Material type 107 is not intended for use in a coupled thermal-mechanical analysis or in a mechanical analysis where temperature is prescribed using *LOAD_THERMAL.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | BETA | XS1 | CP | ALPHA |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| EODOT | TR | TM | TO | FLAG1 | FLAG2 | | |
|---|---|---|---|---|---|---|---|

**Card 3a.1.** This card is included if FLAG1 = 0.

| A | B | N | C | M | | | |
|---|---|---|---|---|---|---|---|

**Card 3a.2.** This card is included if FLAG1 = 0.

| Q1 | C1 | Q2 | C2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 3b.1.** This card is included if FLAG1 = 1.

| SIGA | B | BETA0 | BETA1 | | | | |
|---|---|---|---|---|---|---|---|

**Card 3b.2.** This card is included if FLAG1 = 1.

| A | N | ALPHA0 | ALPHA1 | | | | |
|---|---|---|---|---|---|---|---|

**Card 4a.** This card is included if FLAG2 = 0.

| DC | PD | D1 | D2 | D3 | D4 | D5 | |
|---|---|---|---|---|---|---|---|

**Card 4b.** This card is included if FLAG2 = 1.

| DC | WC | PHI | GAMMA | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| TC | TAUC | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | BETA | XS1 | CP | ALPHA |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\upsilon$ |
| BETA | Damage coupling parameter; see Equation (107.3).<br>EQ.0.0: No coupling between ductile damage and the constitutive relation<br>EQ.1.0: Full coupling between ductile damage and the constitutive relation |
| XS1 | Taylor-Quinney coefficient $\chi$, see Equation (107.21). Gives the portion of plastic work converted into heat (normally taken to be 0.9) |
| CP | Specific heat $C_p$; see Equation (107.21). |
| ALPHA | Thermal expansion coefficient, $\alpha$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0DOT | TR | TM | T0 | FLAG1 | FLAG2 | | |
| Type | F | F | F | F | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E0DOT | Quasi-static threshold strain rate ($\dot{\varepsilon}_0 = \dot{p}_0 = \dot{r}_0$); see Equation (107.12). See description for EPS0 in *MAT_015. |

| VARIABLE | DESCRIPTION |
|---|---|
| TR | Room temperature, see Equation (107.13) |
| TM | Melt temperature, see Equation (107.13) |
| T0 | Initial temperature |
| FLAG1 | Constitutive relation flag:<br>EQ.0: Modified Johnson-Cook constitutive relation; see Equation (107.11).<br>EQ.1: Zerilli-Armstrong constitutive relation, see Equation (107.14). |
| FLAG2 | Fracture criterion flag:<br>EQ.0: Modified Johnson-Cook fracture criterion; see Equation (107.15).<br>EQ.1: Cockcroft-Latham fracture criterion; see Equation (107.19). |

**Modified Johnson-Cook Constitutive Relation.** This card is included when FLAG1 = 0.

| Card 3a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | N | C | M | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A | Johnson-Cook yield stress $A$; see Equation (107.11). |
| B | Johnson-Cook hardening parameter $B$; see Equation (107.11). |
| N | Johnson-Cook hardening parameter $n$; see Equation (107.11). |
| C | Johnson-Cook strain rate sensitivity parameter $C$; see Equation (107.11). |
| M | Johnson-Cook thermal softening parameter $m$; see Equation (107.11). |

**Modified Johnson-Cook Constitutive Relation.** This card is included when FLAG1 = 0.

| Card 3a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | C1 | Q2 | C2 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| Q1 | Voce hardening parameter $Q_1$ (when $B = n = 0$); see Equation (107.11). |
| C1 | Voce hardening parameter $C_1$ (when $B = n = 0$); see Equation (107.11). |
| Q2 | Voce hardening parameter $Q_2$ (when $B = n = 0$); see Equation (107.11). |
| C2 | Voce hardening parameter $C_2$ (when $B = n = 0$); see Equation (107.11). |

**Modified Zerilli-Armstrong Constitutive Relation.** This card is included when FLAG1 = 1.

| Card 3b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGA | B | BETA0 | BETA1 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGA | Zerilli-Armstrong parameter $\alpha_a$; see Equation (107.14). |
| B | Zerilli-Armstrong parameter $B$; see Equation (107.14). |
| BETA0 | Zerilli-Armstrong parameter $\beta_0$; see Equation (107.14). |
| BETA1 | Zerilli-Armstrong parameter $\beta_1$; see Equation (107.14). |

**Modified Zerilli-Armstrong Constitutive Relation.** This card is included when FLAG1 = 1.

| Card 3b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | N | ALPHA0 | ALPHA1 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A | Zerilli-Armstrong parameter $A$; see Equation (107.14). |
| N | Zerilli-Armstrong parameter $n$; see Equation (107.14). |
| ALPHA0 | Zerilli-Armstrong parameter $\alpha_0$, see Equation (107.14). |
| ALPHA1 | Zerilli-Armstrong parameter $\alpha_1$, see Equation (107.14). |

**Modified Johnson-Cook Fracture Criterion.** This card is included when FLAG2 = 0.

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DC | PD | D1 | D2 | D3 | D4 | D5 | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| DC | Critical damage parameter $D_c$; see Equations (107.15) and (107.22). When the damage value $D$ reaches this value, the element is eroded from the calculation. |
| PD | Damage threshold; see Equation (107.15). |
| D1-D5 | Fracture parameters in the Johnson-Cook fracture criterion; see Equation (107.16). |

**Cockcroft Latham Fracture Criterion.** This card is included when FLAG2 = 1.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DC | WC | PHI | GAMMA | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| DC | Critical damage parameter $D_c$; see Equations (107.15) and (107.22). When the damage value $D$ reaches this value, the element is eroded from the calculation. |
| WC | Critical Cockcroft-Latham parameter $W_c$, see Equation (107.19). When the plastic work per volume reaches this value, the element is eroded from the simulation. |
| PHI | Extended Cockcroft-Latham parameter $\phi$, see Equation (107.20). |
| GAMMA | Extended Cockcroft-Latham parameter $\gamma$, see Equation (107.20). |

**Additional Element Erosion Criteria Card.**

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TC | TAUC | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TC | Critical temperature parameter $T_c$; see Equation (107.24). When the temperature, $T$, reaches this value, the element is eroded from the simulation. |
| TAUC | Critical shear stress parameter, $\tau_c$. When the maximum shear stress, $\tau$, reaches this value, the element is eroded from the simulation. |

**Remarks:**

An additive decomposition of the rate-of-deformation tensor $\mathbf{d}$ is assumed, that is,

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p + \mathbf{d}^t \tag{107.1}$$

where $\mathbf{d}^e$ is the elastic part, $\mathbf{d}^p$ is the plastic part and $\mathbf{d}^t$ is the thermal part.

The elastic rate-of-deformation $\mathbf{d}^e$ is defined by a linear hypo-elastic relation

$$\overset{\nabla J}{\tilde{\sigma}} = \left(K - \frac{2}{3}G\right)\mathrm{tr}(\mathbf{d}^e)\mathbf{I} + \mathbf{2}G\mathbf{d}^e \tag{107.2}$$

where $\mathbf{I}$ is the unit tensor, $K$ is the bulk modulus and $G$ is the shear modulus. The effective stress tensor is defined by

$$\tilde{\sigma} = \frac{\sigma}{1-\beta D} \tag{107.3}$$

where $\sigma$ is the Cauchy-stress and $D$ is the damage variable, while the Jaumann rate of the effective stress reads

$$\overset{\nabla J}{\tilde{\sigma}} = \dot{\tilde{\sigma}} - \mathbf{W}\cdot\tilde{\sigma} - \tilde{\sigma}\cdot\mathbf{W}^T \tag{107.4}$$

Here $\mathbf{W}$ is the spin tensor. The parameter $\beta$ is equal to unity for coupled damage and equal to zero for uncoupled damage.

The thermal rate-of-deformation $\mathbf{d}^T$ is defined by

$$\mathbf{d}^T = \alpha\dot{T}\mathbf{I} \tag{107.5}$$

where $\alpha$ is the linear thermal expansion coefficient and $T$ is the temperature.

The plastic rate-of-deformation is defined by the associated flow rule as

$$\mathbf{d}^p = \dot{r}\frac{\partial f}{\partial \sigma} = \frac{3}{2}\frac{\dot{r}}{1-\beta D}\frac{\tilde{\sigma}'}{\tilde{\sigma}_{\mathrm{eq}}} \tag{107.6}$$

where $(\cdot)'$ means the deviatoric part of the tensor, $r$ is the damage-equivalent plastic strain, $f$ is the dynamic yield function, that is,

$$\mathbf{d}^p = \dot{r}\frac{\partial f}{\partial \sigma} = \frac{3}{2}\frac{\dot{r}}{1-\beta D}\frac{\tilde{\sigma}'}{\tilde{\sigma}_{\mathrm{eq}}} \tag{107.6}$$

$$f = \sqrt{\frac{3}{2}\tilde{\sigma}':\tilde{\sigma}'} - \sigma_Y(r,\dot{r},T) \le 0, \qquad \dot{r} \ge 0, \qquad \dot{r}f = 0 \tag{107.7}$$

and $\tilde{\sigma}_{\mathrm{eq}}$ is the damage-equivalent stress,

$$\tilde{\sigma}_{\mathrm{eq}} = \sqrt{\frac{3}{2}\tilde{\sigma}':\tilde{\sigma}'} \tag{107.8}$$

The following plastic work conjugate pairs are identified

$$\dot{W}^p = \sigma:\mathbf{d}^p = \tilde{\sigma}_{\mathrm{eq}}\dot{r} = \sigma_{\mathrm{eq}}\dot{p} \tag{107.9}$$

where $\dot{W}^p$ is the specific plastic work rate, and the equivalent stress $\sigma_{\text{eq}}$ and the equivalent plastic strain $p$ are defined as

$$\sigma_{\text{eq}} = \sqrt{\frac{3}{2}\widetilde{\sigma}' : \widetilde{\sigma}'} = (1-\beta D)\tilde{\sigma}_{\text{eq}} \quad \dot{p} = \sqrt{\frac{2}{3}\mathbf{d}^p : \mathbf{d}^p} = \frac{\dot{r}}{(1-\beta D)} \tag{107.10}$$

The material strength $\sigma_Y$ is defined by:

1. The modified Johnson-Cook constitutive relation

$$\sigma_Y = \left\{A + Br^n + \sum_{i=1}^{2} Q_i[1-\exp(-C_i r)]\right\}(1+\dot{r}^*)^C(1-T^{*m}) \tag{107.11}$$

where $A$, $B$, $C$, $m$, $n$, $Q_1$, $C_1$, $Q_2$, and $C_2$ are material parameters; the normalized damage-equivalent plastic strain rate $\dot{r}^*$ is defined by

$$\dot{r}^* = \frac{\dot{r}}{\dot{\varepsilon}_0} \tag{107.12}$$

in which $\dot{\varepsilon}_0$ is a user-defined reference strain rate; and the homologous temperature reads

$$T^* = \frac{T - T_r}{T_m - T_r} \tag{107.13}$$

in which $T_r$ is the room temperature and $T_m$ is the melting temperature.

2. The Zerilli-Armstrong constitutive relation

$$\sigma_Y = \{\sigma_a + B\exp[-(\beta_0 - \beta_1 \ln\dot{r})T] + Ar^n \exp[-(\alpha_0 - \alpha_1 \ln\dot{r})T]\} \tag{107.14}$$

where $\sigma_a$, $B$, $\beta_0$, $\beta_1$, $A$, $n$, $\alpha_0$, and $\alpha_1$ are material parameters.

Damage evolution is defined by:

1. The extended Johnson-Cook damage evolution rule:

$$\Delta D = \begin{cases} 0 & p \le p_d \\ \dfrac{D_c \Delta p}{p_f - p_d} & p > p_d \end{cases} \tag{107.15}$$

where the current equivalent fracture strain $p_f = p_f(\sigma^*, \Delta p^*, T^*)$ is defined as

$$p_f = [D_1 + D_2\exp(D_3\sigma^*)](1+\Delta p^*)^{D_4}(1+D_5 T^*) \tag{107.16}$$

Here $D_1$, $D_2$, $D_3$, $D_4$, $D_5$, $D_C$, and $p_d$ are material parameters. The normalized equivalent plastic strain increment $\Delta p^*$ is defined by

$$\Delta p^* = \frac{\Delta p}{\dot{\varepsilon}_0} \tag{107.17}$$

and the stress triaxiality $\sigma^*$ reads

$$\sigma^* = \frac{\sigma_H}{\sigma_{\text{eq}}}, \qquad \sigma_H = \frac{1}{3} tr(\sigma) \tag{107.18}$$

2.The Cockcroft-Latham damage evolution rule:

$$\Delta D = \frac{D_C}{W_C} \max(\sigma_1, 0) \Delta p \tag{107.19}$$

where $D_C$ and $W_C$ are material parameters. This assumes that the material parameters $\phi$ or $\gamma$ are zero. If they are not, the uncoupled extended Cockcroft-Latham damage evolution rule is used:

$$\Delta D = \frac{\sigma_{\text{eq}}}{W_C} \max\left( \phi \frac{\sigma_1}{\sigma_{\text{eq}}} + (1 - \phi) \frac{\sigma_1 - \sigma_3}{\sigma_{\text{eq}}}, 0 \right)^{\gamma} \Delta p \tag{107.20}$$

Adiabatic heating is calculated as

$$\dot{T} = \chi \frac{\boldsymbol{\sigma} : \mathbf{d}^p}{\rho C_p} = \chi \frac{\tilde{\sigma}_{eq} \dot{r}}{\rho C_p} \tag{107.21}$$

where $\chi$ is the Taylor-Quinney parameter, $\rho$ is the density and $C_p$ is the specific heat. The initial value of the temperature $T_0$ may be specified by the user.

Element erosion occurs when one of the following several criteria are fulfilled:

1. The damage is greater than the critical value

$$D \geq D_C \tag{107.22}$$

2. The maximum shear stress is greater than a critical value

$$\tau_{\max} = \frac{1}{2} \max\{|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|\} \geq \tau_C \tag{107.23}$$

3. The temperature is greater than a critical value

$$T \geq T_C \tag{107.24}$$

| History Variable # | Description |
|---|---|
| 1 | Evaluation of damage $D$ |
| 2 | Evaluation of stress triaxiality $\sigma^*$ |
| 3 | Evaluation of damaged plastic strain $r$ |
| 4 | Evaluation of temperature $T$ |
| 5 | Evaluation of damaged plastic strain rate $\dot{r}$ |

| History Variable # | Description |
|---|---|
| 8 | Evaluation of plastic work per volume $W$ |
| 9 | Evaluation of maximum shear stress $\tau_{\max}$ |

## *MAT_ORTHO_ELASTIC_PLASTIC

This is Material Type 108. This model combines orthotropic elastic plastic behavior with an anisotropic yield criterion. This model is implemented only for shell elements.

### Card Summary:

**Card 1.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| MID | RO | E11 | E22 | G12 | PR12 | PR23 | PR31 |

**Card 2.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| SIGMA0 | LC | QR1 | CR1 | QR2 | CR2 | | |

**Card 3.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| R11 | R22 | R33 | R12 | | | | |

**Card 4.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| AOPT | BETA | | | | | | |

**Card 5.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | A1 | A2 | A3 | | |

**Card 6.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| V1 | V2 | V3 | D17 | D2 | D3 | | |

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E11 | E22 | G12 | PR12 | PR23 | PR31 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| E11 | Young's modulus in 11-direction |
| E22 | Young's modulus in 22-direction |
| G12 | Shear modulus in 12-direction |
| PR12 | Poisson's ratio 12 |
| PR23 | Poisson's ratio 23 |
| PR31 | Poisson's ratio 31 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGMA0 | LC | QR1 | CR1 | QR2 | CR2 | | |
| Type | F | I | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGMA0 | Initial yield stress, $\sigma_0$ |
| LC | Load curve defining effective stress as a function of effective plastic strain. If defined, QR1, CR1, QR2, and CR2 are ignored. |
| QR1 | Isotropic hardening parameter, $Q_{R1}$ |
| CR1 | Isotropic hardening parameter, $C_{R1}$ |
| QR2 | Isotropic hardening parameter, $Q_{R2}$ |
| CR2 | Isotropic hardening parameter, $C_{R2}$ |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R11 | R22 | R33 | R12 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| R11 | Yield criteria parameter, $R_{11}$ |
| R22 | Yield criteria parameter, $R_{22}$ |
| R33 | Yield criteria parameter, $R_{33}$ |
| R12 | Yield criteria parameter, $R_{12}$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | BETA | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. Nodes 1, 2 and 4 of an element are identical to the node used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector **v** with the normal to the plane of the element<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later. |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

The yield function is defined as

$$f = \bar{f}(\sigma) - [\sigma_0 + R(\varepsilon^p)] \ ,$$

where the equivalent stress $\sigma_{\text{eq}}$ is defined as an anisotropic yield criterion

$$\sigma_{\text{eq}} = \sqrt{F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{31}^2 + 2N\sigma_{12}^2} \ .$$

Here $F$, $G$, $H$, $L$, $M$ and $N$ are constants obtained by testing the material in different orientations. They are defined as

$$F = \frac{1}{2}\left(\frac{1}{R_{22}^2} + \frac{1}{R_{33}^2} - \frac{1}{R_{11}^2}\right), \qquad L = \frac{3}{2R_{23}^2}$$
$$G = \frac{1}{2}\left(\frac{1}{R_{33}^2} + \frac{1}{R_{11}^2} - \frac{1}{R_{22}^2}\right), \qquad M = \frac{3}{2R_{31}^2}$$
$$H = \frac{1}{2}\left(\frac{1}{R_{11}^2} + \frac{1}{R_{22}^2} - \frac{1}{R_{33}^2}\right), \qquad N = \frac{3}{2R_{12}^2}$$

The yield stress ratios are defined as follows

$$R_{11} = \frac{\overline{\sigma}_{11}}{\sigma_0}, \qquad R_{12} = \frac{\overline{\sigma}_{12}}{\tau_0}$$
$$R_{22} = \frac{\overline{\sigma}_{22}}{\sigma_0}, \qquad R_{23} = \frac{\overline{\sigma}_{23}}{\tau_0}$$
$$R_{33} = \frac{\overline{\sigma}_{33}}{\sigma_0}, \qquad R_{31} = \frac{\overline{\sigma}_{31}}{\tau_0}$$

where $\sigma_{ij}$ is the measured yield stress values, $\sigma_0$ is the reference yield stress, and $\tau_0 = \sigma_0/\sqrt{3}$.

The strain hardening, $R$, is either defined by the load curve or by the extended Voce law,

$$R(\varepsilon^p) = \sum_{i=1}^{2} Q_{Ri}[1 - \exp(-C_{Ri}\varepsilon^p)] \ ,$$

where $\varepsilon^p$ is the effective (or accumulated) plastic strain, and $Q_{Ri}$ and $C_{Ri}$ are strain hardening parameters.

## *MAT_JOHNSON_HOLMQUIST_CERAMICS

This is Material Type 110. This Johnson-Holmquist Plasticity Damage Model is useful for modeling ceramics, glass and other brittle materials. A more detailed description can be found in a paper by Johnson and Holmquist [1993].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | A | B | C | M | N |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS0 | T | SFMAX | HEL | PHEL | BETA | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | K1 | K2 | K3 | FS | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Density |
| G | Shear modulus |
| A | Intact normalized strength parameter |
| B | Fractured normalized strength parameter |
| C | Strength parameter (for strain rate dependence) |
| M | Fractured strength parameter (pressure exponent) |

| VARIABLE | DESCRIPTION |
|---|---|
| N | Intact strength parameter (pressure exponent) |
| EPS0 | Quasi-static threshold strain rate. See *MAT_015. |
| T | Maximum tensile pressure strength |
| SFMAX | Maximum normalized fractured strength (defaults to $10^{20}$ when set to 0.0). |
| HEL | Hugoniot elastic limit |
| PHEL | Pressure component at the Hugoniot elastic limit |
| BETA | Fraction of elastic energy loss converted to hydrostatic energy. It affects bulking pressure (history variable 1) that accompanies damage. |
| D1 | Parameter for plastic strain to fracture |
| D2 | Parameter for plastic strain to fracture (exponent) |
| K1 | First pressure coefficient (equivalent to the bulk modulus) |
| K2 | Second pressure coefficient |
| K3 | Third pressure coefficient |
| FS | Element deletion criterion:<br>LT.0.0: Fail if $p^* + t^* < 0$ (tensile failure)<br>EQ.0.0: No failure (default)<br>GT.0.0: Fail if the effective plastic strain > FS |

**Remarks:**

The equivalent stress for a ceramic-type material is given by

$$\sigma^* = \sigma_i^* - D\left(\sigma_i^* - \sigma_f^*\right) \ ,$$

where

$$\sigma_i^* = a(p^* + t^*)^n(1 + c\ln\dot{\varepsilon}^*)$$

represents the intact, undamaged behavior. The superscript, "*", indicates a normalized quantity. The stresses are normalized by the equivalent stress at the Hugoniot elastic limit, the pressures are normalized by the pressure at the Hugoniot elastic limit, and the

strain rate by the reference strain rate defined in the input. In this equation $a$ is the intact normalized strength parameter, $c$ is the strength parameter for strain rate dependence, $\dot{\varepsilon}^*$ is the normalized plastic strain rate, and

$$t^* = \frac{T}{\text{PHEL}}$$
$$p^* = \frac{p}{\text{PHEL}}$$

In the above, $T$ is the maximum tensile pressure strength, PHEL is the pressure component at the Hugoniot elastic limit, and $p$ is the pressure.

$$D = \sum \frac{\Delta\varepsilon^p}{\varepsilon_f^p}$$

represents the accumulated damage (history variable 2) based upon the increase in plastic strain per computational cycle and the plastic strain to fracture

$$\varepsilon_f^p = d_1(p^* + t^*)^{d_2}$$

and

$$\sigma_f^* = b(p^*)^m(1 + c\ln\dot{\varepsilon}^*) \le \text{SFMAX}$$

represents the damaged behavior. The parameter $d_1$ controls the rate at which damage accumulates. If it is set to 0, full damage occurs in one time step, that is, instantaneously. It is also the best parameter to vary when attempting to reproduce results generated by another finite element program.

In undamaged material, the hydrostatic pressure is given by

$$P = k_1\mu + k_2\mu^2 + k_3\mu^3$$

in compression and

$$P = k_1\mu$$

in tension where $\mu = \rho/\rho_0 - 1$. When damage starts to occur, there is an increase in pressure. A fraction, between 0 and 1, of the elastic energy loss, $\beta$, is converted into hydrostatic potential energy (pressure). The details of this pressure increase are given in the reference.

Given HEL and G, $\mu_{\text{hel}}$ can be found iteratively from

$$\text{HEL} = k_1\mu_{\text{hel}} + k_2\mu_{\text{hel}}^2 + k_3\mu_{\text{hel}}^3 + (4/3)g(\mu_{\text{hel}}/(1 + \mu_{\text{hel}})$$

and, subsequently, for normalization purposes,

$$P_{\text{hel}} = k_1\mu_{\text{hel}} + k_2\mu_{\text{hel}}^2 + k_3\mu_{\text{hel}}^3$$

and

$$\sigma_{\text{hel}} = 1.5(\text{hel} - p_{\text{hel}}) \ .$$

These are calculated automatically by LS-DYNA if $p_{hel}$ is zero on input.

## *MAT_JOHNSON_HOLMQUIST_CONCRETE

This is Material Type 111. This model can be used for concrete subjected to large strains, high strain rates and high pressures. The equivalent strength is expressed as a function of the pressure, strain rate, and damage. The pressure is expressed as a function of the volumetric strain and includes the effect of permanent crushing. The damage is accumulated as a function of the plastic volumetric strain, equivalent plastic strain and pressure. A more detailed description of this model can be found in the paper by Holmquist, Johnson, and Cook [1993].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | A | B | C | N | FC |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T | EPS0 | EFMIN | SFMAX | PC | UC | PL | UL |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | K1 | K2 | K3 | FS | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G | Shear modulus |
| A | Normalized cohesive strength |
| B | Normalized pressure hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Strain rate coefficient |
| N | Pressure hardening exponent |
| FC | Quasi-static uniaxial compressive strength |
| T | Maximum tensile hydrostatic pressure |
| EPS0 | Quasi-static threshold strain rate. See *MAT_015. |
| EFMIN | Amount of plastic strain before fracture |
| SFMAX | Normalized maximum strength |
| PC | Crushing pressure |
| UC | Crushing volumetric strain |
| PL | Locking pressure |
| UL | Locking volumetric strain |
| D1 | Damage constant |
| D2 | Damage constant |
| K1 | Pressure constant |
| K2 | Pressure constant |
| K3 | Pressure constant |
| FS | Failure type:<br>LT.0.0: Fail if damage strength < 0<br>EQ.0.0: Fail if $P^* + T^* \leq 0$ (tensile failure)<br>GT.0.0: Fail if the effective plastic strain > FS |

**Remarks:**

The normalized equivalent stress is defined as

$$\sigma^* = \frac{\sigma}{f_c'} \ ,$$

where $\sigma$ is the actual equivalent stress, and $f_c'$ is the quasi-static uniaxial compressive strength. The expression is defined as:

$$\sigma^* = \left[A(1-D) + BP^{*N}\right][1 + C\ln(\dot{\varepsilon}^*)] \ .$$

where $D$ is the damage parameter, $P^* = P/f_c'$ is the normalized pressure and $\dot{\varepsilon}^* = \dot{\varepsilon}/\dot{\varepsilon}_0$ is the dimensionless strain rate. The model incrementally accumulates damage, $D$, both from equivalent plastic strain and plastic volumetric strain, and is expressed as

$$D = \sum \frac{\Delta\varepsilon_p + \Delta\mu_p}{D_1(P^* + T^*)^{D_2}} \ .$$

Here, $\Delta\varepsilon_p$ and $\Delta\mu_p$ are the equivalent plastic strain and plastic volumetric strain, $D_1$and $D_2$ are material constants and $T^* = T/f_c'$ is the normalized maximum tensile hydrostatic pressure.

The damage strength, DS, is defined in compression when $P^* > 0$ as

$$\mathrm{DS} = f_c' \min\left[\mathrm{SFMAX}, A(1-D) + BP^{*N}\right][1 + C * \ln(\dot{\varepsilon}^*)]$$

or in tension if $P^* < 0$, as

$$\mathrm{DS} = f_c' \max\left[0, A(1-D) - A\left(\frac{P^*}{T}\right)\right][1 + C * \ln(\dot{\varepsilon}^*)] \ .$$

The pressure for fully dense material is expressed as

$$P = K_1\bar{\mu} + K_2\bar{\mu}^2 + K_3\bar{\mu}^3 \ ,$$

where $K_1$ , $K_2$ and $K_3$ are material constants and the modified volumetric strain is defined as

$$\bar{\mu} = \frac{\mu - \mu_{\mathrm{lock}}}{1 + \mu_{\mathrm{lock}}} \ ,$$

where $\mu_{\mathrm{lock}}$ is the locking volumetric strain.

## *MAT_FINITE_ELASTIC_STRAIN_PLASTICITY

This is Material Type 112. An elasto-plastic material with an arbitrary stress versus strain curve and arbitrary strain rate dependency can be defined. The elastic response of this model uses a finite strain formulation so that large elastic strains can develop before yielding occurs. This model is available for solid elements only. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | none | 0.0 | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | | | | |
| Type | F | F | I | I | | | | |
| Default | 0.0 | 0.0 | 0 | 0 | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus; ignored if LCSS > 0 |
| C | Strain rate parameter, *C*; see Remarks below. |
| P | Strain rate parameter, *p*; see Remarks below. |
| LCSS | Load curve ID or table ID.<br>**Load Curve ID.** The load curve defines effective stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored.<br>**Table ID.** The table defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see Figure M24-1. The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters, *C* and *p*; the curve ID LCSR; EPS1 - EPS8; and ES1 - ES8 are ignored if a table ID is defined. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1 - EPS8 | Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero, the yield stress is extrapolated to determine the initial yield. If this option is used, SIGY and ETAN are ignored and may be input as zero. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8. |

**Remarks:**

By defining the tangent modulus ETAN, the stress strain behavior is treated using a bilinear stress strain curve. Alternately, a curve similar to that shown in Figure M10-1 is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress as a function of effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p},$$

where $\dot{\varepsilon}$ is the strain rate, $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.

3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in *DEFINE_TABLE must be used; see Figure M24-1.

### *MAT_TRIP

This is Material Type 113. This isotropic elasto-plastic material model applies to shell elements only. It features a special hardening law aimed at modelling the temperature dependent hardening behavior of austenitic stainless TRIP-steels. TRIP stands for Transformation Induced Plasticity. A detailed description of this material model can be found in Hänsel, Hora, and Reissner [1998] and Schedin, Prentzas, and Hilding [2004].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | CP | T0 | TREF | TA0 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | C | D | P | Q | E0MART | VM0 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AHS | BHS | M | N | EPS0 | HMART | K1 | K2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| CP | Adiabatic temperature calculation option:<br>EQ.0.0: Adiabatic temperature calculation is disabled. |

| VARIABLE | DESCRIPTION |
|---|---|
| | GT.0.0: CP is the specific heat $C_p$. Adiabatic temperature calculation is enabled. |
| T0 | Initial temperature $T_0$ of the material if adiabatic temperature calculation is enabled. |
| TREF | Reference temperature for output of the yield stress as history variable 1. |
| TA0 | Reference temperature $T_{A0}$, the absolute zero for the used temperature scale. For example, TA0 is -273.15 if the Celsius scale is used and 0.0 if the Kelvin scale is used. |
| A | Martensite rate equation parameter $A$; see Remarks below. |
| B | Martensite rate equation parameter $B$; see Remarks below. |
| C | Martensite rate equation parameter $C$; see Remarks below. |
| D | Martensite rate equation parameter $D$; see Remarks below. |
| P | Martensite rate equation parameter $p$; see Remarks below. |
| Q | Martensite rate equation parameter $Q$; see Remarks below. |
| E0MART | Martensite rate equation parameter $E_{0(\text{mart})}$; see Remarks below. |
| VM0 | The initial volume fraction of martensite $0.0 < V_{m0} < 1.0$ may be initialised using two different methods:<br>GT.0.0: $V_{m0}$ is set to VM0.<br>LT.0.0: Can be used only when there are initial plastic strains $\varepsilon^p$ present, such as when using *INITIAL_STRESS_SHELL. The absolute value of VM0 is then the load curve ID for a function $f$ that sets $V_{m0} = f(\varepsilon^p)$. The function $f$ must be a monotonically nondecreasing function of $\varepsilon^p$. |
| AHS | Hardening law parameter $A_{\text{HS}}$; see Remarks below. |
| BHS | Hardening law parameter $B_{\text{HS}}$; see Remarks below. |
| M | Hardening law parameter $m$; see Remarks below. |
| N | Hardening law parameter $n$; see Remarks below. |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS0 | Hardening law parameter $\varepsilon_0$; see Remarks below. |
| HMART | Hardening law parameter $\Delta H_{\gamma\rightarrow\alpha'}$; see Remarks below. |
| K1 | Hardening law parameter $K_1$; see Remarks below. |
| K2 | Hardening law parameter $K_2$; see Remarks below. |

**Remarks:**

Here a short description is given of the TRIP-material model. The material model uses the von Mises yield surface in combination with isotropic hardening. The hardening is temperature dependent. Therefore, this material model must be run either in a coupled thermo-mechanical solution, using prescribed temperatures or using the adiabatic temperature calculation option. Setting the parameter CP to the specific heat, $C_p$, of the material activates the adiabatic temperature calculation that calculates the temperature rate from the equation

$$\dot{T} = \sum_{i,j} \frac{\sigma_{ij} D^p_{ij}}{\rho C_p} \ ,$$

where $\sigma : \mathbf{D}^p$ (the numerator) is the plastically dissipated heat. Using the Kelvin scale is recommended, even though other scales may be used without problems.

The hardening behavior is described by the following equations. The Martensite rate equation is

$$\frac{\partial V_m}{\partial \bar{\varepsilon}^p} = \begin{cases} \dfrac{\mathrm{A}}{} 0 & \varepsilon < E_{0(\mathrm{mart})} \\ \dfrac{\mathrm{B}}{A} V_m^p \left(\dfrac{1 - V_m}{V_m}\right)^{\frac{B+1}{B}} \dfrac{[1 - \tanh(\mathrm{C} + \mathrm{D} \times T)]}{2} \exp\left(\dfrac{Q}{T - T_{A0}}\right) & \bar{\varepsilon}^p \geq E_{0(\mathrm{mart})} \end{cases}$$

where $\bar{\varepsilon}^p$ is the effective plastic strain and $T$ is the temperature.

The martensite fraction is integrated from the above rate equation:

$$V_m = \int_0^{\varepsilon} \frac{\partial V_m}{\partial \bar{\varepsilon}^p} d\bar{\varepsilon}^p \ .$$

It always holds that $0.0 < V_m < 1.0$. The initial martensite content is $V_{m0}$ and must be greater than zero and less than 1.0. Note that $V_{m0}$ is not used during a restart or when initializing the $V_{m0}$ history variable using *INITIAL_STRESS_SHELL.

The yield stress is:

$$\sigma_y = \{B_{HS} - (B_{HS} - A_{HS})\exp(-m[\bar{\varepsilon}^p + \varepsilon_0]^n)\}(K_1 + K_2 T) + \Delta H_{\gamma\to\alpha'} V_m \ .$$

The parameters $p$ and $B$ should fulfill the following condition

$$\frac{1+B}{B} < p \ .$$

If the condition is not fulfilled, then the martensite rate will approach infinity as $V_m$ approaches zero. Setting the parameter $\varepsilon_0$ larger than zero (typical range 0.001 - 0.02) is recommended. Apart from the effective true strain a few additional history variables are output; see below.

**Output History Variables:**

| Variable | Description |
|---|---|
| 1 | Yield stress of material at temperature TREF. Useful to evaluate the strength of the material after e.g., a simulated forming operation. |
| 2 | Volume fraction martensite, $V_m$ |
| 3 | CP.EQ.0.0: not used<br>CP.GT.0.0: temperature from adiabatic temperature calculation |

## *MAT_LAYERED_LINEAR_PLASTICITY

This is Material Type 114. It is a layered elastoplastic material with an arbitrary stress as a function of strain curve. An arbitrary strain rate dependency can also be defined. This material must be used with the user-defined integration rules (see *INTEGRATION-SHELL) for modeling laminated composite and sandwich shells where each layer can be represented by elastoplastic behavior with constitutive constants that vary from layer to layer. Lamination theory is applied to correct for the assumption of a uniform constant shear strain through the thickness of the shell. Unless this correction is applied, the stiffness of the shell can be grossly incorrect, leading to poor results. Generally, without the correction, the results are too stiff. This model is available for shell elements only. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | $10^{20}$ | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | | | | |
| Type | F | F | I | I | | | | |
| Default | 0.0 | 0.0 | 0 | 0 | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus; ignored if LCSS > 0 is defined. |
| FAIL | Failure flag:<br>LT.0.0: User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure.<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic element deletion |
| C | Strain rate parameter, *C*; see Remarks below. |
| P | Strain rate parameter, *p*; see Remarks below. |
| LCSS | Load curve ID or Table ID.<br>**Load Curve ID.** The load curve defines effective stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored. |

| VARIABLE | DESCRIPTION |
|---|---|
| | **Table ID.** The table defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see Figure M24-1. The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. C, P, LCSR, EPS1 – EPS8, and ES1 – ES8 are ignored if a table ID is defined. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress |
| EPS1 - EPS8 | Effective plastic strain values (optional if SIGY is defined). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. WARNING: If the first point is nonzero the yield stress is extrapolated to determine the initial yield. If this option is used, SIGY and ETAN are ignored and may be input as zero. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8. |

**Remarks:**

The stress strain behavior may be treated by a bilinear stress strain curve by defining the tangent modulus, ETAN. Alternately, a curve similar to that shown in Figure M10-1 is expected to be defined by (EPS1, ES1) - (EPS8, ES8); however, an effective stress as a function of effective plastic strain curve (LCSS) may be input instead if eight points are insufficient. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p},$$

where $\dot{\varepsilon}$ is the strain rate; $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. This curve defines the scale factor as a function of strain rate.

3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used. Then the table input in *DEFINE_TABLE must be used; see Figure M24-1.

### *MAT_UNIFIED_CREEP

This is Material Type 115. This is an elastic creep model for modeling creep behavior when plastic behavior is not considered.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | A | N | M | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| A | Stress coefficient |
| N | Stress exponent |
| M | Time exponent |

**Remarks:**

The effective creep strain, $\bar{\varepsilon}^c$, given as:

$$\bar{\varepsilon}^c = A\bar{\sigma}^n \bar{t}^m \ ,$$

where $A$, $n$, and $m$ are constants and $\bar{t}$ is the effective time. The effective stress, $\bar{\sigma}$, is defined as:

$$\bar{\sigma} = \sqrt{\frac{3}{2}\sigma_{ij}\sigma_{ij}} \ .$$

The creep strain, therefore, is only a function of the deviatoric stresses. The volumetric behavior for this material is assumed to be elastic. By varying the time constant $m$

primary creep ($m < 1$), secondary creep ($m = 1$), and tertiary creep ($m > 1$) can be modeled. This model is described by Whirley and Henshall [1992].

## *MAT_UNIFIED_CREEP_ORTHO

This is Material Type 115_O. This is an orthotropic elastic creep model for modeling creep behavior when plastic behavior is not considered. This material is available for solid elements, thick shell element formulations 3, 5, and 7, and SPH elements. It is available for both explicit and implicit dynamics.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | E2 | E3 | PR21 | PR31 | PR32 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G12 | G23 | G13 | A | N | M | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E*i* | Young's moduli |
| PR*ij* | Elastic Poisson's ratios |
| G*ij* | Elastic shear moduli |
| A | Stress coefficient |
| N | Stress exponent |
| M | Time exponent |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between |

| VARIABLE | DESCRIPTION |
|---|---|
| | the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP, YP, ZP | Define coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Define components of vector **a** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Define components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Define components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_TSHELL_BETA or *ELEMENT_-SOLID_ORTHO. |

**Remarks:**

The stress-strain relationship is based on an additive split of the strain,

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}_e + \dot{\boldsymbol{\varepsilon}}_c \ .$$

Here, the multiaxial creep strain is given by

$$\dot{\boldsymbol{\varepsilon}}_c = \dot{\bar{\varepsilon}}_c \frac{2\boldsymbol{s}}{3\bar{\sigma}} \ ,$$

and $\bar{\varepsilon}^c$ is the effective creep strain, $\boldsymbol{s}$ the deviatoric stress

$$\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})\mathbf{I} \ .$$

and $\bar{\sigma}$ the effective stress

$$\bar{\sigma} = \sqrt{\frac{3}{2}\boldsymbol{s}:\boldsymbol{s}} \ .$$

The effective creep strain is given by

$$\dot{\bar{\varepsilon}}^c = A\bar{\sigma}^N t^M,$$

where $A$, $N$, and $M$ are constants.

The stress increment is given by

$$\Delta\boldsymbol{\sigma} = \mathbf{C}\Delta\boldsymbol{\varepsilon}_e = \mathbf{C}(\Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}_c),$$

where the constitutive matrix **C** is taken as orthotropic and can be represented in Voigt notation by its inverse as

$$\mathbf{C}^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & & & \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & & & \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & & & \\ & & & \frac{1}{G_{12}} & & \\ & & & & \frac{1}{G_{23}} & \\ & & & & & \frac{1}{G_{13}} \end{bmatrix}.$$

## *MAT_COMPOSITE_LAYUP

This is Material Type 116. This material is for modeling the elastic responses of composite layups that have an arbitrary number of layers through the shell thickness. A pre-integration is used to compute the extensional, bending, and coupling stiffness for use with the Belytschko-Tsay resultant shell formulation. The angles of the local material axes are specified from layer to layer in the *SECTION_SHELL input. This material model must be used with the user defined integration rule for shells (see *INTEGRATION_SHELL) which allows the elastic constants to change from integration point to integration point. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero. Note that this shell *does not use laminated shell theory* and that storage is allocated for just one integration point (as reported in d3hsp) regardless of the layers defined in the integration rule.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | | | | |
| Type | F | F | F | F | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in the $a$-direction |
| EB | $E_b$, Young's modulus in the $b$-direction |
| EC | $E_c$, Young's modulus in the $c$-direction |
| PRBA | $\nu_{ba}$, Poisson's ratio $ba$ |
| PRCA | $\nu_{ca}$, Poisson's ratio $ca$ |
| PRCB | $\nu_{cb}$, Poisson's ratio $cb$ |
| GAB | $G_{ab}$, shear modulus $ab$ |
| GBC | $G_{bc}$, shear modulus $bc$ |
| GCA | $G_{ca}$, shear modulus $ca$ |
| AOPT | Material axes option, see Figure M2-1:<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector, **v**, with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Define coordinates of point $p$ for AOPT = 1 and 4. |
| A1, A2, A3 | Define components of vector **a** for AOPT = 2. |
| V1, V2, V3 | Define components of vector **v** for AOPT = 3 and 4. |
| D1, D2, D3 | Define components of vector **d** for AOPT = 2. |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

This material law is based on standard composite lay-up theory. The implementation, [Jones 1975], allows the calculation of the force, $N$, and moment, $M$, stress resultants from:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_z \end{Bmatrix}$$

where $A_{ij}$ is the extensional stiffness, $D_{ij}$is the bending stiffness, and $B_{ij}$ is the coupling stiffness which is a null matrix for symmetric lay-ups. The mid-surface strains and curvatures are denoted by $\varepsilon_{ij}^0$ and $\kappa_{ij}$, respectively. Since these stiffness matrices are symmetric, 18 terms are needed per shell element in addition to the shell resultants which are integrated in time. This is considerably less storage than would typically be required with through thickness integration which requires a minimum of eight history variables per integration point. For instance, if 100 layers are used, 800 history variables would be stored. Not only is memory much less for this model, but the CPU time required is also considerably reduced.

## *MAT_COMPOSITE_MATRIX

This is Material Type 117. This material is used for modeling the elastic responses of composites where a pre-integration is used to compute the extensional, bending, and coupling stiffness coefficients for use with the Belytschko-Tsay resultant shell formulation. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

**NOTE:** This material does not support specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C11 | C12 | C22 | C13 | C23 | C33 | C14 | C24 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C34 | C44 | C15 | C25 | C35 | C45 | C55 | C16 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C26 | C36 | C46 | C56 | C66 | AOPT | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| CIJ | $C_{ij}$, coefficients of stiffness matrix in the material coordinate system |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |

EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by an angle BETA.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal

LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_CO-

| VARIABLE | DESCRIPTION |
|---|---|
| | ORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

The calculation of the force, $N_{ij}$, and moment, $M_{ij}$, stress resultants is given in terms of the membrane strains, $\varepsilon_i^0$, and shell curvatures, $\kappa_i$, as:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \kappa_x \\ \kappa_y \\ \kappa_z \end{bmatrix}$$

where $C_{ij} = C_{ji}$. In this model this symmetric matrix is transformed into the element local system and the coefficients are stored as element history variables. In *MAT_COMPOSITE_DIRECT, the resultants are already assumed to be given in the element local system which reduces the storage since the 21 coefficients are not stored as history variables as part of the element data.

The shell thickness is built into the coefficient matrix and, consequently, within the part ID, which references this material ID. The thickness must be uniform.

## *MAT_COMPOSITE_DIRECT

This is Material Type 118. This material is used for modeling the elastic responses of composites where a pre-integration is used to compute the extensional, bending, and coupling stiffness coefficients for use with the Belytschko-Tsay resultant shell formulation. Since the stresses are not computed in the resultant formulation, the stresses output to the binary databases for the resultant elements are zero.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C11 | C12 | C22 | C13 | C23 | C33 | C14 | C24 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C34 | C44 | C15 | C25 | C35 | C45 | C55 | C16 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C26 | C36 | C46 | C56 | C66 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| CIJ | Coefficients of the stiffness matrix, $C_{ij}$ |

**Remarks:**

The calculation of the force, $N_{ij}$, and moment, $M_{ij}$, stress resultants is given in terms of the membrane strains, $\varepsilon_i^0$, and shell curvatures, $\kappa_i$, as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_z^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

where $C_{ij} = C_{ji}$. In this model the stiffness coefficients are already assumed to be given in the element local system which reduces the storage. Great care in the element orientation and choice of the local element system, see *CONTROL_ACCURACY, must be observed if this model is used.

The shell thickness is built into the coefficient matrix and, consequently, within the part ID, which references this material ID, the thickness must be uniform.

## *MAT_GENERAL_NONLINEAR_6DOF_DISCRETE_BEAM

This is Material Type 119. It is a very general spring and damper model. This beam is based on *MAT_SPRING_GENERAL_NONLINEAR. Additional unloading options have been included. The two nodes defining the beam may be coincident to give a zero length beam or offset to give a finite length beam. For finite length discrete beams the absolute value of the variable SCOOR in the *SECTION_BEAM input should be set to a value of 2.0 or 3.0 to give physically correct behavior. A triad is used to orient the beam for the directional springs.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | KT | KR | IUNLD | OFFSET | DAMPF | IFLAG |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCIDTR | LCIDTS | LCIDTT | LCIDRR | LCIDRS | LCIDRT | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| LCIDTUR | LCIDTUS | LCIDTUT | LCIDRUR | LCIDRUS | LCIDRUT | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| LCIDTDR | LCIDTDS | LCIDTDT | LCIDRDR | LCIDRDS | LCIDRDT | IVDFLG | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| LCIDTER | LCIDTES | LCIDTET | LCIDRER | LCIDRES | LCIDRET | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| UTFAILR | UTFAILS | UTFAILT | WTFAILR | WTFAILS | WTFAILT | FCRIT | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| UCFAILR | UCFAILS | UCFAILT | WCFAILR | WCFAILS | WCFAILT | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| IUR | IUS | IUT | IWR | IWS | IWT | | |
|---|---|---|---|---|---|---|---|

***Card 9.*** This card is read if IFLAG = 2. It is optional, but if it is included, Cards 10 and 11 must also be included.

| LM1R1S | LM1R2S | LM1R1T | LM1R2T | LM2R1S | LM2R1T | | |
|---|---|---|---|---|---|---|---|

***Card 10.*** This card is read if IFLAG = 2. It is optinal but must be included if Card 9 is included.

| LUM1R1S | LUM1R2S | LUM1R1T | LUM1R2T | LUM2R1S | LUM2R1T | | |
|---|---|---|---|---|---|---|---|

***Card 11.*** This card is read if IFLAG = 2. It is optional but must be included if Card 9 is included.

| KUM1R1S | KUM1R2S | KUM1R1T | KUM1R2T | KUM2R1S | KUM2R1T | KUM2R2S | KUM2R2T |
|---|---|---|---|---|---|---|---|

***Card 12.*** This card is read if IFLAG = 2. It is optional, but if it is included Cards 13 and 14 must be included.

| E1TR | E2TR | E1RR | E2RR | E1RS | E2RS | E1RT | E2RT |
|---|---|---|---|---|---|---|---|

***Card 13.*** This card is read if IFLAG = 2. It is optional but must be included if Card 12 is included.

| E1M1R1S | E2M1R1S | E1M1R2S | E2M1R2S | E1M1R1T | E2M1R1T | E1M1R2T | E2M1R2T |
|---|---|---|---|---|---|---|---|

***Card 14.*** This card is read if IFLAG = 2. It is optional but must be included if Card 12 is included.

| E1M2R1S | E2M2R1S | E1M2R1T | E2M2R1T | | | | |
|---|---|---|---|---|---|---|---|

***Card 15.*** This card is read if IUNLD = 2 and IFLAG = 0 or 1. It is optional.

| KTS | KTT | KRS | KRT | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | KT | KR | IUNLD | OFFSET | DAMPF | IFLAG |
| Type | A | F | F | F | I | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| KT | Translational stiffness along local *r*-axis for IUNLD = 2.0. However, if IFLAG = 2, then it is the translational stiffness for unloading along the local *r*-axis. If left blank, an internally calculated value will be used. |
| KR | Rotational stiffness along local *r*-axis for IUNLD = 2.0. However, if IFLAG = 2, then KR is the rotational stiffness for unloading along the local *r*-axis. If left blank, an internally calculated value will be used. |
| IUNLD | Unloading option (see Figure M119-1):<br>EQ.0.0: Loading and unloading follow loading curve<br>EQ.1.0: Loading follows loading curve, unloading follows unloading curve. The unloading curve ID if undefined is taken as the loading curve.<br>EQ.2.0: Loading follows loading curve, unloading follows unloading stiffness, KT or KR, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes.<br>EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset. |
| OFFSET | Offset factor between 0.0 and 1.0 to determine permanent set upon unloading if the IUNLD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively. |
| DAMPF | Damping factor for stability. Values in the neighborhood of unity are recommended. This damping factor is properly scaled to eliminate time step size dependency. Also, it is active if and only if the local stiffness is defined. |
| IFLAG | Formulation flag:<br>EQ.0: Displacement formulation which is used in all other models<br>EQ.1: Linear strain formulation. The displacements and |

| VARIABLE | DESCRIPTION |
|---|---|
| | velocities are divided by the initial length of the beam. |
| | EQ.2: A displacement formulation to simulate the buckling behavior of crushable frames |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDTR | LCIDTS | LCIDTT | LCIDRR | LCIDRS | LCIDRT | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDTR | Load curve ID defining translational force resultant along local *r*-axis as a function of relative translational displacement. If zero, no stiffness related forces are generated for this degree of freedom. The loading curves must be defined from the most negative displacement to the most positive displacement. The force does not need to increase monotonically. The curves in this input are linearly extrapolated when the displacement range falls outside the curve definition. |
| LCIDTS | Load curve ID defining translational force resultant along local *s*-axis as a function of relative translational displacement (IFLAG = 0 or 1 only). |
| LCIDTT | Load curve ID defining translational force resultant along local *t*-axis as a function of relative translational displacement (IFLAG = 0 or 1 only). |
| LCIDRR | Load curve for rotational moment resultant about the local *r*-axis:<br>IFLAG.NE.2: Load curve ID defining rotational moment resultant about local *r*-axis as a function of relative rotational displacement<br>IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local *r*-axis as a function of relative rotational displacement at node 2 |
| LCIDRS | Load curve for rotational moment resultant about local *s*-axis:<br>IFLAG.NE.2: Load curve ID defining rotational moment resultant about local *s*-axis as a function of relative |

| VARIABLE | DESCRIPTION |
|---|---|
| | rotational displacement |
| | IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local *s*-axis as a function of relative rotational displacement at node 2 |
| LCIDRT | Load curve for rotational moment resultant about local *t*-axis: |
| | IFLAG.NE.2: Load curve ID defining rotational moment resultant about local *t*-axis as a function of relative rotational displacement |
| | IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local *t*-axis as a function of relative rotational displacement at node 2 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDTUR | LCIDTUS | LCIDTUT | LCIDRUR | LCIDRUS | LCIDRUT | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDTUR | Load curve ID defining translational force resultant along local *r*-axis as a function of relative translational displacement during unloading. The force values defined by this curve must increase monotonically from the most negative displacement to the most positive displacement. For IUNLD = 1.0, the slope of this curve must equal or exceed the loading curve for stability reasons. This is not the case for IUNLD = 2.0. For loading and unloading to follow the same path simply set LCIDTUR = LCIDTR. For options IUNLD = 0.0 or 3.0 the unloading curve is not required. For IUNLD = 2.0, if LCIDTUR is left blank or zero, the default is to use the same curve for unloading as for loading. |
| LCIDTUS | Load curve ID defining translational force resultant along local *s*-axis as a function of relative translational displacement during unloading (IFLAG = 0 or 1 only). |
| LCIDTUT | Load curve ID defining translational force resultant along local *t*-axis as a function of relative translational displacement during unloading (IFLAG = 0 or 1 only). |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDRUR | Load curve ID defining rotational moment resultant about local *r*-axis as a function of relative rotational displacement during unloading. |
| LCIDRUS | Load curve for rotational moment resultant about local *s*-axis:<br>IFLAG.NE.2: Load curve ID defining rotational moment resultant about local *s*-axis as a function of relative rotational displacement during unloading<br>IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local *s*-axis as a function of relative rotational displacement during unloading at node 2 |
| LCIDRUT | Load curve ID defining rotational moment resultant about local *t*-axis:<br>IFLAG.NE.2: Load curve ID defining rotational moment resultant about local *t*-axis as a function of relative rotational displacement during unloading. If zero, no viscous forces are generated for this degree of freedom<br>IFLAG.EQ.2: Load curve ID defining rotational moment resultant about local *t*-axis as a function of relative rotational displacement during unloading at node 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDTDR | LCIDTDS | LCIDTDT | LCIDRDR | LCIDRDS | LCIDRDT | IVDFLG | |
| Type | I | I | I | I | I | I | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDTDR | Load curve ID defining translational damping force resultant along local *r*-axis as a function of relative translational velocity. |
| LCIDTDS | Load curve ID defining translational damping force resultant along local *s*-axis as a function relative translational velocity. |
| LCIDTDT | Load curve ID defining translational damping force resultant along local *t*-axis as a function of relative translational velocity. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDRDR | Load curve ID defining rotational damping moment resultant about local *r*-axis as a function of relative rotational velocity. |
| LCIDRDS | Load curve ID defining rotational damping moment resultant about local *s*-axis as a function of relative rotational velocity. |
| LCIDRDT | Load curve ID defining rotational damping moment resultant about local *t*-axis as a function of relative rotational velocity. |
| IVDFLG | Flag for the evaluation of the damping force/moment as a function of velocity curves on Card 4:<br>EQ.0: Use absolute value of velocity (default). The curves only need to be defined in the first positive quadrant.<br>EQ.1: Use signed value of velocity. The curves must define the response in the negative and positive quadrants and pass through point (0,0). This allows the definition of different damping behavior in tension and compression. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDTER | LCIDTES | LCIDTET | LCIDRER | LCIDRES | LCIDRET | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDTER | Load curve ID defining translational damping force scale factor as a function of relative displacement in local *r*-direction. |
| LCIDTES | Load curve ID defining translational damping force scale factor as a function of relative displacement in local *s*-direction. |
| LCIDTET | Load curve ID defining translational damping force scale factor as a function of relative displacement in local *t*-direction. |
| LCIDRER | Load curve ID defining rotational damping moment resultant scale factor as a function of relative displacement in local *r*-rotation. |
| LCIDRES | Load curve ID defining rotational damping moment resultant scale factor as a function of relative displacement in local *s*-rotation. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDRET | Load curve ID defining rotational damping moment resultant scale factor as a function of relative displacement in local $t$-rotation. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UTFAILR | UTFAILS | UTFAILT | WTFAILR | WTFAILS | WTFAILT | FCRIT | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| UTFAILR | Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_r$, is not considered in the failure calculation. |
| UTFAILS | Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_s$, is not considered in the failure calculation. |
| UTFAILT | Optional, translational displacement at failure in tension. If zero, the corresponding displacement, $u_t$, is not considered in the failure calculation. |
| WTFAILR | Optional, rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_r$, is not considered in the failure calculation. |
| WTFAILS | Optional, rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_s$, is not considered in the failure calculation. |
| WTFAILT | Optional rotational displacement at failure in tension. If zero, the corresponding rotation, $\theta_t$, is not considered in the failure calculation. |
| FCRIT | Failure criterion (see Remark 1):<br>EQ.0.0: Two separate criteria, one for negative displacements and rotations, another for positive displacements and rotations<br>EQ.1.0: One criterion that considers both positive and negative displacements and rotations |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UCFAILR | UCFAILS | UCFAILT | WCFAILR | WCFAILS | WCFAILT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| UCFAILR | Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_r$, is not considered in the failure calculation. Define as a positive number. |
| UCFAILS | Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_s$, is not considered in the failure calculation. Define as a positive number. |
| UCFAILT | Optional, translational displacement at failure in compression. If zero, the corresponding displacement, $u_t$, is not considered in the failure calculation. Define as a positive number. |
| WCFAILR | Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_r$, is not considered in the failure calculation. Define as a positive number. |
| WCFAILS | Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_s$, is not considered in the failure calculation. Define as a positive number. |
| WCFAILT | Optional, rotational displacement at failure in compression. If zero, the corresponding rotation, $\theta_t$, is not considered in the failure calculation. Define as a positive number. |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IUR | IUS | IUT | IWR | IWS | IWT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IUR | Initial translational displacement along local *r*-axis |

| VARIABLE | DESCRIPTION |
|---|---|
| IUS | Initial translational displacement along local *s*-axis |
| IUT | Initial translational displacement along local *t*-axis |
| IWR | Initial rotational displacement about the local *r*-axis |
| IWS | Initial rotational displacement about the local *s*-axis |
| IWT | Initial rotational displacement about the local *t*-axis |

**Loading Rotational Moment Card.** This card is read if IFLAG = 2. It is optional. If it is included, Cards 10 and 11 must be included.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LM1R1S | LM1R2S | LM1R1T | LM1R2T | LM2R1S | LM2R1T | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LM1R1S | Load curve ID for loading defining rotational moment resultant at node 1 about local *s*-axis as a function of relative rotational displacement at node 1 |
| LM1R2S | Load curve ID defining rotational moment resultant at node 1 about local *s*-axis as a function of relative rotational displacement at node 2 |
| LM1R1T | Load curve ID defining rotational moment resultant at node 1 about local *t*-axis as a function of relative rotational displacement at node 1 |
| LM1R2T | Load curve ID defining rotational moment resultant at node 1 about local *t*-axis as a function of relative rotational displacement at node 2 |
| LM2R1S | Load curve ID defining rotational moment resultant at node 2 about local *s*-axis as a function of relative rotational displacement at node 1 |
| LM2R1T | Load curve ID defining rotational moment resultant at node 2 about local *t*-axis as a function of relative rotational displacement |

| VARIABLE | DESCRIPTION |
|---|---|
| | at node 1 |

**Unloading Rotational Moment Card.** This card is read if IFLAG = 2. It must be included if Card 9 is included.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LUM1R1S | LUM1R2S | LUM1R1T | LUM1R2T | LUM2R1S | LUM2R1T | | |
| Type | I | I | I | I | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LUM1R1S | Load curve ID for unloading defining rotational moment resultant at node 1 about local *s*-axis as a function of relative rotational displacement at node 1 |
| LUM1R2S | Load curve ID for unloading defining rotational moment resultant at node 1 about local *s*-axis as a function of relative rotational displacement at node 2 |
| LUM1R1T | Load curve ID for unloading defining rotational moment resultant at node 1 about local *t*-axis as a function of relative rotational displacement at node 1 |
| LUM1R2T | Load curve ID for unloading defining rotational moment resultant at node 1 about local *t*-axis as a function of relative rotational displacement at node 2 |
| LUM2R1S | Load curve ID for unloading defining rotational moment resultant at node 2 about local *s*-axis as a function of relative rotational displacement at node 1 |
| LUM2R1T | Load curve ID for unloading defining rotational moment resultant at node 2 about local *t*-axis as a function of relative rotational displacement at node 1 |

**Unload Stiffness for Bending Moment Card.** This card is read if IFLAG = 2. It must be included if Card 9 is included.

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | KUM1R1S | KUM1R2S | KUM1R1T | KUM1R2T | KUM2R1S | KUM2R1T | KUM2R2S | KUM2R2T |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| KUM1R1S | Optional unload stiffness for bending moment about local *s*-axis at node 1 due to relative rotation at node 1. If left blank, this value will be internally calculated. |
| KUM1R2S | Optional unload stiffness for bending moment about local *s*-axis at node 1 due to relative rotation at node 2. If left blank, this value will be internally calculated. |
| KUM1R1T | Optional unload stiffness for bending moment about local *t*-axis at node 1 due to relative rotation at node 1. If left blank, this value will be internally calculated. |
| KUM1R2T | Optional unload stiffness for bending moment about local *t*-axis at node 1 due to relative rotation at node 2. If left blank, this value will be internally calculated. |
| KUM2R1S | Optional unload stiffness for bending moment about local *s*-axis at node 2 due to relative rotation at node 1. If left blank, this value will be internally calculated. |
| KUM2R1T | Optional unload stiffness for bending moment about local *t*-axis at node 2 due to relative rotation at node 1. If left blank, this value will be internally calculated. |
| KUM2R2S | Optional unload stiffness for bending moment about local *s*-axis at node 2 due to relative rotation at node 2. If left blank, this value will be internally calculated. |
| KUM2R2T | Optional unload stiffness for bending moment about local *t*-axis at node 2 due to relative rotation at node 2. If left blank, this value will be internally calculated. |

**Elastic limit of loading curves.** This card is read if IFLAG = 2. It is optional. If not input, the values derived by LS-DYNA based on the related curves will be used. If it is included, Cards 13 and 14 must be included.

| Card 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1TR | E2TR | E1RR | E2RR | E1RS | E2RS | E1RT | E2RT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| E1TR | Negative, compressive, elastic limit of curve LCIDTR |
| E2TR | Positive, tensile, elastic limit of curve LCIDTR |
| E1RR | Negative elastic limit of curve LCIDRR |
| E2RR | Positive elastic limit of curve LCIDRR |
| E1RR | Negative elastic limit of curve LCIDRS |
| E2RR | Positive elastic limit of curve LCIDRS |
| E1RT | Negative elastic limit of curve LCIDRT |
| E2RT | Positive elastic limit of curve LCIDRT |

**Elastic limit of loading curves.** This card is read if IFLAG = 2. If not input, the values derived by LS-DYNA based on the related curves will be used. It must be included if Card 12 is included.

| Card 13 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1M1R1S | E2M1R1S | E1M1R2S | E2M1R2S | E1M1R1T | E2M1R1T | E1M1R2T | E2M1R2T |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| E1M1R1S | Negative, tensile, elastic limit of curve LM1R1S |
| E2M1R1S | Positive, tensile, elastic limit of curve LM1R1S |

| VARIABLE | DESCRIPTION |
|---|---|
| E1M1R2S | Negative elastic limit of curve LM1R2S |
| E2M1R2S | Positive elastic limit of curve LM1R2S |
| E1M1R1T | Negative, tensile, elastic limit of curve LM1R1T |
| E2M1R1T | Positive, tensile, elastic limit of curve LM1R1T |
| E1M1R2T | Negative elastic limit of curve LM1R2T |
| E2M1R2T | Positive elastic limit of curve LM1R2T |

**Elastic limit of loading curves.** This card is read if IFLAG = 2. It is optional. If not input, the values derived by LS-DYNA based on the related curves will be used. It must be included if Card 12 is included.

| Card 14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1M2R1S | E2M2R1S | E1M2R1T | E2M2R1T | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E1M2R1S | Negative, tensile, elastic limit of curve LM2R1S |
| E2M2R1S | Positive, tensile, elastic limit of curve LM2R1S |
| E1M2R1T | Negative elastic limit of curve LM2R1T |
| E2M2R1T | Positive elastic limit of curve LM2R1T |

**Unloading stiffness along local-*s* and local-*t*.** This card is read if IUNLD = 2 and IFLAG = 0 or 1. It is optional. If not input, the values along local *r*-axis, KT and KR, will be used for all axes.

| Card 15 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | KTS | KTT | KRS | KRT | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| KTS | Translational stiffness along local *s*-axis for IUNLD = 2.0. |
| KTT | Translational stiffness along local *t*-axis for IUNLD = 2.0 |
| KRS | Rotational stiffness along local *s*-axis for IUNLD = 2.0 |
| KRT | Rotational stiffness along local *t*-axis for IUNLD = 2.0 |

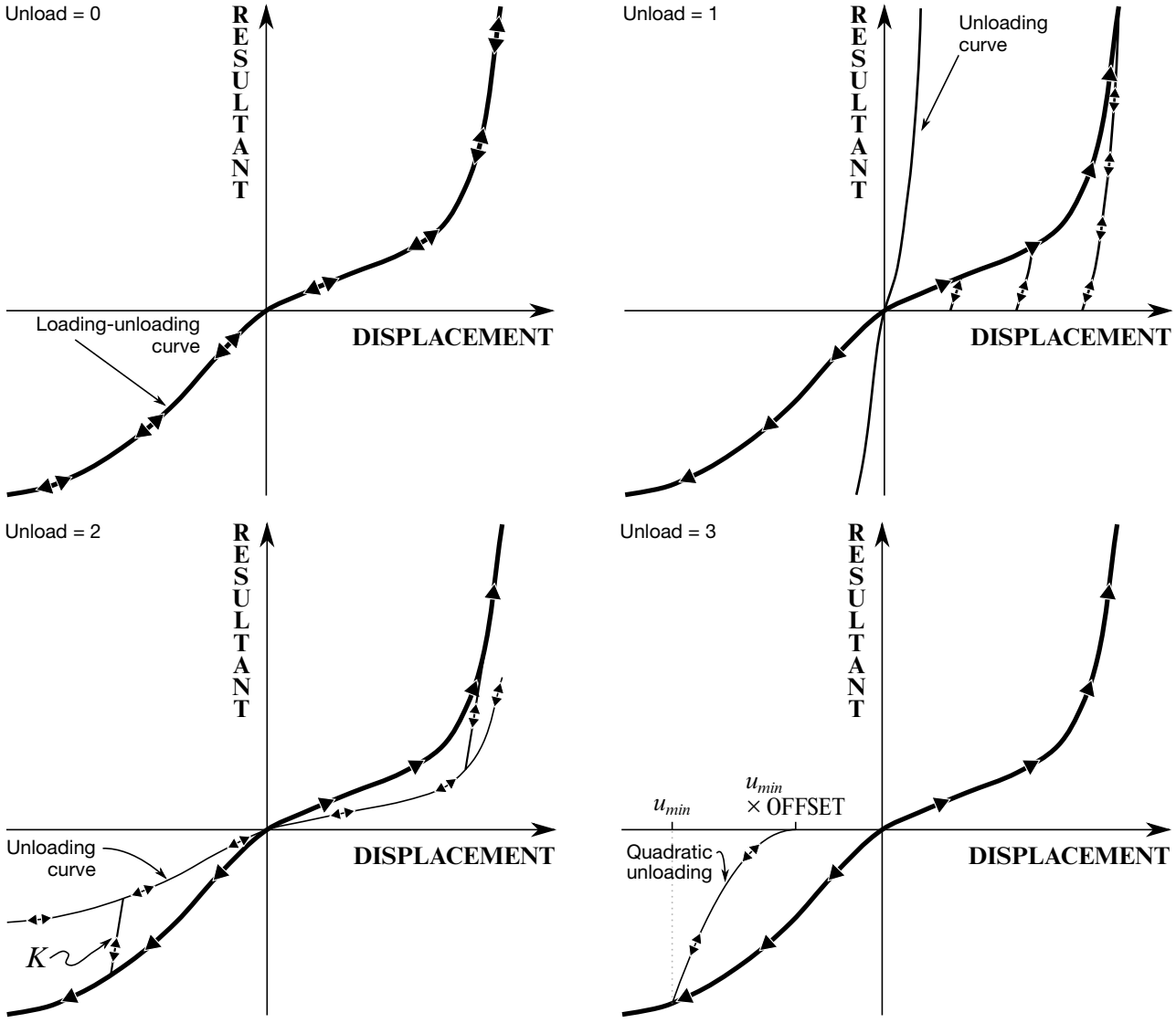



**Figure M119-1.** Load and unloading behavior.

**Remarks:**

1. **Failure criterion.** When the catastrophic failure criterion is satisfied, the discrete element is deleted. Failure for this material depends directly on the displacement resultants. The failure criterion depends on the value of FCRIT.

   If FCRIT = 0.0, failure occurs if <u>either</u> of the following inequalities are satisfied:

   $$A^{\mathrm{t}} - 1 \geq 0$$
   $$A^{\mathrm{c}} - 1 \geq 0$$

   where

   $$A^{\mathrm{t}} = \left[\frac{\max(0, u_{\mathrm{r}})}{u_{\mathrm{r}}^{\mathrm{tfail}}}\right]^2 + \left[\frac{\max(0, u_{\mathrm{s}})}{u_{\mathrm{s}}^{\mathrm{tfail}}}\right]^2 + \left[\frac{\max(0, u_{\mathrm{t}})}{u_{\mathrm{t}}^{\mathrm{tfail}}}\right]^2 + \left[\frac{\max(0, \theta_{\mathrm{r}})}{\theta_{\mathrm{r}}^{\mathrm{tfail}}}\right]^2 + \left[\frac{\max(0, \theta_{\mathrm{s}})}{\theta_{\mathrm{s}}^{\mathrm{tfail}}}\right]^2 + \left[\frac{\max(0, \theta_{\mathrm{t}})}{\theta_{\mathrm{t}}^{\mathrm{tfail}}}\right]^2$$

   $$A^{\mathrm{c}} = \left[\frac{\min(0, u_{\mathrm{r}})}{u_{\mathrm{r}}^{\mathrm{cfail}}}\right]^2 + \left[\frac{\min(0, u_{\mathrm{s}})}{u_{\mathrm{s}}^{\mathrm{cfail}}}\right]^2 + \left[\frac{\min(0, u_{\mathrm{t}})}{u_{\mathrm{t}}^{\mathrm{cfail}}}\right]^2 + \left[\frac{\min(0, \theta_{\mathrm{r}})}{\theta_{\mathrm{r}}^{\mathrm{cfail}}}\right]^2 + \left[\frac{\min(0, \theta_{\mathrm{s}})}{\theta_{\mathrm{s}}^{\mathrm{cfail}}}\right]^2 + \left[\frac{\min(0, \theta_{\mathrm{t}})}{\theta_{\mathrm{t}}^{\mathrm{cfail}}}\right]^2$$

   Positive (tension) values of displacement and rotation are considered in the first criterion and negative (compression) values in the second. Either the tension failure or the compression failure or both may be used. If any of the input failure displacements and rotations (UTFAILR etc) are left as zero, the corresponding terms will be omitted from the equations for $A^{\mathrm{t}}$ and $A^{\mathrm{c}}$ above.

   If FCRIT = 1.0, then a single criterion is used:

   $$A^{\mathrm{t}} + A^{\mathrm{c}} - 1 \geq 0$$

   Thus, the combined effect of all the displacements and rotations is considered, be they positive or negative.

2. **Force.** There are two formulations for calculating the force. The first is the standard displacement formulation, where, for example, the force in a linear spring is

   $$F = -K\Delta L$$

   for a change in length of the beam of $\Delta L$. The second formulation is based on the linear strain, giving a force of

   $$F = -K\frac{\Delta L}{L_0}$$

   for a beam with an initial length of $L_0$. This option is useful when there are springs of different lengths but otherwise similar construction since it

automatically reduces the stiffness of the spring as the length increases, allowing an entire family of springs to be modeled with a single material. Note that all the displacement and velocity components are divided by the initial length, and therefore the scaling applies to the damping and rotational stiffness.

3. **Rotational displacement.** Rotational displacement is measured in radians.

## *MAT_GURSON

This is Material Type 120. This is the Gurson dilatational-plastic model. This model is available for shell and solid elements. A detailed description of this model can be found in the following references: Gurson [1975, 1977], Chu and Needleman [1980] and Tvergaard and Needleman [1984]. The implementation in LS-DYNA is based on the implementation of Feucht [1998] and Faßnacht [1999], which was recoded at LSTC. Strain rate dependency can be defined using a table (see LCSS on Card 6).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | N | Q1 | Q2 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| FC | F0 | EN | SN | FN | ETAN | ATYP | FF0 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| L1 | L2 | L3 | L4 | FF1 | FF2 | FF3 | FF4 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| LCSS | LCFF | NUMINT | LCF0 | LCFC | LCFN | VGTYP | DEXP |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | N | Q1 | Q2 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| N | Exponent for Power law (default = 0.0). This value is only used if ATYP = 1 and LCSS = 0 (see Cards 2 and 6). |
| Q1 | Gurson flow function parameter $q_1$ |
| Q2 | Gurson flow function parameter $q_2$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FC | F0 | EN | SN | FN | ETAN | ATYP | FF0 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| FC | Critical void volume fraction $f_c$ where voids begin to aggregate. This value is only used if LCFC = 0 (see Card 6). |
| F0 | Initial void volume fraction, $f_0$. This value is only used if LCF0 = 0 (see Card 6). |
| EN | Mean nucleation strain $\varepsilon_N$:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-EN) which defines mean nucleation strain $\varepsilon_N$ as a function of element length |
| SN | Standard deviation $s_N$ of the normal distribution of $\varepsilon_N$:<br>GT.0.0: Constant value |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: Load curve ID = (-SN) which defines standard deviation $s_N$ of the normal distribution of $\varepsilon_N$ as a function of element length |
| FN | Void volume fraction of nucleating particles $f_N$. This value is only used if LCFN = 0 (see Card 6). |
| ETAN | Hardening modulus. This value is only used if ATYP = 2 and LCSS = 0 (see Card 6). |
| ATYP | Type of hardening:<br>EQ.0.0: Ideal plastic<br>$\sigma_Y = \text{SIGY}$<br>EQ.1.0: Power law<br>$\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY/E}}{\text{SIGY/E}} \right)^{1/\text{N}}$<br>EQ.2.0: Linear hardening<br>$\sigma_Y = \text{SIGY} + \frac{\text{E} \times \text{ETAN}}{\text{E} - \text{ETAN}} \varepsilon^p$<br>EQ.3.0: 8 points curve |
| FF0 | Failure void volume fraction $f_F$. This value is only used if no curve is given by (L1, FF1) – (L4, FF4) and LCFF = 0 (see Cards 5 and 6). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1 - EPS8 | Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0 (see Cards 2 and 6). |
| ES1 - ES8 | Corresponding yield stress values to EPS1 – EPS8. These values are used if ATYP = 3 and LCSS = 0 (see Cards 2 and 6). |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L1 | L2 | L3 | L4 | FF1 | FF2 | FF3 | FF4 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| L1 - L4 | Element length values. These values are only used if LCFF = 0 (see Card 6). |
| FF1 - FF4 | Corresponding failure void volume fraction. These values are only used if LCFF = 0 (see Card 6). |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSS | LCFF | NUMINT | LCF0 | LCFC | LCFN | VGTYP | DEXP |
| Type | I | I | I | I | I | I | F | F |
| Default | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 3.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSS | Load curve ID or Table ID. If defined, ATYP, EPS1 - EPS8 and ES1 - ES8 are ignored.<br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. |

| VARIABLE | DESCRIPTION |
|---|---|
| | **Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the effective stress as a function effective plastic strain for that rate; see Figure M24-1 and *MAT_024. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used.<br><br>**Logarithmically Defined Tables.** If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. |
| LCFF | Load curve ID defining failure void volume fraction, $f_F$, as a function of element length |
| NUMINT | Number of integration points which must fail before the element is deleted. This option is available for shells and solids.<br><br>LT.0.0: \|NUMINT\| is percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells. |
| LCF0 | Load curve ID defining initial void volume fraction, $f_0$, as a function of element length |
| LCFC | Load curve ID defining critical void volume fraction, $f_c$, as a function of element length |
| LCFN | Load curve ID defining void volume fraction of nucleating particles, $f_N$, as a function of element length |
| VGTYP | Type of void growth behavior:<br><br>EQ.0.0: Void growth in case of tension and void contraction in case of compression, but never below $f_0$ (default)<br><br>EQ.1.0: Void growth only in case of tension<br><br>EQ.2.0: Void growth in case of tension and void contraction in case of compression, even below $f_0$ |

| VARIABLE | DESCRIPTION |
|---|---|
| DEXP | Exponent value for damage history variable 16 |

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2\sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0 \ ,$$

where $\sigma_M$ is the equivalent von Mises stress, $\sigma_Y$ is the yield stress, and $\sigma_H$ is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \le f_c \\ f_c + \dfrac{1/q_1 - f_c}{f_F - f_c}(f - f_c) & f > f_c \end{cases}$$

The growth of void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N \ ,$$

where the growth of existing voids is defined as

$$\dot{f}_G = (1 - f)\dot{\varepsilon}_{kk}^p$$

and nucleation of new voids is defined as

$$\dot{f}_N = A\dot{\varepsilon}_p$$

with function $A$

$$A = \frac{f_N}{S_N\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_p - \varepsilon_N}{S_N}\right)^2\right] \ .$$

Voids are nucleated only in tension.

**History Variables:**

| Shell | Solid | Description |
|---|---|---|
| 1 | 1 | Void volume fraction |
| 4 | 2 | Triaxiality variable $\sigma_H/\sigma_M$ |
| 5 | 3 | Effective strain rate |
| 6 | 4 | Growth of voids |
| 7 | 5 | Nucleation of voids |

| Shell | Solid | Description |
|---|---|---|
| 11 | 11 | Dimensionless material damage value = $\begin{cases} \frac{(f-f_0)}{(f_c-f_0)} & f \le f_c \\ 1+\frac{(f-f_c)}{(f_F-f_c)} & f > f_c \end{cases}$ |
| 13 | 13 | Deviatoric part of microscopic plastic strain |
| 14 | 14 | Volumetric part of macroscopic plastic strain |
| 16 | 16 | Dimensionless material damage value = $\left(\frac{f-f_0}{f_F-f_0}\right)^{1/\mathrm{DEXP}}$ |

## *MAT_GURSON_JC

This is an enhancement of Material Type 120. This is the Gurson model with the additional Johnson-Cook failure criterion (see Card 5). This model is available for shell and solid elements. Strain rate dependency can be defined using a table (see LCSS). An extension for void growth under shear-dominated states and for Johnson-Cook damage evolution is optional.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | N | Q1 | Q2 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| FC | F0 | EN | SN | FN | ETAN | ATYP | FF0 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| SIG1 | SIG2 | SIG3 | SIG4 | SIG5 | SIG6 | SIG7 | SIG8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| LCDAM | L1 | L2 | D1 | D2 | D3 | D4 | LCJC |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| LCSS | LCFF | NUMINT | LCF0 | LCFC | LCFN | VGTYP | DEXP |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is optional.

| KW | BETA | M | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | N | Q1 | Q2 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| N | Exponent for power law. This field is only used if ATYP = 1 and LCSS = 0 (see Cards 2 and 6). |
| Q1 | Gurson flow function parameter $q_1$ |
| Q2 | Gurson flow function parameter $q_2$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FC | F0 | EN | SN | FN | ETAN | ATYP | FF0 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| FC | Critical void volume fraction, $f_c$, where voids begin to aggregate |

| VARIABLE | DESCRIPTION |
|---|---|
| F0 | Initial void volume fraction, $f_0$. This field is only used if LCF0 = 0. |
| EN | Mean nucleation strain, $\varepsilon_N$:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-EN) which defines mean nucleation strain, $\varepsilon_N$, as a function of element length |
| SN | Standard deviation, $s_N$, of the normal distribution of $\varepsilon_N$:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SN) which defines standard deviation, $s_N$, of the normal distribution of $\varepsilon_N$ as a function of element length |
| FN | Void volume fraction of nucleating particles, $f_N$. This field is only used if LCFN = 0. |
| ETAN | Hardening modulus. This field is only used if ATYP = 2 and LCSS = 0 (see Card 6). |
| ATYP | Type of hardening:<br>EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}$<br>EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY/E}}{\text{SIGY/E}} \right)^{1/\text{N}}$<br>EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{\text{E} \times \text{ETAN}}{\text{E} - \text{ETAN}} \varepsilon^p$<br>EQ.3.0: 8 points curve |
| FF0 | Failure void volume fraction, $f_F$. This field is only used if LCFF = 0 (see Card 6). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIG1 | SIG2 | SIG3 | SIG4 | SIG5 | SIG6 | SIG7 | SIG8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1 - EPS8 | Effective plastic strain values. The first point must be zero corresponding to the initial yield stress. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 – EPS8. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCDAM | L1 | L2 | D1 | D2 | D3 | D4 | LCJC |
| Type | I | F | F | F | F | F | F | I |
| Default | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCDAM | Load curve defining the scaling factor, $\Lambda$, as a function of element length. It scales the Johnson-Cook failure strain (see remarks). If LCDAM = 0, no scaling is performed. |
| L1 | Lower triaxiality factor defining failure evolution (Johnson-Cook) |
| L2 | Upper triaxiality factor defining failure evolution (Johnson-Cook) |
| D1 - D4 | Johnson-Cook damage parameters |
| LCJC | Load curve defining the scaling factor for Johnson-Cook failure as a function of triaxiality (see remarks). If LCJC > 0, parameters D1, D2 and D3 are ignored. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSS | LCFF | NUMINT | LCF0 | LCFC | LCFN | VGTYP | DEXP |
| Type | I | I | I | I | I | I | F | F |
| Default | 0 | 0 | 1 | 0 | 0 | 0 | 0.0 | 3.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSS | Load curve ID or Table ID. If defined, ATYP, EPS1 - EPS8 and ES1 - ES8 are ignored.<br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the effective stress as a function effective plastic strain for that rate; see Figure M24-1 and *MAT_024. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used.<br>**Logarithmically Defined Tables.** If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. |
| LCFF | Load curve ID defining failure void volume fraction, $f_F$, as a function of element length |
| NUMINT | Number of through thickness integration points which must fail before the element is deleted. This option is available for shells and solids.<br>LT.0.0: \|NUMINT\| is the percentage of integration points/layers which must fail before shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails and then the given percentage of layers must fail before the element fails. Only available for shells. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCF0 | Load curve ID defining initial void volume fraction, $f_0$, as a function of element length |
| LCFC | Load curve ID defining critical void volume fraction, $f_c$, as a function of element length |
| LCFN | Load curve ID defining void volume fraction of nucleating particles, $f_N$, as a function of element length |
| VGTYP | Type of void growth behavior.<br>EQ.0.0: Void growth in case of tension and void contraction in case of compression, but never below $f_0$ (default)<br>EQ.1.0: Void growth only in case of tension<br>EQ.2.0: Void growth in case of tension and void contraction in case of compression, even below $f_0$ |
| DEXP | Exponent value for damage history variable 16 |

Optional Card (starting with version 971 release R4)

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | KW | BETA | M | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | 0.0 | 1.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| KW | Parameter $k_\omega$ for void growth in shear-dominated states. See remarks. |
| BETA | Parameter $\beta$ in Lode cosine function. See remarks. |
| M | Parameter for generalization of Johnson-Cook damage evolution. See remarks. |

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2\sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0$$

where $\sigma_M$ is the equivalent von Mises stress, $\sigma_Y$ is the yield stress, and $\sigma_H$ is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \le f_c \\ f_c + \dfrac{1/q_1 - f_c}{f_F - f_c}(f - f_c) & f > f_c \end{cases}$$

The growth of void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N$$

where the growth of existing voids is defined as

$$\dot{f}_G = (1-f)\dot{\varepsilon}_{kk}^p + k_\omega \omega(\sigma) f(1-f)\dot{\varepsilon}_M^{pl}\frac{\sigma_Y}{\sigma_M}$$

The second term is an optional extension for shear failure proposed by Nahshon and Hutchinson [2008] with new parameter $k_\omega$ (= 0 by default), effective plastic strain rate in the matrix $\dot{\varepsilon}_M^{pl}$, and Lode cosin function $\omega(\sigma)$:

$$\omega(\sigma) = 1 - \xi^2 - \beta \times \xi(1-\xi), \qquad \xi = \cos(3\theta) = \frac{27}{2}\frac{J_3}{\sigma_M^3}$$

with parameter $\beta$, Lode angle $\theta$ and third deviatoric stress invariant $J_3$.

Nucleation of new voids is defined as

$$\dot{f}_N = A\dot{\varepsilon}_M^{pl}$$

with function $A$

$$A = \frac{f_N}{S_N\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\varepsilon_M^{pl} - \varepsilon_N}{S_N}\right)^2\right].$$

Voids are nucleated only in tension.

The Johnson-Cook failure criterion is added to this material model. Based on the triaxiality ratio $\sigma_H/\sigma_M$ failure is calculated as:

$\sigma_H/\sigma_M > L_1$ : Gurson model

$L_1 \ge \sigma_H/\sigma_M \ge L_2$ : Gurson model and Johnson-Cook failure criteria

$L_2 < \sigma_H/\sigma_M$ : Gurson model

Johnson-Cook failure strain is defined as

$$\varepsilon_f = \left[D_1 + D_2 \exp\left(D_3 \frac{\sigma_H}{\sigma_M}\right)\right](1 + D_4 \ln \dot{\varepsilon})\Lambda \ ,$$

where $D_1$, $D_2$, $D_3$ and $D_4$ are the Johnson-Cook failure parameters and $\Lambda$ is a function for including mesh-size dependency. An alternative expression can be used, where the first term of the above equation (including D1, D2 and D3) is replaced by a general function LCJC which depends on triaxiality

$$\varepsilon_f = \text{LCJC} \times \left(\frac{\sigma_H}{\sigma_M}\right)(1 + D_4 \ln \dot{\varepsilon})\Lambda \ .$$

The Johnson-Cook damage parameter $D_f$ is calculated with the following evolution equation:

$$\dot{D}_f = \frac{\dot{\varepsilon}^{pl}}{\varepsilon_f} \Rightarrow D_f = \sum \frac{\Delta\varepsilon^{pl}}{\varepsilon_f} \ .$$

where $\Delta\varepsilon^{pl}$ is the increment in effective plastic strain. The material fails when $D_f$ reaches 1.0. A more general (non-linear) damage evolution is possible if $M > 1$ is chosen:

$$\dot{D}_f = \frac{M}{\varepsilon_f} D_f^{\left(1 - \frac{1}{M}\right)} \dot{\varepsilon}^{pl}, \quad M \geq 1.0$$

**History variables:**

| Shell | Solid | Description |
|---|---|---|
| 1 | 1 | Void volume fraction |
| 4 | 2 | Triaxiality variable $\sigma_H/\sigma_M$ |
| 5 | 3 | Effective strain rate |
| 6 | 4 | Growth of voids |
| 7 | 5 | Nucleation of voids |
| 8 | 6 | Johnson-Cook failure strain $\varepsilon_f$ |
| 9 | 7 | Johnson-Cook damage parameter $D_f$ |
| 0 | 8 | Domain variable:<br>EQ.0: elastic stress update<br>EQ.1: region (a) Gurson<br>EQ.2: region (b) Gurson + Johnson-Cook<br>EQ.3: region (c) Gurson |

| Shell | Solid | Description |
|---|---|---|
| 11 | 11 | Dimensionless material damage value = $\begin{cases} \frac{(f-f_0)}{(f_c-f_0)} & f \le f_c \\ 1+\frac{(f-f_c)}{(f_F-f_c)} & f > f_c \end{cases}$ |
| 13 | 13 | Deviatoric part of microscopic plastic strain |
| 14 | 14 | Volumetric part of macroscopic plastic strain |
| 16 | 16 | Dimensionless material damage value = $\left(\frac{f-f_0}{f_F-f_0}\right)^{1/\mathrm{DEXP}}$ |

### *MAT_GURSON_RCDC

This is an enhancement of Material Type 120. This is the Gurson model with the addition of the Wilkins Rc-Dc [Wilkins et al., 1977] fracture model. This model is available for shell and solid elements. A detailed description of this model can be found in the following references: Gurson [1975, 1977], Chu and Needleman [1980], and Tvergaard and Needleman [1984].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | N | Q1 | Q2 |
|---|---|---|---|---|---|---|---|

**Card 2.** Description.

| FC | F0 | EN | SN | FN | ETAN | ATYP | FF0 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| L1 | L2 | L3 | L4 | FF1 | FF2 | FF3 | FF4 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| LCSS | LCFF | NUMINT | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| ALPHA | BETA | GAMMA | D0 | B | LAMBDA | DS | L |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | N | Q1 | Q2 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| N | Exponent for Power law. This field is only used if ATYP = 1 and LCSS = 0. See Cards 2 and 6. |
| Q1 | Gurson flow function parameter $q_1$ |
| Q2 | Gurson flow function parameter $q_2$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FC | F0 | EN | SN | FN | ETAN | ATYP | FF0 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| FC | Critical void volume fraction, $f_c$ |

| VARIABLE | DESCRIPTION |
|---|---|
| F0 | Initial void volume fraction, $f_0$ |
| EN | Mean nucleation strain, $\varepsilon_N$ |
| SN | Standard deviation, $S_N$, of the normal distribution of $\varepsilon_N$ |
| FN | Void volume fraction of nucleating particles |
| ETAN | Hardening modulus. This field is only used if ATYP = 2 and LCSS = 0. See Card 6. |
| ATYP | Type of hardening:<br>EQ.0.0: Ideal plastic, $\sigma_Y = \text{SIGY}$<br>EQ.1.0: Power law, $\sigma_Y = \text{SIGY} \times \left( \frac{\varepsilon^p + \text{SIGY/E}}{\text{SIGY/E}} \right)^{1/\text{N}}$<br>EQ.2.0: Linear hardening, $\sigma_Y = \text{SIGY} + \frac{\text{E} \times \text{ETAN}}{\text{E} - \text{ETAN}} \varepsilon^p$<br>EQ.3.0: 8 points curve |
| FF0 | Failure void volume fraction, $f_F$. This field is used if no curve is given by the points (L1, FF1) – (L4, FF4) and LCFF = 0. See Cards 5 and 6. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1 - EPS8 | Effective plastic strain values. The first point must be zero, corresponding to the initial yield stress. This option is only used if ATYP equals 3. At least 2 points should be defined. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8. These values are used if ATYP = 3 and LCSS = 0. See Cards 2 and 6. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L1 | L2 | L3 | L4 | FF1 | FF2 | FF3 | FF4 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| L1 - L4 | Element length values. These fields are only used if LCFF = 0. |
| FF1 - FF4 | Corresponding failure void volume fraction. These values are only used if LCFF = 0. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSS | LCFF | NUMINT | | | | | |
| Type | I | I | I | | | | | |
| Default | 0 | 0 | 1 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSS | Load curve ID defining effective stress as a function of effective plastic strain. ATYP is ignored with this option. |
| LCFF | Load curve ID defining the failure void volume fraction as a function of element length. The values L1 - L4 and FF1 - FF4 are ignored with this option. |

| VARIABLE | DESCRIPTION |
|---|---|
| NUMINT | Number of through-thickness integration points that must fail before the element is deleted |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | BETA | GAMMA | D0 | B | LAMBDA | DS | L |
| Type | F | F | F | F | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Parameter $\alpha$ for the Rc-Dc model |
| BETA | Parameter $\beta$ for the Rc-Dc model |
| GAMMA | Parameter $\gamma$ for the Rc-Dc model |
| D0 | Parameter $D_0$ for the Rc-Dc model |
| B | Parameter $b$ for the Rc-Dc model |
| LAMBDA | Parameter $\lambda$ for the Rc-Dc model |
| DS | Parameter $D_s$ for the Rc-Dc model |
| L | Characteristic element length for this material |

**Remarks:**

The Gurson flow function is defined as:

$$\Phi = \frac{\sigma_M^2}{\sigma_Y^2} + 2q_1 f^* \cosh\left(\frac{3q_2\sigma_H}{2\sigma_Y}\right) - 1 - (q_1 f^*)^2 = 0 \ ,$$

where $\sigma_M$ is the equivalent von Mises stress, $\sigma_Y$ is the yield stress, and $\sigma_H$ is the mean hydrostatic stress. The effective void volume fraction is defined as

$$f^*(f) = \begin{cases} f & f \le f_c \\ f_c + \dfrac{1/q_1 - f_c}{f_F - f_c}(f - f_c) & f > f_c \end{cases}$$

The growth of the void volume fraction is defined as

$$\dot{f} = \dot{f}_G + \dot{f}_N \ ,$$

where the growth of existing voids is given as:

$$\dot{f}_G = (1-f)\dot{\varepsilon}^p_{kk},$$

and nucleation of new voids as:

$$\dot{f}_N = A\dot{\varepsilon}_p$$

in which $A$ is defined as

$$A = \frac{f_N}{S_N\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\varepsilon_p - \varepsilon_N}{S_N}\right)^2\right) \ .$$

The Rc-Dc model is described in the following. The damage $D$ is given by

$$D = \int \omega_1\omega_2 d\varepsilon^p \ ,$$

where $\varepsilon^p$ is the equivalent plastic strain,

$$\omega_1 = \left(\frac{1}{1-\gamma\sigma_m}\right)^\alpha$$

is a triaxial stress weighting term, and

$$\omega_2 = (2 - A_D)^\beta$$

is an asymmetric strain weighting term. In the above $\sigma_m$ is the mean stress and

$$A_D = \max\left(\frac{S_2}{S_3}, \frac{S_2}{S_1}\right)$$

Fracture is initiated when the accumulation of damage is

$$\frac{D}{D_c} > 1 \ ,$$

where $D_c$ is the critical damage given by

$$D_c = D_0\left(1 + b|\nabla D|^\lambda\right) \ .$$

A fracture fraction

$$F = \frac{D - D_c}{D_s}$$

defines the degradations of the material by the Rc-Dc model.

The characteristic element length is used in the calculation of $\nabla D$. This factor is calculated only for elements with a smaller length than this value.

## *MAT_GENERAL_NONLINEAR_1DOF_DISCRETE_BEAM

This is Material Type 121. This is a very general spring and damper model. This beam is based on the MAT_SPRING_GENERAL_NONLINEAR option and is a one-dimensional version of the 6DOF_DISCRETE_BEAM above. The forces generated by this model act along a line between the two connected nodal points. Additional unloading options have been included.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | IUNLD | OFFSET | DAMPF | | |
| Type | A | F | F | I | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDT | LCIDTU | LCIDTD | LCIDTE | | | | |
| Type | I | I | I | I | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UTFAIL | UCFAIL | IU | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| K | Translational stiffness for unloading option 2.0 |
| IUNLD | Unloading option (Also see Figure M119-1):<br>EQ.0.0: Loading and unloading follow loading curve.<br>EQ.1.0: Loading follows loading curve; unloading follows unloading curve. The unloading curve ID if undefined is |

| VARIABLE | DESCRIPTION |
|---|---|
| | taken as the loading curve.<br>EQ.2.0: Loading follows loading curve; unloading follows unloading stiffness, K, to the unloading curve. The loading and unloading curves may only intersect at the origin of the axes.<br>EQ.3.0: Quadratic unloading from peak displacement value to a permanent offset. |
| OFFSET | Offset to determine permanent set upon unloading if the IUNLD = 3.0. The permanent sets in compression and tension are equal to the product of this offset value and the maximum compressive and tensile displacements, respectively. |
| DAMPF | Damping factor for stability. Values in the neighborhood of unity are recommended. This damping factor is properly scaled to eliminate time step size dependency. Also, it is active if and only if the local stiffness is defined. |
| LCIDT | Load curve ID defining translational force resultant along the axis as a function of relative translational displacement. If zero, no stiffness related forces are generated for this degree of freedom. The loading curves must be defined from the most negative displacement to the most positive displacement. The force does not need to increase monotonically for the loading curve. The curves are extrapolated when the displacement range falls outside the curve definition. |
| LCIDTU | Load curve ID defining translational force resultant along the axis as a function of relative translational displacement during unloading. The force values defined by this curve must increase monotonically from the most negative displacement to the most positive displacement. For IUNLD = 1.0, the slope of this curve must equal or exceed the loading curve for stability reasons. This is not the case for IUNLD = 2.0. For loading and unloading to follow the same path simply set LCIDTU = LCIDT. |
| LCIDTD | Load curve ID defining translational damping force resultant along the axis as a function of relative translational velocity. |
| LCIDTE | Load curve ID defining translational damping force scale factor as a function of relative displacement along the axis. |

| VARIABLE | DESCRIPTION |
|---|---|
| UTFAIL | Optional, translational displacement at failure in tension. If zero, failure in tension is not considered. |
| UCFAIL | Optional, translational displacement at failure in compression. If zero, failure in compression is not considered. |
| IU | Initial translational displacement along axis |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_HILL_3R

This is Material Type 122. This is Hill's 1948 planar anisotropic material model with 3 R values.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | HR | P1 | P2 | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| R00 | R45 | R90 | LCID | E0 | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | P1 | P2 | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, *E* |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio, $\nu$ |
| HR | Hardening rule:<br>EQ.1.0: Linear (default)<br>EQ.2.0: Exponential<br>EQ.3.0: Load curve |
| P1 | Material parameter:<br>HR.EQ.1.0: Tangent modulus<br>HR.EQ.2.0: $k$, strength coefficient for exponential hardening |
| P2 | Material parameter:<br>HR.EQ.1.0: Yield stress<br>HR.EQ.2.0: $n$, exponent |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R00 | R45 | R90 | LCID | E0 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| R00 | $R_{00}$, Lankford parameter determined from experiments |
| R45 | $R_{45}$, Lankford parameter determined from experiments |
| R90 | $R_{90}$, Lankford parameter determined from experiments |
| LCID | Load curve ID for the load curve hardening rule |
| E0 | $\varepsilon_0$ for determining initial yield stress for exponential hardening. (default = 0.0) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. The material axes are then rotated about the shell element normal by an angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by the vector **a** defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). Available in R3 version of 971 and later. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1 V2 V3 | Components of vector **v** for AOPT = 3 |
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

The calculated effective stress is stored in history variable #4.

## *MAT_HILL_3R_3D

This is Material Type 122_3D. It combines orthotropic elastic behavior with Hill's 1948 anisotropic plasticity theory. Anisotropic plastic properties are given by 6 material parameters, $F$, $G$, $H$, $L$, $M$, and $N$, which are determined by experiments. This model is implemented for solid elements (defined by *SECTION_SOLID) only.

This keyword can be written either as *MAT_HILL_3R_3D or *MAT_122_3D.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EX | EY | EZ | PRXY | PRYZ | PRXZ |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GXY | GYZ | GXZ | F | G | H | L | M |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| N | HR | P1 | P2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EX | EY | EZ | PRXY | PRYZ | PRXZ |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EX | $E_x$, Young's modulus in the $x$-direction<br>LT.0.0: \|EX\| is a load curve ID defining $E_x$ as a function of temperature. |
| EY | $E_y$, Young's modulus in the $y$-direction<br>LT.0.0: \|EY\| is a load curve ID defining $E_y$ as a function of temperature. |
| EZ | $E_z$, Young's modulus in the $z$-direction<br>LT.0.0: \|EZ\| is a load curve ID defining $E_z$ as a function of temperature. |
| PRXY | $\nu_{xy}$, Poisson's ratio $xy$<br>LT.0.0: \|PRXY\| is a load curve ID defining $\nu_{xy}$ as a function of temperature. |
| PRYZ | $\nu_{yz}$, Poisson's ratio $yz$<br>LT.0.0: \|PRYZ\| is a load curve ID defining $\nu_{yz}$ as a function of temperature. |
| PRXZ | $\nu_{xz}$, Poisson's ratio $xz$<br>LT.0.0: \|PRXZ\| is a load curve ID defining $\nu_{xz}$ as a function of temperature. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GXY | GYZ | GXZ | F | G | H | L | M |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GXY | $G_{xy}$, shear modulus $xy$ |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: \|GXY\| is load curve ID defining $G_{xy}$ as a function of temperature. |
| GYZ | $G_{yz}$, shear modulus $yz$<br>LT.0.0: \|GYZ\| is load curve ID defining $G_{yz}$ as a function of temperature. |
| GXZ | $G_{xz}$, shear modulus $xz$<br>LT.0.0: \|GXZ\| is load curve ID defining $G_{xz}$ as a function of temperature. |
| F | Material constant in Hill's 1948 yield criterion (see Remark 1).<br>LT.0.0: \|F\| is a load curve ID defining $F$ as a function of temperature. |
| G | Material constant in Hill's 1948 yield criterion (see Remark 1).<br>LT.0.0: \|G\| is a load curve ID defining $G$ as a function of temperature. |
| H | Material constant in Hill's 1948 yield criterion (see Remark 1).<br>LT.0.0: \|H\| is a load curve ID defining $H$ as a function of temperature. |
| L | Material constant in Hill's 1948 yield criterion (see Remark 1).<br>LT.0.0: \|L\| is a load curve ID defining $L$ as a function of temperature. |
| M | Material constant in Hill's 1948 yield criterion (see Remark 1).<br>LT.0.0: \|M\| is a load curve ID defining $M$ as a function of temperature. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | N | HR | P1 | P2 | | | | |
| Type | F | I | I/F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| N | Material constant in Hill's 1948 yield criterion (see Remark 1).<br>LT.0.0: \|N\| is a load curve ID defining $N$ as a function of temperature. |
| HR | Hardening rule:<br>EQ.1: Stress-strain relationship is defined by load curve or 2D table ID, P1. P2 is ignored.<br>EQ.2: Stress-strain relationship is defined by strength coefficient $k$ (P1) and strain hardening coefficient $n$ (P2), as in Swift's exponential hardening equation: $\sigma_{\text{yield}} = k(\varepsilon + 0.01)^n$ . |
| P1 | Material parameter:<br>HR.EQ.1: Load curve or 2D table ID defining stress-strain curve. If P1 is a 2D table ID, the table gives stress-strain curves for different temperatures.<br>HR.EQ.2: $k$, strength coefficient in $\sigma_{\text{yield}} = k(\varepsilon + 0.01)^n$ |
| P2 | Material parameter:<br>HR.EQ.1: Not used<br>HR.EQ.2.0: $n$, the exponent in $\sigma_{yield} = k(\varepsilon + 0.01)^n$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | I | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Locally orthotropic with material axes determined by a point *p* in space and the global location of the element center; this is the *a*-direction. |
| | EQ.2.0: Globally orthotropic with material axes determined by the vectors **a** and **d**, as with *DEFINE_COORDINATE_-VECTOR. |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal. |
| | EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **p**, which define the centerline axis. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO. |

**Remarks:**

1. **Hill's 1948 yield criterion.** Hill's yield criterion is based on the assumptions that the material is orthotropic, that hydrostatic stress does not affect yielding, and that there is no Bauschinger effect. According to Hill, when the principal axes of anisotropy are the axes of reference, the yield surface has the form

$$f = \bar{\sigma}(\boldsymbol{\sigma}) - \sigma_{\text{yield}}(\varepsilon_p) = 0,$$

where the effective stress $\bar{\sigma}$ (stored as history variable #2) is given by

$$(F+G)\bar{\sigma}^2 = F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}^2 + 2M\tau_{zx}^2 + 2N\tau_{xy}^2 \ ,$$

and where $F$, $G$, $H$, $L$, $M$, and $N$ are material parameters of the current state of anisotropy, assuming three mutually orthogonal planes of symmetry at every point. The material $z$-direction is the reference direction.

Let $X$, $Y$, $Z$ be the tensile yield stresses in the principal directions of anisotropy, then

$$\frac{{\sigma_{y0}}^2}{X^2} = \frac{G+H}{F+G} \ , \qquad \frac{{\sigma_{y0}}^2}{Y^2} = \frac{H+F}{F+G} \ , \qquad \frac{{\sigma_{y0}}^2}{Z^2} = 1 \ ,$$

where $\sigma_{y0} = \sigma_{\text{yield}}(0)$. $F$, $G$, and $H$ are not uniquely determined, but the choice of $F + G = 1$ gives

$$F = \frac{Z^2}{2}\left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right)$$
$$G = \frac{Z^2}{2}\left(\frac{1}{X^2} + \frac{1}{Z^2} - \frac{1}{Y^2}\right)$$
$$H = \frac{Z^2}{2}\left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)$$

If $R_{xy}$, $S_{zx}$, and $T_{xy}$ are the yield stresses in shear with respect to the principal axes of anisotropy, then

$$L = \frac{Z^2}{2R_{xy}^2} \ , \qquad M = \frac{Z^2}{2S_{zx}^2} \ , \qquad N = \frac{Z^2}{2T_{xy}^2} \ .$$

If $F = G = H$, and, $L = M = N = 3F$, the Hill criterion reduces to the Von-Mises criterion.

The strain hardening in this model can either defined by the load curve or by Swift's exponential hardening equation: $\sigma_{\text{yield}} = k(\varepsilon + 0.01)^n$.

2. **Applications.** This material model is suitable for metal forming application using solid elements to account for anisotropic plasticity. NUMISHEET conferences have provided material constants of Hill's 1948 yield for many commonly used materials.

   It can also be applied to multi-scale simulations of fiberglass and laminated materials, according to CYBERNET SYSTEMS CO., LTD. The elastic coefficients can be calibrated analytically by a homogenization method with tensile tests in the three orthogonal directions and three pure shear tests in the three orthogonal planes.

3. **Material parameter calibration.** The six material parameters required can be calibrated with nonlinear regression analysis (such as those available through LS-OPT) through a series of tensile tests in three orthogonal directions and three shear tests in three orthogonal planes.

### *MAT_HILL_3R_TABULATED

This is Material Type 122. This is Hill's 1948 planar anisotropic material model with 3 R values and yield curves defined in 3 directions as well as biaxial or shear yield. It is implemented for shell elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | | | |
| Type | A | F | F | F | F | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R00 | R45 | R90 | LC00 | ICONV | LC90 | LC45 | LCEX |
| Type | F | F | F | I | I | I | I | I |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| HR | Hardening rule:<br>EQ.1.0: Not applicable<br>EQ.2.0: Not applicable<br>EQ.3.0: Load curve |
| R00 | $R_{00}$, Lankford parameter determined from experiments |
| R45 | $R_{45}$, Lankford parameter determined from experiments |
| R90 | $R_{90}$, Lankford parameter determined from experiments |
| LC00 | Load curve ID for the yield curve in the 0° direction |
| ICONV | Convexity option:<br>EQ.0.0: Convexity of the yield surface is not enforced.<br>EQ.1.0: Convexity of the yield surface is enforced. |
| LC90 | Load curve ID for the yield curve in the 90° direction |
| LC45 | Load curve ID for the yield curve in the 45° direction |
| LCEX | Absolute value is load curve ID for the yield curve in shear or biaxial:<br>GT.0.0: Shear yield is provided.<br>LT.0.0: Biaxial yield is provided. |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES that are then rotated about the shell element normal by an angle BETA |
| | EQ.2.0: Globally orthotropic with material axes determined by the vector **a** defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| XP YP ZP | Coordinates of point *p* for AOPT = 1 |
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |
| V1 V2 V3 | Components of vector **v** for AOPT = 3 |
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

## *MAT_MODIFIED_PIECEWISE_LINEAR_PLASTICITY_{OPTION}

This is Material Type 123, an elasto-plastic material supporting an arbitrary stress as a function of strain curve as well as arbitrary strain rate dependency. This model is available for shell and solid elements. *MAT_PIECEWISE_LINEAR_PLASTICITY is similar but lacks the enhanced failure criteria. Failure is based on effective plastic strain, thinning strain, the major principal in-plane strain component, or a minimum time step size.

Available options include:

<BLANK>

LOG_INTERPOLATION

PRESTRAIN (for shells only)

RATE

RTCL

STOCHASTIC (for shells only)

The LOG_INTERPOLATION keyword option interpolates the strain rate effect in table LCSS with logarithmic interpolation.

The PRESTRAIN option enables the inclusion of prestrain when checking for major strain failure. The RATE option is used to account for the rate dependence of thinning failure or to invoke viscoelasticity (LCEMOD). The RTCL option activates RTCL damage (see Remark 1). One additional card is needed with any of these options.

The STOCHASTIC keyword option allows spatially varying yield and failure behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| C | P | LCSS | LCSR | VP | EPSTHIN | EPSMAJ | NUMINT |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card for the PRESTRAIN, RATE, and RTCL keyword options.

| LCTSRF | EPS0 | TRIAX | IPS | LCEMOD | BETA | RFILTF | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | $10^{20}$ | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus, ignored if LCSS > 0 |
| FAIL | Failure flag:<br>LT.0.0: User-defined failure subroutine, `matusr_24` in dyn21.F, is called to determine failure.<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. The element is deleted from the calculation when the plastic strain reaches this value. |

| VARIABLE | DESCRIPTION |
|---|---|
| TDEL | Minimum time step size for automatic element deletion |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | VP | EPSTHIN | EPSMAJ | NUMINT |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Strain rate parameter, C; see Remark 1 of *MAT_PIECEWISE_LINEAR_PLASTICITY. |
| P | Strain rate parameter, P; see Remark 1 of *MAT_PIECEWISE_LINEAR_PLASTICITY. |
| LCSS | Load curve ID or Table ID.<br>**Load Curve.** When LCSS is a load curve ID, it defines effective stress as a function of effective plastic strain. If defined EPS1 - EPS8 and ES1 - ES8 are ignored.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress versus effective plastic strain for that rate, See Figure M24-1. When the strain rate falls below the minimum value, the stress versus effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the stress versus effective plastic strain curve for the highest value of strain rate is used. Fields C, P, LCSR, EPS1 - EPS8, and ES1 - ES8 are ignored if a Table ID is defined. Linear interpolation between the discrete strain rates is used by default; logarithmic interpolation is used when the LOG_INTERPOLATION option is invoked.<br>**Logarithmically Defined Tables.** An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| | when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated with invoking logarithmic interpolation. |
| LCSR | Load curve ID defining the strain rate scaling effect on yield stress |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation (recommended) |
| EPSTHIN | Thinning strain at failure. To specify the thinning strain to failure as a function of plastic strain rate, see LCTSRF.<br>GT.0.0: Total thinning strain (as in ISTUPD = 1; see *CONTROL_SHELL)<br>LT.0.0: Plastic thinning strain \|EPSTHIN\| (as in ISTUPD = 4) |
| EPSMAJ | Major in plane strain at failure for shells (or) major principal strain at failure for solids (see Remark 1).<br>LT.0: EPSMAJ = \|EPSMAJ\| and filtering is activated. The last twelve values of the major strain are stored at each integration point, and the average value is used to determine failure. |
| NUMINT | Number of integration points that must fail before the element is deleted. (If zero, all points must fail.) For fully integrated shell formulations, each of the 4 × NIP integration points is counted individually in determining a total for failed integration points. NIP is the number of through-thickness integration points. As NUMINT approaches the total number of integration points (NIP for under-integrated shells, 4 × NIP for fully integrated shells), the chance of instability increases.<br>LT.0.0: \|NUMINT\| is the percentage of integration points/layers which must fail before the shell element fails. For fully integrated shells, a methodology is used where a layer fails if one integration point fails, and then the given percentage of layers must fail before the element fails. Only available for shells. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS1 - EPS8 | Effective plastic strain values. At least 2 points should be defined. The first point must be zero, corresponding to the initial yield stress. If this option is used, SIGY and ETAN are ignored. WARNING: If the first point is nonzero, the yield stress is extrapolated to determine the initial yield. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8 |

**RTCL/Rate Card.** Required if the PRESTRAIN, RATE, or RTCL option is active.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCTSRF | EPS0 | TRIAX | IPS | LCEMOD | BETA | RFILTF | |
| Type | I | F | F | I | I | F | F | |
| Default | 0 | 0.0 | 0.0 | 0 | 0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCTSRF | Curve that defines the thinning strain at failure as a function of the plastic strain rate (*The curve should not extrapolate to zero, or failure may occur at low strain.*) If LCTSRF is given, the absolute value of EPSTHIN is unimportant; however, the algebraic sign of EPSTHIN governs whether ordinate values in curve LCTSRF are interpreted as total or plastic thinning strain. For example, if plastic thinning strain should be used, then EPSTHIN should be input as a negative value. |
| EPS0 | EPS0 parameter for RTCL damage (see Remark 2):<br>EQ.0.0: RTCL damage is inactive (default).<br>GT.0.0: RTCL damage is active. |
| TRIAX | RTCL damage triaxiality limit (see Remark 2):<br>EQ.0.0: No limit (default)<br>GT.0.0: Damage does not accumulate when triaxiality exceeds TRIAX. |
| IPS | Flag to add prestrain when checking for major strain failure (see EPSMAJ above on Card 2) for the PRESTRAIN keyword option:<br>EQ.0: No prestrain added (default)<br>EQ.1: Initial strain set with *INITIAL_STRAIN_SHELL will be used as a prestrain when checking for major strain failure (VP = 0 and shells only). |
| LCEMOD | Load curve ID defining Young's modulus as a function of effective strain rate. LCEMOD ≠ 0 activates viscoelasticity. See *MAT_187L for details. The parameters BETA and RFILTF have to be defined too. If the first strain rate is negative, all values are expected to represent the natural logarithm of a strain rate.<br>(If LCEMOD ≠ 0 is used, VP = 1 should be defined, and failure options EPSTHIN, EPSMAJ, NUMINT, and RTCL are currently unavailable. See *DEFINE_ELEMENT_EROSION to define the number of integration points for failure.) |
| BETA | Decay constant in viscoelastic law. BETA has the unit [1/time]. If LCEMOD > 0 is used, a non-zero value for BETA is mandatory. |
| RFILTF | Smoothing factor on the effective strain rate (default is 0.95). The filtered strain rate is used for the viscoelasticity (LCEMOD > 0). |

| VARIABLE | DESCRIPTION |
|---|---|

$$\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$$

**Remarks:**

1. **Major principal strain failure.** The EPSMAJ parameter is compared to the major principal strain in the following senses:

   a) For shells, it is the maximum eigenvalue of the in-plane strain tensor that is incremented by the strain increments. If IPS = 1, then the prestrain set with *INITIAL_STRAIN_SHELL is also included in the strain measure for shells.

   b) For solid elements, it is calculated as the maximum eigenvalue of the logarithmic strain tensor

   $$\boldsymbol{\varepsilon} = \frac{1}{2}\ln\left(\mathbf{F}^{\mathrm{T}}\mathbf{F}\right) \ ,$$

   where **F** is the global deformation gradient.

   In sum, both element types use a natural strain measure to determine failure. The major strain calculated in this way is output as history variable #7.

2. **RTCL damage.** The RTCL option causes the calculation of RTCL damage, and elements are deleted when the damage function exceeds 1.0. During each solution cycle, if the plastic strain increment is greater than zero, an increment of RTCL damage is calculated by

   $$\Delta f_{\text{damage}} = \frac{1}{\varepsilon_0} f\left(\frac{\sigma_H}{\bar{\sigma}}\right)_{\text{RTCL}} d\bar{\varepsilon}^p$$

   where

   $$f\left(\frac{\sigma_H}{\bar{\sigma}}\right)_{\text{RTCL}} = \begin{cases} 0 & \dfrac{\sigma_H}{\bar{\sigma}} \le -\dfrac{1}{3} \\ 2\dfrac{1 + \dfrac{\sigma_H}{\bar{\sigma}}\sqrt{12 - 27\left(\dfrac{\sigma_H}{\bar{\sigma}}\right)^2}}{3\dfrac{\sigma_H}{\bar{\sigma}} + \sqrt{12 - 27\left(\dfrac{\sigma_H}{\bar{\sigma}}\right)^2}} & -\dfrac{1}{3} < \dfrac{\sigma_H}{\bar{\sigma}} < \dfrac{1}{3} \\ \dfrac{1}{1.65}\exp\left(\dfrac{3\sigma_H}{2\bar{\sigma}}\right) & \dfrac{\sigma_H}{\bar{\sigma}} \ge \dfrac{1}{3} \end{cases}$$

   and,

   $\varepsilon_0$ = uniaxial fracture strain / critical damage value
   $\sigma_H$ = hydrostatic stress
   $\bar{\sigma}$ = effective stress

$d\bar{\varepsilon}^p$ = effective plastic strain increment

The increments are summed through time, and the element is deleted when $f_{\text{damage}} \geq 1.0$. For $0.0 < f_{\text{damage}} < 1.0$, the element strength will not be degraded.

The value of $f_{\text{damage}}$ is stored as history variable #9 and can be fringe plotted from d3plot files if the number of extra history variables requested is ≥ 9 on *DATABASE_EXTENT_BINARY.

The optional TRIAX parameter can prevent excessive RTCL damage growth and element erosion for badly shaped elements that might show unrealistically high values for the triaxiality. The triaxiality, $\sigma_H/\overline{\sigma}$, is stored as history variable #11.

3. **Instability indicator.** To get an idea about the probability of failure, an indicator $D$ is computed internally:

$$D = \max\left(\frac{\bar{\varepsilon}^p}{\text{FAIL}}, \frac{-\varepsilon_3}{\text{EPSTHIN}}, \frac{\varepsilon_I}{\text{EPSMAJ}}\right)$$

and stored as history variable #10. $D$ ranges from 0 (intact) to 1 (failed). $\bar{\varepsilon}^p$, $-\varepsilon_3$, and $\varepsilon_I$ are current values of effective plastic strain, thinning strain, and major in-plane strain. This instability measure, including the RTCL damage, can also be retrieved from requesting material histories

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Instability | - - - - | Failure indicator $\max(D, f_{\text{damage}})$ |
| Plastic Strain Rate | - - - - | Effective plastic strain rate $\dot{\varepsilon}^p_{\text{eff}}$ |

4. **Implicit calculations.** For implicit calculations with this material involving severe nonlinear hardening, the radial return method may result in inaccurate stress-strain response. Setting IACC = 1 on *CONTROL_ACCURACY activates a fully iterative plasticity algorithm, which will remedy this. This is not to be confused with the MITER flag on *CONTROL_SHELL, which governs the treatment of the plane stress assumption for shell elements. If any failure model is applied with this option, incident failure will initiate damage, and the stress will continuously degrade to zero before erosion for a deformation of 1% plastic strain. For instance, if the failure strain is FAIL = 0.05, then the element is eroded when $\bar{\varepsilon}^p = 0.06$ and the material goes from intact to completely damaged between $\bar{\varepsilon}^p = 0.05$ and $\bar{\varepsilon}^p = 0.06$. The reason is to enhance implicit performance by maintaining continuity in the internal forces.

## *MAT_PLASTICITY_COMPRESSION_TENSION

This is Material Type 124. An isotropic elastic-plastic material where unique yield stress as a function of plastic strain curves can be defined for compression and tension. Also, failure can occur based on a plastic strain or a minimum time step size. Rate effects on the yield stress are modeled either by using the Cowper-Symonds strain rate model or by using two load curves that scale the yield stress values in compression and tension, respectively. Material rate effects, which are independent of the plasticity model, are based on a 6-term Prony series Maxwell mode that generates an additional stress tensor. The viscous stress tensor is superimposed on the stress tensor generated by the plasticity.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | C | P | FAIL | TDEL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCIDC | LCIDT | LCSRC | LCSRT | SRFLAG | LCFAIL | EC | RPCT |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| PC | PT | PCUTC | PCUTT | PCUTF | | | SRFILT |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| K | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** Include up to 6 instances of this card. The next keyword ("*") card terminates this input.

| G*i* | BETA*i* | | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | C | P | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | 0.0 | 0.0 | $10^{20}$ | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| C | Strain rate parameter, $C$. See Remark 1. |
| P | Strain rate parameter, $P$. See Remark 1. |
| FAIL | Failure flag:<br>LT.0.0: User defined failure subroutine, `matusr_24` in dyn21.F, is called to determine failure.<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic deletion of shell elements |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDC | LCIDT | LCSRC | LCSRT | SRFLAG | LCFAIL | EC | RPCT |
| Type | I | I | I | I | F | I | F | F |
| Default | 0 | 0 | 0 | 0 | 0.0 | 0 | optional | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDC | Load curve ID defining effective stress as a function of effective plastic strain in compression. Enter positive yield stress and plastic strain values when defining this curve. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDT | Load curve ID defining effective stress as a function of effective plastic strain in tension. Enter positive yield stress and plastic strain values when defining this curve. |
| LCSRC | Optional load curve ID defining strain rate scaling factor on yield stress as a function of strain rate when the material is in compression |
| LCSRT | Optional load curve ID defining strain rate scaling factor on yield stress as a function of strain rate when the material is in tension |
| SRFLAG | Formulation for rate effects:<br>EQ.0.0: total strain rate (based on the total strain tensor)<br>EQ.1.0: effective strain rate (based on deviatoric portion of the strain tensor)<br>EQ.2.0: effective plastic strain rate (viscoplastic) |
| LCFAIL | Optional load curve ID defining effective plastic strain at failure as a function of strain rate. If LCFAIL is specified, FAIL is ignored. See Remark 2. |
| EC | Optional Young's modulus for compression, > 0. |
| RPCT | Fraction of PT and PC, used to define mean stress at which Young's modulus is E and EC, respectively. Young's modulus is E when mean stress > RPCT × PT, and EC when mean stress < −RPCT × PC. If the mean stress falls between – RPCT × PC and RPCT × PT, a linearly interpolated value is used. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PC | PT | PCUTC | PCUTT | PCUTF | | | SRFILT |
| Type | F | F | F | F | F | | | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| PC | Compressive mean stress (pressure) at which the yield stress follows load curve ID, LCIDC. If the pressure falls between PC and PT, a weighted average of the two load curves is used. Both PC and PT should be entered as positive values. |
| PT | Tensile mean stress at which the yield stress follows load curve ID, LCIDT |
| PCUTC | Pressure cut-off in compression (PCUTC must be greater than or equal to zero). PCUTC (and PCUTT) apply only to element types that use a 3D stress update, such as solids, tshell formulations 3 and 5, and SPH. When the pressure cut-off is reached the deviatoric stress tensor is set to zero and the pressure remains at its compressive value. Like the yield stress, PCUTC is scaled to account for rate effects. |
| PCUTT | Pressure cut-off in tension (PCUTT must be less than or equal to zero). When the pressure cut-off is reached, the deviatoric stress tensor and tensile pressure is set to zero. Like the yield stress, PCUTT is scaled to account for rate effects. |
| PCUTF | Pressure cut-off flag activation:<br>EQ.0.0: Inactive<br>EQ.1.0: Active |
| SRFILT | Strain rate filtering parameter in exponential moving average with admissible values ranging from 0 to 1 (available for LCSRC ≠ 0 or LCSRT ≠ 0 with SRFLAG = 0 or 1):<br>$\dot{\varepsilon}_n^{\text{avg}} = \text{SRFILT} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{SRFILT}) \times \dot{\varepsilon}_n$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Optional bulk modulus for the viscoelastic material. If nonzero, a Kelvin type behavior will be obtained. Generally, *K* is set to zero. |

**Viscoelastic Constant Cards.** Up to 6 cards may be input. The next keyword ("*") card terminates this input.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional shear decay constant for the $i^{th}$ term |

**Remarks:**

1. **Stress-Strain Behavior.** The stress-strain behavior follows a different curve in compression than it does in tension. Tension is determined by the sign of the mean stress where a positive mean stress (meaning a negative pressure) is indicative of tension. Two curves must be defined giving the yield stress as a function of effective plastic strain for both the tension and compression regimes.

   Mean stress is an invariant which can be expressed as $(\sigma_x + \sigma_y + \sigma_z)/3$. PC and PT define a range of mean stress values within which interpolation is done between the tensile yield surface and compressive yield surface. PC and PT are not true material properties but are just a numerical convenience so that the transition from one yield surface to the other is not abrupt as the sign of the mean stress changes. Both PC and PT are input as positive values as it is implied that PC is a compressive mean stress value and PT is tensile mean stress value.

   Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor:

$$1 + \left[\frac{\dot{\varepsilon}}{C}\right]^{1/p} .$$

   If SRFLAG = 0, $\dot{\varepsilon}$ is the total strain rate,

$$\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}.$$

2. **LCFAIL.** The LCFAIL field is only applicable when at least one of the following four conditions are met:

a) SRFLAG = 2

b) LCSRC is nonzero

c) LCSRT is nonzero

d) G*i* and BETA*i* values are provided.

## *MAT_KINEMATIC_HARDENING_TRANSVERSELY_ANISOTROPIC_{OPTION}

This is Material Type 125. This material model combines Yoshida and Uemori's nonlinear kinematic hardening rule with material type 37. Yoshida and Uemori's theory uses two surfaces to describe the hardening rule: the yield surface and the bounding surface. In the forming process, the yield surface does not change in size, but its center translates with deformation; the bounding surface changes in both size and location. In addition, the change of Young's modulus can be a function of effective plastic strain, as proposed by Yoshida and Uemori [2002]. This material type is available for shells, thick shells, and solid elements.

Available options include:

<BLANK>

NLP

The NLP option estimates necking failure using the Formability Index (F.I.), which accounts for the nonlinear strain paths seen in metal forming applications (see A Failure Criterion for Nonlinear Strain Paths (NLP) in the remarks section). When using this option, specify IFLD in Card 3. Since the NLP option also works with a linear strain path, it is recommended to be used as the default failure criterion in metal forming. The NLP option is also available for *MAT_036, *MAT_037, and *MAT_226.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | RBAR | HLCID | OPT | |
| Type | A | F | F | F | F | I | I | |
| Default | none | none | none | none | none | 0 | 0 | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CB | Y | SC1 | K | RSAT | SB | H | SC2 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | 0.0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EA | COE | IOPT | C1 | C2 | IFLD | | |
| Type | F | F | I | F | F | I | | |
| Default | none | none | 0 | none | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| RBAR | Plastic anisotropic parameter $\bar{r}$ (Lankford coefficient), also commonly referred to as "r-bar value" in sheet metal forming literature. For shell elements, $\bar{r} = R_{00} = R_{45} = R_{90}$ is assumed in the plane of the shell. |
| HLCID | Load curve ID (see *DEFINE_CURVE) giving true strain as a function of true stress. This curve is used with OPT below and should not be referenced or used in other keywords. *Using this parameter is not recommended.* |
| OPT | Error calculation flag. The default value of "0" is recommended.<br>EQ.2: Perform the error calculation based on the true stress-strain curve from uniaxial tension, specified by HLCID. The corrections will be made to the cyclic load curve, both in the loading and unloading portions. Since, in some cases where loading is more complex, the accumulated plastic strain could be large (say more than 30%), the input uniaxial stress-strain curve must have enough strain range to cover the maximum expected plastic strain. Note that this variable must be set to a value of "2" if HLCID is specified and a stress-strain curve is used. |
| CB | Uppercase *B* defined in Yoshida and Uemori [2002]. It is the initial size of the bounding surface. See Equations 5, 7b, 21, and 43a and |

| VARIABLE | DESCRIPTION |
|---|---|
| | section 2.5 in the paper. |
| Y | Hardening parameter appearing in Yoshida and Uemori [2002] and Yoshida and Uemori [2003]. It gives the yield surface's radius in the deviatoric stress space. See Equations 4a, 7b, and 43a and section 2.5 in Yoshida and Uemori [2002]. |
| SC1 | $C_1$ in Equation 43a of Yoshida and Uemori [2002]. Note that SC1 must be greater than SC2. As reproduced from the paper: $$c = \begin{cases} \text{SC1} & \max(\bar{\alpha}_*) < B - Y \\ \text{SC2} & \text{otherwise} \end{cases}$$ Here, $\bar{\alpha}_*$ is the relative kinematic motion of the yield surface. |
| K | Isotropic hardening parameter appearing in Equations 11, 12, 13, and 14 and section 2.4 of Yoshida and Uemori [2003]. In Yoshida and Uemori [2002], this parameter is called $m$ and is in Equations 19 and 20 and section 2.5. |
| RSAT | Hardening parameter, $R_{\text{sat}}$, appearing in Yoshida and Uemori [2002] and Yoshida and Uemori [2003]. See Equations 19 and 21 and section 2.5 of Yoshida and Uemori [2002]. |
| SB | The lowercase $b$ material parameter appearing in Yoshida and Uemori [2002] (see Equation 20 and section 2.5 of the paper) and Yoshida and Uemori [2003] |
| H | Anisotropic parameter associated with work-hardening stagnation. See Equations 26a, 27, and 30a and section 2.5 of Yoshida and Uemori [2002]. |
| SC2 | $C_2$ in Equation 43b of Yoshida and Uemori [2002]. See the description of SC1. If SC2 equals 0.0, is left blank, or equals SC1, then it turns into the basic model (the one $c$ model). |
| EA | Parameter controlling the change of Young's modulus, $E_a$. in Equation 42 of Yoshida and Uemori [2002] |
| COE | Parameter controlling the change of Young's modulus, $\xi$, in Equation 42 of Yoshida and Uemori [2002] |
| IOPT | Modified kinematic hardening rule flag:<br>EQ.0: Original Yoshida and Uemori formulation, |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: Modified formulation. Define C1 and C2 below. |
| C1, C2 | Constants used to modify $\dot{R}$, so strain hardening will not saturate: $$\dot{R} = R_{\text{sat}} \times \left[ (C_1 + \bar{\varepsilon}^p)^{C_2} - {C_1}^{C_2} \right]$$ Note that these variables are not the material parameter $c$ that controls the rate of the kinematic hardening in Yoshida and Uemori [2002]. |
| IFLD | ID of a load curve defining Forming Limit Diagram (FLD) under linear strain paths. In the load curve, abscissas represent minor strains, while ordinates represent major strains. Define only when the option NLP is used. |

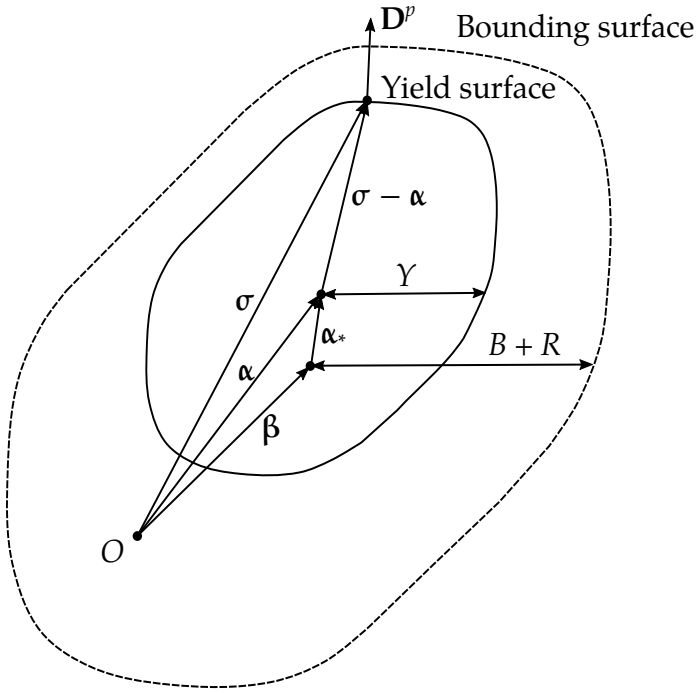



**Figure M125-1.** Schematic illustration of the two-surface model. $O$ is the original center of the yield surface; $\boldsymbol{\alpha}$ is the current center for the yield surface; $\boldsymbol{\beta}$ is the center of the bounding surface; and $\boldsymbol{\alpha}_*$ represents the relative position of the centers of the two surfaces. $Y$ is the size of the yield surface and is constant throughout the deformation process. $B + R$ represents the size of the bounding surface, with $R$ being associated with isotropic hardening. This figure was reproduced from Yoshida and Uemori [2002].

### The Yoshida and Uemori Kinematic Hardening Model:

The following equations give the two-surface model from Yoshida and Uemori [2003] (see Equations 8, 9a, and 9b):

$$\boldsymbol{\alpha}_* = \boldsymbol{\alpha} - \boldsymbol{\beta}$$
$$\overset{\circ}{\boldsymbol{\alpha}}_* = c\left[\left(\frac{a}{Y}\right)(\boldsymbol{\sigma} - \boldsymbol{\alpha}) - \sqrt{\frac{a}{\alpha_*}}\boldsymbol{\alpha}_*\right]\dot{\bar{\varepsilon}}^p$$
$$a = B + R - Y$$

Figure M125-1 illustrates these variables.

The anisotropic hardening parameter, $\dot{R}$, depends on IOPT. The original Yoshida and Uemori model (Yoshida and Uemori [2002]) includes saturation in strain hardening (IOPT = 0). A modified version includes continuous hardening (IOPT = 1). $\dot{R}$ is, thus, given as follows depending on the value of IOPT:

$$\dot{R} = \begin{cases} k(R_{\text{sat}} - R)\dot{\bar{\varepsilon}}^p & \text{if IOPT} = 0 \\ R_{\text{sat}} \times \left[(C_1 + \bar{\varepsilon}^p)^{C_2} - {C_1}^{C_2}\right] & \text{if IOPT} = 1 \end{cases}$$

The following equations (see Equations 12 and 14 of Yoshida and Uemori [2003]) define the change of size and location for the bounding surface, with variable descriptions shown in Figure M125-2:

$$\overset{\circ}{\boldsymbol{\beta}}' = k\left(\frac{2}{3}b\mathbf{D}^p - \boldsymbol{\beta}'\dot{\bar{\varepsilon}}^p\right)$$
$$\sigma_{\text{bound}} = B + R + \beta$$

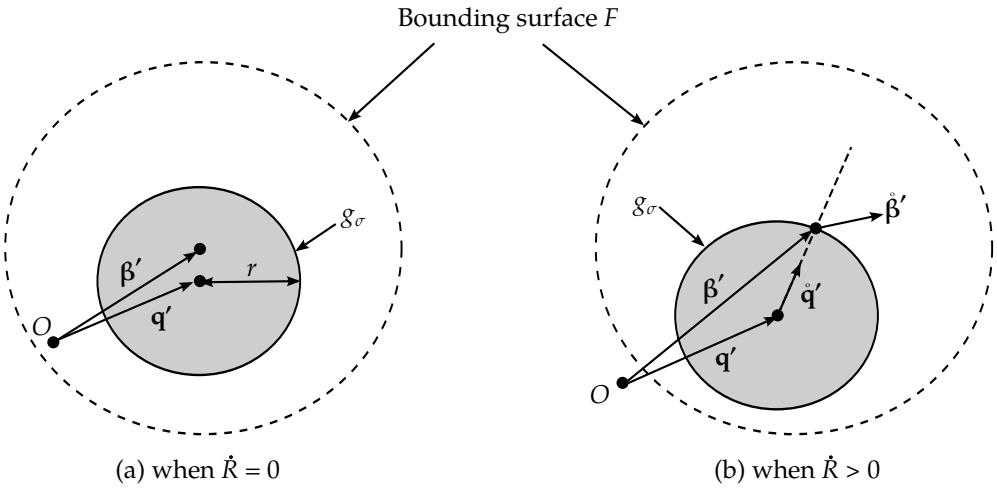



**Figure M125-2.** Change in bounding surface (reproduced from Yoshida and Uemori [2002]).

The unloading process, which follows, includes work-hardening stagnation (see Equations 15, 17, 19a, and 19b of Yoshida and Uemori [2003]):

$$g_\sigma(\boldsymbol{\sigma}', \mathbf{q}', r') = \frac{3}{2}(\boldsymbol{\sigma}' - \mathbf{q}'):(\boldsymbol{\sigma}' - \mathbf{q}') - r^2$$

$$\overset{o}{\mathbf{q}}' = \mu(\boldsymbol{\beta}' - \mathbf{q}')$$

$$\dot{r} = \begin{cases} h\Gamma & \text{if } \dot{R} > 0 \\ 0 & \text{if } \dot{R} = 0 \end{cases}$$

$$\Gamma = \frac{3(\beta' - q'):\overset{o}{\beta}'}{2r}$$

The change in Young's modulus is defined as a function of effective plastic strain (see Equation 24 of Yoshida and Uemori [2003]):

$$E = E_0 - (E_0 - E_a)[1 - \exp(-\xi\bar{\varepsilon}^p)] \ .$$

### A Failure Criterion for Nonlinear Strain Paths (NLP):

The manual pages for *MAT_036 and *MAT_037 describe the NLP failure criterion and corresponding post-processing procedures. With this keyword option, the history variables for every element stored in d3plot files include:

| History Variable # | Description |
|---|---|
| 1 | Formability Index (F.I.) |
| 2 | Strain ratio $\beta$ (in-plane minor strain increment/major strain increment) |
| 3 | Effective strain from the planar isotropic assumption |

To enable the output of these history variables to the d3plot files, NEIPS on the *DATABASE_EXTENT_BINARY card must be set to at least 3.

### References:

Shi, M.F., Zhu, X.H., Xia, Z.C. & Stoughton, T.B. (2008). Determination of nonlinear isotropic/kinematic hardening constitutive parameters for AHSS using tension and compression tests. NUMISHEET. 2008. 137-142.

Yoshida, Fusahito & Uemori, Takeshi. (2002). A model of large-strain cyclic plasticity describing the Bauschinger effect and workhardening stagnation. International Journal of Plasticity. 18. 661-686. 10.1016/S0749-6419(01)00050-X.

Yoshida, Fusahito & Uemori, Takeshi. (2003). A model of large-strain cyclic plasticity and its application to springback simulation. International Journal of Mechanical Sciences. 45. 1687-1702. 10.1016/j.ijmecsci.2003.10.013.

## *MAT_MODIFIED_HONEYCOMB

This is Material Type 126. This material model is usually used for aluminum honeycomb crushable foam materials with anisotropic behavior. Three yield surfaces are available. The first yield surface defines the nonlinear elastoplastic material behavior separately for all normal and shear stresses, which are considered fully uncoupled. The second yield surface considers the effects of off-axis loading. It is transversely isotropic. However, because of this definition of the second yield surface, the material can collapse in a shear mode due to low shear resistance. There was no obvious way of increasing the shear resistance without changing the behavior in purely uniaxial compression. Therefore, with the third yield surface, the model has been modified so that the material's shear and hydrostatic resistance can be prescribed without affecting the uniaxial behavior. The sign of the first load curve ID, LCA, flags the choice of the second yield surface. The third yield surface is flagged by the sign of ECCU, which becomes the initial stress yield limit in simple shear. A description is given below.

The development of the second and third yield surfaces is based on experimental test results of aluminum honeycomb specimens at Toyota Motor Corporation.

The default element for this material is solid type 0, a nonlinear spring-type solid element. The recommended hourglass control is the type 2 viscous formulation for one-point integrated solid elements. When used with this constitutive model, the hourglass control's stiffness form can lead to nonphysical results since it can inhibit strain localization in the shear modes.

This material is available for solid elements and thick shell formulations 3, 5, and 7.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | VF | MU | BULK |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCA | LCB | LCC | LCS | LCAB | LCBC | LCCA | LCSR |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| EAAU | EBBU | ECCU | GABU | GBCU | GCAU | AOPT | MACF |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | RFAC | PRU |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| D1 | D2 | D3 | TSEF | SSEF | VREF | TREF | SHDFLG |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card if AOPT = 3 or 4 (see Card 3).

| V1 | V2 | V3 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** Include this card if LCSR = -1 (see Card 2).

| LCSRA | LCSRB | LCSRC | LCSRAB | LCSRBC | LCSCA | LCSRA | LCSRB |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if PRU = 2.

| PRUAB | PRUAC | PRUBC | PRUBA | PRUCA | PRUCB | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | VF | MU | BULK |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | .05 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus for compacted honeycomb material |
| PR | Poisson's ratio for compacted honeycomb material |
| SIGY | Yield stress for fully compacted honeycomb |
| VF | Relative volume at which the honeycomb is fully compacted. This field is ignored for corotational solid elements, types 0 and 9. |

| VARIABLE | DESCRIPTION |
|---|---|
| MU | Material viscosity coefficient, $\mu$. The default value of 0.05 is recommended. |
| BULK | Bulk viscosity flag:<br>EQ.0.0: Bulk viscosity is not used. This is recommended.<br>EQ.1.0: Bulk viscosity is active and $\mu = 0$. This will give results identical to previous versions of LS-DYNA. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCA | LCB | LCC | LCS | LCAB | LCBC | LCCA | LCSR |
| Type | I | I | I | I | I | I | I | F |
| Default | none | LCA | LCA | LCA | LCS | LCS | LCS | optional |

| VARIABLE | DESCRIPTION |
|---|---|
| LCA | Load curve ID (see *DEFINE_CURVE):<br>LT.0: Yield stress as a function of the angle off the material axis in degrees<br>GT.0: $\sigma_{aa}$ as a function of normal strain component aa, $\varepsilon_{aa}$. Normal strain rate effect can be considered when LCA is defined as a table, see LCSS of MAT_024 for details. Both compressive normal strain and rate are considered positive when defining the curve or table. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a logarithmic strain is expected. See Remarks 1 and 3.<br>Note that LCA < 0 flags using the second yield surface (the transversely isotropic surface) and determines the definition for LCB, LCC, LCS, LCAB, LCBC, and LCCA. |
| LCB | Load curve ID (see *DEFINE_CURVE):<br>LCA.LT.0: Strong axis hardening stress as a function of the volumetric strain<br>LCA.GT.0: $\sigma_{bb}$ as a function of normal strain component bb, $\varepsilon_{bb}$. |

| VARIABLE | DESCRIPTION |
|---|---|
| | Normal strain rate effect can be considered when LCB is defined as a table, see LCSS of MAT_024 for details. Both compressive normal strain and rate are considered positive when defining the curve or table. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a logarithmic strain is expected. See Remarks 1 and 3. |
| LCC | Load curve ID (see *DEFINE_CURVE):<br>LCA.LT.0: Weak axis hardening stress as a function of the volumetric strain<br>LCA.GT.0: $\sigma_{cc}$ as a function of normal strain component cc, $\varepsilon_{cc}$. Normal strain rate effect can be considered when LCC is defined as a table, see LCSS of MAT_024 for details. Both compressive normal strain and rate are considered positive when defining the curve or table. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a logarithmic strain is expected. See Remarks 1 and 3. |
| LCS | Load curve ID (see *DEFINE_CURVE):<br>LCA.LT.0: Damage curve giving the shear stress multiplier as a function of the shear strain component. This curve definition is optional and may be used if damage is desired. IF SHDFLG = 0 (the default), the damage value multiplies the stress every time step and the stress is updated incrementally. The damage curve should be set to unity until failure begins. After failure the value should drop to 0.999 or 0.99 or any number between zero and one depending on how many steps are needed to zero the stress. Alternatively, if SHDFLG = 1, the damage value is treated as a factor that scales the shear stress compared to the undamaged value.<br>LCA.GT.0: Shear stress as a function of shear strain. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. Each |

| VARIABLE | DESCRIPTION |
|---|---|
| | component of shear stress may have its own load curve. See Remarks 1 and 3. |
| LCAB | Load curve ID (see *DEFINE_CURVE):<br>LCA.LT.0: Damage curve giving shear *ab*-stress multiplier as a function of the ab-shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.<br>LCA.GT.0: $\sigma_{ab}$ as a function of the absolute value of shear strain-*ab*, $\varepsilon_{ab}$. Shear strain rate effect can be considered when LCAB is defined as a table, see LCSS of MAT_024 for details. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. See Remarks 1 and 3. |
| LCBC | Load curve ID (see *DEFINE_CURVE):<br>LCA.LT.0: Damage curve giving *bc*-shear stress multiplier as a function of the ab-shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.<br>LCA.GT.0: $\sigma_{bc}$ as a function of the absolute value of shear strain-*bc*, $\varepsilon_{bc}$. Shear strain rate effect can be considered when LCBC is defined as a table, see LCSS of MAT_024 for details. For the corotational solid elements, types 0 and 9, engineering strain is expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. See Remarks 1 and 3. |
| LCCA | Load curve ID (see *DEFINE_CURVE):<br>LCA.LT.0: Damage curve giving *ca*-shear stress multiplier as a function of the ca-shear strain component. This curve definition is optional and may be used if damage is desired. See LCS above.<br>LCA.GT.0: $\sigma_{ca}$ as a function of the absolute value of shear strain-*ca*, $\varepsilon_{ca}$. Shear strain rate effect can be considered when LCCA is defined as a table, see LCSS of *MAT_024 for details. For the corotational solid elements, types 0 and 9, engineering strain is |

| VARIABLE | DESCRIPTION |
|---|---|
| | expected, but for all other solid and thick shell element formulations a shear strain based on the deformed configuration is used. See Remarks 1 and 3. |
| LCSR | Load curve ID (see *DEFINE_CURVE) for strain-rate effects defining the scale factor as a function of effective strain rate $\dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3}(\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij})}$. This is optional. The curves defined above are scaled using this curve. Set LCSR = -1 to define a scale factor in each direction using Card 7. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EAAU | EBBU | ECCU | GABU | GBCU | GCAU | AOPT | MACF |
| Type | F | F | F | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| EAAU | Elastic modulus $E_{aau}$ in uncompressed configuration.<br>LCA.LT.0: Strong axis elastic modulus in uncompressed configuration |
| EBBU | Elastic modulus $E_{bbu}$ in uncompressed configuration.<br>LCA.LT.0: Weak axis elastic modulus in uncompressed configuration |
| ECCU | Elastic modulus $E_{ccu}$ in uncompressed configuration.<br>LT.0.0: \|ECCU\| is the initial stress limit (yield) in simple shear, $\sigma_d^Y$. ECCU < 0 activates the third yield surface if LCA < 0. |
| GABU | Shear modulus $G_{abu}$in uncompressed configuration.<br>LCA.LT.0: Strong-weak shear modulus in uncompressed configuration |
| GBCU | Shear modulus $G_{bcu}$ in uncompressed configuration.<br>LCA.LT.0: Weak-weak shear modulus in uncompressed configuration |

| VARIABLE | DESCRIPTION |
|---|---|
| GCAU | Shear modulus $G_{cau}$ in uncompressed configuration.<br>ECCU.LT.0.0: GCAU is the initial stress limit (yield) in hydrostatic compression, $\sigma_p^Y$. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, $P$, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| VARIABLE | DESCRIPTION |
|---|---|
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF to obtain the final material axes. BETA on *ELEMENT_SOLID_*{OPTION}* is used to rotate the axes. The BETA rotation is optional. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | RFAC | PRU |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| XP YP ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |
| RFAC | Filtering factor for strain rate effects, see MAT_089 for details. |
| PRU | Poisson effect option for the uncompacted status:<br>EQ.0: No Poisson's effect.<br>EQ.1: The Poisson's ratio ramps from 0., when an element is in its un-deformed state, to PR when it is fully compacted.<br>EQ.2: Poisson's ratios are input on Card 8. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | TSEF | SSEF | VREF | TREF | SHDFLG |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| D1 D2 D3 | Components of vector **d** for AOPT = 2. |
| TSEF | Tensile strain at element failure (element will erode)<br>GT.0.0: Constant value<br>LT.0.0: \|TSEF\| is a load curve ID for the curve that defines tensile failure strain as a function of the ratio of compressive to tensile strain. See Sahraei et al. [2016] for details. |
| SSEF | Shear strain at element failure (element will erode)<br>GT.0.0: Constant value<br>LT.0.0: \|SSEF\| is a load curve ID for the curve that defines shear failure strain as a function of the ratio of compressive to tensile strain. |
| VREF | This is an optional input parameter for solid element types 1, 2, 3, 4, and 10 and thick shell formulations 3, 5, and 7. Relative volume at which the reference geometry is stored. At this time, the element behaves like a nonlinear spring. If TREF, below, is reached first, VREF has no effect. |
| TREF | This is an optional input parameter for solid element types 1, 2, 3, 4, and 10 and thick shell formulations 3, 5, and 7. Element time step size at which the reference geometry is stored. When this time step size is reached, the element behaves like a nonlinear spring. If VREF, above, is reached first, TREF has no effect. |
| SHDFLG | Flag defining treatment of damage from curves LCS, LCAB, LCBC, and LCCA (relevant only when LCA < 0):<br>EQ.0.0: Damage reduces shear stress every time step,<br>EQ.1.0: Damage = (shear stress)/(undamaged shear stress) |

Additional card for AOPT = 3 or AOPT = 4.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1 V2 V3 | Components of vector **v** for AOPT = 3 and 4 |

Additional card for LCSR = -1.0

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSRA | LCSRB | LCSRC | LCSRAB | LCSRBC | LCSCA | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSRA | Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the *a*-direction as a function of the *natural logarithm* of the absolute value of deviatoric strain rate in the *a*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor. |
| LCSRB | Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the *b*-direction as a function of the *natural logarithm* of the absolute value of deviatoric strain rate in the *b*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor. |
| LCSRC | Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the *c*-direction as a function of the *natural logarithm* of the absolute |

| VARIABLE | DESCRIPTION |
|---|---|
| | value of deviatoric strain rate in the *c*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor. |
| LCSRAB | Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the *ab*-direction as a function of the *natural logarithm* of the absolute value of strain rate in the *ab*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor. |
| LCSRBC | Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the *bc*-direction as a function of the *natural logarithm* of the absolute value of strain rate in the *bc*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor. |
| LCSRCA | Optional load curve ID if LCSR = -1 (see *DEFINE_CURVE) for strain rate effects defining the scale factor for the yield stress in the *ca*-direction as a function of the *natural logarithm* of the absolute value of strain rate in the *ca*-direction. The scale factor for the lowest value of strain rate defined by the curve is used if the strain rate is zero. The scale factor for the highest value of strain rate defined by the curve also defines the upper limit of the scale factor. |

Additional card for PRU = 2.0

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PRUAB | PRUAC | PRUBC | PRUBA | PRUCA | PRUCB | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PRU*ij* | Poisson's ratios on the *i-j* plane during uncompacted status. The *j*-direction is the direction of transverse strain when the element is stressed in the *i*-direction. |

**Remarks:**

1. **Load curves and efficiency.** For efficiency, the load curves, LCA, LCB, LCC, LCS, LCAB, LCBC, and LCCA, are strongly recommended to contain exactly the same number of points with corresponding strain values on the abscissa. If this recommendation is followed, the cost of the table lookup is insignificant. Conversely, the cost increases significantly if the abscissa strain values are inconsistent between load curves.

2. **Elastic moduli.** For solid element formulations 1 and 2 and thick shell formulations 3, 5, and 7, the behavior before compaction is orthotropic, where the components of the stress tensor are uncoupled, meaning a component of strain will generate resistance in the local $a$-direction with no coupling to the local $b$ and $c$ directions. The elastic moduli vary from their initial values to the fully compacted values linearly with the relative volume:

$$E_{aa} = E_{aau} + \beta(E - E_{aau}) \qquad G_{ab} = E_{abu} + \beta(G - G_{abu})$$
$$E_{bb} = E_{bbu} + \beta(E - E_{bbu}) \qquad G_{bc} = G_{bcu} + \beta(G - G_{bcu})$$
$$E_{cc} = E_{ccu} + \beta(E - E_{ccu}) \qquad G_{ca} = G_{cau} + \beta(G - G_{cau})$$

where

$$\beta = \max\left[\min\left(\frac{1-V}{1-V_f}, 1\right), 0\right]$$

and $G$ is the elastic shear modulus for the fully compacted honeycomb material

$$G = \frac{E}{2(1+v)} \ .$$

The relative volume, $V$, is defined as the ratio of the current volume over the initial volume, and typically, $V = 1$ at the beginning of a calculation.

For corotational solid elements, types 0 and 9, the components of the stress tensor remain uncoupled, and the uncompressed elastic moduli are used; that is, the fully compacted elastic moduli are ignored. However, calculating the element time step size still requires the Young's modulus and Poisson's ratio input on Card 1.

3. **Stress update for uncompacted material.** The load curves define the magnitude of the stress as the material undergoes deformation. The first value in the curve should be less than or equal to zero, corresponding to tension, and increase to full compaction. *Care should be taken when defining the curves so the extrapolated values do not lead to negative yield stresses.*

At the beginning of the stress update, we transform each element's stresses and strain rates into the local element coordinate system. After completing the stress update, we transform the stresses back to the global configuration. For the

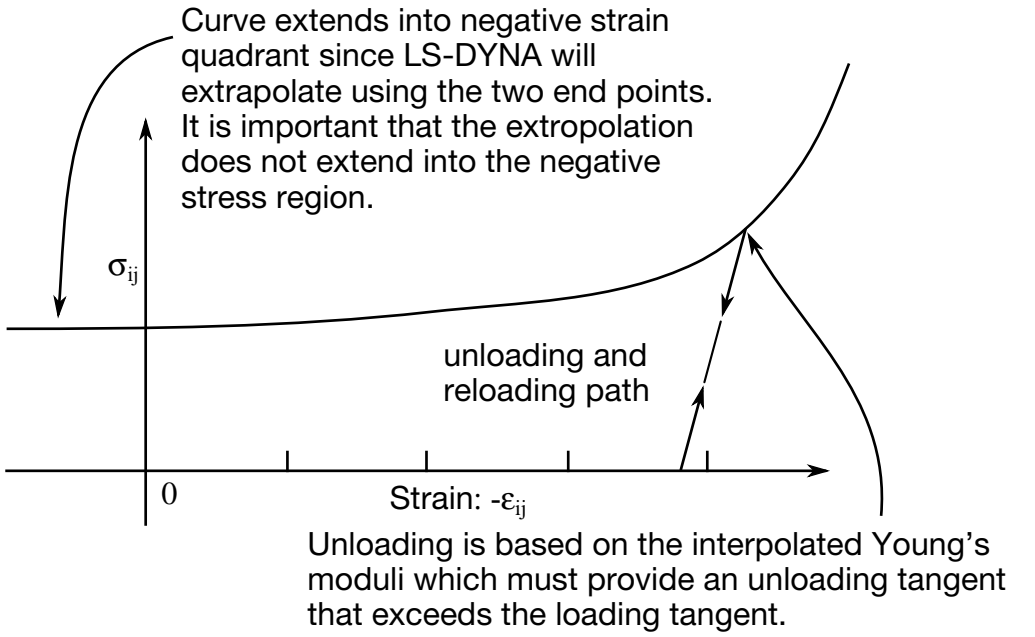



**Figure M126-1.** Stress as a function of strain. Note that the "yield stress" at a strain of zero is nonzero. In the load curve definition the "time" value is the directional strain and the "function" value is the yield stress. Note that for element types 0 and 9 engineering strains are used, but for all other element types the rates are integrated in time.

uncompacted material, the trial stress components are updated using the elastic interpolated moduli according to:

$$\sigma_{aa}^{n+1^{\text{trial}}} = \sigma_{aa}^{n} + E_{aa}\Delta\varepsilon_{aa} \qquad \sigma_{ab}^{n+1^{\text{trial}}} = \sigma_{ab}^{n} + 2G_{ab}\Delta\varepsilon_{ab}$$

$$\sigma_{cc}^{n+1^{\text{trial}}} = \sigma_{cc}^{n} + E_{cc}\Delta\varepsilon_{cc} \qquad \sigma_{bc}^{n+1^{\text{trial}}} = \sigma_{bc}^{n} + 2G_{bc}\Delta\varepsilon_{bc}$$

$$\sigma_{bb}^{n+1^{\text{trial}}} = \sigma_{bb}^{n} + E_{bb}\Delta\varepsilon_{bb} \qquad \sigma_{ca}^{n+1^{\text{trial}}} = \sigma_{ca}^{n} + 2G_{ca}\Delta\varepsilon_{ca}$$

If LCA > 0, each component of the updated stress tensor is checked to ensure that it does not exceed the permissible value determined from the load curves; for example, if

$$\left|\sigma_{ij}^{n+1^{\text{trial}}}\right| > \lambda\sigma_{ij}(\varepsilon_{ij}) \ ,$$

then

$$\sigma_{ij}^{n+1} = \sigma_{ij}(\varepsilon_{ij})\frac{\lambda\sigma_{ij}^{n+1^{\text{trial}}}}{\left|\sigma_{ij}^{n+1^{\text{trial}}}\right|}$$

On Card 3 $\sigma_{ij}(\varepsilon_{ij})$ is defined in the load curve specified in columns 31-40 for the aa stress component, 41-50 for the bb component, 51-60 for the cc component, and 61-70 for the ab, bc, cb shear stress components. The parameter $\lambda$ is either unity or a value taken from the load curve number, LCSR, that defines $\lambda$ as a

function of strain rate. Strain rate is defined here as the Euclidean norm of the deviatoric strain-rate tensor.

If LCA < 0, a transversely isotropic yield surface is obtained where the uniaxial limit stress, $\sigma^y(\varphi, \varepsilon^{\text{vol}})$, can be defined as a function of angle $\varphi$ with the strong axis and volumetric strain, $\varepsilon^{\text{vol}}$. To facilitate the input of data to such a limit stress surface, the limit stress is written as:

$$\sigma^y(\varphi, \varepsilon^{\text{vol}}) = \sigma^b(\varphi) + (\cos\varphi)^2\sigma^s(\varepsilon^{\text{vol}}) + (\sin\varphi)^2\sigma^w(\varepsilon^{\text{vol}})$$

where the functions $\sigma^b$, $\sigma^s$, and $\sigma^w$ are represented by load curves LCA, LCB, LCC, respectively. The latter two curves can be used to include the stiffening effects that are observed as the foam material crushes to the point where it begins to lock up. To ensure that the limit stress decreases with respect to the off-angle, the curves should be defined such that the following equations hold:

$$\frac{\partial\sigma^b(\varphi)}{\partial\varphi} \le 0$$

and

$$\sigma^s(\varepsilon^{\text{vol}}) - \sigma^w(\varepsilon^{\text{vol}}) \ge 0.$$

A drawback of this implementation was that the material often collapsed in shear mode due to low shear resistance. There was no way of increasing the shear resistance without changing the behavior in pure uniaxial compression. We have, therefore, modified the model so that the user can optionally prescribe the shear and hydrostatic resistance in the material without affecting the uniaxial behavior. We introduce the parameters $\sigma_p^Y(\varepsilon^{\text{vol}})$ and $\sigma_d^Y(\varepsilon^{\text{vol}})$ as the *hydrostatic* and *shear limit stresses*, respectively. These are functions of the volumetric strain and are assumed to be given by

$$\begin{aligned}\sigma_p^Y(\varepsilon^{\text{vol}}) &= \sigma_p^Y + \sigma^s(\varepsilon^{\text{vol}}) \\ \sigma_d^Y(\varepsilon^{\text{vol}}) &= \sigma_d^Y + \sigma^s(\varepsilon^{\text{vol}})\end{aligned}\ ,$$

where we have reused the densification function $\sigma^s$. The new parameters are the initial hydrostatic and shear limit stress values, $\sigma_p^Y$ and $\sigma_d^Y$, and are provided by the user as GCAU and |ECCU|, respectively. The negative sign of ECCU flags the third yield surface option whenever LCA < 0. The effect of the third formulation is that (i) for a uniaxial stress the stress limit is given by $\sigma^Y(\phi, \varepsilon^{\text{vol}})$, (ii) for a pressure the stress limit is given by $\sigma_p^Y(\varepsilon^{\text{vol}})$, and (iii) for a simple shear the stress limit is given by $\sigma_d^Y(\varepsilon^{\text{vol}})$. Experiments have shown that the model may give noisy responses and inhomogeneous deformation modes if parameters are not carefully chosen. We, therefore, recommend (i) avoiding large slopes in the function $\sigma^P$, (ii) letting the functions $\sigma^s$ and $\sigma^w$ be slightly increasing, and (iii) avoiding large differences between the stress limit values $\sigma^y(\varphi, \varepsilon^{\text{vol}})$, $\sigma_p^Y(\varepsilon^{\text{vol}})$, and $\sigma_d^Y(\varepsilon^{\text{vol}})$. These guidelines are likely to contradict how one would interpret

test data, and it is up to the user to find a reasonable trade-off between matching experimental results and avoiding the mentioned numerical side effects.

4. **Stress update for fully compacted material.** As in the uncompacted case, we transform each element's stresses and strain rates into the local element coordinate system. For fully compacted material (element formulations 1 and 2), we assume that the material behavior is elastic-perfectly plastic and updated the stress components according to:

$$s_{ij}^{\text{trial}} = s_{ij}^{n} + 2G\Delta\varepsilon_{ij}^{dev^{n+1/2}}$$

where the deviatoric strain increment is defined as

$$\Delta\varepsilon_{ij}^{dev} = \Delta\varepsilon_{ij} - \frac{1}{3}\Delta\varepsilon_{kk}\delta_{ij}.$$

We now check to see if the yield stress for the fully compacted material is exceeded by comparing

$$s_{\text{eff}}^{\text{trial}} = \left(\frac{3}{2}s_{ij}^{\text{trial}}s_{ij}^{\text{trial}}\right)^{1/2}$$

the effective trial stress to the yield stress, $\sigma_y$ (Card 1, field 41-50). If the effective trial stress exceeds the yield stress, we scale back the stress components to the yield surface

$$s_{ij}^{n+1} = \frac{\sigma_y}{s_{\text{eff}}^{\text{trial}}}s_{ij}^{\text{trial}}.$$

We can now update the pressure using the elastic bulk modulus, $K$

$$p^{n+1} = p^{n} - K\Delta\varepsilon_{kk}^{n+1/2}$$

$$K = \frac{E}{3(1-2\nu)}$$

and obtain the final value for the Cauchy stress

$$\sigma_{ij}^{n+1} = s_{ij}^{n+1} - p^{n+1}\delta_{ij}$$

After completing the stress update, we transform the stresses back to the global configuration.

5. **Failure.** For *CONSTRAINED_TIED_NODES_WITH_FAILURE, the failure is based on the volumetric strain instead of the plastic strain.

## *MAT_ARRUDA_BOYCE_RUBBER

This is Material Type 127. This material model provides a hyperelastic rubber model (see [Arruda and Boyce 1993]) combined optionally with linear viscoelasticity as outlined by [Christensen 1980].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | G | N | | | |
| Type | A | F | F | F | F | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID | TRAMP | NT | | | | | |
| Type | F | F | F | | | | | |

**Viscoelastic Constant Cards**. Up to 6 cards may be input. The next keyword ("*") card terminates this input.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus, $K$ |
| G | Shear modulus, $G$ |
| N | Number of statistical links, $N$ |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID | Optional load curve ID of relaxation curve if constants $G_i$ and $\beta_i$ are determined using a least squares fit. This relaxation curve is shown in Figure M76-1. This model ignores the constant stress. |
| TRAMP | Optional ramp time for loading |
| NT | Number of Prony series terms in optional fit. If zero, the default is 6. Currently, the maximum number is 6. Values less than 6, possibly 3-5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Always check the results of the fit in the output file. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved, it is recommended that the coefficients which are written into the output file be input in future runs. |
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term. |
| BETA*i* | Optional decay constant if $i^{th}$ term. |

**Remarks:**

Rubber is generally considered to be fully incompressible since the bulk modulus greatly exceeds the shear modulus in magnitude. To model the rubber as an unconstrained material, a hydrostatic work term, $W_H(J)$, is included in the strain energy functional which is function of the relative volume, $J$, [Ogden 1984]:

$$W(J_1,J) = G\left[\frac{1}{2}(J_1-3)+\frac{1}{20N}\left(J_1^2-9\right)+\frac{11}{1050N^2}\left(J_1^3-27\right)\right] + G\left[\frac{19}{7000N^3}\left(J_1^4-81\right)+\frac{519}{673750N^4}\left(J_1^5-243\right)\right]+W_H(J) \ ,$$

where the hydrostatic work term is in terms of the bulk modulus, $K$, and $J$ as:

$$W_H(J) = \frac{K}{2}(J-1)^2$$

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \ ,$$

or in terms of the second Piola-Kirchhoff stress, $S_{ij}$, and Green's strain tensor, $E_{ij}$,

$$S_{ij} = \int_0^t G_{ijkl}(t-\tau)\frac{\partial E_{kl}}{\partial \tau}d\tau \ ,$$

where $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$ are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by up to six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = \sum_{i=1}^{n} G_i e^{-\beta_i t} \quad .$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. The viscoelastic behavior is optional and an arbitrary number of terms may be used.

### *MAT_HEART_TISSUE

This is Material Type 128. This material model provides a heart tissue model described in the paper by Walker *et al* [2005] as interpreted by Kay Sun. It is backward compatible with an earlier heart tissue model described in the paper by Guccione, McCulloch, and Waldman [1991]. Both models are transversely isotropic.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | C | B1 | B2 | B3 | P | B |
| Type | A | F | F | F | F | F | F | F |

Skip to Card 3 to activate older Guccione, McCulloch, and Waldman [1991] model.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LO | CAOMAX | LR | M | BB | CAO | TMAX | TACT |
| Type | F | I | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | | | | | | |
| Type | F | I | | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| C | Diastolic material coefficient |
| B1 | $b_1$, diastolic material coefficient |
| B2 | $b_2$, diastolic material coefficient |
| B3 | $b_3$, diastolic material coefficient |
| P | Pressure in the muscle tissue |
| B | Systolic material coefficient. Omit for the earlier model. |
| L0 | $l_0$, sarcomere length at which no active tension develops. Omit for the earlier model. |
| CA0MAX | $(Ca_0)_{\max}$, maximum peak intracellular calcium concentrate. Omit for the earlier model. |
| LR | $l_R$, Stress-free sarcomere length. Omit for the earlier model. |
| M | Systolic material coefficient. Omit for the earlier model. |
| BB | Systolic material coefficient. Omit for the earlier model. |
| CA0 | $Ca_0$, peak intracellular calcium concentration. Omit for the earlier model. |
| TMAX | $T_{\max}$, maximum isometric tension achieved at the longest sarcomere length. Omit for the earlier model. |
| TACT | $t_{\text{act}}$, time at which active contraction initiates. Omit for the earlier model |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 5 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| BETA | Material angle in degrees for AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SOLID_ORTHO. |

**Remarks:**

1. **Tissue Model.** The tissue model is described in terms of the energy functional that is transversely isotropic with respect to the local fiber direction,

$$W = \frac{C}{2}(e^Q - 1)$$
$$Q = b_f E_{11}^2 + b_t(E_{22}^2 + E_{33}^2 + E_{23}^2 + E_{32}^2) + b_{fs}(E_{12}^2 + E_{21}^2 + E_{13}^2 + E_{31}^2)$$

Here $C$, $b_f$, $b_t$, and $b_{fs}$ are material parameters and $\mathbf{E}$ is the Lagrange-Green strain.

The systolic contraction is modeled as the sum of the passive stress derived from the strain energy function and an active fiber directional component, $T_0$, which is a function of time, $t$,

$$\mathbf{S} = \frac{\partial W}{\partial \mathbf{E}} - pJ\mathbf{C}^{-1} + T_0\{t, Ca_0, l\}$$
$$\sigma = \frac{1}{J}\mathbf{F}\mathbf{S}\mathbf{F}^T$$

with **S**, the second Piola-Kirchoff stress tensor; **C**, the right Cauchy-Green deformation tensor; $J$, the Jacobian of the deformation gradient tensor **F**; and $\sigma$, the Cauchy stress tensor.

The active fiber directional stress component is defined by a time-varying elastance model, which at end-systole, is reduced to

$$T_0 = T_{\max}\frac{Ca_0^2}{Ca_0^2 + ECa_{50}^2}C_t$$

Here, $T_{\max}$ is the maximum isometric tension achieved at the longest sarcomere length and maximum peak intracellular calcium concentration. The length-dependent calcium sensitivity and internal variable is given by,

$$\begin{aligned} ECa_{50} &= \frac{(Ca_0)_{\max}}{\sqrt{\exp[B(l-l_0]-1}} \\ C_t &= 1/2(1-\cos w) \\ l &= l_R\sqrt{2E_{11}+1} \\ w &= \pi\frac{0.25+t_r}{t_r} \\ t_r &= ml + bb \end{aligned}$$

A cross-fiber, in-plane stress equivalent to 40% of that along the myocardial fiber direction is added.

2. **Older Tissue Model.** The earlier tissue model is described in terms of the energy functional in terms of the Green strain components, $E_{ij}$,

$$\begin{aligned} W(E) &= \frac{C}{2}(e^Q - 1) + \frac{1}{2}P(I_3 - 1) \\ Q &= b_1E_{11}^2 + b_2(E_{22}^2 + E_{33}^2 + E_{23}^2 + E_{32}^2) + b_3(E_{12}^2 + E_{21}^2 + E_{13}^2 + E_{31}^2) \end{aligned}$$

The Green components are modified to eliminate any effects of volumetric work following the procedures of Ogden. See the paper by Guccione *et al* [1991] for more detail.

## *MAT_LUNG_TISSUE

This is Material Type 129. This material model provides a hyperelastic model for heart tissue, see [Vawter 1980] combined optionally with linear viscoelasticity as outlined by [Christensen 1980].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | C | DELTA | ALPHA | BETA | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | LCID | TRAMP | NT | | | |
| Type | F | F | F | F | F | | | |

**Viscoelastic Constant Cards**. Up to 6 cards may be input. A keyword card (with a "*" in column 1) terminates this input if less than 6 cards are used.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GI | BETAI | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus |
| C | Material coefficient. |
| DELTA | $\Delta$, material coefficient. |
| ALPHA | $\alpha$, material coefficient. |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | $\beta$, material coefficient. |
| C1 | Material coefficient. |
| C2 | Material coefficient. |
| LCID | Optional load curve ID of relaxation curve<br>If constants $G_i$ and $\beta_i$ are determined via a least squares fit. This relaxation curve is shown in Figure M76-1. This model ignores the constant stress. |
| TRAMP | Optional ramp time for loading. |
| NT | Number of Prony series terms in optional fit. If zero, the default is 6. Currently, the maximum number is 6. Values less than 6, possibly 3 - 5 are recommended, since each term used adds significantly to the cost. Caution should be exercised when taking the results from the fit. Always check the results of the fit in the output file. Preferably, all generated coefficients should be positive. Negative values may lead to unstable results. Once a satisfactory fit has been achieved it is recommended that the coefficients which are written into the output file be input in future runs. |
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional decay constant if $i^{th}$ term |

**Remarks:**

The material is described by a strain energy functional expressed in terms of the invariants of the Green Strain:

$$W(I_1, I_2) = \frac{C}{2\Delta} e^{(\alpha I_1^2 + \beta I_2)} + \frac{12C_1}{\Delta(1+C_2)}\left[A^{(1+C_2)} - 1\right]$$

$$A^2 = \frac{4}{3}(I_1 + I_2) - 1$$

where the hydrostatic work term is in terms of the bulk modulus, $K$, and the third invariant, $J$, as:

$$W_H(J) = \frac{K}{2}(J-1)^2$$

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}\,(t-\tau)\,\frac{\partial \varepsilon_{kl}}{\partial \tau}\,d\tau$$

or in terms of the second Piola-Kirchhoff stress, $S_{ij}$, and Green's strain tensor, $E_{ij}$,

$$S_{ij} = \int_0^t G_{ijkl}\,(t-\tau)\,\frac{\partial\, E_{kl}}{\partial \tau}\,d\tau$$

where $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$ are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m\, e^{-\beta\, t}$$

given by,

$$g(t) = \sum_{i=1}^{n} G_i e^{-\beta_i\, t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. The viscoelastic behavior is optional and an arbitrary number of terms may be used.

## *MAT_SPECIAL_ORTHOTROPIC

This is Material Type 130. This model is available for Belytschko-Tsay and C0 triangular shell elements. It is based on a resultant stress formulation. In-plane behavior is treated separately from bending in order to model perforated materials, such as television shadow masks. If other shell formulations are specified, the formulation will be automatically switched to Belytschko-Tsay. As implemented, this material model cannot be used with user defined integration rules.

**NOTE:** This material does not support specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | YS | EP | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E11P | E22P | V12P | V21P | G12P | G23P | G31P | |
| Type | F | F | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E11B | E22B | V12B | V21B | G12B | AOPT | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| YS | Yield stress. This parameter is optional and approximates the yield condition. Set to zero if the behavior is elastic. |
| EP | Plastic hardening modulus |
| E11P | $E_{11p}$, for in plane behavior |
| E22P | $E_{22p}$, for in plane behavior |
| V12P | $\nu_{12p}$, for in plane behavior. |
| V11P | $\nu_{11p}$, for in plane behavior |
| G12P | $G_{12p}$, for in plane behavior |
| G23P | $G_{23p}$, for in plane behavior |
| G31P | $G_{31p}$, for in plane behavior |
| E11B | $E_{11b}$, for bending behavior |
| E22B | $E_{22b}$, for bending behavior |
| V12B | $\nu_{12b}$, for bending behavior |
| V21B | $\nu_{21b}$, for bending behavior |
| G12B | $G_{12b}$, for bending behavior |
| AOPT | Material axes option (see MAT_{*OPTION*}TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by |

| VARIABLE | DESCRIPTION |
|---|---|
| | element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| V1 ,V2, V3 | Components of vector **v** for AOPT = 3 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

The in-plane elastic matrix for in-plane, plane stress behavior is given by:

$$\mathbf{C}_{\text{in plane}} = \begin{bmatrix} Q_{11p} & Q_{12p} & 0 & 0 & 0 \\ Q_{12p} & Q_{22p} & 0 & 0 & 0 \\ 0 & 0 & Q_{44p} & 0 & 0 \\ 0 & 0 & 0 & Q_{55p} & 0 \\ 0 & 0 & 0 & 0 & Q_{66p} \end{bmatrix}$$

The terms $Q_{ijp}$ are defined as:

$$Q_{11p} = \frac{E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{22p} = \frac{E_{22p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{12p} = \frac{\nu_{21p}E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{44p} = G_{12p}$$

$$Q_{55p} = G_{23p}$$

$$Q_{66p} = G_{31p}$$

The elastic matrix for bending behavior is given by:

$$\mathbf{C}_{\text{bending}} = \begin{bmatrix} Q_{11b} & Q_{12b} & 0 \\ Q_{12b} & Q_{22b} & 0 \\ 0 & 0 & Q_{44b} \end{bmatrix}$$

The terms $Q_{ijb}$ are similarly defined.

Because this is a resultant formulation, nothing is written to the six stress slots of d3plot. Resultant forces and moments may be written to elout and to dynain in place of the six stresses. The first two extra history variables may be used to complete output of the eight resultants to elout and dynain.

## *MAT_ISOTROPIC_SMEARED_CRACK

This is Material Type 131. This model was developed by Lemmen and Meijer [2001] as a smeared crack model for isotropic materials. This model is available of solid elements only and is restricted to cracks in the *xy*-plane. Users should choose other models unless they have the report by Lemmen and Meijer [2001].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | ISPL | SIGF | GK | SR |
| Type | A | F | F | F | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| ISPL | Failure option:<br>EQ.0: Maximum principal stress criterion<br>EQ.5: Smeared crack model<br>EQ.6: Damage model based on modified von Mises strain |
| SIGF | Peak stress |
| GK | Critical energy release rate |
| SR | Strength ratio |

**Remarks:**

The following documentation is taken nearly verbatim from the documentation of Lemmen and Meijer [2001].

Three methods are offered to model progressive failure. The maximum principal stress criterion detects failure if the maximum (most tensile) principal stress exceeds $\sigma_{\max}$. Upon failure, the material can no longer carry stress.

The second failure model is the smeared crack model with linear softening stress-strain using equivalent uniaxial strains. Failure is assumed to be perpendicular to the principal strain directions. A rotational crack concept is employed in which the crack directions are related to the current directions of principal strain. Therefore, crack directions may rotate in time. Principal stresses are expressed as

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{pmatrix} = \begin{bmatrix}\overline{\mathrm{E}}_1 & 0 & 0\\ 0 & \overline{\mathrm{E}}_2 & 0\\ 0 & 0 & \overline{\mathrm{E}}_3\end{bmatrix}\begin{pmatrix}\tilde{\varepsilon}_1\\ \tilde{\varepsilon}_2\\ \tilde{\varepsilon}_3\end{pmatrix} = \begin{pmatrix}\overline{\mathrm{E}}_1\tilde{\varepsilon}_1\\ \overline{\mathrm{E}}_2\tilde{\varepsilon}_2\\ \overline{\mathrm{E}}_3\tilde{\varepsilon}_3\end{pmatrix} \tag{131.1}$$

with $\overline{\mathrm{E}}_1$, $\overline{\mathrm{E}}_2$ and $\overline{\mathrm{E}}_3$ as secant stiffness in the terms that depend on internal variables.

In the model developed for DYCOSS it has been assumed that there is no interaction between the three directions in which case stresses simply follow from

$$\sigma_j(\tilde{\varepsilon}_j) = \begin{cases}\mathrm{E}\tilde{\varepsilon}_j & \text{if} \quad 0 \le \tilde{\varepsilon}_j \le \tilde{\varepsilon}_{j,\mathrm{ini}}\\ \overline{\sigma}\left(1 - \dfrac{\tilde{\varepsilon}_j - \tilde{\varepsilon}_{j,\mathrm{ini}}}{\tilde{\varepsilon}_{j,\mathrm{ult}} - \tilde{\varepsilon}_{j,\mathrm{ini}}}\right) & \text{if} \quad \tilde{\varepsilon}_{j,\mathrm{ini}} < \tilde{\varepsilon}_j \le \tilde{\varepsilon}_{j,\mathrm{ult}}\\ 0 & \text{if} \quad \tilde{\varepsilon}_j > \tilde{\varepsilon}_{j,\mathrm{ult}}\end{cases} \tag{131.2}$$

with $\overline{\sigma}$ the ultimate stress, $\tilde{\varepsilon}_{j,\mathrm{ini}}$ the damage threshold, and $\tilde{\varepsilon}_{j,\mathrm{ult}}$ the ultimate strain in $j$-direction. The damage threshold is defined as

$$\tilde{\varepsilon}_{j,\mathrm{ini}} = \frac{\overline{\sigma}}{\mathrm{E}} \ . \tag{131.3}$$

The ultimate strain is obtained by relating the crack growth energy and the dissipated energy

$$\int\int \overline{\sigma} d\tilde{\varepsilon}_{j,\mathrm{ult}} dV = GA \tag{131.4}$$

with $G$ as the energy release rate, $V$ as the element volume and $A$ as the area perpendicular to the principal strain direction. The one point elements in LS-DYNA have a single integration point and the integral over the volume may be replaced by the volume. For linear softening it follows

$$\tilde{\varepsilon}_{j,\mathrm{ult}} = \frac{2GA}{V\overline{\sigma}} \ . \tag{131.5}$$

The above formulation may be regarded as a damage equivalent to the maximum principle stress criterion.

The third model is a damage model represented by Brekelmans et. al [1991]. Here the Cauchy stress tensor, $\sigma$, is expressed as

$$\sigma = (1 - D)\mathrm{E}\varepsilon \tag{131.6}$$

where $D$ represents the current damage and the factor $1 - D$ is the reduction factor caused by damage. The scalar damage variable is expressed as function of a so-called damage equivalent strain $\varepsilon_d$

$$D = D(\varepsilon_d) = 1 - \frac{\varepsilon_{\text{ini}}(\varepsilon_{\text{ult}} - \varepsilon_d)}{\varepsilon_d(\varepsilon_{\text{ult}} - \varepsilon_{\text{ini}})} \ , \tag{131.7}$$

where

$$\varepsilon_d = \frac{k-1}{2k(1-2v)} J_1 + \frac{1}{2k}\sqrt{\left(\frac{k-1}{1-2v} J_1\right)^2 + \frac{6k}{(1+v)^2} J_2} \ . \tag{131.8}$$

Here the constant $k$ represents the ratio of the strength in tension over the strength in compression

$$k = \frac{\sigma_{\text{ult ,tension}}}{\sigma_{\text{ult, compression}}} \ , \tag{131.9}$$

$J_1$ and $J_2$ are the first and second invariant of the strain tensor representing the volumetric and the deviatoric straining, respectively

$$\begin{aligned} J_1 &= \text{tr}(\varepsilon) \\ J_2 &= \text{tr}(\varepsilon \cdot \varepsilon) - \frac{1}{3}[tr(\varepsilon)]^2 \end{aligned} \tag{131.10}$$

If the compression and tension strength are equal, the dependency on the volumetric strain vanishes in (131.8) and failure is shear dominated. If the compressive strength is much larger than the strength in tension, $k$ becomes small and the $J_1$ terms in (131.8) dominate the behavior.

## *MAT_ORTHOTROPIC_SMEARED_CRACK

This is Material Type 132. This material is a smeared crack model for orthotropic materials. It is available for solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UINS | UISS | CERRMI | CERRMII | IND | ISD | | |
| Type | F | F | F | F | I | I | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | | | | |
| Type | F | F | F | F | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | I | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. |
| EA | Young's modulus in *a*-direction, $E_a$ |
| EB | Young's modulus in *b*-direction, $E_b$ |
| EC | Young's modulus in *c*-direction, $E_c$ |
| PRBA | Poisson's ratio *ba*, $\nu_{ba}$ |
| PRCA | Poisson's ratio *ca*, $\nu_{ca}$ |
| PRCB | Poisson's ratio *cb*, $\nu_{cb}$ |
| UINS | Ultimate interlaminar normal stress |
| UISS | Ultimate interlaminar shear stress |
| CERRMI | Critical energy release rate mode I |
| CERRMII | Critical energy release rate mode II |
| IND | Interlaminar normal direction:<br>EQ.1.0: Along local *a*-axis<br>EQ.2.0: Along local *b*-axis<br>EQ.3.0: Along local *c*-axis |
| ISD | Interlaminar shear direction :<br>EQ.4.0: Along local *ab*-axis<br>EQ.5.0: Along local *bc*-axis<br>EQ.6.0: Along local *ca*-axis |
| GAB | Shear modulus *ab*, $G_{ab}$ |
| GBC | Shear modulus *bc*, $G_{bc}$ |
| GCA | Shear modulus *ca*, $G_{ca}$ |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. |
| | EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| XP YP ZP | Define coordinates of point *P* for AOPT = 1 and 4. |
| A1 A2 A3 | Define components of vector ***a*** for AOPT = 2. |
| MACF | Material axes change flag for solid elements: |
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation |
| | EQ.-3: Switch material axes *a* and *c* before BETA rotation |
| | EQ.-2: Switch material axes *a* and *b* before BETA rotation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 5 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| V1 V2 V3 | Define components of vector **v** for AOPT = 3 and 4. |
| D1 D2 D3 | Define components of vector **d** for AOPT = 2: |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Remarks:**

This is an orthotropic material with optional delamination failure for brittle composites. The elastic formulation is identical to the DYNA3D model that uses total strain formulation. The constitutive matrix C that relates to global components of stress to the global components of strain is defined as:

$$C = T^T C_L T$$

where T is the transformation matrix between the local material coordinate system and the global system and $C_L$ is the constitutive matrix defined in terms of the material constants of the local orthogonal material axes *a*, *b*, and *c* (see DYNA3D use manual).

Failure is described using linear softening stress strain curves for the interlaminar normal and interlaminar shear directions. The current implementation for failure is essentially two-dimensional. Damage can occur in the interlaminar normal direction and a single

interlaminar shear direction. The orientation of these directions with respect to the principal material directions must be specified by the user.

Based on specified values for the ultimate stress and the critical energy release rate bounding surfaces are defined as

$$f_n = \sigma_n - \overline{\sigma}_n(\varepsilon_n)$$
$$f_s = \sigma_s - \overline{\sigma}_s(\varepsilon_s)$$

where the subscripts $n$ and $s$ refer to the normal and shear component. If stresses exceed the bounding surfaces, inelastic straining occurs. The ultimate strain is obtained by relating the crack growth energy and the dissipated energy. For solid elements with a single integration point it can be derived to obtain

$$\varepsilon_{i,\text{ult}} = \frac{2G_i A}{V\sigma_{i,\text{ult}}}$$

with $G_i$ as the critical energy release rate, $V$ as the element volume, $A$ as the area perpendicular to the active normal direction and $\sigma_{i,\text{ ult}}$ as the ultimate stress. For the normal component failure can only occur under tensile loading. For the shear component the behavior is symmetric around zero. The resulting stress bounds are depicted in Figure M132-1. Unloading is modeled with a Secant stiffness.

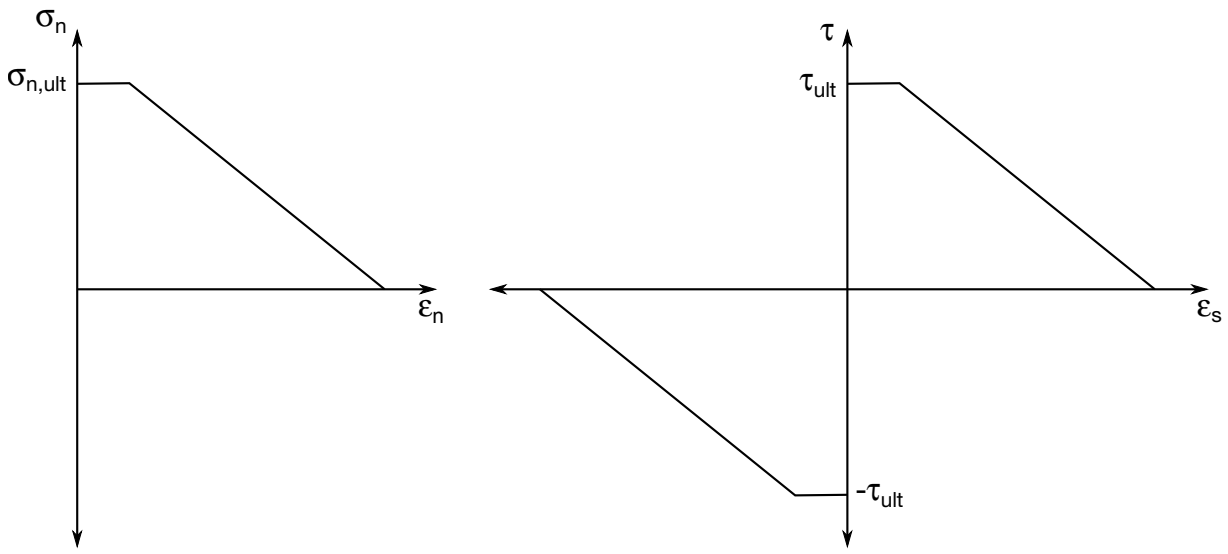



**Figure M132-1.** Shows stress bounds for the active normal component (left) and the archive shear component (right).

## *MAT_BARLAT_YLD2000

This is Material Type 133. This model was developed by Barlat et al. [2003] to overcome some shortcomings of the six-parameter Barlat model implemented as material 33 (*MAT_BARLAT_YLD96). This model is available for shell, thick shell, and solid elements. Support for solid elements started with R12, but only for explicit analysis. The model for solid elements is based on the approach by Dunand et al. [2012].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | FIT | BETA | ITER | ISCALE |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| K | E0 | N | C | P | HARD | A | |
|---|---|---|---|---|---|---|---|

**Card 2.1.** This card is included if A < 0.

| CRC1 | CRA1 | CRC2 | CRA2 | CRC3 | CRA3 | CRC4 | CRA4 |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included if FIT = 0.

| ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
|---|---|---|---|---|---|---|---|

**Card 3b.1.** This card is included if FIT = 1.

| SIG00 | SIG45 | SIG90 | R00 | R45 | R90 | | |
|---|---|---|---|---|---|---|---|

**Card 3b.2.** This card is included if FIT = 1.

| SIGXX | SIGYY | SIGXY | DXX | DYY | DXY | | |
|---|---|---|---|---|---|---|---|

**Card 4.1.** This card is included if HARD = 3.

| CP | T0 | TREF | TA0 | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.2.** This card is included if HARD = 3.

| A | B | C | D | P | Q | E0MART | VM0 |
|---|---|---|---|---|---|---|---|

**Card 4.3.** This card is included if HARD = 3.

| AHS | BHS | M | N | EPS0 | HMART | K1 | K2 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| AOPT | OFFANG | P4 | HTFLAG | HTA | HTB | HTC | HTD |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | USRFAIL | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | FIT | BETA | ITER | ISCALE |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus<br>LE.0: -E is either a load curve ID for Young's modulus as a function of plastic strain or a table ID for Young's modulus as a function of plastic strain and temperature. |
| PR | Poisson's ratio<br>LE.0: -PR is a load curve ID for Poisson's ratio as a function of temperature. |
| FIT | Material parameter fit flag:<br>EQ.0.0: Material parameters are used directly on Card 3a.<br>EQ.1.0: Material parameters are determined from test data on Cards 3b.1 and 3b.2. |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Hardening parameter. Any value ranging from 0 (isotropic hardening) to 1 (kinematic hardening) may be input. This field is ignored if the flow potential exponent A is input as a negative number. |
| ITER | Plastic iteration flag:<br>EQ.0.0: Plane stress algorithm for stress return<br>EQ.1.0: Secant iteration algorithm for stress return<br>ITER provides an option of using three secant iterations for determining the thickness strain increment as experiments have shown that this leads to a more accurate prediction of shell thickness changes for rapid processes. A significant increase in computation time is incurred with this option so it should be used only for applications associated with high rates of loading and/or for implicit analysis. |
| ISCALE | Yield locus scaling flag:<br>EQ.0.0: Scaling on - reference direction is the rolling direction (default)<br>EQ.1.0: Scaling off – reference direction arbitrary |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | E0 | N | C | P | HARD | A | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Material parameter:<br>HARD.EQ.1.0: $k$, strength coefficient for exponential hardening<br>HARD.EQ.2.0: $a$ in Voce hardening law<br>HARD.EQ.4.0: $k$, strength coefficient for Gosh hardening<br>HARD.EQ.5.0: $a$ in Hocket-Sherby hardening law |
| E0 | Material parameter:<br>HARD.EQ.1.0: $e_0$, strain at yield for exponential hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| | HARD.EQ.2.0: $b$ in Voce hardening law<br>HARD.EQ.4.0: $\varepsilon_0$, strain at yield for Gosh hardening<br>HARD.EQ.5.0: $b$ in Hocket-Sherby hardening law |
| N | Material parameter:<br>HARD.EQ.1.0: $n$, exponent for exponential hardening<br>HARD.EQ.2.0: $c$ in Voce hardening law<br>HARD.EQ.4.0: $n$, exponent for Gosh hardening<br>HARD.EQ.5.0: $c$ in Hocket-Sherby hardening law |
| C | Cowper-Symonds strain rate parameter, $C$; see Remark 1. |
| P | Cowper-Symonds strain rate parameter, $p$; see Remark 1.<br>$$\sigma_y^v(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_y(\varepsilon_p)\left(1 + \left[\frac{\dot{\varepsilon}_p}{C}\right]^{1/p}\right)$$ |
| HARD | Hardening law:<br>EQ.1.0: Exponential hardening: $\sigma_y = k(\varepsilon_0 + \varepsilon_p)^n$<br>EQ.2.0: Voce hardening: $\sigma_y = a - be^{-c\varepsilon_p}$<br>EQ.3.0: Hansel hardening (see Remark 4)<br>EQ.4.0: Gosh hardening: $\sigma_y = k(\varepsilon_0 + \varepsilon_p)^n - p$<br>EQ.5.0: Hocket-Sherby hardening: $\sigma_y = a - be^{-c\varepsilon_p^q}$<br>LT.0.0: Absolute value defines load curve ID, table ID or 3D table ID. If it is a load curve, then yield stress is a function of plastic strain. If it is a table, then yield stress is a function of either plastic strain and plastic strain rate in case of a 2D table, or, a function of plastic strain, plastic strain rate, and temperature in case of a 3D table. |
| A | Flow potential exponent. For face-centered cubic (FCC) materials, A = 8 is recommended, and for body-centered cubic (BCC) materials, A = 6 may be used. If the input is negative, then an extra card for Chaboche-Roussilier kinematic hardening is read, the flow potential exponent is taken as the absolute value of what is input, and BETA above is ignored. |

**Chaboche-Roussilier Card.** Additional Card for A < 0.

| Card 2.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CRC1 | CRA1 | CRC2 | CRA2 | CRC3 | CRA3 | CRC4 | CRA4 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CRC*n* | Chaboche-Rousselier kinematic hardening parameters; see Remark 3. |
| CRA*n* | Chaboche-Rousselier kinematic hardening parameters; see Remark 3. |

**Direct Material Parameter Card.** Additional card for FIT = 0.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA*i* | $\alpha_i$, see Remark 2. If ALPHA1 is input as a negative number, then the absolute value is the ID of a load curve giving $\alpha_1$ as a function of temperature. With this choice, *all* ALPHAi must be negative and given by curves. |

**Test Data Card 1.** Additional card for FIT = 1.

| Card 3b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIG00 | SIG45 | SIG90 | R00 | R45 | R90 | | |
| Type | F | F | F | F | F | F | | |

**Test Data Card 2.** Additional Card for FIT = 1.

| Card 3b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGXX | SIGYY | SIGXY | DXX | DYY | DXY | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIG00 | Yield stress in 00 direction<br>LT.0.0: -SIG00 is load curve ID, defining this stress as a function of temperature. |
| SIG45 | Yield stress in 45 direction<br>LT.0.0: -SIG45 is load curve ID, defining this stress as a function of temperature. |
| SIG90 | Yield stress in 90 direction<br>LT.0.0: -SIG90 is load curve ID, defining this stress as a function of temperature. |
| R00 | *R*-value in 00 direction<br>LT.0.0: -R00 is load curve ID, defining this value as a function of temperature. |
| R45 | *R*-value in 45 direction<br>LT.0.0: -R45 is load curve ID, defining this value as a function of temperature. |
| R90 | *R*-value in 90 direction<br>LT.0.0: -R90 is load curve ID, defining this value as a function of temperature. |
| SIGXX | *xx*-component of stress on the yield surface (see Remark 2). |
| SIGYY | *yy*-component of stress on the yield surface (see Remark 2). |
| SIGXY | *xy*-component of stress on the yield surface (see Remark 2). |
| DXX | *xx*-component of tangent to the yield surface (see Remark 2). |

| VARIABLE | DESCRIPTION |
|---|---|
| DYY | $yy$-component of tangent to the yield surface (see Remark 2). |
| DXY | $xy$-component of tangent to the yield surface (see Remark 2). |

**Hansel Hardening Card 1.** Additional card for HARD = 3.

| Card 4.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP | T0 | TREF | TA0 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CP | Adiabatic temperature calculation option:<br>EQ.0.0: Adiabatic temperature calculation is disabled.<br>GT.0.0: CP is the specific heat $C_p$. Adiabatic temperature calculation is enabled. |
| T0 | Initial temperature $T_0$ of the material if adiabatic temperature calculation is enabled |
| TREF | Reference temperature for output of the yield stress as history variable |
| TA0 | Reference temperature $T_{A0}$, the absolute zero for the used temperature scale, such as -273.15 if the Celsius scale is used and 0.0 if the Kelvin scale is used. |

**Hansel Hardening Card 2.** Additional card for HARD = 3.

| Card 4.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | C | D | P | Q | EOMART | VM0 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A | Martensite rate equation parameter $A$, see Remark 4. |

| VARIABLE | DESCRIPTION |
|---|---|
| B | Martensite rate equation parameter $B$, see Remark 4. |
| C | Martensite rate equation parameter $C$, see Remark 4. |
| D | Martensite rate equation parameter $D$, see Remark 4. |
| P | Martensite rate equation parameter $p$, see Remark 4. |
| Q | Martensite rate equation parameter $Q$, see Remark 4. |
| E0MART | Martensite rate equation parameter $E_{0(\text{mart})}$, see Remark 4. |
| VM0 | The initial volume fraction of martensite $0.0 < V_{m0} < 1.0$ may be initialized using two different methods:<br>GT.0.0: $V_{m0}$ is set to VM0.<br>LT.0.0: Can be used only when there are initial plastic strains $\varepsilon^p$ present, such as when using *INITIAL_STRESS_SHELL. The absolute value of VM0 is then the load curve ID for a function, $f$, that sets $V_{m0} = f(\varepsilon^p)$. The function $f$ must be a monotonically nondecreasing function of $\varepsilon^p$. |

**Hansel Hardening Card 3.** Additional card for HARD = 3.

| Card 4.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AHS | BHS | M | N | EPS0 | HMART | K1 | K2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AHS | Hardening law parameter $A_{\text{HS}}$, see Remark 4. |
| BHS | Hardening law parameter $B_{\text{HS}}$, see Remark 4. |
| M | Hardening law parameter $m$, see Remark 4. |
| N | Hardening law parameter $n$, see Remark 4. |
| EPS0 | Hardening law parameter $\varepsilon_0$, see Remark 4. |
| HMART | Hardening law parameter $\Delta H_{\gamma \to \alpha'}$, see Remark 4. |

| VARIABLE | DESCRIPTION |
|---|---|
| K1 | Hardening law parameter $K_1$, see Remark 4. |
| K2 | Hardening law parameter $K_2$, see Remark 4. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | OFFANG | P4 | HTFLAG | HTA | HTB | HTC | HTD |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for more details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M133-1. Nodes 1, 2, and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, OFFANG, from a line determined by taking the cross product of the vector $v$ with the normal to the plane of the element.<br>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| OFFANG | Offset angle for AOPT = 3 |
| P4 | Material parameter:<br>HARD.EQ.4.0: $p$ in Gosh hardening law<br>HARD.EQ.5.0: $q$ in Hocket-Sherby hardening law |
| HTFLAG | Heat treatment flag (see Remark 5): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0: Preforming stage<br>EQ.1: Heat treatment stage<br>EQ.2: Postforming stage |
| HTA | Load curve or table ID for postforming parameter *a* |
| HTB | Load curve or table ID for postforming parameter *b* |
| HTC | Load curve or table ID for postforming parameter *c* |
| HTD | Load curve or table ID for postforming parameter *d* |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | USRFAIL | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| USRFAIL | User defined failure flag:<br>EQ.0: No user subroutine is called.<br>EQ.1: User subroutine `matusr_24` in dyn21.f is called. |

**Remarks:**

1. **Cowper – Symonds strain rate.** Strain rate is accounted for using the Cowper and Symonds model, which scales the yield stress with the factor

$$1+\left(\frac{\dot{\varepsilon}_p}{C}\right)^{1/p},$$

where $\dot{\varepsilon}_p$ is the plastic strain rate. To ignore strain rate effects, set both C and P to zero.

2. **Yield condition.** The yield condition for this material can be written as

$$f(\boldsymbol{\sigma},\boldsymbol{\alpha},\varepsilon_p)=\sigma_{\text{eff}}(\sigma_{xx}-2\alpha_{xx}-\alpha_{yy},\sigma_{yy}-2\alpha_{yy}-\alpha_{xx},\sigma_{xy}-\alpha_{xy}) -\sigma_Y^t(\varepsilon_p,\dot{\varepsilon}_p,\beta)\le 0$$

where

$$\sigma_{\text{eff}}(s_{xx},s_{yy},s_{xy})=\left[\frac{1}{2}(\varphi'+\varphi'')\right]^{1/a}$$

$$\varphi'=|X'_1-X'_2|^a$$

$$\varphi''=|2X''_1+X''_2|^a+|X''_1+2X''_2|^a$$

The $X'_i$ and $X''_i$ are eigenvalues of $X'_{ij}$ and $X''_{ij}$ and are given by

$$X'_1=\frac{1}{2}\left(X'_{11}+X'_{22}+\sqrt{(X'_{11}-X'_{22})^2+4X'^2_{12}}\right)$$
$$X'_2=\frac{1}{2}\left(X'_{11}+X'_{22}-\sqrt{(X'_{11}-X'_{22})^2+4X'^2_{12}}\right)$$

and

$$X''_1=\frac{1}{2}\left(X''_{11}+X''_{22}+\sqrt{(X''_{11}-X''_{22})^2+4X''^2_{12}}\right)$$
$$X'_2=\frac{1}{2}\left(X''_{11}+X''_{22}-\sqrt{(X''_{11}-X''_{22})^2+4X''^2_{12}}\right)$$

respectively. The $X'_{ij}$ and $X''_{ij}$ are given by

$$\begin{pmatrix}X'_{11}\\X'_{22}\\X'_{12}\end{pmatrix}=\begin{pmatrix}L'_{11}&L'_{12}&0\\L'_{21}&L'_{22}&0\\0&0&L'_{33}\end{pmatrix}\begin{pmatrix}s_{xx}\\s_{yy}\\s_{xy}\end{pmatrix}$$
$$\begin{pmatrix}X''_{11}\\X''_{22}\\X''_{12}\end{pmatrix}=\begin{pmatrix}L''_{11}&L''_{12}&0\\L''_{21}&L''_{22}&0\\0&0&L''_{33}\end{pmatrix}\begin{pmatrix}s_{xx}\\s_{yy}\\s_{xy}\end{pmatrix}$$

where,

$$\begin{pmatrix} L'_{11} \\ L'_{12} \\ L'_{21} \\ L'_{22} \\ L'_{33} \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{pmatrix}$$

$$\begin{pmatrix} L''_{11} \\ L''_{12} \\ L''_{21} \\ L''_{22} \\ L''_{33} \end{pmatrix} = \frac{1}{9}\begin{pmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{pmatrix}\begin{pmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{pmatrix}$$

The parameters $\alpha_1$ to $\alpha_8$ determine the shape of the yield surface. $s_{xx}, s_{yy}$, and $s_{xy}$ do not denote the deviatoric stress components, but the arguments are used in the $\sigma_{\text{eff}}$ function.

Three uniaxial tests and a more general test facilitate determining the material parameters. The yield stress and R-values come from the uniaxial tests. The general test provides an arbitrary point on the yield surface, given by the stress components in the material system as

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{pmatrix} ,$$

and a tangent to the yield surface at that particular point. For the latter, the tangential direction should be determined so that

$$d_{xx}\dot{\varepsilon}^p_{xx} + d_{yy}\dot{\varepsilon}^p_{yy} + 2d_{xy}\dot{\varepsilon}^p_{xy} = 0 \ .$$

The data for the general test can be set to zero in the input deck for LS-DYNA to just fit the uniaxial data.

The effective stress (excluding back stress) can be output to the d3plot database through *DEFINE_MATERIAL_HISTORIES.

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Effective Stress | - - - - - | Effective stress $\sigma_{\text{eff}}(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$, see above |

3. **Kinematic hardening model.** A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress, $\boldsymbol{\alpha}$, is introduced such that the effective stress is computed as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12}) \ .$$

The back stress is the sum of up to four terms according to

$$\alpha_{ij} = \sum_{k=1}^{4} \alpha_{ij}^{k} \ ,$$

and the evolution of each back stress component is as follows

$$\delta\alpha_{ij}^{k} = C_k \left( a_k \frac{s_{ij} - \alpha_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^{k} \right) \delta\varepsilon_p \ ,$$

where $C_k$ and $a_k$ are material parameters, $s_{ij}$ is the deviatoric stress tensor, $\sigma_{\text{eff}}$ is the effective stress and $\varepsilon_p$ is the effective plastic strain. The yield condition for this case is modified according to

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon_p) = \sigma_{\text{eff}}(\sigma_{xx} - 2\alpha_{xx} - \alpha_{yy}, \sigma_{yy} - 2\alpha_{yy} - \alpha_{xx}, \sigma_{xy} - \alpha_{xy}) - \left\{ \sigma_Y^t(\varepsilon_p, \dot{\varepsilon}_p, 0) - \sum_{k=1}^{4} a_k \left[ 1 - \exp(-C_k \varepsilon_p) \ \right] \right\} \leq 0$$

in order to get the expected stress-strain response for uniaxial stress.

4. **Hansel hardening law.** The Hansel hardening law is the same as in material 113 but is repeated here for the sake of convenience.

The hardening is temperature dependent; therefore, this material model must be run in a coupled thermo-mechanical solution, with prescribed temperatures, or with the adiabatic temperature calculation option. Setting the parameter CP to the specific heat $C_p$ of the material activates the adiabatic temperature calculation, which calculates the temperature rate from the equation

$$\dot{T} = \sum_{i,j} \frac{\sigma_{\mathbf{ij}} D_{ij}^{\mathrm{p}}}{\rho C_p} \ ,$$

where $\sigma : \mathbf{D}^p$ (the numerator) is the plastically dissipated heat. Using the Kelvin scale is recommended, even though other scales may be used without problems.

The hardening behaviour is described by the following equations. The martensite rate equation is

$$\frac{\partial V_m}{\partial \bar{\varepsilon}^p} = \begin{cases} 0 & \varepsilon < E_{0(\text{mart})} \\ \dfrac{B}{A} {V_m}^p \left( \dfrac{1 - V_m}{V_m} \right)^{\frac{B+1}{B}} \dfrac{[1 - \tanh(C + \mathrm{D} \times T)]}{2} \exp\left( \dfrac{Q}{T - T_{A0}} \right) & \bar{\varepsilon}^p \geq E_{0(\text{mart})} \end{cases}$$

where $\bar{\varepsilon}^p$ is the effective plastic strain and $T$ is the temperature. The martensite fraction is integrated from the above rate equation:

$$V_m = \int_0^{\varepsilon} \frac{\partial V_m}{\partial \bar{\varepsilon}^p} d\bar{\varepsilon}^p .$$

It always holds that $0.0 < V_m < 1.0$. The initial martensite content is $V_{m0}$ and must be greater than zero and less than 1.0. Note that $V_{m0}$ is not used during a

restart or when initializing the $V_m$ history variable using *INITIAL_STRESS_-SHELL.

The yield stress $\sigma_y$ is

$$\sigma_y = \{B_{HS} - (B_{HS} - A_{HS})\exp(-m[\bar{\varepsilon}^p + \varepsilon_0]^n)\}(K_1 + K_2 T) + \Delta H_{\gamma\to\alpha'} V_m.$$

The parameters P and B should fulfill the following condition

$$\frac{1+B}{B} < p \ .$$

If not fulfilled, the martensite rate will approach infinity as $V_m$ approaches zero. A value between 0.001 and 0.02 is recommended for $\varepsilon_0$.

Apart from the effective plastic strain, a few additional history variables are output as described in the table below.

| History Variable # | Description |
|---|---|
| 26 | Yield stress of material at temperature TREF. This variable is useful when evaluating the strength of the material after, for example, a simulated forming operation. |
| 27 | Volume fraction martensite, $V_m$ |
| 28 | If CP = 0.0, it is not used. If CP > 0.0, then it is the temperature from the adiabatic temperature calculation. |

5. **Heat treatment.** Heat treatment for increasing the formability of prestrained aluminum sheets can be simulated through the use of HTFLAG, where the intention is to run a forming simulation in steps involving preforming, springback, heat treatment, and postforming. In each step, the history is transferred to the next using a dynain file (see *INTERFACE_SPRINGBACK). The first two steps are performed with HTFLAG = 0 according to standard procedures, resulting in a plastic strain field $\varepsilon_p^0$ corresponding to the prestrain. The heat treatment step is performed using HTFLAG = 1 in a coupled thermomechanical simulation, where the blank is heated. The coupling between thermal and mechanical processes is only through the maximum temperature $T^0$ being stored as a history variable in the material model, corresponding to the heat treatment temperature. Here, it is important to export all history variables to the dynain file for the postforming step. In the final postforming step, HTFLAG = 2, the yield stress is then augmented by the Hocket-Sherby-like term

$$\Delta\sigma = b - (b-a)\exp\left[-c\left(\varepsilon_p - \varepsilon_p^0\right)^d\right],$$

where $a$, $b$, $c$, and $d$ are given as tables as functions of the heat treatment temperature $T^0$ and prestrain $\varepsilon_p^0$. That is, in the table definitions, each load curve

corresponds to a given prestrain, and the load curve value is with respect to the heat treatment temperature,

$$a = a(T^0, \varepsilon_p^0) \ , \qquad b = b(T^0, \varepsilon_p^0) \ , \qquad c = c(T^0, \varepsilon_p^0) \ , \qquad d = d(T^0, \varepsilon_p^0)$$

The effect of heat treatment is that the material strength decreases, but hardening increases, thus typically,

$$a \le 0 \ , \qquad b \ge a \ , \qquad c > 0 \ , \qquad d > 0 \ .$$

6. **Prestraining.** It is possible to add an initial plastic strain shift, $\Delta\varepsilon_p$, to the yield criterion so that it changes to:

$$f(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon_p) = \sigma_{\text{eff}}(\sigma_{xx} - 2\alpha_{xx} - \alpha_{yy}, \sigma_{yy} - 2\alpha_{yy} - \alpha_{xx}, \sigma_{xy} - \alpha_{xy}) - \left\{ \sigma_Y^t(\varepsilon_p + \Delta\varepsilon_p, \dot{\varepsilon}_p, 0) - \sum_{k=1}^{4} a_k\left[1 - \exp(-C_k\varepsilon_p)\ \right] \right\} \le 0$$

where $\Delta\varepsilon_p$ can be defined using history variable #8 (see *INITIAL_STRESS_... for details). This option is available for explicit analysis with ITER = 0 and HARD < 0 only.

7. **History variables.** The following table lists the available history variables. Set NEIPH for solid elements and NEIPS for shell elements on *DATABASE_EXTENT_BINARY to output history variables to d3plot.

| History Variable # | Description for Shell Elements | Description for Solid Elements |
|---|---|---|
| 1 | Back stress component $\alpha_{11}$ | Back stress component $\alpha_{11}$ |
| 2 | Back stress component $\alpha_{22}$ | Back stress component $\alpha_{22}$ |
| 3 | $\cos\alpha$, where $\alpha$ is the in-plane angle between the material coordinate system and the element coordinate system | Back stress component $\alpha_{12}$ |
| 4 | $-\sin\alpha$, where $\alpha$ is defined in the description of history variable #3 | |
| 5 | Back stress component $\alpha_{12}$ | Strain rate |
| 7 | Strain rate | |
| 9 | Formability Index | |
| 21 | Temperature if HTFLAG = 1 | Temperature if HTFLAG = 1 |

## *MAT_VISCOELASTIC_FABRIC

This is Material Type 134. The viscoelastic fabric model is a variation on the general viscoelastic model of material 76. This model is valid for 3 and 4 node membrane elements only and is strongly recommended for modeling isotropic viscoelastic fabrics where wrinkling may be a problem. For thin fabrics, buckling can result in an inability to support compressive stresses; thus, a flag is included for this option. If bending stresses are important use a shell formulation with model 76.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | BULK | | | | CSE | |
|---|---|---|---|---|---|---|---|

**Card 2.** If fitting is done from a relaxation curve, specify fitting parameters on this card, otherwise if constants are set on Card 3, LEAVE THIS CARD BLANK.

| LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is not needed if Card 2 is defined (not blank). Up to 6 of this card may be input. If fewer than 6 cards are used, then the next keyword ("*") card terminates this input.

| GI | BETAI | KI | BETAKI | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | | | | CSE | |
| Type | I | F | F | | | | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number must be specified. |
| RO | Mass density. |
| BULK | Elastic constant bulk modulus. If the bulk behavior is viscoelastic, then this modulus is used in determining the contact interface stiffness only. |

| VARIABLE | DESCRIPTION |
|---|---|
| CSE | Compressive stress flag (default = 0.0):<br>EQ.0.0: Don't eliminate compressive stresses.<br>EQ.1.0: Eliminate compressive stresses. |

**Relaxation Curve Card.** If fitting is done from a relaxation curve, specify fitting parameters on card 2, *otherwise* if constants are set on Viscoelastic Constant Cards *LEAVE THIS CARD BLANK*.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
| Type | F | I | F | F | F | I | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID | Load curve ID if constants, $G_i$, and $\beta_i$ are determined using a least squares fit. See Figure M134-1. |
| NT | Number of terms in shear fit. If zero, the default is 6. Currently, the maximum number is set to 6. |
| BSTART | In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times $\beta_3$, and so on. If zero, BSTART = 0.01. |
| TRAMP | Optional ramp time for loading. |
| LCIDK | Load curve ID for bulk behavior if constants, $K_i$ and $\beta_{\kappa_i}$ are determined using a least squares fit. See Figure M134-1. |
| NTK | Number of terms desired in bulk fit. If zero, the default is 6. Currently, the maximum number is set to 6. |
| BSTARTK | In the fit, $\beta_{\kappa_1}$ is set to zero, $\beta_{\kappa_2}$ is set to BSTARTK, $\beta_{\kappa_3}$ is 10 times $\beta_{\kappa_2}$, $\beta_{\kappa_4}$ is 10 times $\beta_{\kappa_3}$, and so on. If zero, BSTARTK = 0.01. |
| TRAMPK | Optional ramp time for bulk loading |

**Viscoelastic Constant Cards.** Up to 6 cards may be input. A keyword ("*") card terminates this input if fewer than 6 cards are used. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| GI | Optional shear relaxation modulus for the $i^{th}$ term |
| BETAI | Optional shear decay constant for the $i^{th}$ term |
| KI | Optional bulk relaxation modulus for the $i^{th}$ term |
| BETAKI | Optional bulk decay constant for the $i^{th}$ term |

**Remarks:**

Rate effects are taken into accounted through linear viscoelasticity through a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}\,(t-\tau)\,\frac{\partial \varepsilon_{kl}}{\partial \tau}\,d\tau \ ,$$

where $g_{ijkl}(t-\tau)$ is the relaxation function. If we wish to include only simple rate effects for the deviatoric stresses, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^{N} G_m\, e^{-\beta_m t} \ .$$

We characterize this function by the input shear moduli, $G_i$, and decay constants, $\beta_i$. An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^{N} K_m\, e^{-\beta_{\kappa_m} t} \ .$$

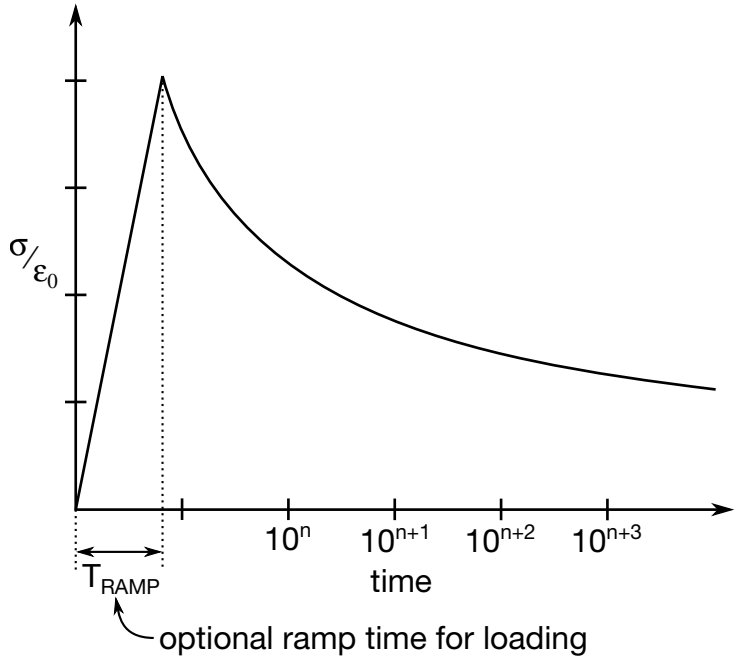



**Figure M134-1.** Stress Relaxation Curve

For an example of a stress relaxation curve see Figure M134-1. This curve defines stress as a function of time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If non-physical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.

## *MAT_WTM_STM

This is Material Type 135. This anisotropic-viscoplastic material model adopts two yield criteria for metals with orthotropic anisotropy proposed by Barlat and Lian [1989] (Weak Texture Model) and Aretz [2004] (Strong Texture Model).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | NUMFI | EPSC | WC | TAUC |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SIGMA0 | QR1 | CR1 | QR2 | CR2 | K | LC | FLG |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included if and only if FLG = 0.

| A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 |
|---|---|---|---|---|---|---|---|

**Card 3b.** This card is included if and only if FLG = 1.

| S00 | S45 | S90 | SBB | R00 | R45 | R90 | RBB |
|---|---|---|---|---|---|---|---|

**Card 3c.** This card is included if and only if FLG = 2.

| A | C | H | P | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| QX1 | CX1 | QX2 | CX2 | EDOT | M | EMIN | S100 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| AOPT | BETA | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | NUMFI | EPSC | WC | TAUC |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| NUMFI | Number of through thickness integration points that must fail before the element is deleted (remember to change this number if switching between full and reduced integration type of elements). |
| EPSC | Critical value $\varepsilon_{tC}$ of the plastic thickness strain (used in the CTS fracture criterion). |
| WC | Critical value $W_c$ for the Cockcroft-Latham fracture criterion |
| TAUC | Critical value $\tau_c$ for the Bressan-Williams shear fracture criterion |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGMA0 | QR1 | CR1 | QR2 | CR2 | K | LC | FLG |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGMA0 | Initial mean value of yield stress $\sigma_0$:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = -SIGMA0 which defines yield stress as a function of plastic strain. Hardening parameters QR1, |

| VARIABLE | DESCRIPTION |
|---|---|
| | CR1, QR2, and CR2 are ignored in that case. |
| QR1 | Isotropic hardening parameter $Q_{R1}$ |
| CR1 | Isotropic hardening parameter $C_{R1}$ |
| QR2 | Isotropic hardening parameter $Q_{R2}$ |
| CR2 | Isotropic hardening parameter $C_{R2}$ |
| K | $k$, equals half YLD2003 exponent $m$. Recommended value for FCC materials is $m = 8$, that is, $k = 4$. |
| LC | Load curve ID giving the relation between the pre-strain and the yield stress $\sigma_0$. Similar curves for $Q_{R1}$, $C_{R1}$, $Q_{R2}$, $C_{R2}$, and $W_c$ must follow consecutively from this number. |
| FLG | Flag to determine the card for defining yield:<br>EQ.0: Use Card 3a for YLD2003 (STM).<br>EQ.1: Use Card 3b for yield surface (STM – alternative input).<br>EQ.2: Use Card 3c forYLD89 (WTM). |

**YLD2003 Card.** This card 3 format is used when FLG = 0.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A1 | YLD2003 parameter $a_1$ |
| A2 | YLD2003 parameter $a_2$ |
| A3 | YLD2003 parameter $a_3$ |
| A4 | YLD2003 parameter $a_4$ |
| A5 | YLD2003 parameter $a_5$ |
| A6 | YLD2003 parameter $a_6$ |

| VARIABLE | DESCRIPTION |
|---|---|
| A7 | YLD2003 parameter $a_7$ |
| A8 | YLD2003 parameter $a_8$ |

**Yield Surface Card.** This card 3 format is used when FLG = 1.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S00 | S45 | S90 | SBB | R00 | R45 | R90 | RBB |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| S00 | Yield stress in 0° direction |
| S45 | Yield stress in 45° direction |
| S90 | Yield stress in 90° direction |
| SBB | Balanced biaxial flow stress |
| R00 | R-ratio in 0° direction |
| R45 | R-ratio in 45° direction |
| R90 | R-ratio in 90° direction |
| RBB | Balance biaxial flow ratio |

**YLD89 Card.** This card 3 format used when FLG = 2.

| Card 3c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | C | H | P | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A | YLD89 parameter *a* |

| VARIABLE | DESCRIPTION |
|---|---|
| C | YLD89 parameter $c$ |
| H | YLD89 parameter $h$ |
| P | YLD89 parameter $p$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QX1 | CX1 | QX2 | CX2 | EDOT | M | EMIN | S100 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| QX1 | Kinematic hardening parameter $Q_{x1}$ |
| CX1 | Kinematic hardening parameter $C_{x1}$ |
| QX2 | Kinematic hardening parameter $Q_{x2}$ |
| CX2 | Kinematic hardening parameter $C_{x2}$ |
| EDOT | Strain rate parameter $\dot{\varepsilon}_0$ |
| M | Strain rate parameter $m$ |
| EMIN | Lower limit of the isotropic hardening rate $\frac{dR}{d\bar{\varepsilon}}$. This feature is included to model a non-zero and linear/exponential isotropic work hardening rate at large values of effective plastic strain. If the isotropic work hardening rate predicted by the utilized Voce-type work hardening rule falls below the specified value it is substituted by the prescribed value or switched to a power-law hardening if S100 ≠ 0. This option should be considered for problems involving extensive plastic deformations. If process dependent material characteristics are prescribed, that is, if LC > 0 the same minimum tangent modulus is assumed for all the prescribed work hardening curves. If instead EMIN < 0 then –EMIN defines the plastic strain value at which the linear or power-law hardening approximation commences. |
| S100 | Yield stress at 100% strain for using a power-law approximation beyond the strain defined by EMIN. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | BETA | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by an angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| BETA | Material angle in degrees for AOPT = 0 or 3. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1 V2 V3 | Components of vector **v** for AOPT = 3 |
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

1. **Material model.** The yield condition for this material can be written

$$t(\boldsymbol{\sigma}, \boldsymbol{\alpha}, \varepsilon^p, \dot{\varepsilon}^p) = \sigma_{\text{eff}}(\boldsymbol{\sigma}, \boldsymbol{\alpha}) - \sigma_Y(\varepsilon^p, \dot{\varepsilon}^p) \ .$$

The yield stress is defined as

$$\sigma_Y = [\sigma_0 + R(\varepsilon^p)]\left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right)^C \ ,$$

where the isotropic hardening reads

$$R(\dot{\varepsilon}^p) = Q_{R1}[1 - \exp(-C_{R1}\varepsilon^p)] + Q_{R2}[1 - \exp(-C_{R2}\varepsilon^p)] \ .$$

For the Weak Texture Model the yield function is defined as

$$\sigma_{\text{eff}} = \left[\frac{1}{2}\{a(k_1 + k_2)^m + a(k_1 - k_2)^m + C(2k_2)^m\}\right]^{1/m}$$

where

$$k_1 = \frac{\sigma_x + h\sigma_y}{2}$$

$$k_2 = \sqrt{\left(\frac{\sigma_x + h\sigma_y}{2}\right)^2 + (p\sigma_{xy})^2}$$

For the Strong Texture Model the yield function is defined as

$$\sigma_{\text{eff}} = \left\{\frac{1}{2}[(\sigma'_+)^m + (\sigma'_-)^m + (\sigma''_+ - \sigma''_-)^m]\right\}^{\frac{1}{m}} \ ,$$

where

$$\sigma'_{\pm} = \frac{a_8\sigma_x + a_1\sigma_y}{2} \pm \sqrt{\left(\frac{a_2\sigma_x - a_3\sigma_y}{2}\right)^2 + a_4^2\sigma_{xy}^2}$$

$$\sigma''_{\pm} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{a_5\sigma_x - a_6\sigma_y}{2}\right)^2 + a_7^2\ \sigma_{xy}}$$

Kinematic hardening can be included by

$$\boldsymbol{\alpha} = \sum_{R=1}^{2} \boldsymbol{\alpha}_R \ ,$$

where each of the kinematic hardening variables $\alpha_R$ is independent and obeys a nonlinear evolutionary equation in the form

$$\dot{\boldsymbol{\alpha}}_R = C_{\alpha i}\left(Q_{\alpha i}\frac{\boldsymbol{\tau}}{\sigma} - \boldsymbol{\alpha}_R\right)\dot{\varepsilon}^p \ .$$

The effective stress $\overline{\sigma}$ is defined as

$$\overline{\sigma} = \sigma_{\text{eff}}(\boldsymbol{\tau}) \ ,$$

where

$$\boldsymbol{\tau} = \boldsymbol{\sigma} - \boldsymbol{\alpha} \ .$$

Critical thickness strain failure in a layer is assumed to occur when

$$\varepsilon_t \le \varepsilon_{tc} \ ,$$

where $\varepsilon_{tc}$ is a material parameter. It should be noted that $\varepsilon_{tc}$ is a negative number (meaning failure is assumed to occur only in the case of thinning).

Cockcraft and Latham fracture is assumed to occur when

$$W = \int \max(\sigma_1, 0)d\varepsilon^p \ge W_C \ ,$$

where $\sigma_1$ is the maximum principal stress and $W_C$ is a material parameter.

2. **Yield surface parameters.** If FLG = 1, that is, if the yield surface parameters $a_1$ through $a_8$ are identified on the basis of prescribed material data internally in the material routine, files with point data for plotting of the identified yield surface, along with the predicted directional variation of the yield stress and plastic flow are generated in the directory where the LS-DYNA analysis is run. Four different files are generated for each specified material.

   These files are named according to the scheme:

   a) Contour_1#

   b) Contour_2#

   c) Contour_3#

d) R_and_S#

where # is a value starting at 1.

The first three files contain contour data for plotting of the yield surface as shown in Figure M135-2. To generate these plots a suitable plotting program should be adopted and for each file/plot, column A should be plotted as a function of column B. Figure M135-3 further shows a plot generated from the final file named R_and_S# showing the directional dependency of the normalized yield stress (column A vs. B) and plastic strain ratio (column B vs. C).

3. **History variables.** The following additional history variables can be included in the output d3plot file.

| History Variable # | Description |
|---|---|
| 1 | Isotropic hardening value $R_1$ |
| 2 | Isotropic hardening value $R_2$ |
| 3 | Increment in effective plastic strain $\Delta\bar{\varepsilon}$ |
| 4 | Not defined, for internal use in the material model |
| 5 | Not defined, for internal use in the material model |
| 6 | Not defined, for internal use in the material model |
| 7 | Failure in integration point<br>EQ.0: No failure<br>EQ.1: Failure due to EPSC, i.e. $\varepsilon_t \geq \varepsilon_{tc}$.<br>EQ.2: Failure due to WC, i.e. $W \geq W_c$.<br>EQ.3: Failure due to TAUC, i.e. $\tau \geq \tau_c$ |
| 8 | Sum of incremental strain in local element $x$-direction: $\varepsilon_{xx} = \sum \Delta\varepsilon_{xx}$ |
| 9 | Sum of incremental strain in local element $y$-direction: $\varepsilon_{yy} = \sum \Delta\varepsilon_{yy}$ |
| 10 | Value of the Cockcroft-Latham failure parameter $W = \sum \sigma_1 \Delta p$ |
| 11 | Plastic strain component in thickness direction $\varepsilon_t$ |
| 12 | Mean value of increments in plastic strain through the thickness. (For use with the non-local instability criterion. Note that constant lamella thickness is assumed, and the instability criterion can give unrealistic results if used with a user-defined integration rule with varying lamella thickness.) |

| History Variable # | Description |
|---|---|
| 13 | Not defined, for internal use in the material model |
| 14 | Nonlocal value $\rho = \frac{\Delta\varepsilon_3}{\Delta\varepsilon_3^{\Omega}}$ |
| 17 | Value of the Bressan-Williams failure parameter $\tau$ |

**Table M135-1.**

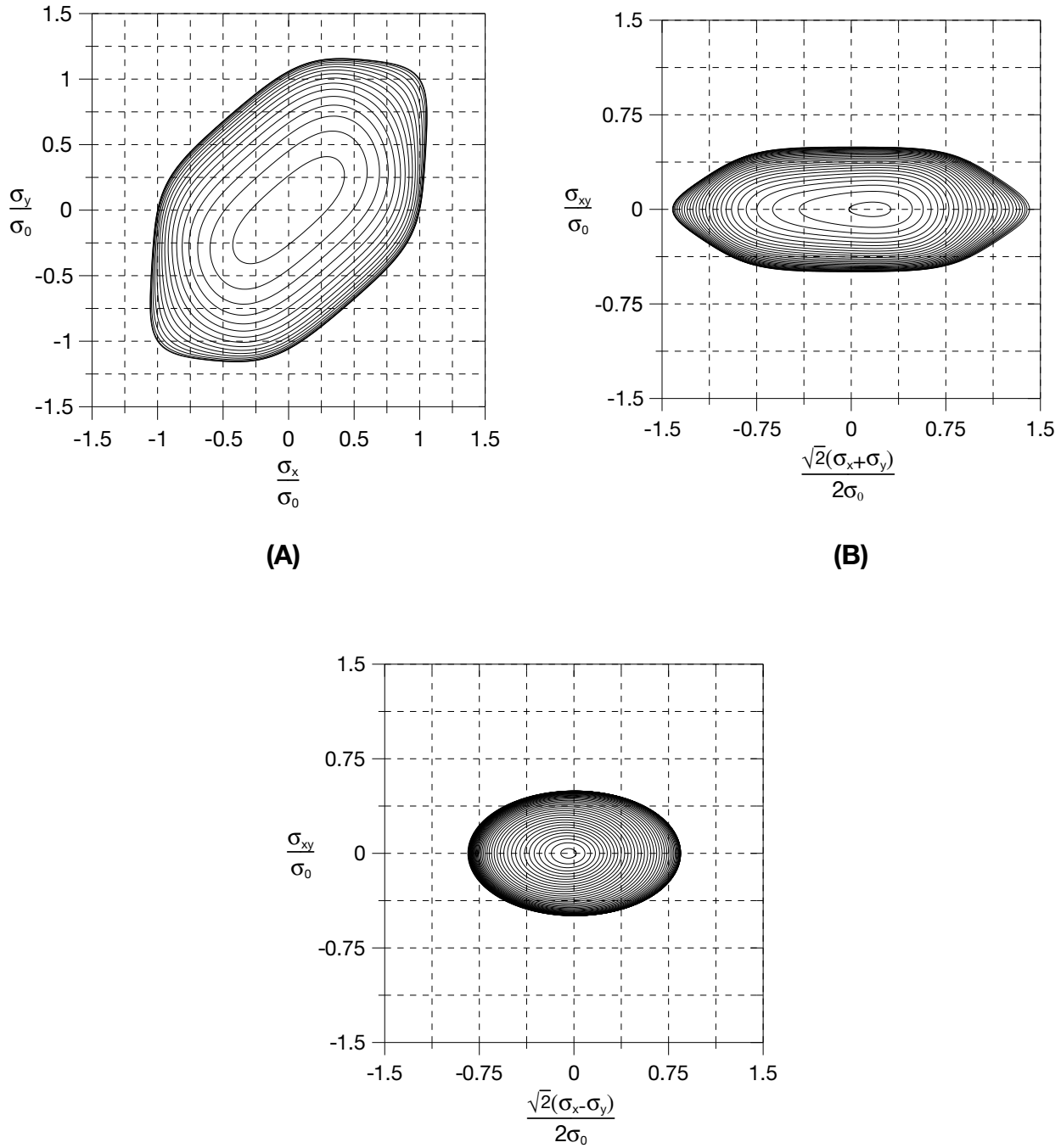



**Figure M135-2.** Contour plots of the yield surface generated from the files (a) ‘Contour_1<#>, (b) Contour_2<#>, and (c) Contour_3<#>.

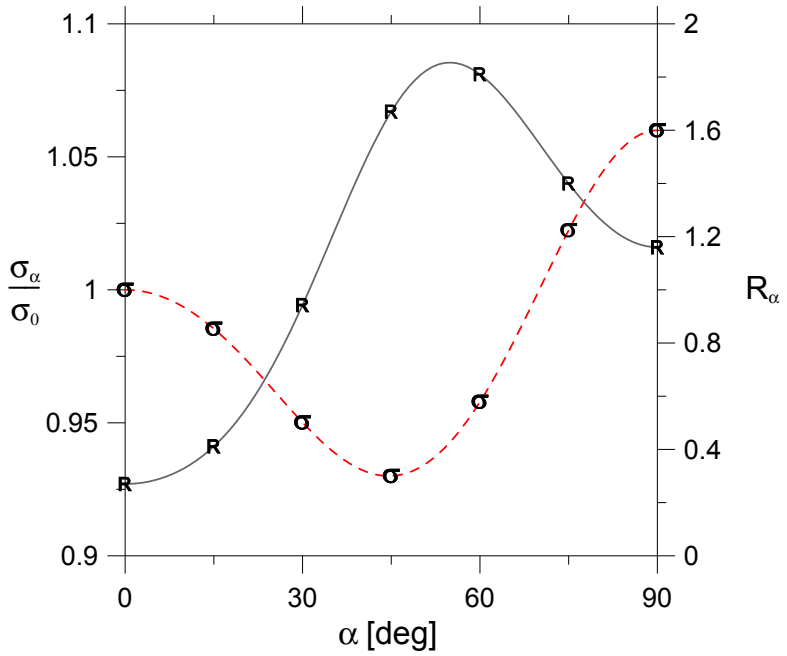



**Figure M135-3.** Predicted directional variation of the yield stress and plastic flow generated from the file R_and_S<#>.

## *MAT_WTM_STM_PLC

This is Material Type 135. This anisotropic material adopts the yield criteria proposed by Aretz [2004]. The material strength is defined by McCormick's constitutive relation for materials exhibiting negative steady-state Strain Rate Sensitivity (SRS). McCormick [1998] and Zhang, McCormick and Estrin [2001].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | NUMFI | EPSC | WC | TAUC |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGMA0 | QR1 | CR1 | QR2 | CR2 | K | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S | H | OMEGA | TD | ALPHA | EPS0 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | BETA | | | | | | |
| Type | F | F | | | | | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| NUMFI | Number of through thickness integration points that must fail before the element is deleted (remember to change this number if switching between full and reduced integration type of elements) |
| EPSC | Critical value, $\varepsilon_{tC}$, of the plastic thickness strain |
| WC | Critical value, $W_c$, for the Cockcroft-Latham fracture criterion. |
| TAUC | Critical value, $\tau_c$, for the shear fracture criterion. |
| SIGMA0 | Initial yield stress, $\sigma_0$ |
| QR1 | Isotropic hardening parameter, $Q_{R1}$ |
| CR1 | Isotropic hardening parameter, $C_{R1}$ |
| QR2 | Isotropic hardening parameter, $Q_{R2}$ |
| CR2 | Isotropic hardening parameter, $C_{R2}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| K | $k$ equals half the exponent $m$ for the yield criterion |
| A1 | Yld2003 parameter, $a_1$ |
| A2 | Yld2003 parameter, $a_2$ |
| A3 | Yld2003 parameter, $a_3$ |
| A4 | Yld2003 parameter, $a_4$ |
| A5 | Yld2003 parameter, $a_5$ |
| A6 | Yld2003 parameter, $a_6$ |
| A7 | Yld2003 parameter, $a_7$ |
| A8 | Yld2003 parameter, $a_8$ |
| S | Dynamic strain aging parameter, $S$ |
| H | Dynamic strain aging parameter, $H$ |
| OMEGA | Dynamic strain aging parameter, $\Omega$ |
| TD | Dynamic strain aging parameter, $t_d$ |
| ALPHA | Dynamic strain aging parameter, $\alpha$ |
| EPS0 | Dynamic strain aging parameter, $\dot{\varepsilon}_0$ |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1, and then rotated about the shell element normal by the angle BETA. Nodes 1, 2 and 4 of an element are identical to the nodes used for the definition of a coordinate system as by *DEFINE_COORDINATE_NODES.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by offsetting the material axes by an angle, BETA, from a line determined by taking the cross product of the vector **v** |

| VARIABLE | DESCRIPTION |
|---|---|
| | with the normal to the plane of the element.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

The yield function is defined as

$$f = \sigma_{\text{eq}}(\boldsymbol{\sigma}) - \left[\sigma_Y(t_a) + R(\varepsilon_p) + \sigma_v(\dot{\varepsilon}^p)\right] \ ,$$

where the equivalent stress $\sigma_{\text{eq}}$ is defined as by an anisotropic yield criterion

$$\sigma_{\text{eq}} = \left[\frac{1}{2}\left(|\sigma'_1|^m + |\sigma'_2|^m + |\sigma''_1 - \sigma''_2|\right)\right]^{\frac{1}{m}} \ .$$

Here

$$\begin{Bmatrix}\sigma'_1\\ \sigma'_2\end{Bmatrix} = \frac{a_8\sigma_{xx} + a_1\sigma_{yy}}{2} \pm \sqrt{\left(\frac{a_2\sigma_{xx} - a_3\sigma_{yy}}{2}\right)^2 + a_4^2\sigma_{xy}^2}$$

and

$$\begin{Bmatrix}\sigma''_1\\ \sigma''_2\end{Bmatrix} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{a_5\sigma_{xx} - a_6\sigma_{yy}}{2}\right)^2 + a_7^2\sigma_{xy}^2} \ .$$

The strain hardening function, $R$, is defined by the extended Voce law

$$R(\varepsilon^p) = \sum_{i=1}^{2} Q_{Ri}\left(1 - \exp(-C_{Ri}\varepsilon^p)\right) \ ,$$

where $\varepsilon^p$ is the effective (or accumulated) plastic strain, and $Q_{Ri}$and $C_{Ri}$ are strain hardening parameters.

Viscous stress, $\sigma_v$, is given by

$$\sigma_v = (\dot{\varepsilon}^p) = s \ln\left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right) ,$$

where $s$ represents the instantaneous strain rate sensitivity (SRS) and $\dot{\varepsilon}_0$ is a reference strain rate. In this model the yield strength, including the contribution from dynamic strain aging (DSA) is defined as

$$\sigma_Y(t_a) = \sigma_0 + \mathrm{SH}\left[1 - \exp\left\{-\left(\frac{t_a}{t_d}\right)^{\alpha}\right\}\right]$$

where $\sigma_0$is the yield strength for vanishing average waiting time, $t_a$ (meaning at high strain rates). $H$, $\alpha$, and $t_d$ are material constants linked to dynamic strain aging. It is noteworthy that $\sigma_Y$ is an increasing function of $t_a$. The average waiting time is defined by the evolution equation

$$\dot{t}_a = 1 - \frac{t_a}{t_{a,ss}} ,$$

where the quasi-steady waiting time $t_{a,ss}$ is given as

$$t_{a,ss} = \frac{\Omega}{\dot{\varepsilon}^p} .$$

Here $\Omega$ is the strain produced by all mobile dislocations moving to the next obstacle on their path.

## *MAT_VEGTER

This is Material Type 136 (formerly named *MAT_CORUS_VEGTER), a plane stress orthotropic material model for metal forming. Yield surface construction is based on the interpolation by second-order Bezier curves, and model parameters are determined directly from a set of mechanical tests conducted for several directions. For each direction, four mechanical tests are carried out: a uniaxial, an equi-biaxial, a plane strain tensile test and a shear test. These test results are used to determine the coefficients of the Fourier directional dependency field. For a more detailed description please see Vegter and Boogaard [2006].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | N | FBI | RBIO | LCID |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SYS | SIP | SHB | SHO | ESH | E0 | ALPHA | LCID2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 6.** Include N+1 of this card.

| FUN-*i* | RUN-*i* | FPS1-*i* | FPS2-*i* | FSH-*i* | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | N | FBI | RBI0 | LCID |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Material density |
| E | Elastic Young's modulus |
| PR | Poisson's ratio |
| N | \|N\| is the order of the Fourier series (meaning number of test groups minus one). The minimum number for \|N\| is 2, and the maximum is 10.<br>GE.0.0: Explicit cutting-plane return mapping algorithm<br>LT.0.0: Fully implicit return mapping algorithm (more robust) |
| FBI | Normalized yield stress $\sigma_{\mathrm{bi}}$ for equi-biaxial test |
| RBI0 | Strain ratio $\sigma_{\mathrm{bi}}(0°) = \dot{\varepsilon}_2(0°)/\dot{\varepsilon}_1(0°)$ for equi-biaxial test in the rolling direction |
| LCID | Load curve ID or Table ID. If defined, SYS, SIP, SHB, SHO, ESH, and E0 are ignored.<br>**Load Curve.** When LCID is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that strain rate. Linear interpolation between the discrete strain rates is used by default.<br>**Logarithmically Defined Tables.** A way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. |

| VARIABLE | DESCRIPTION |
|---|---|
| | There is some additional computational cost associated with invoking logarithmic interpolation. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SYS | SIP | SHB | SHO | ESH | E0 | ALPHA | LCID2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SYS | Static yield stress, $\sigma_0$ |
| SIP | Stress increment parameter, $\Delta\sigma_m$ |
| SHB | Strain hardening parameter for large strain, $\beta$ |
| SHO | Strain hardening parameter for small strain, $\Omega$ |
| ESH | Exponent for strain hardening, $n$ |
| E0 | Initial plastic strain, $\varepsilon_0$ |
| ALPHA | Distribution of hardening used in the curve-fitting, $\alpha$. $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening. |
| LCID2 | Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default, the Young's modulus is assumed to remain constant. Effective value is between 0 and 1. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by the angle BETA |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |

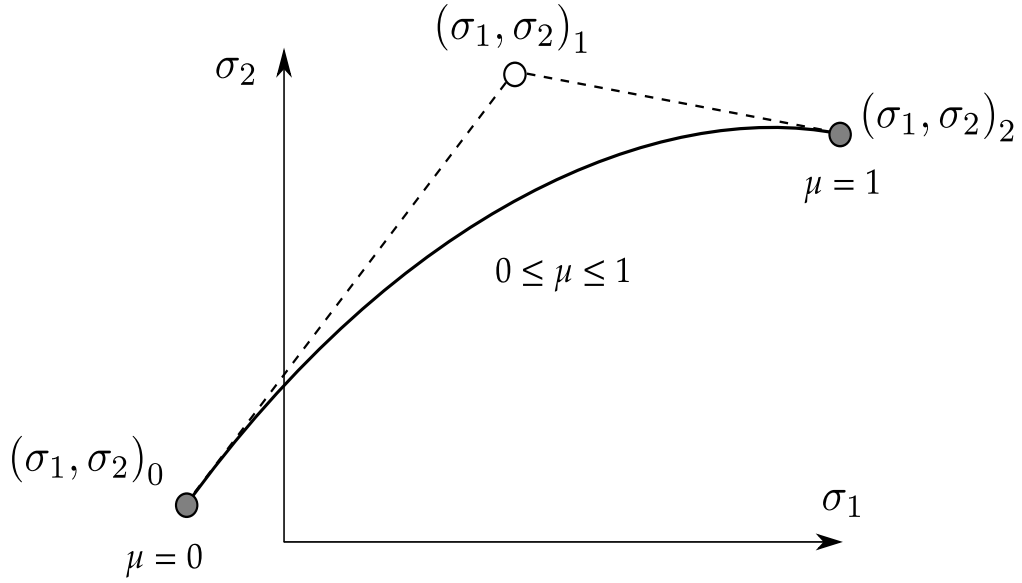



**Figure M136-1.** Bézier interpolation curve.

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |

**Experimental Data Cards.** The next N+1 cards (see N on Card 1) contain experimental data obtained from four mechanical tests for a group of equidistantly placed directions $\theta_i = \frac{i\pi}{2N}, i = 0, 1, 2, \ldots, N$.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FUN-*i* | RUN-*i* | FPS1-*i* | FPS2-*i* | FSH-*i* | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FUN-*i* | Normalized yield stress $\sigma_{\text{un}}$ for uniaxial test for the $i^{\text{th}}$ direction |
| RUN-*i* | Strain ratio (R-value) for uniaxial test for the $i^{\text{th}}$ direction |
| FPS1-*i* | First normalized yield stress $\sigma_{\text{ps1}}$ for plain strain test for the $i^{\text{th}}$ direction |

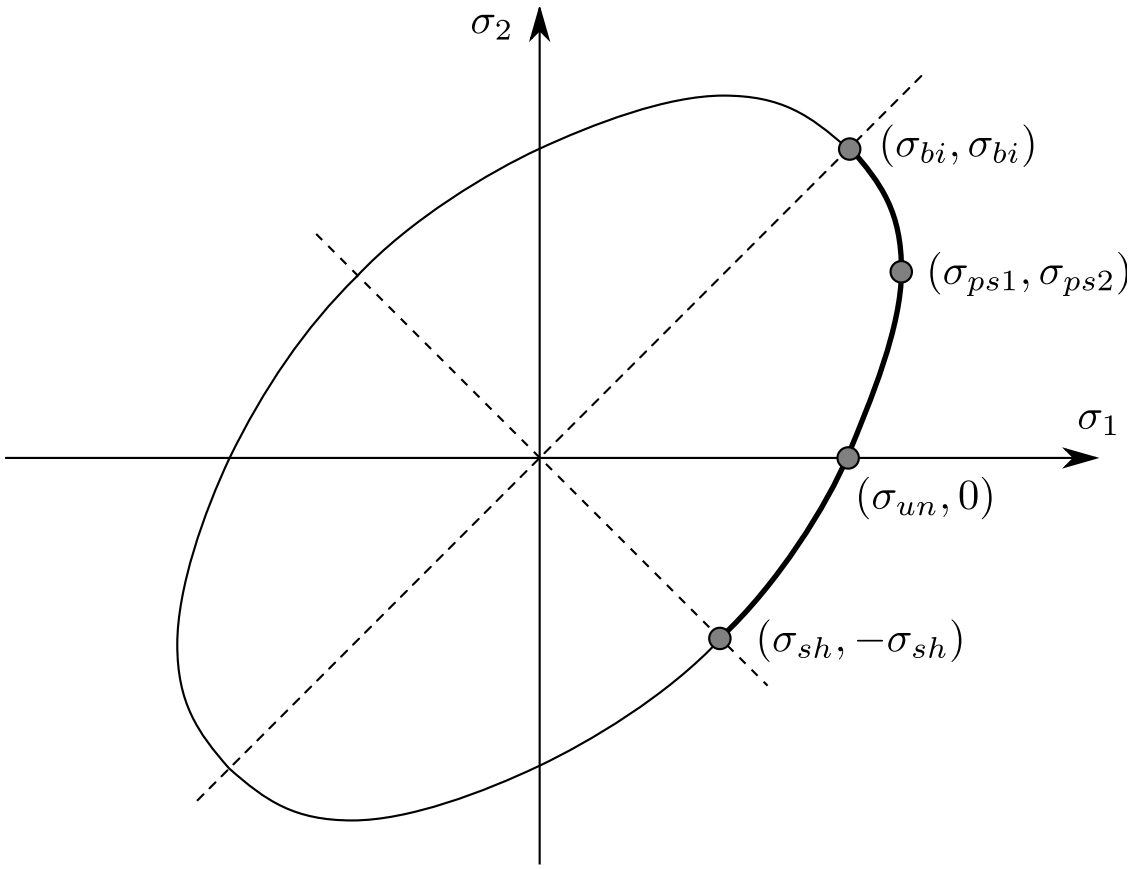



**Figure M136-2.** Vegter yield surface.

| VARIABLE | DESCRIPTION |
|---|---|
| FPS2-*i* | Second normalized yield stress $\sigma_{ps2}$ for plain strain test for the $i^{th}$ direction |
| FSH-*i* | First normalized yield stress $\sigma_{sh}$ for pure shear test for the $i^{th}$ direction |

**Remarks:**

The Vegter yield locus is section-wise defined by quadratic Bézier interpolation functions. Each individual curve uses 2 reference points and a hinge point in the principal plane stress space; see Figure M136-1.

The mathematical description of the Bézier interpolation is given by:

$$\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}_0 + 2\mu\left[\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}_1 - \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}_0\right] + \mu^2\left[\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}_2 + \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}_0 - 2\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}_1\right],$$

where $(\sigma_1, \sigma_2)_0$ is the first reference point, $(\sigma_1, \sigma_2)_1$ is the hinge point, and $(\sigma_1, \sigma_2)_2$ is the second reference point. $\mu$ is a parameter which determines the location on the curve ($0 \leq \mu \leq 1$).

Four characteristic stress states are selected as reference points: the equi-biaxial point $(\sigma_{bi}, \sigma_{bi})$, the plane strain point $(\sigma_{ps1}, \sigma_{ps2})$, the uniaxial point $(\sigma_{un}, 0)$ and the pure shear point $(\sigma_{sh}, -\sigma_{sh})$; see Figure M136-2. Between the 4 stress points, 3 Bézier curves are used to interpolate the yield locus. Symmetry conditions are used to construct the complete surface. The yield locus in Figure M136-2 shows the reference points of experiments for one specific direction. The reference points can also be determined for other angles to the rolling direction (planar angle $\theta$). For example, if N = 2 is chosen, normalized yield stresses for directions 0°, 45°, and 90° should be defined. A Fourier series is used to interpolate intermediate angles between the measured points.

The Vegter yield function with isotropic hardening (ALPHA = 1) is given as:

$$\phi = \sigma_{eq}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\varepsilon}^p)$$

with the equivalent stress $\sigma_{eq}$ obtained from the appropriate Bézier function related to the current stress state. The uniaxial yield stress $\sigma_y$ can be defined as stress-strain curve LCID or alternatively as a functional expression:

$$\sigma_y = \sigma_0 + \Delta\sigma_m\left[\beta(\bar{\varepsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\varepsilon}^p + \varepsilon_0)}\right)^n\right]$$

In case of kinematic hardening (ALPHA < 1), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

To determine the yield stress or reference points of the Vegter yield locus, four mechanical tests have to be performed for different directions. A good description about the material characterization procedure can be found in Vegter et al. (2003).

## *MAT_VEGTER_STANDARD

This is Material Type 136_STD, a plane stress orthotropic material model for metal forming. Yield surface construction is based on the interpolation by second-order Bezier curves, and model parameters are determined directly from a set of mechanical tests conducted for a number of directions. For each direction, four mechanical tests are carried out: a uniaxial, an equi-biaxial, a plane strain tensile test and a shear test. The material formulation is equivalent to MAT_VEGTER, except it requires different parameters for the plane strain tensile test and can use the Bergström-Van Liempt equation to deal with strain rate effects. These test results are used to determine the coefficients of the Fourier directional dependency field. For a more detailed description please see Vegter and Boogaard [2006].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | N | FBI | RBIO | LCID |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SYS | SIP | SHB | SHO | ESH | E0 | ALPHA | LCID2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | | DYS | RATEN | SRNO | EXSR | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 6.** Include N+1 of this card.

| FUN-*i* | RUN-*i* | FPS1-*i* | ALPS-*i* | FSH-*i* | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | N | FBI | RBI0 | LCID |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Material density |
| E | Elastic Young's modulus |
| PR | Poisson's ratio |
| N | Order of the Fourier series (meaning number of test groups minus one). The minimum number for N is 2, and the maximum is 10. |
| FBI | Normalized yield stress $\sigma_{\mathrm{bi}}$ for equi-biaxial test |
| RBI0 | Strain ratio $\sigma_{\mathrm{bi}}(0°) = \dot{\varepsilon}_2(0°)/\dot{\varepsilon}_1(0°)$ for equi-biaxial test in the rolling direction |
| LCID | Load curve ID or table ID. If defined, SYS, SIP, SHB, SHO, ESH, E0, DYS, RATEN, SRN0, and EXSR are ignored.<br>**Load Curve.** When LCID is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that strain rate. Linear interpolation between the discrete strain rates is used by default.<br>**Logarithmically Defined Tables.** A way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. There is some additional computational cost associated with invoking logarithmic interpolation. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SYS | SIP | SHB | SHO | ESH | E0 | ALPHA | LCID2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SYS | Static yield stress, $\sigma_0$ |
| SIP | Stress increment parameter, $\Delta\sigma_m$ |
| SHB | Strain hardening parameter for large strain, $\beta$ |
| SHO | Strain hardening parameter for small strain, $\Omega$ |
| ESH | Exponent for strain hardening, $n$ |
| E0 | Initial plastic strain, $\varepsilon_0$ |
| ALPHA | Distribution of hardening used in the curve-fitting, $\alpha$. $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening. |
| LCID2 | Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default, the Young's modulus is assumed to remain constant. Effective value is between 0 and 1. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | DYS | RATEN | SRNO | EXSR | | |
| Type | F | | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by the angle BETA |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| DYS | Limit dynamic flow stress $\sigma_0^*$ |
| RATEN | Ratio $r_{\text{enth}}$ of Boltzman constant $k$ (8.617E-5 eV/K) and maximum activation enthalpy $\Delta G_0$ (in eV): $r_{\text{enth}} = \frac{k}{\Delta G_0}$ |
| SRN0 | Limit strain rate $\dot{\varepsilon}_0$ |
| EXSR | Exponent $m$ for strain rate behavior |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |

**Experimental Data Cards.** The next N+1 cards (see N on Card 1) contain experimental data obtained from four mechanical tests for a group of equidistantly placed directions $\theta_i = i\pi/(2N), i = 0, 1, 2, \ldots, N$.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FUN-*i* | RUN-*i* | FPS1-*i* | ALPS-*i* | FSH-*i* | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FUN-*i* | Normalized yield stress $\sigma_{\text{un}}$ for uniaxial test for the $i^{\text{th}}$ direction |
| RUN-*i* | Strain ratio (R-value) for uniaxial test for the $i^{\text{th}}$ direction |
| FPS1-*i* | First normalized yield stress $\sigma_{\text{ps1}}$ for plain strain test for the $i^{\text{th}}$ direction |
| ALPS-*i* | Normalized distance $\alpha_{\text{ps}}$ of second component of plain stress point between the hinge points on both sides for the $i^{\text{th}}$ direction. See Remarks for details. |
| FSH-*i* | First normalized yield stress $\sigma_{\text{sh}}$ for pure shear test for the $i^{\text{th}}$ direction |

**Remarks:**

1. **Yield locus.** The yield locus description of this material is the same as for *MAT_VEGTER. The materials share the same Bézier interpolation for the section-wise definition of the yield locus and also use the same four characteristic stress states as reference points. They only differ in the plane-strain point definition in the input. This material MAT_VEGTER_STANDARD does not expect the direct input of the two components $(\sigma_{\text{ps1}}, \sigma_{\text{ps2}})$, but only of the first component $\sigma_{\text{ps1}}$. The second component is assumed to be at a fixed distance between

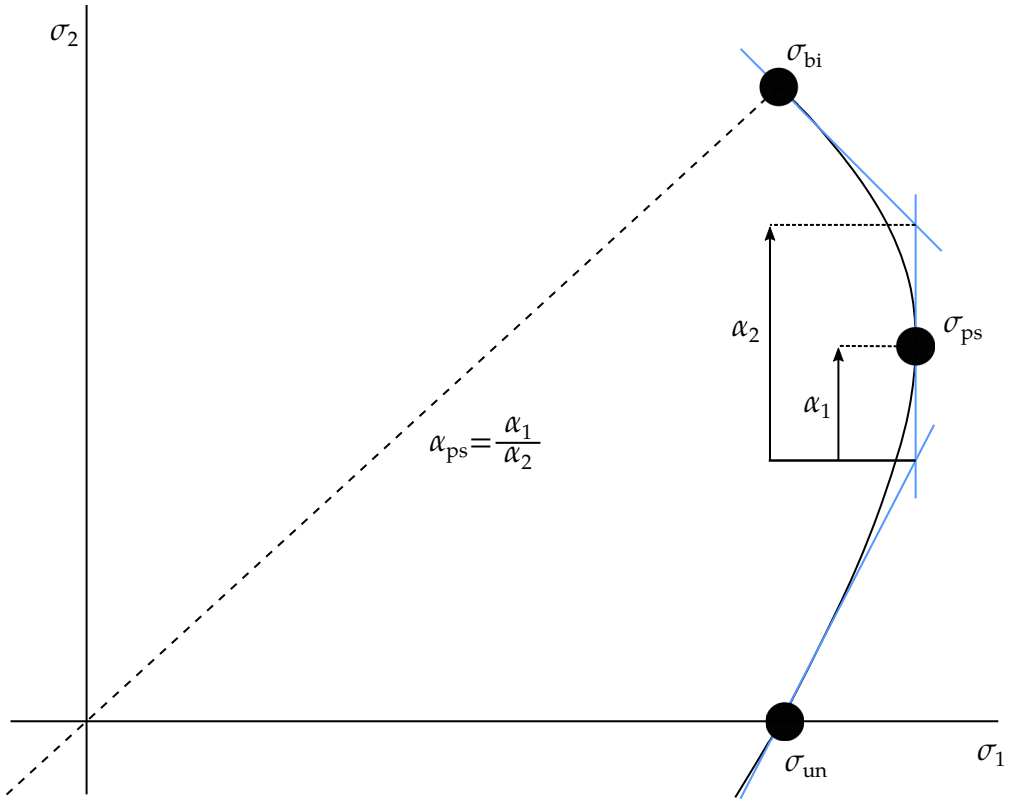



**Figure M136-1.** Vegter yield surface

the hinge points on both sides. This distance is defined by factor $\alpha_{ps} = \alpha_1/\alpha_2$, as shown in Figure M136-1. This approach is favored in most publications and has for example been discussed in the PhD-thesis of Pijlman, H. H. (2001).

To determine the yield stress or reference points of the Vegter yield locus, four mechanical tests must be performed for different directions. A good description about the material characterization procedure can be found in Vegter et al. (2003).

2. **Strain hardening.** The Vegter yield function with isotropic hardening (ALPHA = 1) is given as:

$$\phi = \sigma_{eq}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, \dot{\varepsilon})$$

with the equivalent stress $\sigma_{eq}$ obtained from the appropriate Bézier function related to the current stress state. The uniaxial yield stress $\sigma_y$ can be defined as a yield stress curve, $\sigma_y(\bar{\varepsilon}^p)$, or a yield stress surface, $\sigma_y(\bar{\varepsilon}^p, \dot{\varepsilon})$, with LCID. In contrast to *MAT_VEGTER, this material also provides the temperature-dependent Bergström-Van Liempt equation as a third alternative:

$$\sigma_y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T) = \sigma_0 + \Delta\sigma_m\left[\beta(\bar{\varepsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\varepsilon}^p + \varepsilon_0)}\right)^n\right] + \sigma_0^*\left[1 + r_{enth} T \ln\left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0}\right)\right]^m,$$

where $T$ represents the temperature in K.

In the case of kinematic hardening (ALPHA < 1), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

### *MAT_VEGTER_2017

This is Material Type 136_2017, a plane stress orthotropic material model for metal forming. It features the advanced Vegter yield locus based on the interpolation by second-order Bezier curves. Model parameters are determined from uniaxial test data at 0°, 45° and 90° to the rolling direction. Therefore, the same mechanical tests must be carried out as for Hill's 1948 planar anisotropic material model. For a more detailed description of the yield locus, please see Vegter and Boogaard [2006]. The relationships between the results of uniaxial testing and the advanced yield locus are introduced and discussed in Abspoel et al [2017].

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | E | PR | | FBI | RBIO | LCID |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SYS | SIP | SHB | SHO | ESH | EO | ALPHA | LCID2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | | DYS | RATEN | SRNO | EXSR | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| RM-0 | RM-45 | RM-90 | AG-0 | AG-45 | AG90 | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| R00 | R45 | R90 | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | FBI | RBI0 | LCID |
| Type | A | F | F | F | | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Material density |
| E | Elastic Young's modulus |
| PR | Poisson's ratio |
| FBI | Normalized yield stress $\sigma_{bi}$ for equi-biaxial test. If this value is not defined in the input, it will be approximated based on the uniaxial test result. |
| RBI0 | Strain ratio $\sigma_{bi}(0°) = \dot{\varepsilon}_2(0°)/\dot{\varepsilon}_1(0°)$ for equi-biaxial test in the rolling direction. If this value is not defined in the input, it will be approximated based on the uniaxial test result. |
| LCID | Load curve ID or Table ID. If defined, SYS, SIP, SHB, SHO, ESH, E0, DYS, RATEN, SRN0, and EXSR are ignored.<br>**Load Curve.** When LCID is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that strain rate. Linear interpolation between the discrete strain rates is used by default.<br>**Logarithmically Defined Tables.** A way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. There is some additional computational cost associated with invoking logarithmic interpolation. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SYS | SIP | SHB | SHO | ESH | E0 | ALPHA | LCID2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SYS | Static yield stress, $\sigma_0$ |
| SIP | Stress increment parameter, $\Delta\sigma_m$ |
| SHB | Strain hardening parameter for large strain, $\beta$ |
| SHO | Strain hardening parameter for small strain, $\Omega$ |
| ESH | Exponent for strain hardening, $n$ |
| E0 | Initial plastic strain, $\varepsilon_0$ |
| ALPHA | Distribution of hardening used in the curve-fitting, $\alpha$. $\alpha = 0$ is pure kinematic hardening while $\alpha = 1$ provides pure isotropic hardening. |
| LCID2 | Curve ID. The curve defines Young's modulus scaling factor with respect to the plastic strain. By default, the Young's modulus is assumed to remain constant. Effective value is between 0 and 1. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | DYS | RATEN | SRNO | EXSR | | |
| Type | F | | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES and then rotated about the shell element normal by the angle BETA |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| DYS | Limit dynamic flow stress, $\sigma_0^*$ |
| RATEN | Ratio, $r_{\text{enth}}$, of the Boltzmann constant, $k$, (8.617E-5 eV/K) and maximum activation enthalpy, $\Delta G_0$, (in eV): $$r_{\text{enth}} = \frac{k}{\Delta G_0}$$ |
| SRN0 | Limit strain rate, $\dot{\varepsilon}_0$ |
| EXSR | Exponent, $m$, for strain rate behavior |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RM-0 | RM-45 | RM-90 | AG-0 | AG-45 | AG90 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| RM-*i* | Tensile strength for uniaxial testing at *i*° to rolling direction |
| AG-*i* | Uniform elongation for uniaxial testing at *i*° to rolling direction |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R00 | R45 | R90 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| R00 | Lankford parameter $R_{00}$ |
| R45 | Lankford parameter $R_{45}$ |
| R90 | Lankford parameter $R_{90}$ |

**Remarks:**

1. **Yield Locus.** The yield locus description of this material is the same as for *MAT_VEGTER. The materials share the same Bézier interpolation for the section-wise definition of the yield locus.

   The four characteristics stress states are predicted based on standard parameters from uniaxial tensile tests. This approach has been presented by Abspoel et al. (2017). Test data for 0°, 45°, and 90° to rolling direction must be given to account for anisotropic behavior of the material. The resulting formulation is then equivalent with *MAT_VEGTER_STANDARD and N = 2.

2. **Strain Hardening.** The Vegter yield function with isotropic hardening (ALPHA = 1) is given as:

$$\phi = \sigma_{\text{eq}}(\sigma_1, \sigma_2, \theta) - \sigma_y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, \dot{\varepsilon})$$

   with the equivalent stress $\sigma_{\text{eq}}$ obtained from the appropriate Bézier function related to the current stress state. The uniaxial yield stress $\sigma_y$ can be defined as yield stress curve $\sigma_y(\bar{\varepsilon}^p)$ with LCID or as $\sigma_y(\bar{\varepsilon}^p, \dot{\varepsilon})$ with table LCID. In contrast to *MAT_VEGTER, this material also provides the temperature-dependent Bergström-Van Liempt equation as a third alternative:

$$\sigma_y(\bar{\varepsilon}^p, \dot{\bar{\varepsilon}}^p, T) = \sigma_0 + \Delta\sigma_m\left[\beta(\bar{\varepsilon}^p + \varepsilon_0) + \left(1 - e^{-\Omega(\bar{\varepsilon}^p + \varepsilon_0)}\right)^n\right] + \sigma_0^* \left[1 + r_{\text{enth}} T \ln\left(\frac{\dot{\bar{\varepsilon}}^p}{\dot{\varepsilon}_0}\right)\right]^m,$$

   where $T$ represents the temperature in K.

   In the case of kinematic hardening (ALPHA < 1), the standard stress tensor is replaced by a relative stress tensor, defined as the difference between the stress tensor and a back stress tensor.

## *MAT_COHESIVE_MIXED_MODE

This is Material Type 138. This model is a simplification of *MAT_COHESIVE_GENERAL, restricted to linear softening. It includes a bilinear traction-separation law with a quadratic mixed-mode delamination criterion and a damage formulation. This material model can only be used with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | EN | ET | GIC | GIIC |
| Type | A | F | I | F | F | F | F | F |
| Default | none | none | 0 | 0.0 | none | none | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XMU | T | S | UND | UTD | GAMMA | CTS | |
| Type | F | F | F | F | F | F | F | |
| Default | none | 0.0 | 0.0 | none | none | 1.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| ROFLG | Flag stating whether density is specified per unit area or volume:<br>EQ.0: Specified density is per unit volume (default).<br>EQ.1: Specified density is per unit area for controlling the mass of cohesive elements with an initial volume of zero. |
| INTFAIL | The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4, with 1 being the recommended value. This field also determines the integration scheme. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when \|INTFAIL\| integration points have failed. |
| | EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion. |
| | GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed. |
| EN | The stiffness (units of stress / length) normal to the plane of the cohesive element |
| ET | The stiffness (units of stress / length) in the plane of the cohesive element |
| GIC | Energy release rate for mode I (units of stress × length).<br>LT.0.0: Load curve ID = (-GIC), which defines the energy release rate for mode I as a function of element size. |
| GIIC | Energy release rate for mode II (units of stress × length).<br>LT.0.0: Load curve ID = (-GIIC), which defines the energy release rate for mode II as a function of element size. |
| XMU | Exponent of the mixed mode criteria (see Remark 2) |
| T | Peak traction (stress units) in the normal direction.<br>LT.0.0: Load curve ID = (-T), which defines peak traction in the normal direction as a function of element size. See Remark 4.<br>EQ.0.0: See Remark 1.<br>GT.0.0: Peak traction in the normal direction, $T$ |
| S | Peak traction (stress units) in the tangential direction.<br>LT.0.0: Load curve ID = (-S), which defines peak traction in the tangential direction as a function of element size. See Remark 4.<br>EQ.0.0: See Remark 1.<br>GT.0.0: Peak traction in the tangential direction, $S$ |

| VARIABLE | DESCRIPTION |
|---|---|
| UND | Ultimate displacement in the normal direction |
| UTD | Ultimate displacement in the tangential direction |
| GAMMA | Additional exponent for Benzeggagh-Kenane law (default = 1.0) |
| CTS | Flag to use a consistent tangent stiffness and change the cohesive law formulation in implicit simulations:<br>EQ.0.0: Original cohesive law formulation with an approximate tangent stiffness<br>EQ.1.0: Original cohesive law formulation with a consistent tangent stiffness<br>EQ.2.0: Ortiz and Pandolfi cohesive law (Ortiz and Pandolfi [1999]) with a consistent tangent stiffness. See Remark 6.<br>Using a consistent tangent stiffness improves the numerical convergence. |

**Remarks:**

1. **Ultimate displacements.** The ultimate displacements in the normal and tangential directions are the displacements at the time when the material has entirely failed; that is, the tractions are zero. The linear stiffness for loading followed by the linear softening during the damage provides a straightforward relationship among the energy release rates, peak tractions, and ultimate displacements:

$$\text{GIC} = T \times \frac{\text{UND}}{2}$$
$$\text{GIIC} = S \times \frac{\text{UTD}}{2}$$

If the peak tractions are not specified, the solver calculates them from the ultimate displacements. See Fiolka and Matzenmiller [2005] and Gerlach, Fiolka and Matzenmiller [2005].

2. **Mixed-mode relative displacement.** In this cohesive material model, the total mixed-mode relative displacement, $\delta_m$, is defined as $\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}$, where $\delta_I = \delta_3$ is the separation in the normal direction (mode I) and $\delta_{II} = \sqrt{\delta_1^2 + \delta_2^2}$ is the separation in the tangential direction (mode II). The mixed-mode damage initiation displacement $\delta^0$ (onset of softening) is given by

$$\delta^0 = \delta_I^0 \delta_{II}^0 \sqrt{\frac{1+\beta^2}{(\delta_{II}^0)^2 + (\beta\delta_I^0)^2}}$$

where $\delta_I^0 = T/\text{EN}$ and $\delta_{II}^0 = S/\text{ET}$ are the single mode damage initiation separations and $\beta = \delta_{II}/\delta_I$ is the "mode mixity" (see Figure M138-1). The ultimate mixed-mode displacement $\delta^F$ (total failure) for the power law (XMU > 0) is:

$$\delta^F = \frac{2(1+\beta^2)}{\delta^0}\left[\left(\frac{\text{EN}}{\text{GIC}}\right)^{\text{XMU}} + \left(\frac{\text{ET}\times\beta^2}{\text{GIIC}}\right)^{\text{XMU}}\right]^{-1/\text{XMU}}$$

and, alternatively, for the Benzeggagh-Kenane law [1996] (XMU < 0):

$$\delta^F = \frac{2}{\delta^0\left(\frac{1}{1+\beta^2}\text{EN}^\gamma + \frac{\beta^2}{1+\beta^2}\text{ET}^\gamma\right)^{1/\gamma}}\left[\text{GIC} + (\text{GIIC}-\text{GIC})\left(\frac{\beta^2\times\text{ET}}{\text{EN}+\beta^2\times\text{ET}}\right)^{|\text{XMU}|}\right]$$

A reasonable choice for the exponent $\gamma$ would be GAMMA = 1.0 (default) or GAMMA = 2.0.

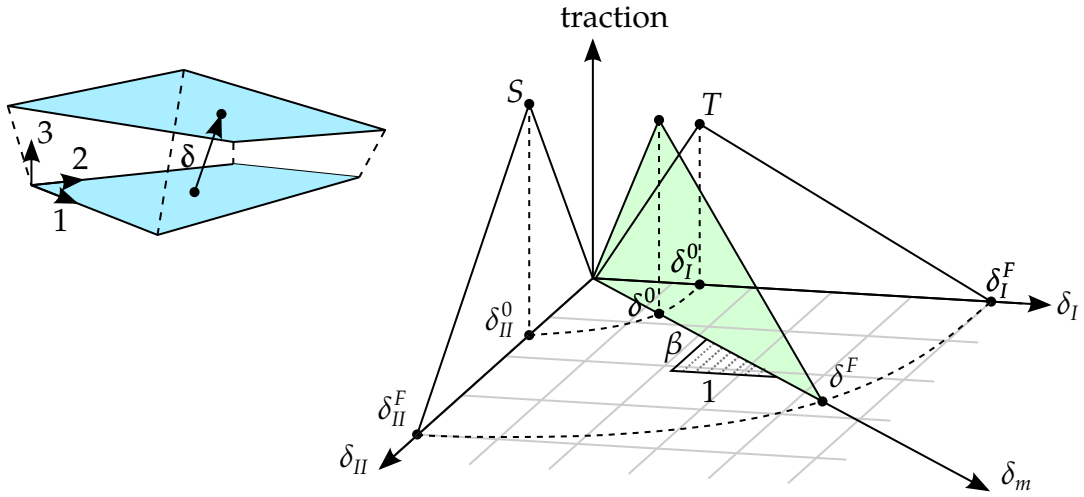



**Figure M138-1.** Mixed-mode traction-separation law

3. **Interface damage.** This model considers damage to the interface. The model enforces irreversible conditions with loading/unloading paths coming from/pointing to the origin.

4. **Peak tractions as load curves.** Peak tractions $T$ and/or $S$ can be defined as functions of characteristic element length (square root of mid-surface area) using a load curve. This option helps obtain the same global responses (e.g., load-

displacement curve) with coarse meshes compared to the solution with a fine mesh. In general, lower peak traction values are needed for coarser meshes.

5. **Error Checks of material data.** We have implemented three error checks for this material model to ensure proper material data. Since the traction as a function of displacement curve is fairly simple (triangular shaped), we can check to ensure that the displacement, $L$, at the peak load (QMAX), is smaller than the ultimate distance for failure, $u$. See Figure M138-2 for the used notation.

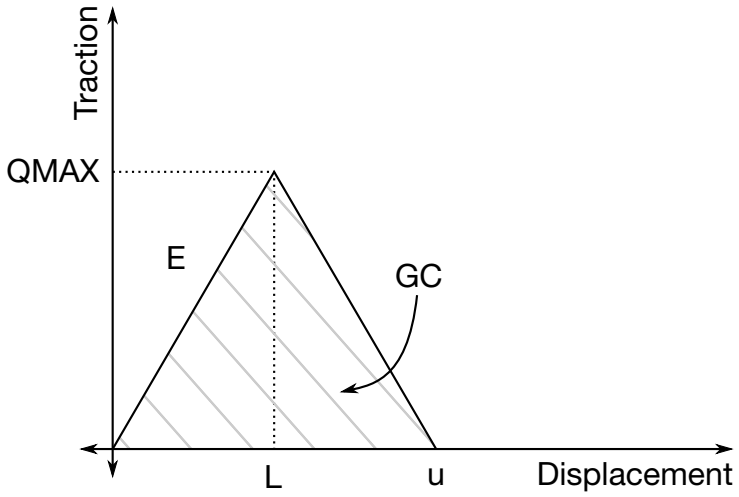



**Figure M138-2.** Bilinear traction-separation

As shown in Figure M138-2,

$$\mathrm{GC} = \frac{1}{2} u \times \mathrm{QMAX}$$

and

$$L = \frac{\mathrm{QMAX}}{E} \ .$$

To ensure the peak is not past the failure point, $u/L$ must be larger than 1. Here,

$$u = \frac{2\mathrm{GC}}{\mathrm{EL}} \ ,$$

where GC is the energy release rate. This gives

$$\frac{u}{L} = \frac{2\mathrm{GC}}{\mathrm{EL} \times L} = \frac{2\mathrm{GC}}{E\left(\frac{\mathrm{QMAX}}{E}\right)^2} > 1 \ .$$

Based on this, the solver performs three error checks, one for tension, one for pure shear, and one for mixed modes:

$$\frac{u}{L} = \frac{\delta_I^F}{\delta_I^0} = \frac{2\text{GIC}}{\text{EN}\left(\frac{T}{\text{EN}}\right)^2} > 1$$

$$\frac{u}{L} = \frac{\delta_{II}^F}{\delta_{II}^0} = \frac{2\text{GIIC}}{\text{ET}\left(\frac{S}{\text{ET}}\right)^2} > 1$$

$$\frac{u}{L} = \frac{\delta^F}{\delta^0} > 1$$

In this last equation, we did not perform the substitution as the equations are complicated and depend on the sign of XMU (see Remark 2). The value of XMU significantly affects $\delta^F$ and should be chosen carefully. Because this check occurs during initialization, the mode-mixity, $\beta$, is computed using the displacements at failure given in input. Thus, it does not reflect any specific loading scenario.

6. **Ortiz and Pandolfi cohesive law**. The Ortiz and Pandolfi cohesive law (Ortiz and Pandolfi [1999]) is often efficient and stable in implicit finite element analyses. It considers the effect of mode mixity on the peak traction but neglects the effect of mode mixity on the critical energy release rate. Thus, it can be considered a simplified version of the original *MAT_138 cohesive law.

## *MAT_MODIFIED_FORCE_LIMITED

This is Material Type 139. This material which is for the Belytschko-Schwer resultant beam is an extension of MAT_029. In addition to the original plastic hinge and collapse mechanisms of MAT_029, yield moments may be defined as a function of axial force. After a hinge forms, the moment transmitted by the hinge is limited by a moment-plastic rotation relationship.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | DF | IAFLC | YTFLAG | ASOFT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| LC1 | LC2 | LC3 | LC4 | LC5 | LC6 | LC7 | LC8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| LPS1 | SFS1 | LPS2 | SFS2 | YMS1 | YMS2 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| LPT1 | SFT1 | LPT2 | SFT2 | YMT1 | YMT2 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| LPR | SFR | YMR | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| LYS1 | SYS1 | LYS2 | SYS2 | LYT1 | SYT1 | LYT2 | SYT2 |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| LYR | SYR | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| HMS1_1 | HMS1_2 | HMS1_3 | HMS1_4 | HMS1_5 | HMS1_6 | HMS1_7 | HMS1_8 |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is required.

| LPMS1_1 | LPMS1_2 | LPMS1_3 | LPMS1_4 | LPMS1_5 | LPMS1_6 | LPMS1_7 | LPMS1_8 |
|---|---|---|---|---|---|---|---|

**Card 11.** This card is required.

| HMS2_1 | HMS2_2 | HMS2_3 | HMS2_4 | HMS2_5 | HMS2_6 | HMS2_7 | HMS2_8 |
|---|---|---|---|---|---|---|---|

**Card 12.** This card is required.

| LPMS2_1 | LPMS2_2 | LPMS2_3 | LPMS2_4 | LPMS2_5 | LPMS2_6 | LPMS2_7 | LPMS2_8 |
|---|---|---|---|---|---|---|---|

**Card 13.** This card is required.

| HMT1_1 | HMT1_2 | HMT1_3 | HMT1_4 | HMT1_5 | HMT1_6 | HMT1_7 | HMT1_8 |
|---|---|---|---|---|---|---|---|

**Card 14.** This card is required.

| LPMT1_1 | LPMT1_2 | LPMT1_3 | LPMT1_4 | LPMT1_5 | LPMT1_6 | LPMT1_7 | LPMT1_8 |
|---|---|---|---|---|---|---|---|

**Card 15.** This card is required.

| HMT2_1 | HMT2_2 | HMT2_3 | HMT2_4 | HMT2_5 | HMT2_6 | HMT2_7 | HMT2_8 |
|---|---|---|---|---|---|---|---|

**Card 16.** This card is required.

| LPMT2_1 | LPMT2_2 | LPMT2_3 | LPMT2_4 | LPMT2_5 | LPMT2_6 | LPMT2_7 | LPMT2_8 |
|---|---|---|---|---|---|---|---|

**Card 17.** This card is required.

| HMR_1 | HMR_2 | HMR_3 | HMR_4 | HMR_5 | HMR_6 | HMR_7 | HMR_8 |
|---|---|---|---|---|---|---|---|

**Card 18.** This card is required.

| LPMR_1 | LPMR_2 | LPMR_3 | LPMR_4 | LPMR_5 | LPMR_6 | LPMR_7 | LPMR_8 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | DF | IAFLC | YTFLAG | ASOFT |
| Type | A | F | F | F | F | I | F | F |
| Default | none | none | none | none | 0.0 | 0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| DF | Damping factor; see Remark 2. *A proper control for the timestep must be maintained by the user.* |
| IAFLC | Axial load curve option:<br>EQ.0: Axial load curves are force as a function of strain.<br>EQ.1: Axial load curves are force as a function of change in length. |
| YTFLAG | Flag to allow beam to yield in tension:<br>EQ.0.0: Beam does not yield in tension.<br>EQ.1.0: Beam can yield in tension. |
| ASOFT | Axial elastic softening factor applied once hinge has formed. When a hinge has formed, the stiffness is reduced by this factor. If zero, this factor is ignored. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M1 | M2 | M3 | M4 | M5 | M6 | M7 | M8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| M1, M2, …, M8 | Applied end moment for force as a function of strain/change in length curve. At least one moment must be defined with a maximum of 8. The values should be in ascending order. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC1 | LC2 | LC3 | LC4 | LC5 | LC6 | LC7 | LC8 |
| Type | I | I | I | I | I | I | I | I |
| Default | none | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LC1, LC2, …, LC8 | Load curve ID (see *DEFINE_CURVE) defining axial force as a function of strain/change in length (see IAFLC) for the corresponding applied end moment. Define the same number as end moments. Each curve must contain the same number of points. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPS1 | SFS1 | LPS2 | SFS2 | YMS1 | YMS2 | | |
| Type | I | F | I | F | F | F | | |
| Default | 0 | 1.0 | LPS1 | 1.0 | $10^{20}$ | YMS1 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LPS1 | Load curve ID for plastic moment as a function of rotation about the *s*-axis at node 1. If zero, this load curve is ignored. |
| SFS1 | Scale factor for plastic moment as a function of rotation curve about the *s*-axis at node 1. |
| LPS2 | Load curve ID for plastic moment as a function of rotation about the s-axis at node 2. The default is LPS1. |
| SFS2 | Scale factor for plastic moment as a function of rotation curve about the *s*-axis at node 2. Default: SFS1. |
| YMS1 | Yield moment about the *s*-axis at node 1 for interaction calculations (default set to $10^{20}$ to prevent interaction) |
| YMS2 | Yield moment about the *s*-axis at node 2 for interaction calculations (default set to YMS1) |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPT1 | SFT1 | LPT2 | SFT2 | YMT1 | YMT2 | | |
| Type | I | F | I | F | F | F | | |
| Default | 0 | 1.0 | LPT1 | 1.0 | $10^{20}$ | YMT1 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LPT1 | Load curve ID for plastic moment as a function of rotation about the *t*-axis at node 1. If zero, this load curve is ignored. |
| SFT1 | Scale factor for plastic moment as a function of rotation curve about the *t*-axis at node 1. Default = 1.0. |
| LPT2 | Load curve ID for plastic moment as a function of rotation about the *t*-axis at node 2. Default: LPT1. |
| SFT2 | Scale factor for plastic moment as a function of rotation curve about the *t*-axis at node 2. Default: SFT1. |
| YMT1 | Yield moment about the *t*-axis at node 1 for interaction calculations (default set to $10^{20}$ to prevent interactions) |

| VARIABLE | DESCRIPTION |
|---|---|
| YMT2 | Yield moment about the *t*-axis at node 2 for interaction calculations (default set to YMT1) |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPR | SFR | YMR | | | | | |
| Type | I | F | F | | | | | |
| Default | 0 | 1.0 | $10^{20}$ | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LPR | Load curve ID for plastic torsional moment as a function of rotation. If zero, this load curve is ignored. |
| SFR | Scale factor for plastic torsional moment as a function of rotation (default = 1.0) |
| YMR | Torsional yield moment for interaction calculations (default set to $10^{20}$ to prevent interaction) |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LYS1 | SYS1 | LYS2 | SYS2 | LYT1 | SYT1 | LYT2 | SYT2 |
| Type | I | F | I | F | I | F | I | F |
| Default | 0 | 1.0 | 0 | 1.0 | 0 | 1.0 | 0 | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LYS1 | ID of curve defining yield moment as a function of axial force for the *s*-axis at node 1 |
| SYS1 | Scale factor applied to load curve LYS1 |
| LYS2 | ID of curve defining yield moment as a function of axial force for the *s*-axis at node 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| SYS2 | Scale factor applied to load curve LYS2 |
| LYT1 | ID of curve defining yield moment as a function of axial force for the *t*-axis at node 1 |
| SYT1 | Scale factor applied to load curve LYT1 |
| LYT2 | ID of curve defining yield moment as a function of axial force for the *t*-axis at node 2 |
| SYT2 | Scale factor applied to load curve LYT2 |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LYR | SYR | | | | | | |
| Type | I | F | | | | | | |
| Default | 0 | 1.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LYR | ID of curve defining yield moment as a function of axial force for the torsional axis. |
| SYR | Scale factor applied to load curve LYR. |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HMS1_1 | HMS1_2 | HMS1_3 | HMS1_4 | HMS1_5 | HMS1_6 | HMS1_7 | HMS1_8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| HMS1_*n* | Hinge moment for the *s*-axis at node 1 |

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPMS1_1 | LPMS1_2 | LPMS1_3 | LPMS1_4 | LPMS1_5 | LPMS1_6 | LPMS1_7 | LPMS1_8 |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LPMS1_*n* | ID of curve defining plastic moment as a function of plastic rotation for the *s*-axis at node 1 for hinge moment HMS1_*n* |

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HMS2_1 | HMS2_2 | HMS2_3 | HMS2_4 | HMS2_5 | HMS2_6 | HMS2_7 | HMS2_8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| HMS2_*n* | Hinge moment for the *s*-axis at node 2 |

| Card 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPMS2_1 | LPMS2_2 | LPMS2_3 | LPMS2_4 | LPMS2_5 | LPMS2_6 | LPMS2_7 | LPMS2_8 |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LPMS2_*n* | ID of curve defining plastic moment as a function of plastic rotation for the *s*-axis at node 2 for hinge moment HMS2_*n* |

| Card 13 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HMT1_1 | HMT1_2 | HMT1_3 | HMT1_4 | HMT1_5 | HMT1_6 | HMT1_7 | HMT1_8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| HMT1_*n* | Hinge moment for the *t*-axis at node 1 |

| Card 14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPMT1_1 | LPMT1_2 | LPMT1_3 | LPMT1_4 | LPMT1_5 | LPMT1_6 | LPMT1_7 | LPMT1_8 |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LPMT1_*n* | ID of curve defining plastic moment as a function of plastic rotation for the *t*-axis at node 1 for hinge moment HMT1_*n* |

| Card 15 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HMT2_1 | HMT2_2 | HMT2_3 | HMT2_4 | HMT2_5 | HMT2_6 | HMT2_7 | HMT2_8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| HMT2_*n* | Hinge moment for the *t*-axis at node 2 |

| Card 16 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPMT2_1 | LPMT2_2 | LPMT2_3 | LPMT2_4 | LPMT2_5 | LPMT2_6 | LPMT2_7 | LPMT2_8 |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LPMT2_*n* | ID of curve defining plastic moment as a function of plastic rotation for the *t*-axis at node 2 for hinge moment HMT2_*n* |

| Card 17 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HMR_1 | HMR_2 | HMR_3 | HMR_4 | HMR_5 | HMR_6 | HMR_7 | HMR_8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| HMR_*n* | Hinge moment for the torsional axis |

| Card 18 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LPMR_1 | LPMR_2 | LPMR_3 | LPMR_4 | LPMR_5 | LPMR_6 | LPMR_7 | LPMR_8 |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LPMR_*n* | ID of curve defining plastic moment as a function of plastic rotation for the torsional axis for hinge moment HMR_*n* |

**Remarks:**

1. **Load Curves.** This material model is available for the Belytschko resultant beam element only. Plastic hinges form at the ends of the beam when the moment reaches the plastic moment. The plastic moment as a function of rotation relationship is specified by the user in the form of a load curve and scale factor. The points of the load curve are (plastic rotation in radians, plastic moment). Both quantities should be positive for all points, with the first point being (zero, initial plastic moment). Within this constraint any form of characteristic may be used, including flat or falling curves. Different load curves and scale factors may be specified at each node and about each of the local $s$ and $t$ axes.

   Axial collapse occurs when the compressive axial load reaches the collapse load. Collapse load as a function of collapse deflection is specified in the form of a load curve. The points of the load curve are either (true strain, collapse force) or (change in length, collapse force). Both quantities should be entered as positive for all points and will be interpreted as compressive. The first point should be (zero, initial collapse load).

   The collapse load may vary with end moment as well as with deflections. In this case several load-deflection curves are defined, each corresponding to a different end moment. Each load curve should have the same number of points and the same deflection values. The end moment is defined as the average of the absolute moments at each end of the beam and is always positive.

2. **Damping.** Stiffness-proportional damping may be added using the damping factor $\lambda$. This is defined as follows:

$$\lambda = \frac{2 \times \xi}{\omega}$$

   where $\xi$ is the damping factor at the reference frequency $\omega$ (in radians per second). For example, if 1% damping at 2Hz is required

$$\lambda = \frac{2 \times 0.01}{2\pi \times 2} = 0.001592$$

   If damping is used, a small time step may be required. LS-DYNA does not check this so to avoid instability it may be necessary to control the time step using a load curve. As a guide, the time step required for any given element is multiplied by $0.3L/c\lambda$ when damping is present ($L$ = element length, $c$ = sound speed).

3. **Moment Interaction.** Plastic hinges can form due to the combined action of moments about the three axes. This facility is activated only when yield moments are defined in the material input. A hinge forms when the following condition is first satisfied.

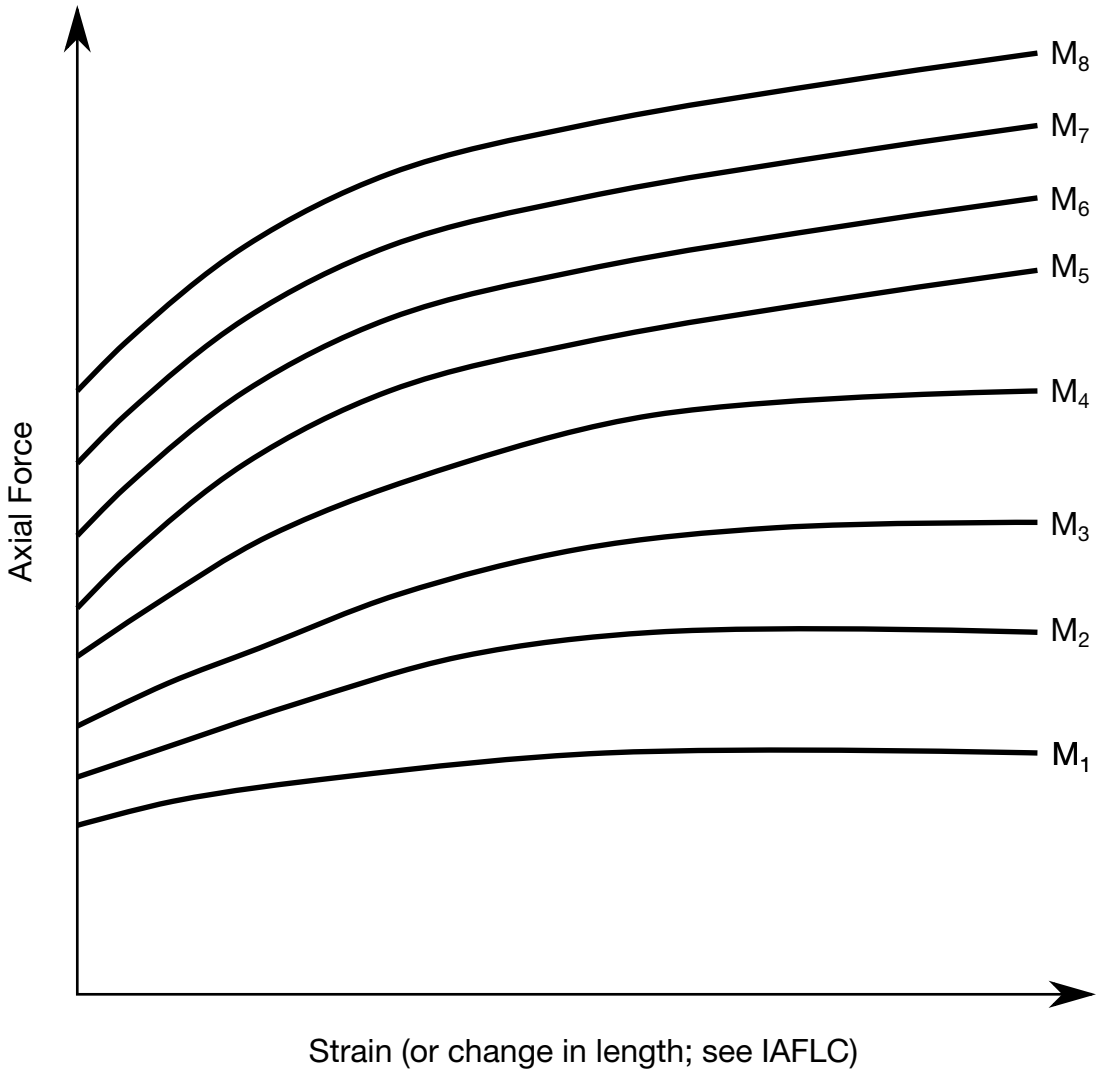



**Figure M139-1.** The force magnitude is limited by the applied end moment. For an intermediate value of the end moment LS-DYNA interpolates between the curves to determine the allowable force value.

$$\left(\frac{M_r}{M_{r_{\text{yield}}}}\right)^2 + \left(\frac{M_s}{M_{s_{\text{yield}}}}\right)^2 + \left(\frac{M_t}{M_{t_{\text{yield}}}}\right)^2 \geq 1 \ ,$$

where

$$M_r,\ M_s,\ M_t = \text{current moment}$$

$$M_{r_{\text{yield}}}, M_{s_{\text{yield}}}, M_{t_{\text{yield}}} = \text{yield moment}$$

Note that scale factors for hinge behavior defined in the input will also be applied to the yield moments: for example, $M_{s_{\text{yield}}}$ in the above formula is given by the input yield moment about the local axis times the input scale factor for the local $s$-axis. For strain-softening characteristics, the yield moment should generally be set equal to the initial peak of the moment-rotation load curve.

On forming a hinge, upper limit moments are set. These are given by

$$M_{r_{\text{upper}}} = \max\left(M_r, \frac{M_{r_{\text{yield}}}}{2}\right)$$

with similar conditions holding for $M_{s_{upper}}$ and $M_{t_{upper}}$. Thereafter the plastic moments will be given by

$$M_{r_p} = \min(M_{r_{\text{upper}}}, M_{r_{\text{curve}}}) \ ,$$

where $M_{r_p}$ is the current plastic moment and $M_{r_{\text{curve}}}$ is the moment from the load curve at the current rotation scaled by the scale factor. $M_{s_p}$ and $M_{t_p}$ satisfy similar conditions. $M_{s_p}$ and $M_{t_p}$ satisfy similar conditions.

This provides an upper limit to the moment that can be generated; it represents the softening effect of local buckling at a hinge site. Thus, if a member is bent about the local *s*-axis, it will then be weaker in torsion and about its local *t*-axis. For moments-softening curves, the effect is to trim off the initial peak (although if the curves subsequently harden, the final hardening will also be trimmed off).

It is not possible to make the plastic moment vary with the current axial load, but it is possible to make hinge formation a function of axial load and subsequent plastic moment a function of the moment at the time the hinge formed. This is discussed in Remark 4.

4. **Independent Plastic Hinge Formation.** In addition to the moment interaction equation, Cards 7 through 18 allow plastic hinges to form independently for the *s*-axis and *t*-axis at each end of the beam as well as for the torsional axis. A plastic hinge is assumed to form if any component of the current moment exceeds the yield moment as defined by the yield moment as a function axial force curves input on cards 7 and 8. If any of the 5 curves is omitted, a hinge will not form for that component. The curves can be defined for both compressive and tensile axial forces. If the axial force falls outside the range of the curve, the first or last point in the curve will be used. A hinge forming for one component of moment does not affect the other components.

   Upon forming a hinge, the magnitude of that component of moment will not be permitted to exceed the current plastic moment. The current plastic moment is obtained by interpolating between the plastic moment as a function of plastic rotation curves input on cards 10, 12, 14, 16, or 18. Curves may be input for up to 8 hinge moments, where the hinge moment is defined as the yield moment at the time that the hinge formed. Curves must be input in order of increasing hinge moment and each curve should have the same plastic rotation values. The first or last curve will be used if the hinge moment falls outside the range of the curves. If no curves are defined, the plastic moment is obtained from the curves

on cards 4 through 6. The plastic moment is scaled by the scale factors on lines 4 to 6.

A hinge will form if either the independent yield moment is exceeded or if the moment interaction equation is satisfied. If both are true, the plastic moment will be set to the minimum of the interpolated value and $M_{r_p}$.

## *MAT_VACUUM

This is Material Type 140. This model is a dummy material representing a vacuum in a multi-material Euler/ALE model. Instead of using ELFORM = 12 (under *SECTION_-SOLID), it is better to use ELFORM = 11 with the void material defined as the vacuum material.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHO | | | | | | |
| Type | A | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RHO | Estimated material density. This is used only as a stability check. |

**Remarks:**

The vacuum density is estimated. It should be small relative compared to air in the model (possibly at least order of magnitude $10^3$ to $10^6$ lighter than air).

## *MAT_RATE_SENSITIVE_POLYMER

This is Material Type 141. This model, called the modified Ramaswamy-Stouffer model, is for the simulation of an isotropic ductile polymer with strain rate effects. See references; Stouffer and Dame [1996] and Goldberg and Stouffer [1999]. Uniaxial test data is used to fit the material parameters. This material model was implemented by Professor Ala Tabiei.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | Do | N | ZO | Q |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OMEGA | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| E | Elastic modulus |
| PR | Poisson's ratio |
| Do | Reference strain rate (= 1000 × max strain rate used in the test) |
| N | Exponent (see inelastic strain rate equation below) |
| ZO | Initial hardness of material, $Z_o$ |
| Q | Material constant, $q$ (see equations below) |
| OMEGA | Maximum internal stress, $\Omega_m$ |

**Remarks:**

The inelastic strain rate is defined as:

$$\dot{\varepsilon}_{ij}^{I} = D_o \exp\left[-0.5\left(\frac{Z_o^2}{3K_2}\right)^{\mathrm{N}}\right]\left(\frac{S_{ij}-\Omega_{ij}}{\sqrt{K_2}}\right)$$

where the $K_2$ term is given as:

$$K_2 = 0.5(S_{ij}-\Omega_{ij})(S_{ij}-\Omega_{ij})$$

and represents the second invariant of the overstress tensor. The elastic components of the strain are added to the inelastic strain to obtain the total strain. The following relationship defines the back stress variable rate:

$$\Omega_{ij} = \frac{2}{3}q\Omega_m\dot{\varepsilon}_{ij}^{I} - q\Omega_{ij}\dot{\varepsilon}_{e}^{I}$$

where $q$ is a material constant, $\Omega_m$ is a material constant that represents the maximum value of the internal stress, and $\dot{\varepsilon}_e^I$ is the effective inelastic strain rate.

## *MAT_TRANSVERSELY_ISOTROPIC_CRUSHABLE_FOAM

This is Material Type 142. This model is for an extruded foam material that is transversely isotropic, crushable, and of low density with no significant Poisson effect. This material is used in energy-absorbing structures to enhance automotive safety in low velocity (bumper impact) and medium high velocity (interior head impact and pedestrian safety) applications. The formulation of this foam is due to Hirth, Du Bois, and Weimar and is documented by Du Bois [2001].

This material is not wholly isotropic since the extrusion direction is preferred. The properties in directions orthogonal to the extrusion direction are, however, the same. In other words, the material is isotropic in all transversal directions to extrusion.

This material is available for solid elements and thick shell formulations 3, 5 and 7.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E11 | E22 | E12 | E23 | G | K |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | I11 | I22 | I12 | I23 | IAA | NSYM | ANG | MU |
| Type | I | I | I | I | I | I | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | ISCL | BETA | MACF | | | | |
| Type | F | I | F | I | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | V1 | V2 | V3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E11 | Elastic modulus in axial direction |
| E22 | Elastic modulus in transverse direction (E22 = E33) |
| E12 | Elastic shear modulus (E12 = E31) |
| E23 | Elastic shear modulus in transverse plane |
| G | Shear modulus |
| K | Bulk modulus for contact stiffness |
| I11 | Load curve for nominal axial stress as a function of volumetric strain |
| I22 | Load curve ID for nominal transverse stresses as a function of volumetric strain (I22 = I33) |
| I12 | Load curve ID for shear stress component 12 and 31 as a function of volumetric strain (I12 = I31) |
| I23 | Load curve ID for shear stress component 23 as a function of volumetric strain |
| IAA | Load curve ID (optional) for nominal stress as a function of volumetric strain for load at angle, ANG, relative to the material *a*-axis |
| NSYM | Set to unity for a symmetric yield surface in volumetric compression and tension direction |
| ANG | Angle corresponding to load curve ID, IAA |

| VARIABLE | DESCRIPTION |
|---|---|
| MU | Damping coefficient for tensor viscosity which acts in both tension and compression. Recommended values vary between 0.05 to 0.10. If zero, bulk viscosity is used instead of tensor viscosity. Bulk viscosity creates a pressure as the element compresses that is added to the normal stresses which can have the effect of creating transverse deformations when none are expected. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| ISCL | Load curve ID for the strain rate scale factor as a function of the volumetric strain rate. The yield stress is scaled by the value specified by the load curve. |
| BETA | Material angle in degrees for AOPT = 0 (tshells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_TSHELL_BETA and *ELEMENT_SOLID_ORTHO. |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP YP ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |
| V1 V2 V3 | Define components of vector **v** for AOPT = 3 and 4 |

**Remarks:**

This model behaves in a more physical way for off axis loading the material than, for example, *MAT_HONEYCOMB which can exhibit nonphysical stiffening for loading

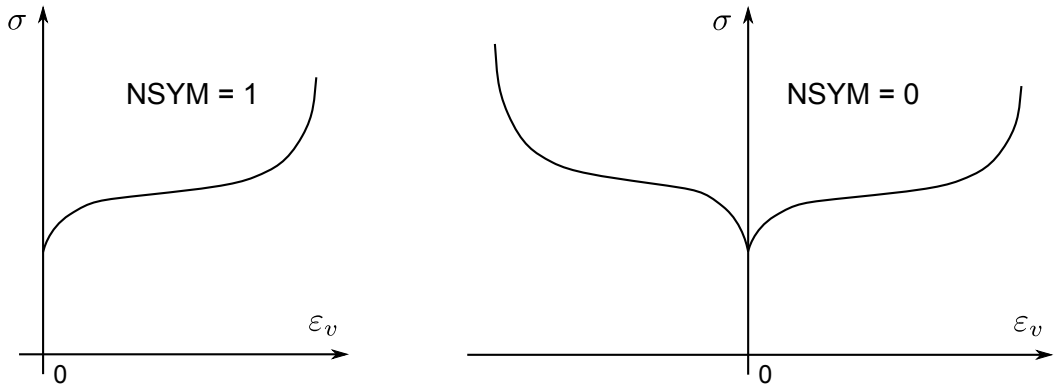



**Figure M142-1.** Differences between options NSYM = 1 and NSYM = 0.

conditions that are off axis. The curves given for I11, I22, I12 and I23 are used to define a yield surface of Tsai-Wu-type that bounds the deviatoric stress tensor. Hence the elastic parameters E11, E12, E22 and E23 as well as G and K must be defined in a consistent way.

For the curve definitions volumetric strain $\varepsilon_v = 1 - V/V_0$ is used as the abscissa parameter. If the symmetric option (NSYM = 1) is used, a curve must be provided for the first quadrant, but may also be defined in both the first and second quadrants. If NSYM = 0 is chosen, the curve definitions for I11, I22, I12 and I23 (and IAA) must be in the first and second quadrant as shown in Figure M142-1.

Tensor viscosity, which is activated by a nonzero value for MU, is generally more stable than bulk viscosity. A damping coefficient less than 0.01 has little effect, and a value greater than 0.10 may cause numerical instabilities.

***MAT_WOOD**_*{OPTION}*

This is Material Type 143. This is a transversely isotropic material. It is available for solid elements, thick shell formulations 3, 5, and 7, and SPH elements. You have the option of inputting your own material properties (<BLANK>) or requesting default material properties for Southern yellow pine (PINE) or Douglas fir (FIR). This model was developed by Murray [2002] under a contract from the FHWA.

Available options include:

<BLANK>

PINE

FIR

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | NPLOT | ITERS | IRATE | GHARD | IFAIL | IVOL |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if the keyword option is set to FIR or PINE.

| MOIS | TEMP | QUAL_T | QUAL_C | UNITS | IQUAL | | |
|---|---|---|---|---|---|---|---|

**Card 2b.1.** This card is included if the keyword option is unset (<BLANK>).

| EL | ET | GLT | GTR | PR | | | |
|---|---|---|---|---|---|---|---|

**Card 2b.2.** This card is included if the keyword option is unset (<BLANK>).

| XT | XC | YT | YC | SXY | SYZ | | |
|---|---|---|---|---|---|---|---|

**Card 2b.3.** This card is included if the keyword option is unset (<BLANK>).

| GF1∥ | GF2∥ | BFIT | DMAX∥ | GF1⊥ | GF2⊥ | DFIT | DMAX⊥ |
|---|---|---|---|---|---|---|---|

**Card 2b.4.** This card is included if the keyword option is unset (<BLANK>).

| FLPAR | FLPARC | POWPAR | FLPER | FLPERC | POWPER | | |
|---|---|---|---|---|---|---|---|

**Card 2b.5.** This card is included if the keyword option is unset (<BLANK>).

| NPAR | CPAR | NPER | CPER | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | MACF | BETA | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| D1 | D2 | D3 | V1 | V2 | V3 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | NPLOT | ITERS | IRATE | GHARD | IFAIL | IVOL |
| Type | A | F | I | I | I | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| NPLOT | Controls what is written as component 7 to the d3plot database. LS-PrePost always blindly labels this component as effective plastic strain.<br>EQ.1: Parallel damage (default)<br>EQ.2: Perpendicular damage |
| ITERS | Number of plasticity algorithm iterations. The default is one iteration.<br>GE.0: Original plasticity iteration developed by Murray [2002]<br>LT.0: Plasticity iteration (return mapping) with non-associated flow direction for perpendicular yielding. The absolute value of ITERS is used as number of plasticity algorithm iterations. |

| VARIABLE | DESCRIPTION |
|---|---|
| IRATE | Rate effects option:<br>EQ.0: Rate effects model turned off (default).<br>EQ.1: Rate effects model turned on with the original rate dependence described by Murray [2002].<br>EQ.2: Rate effects model turned on with Johnson-Cook like rate dependence of the strength parameters, as described below in Remark 2. Only works when ITERS < 0 and the keyword option is unset (<BLANK>). |
| GHARD | Perfect plasticity override. Values greater than or equal to zero are allowed. Positive values model late time hardening in compression (an increase in strength with increasing strain). A zero value models perfect plasticity (no increase in strength with increasing strain). The default is zero. |
| IFAIL | Erosion perpendicular to the grain:<br>EQ.0: No (default)<br>EQ.1: Yes (not recommended except for debugging) |
| IVOL | Flag to invoke erosion based on negative volume or strain increments greater than 0.01:<br>EQ.0: No, do not apply erosion criteria.<br>EQ.1: Yes, apply erosion criteria. |

This card is included for the PINE and FIR keyword options.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MOIS | TEMP | QUAL_T | QUAL_C | UNITS | IQUAL | | |
| Type | F | F | F | F | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MOIS | Percent moisture content. If left blank, the moisture content defaults to saturated at 30%. |
| TEMP | Temperature in °C. If left blank, the temperature defaults to room temperature at 20 °C |

| VARIABLE | DESCRIPTION |
|---|---|
| QUAL_T | Quality factor options (see Remark 1). These quality factors reduce the clear wood tension, shear, and compression strengths as a function of grade. |

EQ.0.0: Grade 1, 1D, 2, 2D. Predefined strength reduction factors are:

| Wood Type | Tension/Shear Factor | Compression Factor |
|---|---|---|
| Pine | 0.47 | 0.63 |
| Fir | 0.40 | 0.73 |

EQ.-1.0: DS-65 or SEl STR (pine and fir). Predefined strength reduction factors are 0.80 in tension/shear and 0.93 in compression.

EQ.-2.0: Clear wood. No strength reduction factors are applied, that is, the reduction factors are 1.0 in tension, shear, and compression.

GT.0.0: User defined quality factor in tension/shear. Values between 0 and 1 are expected. Values greater than one are allowed but may not be realistic.

QUAL_C: User defined quality factor in compression (see Remark 1). This input value is used if QUAL_T > 0. Values between 0 and 1 are expected. Values greater than one are allowed but may not be realistic. If left blank, a default value of QUAL_C = QUAL_T is used.

UNITS: Units options:

EQ.0: GPa, mm, msec, Kg/mm3, kN.

EQ.1: MPa, mm, msec, g/mm3, Nt.

EQ.2: MPa, mm, sec, Mg/mm3, Nt.

EQ.3: Psi, inch, sec, lb-s2/inch4, lb

IQUAL: Apply quality factors perpendicular to the grain:

EQ.0: Yes (default)

EQ.1: No

This card is included if the keyword option is unset (<BLANK>).

| Card 2b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EL | ET | GLT | GTR | PR | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EL | Parallel normal modulus |
| ET | Perpendicular normal modulus |
| GLT | Parallel shear modulus (GLT = GLR) |
| GTR | Perpendicular shear modulus |
| PR | Parallel major Poisson's ratio |

This card is included if the keyword option is unset (<BLANK>).

| Card 2b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XT | XC | YT | YC | SXY | SYZ | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XT | Parallel tensile strength |
| XC | Parallel compressive strength |
| YT | Perpendicular tensile strength |
| YC | Perpendicular compressive strength |
| SXY | Parallel shear strength |
| SYZ | Perpendicular shear strength |

This card is included if the keyword option is unset (<BLANK>).

| Card 2b.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GF1∥ | GF2∥ | BFIT | DMAX∥ | GF1⊥ | GF2⊥ | DFIT | DMAX⊥ |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| XT | Parallel tensile strength |
| GF1∥ | Parallel fracture energy in tension |
| GF2∥ | Parallel fracture energy in shear |
| BFIT | Parallel softening parameter |
| DMAX∥ | Parallel maximum damage |
| GF1⊥ | Perpendicular fracture energy in tension |
| GF2⊥ | Perpendicular fracture energy in shear |
| DFIT | Perpendicular softening parameter |

This card is included if the keyword option is unset (<BLANK>).

| Card 2b.4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FLPAR | FLPARC | POWPAR | FLPER | FLPERC | POWPER | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FLPAR | Rate effects parameter:<br>IRATE.EQ.0: Ignored<br>IRATE.EQ.1: Parallel fluidity parameter for tension and shear<br>IRATE.EQ.2: Dimensionless parallel strain rate parameter for tension and shear (see Remark 2) |
| FLPARC | Rate effects parameter: |

| VARIABLE | DESCRIPTION |
|---|---|
| | IRATE.EQ.0: Ignored<br>IRATE.EQ.1: Parallel fluidity parameter for compression<br>IRATE.EQ.2: Dimensionless parallel strain rate parameter for compression (see Remark 2) |
| POWPAR | Rate effects parameter:<br>IRATE.EQ.0: Ignored<br>IRATE.EQ.1: Parallel power<br>IRATE.EQ.2: Quasi-static threshold strain rate value in the parallel direction (see Remark 2) |
| FLPER | Rate effects parameter:<br>IRATE.EQ.0: Ignored<br>IRATE.EQ.1: Perpendicular fluidity parameter for tension and shear<br>IRATE.EQ.2: Dimensionless perpendicular strain rate parameter for tension and shear (see Remark 2) |
| FLPERC | Rate effects parameter:<br>IRATE.EQ.0: Ignored<br>IRATE.EQ.1: Perpendicular fluidity parameter for compression<br>IRATE.EQ.2: Dimensionless perpendicular strain rate parameter for compression (see Remark 2) |
| POWPER | Rate effects parameter:<br>IRATE.EQ.0: Ignored<br>IRATE.EQ.1: Perpendicular power<br>IRATE.EQ.2: Quasi-static threshold strain rate value in the perpendicular direction (see Remark 2) |

This card is included if the keyword option is unset (<BLANK>).

| Card 2b.5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NPAR | CPAR | NPER | CPER | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| NPAR | Parallel hardening initiation |
| CPAR | Parallel hardening rate |
| NPER | Perpendicular hardening initiation |
| CPER | Perpendicular hardening rate |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | BETA | | | | | |
| Type | F | I | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between |

| VARIABLE | DESCRIPTION |
|---|---|
| | the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO and *ELEMENT_TSHELL_BETA. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP YP ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | V1 | V2 | V3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |
| V1 V2 V3 | Define components of vector **v** for AOPT = 3 and 4 |

**Remarks:**

1. **Quality factors.** Material property data is for clear wood (small samples without defects like knots), whereas real structures are composed of graded wood. Clear wood is stronger than graded wood. Quality factors (strength reduction factors) are applied to the clear wood strengths to account for reductions in strength as a function of grade. One quality factor (QUAL_T) is applied to the tensile and shear strengths. A second quality factor (QUAL_C) is applied to the compressive strengths. As an option, predefined quality factors are provided based on correlations between LS-DYNA calculations and test data for pine and fir posts impacted by bogie vehicles. By default, quality factors are applied to both the parallel and perpendicular to the grain strengths. An option is available (IQUAL) to eliminate application perpendicular to the grain.

2. **Johnson-Cook-like logarithmic rate dependence.** A Johnson-Cook-like logarithmic rate dependence can be invoked by IRATE = 2 when the keyword option is unset and ITERS < 0. In that case, the strength parameters are:

$$\hat{X}_T = X_T\left(1 + \text{FLPAR} \times \ln\left(1 + \frac{\dot{\bar{\varepsilon}}}{\text{POWPAR}}\right)\right)$$
$$\hat{X}_C = X_C\left(1 + \text{FLPARC} \times \ln\left(1 + \frac{\dot{\bar{\varepsilon}}}{\text{POWPAR}}\right)\right)$$
$$\hat{Y}_T = Y_T\left(1 + \text{FLPER} \times \ln\left(1 + \frac{\dot{\bar{\varepsilon}}}{\text{POWPER}}\right)\right)$$
$$\hat{Y}_C = Y_C\left(1 + \text{FLPERC} \times \ln\left(1 + \frac{\dot{\bar{\varepsilon}}}{\text{POWPER}}\right)\right)$$
$$\hat{S}_{XY} = S_{XY}\left(1 + \text{FLPAR} \times \ln\left(1 + \frac{\dot{\bar{\varepsilon}}}{\text{POWPAR}}\right)\right)$$
$$\hat{S}_{YZ} = S_{YZ}\left(1 + \text{FLPER} \times \ln\left(1 + \frac{\dot{\bar{\varepsilon}}}{\text{POWPER}}\right)\right)$$

The strain rate parameters, FLPAR, FLPARC, FLPER, and FLPERC, are dimensionless (factors $\geq 0$ that quantify the strain rate dependence). POWPAR and POWPER are quasi-static threshold strain rate values in the parallel and perpendicular directions with the units of [time]$^{-1}$. Variable $\dot{\bar{\varepsilon}}$ is an effective strain rate.

## *MAT_PITZER_CRUSHABLE_FOAM

This is Material Type 144. This model is for the simulation of isotropic crushable forms with strain rate effects. Uniaxial and triaxial test data have to be used. For the elastic response, the Poisson ratio is set to zero.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | G | PR | TY | SRTV | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCPY | LCUYS | LCSR | VC | DFLG | | | |
| Type | I | I | I | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus |
| G | Shear modulus |
| PR | Poisson's ratio |
| TY | Tension yield |
| SRTV | Young's modulus ($E$) |
| LCPY | Load curve ID giving pressure as a function of volumetric strain; see Figure M75-1. |
| LCUYS | Load curve ID giving uniaxial stress as a function of volumetric strain; see Figure M75-1. |
| LCSR | Load curve ID giving strain rate scale factor as a function of volumetric strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| VC | Viscous damping coefficient (.05 < recommended value < .50) |
| DFLG | Density flag:<br>EQ.0.0: Use initial density.<br>EQ.1.0: Use current density (larger step size with less mass scaling). |

**Remarks:**

The logarithmic volumetric strain is defined in terms of the relative volume, $V$, as:

$$\gamma = -\ln(V) .$$

When defining the curves, the stress and strain pairs should be positive values starting with a volumetric strain value of zero.

## *MAT_SCHWER_MURRAY_CAP_MODEL

This is Material Type 145. *MAT_145 is a Continuous Surface Cap Model and is a three invariant extension of *MAT_GEOLOGIC_CAP_MODEL (*MAT_025) that includes viscoplasticity for rate effects and damage mechanics to model strain softening. The primary references for the model are Schwer and Murray [1994], Schwer [1994], and Murray and Lewis [1994]. *MAT_145 was developed for geomaterials including soils, concrete, and rocks. We recommend using an updated version of a Continuous Surface Cap Model, *MAT_CSCM (*MAT_159), rather than this model, *MAT_SCHWER_MURRAY_CAP_-MODEL (*MAT_145).

> **WARNING**: No default input parameter values are assumed, but recommendations for the more obscure parameters are provided in the descriptions that follow.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | SHEAR | BULK | GRUN | SHOCK | PORE | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| ALPHA | THETA | GAMMA | BETA | EFIT | FFIT | ALPHAN | CALPHA |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| RO | XO | IROCK | SECP | AFIT | BFIT | RDAMO | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| W | D1 | D2 | NPLOT | EPSMAX | CFIT | DFIT | TFAIL |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| FAILFL | DBETA | DDELTA | VPTAU | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| ALPHA1 | THETA1 | GAMMA1 | BETA1 | ALPHA2 | THETA2 | GAMMA2 | BETA2 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | SHEAR | BULK | GRUN | SHOCK | PORE | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| SHEAR | Shear modulus, $G$ |
| BULK | Bulk modulus, $K$ |
| GRUN | Gruneisen ratio (typically = 0), $\Gamma$ |
| SHOCK | Shock velocity parameter (typically 0), $S_l$ |
| PORE | Flag for pore collapse<br>EQ.0.0: Pore collapse<br>EQ.1.0: Constant bulk modulus (typical) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | THETA | GAMMA | BETA | EFIT | FFIT | ALPHAN | CALPHA |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Shear failure parameter, $\alpha$ |
| THETA | Shear failure parameter, $\theta$ |
| GAMMA | Shear failure parameter, $\gamma$ |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Shear failure parameter, $\beta$<br>$\sqrt{J'_2} = F_e(J_1) = \alpha - \gamma\exp(-\beta J_1) + \theta J_1$ |
| EFIT | Dilation damage mechanics parameter (no damage = 1) |
| FFIT | Dilation damage mechanics parameter (no damage = 0) |
| ALPHAN | Kinematic strain hardening parameter, $N^\alpha$ |
| CALPHAN | Kinematic strain hardening parameter, $c^\alpha$ |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R0 | X0 | IROCK | SECP | AFIT | BFIT | RDAM0 | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| R0 | Initial cap surface ellipticity, $R$ |
| X0 | Initial cap surface $J_1$ (mean stress) axis intercept, $X(\kappa_0)$ |
| IROCK | Material flag:<br>EQ.0: Soils (cap can contract)<br>EQ.1: Rock/concrete |
| SECP | Shear enhanced compaction |
| AFIT | Ductile damage mechanics parameter (=1 no damage) |
| BFIT | Ductile damage mechanics parameter (=0 no damage) |
| RDAM0 | Ductile damage mechanics parameter |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | W | D1 | D2 | NPLOT | EPSMAX | CFIT | DFIT | TFAIL |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| W | Plastic Volume Strain parameter, $W$ |
| D1 | Plastic Volume Strain parameter, $D_1$ |
| D2 | Plastic Volume Strain parameter, $D_2$ $\varepsilon_V^P = W\left\{1 - \exp\left\{-D_1[X(\kappa) - X(\kappa_0)] - D_2[(X(\kappa) - X(\kappa_0)]^2\right\}\right\}$ |
| NPLOT | History variable post-processed as effective plastic strain. (See Table M145-1 for history variables available for plotting.) |
| EPSMAX | Maximum permitted strain increment:<br>EQ.0.0: $\Delta\varepsilon_{\max} = 0.05(\alpha - N^\alpha - \gamma)\min\left(\frac{1}{G'}, \frac{R}{9K}\right)$ (calculated default) |
| CFIT | Brittle damage mechanics parameter (= 1 no damage) |
| DFIT | Brittle damage mechanics parameter (= 0 no damage) |
| TFAIL | Tensile failure stress |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FAILFL | DBETA | DDELTA | VPTAU | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FAILFL | Flag controlling element deletion and effect of damage on stress (see Remark 1):<br>EQ.1: $\sigma_{ij}$ reduces with increasing damage; element is deleted when fully damaged (default). |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-1: $\sigma_{ij}$ reduces with increasing damage; element is not deleted. |
| | EQ.2: $S_{ij}$ reduces with increasing damage; element is deleted when fully damaged. |
| | EQ.-2: $S_{ij}$ reduces with increasing damage; element is not deleted. |
| DBETA | Rounded vertices parameter, $\Delta\beta_0$ |
| DDELTA | Rounded vertices parameter, $\delta$ |
| VPTAU | Viscoplasticity relaxation time parameter, $\tau$ |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | THETA1 | GAMMA1 | BETA1 | ALPHA2 | THETA2 | GAMMA2 | BETA2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA1 | Torsion scaling parameter, $\alpha_1$ |
| | LT.0.0: \|ALPHA1\| is the friction angle in degrees. |
| THETA1 | Torsion scaling parameter, $\theta_1$ |
| GAMMA1 | Torsion scaling parameter, $\gamma_1$ |
| BETA1 | Torsion scaling parameter, $\beta_1$ $$Q_1 = \alpha_1 - \gamma_1 \exp(-\beta_1 J_1) + \theta_1 J_1 \theta_2$$ |
| ALPHA2 | Tri-axial extension scaling parameter, $\alpha_2$ |
| THETA2 | Tri-axial extension scaling parameter, $\theta_2$ |
| GAMMA2 | Tri-axial extension scaling parameter, $\gamma_2$ |
| BETA2 | Tri-axial extension scaling parameter, $\beta_2$ $$Q_2 = \alpha_2 - \gamma_2 \exp(-\beta_2 J_1) + \theta_2 J_1$$ |

**Remarks:**

1. **Damage Accumulation and Element Deletion.** FAILFL controls whether the damage accumulation applies to either the total stress tensor, $\sigma_{ij}$, or the deviatoric stress tensor, $S_{ij}$. When FAILFL = 2, damage does not diminish the ability of the material to support hydrostatic stress.

   FAILFL also serves as a flag to control element deletion. Fully damaged elements are deleted only if FAILFL is a positive value. When *MAT_145 is used with the ALE or EFG solvers, failed elements should not be eroded and so a negative value of FAILFL should be used.

2. **History Variables.** All the output parameters listed in Table M145-1 are available for post-processing using LS-PrePost and its displayed list of history variables. The LS-DYNA input parameter NEIPH should be set to 22 on *DATABASE_EXTENT_BINARY.

| History Variable # | Function | Description |
|---|---|---|
| 1 | $X(\kappa)$ | $J_1$ intercept of cap surface |
| 2 | $L(\kappa)$ | $J_1$ value at cap-shear surface intercept |
| 3 | $R$ | Cap surface ellipticity |
| 4 | $\tilde{R}$ | Rubin function |
| 5 | $\varepsilon_v^p$ | Plastic volume strain |
| 6 | | Yield flag (= 0 elastic) |
| 7 | | Number of strain sub-increments |
| 8 | $G^\alpha$ | Kinematic hardening parameter |
| 9 | $J_2^\alpha$ | Kinematic hardening back stress |
| 10 | | Effective strain rate |
| 11 | | Ductile damage |
| 12 | | Ductile damage threshold |
| 13 | | Strain energy |

| History Variable # | Function | Description |
|---|---|---|
| 14 | | Brittle damage |
| 15 | | Brittle damage threshold |
| 16 | | Brittle energy norm |
| 17 | | $J_1$ (without visco-damage/plastic) |
| 18 | | $J_2'$ (without visco-damage/plastic) |
| 19 | | $J_3'$ (without visco-damage/plastic) |
| 20 | | $\hat{J}_3$ (without visco-damage/plastic) |
| 21 | $\beta$ | Lode angle |
| 22 | $d$ | Maximum damage parameter |

**Table M145-1.** Output variables for post-processing using NPLOT parameter.

3. **Sample Input for Concrete.** Gran and Senseny [1996] report the axial stress as a function of strain response for twelve unconfined compression tests of concrete, used in scale-model reinforced-concrete wall tests. The Schwer & Murray Cap Model parameters provided below were used, see Schwer [2001], to model the unconfined compression test stress-strain response for the nominal 40 MPa strength concrete reported by Gran and Senseny. The basic units for the provided parameters are length in millimeters (mm), time in milliseconds (msec), and mass in grams (g). This base unit set yields units of force in Newtons (N) and pressure in Mega-Pascals (MPa).

```
*MAT_SCHWER_MURRAT_CAP_MODEL
$      MID        RO     SHEAR      BULK      GRUN     SHOCK      PORE
         1    2.3E-3   1.048E4   1.168E4       0.0       0.0       1.0
$    ALPHA     THETA     GAMMA      BETA      EFIT      FFIT    ALPHAN    CALPHA
     190.0       0.0     184.2    2.5E-3     0.999       0.7       2.5     2.5E3
$       R0        X0     IROCK      SECP      AFIT      BFIT     RDAM0
       5.0     100.0       1.0       0.0     0.999       0.3      0.94
$        W        D1        D2     NPLOT    EPSMAX      CFIT      DFIT     TFAIL
    5.0E-2    2.5E-4    3.5E-7      23.0       0.0       1.0     300.0       7.0
$   FAILFG     DBETA    DDELTA     VPTAU
       1.0       0.0       0.0       0.0
$   ALPHA1    THETA1    GAMMA1     BETA1    ALPHA2    THETA2    GAMMA2     BETA2
     0.747    3.3E-4      0.17    5.0E-2      0.66    4.0E-4      0.16    5.0E-2
```

4. **User Input Parameters and System of Units.** Consider the following basic units:

   Length: $L$ (e.g. millimeters - mm )

Mass: M (e.g. grams - g )

Time: T (e.g. milliseconds - ms )

The following consistent unit systems can then be derived using Newton's Law, $F = Ma$:

Force: $F = ML/T^2$ [ g-mm/ms $^2$= Kg-m/s $^2$= Newton - N ]

Stress: $\sigma = F/L^2$ [ N/mm $^2$ = 10 $^6$N/m $^2$ = 10 $^6$ Pascals = MPa ]

Density: $\rho = M/L^3$ [ g/mm $^3$ = 10 $^6$ Kg/m $^3$ ]

| Variable | MID | RO | SHEAR | BULK | GRUN | SHOCK | PORE | |
|---|---|---|---|---|---|---|---|---|
| Units | | Density: $M/L^3$ | Stress: $F/L^2$ | Stress: $F/L^2$ | | | | |
| Variable | ALPHA | THETA | GAMMA | BETA | EFIT | FFIT | ALPHAN | CALPHA |
| Units | Stress: $F/L^2$ | | Stress: $F/L^2$ | Stress$^{-1}$: $L^2/F$ | | Stress$^{-½}$: $L/F^{½}$ | Stress: $F/L^2$ | Stress: $F/L^2$ |
| Variable | R0 | X0 | IROCK | SECP | AFIT | BFIT | RDAM0 | |
| Units | | Stress: $F/L^2$ | | | | Stress$^{-½}$: $L/F^{½}$ | Stress$^{½}$: $F^{½}/L$ | |
| Variable | W | D1 | D2 | NPLOT | MAXEPS | CFIT | DFIT | TFAIL |
| Units | | Stress$^{-1}$: $L^2/F$ | Stress$^{-2}$: $L^4/F^2$ | | | | Stress$^{-½}$: $L/F^{½}$ | Stress: $F/L^2$ |
| Variable | FAILFG | DBETA | DDELTA | VPTAU | | | | |
| Units | | Angle: degrees | | Time: T | | | | |
| Variable | ALPHA1 | THETA1 | GAMMA1 | BETA1 | ALPHA2 | THETA2 | GAMMA2 | BETA2 |
| Units | Stress: $F/L^2$ | | Stress: $F/L^2$ | Stress$^{-1}$: $L^2/F$ | Stress: $F/L^2$ | | Stress: $F/L^2$ | Stress$^{-1}$: $L^2/F$ |

## *MAT_1DOF_GENERALIZED_SPRING

This is Material Type 146. This is a linear spring or damper that allows different degrees-of-freedom at two nodes to be coupled.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | C | SCLN1 | SCLN2 | DOFN1 | DOFN2 |
| Type | A | F | F | F | F | F | I | I |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CID1 | CID2 | | | | | | |
| Type | I | I | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| K | Spring stiffness |
| C | Damping constant |
| SCLN1 | Scale factor on force at node 1. Default = 1.0. |
| SCLN2 | Scale factor on force at node 2. Default = 1.0. |
| DOFN1 | Active degree-of-freedom at node 1, a number between 1 to 6 where 1, 2 and 3 are the $x$, $y$, and $z$-translations and 4, 5, and 6 are the $x$, $y$, and $z$-rotations. If this parameter is defined in the *SECTION_BEAM definition or on the *ELEMENT_BEAM_SCALAR card, then the value here, if defined, is ignored. |
| DOFN2 | Active degree-of-freedom at node 2, a number between 1 to 6 where 1, 2 and 3 are the $x$, $y$, and $z$-translations and 4, 5, and 6 are the $x$, $y$, and $z$-rotations. If this parameter is defined in the *SECTION_BEAM definition or on the *ELEMENT_BEAM_SCALAR card, then the value here, if defined, is ignored. |

| VARIABLE | DESCRIPTION |
|---|---|
| CID1 | Local coordinate system at node 1. This coordinate system can be overwritten by a local system specified on the *ELEMENT_-BEAM_SCALAR or *SECTION_BEAM keyword input. If no coordinate system is specified, the global system is used. |
| CID2 | Local coordinate system at node 2. If CID2 = 0, CID2 = CID1. |

## *MAT_FHWA_SOIL

This is Material Type 147. This is an isotropic material with damage and is available for solid elements. The model has a modified Mohr-Coulomb surface to determine the pressure dependent peak shear strength. It was developed for applications involving roadbase soils by Lewis [1999] for the FHWA, who extended the work of Abbo and Sloan [1995] to include excess pore water effects.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | NPLOT | SPGRAV | RHOWAT | VN | GAMMAR | ITERMX |
| Type | A | F | I | F | F | F | F | I |
| Default | none | none | 1 | none | 1.0 | 0.0 | 0.0 | 1 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | G | PHIMAX | AHYP | COH | ECCEN | AN | ET |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | none | none |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MCONT | PWD1 | PWKSK | PWD2 | PHIRES | DINT | VDFM | DAMLEV |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | 0.0 | none | none | none |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSMAX | | | | | | | |
| Type | F | | | | | | | |
| Default | none | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| NPLOT | Controls what is written as component 7 to the d3plot database. LS-PrePost always blindly labels this component as effective plastic strain.<br>EQ.1: Effective strain<br>EQ.2: Damage criterion threshold<br>EQ.3: Damage (diso)<br>EQ.4: Current damage criterion<br>EQ.5: Pore water pressure<br>EQ.6: Current friction angle (phi) |
| SPGRAV | Specific gravity of soil used to get porosity. |
| RHOWAT | Density of water in model units - used to determine air void strain (saturation) |
| VN | Viscoplasticity parameter (strain-rate enhanced strength) |
| GAMMAR | Viscoplasticity parameter (strain-rate enhanced strength) |
| ITERMX | Maximum number of plasticity iterations (default 1) |
| K | Bulk modulus (non-zero) |
| G | Shear modulus (non-zero) |
| PHIMAX | Peak shear strength angle (friction angle in radians) |

| VARIABLE | DESCRIPTION |
|---|---|
| AHYP | Coefficient A for modified Drucker-Prager Surface |
| COH | Cohesion $\tilde{n}$ shear strength at zero confinement (overburden) |
| ECCEN | Eccentricity parameter for third invariant effects |
| AN | Strain hardening percent of PHIMAX where non-linear effects start |
| ET | Strain hardening amount of non-linear effects |
| MCONT | Moisture content of soil. It determines the amount of air voids and should be a value between 0.0 and 1.0. |
| PWD1 | Parameter for pore water effects on bulk modulus |
| PWKSK | Skeleton bulk modulus. Pore water parameter, $\tilde{n}$, set to zero to eliminate effects. |
| PWD2 | Parameter for pore water effects on the effective pressure (confinement) |
| PHIRES | The minimum internal friction angle in radians (residual shear strength) |
| DINT | Volumetric strain at initial damage threshold |
| VDFM | Void formation energy (like fracture energy) |
| DAMLEV | Level of damage that will cause element deletion (0.0 - 1.00) |
| EPSMAX | Maximum principle failure strain |

## *MAT_FHWA_SOIL_NEBRASKA

This is an option to use the default properties determined for soils used at the University of Nebraska (Lincoln). The default units used for this material are millimeter, millisecond, and kilograms. If different units are desired, the conversion factors must be input.

This is Material Type 147. This is an isotropic material with damage and is available for solid elements. The model has a modified Mohr-Coulomb surface to determine the pressure dependent peak shear strength. It was developed for applications involving road base soils.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | FCTIM | FCTMAS | FCTLEN | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| FCTIM | Factor to multiply milliseconds by to get desired time units |
| FCTMAS | Factor to multiply kilograms by to get desired mass units |
| FCTLEN | Factor to multiply millimeters by to get desired length units |

**Remarks:**

As an example, if units of seconds are desired for time, then FCTIM = 0.001.

## *MAT_GAS_MIXTURE

This is Material Type 148. This model is for the simulation of thermally equilibrated ideal gas mixtures. This model only works with the multi-material ALE formulation (ELFORM = 11 in *SECTION_SOLID). This keyword must be used together with *INITIAL_GAS_MIXTURE for the initialization of gas densities and temperatures. When applied in the context of ALE airbag modeling, the injection of inflator gas is done with a *SECTION_POINT_SOURCE_MIXTURE command which controls the injection process. *MAT_ALE_GAS_MIXTURE (*MAT_ALE_02) is identical to this model and is another name for this material model.

### Card Summary:

**Card 1.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| MID | IADIAB | RUNIV | PDV | | | | |

**Card 2a.1.** Include this card if RUNIV is blank or zero.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| CVMASS1 | CVMASS2 | CVMASS3 | CVMASS4 | CVMASS5 | CVMASS6 | CVMASS7 | CVMASS8 |

**Card 2a.2.** Include this card if RUNIV is blank or zero.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| CPMASS1 | CPMASS2 | CPMASS3 | CPMASS4 | CPMASS5 | CPMASS6 | CPMASS7 | CPMASS8 |

**Card 2b.1.** Include this card if RUNIV is nonzero.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| MOLWT1 | MOLWT2 | MOLWT3 | MOLWT4 | MOLWT5 | MOLWT6 | MOLWT7 | MOLWT8 |

**Card 2b.2.** Include this card if RUNIV is nonzero.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| CPMOLE1 | CPMOLE2 | CPMOLE3 | CPMOLE4 | CPMOLE5 | CPMOLE6 | CPMOLE7 | CPMOLE8 |

**Card 2b.3.** Include this card if RUNIV is nonzero.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 |

**Card 2b.4.** Include this card if RUNIV is nonzero.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | IADIAB | RUNIV | PDV | | | | |
| Type | A | I | F | I | | | | |
| Default | none | 0 | 0.0 | 0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| IADIAB | Flag to turn on/off adiabatic compression logic for an ideal gas. See Remark 5.<br>EQ.0: Off (default)<br>EQ.1: On |
| RUNIV | Universal gas constant in per-mole unit (8.31447 J/(mole × K)). See Remark 1. |
| PDV | Element energy update method (see Remark 6):<br>EQ.0: Ideal gas gamma law<br>EQ.1: Pressure work |

**Card 2 for Per Mass Calculation.** Method (A) RUNIV = blank or 0.0.

| Card 2a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CVMASS1 | CVMASS2 | CVMASS3 | CVMASS4 | CVMASS5 | CVMASS6 | CVMASS7 | CVMASS8 |
| Type | F | F | F | F | F | F | F | F |

**Card 3 for Per Mass Calculation.** Method (A) RUNIV = blank or 0.0.

| Card 2a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPMASS1 | CPMASS2 | CPMASS3 | CPMASS4 | CPMASS5 | CPMASS6 | CPMASS7 | CPMASS8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CVMASS1 - CVMASS8 | Heat capacity at constant volume for up to eight different gases in per-mass unit. |
| CPMASS1 - CPMASS8 | Heat capacity at constant pressure for up to eight different gases in per-mass unit. |

**Card 2 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

| Card 2b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MOLWT1 | MOLWT2 | MOLWT3 | MOLWT4 | MOLWT5 | MOLWT6 | MOLWT7 | MOLWT8 |
| Type | F | F | F | F | F | F | F | F |

**Card 3 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

| Card 2b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPMOLE1 | CPMOLE2 | CPMOLE3 | CPMOLE4 | CPMOLE5 | CPMOLE6 | CPMOLE7 | CPMOLE8 |
| Type | F | F | F | F | F | F | F | F |

**Card 4 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

| Card 2b.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 |
| Type | F | F | F | F | F | F | F | F |

**Card 5 for Per Mole Calculation.** Method (B) RUNIV is nonzero.

| Card 2b.4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MOLWT1 - MOLWT8 | Molecular weight of each ideal gas in the mixture (mass-unit/mole). See Remark 2. |
| CPMOLE1 - CPMOLE8 | Heat capacity at constant pressure for up to eight different gases in per-mole unit. These are nominal heat capacity values typically at STP. These are denoted by the variable $A$ in the equation in Remark 2. |
| B1 - B8 | First order coefficient for a temperature dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable $B$ in the equation in Remark 2. |
| C1 - C8 | Second-order coefficient for a temperature-dependent heat capacity at constant pressure for up to eight different gases. These are denoted by the variable $C$ in the equation in Remark 2. |

**Remarks:**

1. **Methods for defining gas properties.** There are 2 methods of defining the gas properties for the mixture. If RUNIV is BLANK or ZERO, Method (A) is used to define constant heat capacities where per-mass unit values of $C_v$ and $C_p$ are input. Only Cards 2a.1 and 2a.2 are required for this method. Method (B) is used to define constant or temperature dependent heat capacities where per-mole unit values of $C_p$ are input. Cards 2b.1 through 2b.4 are required for this method.

2. **Temperature-dependent heat capacity.** The per-mass-unit, temperature-dependent, constant-pressure heat capacity is

$$C_p(T) = \frac{[\text{CPMOLE} + B \times T + C \times T^2]}{\text{MOLWT}}$$

Table M148-1 shows standard SI units for these quantities.

**Table M148-1.** Standard SI units.

| $C_p(T)$ | CPMOLE | $B$ | $C$ |
|---|---|---|---|
| $\frac{\text{J}}{\text{kg K}}$ | $\frac{\text{J}}{\text{mole K}}$ | $\frac{\text{J}}{\text{mole K}^2}$ | $\frac{\text{J}}{\text{mole K}^3}$ |

3. **Initial temperature and density.** *INITIAL_GAS_MIXTURE specifies the initial temperature and the density of the gas species present in a mesh or part at time zero.

4. **Temperature and energy conservation.** The ideal gas mixture is assumed to be thermal equilibrium, that is, all species are at the same temperature ($T$). The gases in the mixture are also assumed to follow Dalton's Partial Pressure Law, $P = \sum_i^{\text{ngas}} P_i$. The partial pressure of each gas is then $P_i = \rho_i R_{\text{gas}_i} T$ where $R_{\text{gas}_i} = \frac{R_{\text{univ}}}{MW}$. The individual gas species temperature equals the mixture temperature. The temperature is computed from the internal energy where the *mixture internal energy per unit volume* is used,

$$e_V = \sum_i^{\text{ngas}} \rho_i C_{V_i} T_i = \sum_i^{\text{ngas}} \rho_i C_{V_i} T$$

$$T = T_i = \frac{e_V}{\sum_i^{\text{ngas}} \rho_i C_{V_i}}$$

In general, the advection step conserves *momentum* and *internal energy*, but not *kinetic energy*. This can result in energy lost in the system and lead to a pressure drop. In *MAT_GAS_MIXTURE the dissipated kinetic energy is automatically converted into heat (internal energy). Thus, in effect the total energy is conserved instead of conserving just the internal energy. This numerical scheme has been shown to improve accuracy in some cases. However, the user should always be vigilant and check the physics of the problem closely.

5. **IADIAB.** As an example, consider an airbag surrounded by ambient air. As the inflator gas flows into the bag, the ALE elements cut by the airbag fabric shell elements will contain some inflator gas inside and some ambient air outside. The multi-material element treatment is not perfect. Consequently the temperature of the outside air may be made artificially high after the multi-material element treatment. To prevent the outside ambient air from getting artificially high $T$, set IADIAB = 1 for the ambient air outside. A simple adiabatic compression equation is then assumed for the outside air. The use of this flag may be needed, but only when that air is modeled by the *MAT_GAS_MIXTURE card.

6. **PDV.** In the original implementation, the ideal gas gamma law, $e_{n+1}/e_n = (\rho_{n+1}/\rho_n)^\gamma$, gave the element energy update. While this approach better-preserved accuracy, it violated energy balance in cases where $\gamma$ differed across elements. For temperature-dependent cases, we recommend PDV = 1 which uses

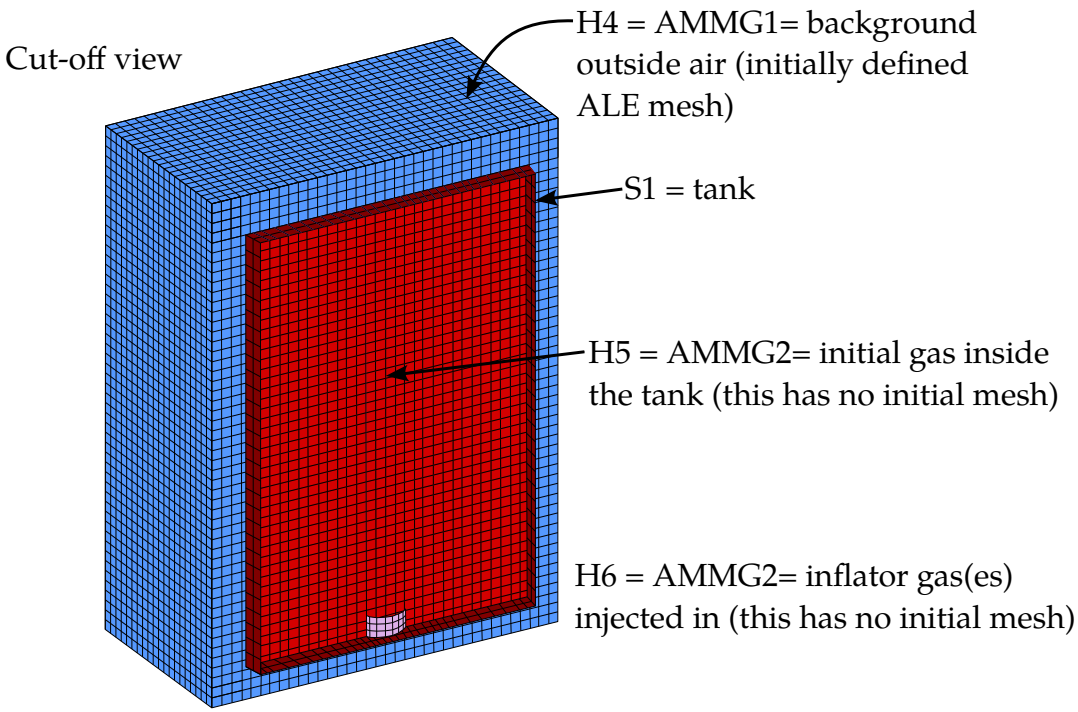
pressure work instead of the ideal gas gamma law. PDV = 0 maintains the original behavior and is only for constant $C_p/C_v$ cases.

**Example:**

Consider a tank test model where the Lagrangian tank (Part S1) is surrounded by an ALE air mesh (Part H4 = AMMGID 1). There are 2 ALE parts which are defined but initially have no corresponding mesh: part 5 (H5 = AMMGID 2) is the resident gas inside the tank at $t = 0$, and part 6 (H6 = AMMGID 2) is the inflator gas(es) which is injected into the tank when $t > 0$. AMMGID stands for ALE Multi-Material Group ID. Please see the figure and input below. The *MAT_GAS_MIXTURE input defines the gas properties of ALE parts H5 & H6. The *MAT_GAS_MIXTURE card input for both methods (A) and (B) are shown below.

The *INITIAL_GAS_MIXTURE keyword input is also shown below. It basically specifies that "AMMGID 2 may be present in part or mesh H4 at $t = 0$, and the initial density of this gas is defined in the rho1 position which corresponds to the 1st material in the mixture (or H5, the resident gas)."

**Example Configuration:**




**Sample Input:**

```
$---------------------------------------------------------------------------
*PART
```

```
H5 = initial gas inside the tank
$      PID     SECID       MID     EOSID      HGID      GRAV    ADPOPT      TMID
         5         5         5         0         5         0         0
*SECTION_SOLID
         5        11         0
$---------------------------------------------------------------------------------
$ Example 1:  Constant heat capacities using per-mass unit.
$*MAT_GAS_MIXTURE
$      MID    IADIAB    R_univ
$        5         0         0
$  Cv1_mas   Cv2_mas   Cv3_mas   Cv4_mas   Cv5_mas   Cv6_mas   Cv7_mas   Cv8_mas
$718.7828911237.56228
$  Cp1_mas   Cp2_mas   Cp3_mas   Cp4_mas   Cp5_mas   Cp6_mas   Cp7_mas   Cp8_mas
$1007.00058 1606.1117
$---------------------------------------------------------------------------------
$ Example 2:  Variable heat capacities using per-mole unit.
*MAT_GAS_MIXTURE
$      MID    IADIAB    R_univ
         5         0  8.314470
$      MW1       MW2       MW3       MW4       MW5       MW6       MW7       MW8
 0.0288479   0.02256
$  Cp1_mol   Cp2_mol   Cp3_mol   Cp4_mol   Cp5_mol   Cp6_mol   Cp7_mol   Cp8_mol
 29.049852  36.23388
$       B1        B2        B3        B4        B5        B6        B7        B8
  7.056E-3  0.132E-1
$       C1        C2        C3        C4        C5        C6        C7        C8
 -1.225E-6 -0.190E-5
$---------------------------------------------------------------------------------
$ One card is defined for each AMMG that will occupy some elements of a mesh set
*INITIAL_GAS_MIXTURE
$      SID     STYPE     MMGID        T0
         4         1         1    298.15
$     RHO1      RHO2      RHO3      RHO4      RHO5      RHO6      RHO7      RHO8
1.17913E-9
*INITIAL_GAS_MIXTURE
$      SID     STYPE     MMGID        T0
         4         1         2    298.15
$     RHO1      RHO2      RHO3      RHO4      RHO5      RHO6      RHO7      RHO8
1.17913E-9
$---------------------------------------------------------------------------------
```

## *MAT_EMMI

This is Material Type 151. The Evolving Microstructural Model of Inelasticity (EMMI) is a temperature and rate-dependent state variable model developed to represent the large deformation of metals under diverse loading conditions [Marin et al. 2006]. It includes various state variables to characterize effects of microstructural features, such as dislocation creation or annihilation. This model is available for 3D solid elements, 2D solid elements and thick shell forms 3 and 5.

### Card Summary:

**Card 1.** This card is required.

| MID | RHO | E | PR | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| RGAS | BVECT | D0 | QD | CV | ADRAG | BDRAG | DMTHTA |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| DMPHI | DNTHTA | DNPHI | THETA0 | THETAM | BETA0 | BTHETA | DMR |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| DNUC1 | DNUC2 | DNUC3 | DNUC4 | DM1 | DM2 | DM3 | DM4 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| DM5 | Q1ND | Q2ND | Q3ND | Q4ND | CALPHA | CKAPPA | C1 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| C2ND | C3 | C4 | C5 | C6 | C7ND | C8ND | C9ND |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| C10 | A1 | A2 | A3 | A4 | A_XX | A_YY | A_ZZ |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| A_XY | A_YZ | A_XZ | ALPHXX | ALPHYY | ALPHZZ | ALPHXY | ALPHYZ |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| ALPHXZ | DKAPPA | PHI0 | PHICR | DLBDAG | FACTOR | RSWTCH | DMGOPT |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is required.

| DELASO | DIMPLO | ATOL | RTOL | DINTER | | | |
|---|---|---|---|---|---|---|---|

**Card 11.** This card is required. *Leave this card blank.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHO | E | PR | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RHO | Material density |
| E | Young's modulus |
| PR | Poisson's ratio |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RGAS | BVECT | D0 | QD | CV | ADRAG | BDRAG | DMTHTA |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| RGAS | Universal gas constant |
| BVECT | Burger's vector |
| D0 | Pre-exponential diffusivity coefficient |
| QD | Activation energy |

| VARIABLE | DESCRIPTION |
|---|---|
| CV | Specific heat at constant volume |
| ADRAG | Drag intercept |
| BDRAG | Drag coefficient |
| DMTHTA | Shear modulus temperature coefficient |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DMPHI | DNTHTA | DNPHI | THETA0 | THETAM | BETA0 | BTHETA | DMR |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| DMPHI | Shear modulus damage coefficient |
| DNTHTA | Bulk modulus temperature coefficient |
| DNPHI | Bulk modulus damage coefficient |
| THETA0 | Reference temperature |
| THETAM | Melt temperature |
| BETA0 | Coefficient of thermal expansion at reference temperature |
| BTHETA | Thermal expansion temperature coefficient |
| DMR | Damage rate sensitivity parameter |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DNUC1 | DNUC2 | DNUC3 | DNUC4 | DM1 | DM2 | DM3 | DM4 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| DNUC1 | Nucleation coefficient 1 |

| VARIABLE | DESCRIPTION |
|---|---|
| DNUC2 | Nucleation coefficient 2 |
| DNUC3 | Nucleation coefficient 3 |
| DNUC4 | Nucleation coefficient 4 |
| DM1 | Coefficient of yield temperature dependence |
| DM2 | Coefficient of yield temperature dependence |
| DM3 | Coefficient of yield temperature dependence |
| DM4 | Coefficient of yield temperature dependence |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DM5 | Q1ND | Q2ND | Q3ND | Q4ND | CALPHA | CKAPPA | C1 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| DM5 | Coefficient of yield temperature dependence |
| Q1ND | Dimensionless activation energy, $Q_1$, for $f$ |
| Q2ND | Dimensionless activation energy, $Q_2$, for $r_d$ |
| Q3ND | Dimensionless activation energy, $Q_3$, for $R_d$ |
| Q4ND | Dimensionless activation energy, $Q_4$, for $R_s$ |
| CALPHA | Coefficient for backstress, $\alpha$ |
| CKAPPA | Coefficient for internal stress, $\kappa$ |
| C1 | Parameter, $c_1$, for flow rule exponent, $n$ |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C2ND | C3 | C4 | C5 | C6 | C7ND | C8ND | C9ND |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C2ND | Parameter, $c_2$, for transition rate $f$ |
| C3 | Parameter, $c_3$, for alpha dynamic recovery, $r_d$ |
| C4 | Parameter, $c_4$, for alpha hardening, $h$ |
| C5 | Parameter, $c_5$, for kappa dynamic recovery, $R_d$ |
| C6 | Parameter, $c_6$, for kappa hardening, $H$ |
| C7ND | Parameter, $c_7$, kappa static recovery, $R_s$ |
| C8ND | Parameter, $c_8$, for yield |
| C9ND | Parameter, $c_9$, for temperature dependence of flow rule exponent, $n$ |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C10 | A1 | A2 | A3 | A4 | A_XX | A_YY | A_ZZ |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C10 | Parameter, $c_{10}$, for static recovery (set to 1) |
| A1 | Plastic anisotropy parameter |
| A2 | Plastic anisotropy parameter |
| A3 | Plastic anisotropy parameter |
| A4 | Plastic anisotropy parameter |
| A_XX | Initial structure tensor component |

| VARIABLE | DESCRIPTION |
|---|---|
| A_YY | Initial structure tensor component |
| A_ZZ | Initial structure tensor component |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A_XY | A_YZ | A_XZ | ALPHXX | ALPHYY | ALPHZZ | ALPHXY | ALPHYZ |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A_XY | Initial structure tensor component |
| A_YZ | Initial structure tensor component |
| A_XZ | Initial structure tensor component |
| ALPHXX | Initial backstress component |
| ALPHYY | Initial backstress component |
| ALPHZZ | Initial backstress component |
| ALPHXY | Initial backstress component |
| ALPHYZ | Initial backstress component |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHXZ | DKAPPA | PHI0 | PHICR | DLBDAG | FACTOR | RSWTCH | DMGOPT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHXZ | Initial backstress component |
| DKAPPA | Initial isotropic internal stress |
| PHI0 | Initial isotropic porosity |

| VARIABLE | DESCRIPTION |
|---|---|
| PHICR | Critical cutoff porosity |
| DLBDAG | Slip system geometry parameter |
| FACTOR | Fraction of plastic work converted to heat, adiabatic |
| RSWTCH | Rate sensitivity switch |
| DMGOPT | Damage model option parameter:<br>EQ.1.0: Pressure independent Cocks/Ashby 1980<br>EQ.2.0: Pressure dependent Cocks/Ashby 1980<br>EQ.3.0: Pressure dependent Cocks 1989 |

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DELASO | DIMPLO | ATOL | RTOL | DINTER | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| DELASO | Temperature option:<br>EQ.0.0: Driven externally<br>EQ.1.0: Adiabatic |
| DIMPLO | Implementation option flag:<br>EQ.1.0: Combined viscous drag and thermally activated dislocation motion<br>EQ.2.0: Separate viscous drag and thermally activated dislocation motion |
| ATOL | Absolute error tolerance for local Newton iteration |
| RTOL | Relative error tolerance for local Newton iteration |
| DNITER | Maximum number of iterations for local Newton iteration |

*Leave this card blank (but include it!).*

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |

**Remarks:**

1. **EMMI Plasticity Model.** The following equations summarize the evolution equations and material functions for the EMMI model. See [Marin et al 2006] for more details.

$$\overset{\nabla}{\boldsymbol{\alpha}} = h\,\mathbf{d}^p - r_d\,\dot{\bar{\varepsilon}}^p\bar{\alpha}\,\boldsymbol{\alpha}$$

$$\dot{\kappa} = (H - R_d\kappa)\dot{\bar{\varepsilon}}^p - R_s\kappa\sinh(Q_s\kappa)$$

$$\mathbf{d}^{\mathrm{p}} = \sqrt{\frac{3}{2}}\,\dot{\bar{\varepsilon}}^p\mathbf{n}, \dot{\bar{\varepsilon}}^p = f\sinh^n\left[\left\langle\frac{\overline{\sigma}}{\kappa + Y} - 1\right\rangle\right]$$

| $\dot{\bar{\varepsilon}}^p -$ equation | $\boldsymbol{\alpha} -$ equation | $\kappa -$ equation |
|---|---|---|
| $f = c_2\exp\left(\frac{Q_1}{\theta}\right)$ | $r_d = c_3\exp\left(\frac{-Q_2}{\theta}\right)$ | $R_d = c_5\exp\left(\frac{-Q_3}{\theta}\right)$ |
| $n = \frac{c_9}{\theta} - c_1$ | $h = c_4\hat{\mu}(\theta)$ | $H = c_6\hat{\mu}(\theta)$ |
| $Y = c_8\hat{Y}(\theta)$ | | $R_s = c_7\exp\left(\frac{-Q_4}{\theta}\right)$ |
| | | $Q_s = c_{10}\exp\left(\frac{-Q_5}{\theta}\right)$ |

**Table M151-1.** Plasticity Material Functions of EMMI Model.

2. **Void Growth.** The following equations extend the EMMI material model for void growth. See [Marin et al 2006] for more details

$$\dot{\varphi} = \frac{3}{\sqrt{2}}(1 - \varphi)\hat{G}(\overline{\sigma}_{eq}, \bar{p}, \varphi)\dot{\bar{\varepsilon}}^p$$

$$\hat{G}(\overline{\sigma}_{eq}, \bar{p}_\tau, \varphi) = \frac{3}{\sqrt{3}}\left[\frac{1}{(1 - \varphi)m + 1} - 1\right]\sinh\left[\frac{2(2m - 1)}{2m + 1}\frac{\langle\bar{p}\rangle}{\overline{\sigma}_{eq}}\right]$$

## *MAT_DAMAGE_3

This is Material Type 153. This model has up to 10 back stress terms for kinematic hardening combined with isotropic hardening and a damage model for modeling low cycle fatigue and failure. The model is based on Huang [2009]. It is available for solid, shell, thick shell, and beam elements. This model is supported for both explicit and implicit analysis. For beams the model is restricted to 3 back stress terms, temperature independent data and KHFLG = 0; while for solids, shells, and thick shells up to 10 back stress terms can be used, including temperature effects and parameter fit from uniaxial cyclic stress-strain tests (KHFLG > 0).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | HARDI | BETA | LCSS |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| HARDK1 | GAMMA1 | HARDK2 | GAMMA2 | SRC | SRP | HARDK3 | GAMMA3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| IDAM | IDS | IDEP | EPSD | S | T | DC | KHFLG |
|---|---|---|---|---|---|---|---|

***Card 4a.*** This card is only read when KHFLG = 0. It is optional.

| HARDK4 | GAMMA4 | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4b.** This card is included if KHFLG > 0.

| LCKH | NKH | | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | HARDI | BETA | LCSS |
| Type | A | F | F | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, $\rho$ |
| E | Young's modulus, $E$<br>LT.0: -E gives the curve ID for $E$ as a function of temperature. |
| PR | Poisson's ratio, $\nu$<br>LT.0: -PR gives the curve ID for $\nu$ as a function of temperature. |
| SIGY | Initial yield stress, $\sigma_{y0}$ (ignored if LCSS > 0) |
| HARDI | Isotropic hardening modulus, $H$ (ignored if LCSS > 0) |
| BETA | Isotropic hardening parameter, $\beta$. Set $\beta = 0$ for linear isotropic hardening. (Ignored if LCSS > 0 or if HARDI = 0.) |
| LCSS | Load curve or table ID defining effective stress as a function of effective plastic strain (and temperature in the table case) for isotropic hardening. For a table each curve corresponds to a temperature. The first abscissa value (effective plastic strain) in each curve must be zero corresponding to the initial yield stress. The first ordinate value in each curve is the initial yield stress. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HARDK1 | GAMMA1 | HARDK2 | GAMMA2 | SRC | SRP | HARDK3 | GAMMA3 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| HARDK$j$ | Kinematic hardening modulus, $C_j$<br>LT.0: -HARDK$j$ gives the curve ID for $C_j$ as a function of temperature. |
| GAMMA$j$ | Kinematic hardening parameter, $\gamma_j$. Set $\gamma_j = 0$ for linear kinematic hardening. Ignored if HARDK$j$ = 0. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0: -GAMMA$j$ gives the curve ID for $\gamma_j$ as a function of temperature. |
| SRC | Strain rate parameter, $C$, for Cowper Symonds strain rate model; see remarks below. If zero, rate effects are not considered.<br>LT.0: -SRC gives the curve ID for $C$ as a function of temperature. |
| SRP | Strain rate parameter, $p$, for Cowper Symonds strain rate model; see remarks below. If zero, rate effects are not considered.<br>LT.0: -SRP gives the curve ID for $p$ as a function of temperature. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDAM | IDS | IDEP | EPSD | S | T | DC | KHFLG |
| Type | I | I | I | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| IDAM | Isotropic damage flag:<br>EQ.0: Damage is inactivated. IDS, IDEP, EPSD, S, T, and DC are ignored.<br>EQ.1: Damage is activated. |
| IDS | Output stress flag:<br>EQ.0: Undamaged stress, $\tilde{\sigma}$, is output.<br>EQ.1: Damaged stress, $\tilde{\sigma}(1-D)$, is output. |
| IDEP | Damaged plastic strain:<br>EQ.0: Plastic strain is accumulated, $r = \int \dot{\bar{\varepsilon}}^{pl}$.<br>EQ.1: Damaged plastic strain is accumulated, $r = \int (1-D)\dot{\bar{\varepsilon}}^{pl}$. |
| EPSD | Damage threshold, $r_d$. Damage accumulation begins when $r > r_d$. |
| S | Damage material constant, $S$. Default = $\sigma_{y0}/200$. |
| T | Damage material constant, $t$. Default = 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| DC | Critical damage value, $D_c$. When damage value reaches the critical value, the element is deleted from calculation. Default = 0.5. |
| KHFLG | Kinematic hardening flag: |

EQ.0: Use kinematic hardening parameters HARDK*j* and GAMMA*j* (default).

EQ.1: Kinematic hardening parameters $(C_j, \gamma_j)$ given by load curve or table if temperature is considered. NKH data points used (with a maximum of 10) in each curve. HARDK*j* and GAMMA*j* fields are ignored.

EQ.2: Fits NKH kinematic hardening parameters $(C_j, \gamma_j)$ to uniaxial stress-strain data at constant temperature for a half-cycle, meaning it fits

$$\sigma_i = \sigma_y(\varepsilon_i^p) + \sum_{j=1}^{NKH} \frac{C_j}{\gamma_j}\left(1 - \exp\left(-\gamma_j \varepsilon_i^p\right)\right)$$

to stress as a function of plastic strain data. The stress, $\sigma_i$, can be given in field LCKH as a function of strain, $\varepsilon_i^p$, in a load curve or as a function of strain and temperature, $T$, in a table. HARDK*j* and GAMMA*j* fields are ignored.

EQ.3: Fits NKH kinematic hardening parameters $(C_j, \gamma_j)$ to uniaxial stress-strain data for the tensile part of a stabilized cycle, meaning it fits

$$\sigma_i = \frac{\sigma_1 + \sigma_N}{2} + \sum_{j=1}^{NKH} \frac{C_j}{\gamma_j}\left(1 - 2\exp\left(-\gamma_j \varepsilon_i^p\right)\right)$$

to $N$ stress as a function of plastic strain data. This data is given by LCKH as either a load curve or table depending on if temperature is included. Here the first data point is chosen such that $\varepsilon_1^p = 0$. HARDK*j* and GAMMA*j* fields are ignored.

EQ.4: Fits NKH kinematic hardening parameters $(C_j, \gamma_j)$ to uniaxial stress-strain data for different stabilized cycles, that is, it fits

$$\sigma_i = \sigma_y(\varepsilon_i^p) + \sum_{j=1}^{NKH} \frac{C_j}{\gamma_j}\tanh\left(\gamma_j \varepsilon_i^p\right),$$

to max stress as a function of max plastic strain data over $N$ cycles. This data is given by LCKH as either a load

| VARIABLE | DESCRIPTION |
|---|---|
| | curve or table depending on if temperature is defined. HARDK*j* and GAMMA*j* fields are ignored. |

Optional Card 4 (read only if KHFLG = 0)

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HARDK4 | GAMMA4 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| HARDK4 | Kinematic hardening modulus, $C_4$<br>LT.0: -HARDK4 gives the curve ID for $C_4$ as a function of temperature. |
| GAMMA4 | Kinematic hardening parameter, $\gamma_4$. Set $\gamma_4 = 0$ for linear kinematic hardening. Ignored if HARDK4 = 0.<br>LT.0: -GAMMA4 gives the curve ID for $\gamma_4$ as a function of temperature. |

Card 4 (included if and only if KHFLG > 0)

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCKH | NKH | | | | | | |
| Type | I | I | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCKH | Load curve or table ID defining kinematic hardening when KHFLG > 0. A table is used when temperature dependence is considered. Depending on KHFLG, it gives either $(C_j, \gamma_j)$ values or stress as a function of plastic strain with optional temperature dependence. |
| NKH | Number of kinematic hardening parameters when KHFLG > 0. Up to 10 back stresses can be used. |

**Model Description:**

This model is based on the work of Lemaitre [1992], and Dufailly and Lemaitre [1995]. It is a pressure-independent plasticity model with the yield surface defined by the function

$$F = \bar{\sigma} - \sigma_y = 0 \ ,$$

where $\sigma_y$ is uniaxial yield stress,

$$\sigma_y = \sigma_{y0} + \frac{H}{\beta}[1 - \exp(-\beta r)] \ .$$

By setting $\beta = 0$, a linear isotropic hardening is obtained

$$\sigma_y = \sigma_{y0} + Hr \ ,$$

where $\sigma_{y0}$ is the initial yield stress. In the above, $\bar{\sigma}$ is the equivalent von Mises stress, with respect to the deviatoric effective stress,

$$\mathrm{s_e} = \mathrm{dev}[\tilde{\boldsymbol{\sigma}}] - \boldsymbol{\alpha} = \mathbf{s} - \boldsymbol{\alpha} \ .$$

Here $\mathbf{s}$ is deviatoric stress and $\boldsymbol{\alpha}$ is the back stress, which is the sum of up to four terms according to:

$$\boldsymbol{\alpha} = \sum_{\mathrm{j}} \boldsymbol{\alpha}_{\mathrm{j}} \ .$$

$\tilde{\boldsymbol{\sigma}}$ is effective stress (undamaged stress), based on Continuum Damage Mechanics model [Lemaitre 1992],

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{1 - D} \ .$$

Here $D$ is the isotropic damage scalar, which is bounded by 0 and 1

$$0 \le D \le 1 \ .$$

$D = 0$ represents a damage-free material RVE (representative volume element), while $D = 1$ represents a fully broken material RVE in two parts. In fact, fracture occurs when $D = D_c < 1$, modeled as element removal. The evolution of the isotropic damage value related to ductile damage and fracture (the case where the plastic strain or dissipation is much larger than the elastic one, [Lemaitre 1992]) is defined as

$$\dot{D} = \begin{cases} \left(\dfrac{Y}{S}\right)^t \dot{\bar{\varepsilon}}^{\mathrm{pl}} & \text{when } r > r_d \text{ and } \dfrac{\sigma_m}{\sigma_{\mathrm{eq}}} > -\dfrac{1}{3} \\ 0 & \text{otherwise} \end{cases}$$

where $\sigma_m/\sigma_{eq}$ is the stress triaxiality, $r_d$ is damage threshold, $S$ is a material constant, and $Y$ is strain energy density release rate:

$$Y = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{el}} : \mathbf{D}^{\mathrm{el}} : \boldsymbol{\varepsilon}^{\mathrm{el}} \ .$$

Here $\mathbf{D}^{\mathrm{el}}$ represents the fourth-order elasticity tensor and $\boldsymbol{\varepsilon}^{\mathrm{el}}$ is elastic strain. In the above, $t$ is a material constant, introduced by Dufailly and Lemaitre [1995], to provide an

additional degree of freedom for modeling low-cycle fatigue ($t = 1$ in Lemaitre [1992]). Dufailly and Lemaitre [1995] also proposed a simplified method to fit experimental results and get $S$ and $t$.

The equivalent Mises stress is defined as

$$\overline{\sigma}(\mathbf{s}_e) = \sqrt{\frac{3}{2}\mathbf{s}_e : \mathbf{s}_e} = \sqrt{\frac{3}{2}}\|\mathbf{s}_e\| \ .$$

The model assumes associated plastic flow

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}} = \frac{\partial F}{\partial \boldsymbol{\sigma}} d\lambda = \frac{3}{2}\frac{\mathbf{s}_e}{\overline{\sigma}} d\lambda \ ,$$

where $d\lambda$ is the plastic consistency parameter. The evolution of the kinematic component of the model is defined as [Armstrong and Frederick 1966]:

$$\dot{\boldsymbol{\alpha}}_j = \begin{cases} \dfrac{2}{3} C_j \dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}} - \gamma_j \boldsymbol{\alpha}_j \dot{\bar{\varepsilon}}^{\mathrm{pl}} & \text{if IDEP} = 0 \\ (1 - D)\left(\dfrac{2}{3} C_j \dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}} - \gamma_j \boldsymbol{\alpha}_j \dot{\bar{\varepsilon}}^{\mathrm{pl}}\right) & \text{if IDEP} = 1 \end{cases}$$

The damaged plastic strain is accumulated as

$$r = \begin{cases} \displaystyle\int \dot{\bar{\varepsilon}}^{\mathrm{pl}} & \text{if IDEP} = 0 \\ \displaystyle\int (1 - D)\dot{\bar{\varepsilon}}^{\mathrm{pl}} & \text{if IDEP} = 1 \end{cases}$$

where $\dot{\bar{\varepsilon}}^{\mathrm{pl}}$ is the equivalent plastic strain rate

$$\dot{\bar{\varepsilon}}^{\mathrm{pl}} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}} : \dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}}} \ .$$

$\dot{\boldsymbol{\varepsilon}}^{\mathrm{pl}}$ represents the rate of plastic flow.

Strain rate is accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p}$$

where $\dot{\varepsilon}$ is the strain rate.

### Uniaxial cyclic tension and compression:

This material can be used to model cyclic hardening plasticity, including effects known as *plastic shakedown* and *strain racheting*. To understand how the plasticity parameters qualitatively influence the behavior in uniaxial tension and compression, we restrict ourselves to a discussion concerning linear isotropic hardening with initial yield $\sigma_Y$ and hardening modulus $H$. We also only include two kinematic hardening terms. For the

kinematic part, we use one linear term with hardening $C_0$ (and decay coefficient $\gamma_0 = 0$) and one combined term with hardening $C$ and decay coefficient $\gamma$. The elastic Young's modulus is denoted $E$, and we neglect any forms of temperature or rate effects.

While this is merely an attempt to explain the phenomena, estimating the parameters that reflect the actual behavior of the physical material may be difficult. Because of this, we recommend the fitting options provided by KHFLG, where even the effects of temperature can be accounted for.

***Strain induced deformation***

Consider the cyclic deformation depicted in Figure M153-1 in which the uniaxial strain ranges between $\underline{\varepsilon}$ and $\bar{\varepsilon}$. Two subsequent stress-strain cycles are shown.

If the isotropic hardening modulus $H = 0$, then the cycles are identical. For nonzero hardening $H$, the stress level increases with each cycle and the strain width indicated by $\tilde{\varepsilon}$ decreases. As the yield surface expands, the isotropic hardening effect diminishes, and we tend towards a stable cycle; this phenomenon is called *plastic shakedown*.

If both $H$ and $C_0$ are zero, then the end of each cycle tends towards ideal plastic since the presence of a nonzero $\gamma$ saturates the level of back stress and consequently the stress $\sigma$ itself. This physical phenomenon is unlikely. Therefore, we recommend having at least one linear kinematic hardening term present in combination with some isotropic hardening for a realistic behavior.

***Stress induced deformation***

For stress induced deformation, we impose a cyclic stress between $\underline{\sigma}$ and $\bar{\sigma}$ as shown in Figure M153-2 and investigate two subsequent stress-strain cycles. In this case a combination of isotropic hardening and nonlinear kinematic hardening may cause a drift in strain. This drift is referred to as *ratcheting strain* and may be considered a creep phenomenon. Even without the isotropic hardening $H$, a nonzero mean stress, $(\bar{\sigma} + \underline{\sigma})/2$, in the cycle causes a ratcheting effect. We again recommend using combinations of isotropic, linear and nonlinear kinematic hardening for accurate predictions of this creep behavior.

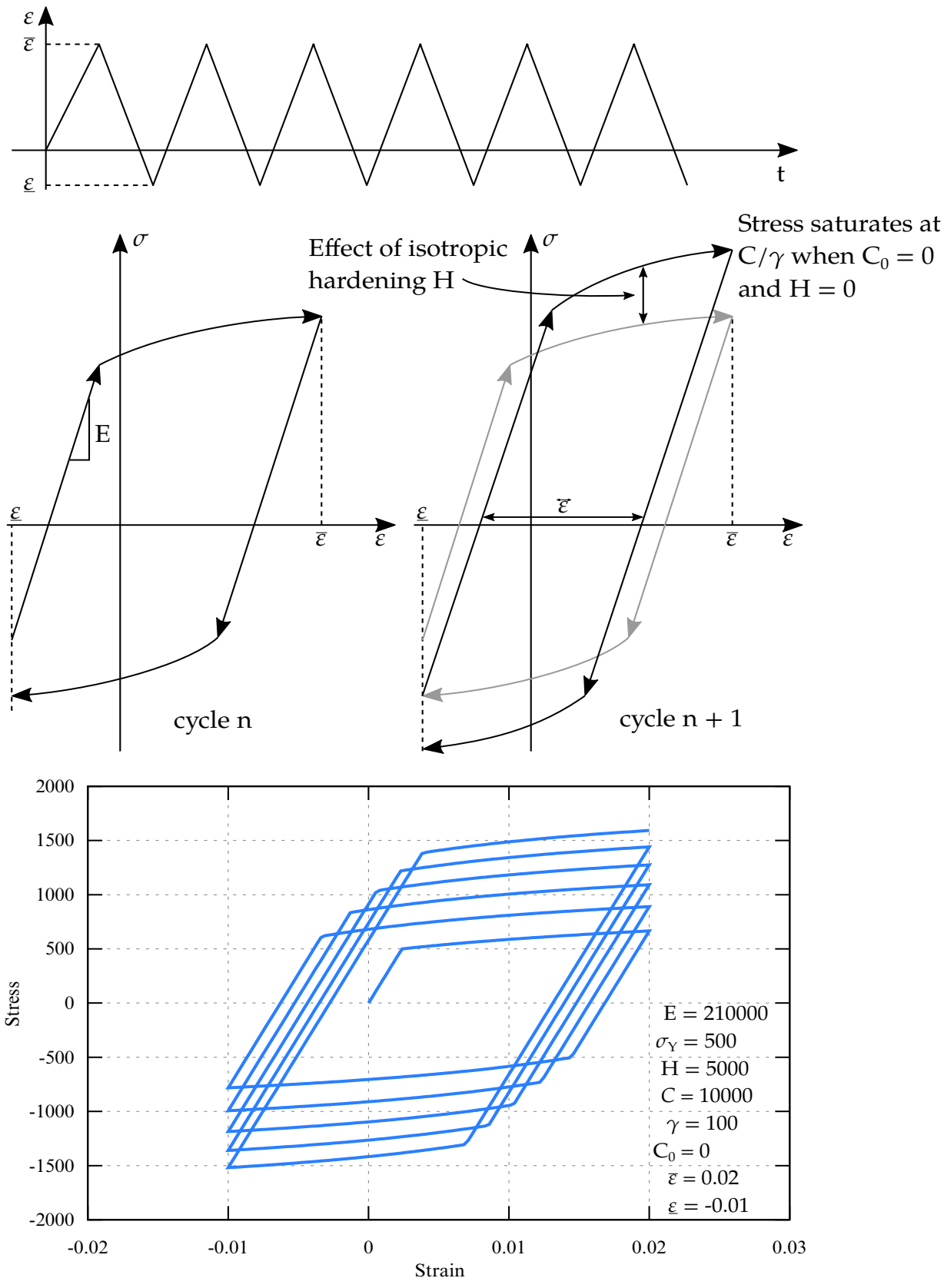
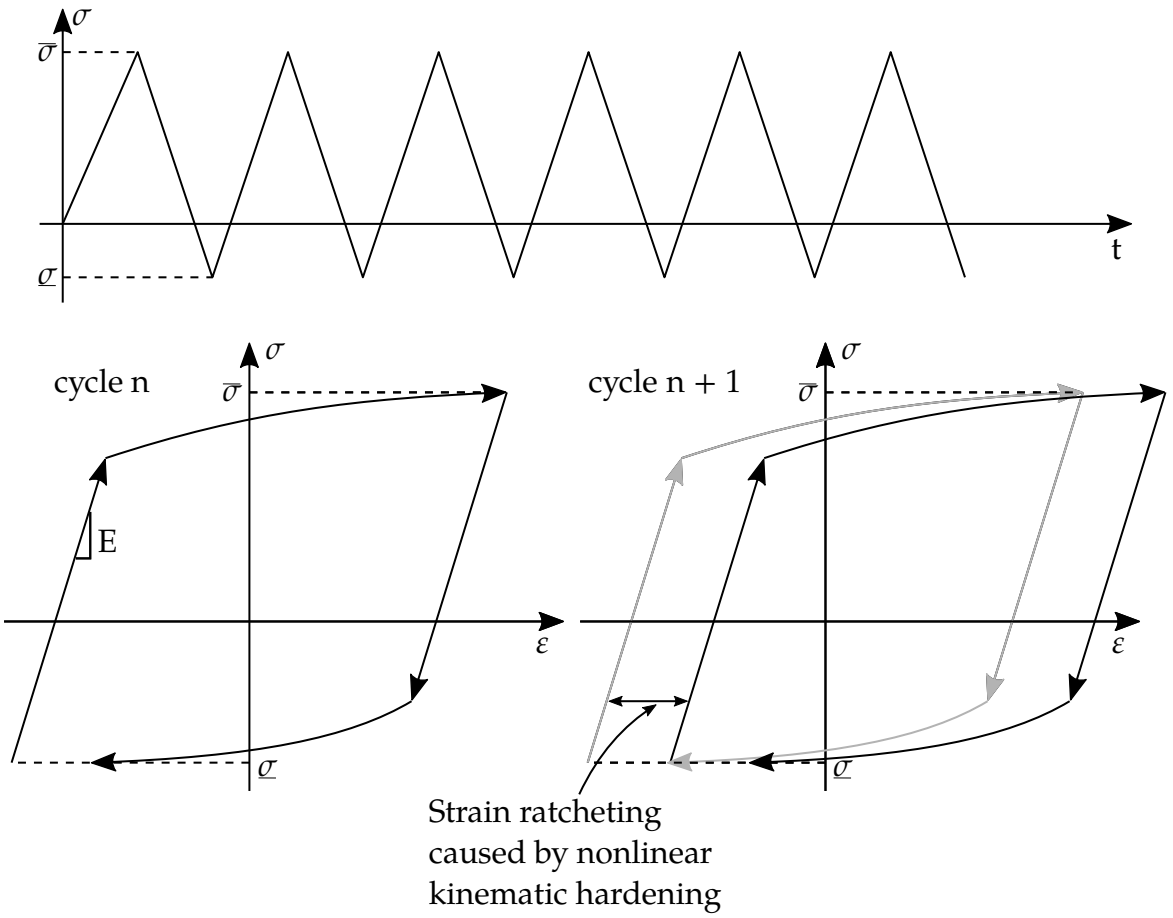



**Figure M153-1.** Schematic of Uniaxial Plastic Shakedown Phenomenon



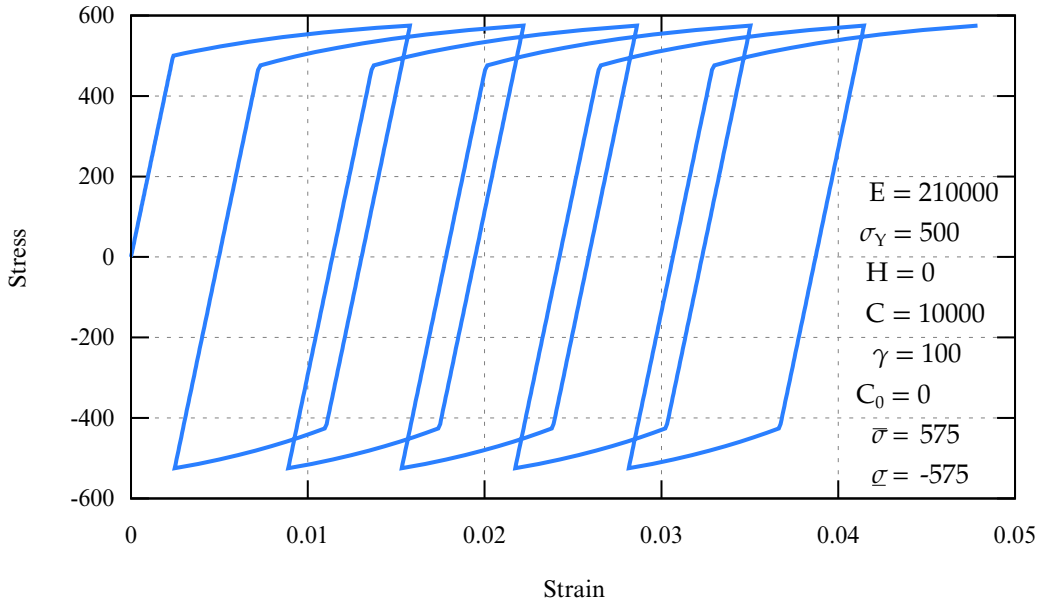




**Figure M153-2.** Schematic of Uniaxial Strain Ratcheting Phenomenon

### Material Model Comparison:

Table M153-1 below shows the difference between MAT 153 (for KHFLG = 0) and MAT 104/105. MAT 153 is less computationally expensive than MAT 104/105. Kinematic hardening, which already exists in MAT 103, is included in MAT 153 but not in MAT 104/105.

| | MAT 153 | MAT 104 | MAT 105 |
|---|---|---|---|
| Computational cost | 1.0 | 3.0 | 3.0 |
| Isotropic hardening | One component | Two components | One component |
| Kinematic hardening | Four components | N/A | N/A |
| Output stress | IDS = 0 $\Rightarrow \tilde{\sigma}$<br>IDS = 1 $\Rightarrow \tilde{\sigma}(1-D)$ | $\tilde{\sigma}(1-D)$ | $\tilde{\sigma}(1-D)$ |
| Damaged plastic strain | IDEP = 0 $\Rightarrow$ $r = \int \dot{\bar{\varepsilon}}^{\mathrm{pl}}$<br>IDEP = 1 $\Rightarrow$ $r = \int (1-D)\dot{\bar{\varepsilon}}^{\mathrm{pl}}$ | $r = \int (1-D)\dot{\bar{\varepsilon}}^{\mathrm{pl}}$ | $r = \int (1-D)\dot{\bar{\varepsilon}}^{\mathrm{pl}}$ |
| Accumulation when | $\frac{\sigma_m}{\sigma_{eq}} > -\frac{1}{3}$ | $\sigma_1 > 0$ | $\sigma_1 > 0$ |
| Isotropic plasticity | Yes | Yes | Yes |
| Anisotropic plasticity | No | Yes | No |
| Isotropic damage | Yes | Yes | Yes |
| Anisotropic damage | No | Yes | No |

**Table M153-1.** Differences between MAT 153 and MAT 104/105

### History Variables:

Additional history variables, which can be written by using variables NEIPH and NEIPS in *DATABASE_EXTENT_BINARY, are as follows:

| History Variable # | Description |
|---|---|
| 1 | Damage, $D$ |

| History Variable # | Description |
|---|---|
| 2 | Back stress term 1 in the 11-direction |
| 3 | Back stress term 1 in the 22-direction |
| 4 | Back stress term 1 in the 12-direction |
| 5 | Back stress term 1 in the 23-direction |
| 6 | Back stress term 1 in the 31-direction |
| 7 | Back stress term 2 in the 11-direction |
| 8 | Back stress term 2 in the 22-direction |
| 9 | Back stress term 2 in the 12-direction |
| 10 | Back stress term 2 in the 23-direction |
| 11 | Back stress term 2 in the 31-direction |
| 12 | Back stress term 3 in the 11-direction |
| 13 | Back stress term 3 in the 22-direction |
| 14 | Back stress term 3 in the 12-direction |
| 15 | Back stress term 3 in the 23-direction |
| 16 | Back stress term 3 in the 31-direction |
| 17 | Back stress term 4 in the 11-direction |
| 18 | Back stress term 4 in the 22-direction |
| 19 | Back stress term 4 in the 12-direction |
| 20 | Back stress term 4 in the 23-direction |
| 21 | Back stress term 4 in the 31-direction |

## *MAT_DESHPANDE_FLECK_FOAM

This is Material Type 154 for solid elements. This material is for modeling aluminum foam used as a filler material in aluminum extrusions to enhance the energy absorbing capability of the extrusion. Such energy absorbers are used in vehicles to dissipate energy during impact. This model was developed by Reyes, Hopperstad, Berstad, and Langseth [2002] and is based on the foam model by Deshpande and Fleck [2000].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHO | E | PR | ALPHA | GAMMA | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSD | ALPHA2 | BETA | SIGP | DERFI | CFAIL | PFAIL | NUM |
| Type | F | F | F | F | F | F | F | I |
| Default | none | none | none | none | none | ↓ | ↓ | 1000 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RHO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| ALPHA | Controls shape of yield surface |
| GAMMA | Material parameter, $\gamma$; see Remarks. |
| EPSD | Densification strain |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA2 | Material parameter, $\alpha_2$; see Remarks. |
| BETA | Material parameter, $\beta$; see Remarks. |
| SIGP | Material parameter, $\sigma_p$; see Remarks. |
| DERFI | Type of derivation used in material subroutine:<br>EQ.0: Numerical derivation<br>EQ.1: Analytical derivation |
| CFAIL | Tensile volumetric strain at failure. Default is no failure due to tensile volumetric strain. |
| PFAIL | Maximum principal stress at failure. Must be sustained NUM ( > 0) timesteps to fail element. Default is no failure due to maximum principal stress. |
| NUM | Number of timesteps at or above PFAIL to trigger element failure |

**Remarks:**

The yield stress function, $\Phi$, is defined by:

$$\Phi = \hat{\sigma} - \sigma_y \ .$$

The equivalent stress, $\hat{\sigma}$, is given by:

$$\hat{\sigma}^2 = \frac{\sigma_{VM}^2 + \alpha^2 \sigma_m^2}{1 + \left(\frac{\alpha}{3}\right)^2} \ ,$$

where, $\sigma_{VM}$, is the von Mises effective stress:

$$\sigma_{VM} = \sqrt{\frac{2}{3}\boldsymbol{\sigma}^{\text{dev}} : \boldsymbol{\sigma}^{\text{dev}}} \ .$$

In this equation $\sigma_m$ and $\boldsymbol{\sigma}^{\text{dev}}$ are the mean and deviatoric stress:

$$\boldsymbol{\sigma}^{\text{dev}} = \boldsymbol{\sigma} - \sigma_m \mathbf{I} \ .$$

The yield stress, $\sigma_y$, can be expressed as:

$$\sigma_y = \sigma_p + \gamma \frac{\hat{\varepsilon}}{\varepsilon_D} + \alpha_2 \ln\left[\frac{1}{1 - \left(\frac{\hat{\varepsilon}}{\varepsilon_D}\right)^\beta}\right] \ .$$

Here, $\sigma_p$, $\alpha_2$, $\gamma$, and $\beta$ are material parameters. The densification strain $\varepsilon_D$ is defined as:

$$\varepsilon_D = -\ln\left(\frac{\rho_f}{\rho_{f0}}\right) ,$$

where $\rho_f$ is the foam density and $\rho_{f0}$ is the density of the virgin material.

## *MAT_PLASTICITY_COMPRESSION_TENSION_EOS

This is Material Type 155. An isotropic elastic-plastic material where unique yield stress as a function of plastic strain curves can be defined for compression and tension. Also, failure can occur based on a plastic strain or a minimum time step size. Rate effects on the yield stress are modeled either by using the Cowper-Symonds strain rate model or by using two load curves that scale the yield stress values in compression and tension, respectively. Material rate effects, which are independent of the plasticity model, are based on a 6-term Prony series Maxwell mode that generates an additional stress tensor. The viscous stress tensor is superimposed on the stress tensor generated by the plasticity. Pressure is defined by an equation of state, which is required to utilize this model. This model is applicable to solid elements and SPH.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | C | P | FAIL | TDEL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCIDC | LCIDT | LCSRC | LCSRT | SRFLAG | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| PC | PT | PCUTC | PCUTT | PCUTF | SCALEP | SCALEE | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| K | | | | | | | |
|---|---|---|---|---|---|---|---|

***Card 5.*** This card is optional. Up to six cards may be input. The next keyword ("*") card terminates this input.

| G*i* | BETA*i* | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | C | P | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | 0.0 | 0.0 | $10^{20}$ | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| C | Strain rate parameter, $C$; see Remarks below. |
| P | Strain rate parameter, $p$; see Remarks below. |
| FAIL | Failure flag:<br>LT.0.0: User defined failure subroutine, `matusr_24` in dyn21.F, is called to determine failure<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic element deletion |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDC | LCIDT | LCSRC | LCSRT | SRFLAG | | | |
| Type | I | I | I | I | F | | | |
| Default | 0 | 0 | 0 | 0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDC | Load curve ID defining effective yield stress as a function of effective plastic strain in compression |
| LCIDT | Load curve ID defining effective yield stress as a function of effective plastic strain in tension. |
| LCSRC | Optional load curve ID defining strain rate scaling effect on yield stress when the material is in compression (compressive yield stress scaling factor as a function of strain rate). |
| LCSRT | Optional load curve ID defining strain rate scaling effect on yield stress when the material is in tension (tensile yield stress scaling factor as a function of strain rate). |
| SRFLAG | Formulation for rate effects:<br>EQ.0.0: Total strain rate<br>EQ.1.0: Deviatoric strain rate |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PC | PT | PCUTC | PCUTT | PCUTF | SCALEP | SCALEE | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| PC | Compressive mean stress at which the yield stress follows load curve ID LCIDC. If the pressure falls between PC and PT, a weighted average of the two load curves is used. |
| PT | Tensile mean stress at which the yield stress follows load curve ID LCIDT. |
| PCUTC | Pressure cut-off in compression. When the pressure cut-off is reached, the deviatoric stress tensor is set to zero. The compressive pressure is not, however, limited to PCUTC. Like the yield stress, PCUTC is scaled to account for rate effects. |
| PCUTT | Pressure cut-off in tension. When the pressure cut-off is reached, the deviatoric stress tensor and tensile pressure is set to zero. Like the yield stress, PCUTT is scaled to account for rate effects. |
| PCUTF | Pressure cut-off flag:<br>EQ.0.0: inactive<br>EQ.1.0: active |
| SCALEP | Scale factor applied to the yield stress after the pressure cut-off is reached in either compression or tension. If SCALEP = 0.0 (default), the deviatoric stress is set to zero after the cut-off is reached. |
| SCALEE | Scale factor applied to the yield stress after the strain exceeds the failure strain set by FAIL. If SCALEE = 0.0 (default), the deviatoric strain is set to zero if the failure strain is exceeded. *If both SCALEP > 0 and SCALEE > 0 and both failure conditions are met, then the minimum scale factor is used.* |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Optional bulk modulus for the viscoelastic material. If nonzero a Kelvin type behavior will be obtained. Generally, K is set to zero. |

**Viscoelastic Constant Cards.** Card format for viscoelastic constants. Up to 6 cards may be input. The next keyword ("*") cards terminates this input.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional shear decay constant for the $i^{th}$ term |

**Remarks:**

The effective yield stress as a function of effective plastic strain behavior follows a different curve in compression than it does in tension. Tension is determined by the sign of the mean stress where a positive mean stress (meaning a negative pressure) is indicative of tension. Two curves must be defined giving the yield stress as a function of effective plastic strain. One curve is for the tensile regime and the other curve is for the compressive regime.

Mean stress is an invariant which can be expressed as $(\sigma_x + \sigma_y + \sigma_z)/3$. PC and PT define a range of mean stress values within which interpolation is done between the tensile yield surface and compressive yield surface. PC and PT are not true material properties but are just a numerical convenience so that the transition from one yield surface to the other is not as abrupt as the sign of the mean stress changes. Both PC, PT, PCUTC, and PCUTT may all be input as positive values. It is implied that PC and PCUTC are compressive values and that PT and PCUTT are tensile values. The algebraic sign given these variables by the user is inconsequential.

Strain rate may be accounted for by using either two curves of yield stress scaling factor as a function of strain rate or a Cowper and Symonds model. The two curves in the former approach are used directly, that is, the curves are not rediscretized before being used by the material model. The Cowper and Symonds model scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p},$$

where $\dot{\varepsilon}$ is the strain rate,

$$\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} \ .$$

**History Variables:**

| History Variable | Description |
|---|---|
| 4 | Tensile pressure cutoff (set to zero if tensile or compressive failure occurs) |
| 5 | The cutoff flag, initially equals 1; set to 0 if tensile or compressive failure occurs. |
| 6 | The failure mode flag<br>EQ.0: No failure<br>EQ.1: Compressive failure<br>EQ.2: Tensile failure<br>EQ.3: Failure by plastic strain |
| 7 | The current flow stress |

## *MAT_MUSCLE

This is Material Type 156 for truss elements. This material is a Hill-type muscle model with activation and a parallel damper. Also, see *MAT_SPRING_MUSCLE (*MAT_S15) where a description of the theory is available.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | SNO | SRM | PIS | SSM | CER | DMP |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALM | SFR | SVS | SVR | SSP | SVRST | | |
| Type | F | F | F | F | F | I | | |
| Default | 0.0 | 1.0 | 1.0 | 1.0 | 0.0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Material density in the initial undeformed configuration |
| SNO | Initial stretch ratio, $l_0/l_{\text{orig}}$, meaning the length as defined by the nodal points at $t = 0$ divided by the original initial length. The density for the nodal mass calculation is RO/SNO, or $\rho l_{\text{orig}}/l_0$. |
| SRM | Maximum strain rate |
| PIS | Peak isometric stress corresponding to the dimensionless value of unity in the dimensionless stress as a function of strain; see SSP below. |
| SSM | Strain when the dimensionless stress as a function of strain, SSP below, reaches its maximum stress value |
| CER | Constant, governing the exponential rise of SSP. Required if SSP = 0.0. |

| VARIABLE | DESCRIPTION |
|---|---|
| DMP | Damping constant (stress × time units) |
| ALM | Activation level as a function of time:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of ALM is used. |
| SFR | Scale factor for strain rate maximum as a function of activation level, $a(t)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| SVS | Active dimensionless tensile stress as a function of the stretch ratio, $l/l_{\text{orig}}$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| SVR | Active dimensionless tensile stress as a function of the normalized strain rate, $\dot{\varepsilon}$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| SSP | Isometric dimensionless stress as a function of the stretch ratio, $l/l_{\text{orig}}$, for the parallel elastic element:<br>LT.0.0: Absolute value gives load curve ID or table ID (see Remarks).<br>EQ.0.0: Exponential function is used (see Remarks).<br>GT.0.0: Constant value of 0.0 is used. |
| SVRST | A non-constant value of the SVR function affects the stiffness of the element (see $\sigma_1$ in Remarks). This stiffness is both a function of the strain rate and the stretch ratio. By default, the SVR stiffness is not included when estimating the element time step. This parameter can change this behavior:<br>EQ.0: Do not take SVR stiffness into account when estimating the element time step (default).<br>EQ.1: Take SVR stiffness into account when estimating the element time step. |

**Remarks:**

The material behavior of the muscle model is adapted from *MAT_S15 (the spring muscle model) and treated here as a standard material. The initial length of muscle is calculated automatically. The force, relative length and shortening velocity are replaced by stress, strain, and strain rate. A new parallel damping element is added.

The strain $\varepsilon$ and normalized strain rate $\dot{\bar{\varepsilon}}$ are defined respectively as

$$\begin{aligned}\varepsilon &= \frac{l}{l_{\text{orig}}} - 1 \\ &= \text{SNO} \times \frac{l}{l_0} - 1\end{aligned}$$

and,

$$\begin{aligned}\dot{\bar{\varepsilon}} &= \frac{l}{l_{\text{orig}}} \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{\max}} \\ &= \text{SNO} \times \frac{l}{l_0} \times \frac{\dot{\varepsilon}}{\text{SFR} \times \text{SRM}}\end{aligned}$$

where $\dot{\varepsilon} = \Delta\varepsilon/\Delta t$ (current strain increment divided by current time step), $l$ is the current muscle length, and $l_{\text{orig}}$ is the original muscle length.

From the relation above, it is known:

$$l_{\text{orig}} = \frac{l_0}{1 + \varepsilon_0}$$

where $\varepsilon_0 = \text{SNO} - 1$ and $l_0$ is the muscle length at $t = 0$.

Stress of Contractile Element is:

$$\sigma_1 = \sigma_{\max} a(t) f\left(\frac{l}{l_{\text{orig}}}\right) g(\dot{\bar{\varepsilon}}) \ ,$$

where $\sigma_{\max} = \text{PIS}$, $a(t) = \text{ALM}$, $f(l/l_{\text{orig}}) = \text{SVS}$, and $g(\dot{\bar{\varepsilon}}) = \text{SVR}$.

Stress of Passive Element is:

$$\sigma_2 = \begin{cases} \sigma_{\max} h\left(\dfrac{l}{l_{\text{orig}}}\right) & \text{for curve} \\ \sigma_{\max} h\left(\dot{\bar{\varepsilon}}, \dfrac{l}{l_{\text{orig}}}\right) & \text{for table} \end{cases}$$

where $h$ = SSP. For SSP < 0, the absolute value gives a load curve ID or table ID. The load curve defines isometric dimensionless stress $h$ as a function of stretch ratio $l/l_{\text{orig}}$. The table defines for each normalized strain rate $\dot{\bar{\varepsilon}}$ a load curve giving the isometric dimensionless stress $h$ as a function of stretch ratio $l/l_{\text{orig}}$ for that rate.

For the exponential relationship (SSP = 0):

$$h\left(l/l_{\text{orig}}\right) = \begin{cases} 0 & l/l_{\text{orig}} < 1 \\ \dfrac{1}{\exp(\text{CER}) - 1}\left[\exp\left(\dfrac{\text{CER}}{\text{SSM}}\varepsilon\right) - 1\right] & l/l_{\text{orig}} \geq 1 \quad \text{CER} \neq 0 \\ \dfrac{\varepsilon}{\text{SSM}} & l/l_{\text{orig}} \geq 1 \quad \text{CER} = 0 \end{cases}$$

Stress of Damping Element is:

$$\sigma_3 = \text{DMP} \times \frac{l}{l_{\text{orig}}}\dot{\varepsilon} \ .$$

Total Stress is:

$$\sigma = \sigma_1 + \sigma_2 + \sigma_3 \ .$$

## *MAT_ANISOTROPIC_ELASTIC_PLASTIC

This is Material Type 157. This material model is a combination of the anisotropic elastic material model (*MAT_002) and the anisotropic plastic material model (*MAT_103_P). Brittle orthotropic failure based on a phenomenological Tsai-Wu or Tsai-Hill criterion can be defined. This material is available for solid, shell, and thick shell (formulations 1, 2, and 6) elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | SIGY | LCSS | QR1 | CR1 | QR2 | CR2 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| C11 | C12 | C13 | C14 | C15 | C16 | C22 | C23 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C24 | C25 | C26 | C33 | C34 | C35 | C36 | C44 |
|---|---|---|---|---|---|---|---|

**Card 4a.** Include this card if the element type is shells or thick shells.

| C45 | C46 | C55 | C56 | C66 | R00 | R45 | R90 |
|---|---|---|---|---|---|---|---|

**Card 4b.** Include this card if the element type is solids.

| C45 | C46 | C55 | C56 | C66 | F | G | H |
|---|---|---|---|---|---|---|---|

**Card 5a.** Include this card if the element type is shells or thick shells.

| S11 | S22 | S33 | S12 | AOPT | VP | | |
|---|---|---|---|---|---|---|---|

**Card 5b.** Include this card if the element type is solids.

| L | M | N | | AOPT | VP | | MACF |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | ID3UPD | EXTRA |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | IHIS |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if EXTRA > 0.

| XT | XC | YT | YC | SXY | FF12 | | NCFAIL |
|---|---|---|---|---|---|---|---|

**Card 9.** Include this card if EXTRA > 0.

| ZT | ZC | SYZ | SZX | FF23 | FF31 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | SIGY | LCSS | QR1 | CR1 | QR2 | CR2 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| SIGY | Initial yield stress |
| LCSS | Load curve ID or Table ID:<br><br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining effective stress as a function of effective plastic strain. If defined, QR1, CR1, QR2, and CR2 are ignored.<br><br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate. See Figure M24-1. When the strain rate falls below the minimum value, the load curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the load curve for the highest value of strain rate is used.<br><br>**Logarithmically Defined Tables.** An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| | when the lowest strain rate and highest strain rate differ by several orders of magnitude. Logarithmic interpolation has some additional computational cost. |
| QR1 | Isotropic hardening parameter |
| CR1 | Isotropic hardening parameter |
| QR2 | Isotropic hardening parameter |
| CR2 | Isotropic hardening parameter |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C11 | C12 | C13 | C14 | C15 | C16 | C22 | C23 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C24 | C25 | C26 | C33 | C34 | C35 | C36 | C44 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C*ij* | The $ij^{th}$ term in the 6 × 6 anisotropic constitutive matrix. Note that 1 corresponds to the *a* material direction, 2 to the *b* material direction, and 3 to the *c* material direction. |

**Anisotropic Constants Card for Shells.** Include this card if the element type is shells or thick shells.

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C45 | C46 | C55 | C56 | C66 | R00 | R45 | R90 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C*ij* | The $ij^{th}$ term in the 6 × 6 anisotropic constitutive matrix |
| R00 | $R_{00}$ for shell (default = 1.0) |
| R45 | $R_{45}$ for shell (default = 1.0) |
| R90 | $R_{90}$ for shell (default = 1.0) |

**Anisotropic Constants Card for Solids.** Include this card if the element type is solids.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C45 | C46 | C55 | C56 | C66 | F | G | H |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C*ij* | The $ij^{th}$ term in the 6 × 6 anisotropic constitutive matrix |
| F | $F$ for solid (default = 1/2) |
| G | $G$ for solid (default = 1/2) |
| H | $H$ for solid (default = 1/2) |

**Shell Yield Stress Card.** Include this card if the element type is shells or thick shells.

| Card 5a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S11 | S22 | S33 | S12 | AOPT | VP | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| S11 | Yield stress in local-$x$ direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0. |
| S22 | Yield stress in local-$y$ direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0. |

| VARIABLE | DESCRIPTION |
|---|---|
| S33 | Yield stress in local-$z$ direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0. |
| S12 | Yield stress in local $xy$-direction (shells only). This input is ignored when R00, R45, and R90 are greater than 0. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |

**Anisotropic Constants Card for Solids.** Include this card if the element type is solids.

| Card 5b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L | M | N | | AOPT | VP | | MACF |
| Type | F | F | F | | F | F | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| L | *L* for solid (default = 3/2) |
| M | *M* for solid (default = 3/2) |
| N | *N* for solid (default = 3/2) |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| VARIABLE | DESCRIPTION |
|---|---|
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | ID3UPD | EXTRA |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| EXTRA | Flag to input further data to include failure with Cards 8 and 9:<br>EQ.1.0: Tsai-Wu (stress-based) parameters. See Remark 3.<br>EQ.2.0: Tsai-Hill (stress-based) parameters See Remark 4.<br>EQ.3.0: Tsai-Wu (strain-based) parameters. See Remark 5. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4.0: Tsai-Hill (strain-based) parameters. See Remark 6. |
| ID3UPD | Flag for transverse through-thickness strain update (thin shells only):<br>EQ.0.0: Reflects *R*-values by splitting the strain tensor into elastic and plastic components<br>EQ.1.0: Elastic update using total strain tensor |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | IHIS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA and *ELEMENT_SOLID_ORTHO. |
| IHIS | Flag for material properties initialization:<br>EQ.0: Material properties defined in Cards 1 - 5 are used<br>GE.1: Use *INITIAL_STRESS_SOLID/SHELL to initialize material properties on an element-by-element basis for solid or shell elements, respectively (see Remarks 1 and 2 below). |

Two additional cards for EXTRA > 0.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XT | XC | YT | YC | SXY | FF12 | | NCFAIL |
| Type | F | F | F | F | F | F | | I |
| Default | $10^{20}$ | $10^{20}$ | $10^{20}$ | $10^{20}$ | $10^{20}$ | 0.0 | | 10 |

| VARIABLE | DESCRIPTION |
|---|---|
| XT | Longitudinal tensile strength, *a*-axis, for EXTRA = 1 and 2 or longitudinal tensile strain at failure, *a*-axis, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XT) which defines either the longitudinal tensile strength (EXTRA = 1 and 2) or the longitudinal tensile strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. |
| XC | Longitudinal compressive strength, *a*-axis, for EXTRA = 1 and 2 or longitudinal compressive strain at failure, *a*-axis, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XC) which defines either the longitudinal compressive strength (EXTRA = 1 and 2) or the longitudinal compressive strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.<br>Longitudinal compressive strengths and longitudinal compressive strains at failure should be positive. |
| YT | Transverse tensile strength, *b*-axis, for EXTRA = 1 and 2 or transverse tensile strain at failure, *b*-axis, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-YT) which defines either the transverse tensile strength (EXTRA = 1 and 2) or the |

| VARIABLE | DESCRIPTION |
|---|---|
| | transverse tensile strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. |
| YC | Transverse compressive strength, *b*-axis, for EXTRA = 1 and 2 or transverse compressive strain at failure, *b*-axis, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-YC) which defines either the transverse compressive strength (EXTRA = 1 and 2) or the transverse compressive strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate.<br>Transverse compressive strengths and transverse compressive strains at failure should be positive. |
| SXY | Shear strength, *ab*-plane, for EXTRA = 1 and 2 or shear strain at failure, *ab*-plane, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SXY) which defines the shear strength (EXTRA = 1 and 2) or the shear strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. |
| FF12 | Scale factor between -1 and +1 for interaction term F12. See Remark 3. It applies for EXTRA = 1 and 3. |
| NCFAIL | Number of time steps to reduce stresses until element deletion. |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ZT | ZC | SYZ | SZX | FF23 | FF31 | | |
| Type | F | F | F | F | F | F | | |
| Default | $10^{20}$ | $10^{20}$ | $10^{20}$ | $10^{20}$ | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ZT | This field applies to *solid elements only*. Transverse tensile strength, *c*-axis, for EXTRA = 1 and 2 or transverse tensile strain at failure, *c*-axis, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-ZT) which defines either the transverse tensile strength (EXTRA = 1 and 2) or the transverse tensile strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, all strain rate values are assumed to be given as a natural logarithm of the strain rate. |
| ZC | This field applies to *solid elements only*. Transverse compressive strength, *c*-axis, for EXTRA = 1 and 2 or transverse compressive strain at failure, *c*-axis, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-ZC) which defines either the transverse compressive strength (EXTRA = 1 and 2) or the transverse compressive strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, all strain rate values are assumed to be given as a natural logarithm of the strain rate.<br>Transverse compressive strengths and transverse compressive strains at failure should be positive. |
| SYZ | This field applies to *solid elements only*. Shear strength, *bc*-plane, for EXTRA = 1 and 2 or shear strain at failure, *bc*-plane, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SYZ) which defines the shear strength |

| VARIABLE | DESCRIPTION |
|---|---|
| | (EXTRA = 1 and 2) or the shear strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. |
| SZX | This field applies to *solid elements only*. Shear strength, *ca*-plane, for EXTRA = 1 and 2 or shear strain at failure, *ca*-plane, for EXTRA = 3 and 4:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SZX) which defines the shear strength (EXTRA = 1 and 2) or the shear strain at failure (EXTRA = 3 and 4) as a function of strain rate. If the first strain rate value in the curve is negative, it is assumed that all strain rate values are given as a natural logarithm of the strain rate. |
| FF23 | Scale factor between -1 and +1 for interaction term F23. See Remark 3. This field applies to *solid elements only*. It applies for EXTRA = 1 and 3. |
| FF31 | Scale factor between -1 and +1 for interaction term F31. See Remark 3. This field applies to *solid elements only*. It applies for EXTRA = 1 and 3. |

**Remarks:**

1. **Description of IHIS (Solid Elements).** Several of this material's parameters may be overwritten on an element-by-element basis through history variables using the *INITIAL_STRESS_SOLID keyword. Bitwise (binary) expansion of IHIS determines which material properties are to be read:

$$\text{IHIS} = a_4 \times 16 + a_3 \times 8 + a_2 \times 4 + a_1 \times 2 + a_0,$$

where each $a_i$ is a binary flag set to either 1 or 0. The $a_i$ are interpreted according to the following table.

| Flag | Description | Variables | # |
|---|---|---|---|
| $a_0$ | Material directions | $q_{11}, q_{12}, q_{13}, q_{31}, q_{32}, q_{33}$ | 6 |
| $a_1$ | Anisotropic stiffness | C*ij* | 21 |
| $a_2$ | Anisotropic constants | F, G, H, L, M, N | 6 |

| Flag | Description | Variables | # |
|---|---|---|---|
| $a_3$ | Stress-strain curve | LCSS | 1 |
| $a_4$ | Strength limits | XT, XC, YT, YC, ZT, ZC, SXY, SYZ, SZX | 9 |

The NHISV field on *INITIAL_STRESS_SOLID must be set equal to the sum of the number of variables to be read in, which depends on IHIS (and the $a_i$):

$$\text{NHISV} = 6a_0 + 21a_1 + 6a_2 + a_3 + 9a_4.$$

Then, in the following order, *INITIAL_STRESS_SOLID processes the history variables, HISV*i*, as:

a) 6 material direction parameters when $a_0 = 1$

b) 21 anisotropic stiffness parameters when $a_1 = 1$

c) 6 anisotropic constants when $a_2 = 1$

d) 1 parameter when $a_3 = 1$

e) 9 strength parameters when $a_4 = 1$

The $q_{ij}$ terms are the first and third rows of a rotation matrix for the rotation from a co-rotational element's system and the *a*-*b*-*c* material directions. The $c_{ij}$ terms are the upper triangular terms of the symmetric stiffness matrix, $c_{11}$, $c_{12}$, $c_{13}$, $c_{14}$, $c_{15}$, $c_{16}$, $c_{22}$, $c_{23}$, $c_{24}$, $c_{25}$, $c_{26}$, $c_{33}$, $c_{34}$, $c_{35}$, $c_{36}$, $c_{44}$, $c_{45}$, $c_{46}$, $c_{55}$, $c_{56}$, and $c_{66}$.

2. **Description of IHIS (Shell Elements).** Several of this material's parameters may be overwritten on an element-by-element basis through history variables using the *INITIAL_STRESS_SHELL keyword. Bitwise (binary) expansion of IHIS determines which material properties are to be read:

$$\text{IHIS} = a_4 \times 16 + a_3 \times 8 + a_2 \times 4 + a_1 \times 2 + a_0,$$

where each $a_i$ is a binary flag set to either 1 or 0. The $a_i$ are interpreted according to the following table.

| Flag | Description | Variables | # |
|---|---|---|---|
| $a_0$ | Material directions | $q_1$, $q_2$ | 2 |
| $a_1$ | Anisotropic stiffness | C*ij* | 21 |
| $a_2$ | Anisotropic constants | $r_{00}$, $r_{45}$, $r_{90}$ | 3 |
| $a_3$ | Stress-strain curve | LCSS | 1 |
| $a_4$ | Strength limits | XT, XC, YT, YC, SXY | 5 |

The NHISV field on *INITIAL_STRESS_SHELL must be set equal to the sum of the number of variables to be read in, which depends on IHIS (and the $a_i$):

$$\text{NHISV} = 2a_0 + 21a_1 + 3a_2 + a_3 + 5a_4.$$

Then, in the following order, *INITIAL_STRESS_SHELL processes the history variables, HISV*i*, as:

a) 2 material direction parameters when $a_0 = 1$

b) 21 anisotropic stiffness parameters when $a_1 = 1$

c) 3 anisotropic constants when $a_2 = 1$

d) 1 parameter when $a_3 = 1$

e) 5 strength parameters when $a_4 = 1$

The $q_i$ terms are the material direction cosine and sine for the rotation from a co-rotational element's system to the *a-b-c* material directions. The $c_{ij}$ terms are the upper triangular terms of the symmetric stiffness matrix, $c_{11}$, $c_{12}$, $c_{13}$, $c_{14}$, $c_{15}$, $c_{16}$, $c_{22}$, $c_{23}$, $c_{24}$, $c_{25}$, $c_{26}$, $c_{33}$, $c_{34}$, $c_{35}$, $c_{36}$, $c_{44}$, $c_{45}$, $c_{46}$, $c_{55}$, $c_{56}$, and $c_{66}$.

3. **Tsai-Wu failure criterion (EXTRA = 1, stress-based).** EXTRA = 1 with the definition of corresponding parameters on Cards 8 and 9 invokes brittle failure with different strengths in tension and compression in all main material directions. The model used is the phenomenological Tsai-Wu failure criterion which requires that

$$\left(\frac{1}{\text{XT}} - \frac{1}{\text{XC}}\right)\sigma_{aa} + \left(\frac{1}{\text{YT}} - \frac{1}{\text{YC}}\right)\sigma_{bb} + \left(\frac{1}{\text{ZT}} - \frac{1}{\text{ZC}}\right)\sigma_{cc} + \frac{1}{\text{XT} \times \text{XC}}\sigma_{aa}^2 + \frac{1}{\text{YT} \times \text{YC}}\sigma_{bb}^2 + \frac{1}{\text{ZT} \times \text{ZC}}\sigma_{cc}^2 + \frac{1}{\text{SXY}^2}\sigma_{ab}^2 + \frac{1}{\text{SYZ}^2}\sigma_{bc}^2 + \frac{1}{\text{SZX}^2}\sigma_{ca}^2 + 2 \times F_{12} \times \sigma_{aa}\sigma_{bb} + 2 \times F_{23} \times \sigma_{bb}\sigma_{cc} + 2 \times F_{31} \times \sigma_{cc}\sigma_{aa} < 1$$

for the 3-dimensional case (solid elements) with three planes of symmetry with respect to the material coordinate system. The interaction terms $F_{12}$, $F_{23}$, and $F_{31}$ are given by

$$F_{12} = \text{FF12} \times \sqrt{\frac{1}{\text{XT} \times \text{XC} \times \text{YT} \times \text{YC}}}$$

$$F_{23} = \text{FF23} \times \sqrt{\frac{1}{\text{YT} \times \text{YC} \times \text{ZT} \times \text{ZC}}}$$

$$F_{31} = \text{FF31} \times \sqrt{\frac{1}{\text{ZT} \times \text{ZC} \times \text{XT} \times \text{XC}}}$$

For the 2-dimensional case of plane stress (shell elements), this expression reduces to:

$$\left(\frac{1}{\mathrm{XT}}-\frac{1}{\mathrm{XC}}\right)\sigma_{aa}+\left(\frac{1}{\mathrm{YT}}-\frac{1}{\mathrm{YC}}\right)\sigma_{bb}+\frac{1}{\mathrm{XT}\times\mathrm{XC}}\sigma_{aa}^2+\frac{1}{\mathrm{YT}\times\mathrm{YC}}\sigma_{bb}^2 + \frac{1}{\mathrm{SXY}^2}\sigma_{ab}^2+2\times F_{12}\times\sigma_{aa}\sigma_{bb}<1$$

If these conditions are violated, then the stress tensor reduces to zero over NCFAIL time steps, and then the element erodes. A small value for NCFAIL (< 50) is recommended to avoid unphysical behavior; the default is 10.

4. **Tsai-Hill failure criterion (EXTRA = 2, stress-based).** EXTRA = 2 with the definition of corresponding parameters on Cards 8 and 9 (FF12, FF23, and FF31 are not used in this model) invokes brittle failure with different strengths in tension and compression in all main material directions. The model is based on the HILL criterion which can be written as

$$(G+H)\sigma_{aa}^2+(F+H)\sigma_{bb}^2+(F+G)\sigma_{cc}^2-2H\sigma_{aa}\sigma_{bb} -2F\sigma_{bb}\sigma_{cc}-2G\sigma_{cc}\sigma_{aa}+2N\sigma_{ab}^2+2L\sigma_{bc}^2+2M\sigma_{ca}^2<1$$

for the 3-dimensional case. The constants $H$, $F$, $G$, $N$, $L$, and $M$ can be expressed in terms of the strength limits (which then becomes the TSAI-HILL criterion) as

$$G+H=\frac{1}{X_i^2}\quad 2N=\frac{1}{\mathrm{SXY}^2}\quad H=0.5\times\left(\frac{1}{X_i^2}+\frac{1}{Y_i^2}-\frac{1}{Z_i^2}\right)$$
$$F+H=\frac{1}{Y_i^2}\quad 2L=\frac{1}{\mathrm{SYZ}^2}\quad F=0.5\times\left(\frac{1}{Y_i^2}+\frac{1}{Z_i^2}-\frac{1}{X_i^2}\right)$$
$$F+G=\frac{1}{Z_i^2}\quad 2M=\frac{1}{\mathrm{SZX}^2}\quad G=0.5\times\left(\frac{1}{X_i^2}+\frac{1}{Z_i^2}-\frac{1}{Y_i^2}\right)$$

where the current stress state defines whether the compressive or the tensile strength limit will enter into the equation:

$$X_i=\begin{cases}\mathrm{XT} & \text{if}\quad \sigma_{aa}>0\\ \mathrm{XC} & \text{if}\quad \sigma_{aa}<0\end{cases}$$
$$Y_i=\begin{cases}\mathrm{YT} & \text{if}\quad \sigma_{bb}>0\\ \mathrm{YC} & \text{if}\quad \sigma_{bb}<0\end{cases}$$
$$Z_i=\begin{cases}\mathrm{ZT} & \text{if}\quad \sigma_{cc}>0\\ \mathrm{ZC} & \text{if}\quad \sigma_{cc}<0\end{cases}$$

For the 2-dimensional case of plane stress (shell elements) the TSAI-HILL criterion reduces to:

$$(G+H)\sigma_{aa}^2+(F+H)\sigma_{bb}^2-2H\sigma_{aa}\sigma_{bb}+2N\sigma_{ab}^2<1$$

with

$$G + H = \frac{1}{X_i^2}$$

$$F + H = \frac{1}{Y_i^2}$$

$$H = 0.5 \times \frac{1}{X_i^2}$$

$$2N = \frac{1}{SXY^2}$$

If these conditions are violated, then the stress tensor will be reduced to zero over NCFAIL time steps and the element will be eroded. A small value for NCFAIL (< 50) is recommended to avoid unphysical behavior; the default is 10.

5. **Tsai-Wu failure criterion (EXTRA = 3, strain-based).** EXTRA = 3 invokes brittle failure with different strain limits in tension and compression in all main material directions. The failure criterion is like that of EXTRA = 1 as described in Remark 3, but instead of using the stress tensor, the criterion is evaluated based on the current strain tensor. Consequently, the material parameters XT, XC, YT, … give the limit strains at failure in the various directions.

6. **Tsai-Hill failure criterion (EXTRA = 4, strain-based).** EXTRA = 4 invokes brittle failure with different strain limits in tension and compression in all main material directions. The failure criterion is like that of EXTRA = 2 as described in Remark 4, but instead of using the stress tensor, the criterion is evaluated based on the current strain tensor. Consequently, the material parameters XT, XC, YT, … give the limit strains at failure in the various directions.

## *MAT_RATE_SENSITIVE_COMPOSITE_FABRIC

This is Material Type 158. Depending on the type of failure surface, this model may be used to model rate sensitive composite materials with unidirectional layers, complete laminates, and woven fabrics. A viscous stress tensor, based on an isotropic Maxwell model with up to six terms in the Prony series expansion, is superimposed on the rate independent stress tensor of the composite fabric. The viscous stress tensor approach should work reasonably well if the stress increases due to rate affects are up to 15% of the total stress. This model is implemented for both shell and thick shell elements. The viscous stress tensor is effective at eliminating spurious stress oscillations.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | (EC) | PRBA | TAU1 | GAMMA1 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | TSIZE | ERODS | SOFT | FS | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| E11C | E11T | E22C | E22T | GMS | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| XC | XT | YC | YT | SC | | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| K | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 9.** Include up to 6 of this card. This input ends with the next keyword ("*") card.

| G*i* | BETA*i* | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | (EC) | PRBA | TAU1 | GAMMA1 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus - longitudinal direction |
| EB | $E_b$, Young's modulus - transverse direction |
| (EC) | $E_c$, Young's modulus - normal direction (not used) |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| TAU1 | $\tau_1$, stress limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve. The values $\tau_1$ and $\gamma_1$ are used to define a curve of shear stress as a function of shear strain. These values are input if FS, defined in Card 3, is set to -1. |
| GAMMA1 | $\gamma_1$, strain limit of the first slightly nonlinear part of the shear stress as a function of shear strain curve |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | SLIMT1 | SLIMC1 | SLIMT2 | SLIMC2 | SLIMS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus *ab* |
| GBC | $G_{bc}$, shear modulus *bc* |
| GCA | $G_{ca}$, shear modulus *ca* |
| SLIMT1 | Factor to determine the minimum stress limit after stress maximum (fiber tension) |
| SLIMC1 | Factor to determine the minimum stress limit after stress maximum (fiber compression) |
| SLIMT2 | Factor to determine the minimum stress limit after stress maximum (matrix tension) |
| SLIMC2 | Factor to determine the minimum stress limit after stress maximum (matrix compression) |
| SLIMS | Factor to determine the minimum stress limit after stress maximum (shear) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | TSIZE | ERODS | SOFT | FS | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (BETA) from a line in the plane of the element |

| VARIABLE | DESCRIPTION |
|---|---|
| | defined by the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR. |
| TSIZE | Time step for automatic element deletion |
| ERODS | Maximum effective strain for element layer failure. A value of unity would equal 100% strain. |
| SOFT | Softening reduction factor for strength in the crashfront. |
| FS | Failure surface type: |
| | EQ.1.0: Smooth failure surface with a quadratic criterion for both the fiber (*a*) and transverse (*b*) directions. This option can be used with complete laminates and fabrics. |
| | EQ.0.0: Smooth failure surface in the transverse (*b*) direction with a limiting value in the fiber (*a*) direction. This model is appropriate for unidirectional (UD) layered composites only. |
| | EQ.-1: Faceted failure surface. When the strength values are reached, then damage evolves in tension and compression for both the fiber and transverse direction. Shear behavior is also considered. This option can be used with complete laminates and fabrics. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | PRCA | PRCB |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* (default = PRBA) |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* (default = PRBA) |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E11C | E11T | E22C | E22T | GMS | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E11C | Strain at longitudinal compressive strength, *a*-axis |
| E11T | Strain at longitudinal tensile strength, *a*-axis |
| E22C | Strain at transverse compressive strength, *b*-axis |
| E22T | Strain at transverse tensile strength, *b*-axis |
| GMS | Strain at shear strength, *ab*-plane |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SC | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength; see Remark 2 of *MAT_058. |
| XT | Longitudinal tensile strength; see Remark 2 of *MAT_058. |
| YC | Transverse compressive strength, *b*-axis, see Remark 2 of *MAT_-058. |
| YT | Transverse tensile strength, *b*-axis; see Remark 2 of *MAT_058. |
| SC | Shear strength, *ab*-plane; see Remark 2 of *MAT_058. |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Optional bulk modulus for the viscoelastic material. If nonzero, a Kelvin type behavior will be obtained. Generally, K is set to zero. |

**Viscoelastic Cards**. Up to 6 cards may be input. The next keyword ("*") card terminates this input.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional shear decay constant for the $i^{th}$ term |

**Remarks:**

1. **Related material.** See the Remarks for material type 58, *MAT_LAMINATED_COMPOSITE_FABRIC, for the treatment of the composite material.

2. **Rate effects.** Rate effects are taken into account through a Maxwell model using linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \ ,$$

where $g_{ijkl(t-\tau)}$ is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional. Since we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t}$$

We characterize this in the input by the shear moduli, $G_i$, and decay constants, $\beta_i$. An arbitrary number of terms, not exceeding 6, may be used when applying the viscoelastic model. The composite failure is not directly affected by the presence of the viscous stress tensor.

***MAT_CSCM**_*{OPTION}*

This is Material Type 159. This material model is a smooth or continuous surface cap model and is available for solid elements. This model permits inputting your own material properties (<BLANK> option) or requesting default material properties for normal strength concrete (CONCRETE). See [Murray 2007] for a more complete model description.

Available options include:

<BLANK>

CONCRETE

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | NPLOT | INCRE | IRATE | ERODE | RECOV | ITRETRC |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| PRED | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if the CONCRETE keyword option is used.

| FPC | DAGG | UNITS | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if the keyword option is unused (<OPTION>).

| G | K | ALPHA | THETA | LAMBDA | BETA | NH | CH |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card if the keyword option is unused (<OPTION>).

| ALPHA1 | THETA1 | LAMBDA1 | BETA1 | ALPHA2 | THETA2 | LAMBDA2 | BETA2 |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card if the keyword option is unused (<OPTION>).

| R | X0 | W | D1 | D2 | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** Include this card if the keyword option is unused (<OPTION>).

| B | GFC | D | GFT | GFS | PWRC | PWRT | PMOD |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if the keyword option is unused (<OPTION>).

| ETA0C | NC | ETA0T | NT | OVERC | OVERT | SRATE | REPOW |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is optional.

| SSCC | PPSH | PNCF | COEC | CLCH | CRUL | UNIT | FBCR |
|---|---|---|---|---|---|---|---|

**Card 10.** Include this card if Card 9 is included.

| NUMFIP | VOLFRAC | FLAGCL | FLAGCC | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | NPLOT | INCRE | IRATE | ERODE | RECOV | ITRETRC |
| Type | A | F | I | F | I | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| NPLOT | Controls what is written as component 7 to the d3plot database. LS-PrePost always labels this component as effective plastic strain:<br>EQ.1: Maximum of brittle and ductile damage (default)<br>EQ.2: Maximum of brittle and ductile damage, with recovery of brittle damage<br>EQ.3: Brittle damage<br>EQ.4: Ductile damage<br>EQ.5: $\kappa$ (intersection of cap with shear surface)<br>EQ.6: $X_0$ (intersection of cap with pressure axis)<br>EQ.7: $\varepsilon_v^p$ (plastic volume strain). |
| INCRE | Maximum strain increment for subincrementation. If left blank, a default value is set during initialization based upon the shear strength and stiffness. |
| IRATE | Rate effects options: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0: Rate effects model turned off (default).<br>EQ.1: Rate effects model turned on. |
| ERODE | Elements erode when damage exceeds 0.99 and the maximum principal strain exceeds ERODE − 1.0. For erosion that is independent of strain, set ERODE equal to 1.0. Erosion does not occur if ERODE is less than 1.0. |
| RECOV | The modulus is recovered in compression when RECOV is equal to 0.0 (default). The modulus remains at the brittle damage level when RECOV is equal to 1.0. Partial recovery is modeled for values of RECOV between 0.0 and 1.0. Two options are available:<br>1. If RECOV is a value between 0.0 and 1.0, then recovery is based upon the sign of the pressure invariant only.<br>2. If RECOV is a value between 10.0 and 11.0, then recovery is based upon the sign of both the pressure and volumetric strain. In this case, RECOV = RECOV − 10, and a flag is set to request the volumetric strain check. |
| IRETRC | Cap retraction option:<br>EQ.0: Cap does not retract (default).<br>EQ.1: Cap retracts. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PRED | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PRED | Pre-existing damage ($0 \le PRED < 1$). If left blank, the default is zero (no pre-existing damage). |

**Concrete Properties Card.** Include this card if the CONCRETE keyword option is used.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FPC | DAGG | UNITS | | | | | |
| Type | F | F | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FPC | Unconfined compression strength, $f_c'$. Material parameters are internally fit to data for unconfined compression strengths between about 20 and 58 MPa (2,901 to 8,412 psi), with emphasis on the mid-range between 28 and 48 MPa (4,061 and 6,962 psi). If left blank, the default for FPC is 30 MPa. |
| DAGG | Maximum aggregate size, $D_{agg}$. Softening is fit to data for aggregate sizes between 8 and 32 mm (0.3 and 1.3 inches). If left blank, the default for DAGG is 19 mm (3/4 inch). |
| UNITS | Units options:<br>EQ.0: GPa, mm, msec, kg/mm$^3$, kN<br>EQ.1: MPa, mm, msec, g/mm$^3$, N<br>EQ.2: MPa, mm, sec, Mg/mm$^3$, N<br>EQ.3: Psi, inch, sec, lbf-s$^2$/in$^4$, lbf<br>EQ.4: Pa, m, sec, kg/m$^3$, N |

**User Defined Properties Card.** Include this card if the keyword option is left blank.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G | K | ALPHA | THETA | LAMBDA | BETA | NH | CH |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| G | Shear modulus |
| K | Bulk modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Tri-axial compression surface constant term, $\alpha$ |
| THETA | Tri-axial compression surface linear term, $\theta$ |
| LAMBDA | Tri-axial compression surface nonlinear term, $\lambda$ |
| BETA | Tri-axial compression surface exponent, $\beta$ |
| NH | Hardening initiation, $N_H$ |
| CH | Hardening rate, $C_H$ |

**User-Defined Properties Card.** Include this card if the keyword option is left blank.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | THETA1 | LAMBDA1 | BETA1 | ALPHA2 | THETA2 | LAMBDA2 | BETA2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA1 | Torsion surface constant term, $\alpha_1$ |
| THETA1 | Torsion surface linear term, $\theta_1$ |
| LAMBDA1 | Torsion surface nonlinear term, $\lambda_1$ |
| BETA1 | Torsion surface exponent, $\beta_1$ |
| ALPHA2 | Tri-axial extension surface constant term, $\alpha_2$ |
| THETA2 | Tri-axial extension surface linear term, $\theta_2$ |
| LAMBDA2 | Tri-axial extension surface nonlinear term, $\lambda_2$ |
| BETA2 | Tri-axial extension surface exponent, $\beta_2$ |

**User-Defined Properties Card.** Include this card if the keyword option is left blank.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R | X0 | W | D1 | D2 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| R | Cap aspect ratio, $R$ |
| X0 | Cap initial location, $X_0$ |
| W | Maximum plastic volume compaction, $W$ |
| D1 | Linear shape parameter, $D_1$ |
| D2 | Quadratic shape parameter, $D_2$ |

**User-Defined Properties Card.** Include this card if the keyword option is left blank.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B | GFC | D | GFT | GFS | PWRC | PWRT | PMOD |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| B | Ductile shape softening parameter, $B$ |
| GFC | Fracture energy in uniaxial stress, $G_{fc}$ |
| D | Brittle shape softening parameter, $D$ |
| GFT | Fracture energy in uniaxial tension, $G_{ft}$ |
| GFS | Fracture energy in pure shear stress, $G_{fs}$ |
| PWRC | Shear-to-compression transition parameter |
| PWRT | Shear-to-tension transition parameter |
| PMOD | Modify moderate pressure softening parameter |

**User-Defined Properties Card.** Include this card if the keyword option is left blank.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ETA0C | NC | ETA0T | NT | OVERC | OVERT | SRATE | REPOW |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ETA0C | Rate effects parameter for uniaxial compressive stress, $\eta_{0c}$ |
| NC | Rate effects power for uniaxial compressive stress, $N_C$ |
| ETA0T | Rate effects parameter for uniaxial tensile stress, $\eta_{0t}$ |
| NT | Rate effects power for uniaxial tensile stress, $N_T$ |
| OVERC | Maximum overstress allowed in compression |
| OVERT | Maximum overstress allowed in tension |
| SRATE | Ratio of effective shear stress to tensile stress fluidity parameters |
| REPOW | Power which increases fracture energy with rate effects |

**Optional Failure Model Card.** This card is optional.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SSCC | PPSH | PNCF | COEC | CLCH | CRUL | UNIT | FBCR |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 1.0 | 0.75 | 0.4 | 0.2 | optional | 1.0 | 1.16 |

| VARIABLE | DESCRIPTION |
|---|---|
| SSCC | Cutoff value for pure shear strain. See Remark 12. |
| PPSH | Safety coefficient of the cutoff strain criterion |
| PNCF | Safety coefficient of the confinement criterion |

| VARIABLE | DESCRIPTION |
|---|---|
| COEC | Coefficient of the confinement ratio relaxation |
| CLCH | Coefficient of the characteristic length in meters |
| CRUL | Upper limit to the confinement ratio. If CRUL is not defined, it is internally determined using FBCR. See Remark 12. |
| UNIT | Model length unit in meters |
| FBCR | Ratio of equibiaxial to uniaxial compression strength. See Remark 12. |

**Optional Failure Model Card.** Include this card if Card 9 is included.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NUMFIP | | FLAGCL | FLAGCC | | | | |
| Type | F | | F | F | | | | |
| Default | 1.0 | | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| NUMFIP | Number of failed integration points prior to element deletion |
| FLAGCL | Flag controlling the computation of the characteristic length:<br>EQ.0.0: The characteristic length is computed as the cube root of the initial element volume.<br>EQ.1.0: The characteristic length is computed as the minimum over time of the maximum distance between any 2 nodes of the element. |
| FLAGCC | Flag controlling checking the shear strain criterion:<br>EQ.0.0: The shear strain criterion is checked every time step until it is satisfied. Once satisfied, the integration point is flagged and remains flagged for the rest of the simulation.<br>EQ.1.0: The shear strain criterion is checked every time step, even if the integration point was previously flagged. If the criterion is no longer satisfied, the integration point |

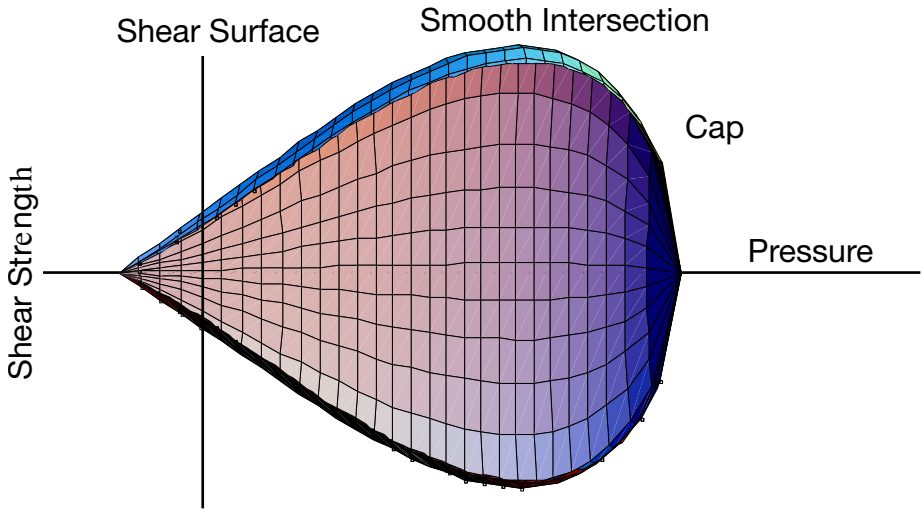



**Figure M159-1.** General shape of concrete model yield surface in two dimensions.

| VARIABLE | DESCRIPTION |
|---|---|
| | becomes unflagged until it is satisfied again. |

**Remarks:**

1. **Model overview.** This is a cap model with a smooth intersection between the shear yield surface and hardening cap, as shown in Figure M159-1. The initial damage surface coincides with the yield surface. Rate effects are modeled with viscoplasticity. For a complete theoretical description, with references and example problems see [Murray 2007] and [Murray, Abu-Odeh and Bligh 2007].

2. **Stress invariants.** The yield surface is formulated in terms of three stress invariants: $J_1$ which is the first invariant of the stress tensor, $J_2'$ which is the second invariant of the deviatoric stress tensor, and $J_3'$ which is the third invariant of the deviatoric stress tensor. The invariants are defined in terms of the deviatoric stress tensor, $S_{ij}$, and pressure, $P$, as follows:

$$J_1 = 3P$$
$$J_2' = \frac{1}{2} S_{ij} S_{ij}$$
$$J_3' = \frac{1}{3} S_{ij} S_{ik} S_{ki}$$

3. **Plasticity surface.** The three invariant yield function is based on these three invariants, and the cap hardening parameter, $\kappa$, as follows:

$$f(J_1, J_2', J_3', \kappa) = J_2' - \Re^2 F_f^2 F_c \, .$$

Here $F_f$ is the shear failure surface, $F_c$ is the hardening cap, and $\mathfrak{R}$ is the Rubin three-invariant reduction factor. The cap hardening parameter $\kappa$ is the value of the pressure invariant at the intersection of the cap and shear surfaces.

Trial elastic stress invariants are temporarily updated using the trial elastic stress tensor, $\sigma^T$. These are denoted $J_1^T$, $J_2'^T$, and $J_3'^T$. Elastic stress states are modeled when $f(J_1^T, J_2'^T, J_3'^T, \kappa^T) \le 0$. Elastic-plastic stress states are modeled when $f(J_1^T, J_2'^T, J_3'^T, \kappa^T) \le 0$. In this case, the plasticity algorithm returns the stress state to the yield surface such that $f(J_1^P, J_2'^P, J_3'^P, \kappa^P) = 0$. This is accomplished by enforcing the plastic consistency condition with associated flow.

4. **Shear failure surface.** The strength of concrete is modeled by the shear surface in the tensile and low confining pressure regimes:

$$F_f(J_1) = \alpha - \lambda \exp(-\beta J_1) + \theta J_1 \quad .$$

Here the values of $\alpha$, $\beta$, $\lambda$, and $\theta$ are selected by fitting the model surface to strength measurements from triaxial compression (TXC) tests conducted on plain concrete cylinders.

5. **Rubin scaling function.** Concrete fails at lower values of $\sqrt{3J_2'}$ (principal stress difference) for triaxial extension (TXE) and torsion (TOR) tests than it does for TXC tests conducted at the same pressure. The Rubin scaling function, $\mathfrak{R}$, determines the strength of concrete for any state of stress relative to the strength for TXC, using $\mathfrak{R}F_f$. Strength in torsion is modeled as $Q_1 F_f$. Strength in TXE is modeled as $Q_2 F_f$, where:

$$Q_1 = \alpha_1 - \lambda_1 \exp(-\beta_1 J_1) + \theta_1 J_1$$
$$Q_2 = \alpha_2 - \lambda_2 \exp(-\beta_2 J_1) + \theta_2 J_1$$

6. **Cap hardening surface.** The strength of concrete is modeled by a combination of the cap and shear surfaces in the low to high confining pressure regimes. The cap is used to model plastic volume change related to pore collapse (although the pores are not explicitly modeled). The isotropic hardening cap is a two-part function that is either unity or an ellipse:

$$F_c(J_1, \kappa) = 1 - \frac{[J_1 - L(\kappa)][\,|J_1 - L(\kappa)| + J_1 - L(\kappa)\,]}{2\,[X(\kappa) - L(\kappa)]^2} \quad ,$$

where $L(\kappa)$ is defined as:

$$L(\kappa) = \begin{cases} \kappa & \text{if} \quad \kappa > \kappa_0 \\ \kappa_0 & \text{otherwise} \end{cases}$$

The equation for $F_c$ is equal to unity for $J_1 \le L(\kappa)$. It describes the ellipse for $J_1 > L(\kappa)$. The intersection of the shear surface and the cap is at $J_1 = \kappa$. $\kappa_0$ is the value of $J_1$ at the *initial* intersection of the cap and shear surfaces before

hardening is engaged (before the cap moves). The equation for $L(\kappa)$ restrains the cap from retracting past its initial location at $\kappa_0$.

The intersection of the cap with the $J_1$ axis is at $J_1 = X(\kappa)$. This intersection depends upon the cap ellipticity ratio $R$, where $R$ is the ratio of its major to minor axes:

$$X(\kappa) = L(\kappa) + RF_f[L(\kappa)] \, .$$

The cap moves to simulate plastic volume change. The cap expands ($X(\kappa)$ and $\kappa$ increase) to simulate plastic volume compaction. The cap contracts ($X(\kappa)$ and $\kappa$ decrease) to simulate plastic volume expansion, called dilation. The motion (expansion and contraction) of the cap is based upon the hardening rule:

$$\varepsilon_v^p = W\left[1 - e^{-D_1(X-X_0)-D_2(X-X_0)^2}\right] \, .$$

Here $\varepsilon_v^p$ is the plastic volume strain, $W$ is the maximum plastic volume strain, and $D_1$ and $D_2$ are model input parameters. $X_0$ is the initial location of the cap when $\kappa = \kappa_0$.

The five input parameters ($X_0$, $W$, $D_1$, $D_2$, and $R$) are obtained from fits to the pressure-volumetric strain curves in isotropic compression and uniaxial strain. $X_0$ determines the pressure at which compaction initiates in isotropic compression. $R$, combined with $X_0$, determines the pressure at which compaction initiates in uniaxial strain. $D_1$ and $D_2$ determine the shape of the pressure-volumetric strain curves. $W$ determines the maximum plastic volume compaction.

7. **Shear hardening surface.** In unconfined compression, the stress-strain behavior of concrete exhibits nonlinearity and dilation prior to the peak. Such behavior is modeled with an initial shear yield surface, $N_H F_f$, which hardens until it coincides with the ultimate shear yield surface, $F_f$. Two input parameters are required. One parameter, $N_H$, initiates hardening by setting the location of the initial yield surface. A second parameter, $C_H$, determines the rate of hardening (amount of nonlinearity).

8. **Damage.** Concrete exhibits softening in the tensile and low to moderate compressive regimes.

$$\sigma_{ij}^d = (1 - d)\sigma_{ij}^{vp}$$

A scalar damage parameter, $d$, transforms the viscoplastic stress tensor without damage, denoted $\boldsymbol{\sigma}^{vp}$, into the stress tensor with damage, denoted $\boldsymbol{\sigma}^d$. Damage accumulation is based upon two distinct formulations, which we call brittle damage and ductile damage. The initial damage threshold is coincident with the shear plasticity surface, so the threshold does not have to be specified by the user.

a) *Ductile Damage.* Ductile damage accumulates when the pressure, $P$, is compressive and an energy-type term, $\tau_c$, exceeds the damage threshold, $\tau_{0c}$. Ductile damage accumulation depends upon the total strain components, $\varepsilon_{ij}$, as follows:

$$\tau_c = \sqrt{\frac{1}{2}\sigma_{ij}\varepsilon_{ij}}$$

The stress components, $\sigma_{ij}$ are the elasto-plastic stresses (with kinematic hardening) calculated before application of damage and rate effects.

b) *Brittle Damage.* Brittle damage accumulates when the pressure is tensile and an energy-type term, $\tau_t$, exceeds the damage threshold, $\tau_{0t}$. Brittle damage accumulation depends upon the maximum principal strain, $\varepsilon_{max}$, as follows:

$$\tau_t = \sqrt{E\varepsilon_{max}^2} \ .$$

As damage accumulates, the damage parameter, $d$, increases from an initial value of zero, towards a maximum value of one, using the following formulations:

$$\text{Brittle Damage:} \quad d(\tau_t) = \frac{0.999}{D}\left[\frac{1+D}{1+D\,e^{-C(\tau_t-\tau_{0t})}} - 1\right]$$

$$\text{Ductile Damage:} \quad d(\tau_c) = \frac{d_{max}}{B}\left[\frac{1+B}{1+Be^{-A(\tau_c-\tau_{0c})}} - 1\right]$$

The damage parameter that is applied to the six stresses is equal to the current maximum of the brittle or ductile damage parameter. The parameters $A$ and $B$ or $C$ and $D$ set the shape of the softening curve plotted as stress-displacement or stress-strain. The parameter $d_{max}$ is the maximum damage level that can be attained. It is internally calculated and is less than one at moderate confining pressures. See [Murray 2007] for a description of how $d_{max}$ is calculated for different loading regimes. The compressive softening parameter, $A$, may also be reduced with confinement, using the input field PMOD, as follows:

$$A = A(d_{max} + 0.001)^{PMOD}$$

9. **Regulating mesh size sensitivity.** The concrete model maintains constant fracture energy, regardless of element size. The fracture energy is defined here as the area under the stress-displacement curve from peak strength to zero strength. This is done by internally formulating the softening parameters $A$ and $C$ (see Remark 8) in terms of the element length, $l$ (cube root of the element volume), the fracture energy, $G_f$, the initial damage threshold, $\tau_{0t}$ or $\tau_{0c}$, and the softening shape parameters, $D$ or $B$.

The fracture energy is calculated from up to five user-specified input fields: GFC, GFS, GFT, PWRC, and PWRT. The user specifies three distinct fracture energy

values. These are the fracture energy in uniaxial tensile stress, GFT; pure shear stress, GFS; and uniaxial compressive stress, GFC. The model internally selects the fracture energy from equations which interpolate between the three fracture energy values as a function of the stress state (expressed using two stress invariants). The interpolation equations depend upon the user-specified input powers PWRC and PWRT, as follows:

$$\text{Tensile Pressure:} \quad G_f = \text{GFS} + \overbrace{\left(\frac{-J_1}{\sqrt{3J_2'}}\right)^{\text{PWRT}}}^{k_t} [\text{GFT} - \text{GFS}]$$

$$\text{Compressive Pressure:} \quad G_f = \text{GFS} + \overbrace{\left(\frac{J_1}{\sqrt{3J_2'}}\right)^{\text{PWRC}}}^{k_c} [\text{GFC} - \text{GFS}]$$

The internal parameters $k_c$ and $k_t$ are restricted to the interval [0,1].

10. **Element erosion.** An element loses all strength and stiffness as $d \to 1$. To prevent computational difficulties with very low stiffness, element erosion is available as a user option. An element erodes when $d > 0.99$ and the maximum principal strain is greater than a user supplied input value, ERODE − 1.0.

11. **Viscoplastic rate effects.** At each time step, the viscoplastic algorithm interpolates between the elastic trial stress, $\sigma_{ij}^{\text{T}}$, and the inviscid stress (without rate effects), $\sigma_{ij}^{\text{P}}$, to set the viscoplastic stress (with rate effects), $\sigma_{ij}^{\text{VP}}$:

$$\sigma_{ij}^{\text{VP}} = (1-\gamma)\sigma_{ij}^{\text{T}} + \gamma\sigma_{ij}^{\text{P}} \ ,$$

where

$$\gamma = \frac{\Delta t/\eta}{1 + \Delta t/\eta} \ .$$

This interpolation depends upon the effective fluidity coefficient, $\eta$, and the time step, $\Delta t$. The effective fluidity coefficient is internally calculated from five user-supplied input parameters and interpolation equations:

$$\text{Tensile Pressure:} \quad \eta = \eta_s + \left(\frac{-J_1}{\sqrt{3J_2'}}\right)^{\text{PWRT}} [\eta_t - \eta_s]$$

$$\text{Compressive Pressure:} \quad \eta = \eta_s + \left(\frac{J_1}{\sqrt{3J_2'}}\right)^{\text{PWRC}} [\eta_c - \eta_s]$$

where

$$\eta_s = \text{SRATE} \times \eta_t$$
$$\eta_t = \frac{\text{ETA0T}}{\dot{\varepsilon}^{\text{NT}}}$$
$$\eta_c = \frac{\text{ETA0C}}{\dot{\varepsilon}^{\text{NC}}}$$

The input parameters are ETA0T and NT for fitting uniaxial tensile stress data, ETA0X and NC for fitting the uniaxial compressive stress data, and SRATE for fitting shear stress data. The effective strain rate is $\dot{\varepsilon}$.

This viscoplastic model may predict substantial rate effects at high strain rates ($\dot{\varepsilon} > 100$). To limit rate effects at high strain rates, the user may input overstress limits in tension OVERT and compression OVERC. These input fields limit calculation of the fluidity parameter, as follows:

$$\text{if } E\dot{\varepsilon}\eta > \text{OVER}, \text{ then } \eta = \frac{m}{E\dot{\varepsilon}} .$$

Here $m$ = OVERT when the pressure is tensile, and $m$ = OVERC when the pressure is compressive.

You have the option of increasing the fracture energy as a function of effective strain rate using the REPOW input parameter, as follows:

$$G_f^{\text{rate}} = G_f \left[ 1 + \frac{E\dot{\varepsilon}\eta}{r^s \sqrt{E}} \right]^{\text{REPOW}}$$

Here $G_f^{\text{rate}}$ is the fracture energy enhanced by rate effects, and $r^s$ is an internally calculated damage threshold determined before applying viscoplasticity (see [Murray 2007] for more details). The term in brackets is only applied if it is greater than or equal to one and is the approximate ratio of the dynamic to static strength.

12. **Optional failure criterion.** A nonzero value of SSCC activates an element erosion algorithm based on shear and confinement proposed by Fedoroff and Calonius [2020] and Özen [2021]; see illustration of the principle in Figure M159-2. An integration point is flagged (and keeps being flagged or not depending on FLAGCC) when the pure shear strain, $\varepsilon_{\text{psh}}$, is equal to or exceeds the product of SSCC with PPSH:

$$\varepsilon_{\text{psh}} = \delta \sqrt{2} \varepsilon_{\text{oct}} \frac{l_{\text{ch}}}{\text{CLCH}} \geq \text{SSCC} \times \text{PPSH} .$$

Here, $\varepsilon_{\text{oct}}$ is the octahedral shear strain, and $l_{\text{ch}}$ is the characteristic element length (see FLAGCL). The variable $\delta$ differentiates the strain conditions having high octahedral strains from the pure-shear-strain conditions as defined in Fedoroff and Calonius [2020]. If flagged, the following confinement criterion must be evaluated:

$$\mathrm{CR} \leq \mathrm{PNCF} \times \mathrm{CRUL} \ .$$

The confinement ratio, CR, is given by:

$$\mathrm{CR} = \frac{[\mathrm{COEC} \times \sigma_c + (1 - \mathrm{COEC}) \times \sigma_{c,\max}]}{f'_c} \ .$$

In the above, $\sigma_c$ is the effective confinement stress:

$$\sigma_c = \langle 0, -p - q/3 \rangle \ ,$$

where $p$ is the pressure and $q$ is the von Mises stress. When the integration point is flagged under pure shear strain and the confinement criterion threshold is crossed, the integration point fails. Depending on the definition of NUMFIP, the element is eroded if enough integration points failed.

If you do not provide CRUL, the upper limit for the confinement ratio is determined internally using the value of FBCR as described in the above-mentioned papers. If the CONCRETE keyword option is not used, the unconfined compressive strength $f'_c$ and the parameter FBCR are computed automatically from other material model variables. The value of FBCR on Card 9 is then ignored, except in the case where the internal calculation fails. Otherwise, with keyword option CONCRETE, the compressive strength $f'_c$ (FPC) on Card 3 and the parameter FBCR on Card 9 are used.

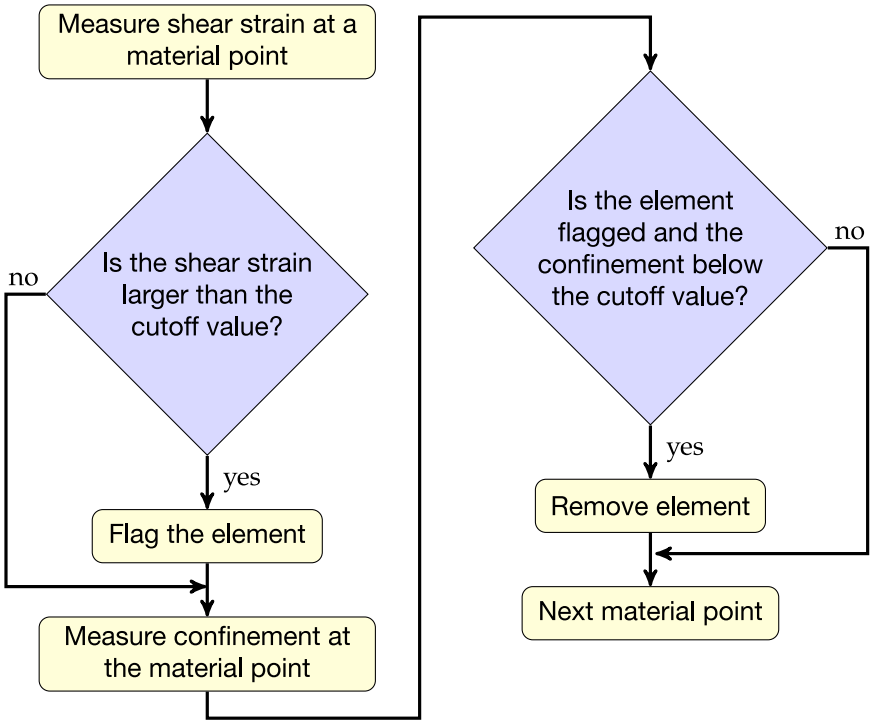



**Figure M159-2.** Steps for determining if an element is removed due to failure

## *MAT_ALE_INCOMPRESSIBLE

This is Material Type 160. This material is for modeling incompressible flows with the ALE solver. It should be used with solid element formulations 6 or 12 (see *SECTION_-SOLID). A projection method enforces the incompressibility condition.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MU | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | 0.0 | 0.0 | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TOL | DTOUT | NCG | METH | | | | |
| Type | F | F | I | I | | | | |
| Default | $10^{-8}$ | $10^{10}$ | 50 | -7 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART) |
| RO | Material density |
| PC | Pressure cutoff (≤ 0.0) |
| MU | Dynamic viscosity coefficient |
| TOL | Tolerance for the convergence of the conjugate gradient |
| DTOUT | Time interval between screen outputs |
| NCG | Maximum number of loops in the conjugate gradient |
| METH | Conjugate gradient methods:<br>EQ.-6: Solves Poisson's equation for the pressure. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-7: Solves Poisson's equation for the pressure increment. |

## *MAT_COMPOSITE_MSC_{OPTION}

Available options include:

<BLANK>

DMG

These are Material Types 161 and 162. These models may be used to model the progressive failure analysis for composite materials consisting of unidirectional and woven fabric layers. The progressive layer failure criteria have been established by adopting the methodology developed by Hashin [1980] with a generalization to include the effect of highly constrained pressure on composite failure. These failure models can be used to effectively simulate fiber failure, matrix damage, and delamination behavior under all conditions - opening, closing, and sliding of failure surfaces. The model with the DMG keyword option (material 162) is a generalization of the basic layer failure model of Material 161 by adopting the damage mechanics approach for characterizing the softening behavior after damage initiation. These models require an additional license from Materials Sciences Corporation, which developed and supports these models. These models are supported for solid elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | AOPT | MACF | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| SAT | SAC | SBT | SBC | SCT | SFC | SFS | SAB |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| SBC | SCA | SFFC | AMODEL | PHIC | E_LIMT | S_DELM | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| OMGMX | ECRSH | EEXPN | CERATE1 | AM1 | | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is included if the DMG keyword option is used.

| AM2 | AM3 | AM4 | CERATE2 | CERATE3 | CERATE4 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material ID. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus - longitudinal direction |
| EB | $E_b$, Young's modulus - transverse direction |
| EC | $E_c$, Young's modulus - through thickness direction |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | MACF | | | |
| Type | F | F | F | F | I | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus *ab* |
| GBC | $G_{bc}$, shear modulus *bc* |
| GCA | $G_{ca}$, shear modulus *ca* |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES..<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| VARIABLE | DESCRIPTION |
|---|---|
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Layer in-plane rotational angle in degrees. It may be override |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SAT | SAC | SBT | SBC | SCT | SFC | SFS | SAB |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SAT | Longitudinal tensile strength |
| SAC | Longitudinal compressive strength |
| SBT | Transverse tensile strength |
| SBC | Transverse compressive strength |
| SCT | Through thickness tensile strength |
| SFC | Crush strength |
| SFS | Fiber mode shear strength |
| SAB | Matrix mode shear strength, *ab* plane; see remarks. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SBC | SCA | SFFC | AMODEL | PHIC | E_LIMT | S_DELM | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| SBC | Matrix mode shear strength, *bc* plane; see remarks. |
| SCA | Matrix mode shear strength, *ca* plane; see remarks. |

| VARIABLE | DESCRIPTION |
|---|---|
| SFFC | Scale factor for residual compressive strength |
| AMODEL | Material models:<br>EQ.1.0: Unidirectional layer model<br>EQ.2.0: Fabric layer model |
| PHIC | Coulomb friction angle for matrix and delamination failure, < 90 |
| E_LIMT | Element eroding axial strain |
| S_DELM | Scale factor for delamination criterion |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OMGMX | ECRSH | EEXPN | CERATE1 | AM1 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| OMGMX | Limit damage parameter for elastic modulus reduction |
| ECRSH | Limit compressive volume strain for element eroding |
| EEXPN | Limit tensile volume strain for element eroding |
| CERATE1 | Coefficient for strain rate dependent strength properties |
| AM1 | Coefficient for strain rate softening property for fiber damage in $a$-direction |

**Failure Card.** Additional card for DMG keyword option.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AM2 | AM3 | AM4 | CERATE2 | CERATE3 | CERATE4 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AM2 | Coefficient for strain rate softening property for fiber damage in *b*-direction |
| AM3 | Coefficient for strain rate softening property for fiber crush and punch shear damage |
| AM4 | Coefficient for strain rate softening property for matrix and delamination damage |
| CERATE2 | Coefficient for strain rate dependent axial moduli |
| CERATE3 | Coefficient for strain rate dependent shear moduli |
| CERATE4 | Coefficient for strain rate dependent transverse moduli |

**Material Models:**

The unidirectional and fabric layer failure criteria and the associated property degradation models for material 161 are described as follows. All the failure criteria are expressed in terms of stress components based on ply level stresses $(\sigma_a, \sigma_b, \sigma_c, \tau_{ab}, \tau_{bc}, \tau_{ca})$ and the associated elastic moduli are $(E_a, E_b, E_c, G_{ab}, G_{bc}, G_{ca})$. Note that for the unidirectional model, *a*, *b* and *c* denote the fiber, in-plane transverse and out-of-plane directions, respectively, while for the fabric model, *a*, *b* and *c* denote the in-plane fill, in-plane warp and out-of-plane directions, respectively.

**Unidirectional Lamina Model:**

Three criteria are used for fiber failure, one in tension/shear, one in compression and another one in crush under pressure. They are chosen in terms of quadratic stress forms as follows:

1. Tensile/shear fiber mode:

$$f_1 = \left(\frac{\langle \sigma_a \rangle}{S_{aT}}\right)^2 + \left(\frac{\tau_{ab}^2 + \tau_{ca}^2}{S_{FS}^2}\right) - 1 = 0$$

2. Compression fiber mode:

$$f_2 = \left(\frac{\langle \sigma_a' \rangle}{S_{aC}}\right)^2 - 1 = 0, \qquad \sigma_a' = -\sigma_a + \left\langle -\frac{\sigma_b + \sigma_c}{2} \right\rangle$$

3. Crush mode:

$$f_3 = \left(\frac{\langle p \rangle}{S_{FC}}\right)^2 - 1 = 0, \qquad p = -\frac{\sigma_a + \sigma_b + \sigma_c}{3}$$

Here $\langle \quad \rangle$ are Macaulay brackets, $S_{aT}$ and $S_{aC}$ are the tensile and compressive strengths in the fiber direction, and $S_{FS}$ and $S_{FC}$ are the layer strengths associated with the fiber shear and crush failure, respectively.

Matrix mode failures must occur without fiber failure, and hence they will be on planes parallel to fibers. For simplicity, only two failure planes are considered: one is perpendicular to the planes of layering and the other one is parallel to them. The matrix failure criteria for the failure plane perpendicular and parallel to the layering planes, respectively, have the forms:

1. Perpendicular matrix mode:

$$f_4 = \left(\frac{\langle \sigma_b \rangle}{S_{bT}}\right)^2 + \left(\frac{\tau_{bc}}{S'_{bc}}\right)^2 + \left(\frac{\tau_{ab}}{S_{ab}}\right)^2 - 1 = 0$$

2. Parallel matrix mode (Delamination):

$$f_5 = S^2 \left\{ \left(\frac{\langle \sigma_c \rangle}{S_{bT}}\right)^2 + \left(\frac{\tau_{bc}}{S''_{bc}}\right)^2 + \left(\frac{\tau_{ca}}{S_{ca}}\right)^2 \right\} - 1 = 0$$

Here $S_{bT}$ is the transverse tensile strength. Based on the Coulomb-Mohr theory, the shear strengths for the transverse shear failure and the two axial shear failure modes are assumed to be the forms,

$$\begin{aligned} S_{ab} &= S_{ab}^{(0)} + \tan(\varphi)\langle -\sigma_b \rangle \\ S'_{bc} &= S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_b \rangle \\ S_{ca} &= S_{ca}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle \\ S''_{bc} &= S_{bc}^{(0)} + \tan(\varphi)\langle -\sigma_c \rangle \end{aligned}$$

where φ is a material constant as $\tan(\varphi)$ is similar to the coefficient of friction, and $S_{ab}^{(0)}$, $S_{ca}^{(0)}$ and $S_{bc}^{(0)}$ are the shear strength values of the corresponding tensile modes.

Failure predicted by the criterion of $f_4$ can be referred to as transverse matrix failure, while the matrix failure predicted by $f_5$, which is parallel to the layer, can be referred as the delamination mode when it occurs within the elements that are adjacent to the ply interface. Note that a scale factor $S$ is introduced to provide better correlation of delamination area with experiments. The scale factor $S$ can be determined by fitting the analytical prediction to experimental data for the delamination area.

When fiber failure in tension/shear mode is predicted in a layer by $f_1$, the load carrying capacity of that layer is completely eliminated. All the stress components are reduced to zero instantaneously (100 time steps to avoid numerical instability). For compressive fiber failure, the layer is assumed to carry a residual axial load, while the transverse load carrying capacity is reduced to zero. When the fiber compressive failure mode is reached due to $f_2$, the axial layer compressive strength stress is assumed to reduce to a residual value $S_{RC}$ (= SFFC × $S_{AC}$). The axial stress is then assumed to remain constant, meaning

$\sigma_a = -S_{RC}$, for continuous compressive loading, while the subsequent unloading curve follows a reduced axial modulus to zero axial stress and strain state. When the fiber crush failure occurs, the material is assumed to behave elastically for compressive pressure, $p > 0$, and to carry no load for tensile pressure, $p < 0$.

When a matrix failure (delamination) in the *ab*-plane is predicted, the strength values for $S_{ca}^{(0)}$ and $S_{bc}^{(0)}$ are set to zero. This results in reducing the stress components $\sigma_c$, $\tau_{bc}$ and $\tau_{ca}$ to the fractured material strength surface. For tensile mode, $\sigma_c > 0$, these stress components are reduced to zero. For compressive mode, $\sigma_c < 0$, the normal stress $\sigma_c$ is assumed to deform elastically for the closed matrix crack. Loading on the failure envelop, the shear stresses are assumed to 'slide' on the fractured strength surface (frictional shear stresses) like in an ideal plastic material, while the subsequent unloading shear stress-strain path follows reduced shear moduli to the zero shear stress and strain state for both $\tau_{bc}$ and $\tau_{ca}$ components.

The post failure behavior for the matrix crack in the a-c plane due to $f_4$ is modeled in the same fashion as that in the *ab*-plane as described above. In this case, when failure occurs, $S_{ab}^{(0)}$ and $S_{bc}^{(0)}$ are reduced to zero instantaneously. The post fracture response is then governed by failure criterion of $f_5$ with $S_{ab}^{(0)} = 0$ and $S_{bc}^{(0)} = 0$. For tensile mode, $\sigma_b > 0$, $\sigma_b$, $\tau_{ab}$ and $\tau_{bc}$ are zero. For compressive mode, $\sigma_b < 0$, $\sigma_b$ is assumed to be elastic, while $\tau_{ab}$ and $\tau_{bc}$ 'slide' on the fracture strength surface as in an ideal plastic material, and the unloading path follows reduced shear moduli to the zero shear stress and strain state. It should be noted that $\tau_{bc}$ is governed by both the failure functions and should lie within or on each of these two strength surfaces.

### Fabric Lamina Model:

The fiber failure criteria of Hashin for a unidirectional layer are generalized to characterize the fiber damage in terms of strain components for a plain weave layer. The fill and warp fiber tensile/shear failure are given by the quadratic interaction between the associated axial and shear stresses, that is,

$$f_6 = \left(\frac{\langle \sigma_a \rangle}{S_{aT}}\right)^2 + \frac{\left(\tau_{ab}^2 + \tau_{ca}^2\right)}{S_{aFS}^2} - 1 = 0$$

$$f_7 = \left(\frac{\langle \sigma_b \rangle}{S_{bT}}\right)^2 + \frac{\left(\tau_{ab}^2 + \tau_{bc}^2\right)}{S_{bFS}^2} - 1 = 0$$

where $S_{aT}$ and $S_{bT}$ are the axial tensile strengths in the fill and warp directions, respectively, and $S_{aFS}$ and $S_{bFS}$ are the layer shear strengths due to fiber shear failure in the fill and warp directions. These failure criteria are applicable when the associated $\sigma_a$ or $\sigma_b$ is positive. It is assumed $S_{aFS} = \text{SFS}$, and

$$S_{bFS} = \text{SFS} \times \frac{S_{bT}}{S_{aT}} .$$

When $\sigma_a$ or $\sigma_b$is compressive, it is assumed that the in-plane compressive failure in both the fill and warp directions are given by the maximum stress criterion, that is,

$$f_8 = \left[\frac{\langle \sigma_a' \rangle}{S_{aC}}\right]^2 - 1 = 0, \quad \sigma_a' = -\sigma_a + \langle -\sigma_c \rangle$$
$$f_9 = \left[\frac{\langle \sigma_b' \rangle}{S_{bC}}\right]^2 - 1 = 0, \quad \sigma_b' = -\sigma_b + \langle -\sigma_c \rangle$$

where $S_{aC}$and $S_{bC}$ are the axial compressive strengths in the fill and warp directions, respectively. The crush failure under compressive pressure is

$$f_{10} = \left(\frac{\langle p \rangle}{S_{FC}}\right)^2 - 1 = 0, \quad p = -\frac{\sigma_a + \sigma_b + \sigma_c}{3} .$$

A plain weave layer can fail under in-plane shear stress without the occurrence of fiber breakage. This in-plane matrix failure mode is given by

$$f_{11} = \left(\frac{\tau_{ab}}{S_{ab}}\right)^2 - 1 = 0 ,$$

where $S_{ab}$ is the layer shear strength due to matrix shear failure.

Another failure mode, which is due to the quadratic interaction between the thickness stresses, is expected to be mainly a matrix failure. This through the thickness matrix failure criterion is

$$f_{12} = S^2 \left\{ \left(\frac{\langle \sigma_c \rangle}{S_{cT}}\right)^2 + \left(\frac{\tau_{bc}}{S_{bc}}\right)^2 + \left(\frac{\tau_{ca}}{S_{ca}}\right)^2 \right\} - 1 = 0 ,$$

where $S_{cT}$ is the through the thickness tensile strength, and $S_{bc}$ and $S_{ca}$ are the shear strengths assumed to depend on the compressive normal stress $\sigma_c$, meaning

$$\begin{Bmatrix} S_{ca} \\ S_{bc} \end{Bmatrix} = \begin{Bmatrix} S_{ca}^{(0)} \\ S_{bc}^{(0)} \end{Bmatrix} + \tan(\varphi)\langle -\sigma_c \rangle .$$

When failure predicted by this criterion occurs within elements that are adjacent to the ply interface, the failure plane is expected to be parallel to the layering planes, and, thus, can be referred to as the delamination mode. Note that a scale factor $S$ is introduced to provide better correlation of delamination area with experiments. The scale factor $S$ can be determined by fitting the analytical prediction to experimental data for the delamination area.

Similar to the unidirectional model, when fiber tensile/shear failure is predicted in a layer by $f_6$ or $f_7$, the load carrying capacity of that layer in the associated direction is completely eliminated. For compressive fiber failure due to by $f_8$ or $f_9$, the layer is assumed to carry a residual axial load in the failed direction, while the load carrying capacity transverse to the failed direction is assumed unchanged. When the compressive axial stress in a layer reaches the compressive axial strength $S_{aC}$ or $S_{bC}$, the axial layer stress is assumed to be reduced to the residual strength $S_{aRC}$ or $S_{bRC}$ where $S_{aRC} = \text{SFFC} \times S_{aC}$ and $S_{bRC} =$

SFFC × $S_{bC}$. The axial stress is assumed to remain constant, that is, $\sigma_a = -S_{aCR}$ or $\sigma_b = -S_{bCR}$, for continuous compressive loading, while the subsequent unloading curve follows a reduced axial modulus. When the fiber crush failure is occurred, the material is assumed to behave elastically for compressive pressure, $p > 0$, and to carry no load for tensile pressure, $p < 0$.

When the in-plane matrix shear failure is predicted by $f_{11}$ the axial load carrying capacity within a failed element is assumed unchanged, while the in-plane shear stress is assumed to be reduced to zero.

For through the thickness matrix (delamination) failure given by equation $f_{12}$, the in-plane load carrying capacity within the element is assumed to be elastic, while the strength values for the tensile mode, $S_{ca}^{(0)}$ and $S_{bc}^{(0)}$, are set to zero. For tensile mode, $\sigma_c > 0$, the through the thickness stress components are reduced to zero. For compressive mode, $\sigma_c < 0$, $\sigma_c$ is assumed to be elastic, while $\tau_{bc}$ and $\tau_{ca}$ 'slide' on the fracture strength surface as in an ideal plastic material, and the unloading path follows reduced shear moduli to the zero shear stress and strain state.

The effect of strain-rate on the layer strength values of the fiber failure modes is modeled by the strain-rate dependent functions for the strength values $\{S_{RT}\}$ as

$$\{S_{RT}\} = \{S_0\}\left(1 + C_{\text{rate1}} \ln \frac{\{\dot{\bar{\varepsilon}}\}}{\dot{\varepsilon}_0}\right)$$

$$\{S_{RT}\} = \begin{Bmatrix} S_{aT} \\ S_{aC} \\ S_{bT} \\ S_{bC} \\ S_{FC} \\ S_{FS} \end{Bmatrix}, \qquad \{\dot{\bar{\varepsilon}}\} = \begin{Bmatrix} |\dot{\varepsilon}_a| \\ |\dot{\varepsilon}_a| \\ |\dot{\varepsilon}_b| \\ |\dot{\varepsilon}_b| \\ |\dot{\varepsilon}_c| \\ \left(\dot{\varepsilon}_{ca}^2 + \dot{\varepsilon}_{bc}^2\right)^{1/2} \end{Bmatrix}$$

where $C_{\text{rate1}}$ is the strain-rate constants, and $\{S_0\}$ are the strength values of $\{S_{RT}\}$ at the reference strain-rate $\dot{\varepsilon}_0$.

### Damage Model:

The damage model is a generalization of the layer failure model of Material 161 by adopting the MLT damage mechanics approach, Matzenmiller et al. [1995], for characterizing the softening behavior after damage initiation. Complete model description is given in Yen [2002]. The damage functions, which are expressed in terms of ply level engineering strains, are converted from the above failure criteria of fiber and matrix failure modes by neglecting the Poisson's effect. Elastic moduli reduction is expressed in terms of the associated damage parameters $\omega_i$:

$$E_i' = (1 - \omega_i) E_i$$

where $\omega_i$ is given by

$$\varpi_i = 1 - \exp\left(-\frac{r_i^{m_i}}{m_i}\right), \qquad r_i \geq 0, \qquad i = 1,\ldots,6 \ .$$

In the above $E_i$ are the initial elastic moduli, $E_i'$ are the reduced elastic moduli, $r_i$ are the damage thresholds computed from the associated damage functions for fiber damage, matrix damage and delamination, and $m_i$ are material damage parameters, which are currently assumed to be independent of strain-rate. The damage function is formulated to account for the overall nonlinear elastic response of a lamina including the initial 'hardening' and the subsequent softening beyond the ultimate strengths.

In the damage model (material 162), the effect of strain-rate on the nonlinear stress-strain response of a composite layer is modeled by the strain-rate dependent functions for the elastic moduli $\{E_{RT}\}$ as

$$\{E_{RT}\} = \{E_0\}\left(1 + \{C_{\text{rate}}\} \ln \frac{\{\dot{\bar{\varepsilon}}\}}{\dot{\varepsilon}_0}\right)$$

$$\{E_{RT}\} = \begin{Bmatrix} E_a \\ E_b \\ E_c \\ G_{ab} \\ G_{bc} \\ G_{ca} \end{Bmatrix} \qquad \{\dot{\bar{\varepsilon}}\} = \begin{Bmatrix} |\dot{\varepsilon}_a| \\ |\dot{\varepsilon}_b| \\ |\dot{\varepsilon}_c| \\ |\dot{\varepsilon}_{ab}| \\ |\dot{\varepsilon}_{bc}| \\ |\dot{\varepsilon}_{ca}| \end{Bmatrix} \qquad \{C_{\text{rate}}\} = \begin{Bmatrix} C_{\text{rate2}} \\ C_{\text{rate2}} \\ C_{\text{rate4}} \\ C_{\text{rate3}} \\ C_{\text{rate3}} \\ C_{\text{rate3}} \end{Bmatrix}$$

where $\{C_{\text{rate}}\}$ are the strain-rate constants. $\{E_0\}$ are the modulus values of $\{E_{RT}\}$ at the reference strain-rate $\dot{\varepsilon}_0$.

### Element Erosion:

A failed element is eroded in any of three different ways:

1. If fiber tensile failure in a unidirectional layer is predicted in the element and the axial tensile strain is greater than E_LIMT. For a fabric layer, both in-plane directions are failed and exceed E_LIMT.

2. If compressive relative volume in a failed element is smaller than ECRSH.

3. If tensile relative volume in a failed element is greater than EEXPN.

### Damage History Parameters:

Information about the damage history variables for the associated failure modes can be plotted in LS-PrePost. These additional history variables are tabulated below:

| History Variable | Description | Value | History Variable # |
|---|---|---|---|
| efa($I$) | Fiber mode in $a$ | | 7 |
| efb($I$) | Fiber mode in $b$ | | 8 |
| efp($I$) | Fiber crush mode | EQ.0: elastic | 9 |
| em($I$) | Perpendicular matrix mode | GE.1: failed | 10 |
| ed($I$) | Parallel matrix/delamination mode | | 11 |
| delm($I$) | Delamination mode | | 12 |

## *MAT_MODIFIED_CRUSHABLE_FOAM

This is Material Type 163 which is dedicated to modeling crushable foam with optional damping, tension cutoff, and strain rate effects. Unloading is fully elastic. Tension is treated as elastic-perfectly-plastic at the tension cut-off value.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | TID | TSC | DAMP | NCYCLE |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.10 | 12. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SRCLMT | SFLAG | | | | | | |
| Type | F | I | | | | | | |
| Default | $10^{20}$ | 0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| TID | Table ID defining yield stress as a function of volumetric strain, $\gamma$, at different strain rates. |
| TSC | Tensile stress cutoff. A nonzero, positive value is strongly recommended for realistic behavior. |
| DAMP | Rate sensitivity via damping coefficient (.05 < recommended value < .50). |

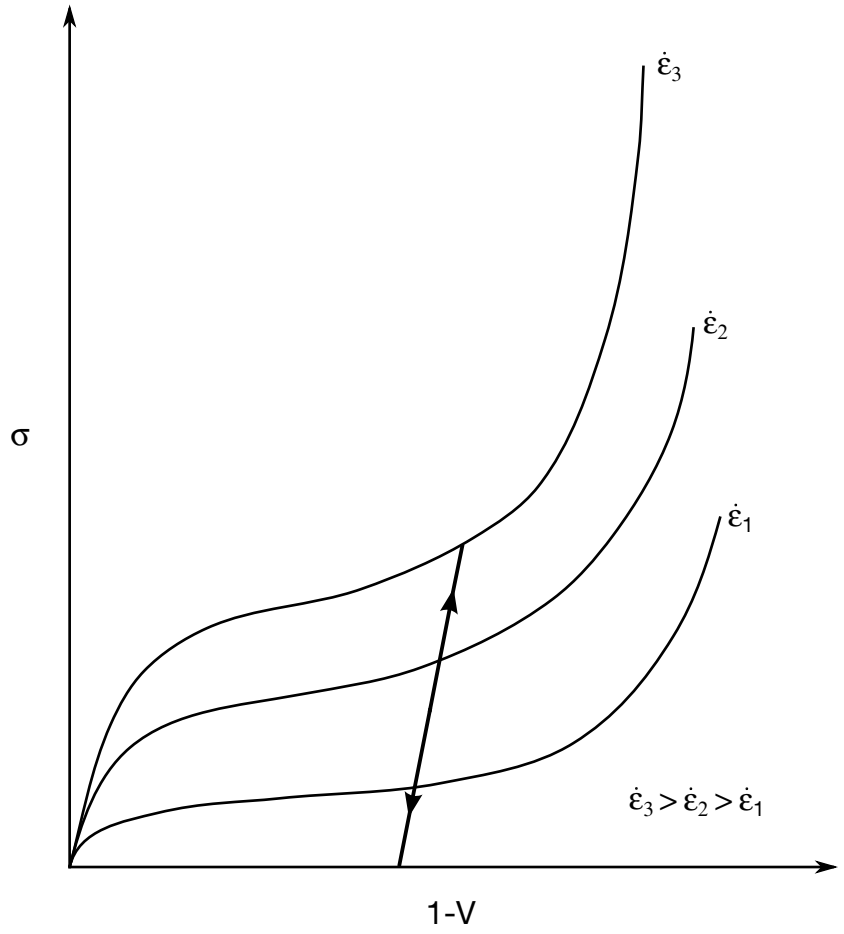



**Figure M163-1.** Rate effects are defined by a family of curves giving yield stress as a function of volumetric strain where $V$ is the relative volume.

| VARIABLE | DESCRIPTION |
|---|---|
| NCYCLE | Number of cycles to determine the average volumetric strain rate. |
| SRCLMT | Strain rate change limit. |
| SFLAG | The strain rate in the table may be the true strain rate (SFLAG = 0) or the engineering strain rate (SFLAG = 1). |

**Remarks:**

The volumetric strain is defined in terms of the relative volume, $V$, as:

$$\gamma = 1 - V$$

The relative volume is defined as the ratio of the current to the initial volume. In place of the effective plastic strain in the d3plot database, the integrated volumetric strain (natural logarithm of the relative volume) is output.

This material is an extension of material 63, *MAT_CRUSHABLE_FOAM. It allows the yield stress to be a function of both volumetric strain rate and volumetric strain. Rate effects are accounted for by defining a table of curves using *DEFINE_TABLE. Each curve defines the yield stress as a function volumetric strain for a different strain rate. The yield stress is obtained by interpolating between the two curves that bound the strain rate.

To prevent high frequency oscillations in the strain rate from causing similar high frequency oscillations in the yield stress, a modified volumetric strain rate is used when interpolating to obtain the yield stress. The modified strain rate is obtained as follows. If NYCLE is > 1, then the modified strain rate is obtained by a time average of the actual strain rate over NCYCLE solution cycles. For SRCLMT > 0, the modified strain rate is capped so that during each cycle, the modified strain rate is not permitted to change more than SRCLMT multiplied by the solution time step.

## *MAT_BRAIN_LINEAR_VISCOELASTIC

This is Material Type 164. This material is a Kelvin-Maxwell model for modeling brain tissue, which is valid for solid elements only. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | G0 | GI | DC | FO | SO |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Bulk modulus (elastic) |
| G0 | Short-time shear modulus, $G_0$ |
| GI | Long-time (infinite) shear modulus, $G_\infty$ |
| DC | Constant depending on formulation option (FO) below:<br>FO.EQ.0.0: Maxwell decay constant, $\beta$<br>FO.EQ.1.0: Kelvin relaxation constant, $\tau$ |
| FO | Formulation option:<br>EQ.0.0: Maxwell<br>EQ.1.0: Kelvin |
| SO | Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step:<br>EQ.0.0: Maximum principal strain that occurs during the calculation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Maximum magnitude of the principal strain values that occurs during the calculation |
| | EQ.2.0: Maximum effective strain that occurs during the calculation |

**Remarks:**

1. **Maxwell Model.** The shear relaxation behavior is described for the Maxwell model by:

$$G(t) = G + (G_0 - G_\infty)e^{-\beta t} \ .$$

A Jaumann rate formulation is used

$$\overset{\nabla}{\sigma}_{ij} = 2\int_0^t G(t-\tau)D'_{ij}(\tau)dt \ ,$$

where the prime denotes the deviatoric part of the stress rate, $\overset{\nabla}{\sigma}_{ij}$, and the strain rate $D_{ij}$.

2. **Kelvin Model.** For the Kelvin model the stress evolution equation is defined as:

$$\dot{s}_{ij} + \frac{1}{\tau}s_{ij} = \left(1+\delta_{ij}\right)G_0\dot{e}_{ij} + \left(1+\delta_{ij}\right)\frac{G_\infty}{\tau}\dot{e}_{ij} \ .$$

The strain data as written to the d3plot database may be used to predict damage, see [Bandak 1991].

## *MAT_PLASTIC_NONLINEAR_KINEMATIC

This is Material Type 165. This relatively simple model, based on a material model by Lemaitre and Chaboche [1990], is suited to model nonlinear kinematic hardening plasticity. The model accounts for the nonlinear Bauschinger effect, cyclic hardening, and ratcheting. Huang [2009] programmed this model and provided it as a user subroutine. It is a very cost effective model and is available for shell and solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | H | C | GAMMA |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FS | | | | | | | |
| Type | F | | | | | | | |
| Default | $10^{16}$ | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress, $\sigma_{y0}$ |
| H | Isotropic plastic hardening modulus |
| C | Kinematic hardening modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| GAMMA | Kinematic hardening parameter, $\gamma$ |
| FS | Failure strain for eroding elements |

**Remarks:**

If the isotropic hardening modulus, $H$, is nonzero, the size of the surface increases as a function of the equivalent plastic strain, $\varepsilon^p$:

$$\sigma_y = \sigma_{y0} + H\varepsilon^p \quad .$$

The rate of evolution of the kinematic component is a function of the plastic strain rate:

$$\dot{\alpha} = [Cn - \gamma\alpha]\dot{\varepsilon}^p \quad ,$$

where $n$ is the flow direction. The term, $\gamma\alpha\dot{\varepsilon}^p$, introduces the nonlinearity into the evolution law, which becomes linear if the parameter, $\gamma$, is set to zero.

## *MAT_MOMENT_CURVATURE_BEAM

This is Material Type 166. This material is for performing nonlinear elastic or multi-linear plastic analysis of Belytschko-Schwer beams with user-defined axial force-strain, moment curvature and torque-twist rate curves. If strain, curvature or twist rate is located outside the curves, use extrapolation to determine the corresponding rigidity. For multi-linear plastic analysis, the user-defined curves are used as yield surfaces.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | ELAF | EPFLG | CTA | CTB | CTT |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | N1 | N2 | N3 | N4 | N5 | N6 | N7 | N8 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | 0.0 / none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCMS1 | LCMS2 | LCMS3 | LCMS4 | LCMS5 | LCMS6 | LCMS7 | LCMS8 |
| Type | I | I | I | I | I | I | I | I |
| Default | none | none | 0 / none | 0 | 0 | 0 | 0 | 0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCMT1 | LCMT2 | LCMT3 | LCMT4 | LCMT5 | LCMT6 | LCMT7 | LCMT8 |
| Type | I | I | I | I | I | I | I | I |
| Default | none | none | 0 / none | 0 | 0 | 0 | 0 | 0 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCT1 | LCT2 | LCT3 | LCT4 | LCT5 | LCT6 | LCT7 | LCT8 |
| Type | I | I | I | I | I | I | I | I |
| Default | none | none | 0 / none | 0 | 0 | 0 | 0 | 0 |

**Multilinear Plastic Analysis Card.** Additional card for EPFLG = 1.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CFA | CFB | CFT | HRULE | REPS | RBETA | RCAPAY | RCAPAZ |
| Type | F | F | F | F | F | F | F | F |
| Default | 1.0 | 1.0 | 1.0 | 0.0 | $10^{20}$ | $10^{20}$ | $10^{20}$ | $10^{20}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus. This variable controls the time step size and must be chosen carefully. Increasing the value of E will decrease the time step size. |
| ELAF | Load curve ID for the axial force-strain curve |

| VARIABLE | DESCRIPTION |
|---|---|
| EPFLG | Function flag:<br>EQ.0.0: Nonlinear elastic analysis<br>EQ.1.0: Multi-linear plastic analysis |
| CTA, CTB, CTT | Type of axial force-strain, moment-curvature, and torque-twist rate curves (see Remarks):<br>EQ.0.0: Curve is symmetric.<br>EQ.1.0: Curve is asymmetric. |
| N1 - N8 | Axial forces at which moment-curvature curves are given. The axial forces must be ordered monotonically increasing. At least two axial forces must be defined if the curves are symmetric. At least three axial forces must be defined if the curves are asymmetric. |
| LCMS1 - LCMS8 | Load curve IDs for the moment-curvature curves about axis *S* under corresponding axial forces. |
| LCMT1 - LCMT8 | Load curve IDs for the moment-curvature curves about axis *T* under corresponding axial forces. |
| LCT1 - LCT8 | Load curve IDs for the torque-twist rate curves under corresponding axial forces. |
| CFA, CFB, CFT | For multi-linear plastic analysis only. Ratio of axial, bending and torsional elastic rigidities to their initial values, no less than 1.0 in value. |
| HRULE | Hardening rule, for multi-linear plastic analysis only.<br>EQ.0.0: Isotropic hardening<br>GT.0.0.AND.LT.1.0: Mixed hardening<br>EQ.1.0: Kinematic hardening |
| REPS | Rupture effective plastic axial strain |
| RBETA | Rupture effective plastic twist rate |
| RCAPAY | Rupture effective plastic curvature about axis *S* |
| RCAPAZ | Rupture effective plastic curvature about axis *T* |

**Remarks:**

For symmetric curves (see fields CTA, CTB, and CTT above), all data points must be in the first quadrant, and at least three data points need to be given, starting from the origin, followed by the yield point.

For asymmetric curves, at least five data points are needed and exactly one point must be at the origin. The two points on both sides of the origin record the positive and negative yield points.

The last data point(s) has no physical meaning: it serves only as a control point for inter or extrapolation.

The curves are input by the user and treated in LS-DYNA as linearly piecewise functions. The curves must be monotonically increasing while the slopes must be monotonically decreasing.

## *MAT_MCCORMICK

This is Material Type 167. This material is intended for finite plastic deformations. McCormick's constitutive relation for materials exhibiting negative steady-state Strain Rate Sensitivity (SRS) defines this material's strength. See McCormick [1988] and Zhang, McCormick and Estrin [2001].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | | | |
| Type | A | F | F | F | F | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | C1 | Q2 | C2 | | | | |
| Type | F | F | F | F | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S | H | OMEGA | TD | ALPHA | EPS0 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress |
| Q1 | Isotropic hardening parameter, $Q_1$ |
| C1 | Isotropic hardening parameter, $C_1$ |

| VARIABLE | DESCRIPTION |
|---|---|
| Q2 | Isotropic hardening parameter, $Q_2$ |
| C2 | Isotropic hardening parameter, $C_2$ |
| S | Dynamic strain aging parameter, $S$ |
| H | Dynamic strain aging parameter, $H$ |
| OMEGA | Dynamic strain aging parameter, $\Omega$ |
| TD | Dynamic strain aging parameter, $t_d$ |
| ALPHA | Dynamic strain aging parameter, $\alpha$ |
| EPS0 | Reference strain rate, $\dot{\varepsilon}_0$ |

**Remarks:**

The uniaxial stress-strain curve is given in the following form:

$$\sigma(\varepsilon^p, \dot{\varepsilon}^p) = \sigma_Y(t_a) + R(\varepsilon^p) + \sigma_v(\dot{\varepsilon}^p) \ .$$

Viscous stress $\sigma_v$ is given by

$$\sigma_v(\dot{\varepsilon}^p) = \mathrm{S} \times \ln\left(1 + \frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_o}\right) \ ,$$

where $S$ represents the instantaneous strain rate sensitivity and $\dot{\varepsilon}_o$ is a reference strain rate.

In the McCormick model the yield strength includes a dynamic strain aging (DSA) contribution. The yield strength is defined as

$$\sigma_Y(t_a) = \sigma_o + \mathrm{S} \times \mathrm{H} \times \left[1 - \exp\left\{-\left(\frac{t_a}{t_d}\right)^{\alpha}\right\}\right] \ ,$$

where $\sigma_o$ is the yield strength for vanishing average waiting time $t_a$, and $H$, $\alpha$, and $t_d$ are material constants linked to dynamic strain aging.

The average waiting time is defined by the evolution equation

$$\dot{t}_a = 1 - \frac{t_a}{t_{a,ss}} \ ,$$

where the quasi-steady state waiting time $t_{a,ss}$ is given as

$$t_{a,ss} = \frac{\Omega}{\dot{\varepsilon}^p} \ .$$

The strain hardening function $R$ is defined by the extended Voce Law

$$R(\varepsilon^p) = Q_1[1 - \exp(-C_1\varepsilon^p)] + Q_2[1 - \exp(-C_2\varepsilon^p)] \ .$$

## *MAT_POLYMER

This is Material Type 168. This model is implemented for brick elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | GAMMA0 | DG | SC | ST |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TEMP | K | CR | N | C | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| GAMMA0 | Pre-exponential factor, $\dot{\gamma}_{0A}$ |
| DG | Energy barrier to flow, $\Delta G$ |
| SC | Shear resistance in compression, $S_c$ |
| ST | Shear resistance in tension, $S_t$ |
| TEMP | Absolute temperature, $\theta$ |
| K | Boltzmann constant, $k$ |
| CR | Product, $C_r = nk\theta$ |
| N | Number of "rigid links" between entanglements, $N$ |
| C | Relaxation factor, $C$ |

**Remarks:**

The polymer is assumed to have two basic resistances to deformation:

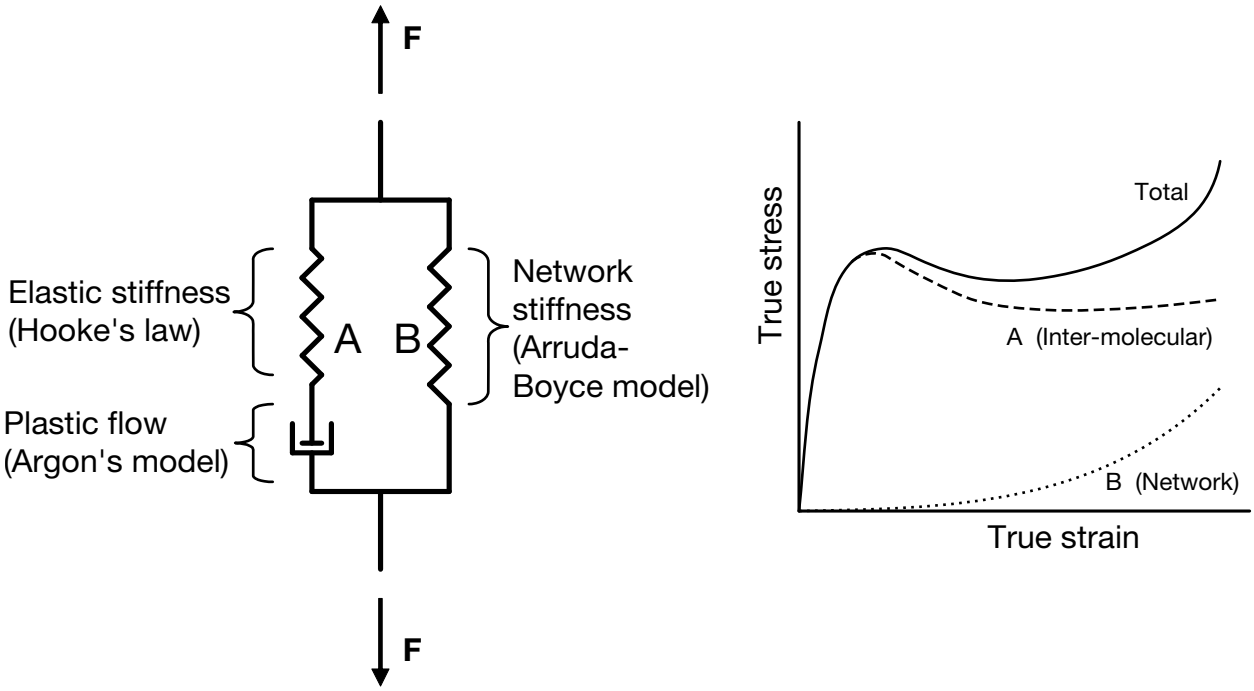



**Figure M168-1.** Stress decomposition in inter-molecular and network contributions.

1. An intermolecular barrier to deformation related to relative movement between molecules.

2. An evolving anisotropic resistance related to straightening of the molecule chains.

The model which is implemented and presented in this paper is mainly based on the framework suggested by Boyce et al. [2000]. Going back to the original work by Haward and Thackray [1968], they considered the uniaxial case only. The extension to a full 3D formulation was proposed by Boyce et al. [1988]. Moreover, Boyce and co-workers have during a period of 20 years changed or further developed the parts of the original model. Haward and Thackray [1968] used an Eyring model to represent the dashpot in Fig. M168-1, while Boyce et al. [2000] employed the double-kink model of Argon [1973] instead. Part B of the model, describing the resistance associated with straightening of the molecules, contained originally a one-dimensional Langevin spring [Haward and Thackray, 1968], which was generalized to 3D with the eight-chain model by Arruda and Boyce [1993].

The main structure of the model presented by Boyce et al. [2000] is kept for this model. Recognizing the large elastic deformations occurring for polymers, a formulation based

on a Neo-Hookean material is here selected for describing the spring in resistance A in Figure M168-1.

Referring to Figure M168-1, it is assumed that the deformation gradient tensor is the same for the two resistances (Part A and B)

$$\mathbf{F} = \mathbf{F}_A = \mathbf{F}_B$$

while the Cauchy stress tensor for the system is assumed to be the sum of the Cauchy stress tensors for the two parts

$$\sigma = \sigma_A + \sigma_B \ .$$

**Part A: Inter-Molecular Resistance:**

The deformation is decomposed into elastic and plastic parts, $\mathbf{F}_A = \mathbf{F}_A^e \mathbf{F}_A^p$, where it is assumed that the intermediate configuration $\overline{\Omega}_A$ defined by $\mathbf{F}_A^p$ is invariant to rigid body rotations of the current configuration. The velocity gradient in the current configuration $\Omega$ is defined by

$$\mathbf{L}_A = \dot{\mathbf{F}}_A \mathbf{F}_A^{-1} = \mathbf{L}_A^e + \mathbf{L}_A^p$$

Owing to the decomposition, $\mathbf{F}_A = \mathbf{F}_A^e \mathbf{F}_A^p$, the elastic and plastic rate-of-deformation and spin tensors are defined by

$$\begin{aligned} \mathbf{L}_A^e &= \mathbf{D}_A^e + \mathbf{W}_A^e = \dot{\mathbf{F}}_A^e (\mathbf{F}_A^e)^{-1} \\ \mathbf{L}_A^p &= \mathbf{D}_A^p + \mathbf{W}_A^p = \mathbf{F}_A^e \dot{\mathbf{F}}_A^p (\mathbf{F}_A^p)^{-1} (\mathbf{F}_A^e)^{-1} = \mathbf{F}_A^e \bar{\mathbf{L}}_A^p (\mathbf{F}_A^e)^{-1} \end{aligned}$$

where $\bar{\mathbf{L}}_A^p = \dot{\mathbf{F}}_A^p (\mathbf{F}_A^p)^{-1}$. The Neo-Hookean material represents an extension of Hooke's law to large elastic deformations and may be chosen for the elastic part of the deformation when the elastic behavior is assumed to be isotropic.

$$\boldsymbol{\tau}_A = \lambda_0 \ln J_A^e \ \mathbf{I} + \mu_0 (\mathbf{B}_A^e - \mathbf{I})$$

where $\boldsymbol{\tau}_A = J_A \sigma_A$ is the Kirchhoff stress tensor of Part A and $J_A^e = \sqrt{\det \mathbf{B}_A^e} = J_A$ is the Jacobian determinant. The elastic left Cauchy-Green deformation tensor is given by $\mathbf{B}_A^e = \mathbf{F}_A^e \mathbf{F}_A^{e\,T}$.

The flow rule is defined by

$$\mathbf{L}_A^p = \dot{\gamma}_A^p \mathbf{N}_A$$

where

$$\mathbf{N}_A = \frac{1}{\sqrt{2}\,\tau_A} \boldsymbol{\tau}_A^{\text{dev}}, \qquad \tau_A = \sqrt{\frac{1}{2} \text{tr}\left(\boldsymbol{\tau}_A^{\text{dev}}\right)^2}$$

and $\tau_A^{\text{dev}}$ is the stress deviator. The rate of flow is taken to be a thermally activated process

$$\dot{\gamma}_A^p = \dot{\gamma}_{0A}\exp\left[-\frac{\Delta G(1-\tau_A/s)}{k\theta}\right]$$

where $\dot{\gamma}_{0A}$ is a pre-exponential factor, $\Delta G$ is the energy barrier to flow, $s$ is the shear resistance, $k$ is the Boltzmann constant and $\theta$ is the absolute temperature. The shear resistance, $s$, is assumed to depend on the stress triaxiality, $\sigma^*$:

$$s = s(\sigma^*), \quad \sigma^* = \frac{\text{tr}\,\boldsymbol{\sigma}_A}{3\sqrt{3}\tau_A} \ .$$

The exact dependence is given by a user-defined load curve, which is linear between the shear resistances in compression and tension. These resistances are denoted $s_c$ and $s_t$, respectively.

### Part B: Network Resistance:

The network resistance is assumed to be nonlinear elastic with deformation gradient $\mathbf{F}_B = \mathbf{F}_B^N$, meaning, any viscoplastic deformation of the network is neglected. The stress-stretch relation is defined by

$$\boldsymbol{\tau}_B = \frac{nk\theta}{3}\frac{\sqrt{N}}{\bar{\lambda}_N}\mathcal{L}^{-1}\left(\frac{\bar{\lambda}_N}{\sqrt{N}}\right)(\bar{\mathbf{B}}_B^N - \bar{\lambda}_N^2\mathbf{I})$$

where $\boldsymbol{\tau}_B = J_B\boldsymbol{\sigma}_B$ is the Kirchhoff stress for Part B, $n$ is the chain density and $N$ the number of "rigid links" between entanglements. In accordance with Boyce et. al [2000], the product, $nk\theta$, is denoted $C_R$ herein. Moreover, $\mathcal{L}^{-1}$ is the inverse Langevin function, $\mathcal{L}(\beta) = \coth\beta - 1/\beta$, and further

$$\bar{\mathbf{B}}_B^N = \bar{\mathbf{F}}_B^N\bar{\mathbf{F}}_B^{N^T}, \quad \bar{\mathbf{F}}_B^N = J_B^{-1/3}\,\mathbf{F}_B^N, \quad J_B = \det\mathbf{F}_B^N, \quad \bar{\lambda}_N = \left[\frac{1}{3}\text{tr}\,\bar{\mathbf{B}}_B^N\right]^{\frac{1}{2}}$$

The flow rule defining the rate of molecular relaxation reads

$$\mathbf{L}_B^F = \dot{\gamma}_B^F\mathbf{N}_B$$

where

$$\mathbf{N}_B = \frac{1}{\sqrt{2}\,\tau_B}\boldsymbol{\tau}_B^{\text{dev}}, \qquad \tau_B = \sqrt{\frac{1}{2}\boldsymbol{\tau}_B^{\text{dev}}:\boldsymbol{\tau}_B^{\text{dev}}}$$

The rate of relaxation is taken equal to

$$\dot{\gamma}_B^F = C\left(\frac{1}{\bar{\lambda}_F - 1}\right)\tau_B$$

where

$$\bar{\lambda}_F = \left[\frac{1}{3}\text{tr}\left(\mathbf{F}_B^F\{\mathbf{F}_B^F\}^T\right)\right]^{\frac{1}{2}}$$

The model has been implemented into LS-DYNA using a semi-implicit stress-update scheme [Moran et. al 1990], and is available for the explicit solver only.

## *MAT_ARUP_ADHESIVE

This is Material Type 169. This material model was created for adhesive bonding in aluminum structures. The plasticity model is not volume-conserving, so it avoids the spuriously high tensile stresses that can develop when modeling adhesive with traditional elasto-plastic material models. It is available *only* for solid elements of formulations 1, 2 and 15. Unless THKDIR = 1, the smallest dimension of the element is assumed to be the through-thickness dimension of the bond.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | TENMAX | GCTEN | SHRMAX | GCSHR |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| PWRT | PWRS | SHRP | SHT_SL | EDOT0 | EDOT2 | THKDIR | EXTRA |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is included if EXTRA = 1 or 3.

| TMAXE | GCTE | SMAXE | GCSE | PWRTE | PWRSE | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is included if EXTRA = 1 or 3.

| FACET | FACCT | FACES | FACCS | SOFTT | SOFTS | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is included when EDOT2 ≠ 0.

| SDFAC | SGFAC | SDEFAC | SGEFAC | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is included if EXTRA = 2 or 3.

| BTHK | OUTFAIL | FSIP | FBR713 | ELF2NS | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | TENMAX | GCTEN | SHRMAX | GCSHR |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | $10^{20}$ | $10^{20}$ | $10^{20}$ | $10^{20}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| TENMAX | Maximum through-thickness tensile stress (see Remark 7):<br>GT.0.0: Constant value<br>LT.0.0: \|TENMAX\| is a function ID. |
| GCTEN | Energy per unit area to fail the bond in tension (see Remark 7):<br>GT.0.0: Constant value<br>LT.0.0: \|GCTEN\| is a function ID. |
| SHRMAX | Maximum through-thickness shear stress (see Remark 7):<br>GT.0.0: Constant value<br>LT.0.0: \|SHRMAX\| is a function ID. |
| GCSHR | Energy per unit area to fail the bond in shear (see Remark 7):<br>GT.0.0: Constant value<br>LT.0.0: \|GCSHR\| is a function ID. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PWRT | PWRS | SHRP | SHT_SL | EDOT0 | EDOT2 | THKDIR | EXTRA |
| Type | F | F | F | F | F | F | F | F |
| Default | 2.0 | 2.0 | 0.0 | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| PWRT | Power law term for tension |
| PWRS | Power law term for shear |
| SHRP | Shear plateau ratio (optional):<br>GT.0.0: Constant value<br>LT.0.0: \|SHRP\| is a function ID (see Remark 7). |
| SHT_SL | Slope (non-dimensional) of yield surface at zero tension (see Remark 3) |
| EDOT0 | Strain rate at which the "static" properties apply |
| EDOT2 | Strain rate at which the "dynamic" properties apply (Card 5) |
| THKDIR | Through-thickness direction flag (see Remark 1):<br>EQ.0.0: Smallest element dimension (default)<br>EQ.1.0: Direction from nodes 1-2-3-4 to nodes 5-6-7-8 |
| EXTRA | Flag to input further data:<br>EQ.1.0: Interfacial failure properties (Cards 3 and 4)<br>EQ.2.0: Bond thickness and more (Card 6)<br>EQ.3.0: Both of the above |

**Interfacial Failure Properties Card.** Additional card for EXTRA = 1 or 3.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMAXE | GCTE | SMAXE | GCSE | PWRTE | PWRSE | | |
| Type | F | F | F | F | F | F | | |
| Default | $10^{20}$ | $10^{20}$ | $10^{20}$ | $10^{20}$ | 2.0 | 2.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMAXE | Maximum tensile force per unit length on edges of joint |
| GCTE | Energy per unit length to fail the edge of the bond in tension |
| SMAXE | Maximum shear force per unit length on edges of joint |
| GCSE | Energy per unit length to fail the edge of the bond in shear |
| PWRTE | Power law term for tension |
| PWRSE | Power law term for shear |

**Interfacial Failure Properties Card.** Additional card for EXTRA = 1 or 3.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FACET | FACCT | FACES | FACCS | SOFTT | SOFTS | | |
| Type | F | F | F | F | F | F | | |
| Default | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FACET | Stiffness scaling factor for edge elements – tension |
| FACCT | Stiffness scaling factor for interior elements – tension |
| FACES | Stiffness scaling factor for edge elements – shear |
| FACCS | Stiffness scaling factor for interior elements – shear |

| VARIABLE | DESCRIPTION |
|---|---|
| SOFTT | Tensile strength reduction factor applied when a neighbor fails |
| SOFTS | Shear strength reduction factor applied when a neighbor fails |

**Dynamic Strain Rate Card.** Additional card for EDOT2 ≠ 0.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SDFAC | SGFAC | SDEFAC | SGEFAC | | | | |
| Type | F | F | F | F | | | | |
| Default | 1.0 | 1.0 | 1.0 | 1.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SDFAC | Factor on TENMAX and SHRMAX at strain rate EDOT2:<br>GT.0.0: Constant value<br>LT.0.0: \|SDFAC\| is a function ID (see Remark 7). |
| SGFAC | Factor on GCTEN and GCSHR at strain rate EDOT2:<br>GT.0.0: Constant value<br>LT.0.0: \|SGFAC\| is a function ID (see Remark 7). |
| SDEFAC | Factor on TMAXE and SMAXE at strain rate EDOT2 |
| SGEFAC | Factor on GCTE and GCSE at strain rate EDOT2 |

**Bond Thickness Card.** Additional card for EXTRA = 2 or 3.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | BTHK | OUTFAIL | FSIP | FBR713 | ELF2NS | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| BTHK | Bond thickness (overrides thickness from element dimensions; see Remark 1):<br>LT.0.0: \|BTHK\| is bond thickness, but critical time step remains unaffected. Helps to avoid very small time steps, but it can affect stability. |
| OUTFAIL | Flag for additional output to messag file which includes information about damage initiation time, failure function terms and forces:<br>EQ.0.0: Off<br>EQ.1.0: On |
| FSIP | Effective in-plane strain at failure |
| FBR713 | Fallback option to get results from previous version. See Remark 8.<br>EQ.0.0: Off<br>EQ.1.0: LS-DYNA release R7.1.3 |
| ELF2NS | Volumetric smearing option for ELFORM = 2. See Remark 9.<br>EQ.0.0: Usual ELFORM = 2 behavior with volumetric smearing<br>EQ.1.0: Volumetric smearing is turned off. |

**Remarks:**

1. **Through-thickness direction and bond thickness.** The through-thickness direction is identified from the smallest dimension of each element by default (THKDIR = 0.0). It is expected that this dimension will be smaller than in-plane dimensions (typically 1 - 2 mm compared with 5 - 10 mm). If this is not the case, one can set the through-thickness direction using element numbering (THKDIR = 1.0). Then the thickness direction is expected to point from lower face (nodes 1-2-3-4) to upper face (nodes 5-6-7-8). For wedge elements, these faces are the two triangular faces (nodes 1-2-5) and (nodes 3-4-6).

   The bond thickness is assumed to be the element size in the thickness direction. This may be overridden using BTHK. In this case the behavior becomes independent of the element thickness. The elastic stiffness is affected by BTHK, so it is necessary to set the characteristic element length to a smaller value

$$l_e^{\text{new}} = \sqrt{\text{BTHK} \times l_e^{\text{old}}} \ .$$

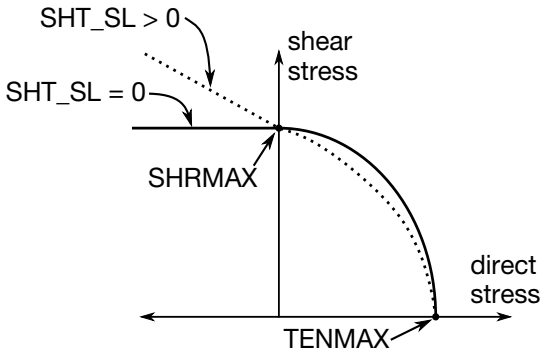



**Figure M169-1.** Figure illustrating the yield surface

This again affects the critical time step of the element, that is, a small BTHK can decrease the element time step significantly.

2. **Bond stiffness and strength.** In-plane stresses are set to zero: it is assumed that the stiffness and strength of the substrate are large compared with that of the adhesive, given the relative thicknesses.

   If the substrate is modeled with shell elements, it is expected that these will lie at the mid-surface of the substrate geometry. Therefore, the solid elements representing the adhesive will be thicker than the actual bond. If the elastic compliance of the bond is significant, this can be corrected by increasing the elastic stiffness property $E$.

3. **Stress and failure.** The yield and failure surfaces are treated as a power-law combination of direct tension and shear across the bond:

$$\left(\frac{\sigma}{\sigma_{\max}}\right)^{\text{PWRT}} + \left(\frac{\tau}{\tau_{\max} - \text{SHT_SL} \times \sigma}\right)^{\text{PWRS}} = 1.0$$

   At yield SHT_SL is the slope of the yield surface at $\sigma = 0$. See Figure M169-1.

   The stress-displacement curves for tension and shear are shown in Figure M169-2. In both cases, GC is the area under the curve. The displacement to failure in tension is given by

$$d_{\text{ft}} = 2\left(\frac{\text{GCTEN}}{\text{TENMAX}}\right) \ ,$$

   subject to a lower limit

$$d_{\text{ft, min}} = \left(\frac{2L_0}{E'}\right)\text{TENMAX} \ ,$$

   where $L_0$ is the initial element thickness (or BTHK if used) and

$$E' = \frac{E(1-\nu)}{(1-2\nu)(1+\nu)} \ .$$

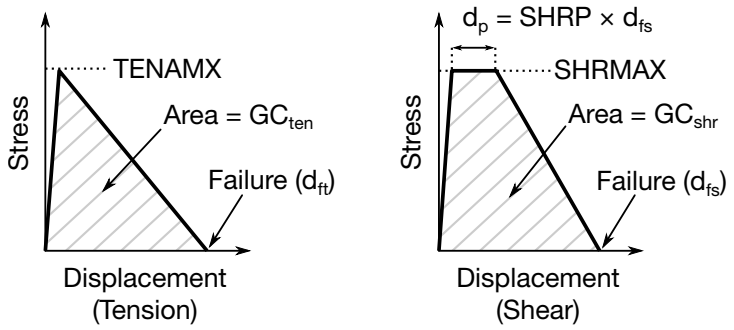



**Figure M169-2.** Stress-Displacement Curves for Tension and Shear

If GCTEN is input such that $d_{\mathrm{ft}} < d_{\mathrm{ft,\,min}}$, LS-DYNA will automatically increase GCTEN to make $d_{\mathrm{ft}} = d_{\mathrm{ft,\,min}}$. Therefore, GCTEN has a minimum value of

$$\mathrm{GCTEN} \geq \frac{L_0}{E'}(\mathrm{TENMAX})^2$$

Similarly, the minimum value for GCSHR is

$$\mathrm{GCSHR} \geq \frac{L_0}{G}(\mathrm{SHRMAX})^2$$

where $G$ is the elastic shear modulus.

Because of the algorithm used, yielding in tension across the bond does not require strains in the plane of the bond – unlike the plasticity models, plastic flow is not treated as volume-conserving.

4. **Output variables.** The plastic strain output variable, PS, has a special meaning:

   0 < PS < 1: PS is the maximum value of the yield function experienced since time zero.

   1 < PS < 2: The element has yielded, and the strength is reducing towards failure – yields at PS = 1, fails at PS = 2.

Extra history variables may be requested for solid elements (NEIPH on *DATABASE_EXTENT_BINARY). They are described in the following table.

| History Variable # | Description |
|---|---|
| 1 | Damage caused by cohesive deformation on a scale of 0 at first yield to 1 at failure |
| 2 | Damage caused by interfacial deformation (see Remark 6) on a scale of 0 at first yield to 1 at failure |
| 5 | Current thickness dimension of the element |
| 6 | Current strain rate (relevant if EDOT0 and EDOT2 are defined, see Remark 5) |

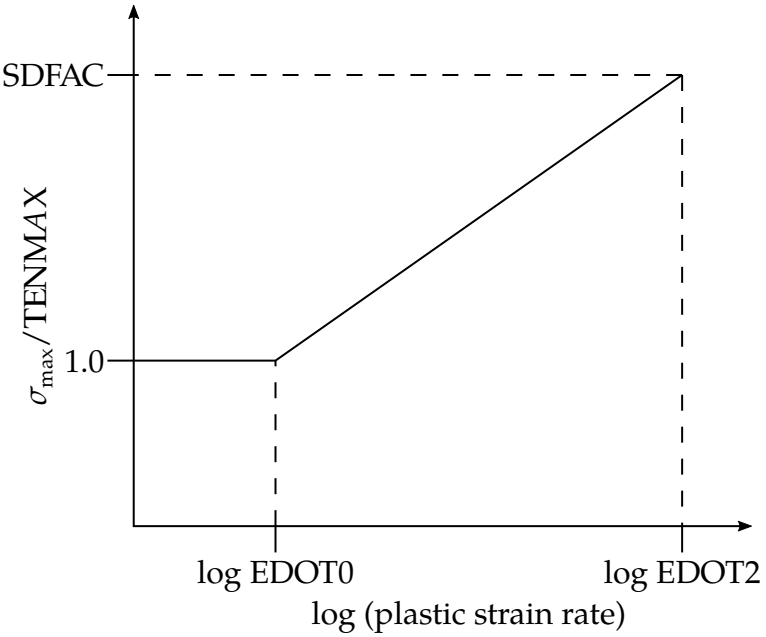



**Figure M169-3**. Figure illustrating rate effects

| History Variable # | Description |
|---|---|
| 10 | Direct stress in the local $z$-direction (bond tensile stress) |
| 12 | Through-thickness shear stress in the local $yz$-direction |
| 13 | Through-thickness shear stress in the local $zx$-direction |

5. **Rate effects.** When the plastic strain rate rises above EDOT0, rate effects are assumed to scale with the logarithm of the plastic strain rate, as in the example shown in Figure M169-3 for cohesive tensile strength with dynamic factor SDFAC. The same form of relationship is applied for the other dynamic factors. If EDOT0 is zero or blank, no rate effects are applied. Rate effects are applied using the viscoplastic method.

6. **Interfacial failure.** Interfacial failure is assumed to arise from stress concentrations at the edges of the bond – typically the strength of the bond becomes almost independent of bond length. This type of failure is usually more brittle than cohesive failure. To simulate this, LS-DYNA identifies the free edges of the bond (made up of element faces that are not shared by other elements of material type *MAT_ARUP_ADHESIVE, excluding the faces that bond to the substrate). Only these elements can fail initially. The neighbors of failed elements can then develop free edges and fail in turn.

   In real adhesive bonds, the stresses at the edges can be concentrated over very small areas; in typical finite element models the elements are much too large to capture this. Therefore, the concentration of loads onto the edges of the bond is accomplished artificially by stiffening elements containing free edges (e.g.,

FACET, FACES > 1) and reducing the stiffness of interior elements (e.g., FACCT, FACCS < 1). Interior elements are allowed to yield at reduced loads (equivalent to TMAXE × FACET/FACCT and SMAXE × FACES/FACCS) to prevent excessive stresses from developing before the edge elements have failed - but cannot be damaged until they become edge elements after the failure of their neighbors.

7. **Function arguments.** Parameters TENMAX, GCTEN, SHRMAX, GCSHR, SHRP, SDFAC, and SGFAC can be defined as negative values. In that case, the absolute values refer to *DEFINE_FUNCTION IDs. The arguments of those functions include several properties of both connection partners if corresponding solid elements are in a tied contact with shell elements.

   These functions depend on:

   (t1, t2) = thicknesses of both bond partners
   (sy1, sy2) = initial yield stresses at plastic strain of 0.002
   (sm1, sm2) = maximum engineering yield stresses (necking points)
   r = strain rate
   a = element area
   (e1, e2) = Young's moduli

   For TENMAX = -100 such a function could look like:

```
*DEFINE_FUNCTION
       100
func(t1,t2,sy1,sy2,sm1,sm2,r,a,e1,e2)=0.5*(sy1+sy2)
```

   Since material parameters must be identified from both bond partners during initialization, this feature is only available for a subset of material models at the moment, namely material models 24, 36, 120, 123, 124, 251, and 258.

8. **Older versions.** Some corrections were made to this material model that can cause results to be different in R8 and later versions compared to R7.1.3 and earlier versions. To avoid recalibration of old material data, it is possible to recover previous results with option FBR713 = 1. The corrections were related to the post-yield stress-strain response not matching the description in the manual, with the difference being most noticeable when (a) the elastic stiffness was low, such that the elastic displacement to yield was of the same order as the element thickness; or (b) when the power-law terms PWRS, PWRT were not both equal to 2, and strain rate effects were specified (EDOT2, SDFAC).

9. **Volumetric smearing.** The element formulation given by ELFORM = 2 on *SECTION_SOLID smears the volumetric strain across the eight integration points. This smearing can sometimes cause an unstable dynamic response with *MAT_ARUP_ADHESIVE. The smearing can be turned off by setting ELF2NS to 1. ELFNS has no effect for other element formulations or when FBR713 is nonzero.

## *MAT_RESULTANT_ANISOTROPIC

This is Material Type 170. This model is available for the Belytschko-Tsay and the C0 triangular shell elements. It is based on a resultant stress formulation. In-plane behavior is treated separately from bending for modeling perforated materials, such as television shadow masks. The plastic behavior of each resultant is specified with a load curve and is completely uncoupled from the other resultants. If other shell formulations are specified, the formulation will be automatically switched to Belytschko-Tsay. As implemented, this material model cannot be used with user defined integration rules.

**NOTE:** This material does not support specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E11P | E22P | V12P | V21P | G12P | G23P | G31P | |
| Type | F | F | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E11B | E22B | V12B | V21B | G12B | AOPT | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LN11 | LN22 | LN12 | LQ1 | LQ2 | LM11 | LM22 | LM12 |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| RO | Mass density |
| E11P | $E_{11p}$, for in-plane behavior |
| E22P | $E_{22p}$, for in-plane behavior |
| V12P | $\nu_{12p}$, for in-plane behavior |
| V11P | $\nu_{11p}$, for in-plane behavior |
| G12P | $G_{12p}$, for in-plane behavior |
| G23P | $G_{23p}$, for in-plane behavior |
| G31P | $G_{31p}$, for in-plane behavior |
| E11B | $E_{11b}$, for bending behavior |
| E22B | $E_{22b}$, for bending behavior |
| V12B | $\nu_{12b}$, for bending behavior |
| V21B | $\nu_{21b}$, for bending behavior |
| G12B | $G_{12b}$, for bending behavior |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| LN11 | Yield curve ID for $N_{11}$, the in-plane force resultant |
| LN22 | Yield curve ID for $N_{22}$, the in-plane force resultant |
| LN12 | Yield curve ID for $N_{12}$, the in-plane force resultant |
| LQ1 | Yield curve ID for $Q_1$, the transverse shear resultant |
| LQ2 | Yield curve ID for $Q_2$, the transverse shear resultant |
| LM11 | Yield curve ID for $M_{11}$, the moment |
| LM22 | Yield curve ID for $M_{22}$, the moment |
| LM12 | Yield curve ID for $M_{12}$, the moment |
| A1, A2, A3 | $(a_1, a_2, a_3)$, components of vector **a** for AOPT = 2 |
| V1, V2, V3 | $(v_1, v_2, v_3)$, components of vector **v** for AOPT = 3 |
| D1, D2, D3 | $(d_1, d_2, d_3)$, components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

The in-plane elastic matrix for in-plane, plane stress behavior is given by:

$$\mathbf{C}_{\text{in plane}} = \begin{bmatrix} Q_{11p} & Q_{12p} & 0 & 0 & 0 \\ Q_{12p} & Q_{22p} & 0 & 0 & 0 \\ 0 & 0 & Q_{44p} & 0 & 0 \\ 0 & 0 & 0 & Q_{55p} & 0 \\ 0 & 0 & 0 & 0 & Q_{66p} \end{bmatrix}$$

The terms $Q_{ijp}$ are defined as:

$$Q_{11p} = \frac{E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{22p} = \frac{E_{22p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{12p} = \frac{\nu_{12p}E_{11p}}{1 - \nu_{12p}\nu_{21p}}$$

$$Q_{44p} = G_{12p}$$

$$Q_{55p} = G_{23p}$$

$$Q_{66p} = G_{31p}$$

The elastic matrix for bending behavior is given by:

$$\mathbf{C}_{\text{bending}} = \begin{bmatrix} Q_{11b} & Q_{12b} & 0 \\ Q_{12b} & Q_{22b} & 0 \\ 0 & 0 & Q_{44b} \end{bmatrix}$$

The terms $Q_{ijp}$ are similarly defined.

Because this is a resultant formulation, no stresses are output to d3plot, and forces and moments are reported to elout in place of stresses.

## *MAT_STEEL_CONCENTRIC_BRACE

This is Material Type 171. It represents the cyclic buckling and tensile yielding behavior of steel braces and is intended primarily for seismic analysis. Use only for beam elements with ELFORM = 2 (Belytschko-Schwer beam).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | YM | PR | SIGY | LAMDA | FBUCK | FBUCK2 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | optional | optional | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CCBRF | BCUR | EPTCRIT | DAMF1 | DAMF2 | DAMEP1 | DAMEP2 | |
| Type | F | F | F | F | F | F | F | |
| Default | optional | optional | 0.01 | optional | optional | optional | optional | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TS1 | TS2 | TS3 | TS4 | CS1 | CS2 | CS3 | CS4 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | TS1 | TS2 | TS3 | TS4 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| YM | Young's modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio |
| SIGY | Yield stress |
| LAMDA | Slenderness ratio, $\lambda$ (optional – see remarks) |
| FBUCK | Initial buckling load (optional – see remarks. If used, should be positive) |
| FBUCK2 | Optional extra term in initial buckling load – see remarks |
| CCBRF | Reduction factor on initial buckling load for cyclic behavior |
| BCUR | Optional load curve giving compressive buckling load ($y$-axis) as a function of compressive strain ($x$-axis - both positive) |
| EPTCRIT | Tensile plastic strain to reduce buckling strength to cyclic value |
| DAMF1 | FEMA threshold at which damage begins (see Remark 5).<br>EQ.0: No damage or failure based on FEMA thresholds |
| DAMF2 | FEMA threshold at which element is eroded, applicable only if DAMF1 > 0 |
| DAMEP1 | Cumulative plastic strain at which damage begins (see Remark 5).<br>EQ.0: No damage or failure based on plastic strain |
| DAMEP2 | Cumulative plastic strain at which element is eroded, applicable only if DAMEP1 > 0 |
| TS1 - TS4 | Tensile axial strain FEMA thresholds 1 to 4 (see Remark 3) |
| CS1 - CS4 | Compressive axial strain FEMA thresholds 1 to 4 (see Remark 3) |

**Remarks:**

1. **General.** The brace element is intended to represent the buckling, yielding and cyclic behavior of steel elements, such as tubes or I-sections, that carry only axial loads. A single beam element should be used to represent each structural element. Empirical relationships are used to determine the buckling and cyclic load-deflection behavior. Details of the axial response are given after the Remarks.

2. **Strain Definitions.** Output variables, and the damage and failure treatment, refer to the following strain definitions, all of which relate to the axial direction of the beam element:

   a) Total strain: Change of length divided by initial length, positive in tension.

   b) Plastic strain: Current inelastic strain, defined as total strain minus elastic strain. It is positive for tensile strains, negative for compressive strains. The term "plastic strain" is used here irrespective of whether the inelastic behavior represents yielding or buckling.

   c) High-tide plastic strain: Maximum plastic strain that has occurred during the analysis. Separate values are recorded for tensile and compressive plastic strains.

   d) Cumulative plastic strain: The sum of the absolute values of the plastic strain increments. The cumulative plastic strain increases whenever yielding or buckling occurs. For cyclic loading in the plastic or buckling regimes, this strain measure increases with each cycle.

3. **FEMA Thresholds.** FEMA thresholds are used in performance-based earthquake engineering to classify the response into categories such as "Elastic", "Immediate Occupancy", "Life Safe", etc., according to the level of deformation of each structural element. During the analysis, the maximum high-tide tensile and compressive plastic strains are recorded. These are checked against the user-defined limits TS1 to TS4 and CS1 to CS4. The output flag is then set to 0, 1, 2, 3, or 4 according to which limits have been passed. The value in the output files is the highest such flag from tensile or compressive strains.

4. **Output.** In addition to the six resultants written for all beam elements, this material model writes further extra history variables to the d3plot and d3thdt files, given in the table below. The data is written in the same position in these files as where integrated beams write the stresses and strains at integration points requested by BEAMIP on *DATABASE_EXTENT_BINARY. Therefore, some post-processors may interpret this data as if the elements were integrated beams with 4 integration points, and in that case the data may be accessed by selecting the appropriate integration point and data component:

| Int. Point | Data component in post-processor | Description |
|---|---|---|
| 1 | XX(RR) axial stress | Total axial deformation/strain |
| 3 | ZX(TR) shear stress | Internal energy |

| Int. Point | Data component in post-processor | Description |
|---|---|---|
| 4 | XX(RR) axial stress | Current buckling/yield force in compression |
| 4 | XY(RS) shear stress | Tensile high-tide plastic strain |
| 4 | ZX(TR) shear stress | Compressive high-tide plastic strain |
| 4 | Equivalent plastic strain | Cumulative plastic strain |
| 4 | XX(RR) axial strain | FEMA flag |

5. **Damage and failure.** Optionally, damage and failure (element erosion) can be modelled. DAMF1 and DAMF2 control damage and failure based on high tide plastic strains. DAMEP1 and DAMEP2 control damage and failure based on cumulative plastic strain (fatigue-type damage). A combination of both of the above is obtained if all four input parameters are defined.

   DAMF1 and DAMF2 refer to the "FEMA" output flag (see Remark 3 above). Only integer values 0, 1, 2, 3 or 4 are meaningful because those are the possible values of the FEMA output flag. DAMEP1 and DAMEP2 refer to values of cumulative plastic strain.

   Damage is modelled with a scaling factor, $D$, that multiplies the stiffness and strength of the element. DAMF1 and DAMEP1 define thresholds at which damage begins. Until that point is reached, the damage algorithm has no effect and $D = 1$. DAMF2 and DAMEP2 define the threshold at which damage is complete. At that point, $D = 0$, meaning the element has no remaining stiffness or strength and is deleted. Between DAMF1 and DAMF2 and between DAMEP1 and DAMEP2, $D$ ramps down linearly from $D = 1.0$, when damage begins, to $D = 0.0$, when damage is complete.

   If both damage mechanisms are modelled (i.e., DAMF1, DAMF2, DAMEP1, DAMEP2 are all nonzero), the damage scaling factors for the two mechanisms are multiplied together. Thus, damage and failure can occur by either mechanism depending on which thresholds are reached first.

**Axial response:**

The cyclic behavior is shown in Figure M171-1 (compression shown as negative force and displacement). The initial buckling load (point 2) is:

$$F_{b,\,\text{initial}} = \text{FBUCK} + \frac{\text{FBUCK2}}{L^2} \ ,$$

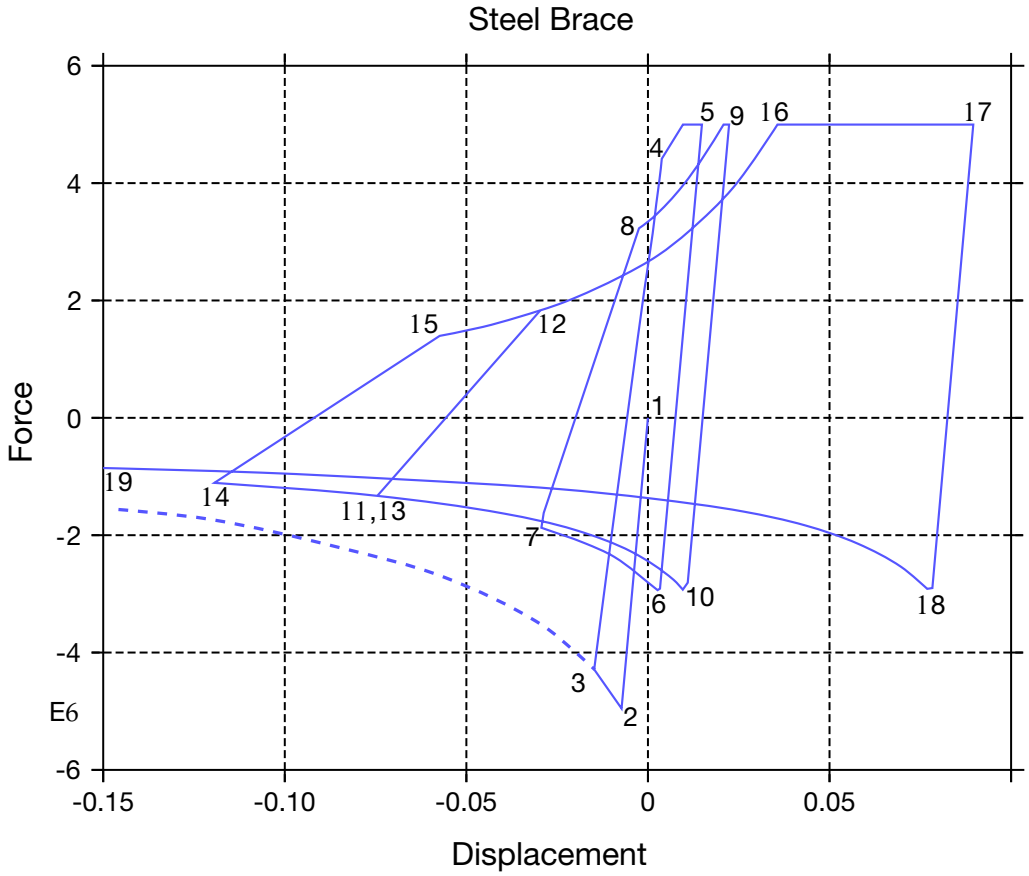



**Figure M171-1.** Cyclic Behavior of a Steel Brace

where FBUCK and FBUCK2 are input parameters, and $L$ is the length of the beam element. If neither FBUCK nor FBUCK2 is defined, the default is that the initial buckling load is

$$\text{SIGY} \times \text{A},$$

where $A$ is the cross sectional area. The buckling curve (shown dashed) has the form:

$$F(d) = \frac{F_{b,\,\text{initial}}}{\sqrt{A\delta + B}}$$

where $\delta$ is |strain/yield strain|, and $A$ and $B$ are internally calculated functions of slenderness ratio ($\lambda$) and loading history.

The member slenderness ratio, $\lambda$, is defined as $\frac{kL}{r}$, where $k$ depends on end conditions, $L$ is the element length, and $r$ is the radius of gyration such that $Ar^2 = I$ (and $I = \min(I_{yy}, I_{zz})$); $\lambda$ will by default be calculated from the section properties and element length using $k = 1$. Optionally, this may be overridden by input parameter LAMDA to allow for different end conditions.

Optionally, you may provide a buckling curve BCUR. The points of the curve give compressive displacement ($x$-axis) as a function of force ($y$-axis); the first point should have

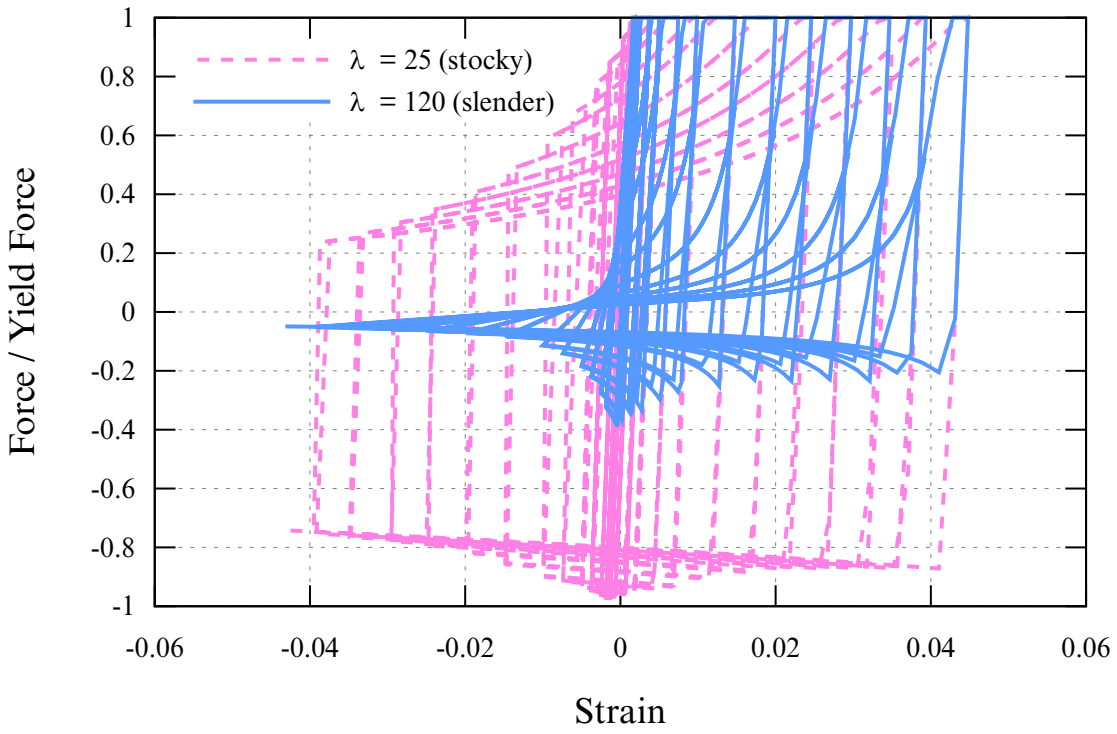



**Figure M171-2.** Comparing the stress-strain response for two values of $\lambda$

zero displacement and the initial buckling force. Displacement and force should both be positive. The initial buckling force must not be greater than the yield force.

The tensile yield force (point 5 and segment 16-17 in Figure M171-1) is defined by

$$F_y = \text{SIGY} \times A,$$

where yield stress SIGY is an input parameter and $A$ is the cross-sectional area.

Following initial buckling and subsequent yield in tension, the member is assumed to be damaged. The initial buckling curve is then scaled by input parameter CCBRF, leading to reduced strength curves such as segments 6-7, 10-14 and 18-19. This reduction factor is typically in the range 0.6 to 1.0 (smaller values for more slender members). By default, CCBRF is calculated using SEAOC 1990:

$$\text{CCBRF} = \frac{1}{\left(1 + \dfrac{0.5\lambda}{\pi\sqrt{\dfrac{E}{0.5\sigma_y}}}\right)}$$

When tensile loading is applied after buckling, the member must first be straightened before the full tensile yield force can be developed. This is represented by a reduced unloading stiffness (such as segment 14-15) and the tensile reloading curve (segments 8-9 and 15-16). Further details can be found in Bruneau, Uang, and Whittaker [1998] and Structural Engineers Association of California [1974, 1990, 1996].

The response of stocky (low $\lambda$) and slender (high $\lambda$) braces are compared in Figure M171-2. These differences are achieved by altering the input value LAMDA (or the section properties of the beam) and FBUCK.

## *MAT_CONCRETE_EC2

This is Material Type 172. This model is available for shell, thick shell (formulations 1, 2, and 6), and Hughes-Liu beam elements. The material model can represent plain concrete only, reinforcing steel only, or a smeared combination of concrete and reinforcement. The model includes concrete cracking in tension and crushing in compression and reinforcement yield, hardening, and failure. Properties are thermally sensitive; the material model can be used for fire analysis. Material data and equations governing the behavior (including thermal properties) are taken from Eurocode 2 (EC2). See the remarks below for more details on how the standard is applied in the material model.

Although the material model offers many options, a reasonable response may be obtained by entering only RO, FC, and FT for plain concrete. If reinforcement is present, YMREINF, SUREINF, FRACRX, and FRACRY must be defined, or for an alternative way to model the reinforcement, see *MAT_203/*MAT_HYSTERETIC_REINFORCEMENT. Note that, from release R10 onwards, the number of possible cracks has been increased from 2 (0 and 90 degrees) to 4 (see TYPEC on Card 1 and Tensile response under the Material Behavior of Concrete section).

> **NOTE:** This material does not support the specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell (ICOMP = 1 on *SECTION_SHELL, B*i* on *PART_COMPOSITE or B*i* on *ELEMENT_SHELL_COMPOSITE).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | FC | FT | TYPEC | UNITC | ECUTEN | FCC |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| ESOFT | LCHAR | MU | TAUMXF | TAUMXC | ECRAGG | AGGSZ | UNITL |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| YMREINF | PRREINF | SUREINF | TYPER | FRACRX | FRACRY | LCRSU | LCALPS |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | ET36 | PRT36 | ECUT36 | LCALPC | DEGRAD | ISHCHK | UNLFAC |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card if AOPT > 0.0.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card if AOPT > 0.0.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 7.** Include this card if ISHCHK ≠ 0.

| TYPESC | P_OR_F | EFFD | GAMSC | ERODET | ERODEC | ERODER | TMPOFF |
|---|---|---|---|---|---|---|---|

**Card 7.1.** Include this card if ISHCHK ≠ 0 and TYPESC = 5, 6, 15, or 16.

| ASW1S | ASW1T | ASW2S | ASW2T | FCK | FYWD | BW | THETA |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if TYPEC = 6 or 9.

| EC1_6 | ECSP69 | GAMCE9 | PHIEF9 | LCRATE | UNITS | | |
|---|---|---|---|---|---|---|---|

**Card 9.** Include this card if FT < 0.0.

| FT2 | FTSHR | LCFTT | WRO_G | ZSURF | LCFIB | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | FC | FT | TYPEC | UNITC | ECUTEN | FCC |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | 0.0 | 1.0 | 1.0 | 0.0025 | ↓ |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| FC | Compressive strength of concrete (stress units). Its meaning depends on TYPEC. |

| VARIABLE | DESCRIPTION |
|---|---|
| | TYPEC.EQ.1,2,3,4,5,7,8: FC is the actual compressive strength.<br>TYPEC.EQ.6: FC is the unconfined compressive strength used in Mander equations.<br>TYPEC.EQ.9: FC is the characteristic compressive strength ($f_{ck}$ in EC2 1-1). See also FCC and the remarks below. |
| FT | Tensile stress to cause cracking. If FT < 0.0, then Card 9 is read. |
| TYPEC | Concrete aggregate type for stress-strain-temperature relationships (see Remark 3):<br>EQ.1.0: Siliceous (default), Draft EC2 Annex (fire engineering)<br>EQ.2.0: Calcareous, Draft EC2 Annex (fire engineering)<br>EQ.3.0: Non-thermally-sensitive using ET3, ECU3<br>EQ.4.0: Lightweight<br>EQ.5.0: Fiber-reinforced<br>EQ.6.0: Non-thermally-sensitive, Mander algorithm<br>EQ.7.0: Siliceous, EC2 1-2:2004 (fire engineering)<br>EQ.8.0: Calcareous, EC2 1-2:2004 (fire engineering)<br>EQ.9.0: EC2 1-1:2004 (general and buildings)<br>To obtain the pre-R10 behavior, that is, a maximum of 2 cracks, add 100 to TYPEC. For example, 109 means two cracks and EC2 1-1:2004 (general and buildings). |
| UNITC | Factor to convert stress units to MPa (used in shear capacity checks and for application of EC2 formulae when TYPEC = 9). For example, if the model units are Newtons and meters, UNITC = $10^{-6}$. |
| ECUTEN | Strain to fully open a crack |
| FCC | Relevant only if TYPEC = 6 or 9.<br>TYPEC.EQ.6: FCC is the compressive strength of confined concrete used in Mander equations. Default: unconfined properties are assumed (FCC = FC).<br>TYPEC.EQ.9: FCC is the actual compressive strength. If blank, this will be set equal to the mean compressive strength ($f_{cm}$ in EC2 1-1) as required for serviceability calculations (8MPa greater than FC). For |

| VARIABLE | DESCRIPTION |
|---|---|
| | ultimate load calculations, you can set FCC to a factored characteristic compressive strength. See remarks below. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ESOFT | LCHAR | MU | TAUMXF | TAUMXC | ECRAGG | AGGSZ | UNITL |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.4 | $10^{20}$ | 1.161 × FT | 0.001 | 0.0 | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ESOFT | Tension stiffening (slope of stress-strain curve post-cracking in tension). See Figure M172-1. |
| LCHAR | Characteristic length at which ESOFT applies. It is also used as crack spacing in aggregate-interlock calculations. |
| MU | Friction on crack planes (max shear = $\mu$ × compressive stress) |
| TAUMXF | Maximum friction shear stress on crack planes (ignored if AGGSZ > 0.0 - see remarks). |
| TAUMXC | Maximum through-thickness shear stress after cracking (see remarks). |
| ECRAGG | Strain parameter for aggregate interlock (ignored if AGGSZ > 0.0 - see remarks). |
| AGGSZ | Aggregate size (length units - used in NS3473 aggregate interlock formula - see remarks). |
| UNITL | Factor to convert length units to millimeters (used only if AGGSZ > 0.0 - see remarks). For example, if the model unit is meters, UNITL = 1000. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | YMREINF | PRREINF | SUREINF | TYPER | FRACRX | FRACRY | LCRSU | LCALPS |
| Type | F | F | F | F | F | F | I | I |
| Default | none | 0.0 | 0.0 | 2.0 | 0.0 | 0.0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| YMREINF | Young's modulus of reinforcement |
| PRREINF | Poisson's ratio of reinforcement |
| SUREINF | Ultimate stress of reinforcement |
| TYPER | Type of reinforcement for stress-strain-temperature relationships (see Remark 3):<br>EQ.1.0: Hot rolled reinforcing steel, Draft EC2 Annex (fire)<br>EQ.2.0: Cold worked reinforcing steel (default), Draft EC2 Annex (fire)<br>EQ.3.0: Quenched/tempered prestressing steel, Draft EC2 Annex (fire)<br>EQ.4.0: Cold worked prestressing steel, Draft EC2 Annex (fire)<br>EQ.5.0: Non-thermally sensitive using load curve LCRSU<br>EQ.7.0: Hot rolled reinforcing steel, EC2 1-2:2004 (fire)<br>EQ.8.0: Cold worked reinforcing steel, EC2 1-2:2004 (fire) |
| FRACRX | Fraction of reinforcement ($x$-axis). For example, to obtain 1% reinforcement set FRACRX = 0.01. See Remark 1. |
| FRACRY | Fraction of reinforcement ($y$-axis). For example, to obtain 1% reinforcement set FRACRY = 0.01. See Remark 1. |
| LCRSU | Load curve for TYPER = 5 giving non-dimensional factor on SUREINF as a function of plastic strain (overrides stress-strain function from EC2). |
| LCALPS | Optional load curve giving thermal expansion coefficient of reinforcement as a function of temperature (overrides function from EC2). |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | ET36 | PRT36 | ECUT36 | LCALPC | DEGRAD | ISHCHK | UNLFAC |
| Type | F | F | F | F | I | F | I | F |
| Default | 0.0 | 0.0 | 0.25 | ↓ | none | 0.0 | 0 | 0.5 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see *MAT_002 for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal. |
| ET36 | Young's modulus of concrete (TYPEC = 3 and 6). For other values of TYPEC, the Young's modulus is calculated internally (see remarks). |
| PRT36 | Poisson's ratio of concrete. Applies to all values of TYPEC. |
| ECUT36 | Strain to failure of concrete in compression (TYPEC = 3 and 6). See "Compressive response…" in the Material Behavior of Concrete section below. Default is 0.02 for TYPEC = 3 and 1.1 × EC1_6 for TYPEC = 6. |
| LCALPC | Optional load curve giving thermal expansion coefficient of concrete as a function of temperature – overrides relationship from EC2. |
| DEGRAD | If non-zero, the compressive strength of concrete parallel to an open crack will be reduced (see remarks). |

| VARIABLE | DESCRIPTION |
|---|---|
| ISHCHK | Set this flag to 1 to include Card 7 (shear capacity check and other optional input data). |
| UNLFAC | Stiffness degradation factor after crushing (0.0 to 1.0 – see Figure M172-4). |

Additional card for AOPT > 0.0.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Not used |
| A1, A2, A3 | Components of vector **a** for AOPT = 2.0 |

Additional card for AOPT > 0.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3.0 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2.0 |
| BETA | Material angle in degrees for AOPT = 3.0. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA |

Include if ISHCHK ≠ 0.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TYPESC | P_OR_F | EFFD | GAMSC | ERODET | ERODEC | ERODER | TMPOFF |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 2.0 | 0.01 | 0.05 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| TYPESC | Type of shear capacity check (see Remark 5):<br>EQ.1: BS 8110, no failure even if capacity is exceeded<br>EQ.2: ACI 318-05M, no failure even if capacity is exceeded<br>EQ.5: EC2 1-1:2004, for members without shear reinforcement, no failure even if capacity is exceeded<br>EQ.6: EC2 1-1:2004, for members with shear reinforcement, no failure even if capacity is exceeded<br>EQ.11: BS 8110, failure occurs if capacity is exceeded<br>EQ.12: ACI 318-05M, failure occurs if capacity is exceeded<br>EQ.15: EC2 1-1:2004, for members without shear reinforcement, failure occurs if capacity is exceeded<br>EQ.16: EC2 1-1:2004, for members with shear reinforcement, failure occurs if capacity is exceeded |
| P_OR_F | Meaning depends on the type of shear capacity check (see TYPESC). For TYPESC = 1 and 11, it is the percent reinforcement. For example, input 0.5 for 0.5% reinforcement. For TYPESC = 2 and 12, it is the ratio of cylinder strength to FC. The default value for this case is 1.0. For all other values of TYPESC, this field is not used. |
| EFFD | If used with shell elements, EFFD is the effective section depth $d$ (length units) used by all the types of shear capacity check. This is usually the section depth excluding the cover concrete. If used with beam elements, the meaning depends on the shear capacity check type. For beam elements with a BS8110 or ACI shear capacity check, EFFD is the product $b_w d$, where $b_w$ is the section width. For beam elements with EC2 shear capacity checks, EFFD is the effective depth $d$, while $b_w$ is input separately on Card 7.1. |

| VARIABLE | DESCRIPTION |
|---|---|
| GAMSC | Meaning depends on the type of shear capacity check (see TYPESC). For TYPESC = 1 and 11, it is a load factor used in the BS8110 shear capacity check. For TYPESC = 5, 6, 15, and 16, it is the material strength factor for concrete, $\gamma_c$. For all other values of TYPESC, this field is not used. |
| ERODET | Crack-opening strain at which element is deleted; see Remark 7. |
| ERODEC | Compressive strain used in erosion criteria; see Remark 7. |
| ERODER | Reinforcement plastic strain used in erosion criteria; see Remark 7. |
| TMPOFF | Constant to be added to the model's temperature unit to convert into degrees Celsius. For example, if the model's temperature unit is degrees Kelvin, set TMPOFF to -273. Degrees Celsius temperatures are then used throughout the material model for LCALPC and the default thermally-sensitive properties. |

Include if ISHCHK ≠ 0 and TYPESC = 5, 6, 15, or 16.

| Card 7.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ASW1S | ASW1T | ASW2S | ASW2T | FCK | FYWD | BW | THETA |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | ASW1S | 0.0 | 0.0 | FC | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ASW1S | The meaning of ASW1S depends on the value of TYPESC.<br>TYPESC.EQ.5: Percent tensile reinforcement, $\rho_l$. For example, if input 0.2 for 0.2%. This is the longitudinal reinforcement resisting shear in local $s$-direction (beams) or local $XZ$-plane (shells). See Remark 5.<br>TYPESC.EQ.6: Vertical (90°) shear reinforcement parameter $A_{sw}/s_h$ (see Remark 5) resisting shear in the local $s$-direction (beams) or $XZ$-plane (shells). |
| ASW1T | The meaning of ASW1T depends on the value of TYPESC. |

| VARIABLE | DESCRIPTION |
|---|---|
| | TYPESC.EQ.5: Percent tensile reinforcement $\rho_l$. For example, if 0.2%, input 0.2. This is the longitudinal reinforcement resisting shear in the local $t$-direction (beams) or $YZ$-plane (shells). See Remark 5. |
| | TYPESC.EQ.6: Vertical (90°) shear reinforcement parameter $A_{sw}/s_h$ (see Remark 5) resisting shear in the local $t$-direction (beams) or $YZ$-plane (shells). |
| ASW2S | \|ASW2S\| is the inclined (45°) shear reinforcement parameter $A_{sw}/s_h$ (see Remark 5) resisting shear in the local $s$-direction (beams) or $XZ$-plane (shells). It is applicable only for TYPESC = 6 or 16. |
| | GT.0.0: Inclined bars work for positive shear force only. |
| | LT.0.0: Inclined bars work for negative shear force only. |
| ASW2T | \|ASW2T\| is the inclined (45°) shear reinforcement parameter $A_{sw}/s_h$ (see Remark 5) resisting shear in the local $t$-direction (beams) or $YZ$-plane (shells). It is applicable only for TYPESC = 6 or 16. |
| | GT.0.0: Inclined bars work for positive shear force only. |
| | LT.0.0: Inclined bars work for negative shear force only. |
| FCK | Concrete characteristic cylinder strength $f_{ck}$ (stress units) to be used in shear capacity check |
| FYWD | Design yield strength of stirrups $f_{ywd}$ (stress units) (TYPESC = 6 or 16). |
| BW | Cross-section width $b_w$ (length units). Relevant to beam elements only. |
| THETA | Calculation method for the inclination of concrete compression strut (TYPESC = 6 or 16): |
| | EQ.0.0: Fixed inclination method with θ = 45° (default) |
| | EQ.1.0: Variable inclination method (see Remark 5) |

Additional card for TYPEC = 6 or 9.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EC1_6 | ECSP69 | GAMCE9 | PHIEF9 | LCRATE | UNITS | | |
| Type | F | F | F | F | I | F | | |
| Default | see remarks | see remarks | 0.0 | 0.0 | 0 | 1.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EC1_6 | Strain at maximum compressive stress for Type 6 concrete |
| ECSP69 | Spalling strain in compression for TYPEC = 6 and 9 |
| GAMCE9 | Material factor that divides the Youngs Modulus (TYPEC = 9) |
| PHIEF9 | Effective creep ratio (TYPEC = 9) |
| LCRATE | Optional load curve ID for a curve giving the strength enhancement factor as a function of strain rate. See Strain Rate Effects below. |
| UNITS | Factor to convert time units to seconds (used only if LCRATE is defined). For example, if the model's time unit is milliseconds, UNITS = $10^{-3}$. |

Define this card only if FT < 0.0.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FT2 | FTSHR | LCFTT | WRO_G | ZSURF | LCFIB | | |
| Type | F | F | F | F | F | I | | |
| Default | \|FT\| | \|FT2\| | 0.0 | 0.0 | 0.0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FT2 | Tensile strength used for calculating tensile response |

| VARIABLE | DESCRIPTION |
|---|---|
| FTSHR | Tensile strength used for calculating post-crack shear response |
| LCFTT | Load curve defining factor on tensile strength as a function of time |
| WRO_G | Density times gravity for water pressure in cracks |
| ZSURF | *Z*-coordinate of water surface (for water pressure in cracks) |
| LCFIB | Optional load curve defining the tensile response. It is intended for fiber-reinforced concrete. The *x*-axis of the curve is tensile strain. The *y*-axis of the curve is a non-dimensional scale factor on the tensile strength FT2. If defined, this curve overrides ECUTEN. |

**Remarks:**

1. **Material types.** This material model can be used to represent unreinforced concrete (FRACR = 0.0 where FRACR = max(FRACX, FRACY)), reinforcing steel (FRACR = 1.0), or a smeared combination of reinforced concrete with evenly distributed reinforcement (0.0 < FRACR < 1.0). Concrete is modeled as an initially isotropic material with a non-rotating smeared crack approach in tension, together with a plasticity model for compressive loading. Reinforcement is treated as separate sets of bars in the local material *x*- and *y*-axes. The reinforcement is assumed not to carry through-thickness shear or in-plane shear. Therefore, this material model should not be used to model steel-only sections; that is, do not create a section in which all the integration points are of *MAT_172 with both FRACRX and FRACRY set to 1.0.

2. **Creating reinforced concrete sections.** Reinforced concrete sections for shell or beam elements may be created using *PART_COMPOSITE (for shells) or *INTEGRATION_BEAM (for beams). Create one material definition representing the concrete using *MAT_CONCRETE_EC2 with FRACR = 0.0. Create another material definition representing the reinforcement using *MAT_CONCRETE_EC2 with FRACRX and/or FRACRY = 1.0. The material ID of each integration point is then set to represent either concrete or steel. The position of each integration point within the cross-section and its cross-sectional area are chosen to represent the actual distribution of reinforcement. If desired, *MAT_HYSTERETIC_REINFORCEMENT can be used for the reinforcement layers instead of *MAT_CONCRETE_EC2.

3. **Eurocode 2.** Eurocode 2 (EC2) contains different sections applicable to general structural engineering versus fire engineering. The latter contains different data for different types of concrete and steel and has been revised during its history. TYPEC and TYPER control the version and section of the EC2 document from

which the material data is taken and the types of concrete and steel represented. In the descriptions of TYPEC and TYPER above, "Draft EC2 Annex (fire engineering)" means data taken from the 1995 draft Eurocode 2 Part 1-2 (for fire engineering), ENV 1992-1-2:1995. These defaults are suitable for general use where elevated temperatures are not considered.

EC2 was then issued in 2004 (described above as EC2 1-2:2004 (fire)) with revised stress-strain data at elevated temperatures (TYPEC and TYPER = 7 or 8). These settings are recommended for analyses with elevated temperatures.

Meanwhile, Eurocode 2 Part 1-1 (for general structural engineering), EC2 1-1:2004, contains material data and formulae that differ from Part 1-2; these are obtained by setting TYPEC = 9. This setting is recommended where compatibility is required with the structural engineering data and assumptions of Part 1-1 of the Eurocode.

A further option for modeling concrete, TYPEC = 6, is provided for applications, such as seismic engineering, in which the different stress-strain behaviors of confined versus unconfined concrete must be captured. This option uses equations by Mander et al. and does not relate directly to Eurocode 2.

4. **Local material axes.** The local material axes define the directions of the reinforcement bars. If the reinforcement directions are inconsistent across neighboring elements, the response may be less stiff than intended – this is equivalent to the bars being bent at the element boundaries. Local material axes default to the same as the element axes, with the local $x$-direction pointing from Node 1 to Node 2. The local material axes can be controlled using the angle BETA on *ELEMENT_SHELL_BETA or AOPT and associated input parameters in the material definition. See material type 2 for a description of the different AOPT settings.

   Only the reinforcement response depends on the local material axes, not the concrete response. Therefore, it is not usually necessary to set the local material axes for material definitions that do not have reinforcement (i.e., FRACRX = 0 and FRACRY = 0). However, when a reinforced concrete section is defined using *PART_COMPOSITE, and either the shear capacity check is invoked (TYPESC > 0, see Remark 5) or CMPFLG is set on *DATABASE_EXTENT_BINARY, all layers in the *PART_COMPOSITE definition need to have identical material axes. This can be achieved by using the BETA angle on *ELEMENT_SHELL_BETA or inputting identical AOPT parameters for all the material definitions referenced by the *PART_COMPOSITE card.

5. **Through-thickness shear.** In this material model, cracks are initiated only by in-plane stresses caused by axial and bending effects. Once a crack has formed, the through-thickness shear stress is limited by considerations of aggregate interlock or friction on the crack surfaces. If the in-plane stresses are insufficient

to cause cracks, the through-thickness shear strength is, by default, unlimited. Thus, failures caused primarily by through-thickness shear will not be predicted. The Shear Capacity Check option (see TYPESC) addresses this limitation. In versions up to and including R16, TYPESC is available for shell elements only. In versions from R17 onwards, TYPESC is available for shells and beams. Two classes of behavior are available (described in a and b), together with different options for the calculation of shear capacity (described in c, d, e, and f):

a) TYPESC < 10 represents the situation where sufficient shear reinforcement will be provided to prevent any through-thickness failure. LS-DYNA generates extra history variables for comparing the shear demand to shear capacity to assess the requirements for shear reinforcement (see the list of additional history variables in Remark 8). Furthermore, we assume that the shear reinforcement prevents inelastic through-thickness shear deformation. Thus, through-thickness slipping on crack planes is automatically disabled.

b) TYPESC > 10 represents the situation where failure occurs if the shear capacity is exceeded. Note, however, that the shear capacity is calculated from equations in design codes and may be quite conservative.

c) If TYPESC = 1 or 11, the shear capacity calculation is based on BS 8110-1:1997. These values of TYPESC require supplying the percentage reinforcement P_OR_F, the effective depth of section EFFD (this typically excludes the cover concrete), and the load factor GAMSC. These quantities are used in Table 3.8 of BS 8110-1:1997 to determine the design shear stress. The "shear capacity" is this design shear stress times the total section thickness (force per unit width), modified according to Equation 6b of BS 8110 to allow axial load.

d) If TYPESC = 2 or 12, the shear capacity calculation is based on ACI 318-05M. The shear capacity, $\emptyset V_n$, is calculated assuming $\emptyset = 0.75$ and taking $V_n$ from ACI 318-05M Equations 11-4 (for compressive axial load) or 11-8 (for tensile axial load). In these equations, $f'_c$ is taken as P_OR_F × FC, $d$ as EFFD, and $b_w$ as 1 to give shear capacity as a force per unit width. Note that in versions prior to R13, Equation 11-4 was incorrectly implemented, so results from the TYPESC = 2 check before R13 should not be used.

e) If TYPESC = 5 or 15, the shear capacity calculation is based on Equations 6.2.a and 6.2.b of EN 1992-1-1:2004. This check is intended for members without shear reinforcement (stirrups). In these equations, $f_{ck}$ is taken as FCK, $f_{cd}$ as FCK/GAMSC, $d$ as EFFD, and $b_w$ as BW for beams and 1 for shells. We also assume $C_{Rd,c} = 0.18$ and $k_1 = 0.15$. The tensile reinforcement percentage $\rho_l$ can optionally be assigned with different values for shear in the $s$- and $t$-directions (beams), or $XZ$ and $YZ$ planes (shells). Setting

different values is appropriate when modelling walls with different longitudinal rebar spacing in the two directions. However, if either ASW1S or ASW1T are left blank, the same percentage applies in both directions. Allowance for axial force is included with the product $k_1\sigma_{cp}$ in the EC2 equations, where $\sigma_{cp}$ is the average compressive stress in the section calculated at each time step.

f) If TYPESC = 6 or 16, the shear capacity calculation is based on Equations 6.8 to 6.14 of EN 1992-1-1:2004. This type of check is intended for members provided with shear reinforcement ("stirrups"). In these equations, input fields FCK, EFFD, and BW have the same meaning as for TYPESC = 5, $z$ is 0.9 × EFFD, $f_{ywd}$ is taken as FYWD, $v_1$ is based on Equation 6.6N, and the coefficient $a_{cw}$ to allow for axial load is internally calculated as per Equation 6.11.

Input fields ASW1S and ASW1T provide the shear reinforcement parameters $A_{sw}/s_h$ for the vertical (90°) stirrups.

i) When used with beam elements, $A_{sw}$ is the cross-sectional area of each set of stirrups, $s_h$ is the spacing between the sets of stirrups in the direction along the beam axis, and $A_{sw}/s_h$ has dimensions of length. It is generally expected that ASW1S and ASW1T will have the same value, as stirrups are formed of closed loops.

ii) When used with shell elements, $A_{sw}$ is the cross-sectional area of stirrups per unit width (where "width" is perpendicular to the plane in which the shear capacity is calculated), $s_h$ is the stirrup spacing in the plane in which the shear capacity is calculated, and $A_{sw}/s_h$ is dimensionless. ASW1S and ASW1T may have different values because the spacing can be different in the two directions.

Similarly, assigning nonzero values to ASW2S and ASW2T adds inclined shear reinforcement at 45°. These fields represent $A_{sw}/s_h$ for the inclined bars in the two local directions. The sign of these two inputs dictates whether the inclined bars contribute to the shear capacity of the section for the positive or negative direction of the shear force. By default, the shear resistance is calculated assuming a fixed inclination, $\theta$, of the concrete compression strut at 45° ($\cot\theta = 1$). Setting THETA = 1 switches the calculation to a variable $\theta$ method. This method searches for the inclination $\theta$ that maximizes the overall shear capacity, within the allowed range $1 \le \cot\theta \le 2.5$.

When both vertical and inclined reinforcement are present, the total shear resistance of the stirrups is the summation of the two contributions

$$V_{Rds,\,tot} = V_{Rds,90°} + V_{Rds,45°},$$

and the total concrete shear resistance is obtained as a weighted average from the two separate contributions

$$V_{\text{Rd, max}} = \frac{V_{\text{Rds,90°}} V_{\text{Rd,max,90°}} + V_{\text{Rds,45°}} V_{\text{Rd,max,45°}}}{V_{\text{Rds, tot}}}.$$

The overall shear capacity of the element is always taken as the smaller value between $V_{\text{Rds, tot}}$ and $V_{\text{Rd, max}}$.

The "shear demand" (actual shear force per unit width) is compared to the shear capacity for all the above options. This process is performed separately for each element's two local reinforcement directions. When defining sections using *PART_COMPOSITE or integration rules with multiple sets of material properties, each set of material properties referenced must have the same local material axes (see Remark 4). The shear demand and axial load (used in calculating the shear capacity) are summed across the integration points within the section. The extra history variables for capacity, demand, and the difference between capacity and demand relate to the whole section (not each integration point separately). Thus, the same values are written to all the integration points within an element.

6. **Thermal expansion.** By default, thermal expansion properties from EC2 are used. If no temperatures are defined in the model, properties for 20°C are used. For TYPEC = 3, 6, or 9, and TYPER = 5, there is no thermal expansion by default, and the properties do not vary with temperature. Defining curves of thermal expansion coefficient as a function of temperature (LCALPC, LCALPR) overrides the default thermal expansion behavior. These apply no matter the selected types of TYPEC and TYPER.

7. **Erosion criteria.** Elements are deleted from the calculation when all their integration points have reached the erosion criteria. Because this material model can represent plain concrete without reinforcement, pure reinforcement without concrete, or a smeared combination, the criteria depend on the modeled type (see Remark 1). There are three criteria:

   a) *Concrete Tensile Strain Limit*. The concrete tensile (crack-opening) strain limit ERODET has been exceeded.

   b) *Concrete Compressive Strain Limit*. The concrete compressive strain limit ERODEC + $\varepsilon_{\text{csp}}$ has been exceeded. $\varepsilon_{\text{csp}}$ is the strain at which the stress-strain relation falls to zero.

   c) *Reinforcement Strain Limit.* The reinforcement strain limit ERODER + $\varepsilon_{\text{rsp}}$ has been exceeded. $\varepsilon_{\text{rsp}}$ is the strain at which the stress-strain relation falls to zero. However, if LCRSU > 0, $\varepsilon_{\text{rsp}}$ is assumed to be 2.0.

The table below indicates which criteria apply to each of the variations of material type. Note that FRACR = max(FRACX, FRACY) as discussed in Remark 1.

| FRACR | Material Type | Erosion Criteria | Erosion Criteria (in plain English) |
|---|---|---|---|
| FRACR = 0.0 | Pure concrete | (a).OR.(b) | The concrete tensile strain limit or concrete compressive strain limit conditions are satisfied. |
| 0.0 < FRACR < 1.0 | Smeared combination | ((a).OR.(b)) .AND.(c) | The reinforcement strain limit and either the concrete tensile strain limit or the concrete compressive strain limit are satisfied. |
| FRACR = 1.0 | Pure steel reinforcement | (c) | The reinforcement strain limit is satisfied. |

If both FRACRX and FRACRY are nonzero, the reinforcement erosion criterion is applied as follows: in LS-DYNA versions up to and including R14, the reinforcement erosion criterion must be met in both local directions ($X$ and $Y$) before erosion occurs. In versions from R15 onwards, erosion occurs when either direction $X$ or direction $Y$ reaches the erosion criterion. The R14 treatment had the counterintuitive side-effect of preventing erosion of elements under large uniaxial strains because the reinforcement in the low-strain direction had not reached its erosion limit.

8. **Output.** "Plastic Strain" is the maximum of the plastic strains in the reinforcement in the two local directions. Extra history variables may be requested for shell elements (NEIPS on *DATABASE_EXTENT_BINARY). They are described in the following table.

| History Variable # | Description |
|---|---|
| 1 | Current crack opening strain (if two cracks are present, max of two) |
| 2 | Equivalent uniaxial strain for concrete compressive behavior |
| 3 | Number of cracks (0, 1, 2, 3, or 4) |
| 4 | Temperature |
| 5 | Thermal strain |
| 6 | Current crack opening strain for the first crack to form |
| 7 | Current crack opening strain for the crack at 90 degrees to the first crack |

| History Variable # | Description |
|---|---|
| 8 | Max crack opening strain for the first crack to form |
| 9 | Max crack opening strain for the crack at 90 degrees to the first crack |
| 10 | Maximum through-thickness shear stress (resultant of local $YZ$ and $ZX$ shear stresses) if TYPESC = 0 or maximum difference (shear demand minus capacity) that has occurred so far in either of the two reinforcement directions if TYPESC ≥ 1 |
| 11 | Maximum through-thickness $YZ$ shear stress (element axes) if TYPESC = 0 or maximum difference (shear demand minus capacity) that has occurred so far in reinforcement $x$-direction if TYPESC ≥ 1 |
| 12 | Maximum through-thickness $ZX$ shear stress (element axes) if TYPESC = 0 or maximum difference (shear demand minus capacity) that has occurred so far in reinforcement $y$-direction if TYPESC ≥ 1 |
| 13 | Current shear demand minus capacity in reinforcement $x$-direction if TYPESC ≥ 1 |
| 14 | Current shear demand minus capacity in reinforcement $y$-direction if TYPESC ≥ 1 |
| 15 | Current shear capacity, $V_{cx}$, in reinforcement $x$-direction if TYPESC ≥ 1 |
| 16 | Current shear capacity, $V_{cy}$, in reinforcement $y$-direction if TYPESC ≥ 1 |
| 17 | Current shear demand, $V_x$, in reinforcement $x$-direction if TYPESC ≥ 1 |
| 18 | Current shear demand, $V_y$, in reinforcement $y$-direction if TYPESC ≥ 1 |
| 19 | Maximum shear demand that has occurred so far in reinforcement $x$-direction if TYPESC > 0 |
| 20 | Maximum shear demand that has occurred so far in reinforcement $y$-direction if TYPESC > 0 |
| 21 | Current strain in reinforcement ($x$-direction) |
| 22 | Current strain in reinforcement ($y$-direction) |
| 23 | Engineering shear strain (slip) across the first crack |
| 24 | Engineering shear strain (slip) across the second crack |
| 25 | $x$-stress in concrete (element local axes) |

| History Variable # | Description |
|---|---|
| 26 | $y$-stress in concrete (element local axes) |
| 27 | $xy$-stress in concrete (element local axes) |
| 28 | $yz$-stress in concrete (element local axes) |
| 29 | $xz$-stress in concrete (element local axes) |
| 30 | Reinforcement stress ($a$-direction) |
| 31 | Reinforcement stress ($b$-direction) |
| 32 | Current shear demand, $V_{\max}$, if TYPESC > 0 |
| 33 | Maximum $V_{\max}$ that has occurred so far if TYPESC > 0 |
| 34 | Current shear capacity, $V_{c\theta}$, if TYPESC > 0 |
| 35 | Excess shear, $V_{\max} - V_{c\theta}$, if TYPESC > 0 |
| 36 | Maximum excess shear that has occurred so far if TYPESC > 0 |
| 56 | Max crack opening strain for the crack at 45 degrees to the first crack |
| 57 | Max crack opening strain for the crack at -45 degrees to the first crack |
| 58 | Current crack opening strain for the crack at 45 degrees to the first crack |
| 59 | Current crack opening strain for the crack at -45 degrees to the first crack |
| 66 | Effective deviatoric strain rate (calculated only if LCRATE is defined) |
| 71 | Dynamic enhancement factor of compressive strength (see LCRATE) |
| 72 | Dynamic enhancement factor of tensile strength (see LCRATE) |

In the above table $V_{\max}$ is given by

$$V_{\max} = \sqrt{V_x^2 + V_y^2} \ ,$$

where $V_x$ and $V_y$ are the shear demand reinforcement in the $x$ and $y$ directions, respectively. Additionally,

$$V_{c\theta} = \frac{V_{\max}}{\sqrt{\left(\frac{V_x}{V_{cx}}\right)^2 + \left(\frac{V_y}{V_{cy}}\right)^2}} \ ,$$

where $V_{cx}$ and $V_{cy}$ are the shear capacities in the $x$- and $y$-directions, respectively.

Note that the concrete stress history variables are stored in element local axes irrespective of AOPT; that is, local $x$ is always the direction from node 1 to node 2. The reinforcement stresses are in the reinforcement directions; these do take account of AOPT.

**Material Behavior of Concrete:**

Thermal sensitivity

For TYPEC = 1, 2, 4, 5, 7, and 8, the material properties are thermally-sensitive. If no temperatures are defined in the model, it behaves as if at 20°C. Pre-programmed relationships between temperature and concrete properties are taken from the EC2 document. The thermal expansion coefficient is as defined in EC2, is nonzero by default, and is a function of temperature. This coefficient may be overridden by inputting the curve LCALPC. TYPEC = 3, 6, and 9 are not thermally sensitive and have no thermal expansion coefficient by default.

Tensile response

The concrete is assumed to crack in tension when the maximum in-plane principal stress reaches FT. A non-rotating smeared crack approach is used. Cracks can open and close repeatedly under hysteretic loading. When a crack is closed, it can carry compression according to the normal compressive stress-strain relationships. The direction of the crack relative to the element coordinate system is stored when the crack first forms. The material can carry compression parallel to the crack even when the crack is open. Further cracks may then form at pre-determined angles to the first crack if the tensile stress in that direction reaches FT. In versions up to R9, the number of further cracks is limited to one, at 90 degrees to the first crack. In versions starting from R10, up to three additional cracks can form at 45, 90, and 135 degrees to the first crack. The tensile stress is limited to FT only in the available crack directions. The tensile stress in other directions is unlimited and could exceed FT. This is a limitation of the non-rotating crack approach and may lead to models being non-conservative; that is, the response is stronger than implied by the input. Increasing the possible number of cracks from two to four significantly reduces this error and may cause models to seem "weaker" in R10 than in R9 under some loading conditions. An option to revert to the previous two-crack behavior is available in R10 and later – add 100 to TYPEC.

After initial cracking, the tensile stress reduces with increasing tensile strain. A finite amount of energy must be absorbed to create a fully open crack. In practice, the reinforcement holds the concrete together, allowing it to continue to take some tension (this effect is known as tension-stiffening). The options available for the stress-strain curve are shown in Figure M172-1. The piecewise curve is used by default. The simple linear curve

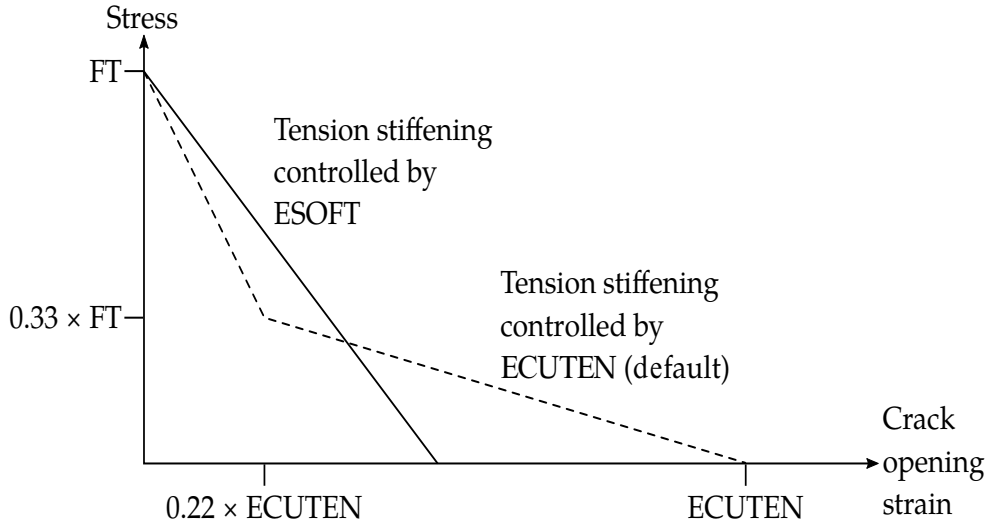



**Figure M172-1.** Tensile behavior of concrete

applies only if ESOFT > 0.0 and ECUTEN = 0.0. A further option is to define the stress-strain response through a load curve; see LCFIB (intended for fiber-reinforced concrete).

LCHAR can optionally be used to maintain constant energy per unit area of crack irrespective of mesh size; that is, the crack opening displacement is fixed rather than the crack opening strain. LCHAR × ECUTEN is then the displacement to open a crack fully. For the actual elements, crack opening displacement is estimated by strain × $\sqrt{\text{area}}$. Note that if LCHAR is defined, it is also used as the crack spacing in the NS 3473 aggregate interlock calculation.

For the thermally-sensitive values of TYPEC, the relationship of FT with temperature is taken from EC2 – there is no input option to change this. FT is assumed to remain at its input value at temperatures up to 100°C, then to reduce linearly with temperature to zero at 600°C. Up to 500°C, the crack opening strain ECUTEN increases with temperature such that the fracture energy to open the crack remains constant. Above 500°C, the crack opening strain does not increase further.

Some concrete design codes and standards stipulate that the tensile strength of concrete should be assumed to be zero. However, for MAT_CONCRETE_EC2, we do not recommend setting FT to zero because:

- Cracks will form at random orientations caused by small dynamic tensile stresses, leading to unexpected behavior when the loading increases because the crack orientations are fixed when the cracks first form;
- The shear strength of cracked concrete may also become zero in the analysis (according to the aggregate interlock formula, the post-crack shear strength is assumed proportional to FT).

These problems may be tackled by using the inputs on Card 9. Firstly, separate tensile strengths may be input for the tensile response and for calculating the shear strength of

cracked concrete. Secondly, by using the load curve LCFTT, the tensile strength may be ramped gradually down to zero after the static loads have been applied, ensuring that the cracks will form in the correct orientation.

Compressive response for TYPEC = 1, 2, 4, 5, 7, and 8

For TYPEC = 1, 2, 4, 5, 7, and 8, the compressive behavior of the concrete initially follows a stress-strain curve defined in EC2 as:

$$\text{Stress} = \text{FC}_{\max} \times \left[ \left( \frac{\varepsilon}{\varepsilon_{c1}} \right) \times \frac{3}{2 + \left( \frac{\varepsilon}{\varepsilon_{c1}} \right)^3} \right],$$

where $\varepsilon_{c1}$ is the strain at which the ultimate compressive strength, $\text{FC}_{\max}$, is reached, and $\varepsilon$ is the current equivalent uniaxial compressive strain.

The initial elastic modulus is given by $E = 3 \times \text{FC}_{\max}/2\varepsilon_{c1}$. Upon reaching $\text{FC}_{\max}$, the stress decreases linearly with increasing strain, reaching zero at a strain $\varepsilon_{cu}$. Strains $\varepsilon_{c1}$ and $\varepsilon_{cu}$ are by default taken from EC2 and are functions of temperature. At 20°C, they are values 0.0025 and 0.02, respectively. $\text{FC}_{\max}$ is also a function of temperature, given by the input parameter FC (which applies at 20°C) times a temperature-dependent softening factor taken from EC2. The differences among TYPEC = 1, 2, 4, 5, 7, and 8 are limited to (a) different reductions of FC at elevated temperatures and (b) different values of $\varepsilon_{c1}$ at elevated temperatures.

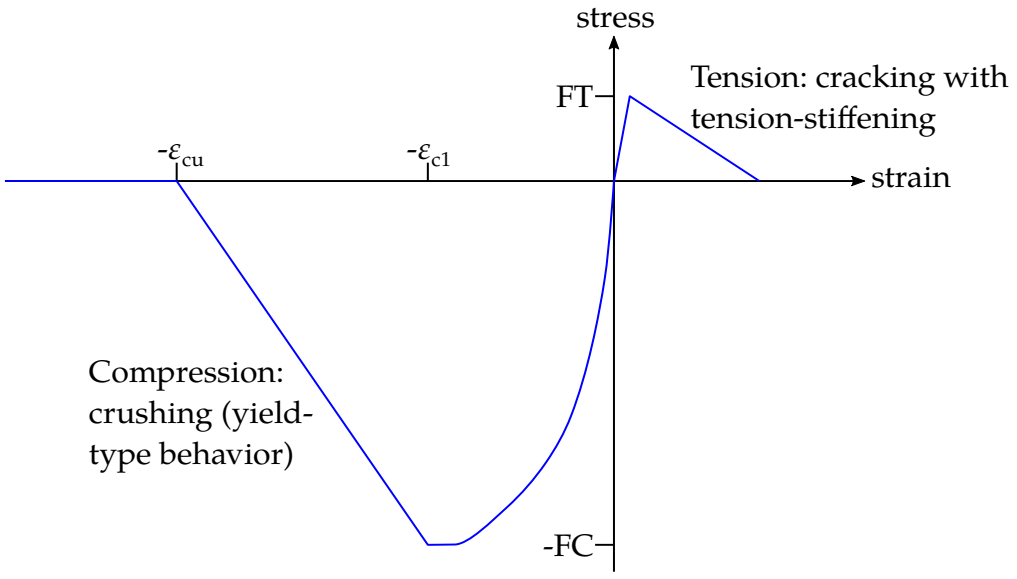



**Figure M172-2.** Concrete stress-strain behavior

Compressive response for TYPEC = 3

For TYPEC = 3, the stress-strain behavior follows the same form described above. To override the default values of the Young's modulus and $\varepsilon_{cu}$, set ET36 and ECUT36, respectively. In this case, the strain, $\varepsilon_{c1}$, is calculated from the elastic stiffness, and there is no thermal sensitivity.

Compressive response for TYPEC = 6

For TYPEC = 6, the above compressive crushing behavior is replaced with the equations proposed by Mander. This algorithm can model unconfined or confined concrete; for unconfined, leave FCC blank. For confined concrete, input the confined compressive strength as FCC.

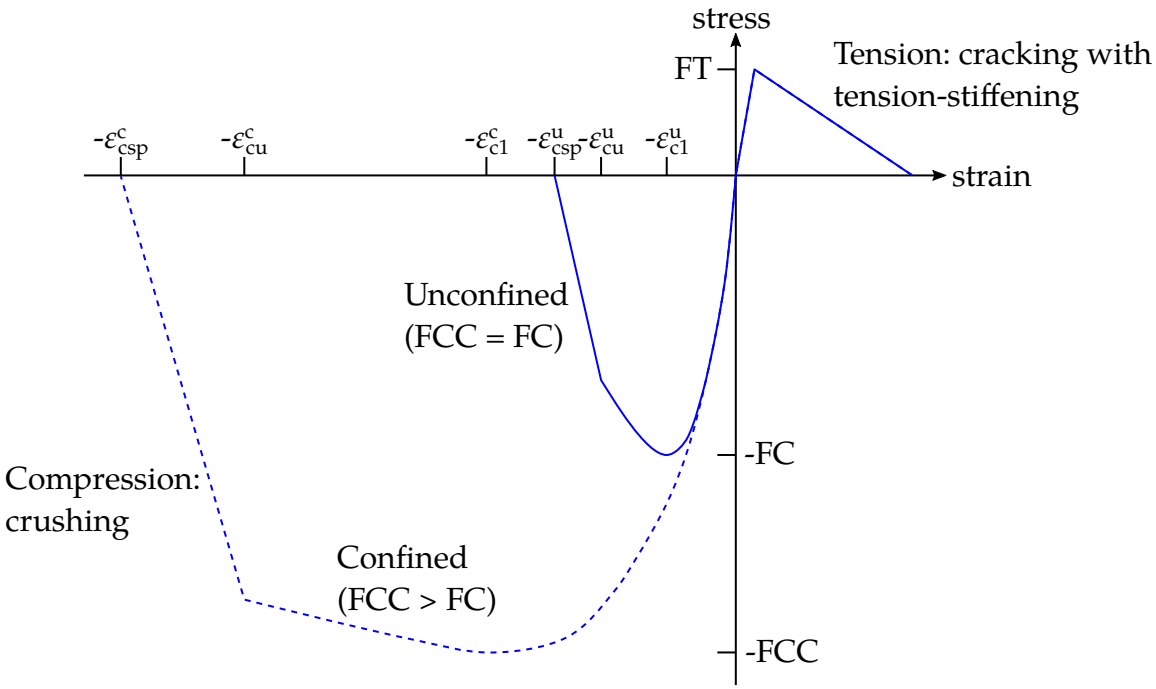



**Figure M172-3.** Type 6 concrete. Values with superscripts *u* and *c* specify they are for the unconfined and confined curves, respectively.

As indicated in Figure M172-3, $\varepsilon_{c1}$ is the strain at maximum compressive stress, $\varepsilon_{cu}$ is the ultimate compressive strain, and $\varepsilon_{csp}$ is the spalling strain. Default values for these quantities for both confined and unconfined concrete are calculated as follows:

$$\varepsilon_{c1} = 0.002 \times \left[1 + 5\left(\frac{FCC}{FC} - 1\right)\right]$$

$$\varepsilon_{cu} = 1.1 \times \varepsilon_{c1}$$

$$\varepsilon_{csp} = \varepsilon_{cu} + 2\frac{FCC}{E}$$

Note that for unconfined concrete, FCC = FC causing $\varepsilon_{c1}$ to default to 0.002. To override the default values $\varepsilon_{c1}$, $\varepsilon_{cu}$, and $\varepsilon_{csp}$, set EC1_6, ECUT36, and ECSP69, respectively.

Compressive response: TYPEC = 9

For TYPEC = 9, the input parameter FC is the characteristic cylinder strength in the stress units of the model. FC × UNITC is assumed to be $f_{ck}$, the strength class in MPa units. The mean tensile strength $f_{ctm}$, mean Young's modulus $E_{cm}$, and the strains used to construct

the stress-strain curve, such as $\varepsilon_{c1}$, are by default evaluated automatically from tabulated functions of $f_{ck}$ given in Table 3.1 of EC2. Input parameter FCC provides the material's compressive strength of the material. It defaults to the mean compressive strength $f_{cm}$ defined in EC2 as $f_{ck}$ + 8MPa. Inputting FCC explicitly overrides the default compressive strength. The stress-strain curve follows this form:

$$\frac{\text{Stress}}{\text{FCC}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta},$$

where FCC is the input parameter FCC (default: = ($f_{ck}$ + 8MPa)/UNITC), $\eta$ = strain/$\varepsilon_{c1}$, $k = 1.05E \times \varepsilon_{c1}$/FCC, and $E$ is the Young's modulus.

The default parameters are intended to be appropriate for a serviceability analysis (mean properties), so default FT = $f_{ctm}$ and default $E = E_{cm}$. For an ultimate load analysis, FCC should be the "design compressive strength" (normally the factored characteristic strength, including any appropriate material factors); FT should be input as the factored characteristic tensile strength; GAMCE9 may be input (a material factor that divides the Young's Modulus so $E = E_{cm}$/GAMCE9); and a creep factor PHIEF9 may be input that scales $\varepsilon_{c1}$ by (1 + PHIEF9).

### Unload/Reload Stiffness (All Concrete Types):

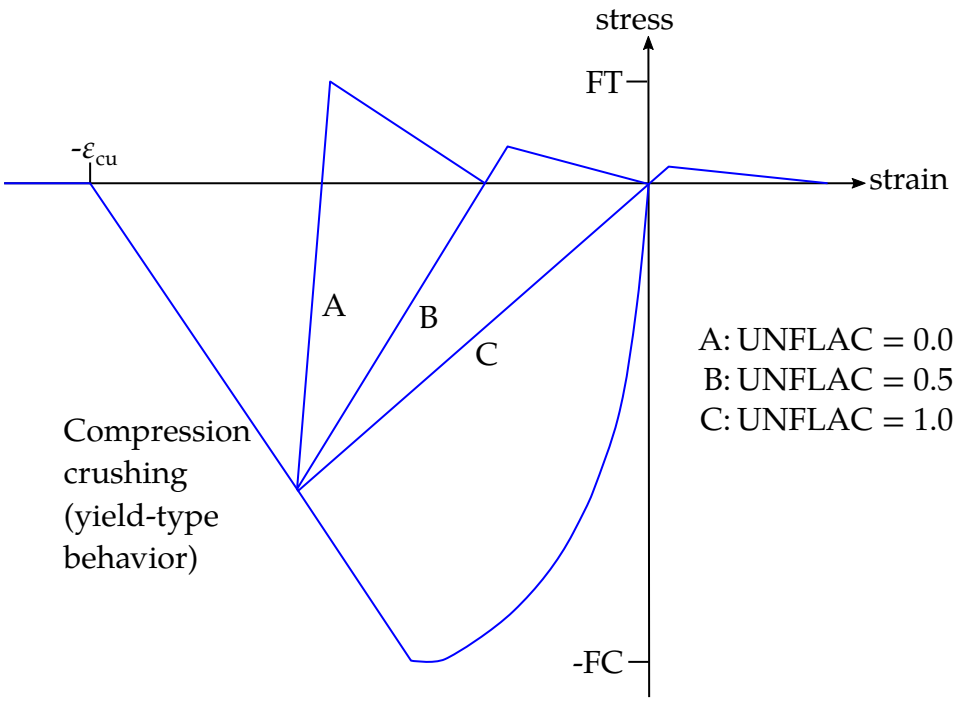



**Figure M172-4.** Concrete unloading behavior

The parameter UNLFAC (default = 0.5) determines the reduction of the elastic modulus during compressive loading. See Figure M172-4. UNLFAC = 0.0 means no reduction; the initial elastic modulus applies during unloading and reloading. UNLFAC =1.0 means

that unloading results in no permanent strain. Intermediate values imply a permanent strain linearly interpolated between these extremes. The same factor reduces the tensile strength and the elastic modulus.

### Optional Compressive Strength Degradation due to Cracking:

By default, the compressive strength of cracked and uncracked elements is the same. If DEGRAD is non-zero, the formula from BS8110 reduces compressive strength during or after crack opening has occurred:

$$\text{Reduction factor} = \min\left(1.0, \frac{1.0}{0.8 + 100\varepsilon_{\text{tmax}}}\right),$$

where $\varepsilon_{\text{tmax}}$ is the maximum (tensile) crack-opening strain that has occurred up to the current time.

### Shear Strength on Cracking Planes:

Before cracking, the through-thickness shear stress in the concrete is unlimited., unless TYPESC > 10 (see Remark 5). For cracked elements, shear stress on the crack plane (magnitude of shear stress including element-plane and through-thickness terms) is treated in one of two ways:

- If AGGSZ > 0.0, the relationship from Modified Compression Field Theory is used to model the aggregate-interlock that allows cracked concrete to carry shear loading. The maximum shear stress that can be carried on the crack plane, $\tau_{max}$, depends on compressive stress on the crack $\sigma_c$(if the crack is closed) or on crack opening width $w$ (if the crack is open):

$$\tau_{\text{max}} = 0.18\tau_{\text{rm}} + 1.64\sigma_c - 0.82\frac{\sigma_c^2}{\tau_{\text{rm}}}$$

$$\tau_{\text{rm}} = \frac{2\text{FTSHR}}{0.31 + \frac{24w}{(\text{D}_0 + 16)}}$$

  FTSHR is defined on Card 9 and defaults to FT on Card 1.

  UNITL is compulsory when AGGSZ is non-zero. This is the factor that converts model length units to millimeters; that is, the aggregate size in millimeters $\text{D}_0$ = AGGSZ × UNITL.

  The crack width is estimated from $w$ = UNITL × $\varepsilon_{\text{cro}} \times L_e$, where $\varepsilon_{\text{cro}}$ is the crack opening strain and $L_e$ is the crack spacing. $L_e$ is taken as LCHAR if non-zero or is equal to element size if LCHAR is zero.

  Optionally, TAUMXC may be used to set the maximum shear stress when the crack is closed, and the normal stress is zero – by default, this works out as

1.161 × FT from the above equations. If TAUMXC is defined, the shear stress from the NS3473 formula, $\tau_{max}$, is scaled by TAUMXC / 1.161 × FT.

- If AGGSZ = 0.0, the aggregate interlock is modeled by this formula:

$$\tau_{max} = \frac{\text{TAUMXC}}{1.0 + \frac{\varepsilon_{cro}}{\text{ECRAGG}}} + \min\left(\text{MU} \times \sigma_{comp}, \text{TAUMXF}\right) ,$$

where $\tau_{max}$ is the maximum shear stress carried across a crack; $\sigma_{comp}$is the compressive stress across the crack (this is zero if the crack is open); and ECRAGG is the crack opening strain at which the input shear strength TAUMXC is halved. Again, TAUMXC defaults to 1.161 × FT.

Note that if a shear capacity check is specified, the above applies only to in-plane shear, while the through-thickness shear is unlimited.

### Strain Rate Effects:

Strain rate effects are available only for TYPEC = 6 and 9. The definition of the load curve LCRATE follows the convention adopted in *MAT_072R3, where negative strain rate values (on the $x$-axis) denote the tensile strength enhancement factor (on the $y$-axis), and positive strain rate values denote the compressive strength enhancement factor. The $x$-axis strain rate, $\dot{\varepsilon}$, is the effective deviatoric strain rate. The curve must include a point at zero strain rate with an enhancement factor of 1.0.

In compression, the default stress-strain curve of concrete is modified to account for strain rate effects as follows:

- The static compressive strength FCC is scaled to obtain the dynamic compressive strength:

$$\text{FCC}_{dyn} = \text{FCC} \times \text{DIF}_c(\dot{\varepsilon})$$

where $\text{DIF}_c(\dot{\varepsilon})$ is the compressive strength enhancement factor interpolated from the compressive portion of LCRATE at the current effective strain rate.

- The strain at maximum compressive stress, $\varepsilon_{c1}$, is scaled according to fib MC2010 Equation 5.1-120 to account for the increased ductility observed at high strain rates:

$$\varepsilon_{c1}^{dyn} = \varepsilon_{c1} \times \left(\frac{\dot{\varepsilon}}{30 \times 10^{-6} \times \text{UNITS}}\right)^{0.02}$$

The same scaling factor is also applied to both the ultimate compressive strain, $\varepsilon_{cu}$, and the spalling strain, $\varepsilon_{csp}$.

- Because the above stress and strain scaling factors are different from each other, applying them to the whole stress-strain curve would alter the initial slope and therefore change the concrete Young's modulus. The curve obtained by using

such scaling rules is represented by the grey line in Figure M172-5. To preserve the initial elastic stiffness, LS-DYNA performs an additional adjustment, resulting in the final dynamic stress-strain curve represented by the dashed blue line. Consequently, the pre-peak portion of the dynamic curve may not strictly follow the original TYPEC = 6 or TYPEC = 9 equations.

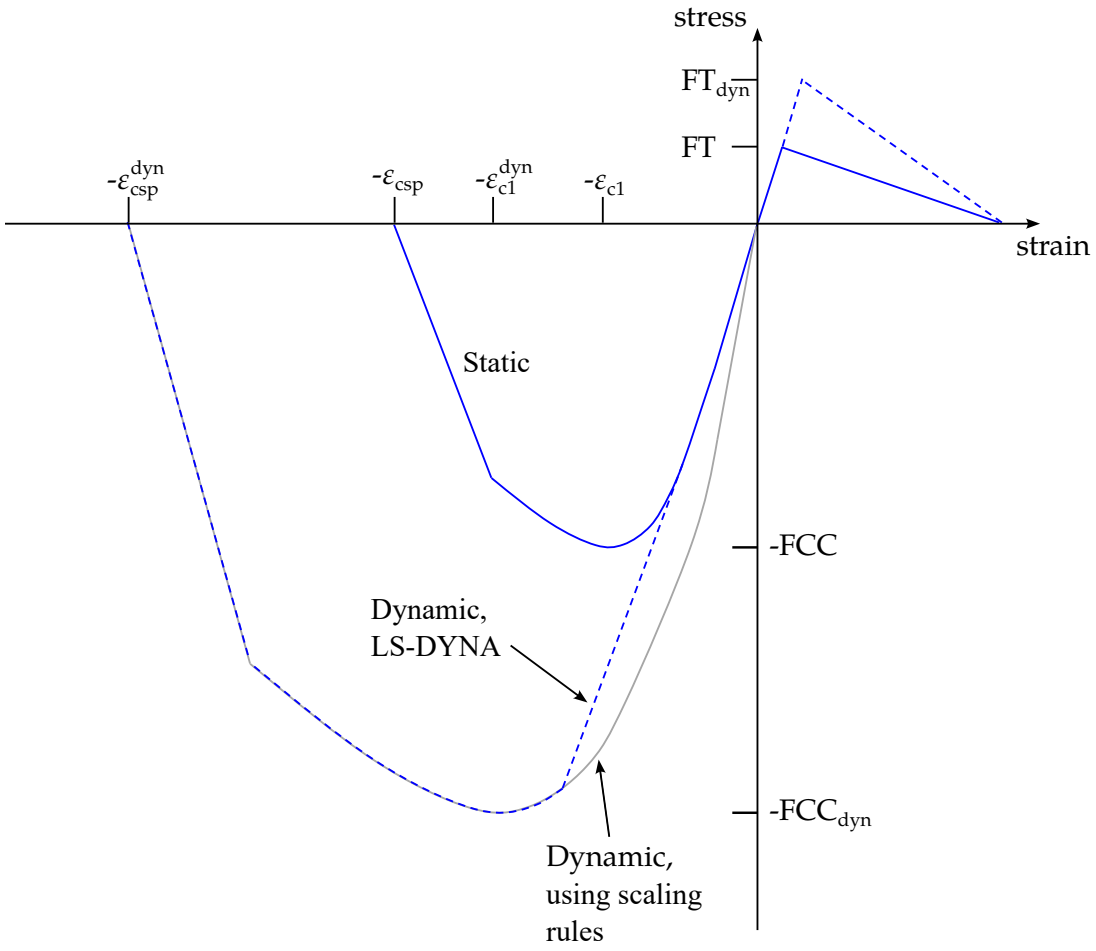



**Figure M172-5.** Strain rate effects on concrete stress strain behavior

In tension, the static tensile strength FT is scaled to obtain the dynamic tensile strength:

$$FT_{dyn} = FT \times DIF_t(\dot{\varepsilon})$$

where $DIF_t(\dot{\varepsilon})$ is the tensile strength enhancement factor interpolated from the tensile portion of LCRATE at the current effective strain rate. When strain rate effects are active, the dynamic tensile strength, $FT_{dyn}$, replaces the static strength, FT, in the formulas for calculating the shear strength on a cracking plane.

Additional extra history variables are available for monitoring the effective strain rate and the dynamic enhancement factor of the compressive and tensile strength (see Remark 8).

**Reinforcement:**

The reinforcement is treated as separate bars providing resistance only in the local $x$- and $y$-directions – it does not carry shear in-plane or out-of-plane.

For TYPER = 1, 2, 3, 4, 7, and 8, the behavior is thermally sensitive and follows stress-strain relationships of a form defined in EC2. At 20°C (or if no thermal input is specified), the behavior is elastic-perfectly-plastic with Young's Modulus EREINF and ultimate stress SUREINF, up to the onset of failure, after which the stress reduces linearly with increasing strain until final failure. At elevated temperatures, a nonlinear transition between the elastic and the perfectly plastic phases exists, and temperature-dependent factors defined in EC2 scale down EREINF and SUREINF. The strain at which failure occurs depends on the reinforcement type (TYPER) and the temperature. For example, for hot-rolled reinforcing steel at 20°C, failure begins at 15% strain and is complete at 20%. The thermal expansion coefficient is as defined in EC2 and is a function of temperature. This may be overridden by inputting the curve LCAPLS. The differences between TYPER = 1, 2, 4, 7, 8 are limited to (a) different reductions of EREINF and SUREINF at elevated temperatures, (b) different nonlinear transitions between elastic and plastic phases and (c) the strains at which softening begins and is complete.

The default stress-strain curve for reinforcement may be overridden using TYPER = 5 and LCRSU. In this case, the reinforcement properties are not temperature-sensitive, and SUREINF $\times f(\varepsilon_p)$ gives the yield stress, where $f(\varepsilon_p)$ is the load curve value at the current plastic strain. To include failure of the reinforcement, the curve should reduce to zero at the desired failure strain and remain zero for higher strains. Note that by default, the solver re-interpolates the input curve to have 100 equally-spaced points; if the last point on the curve is at very high strain, then the initial part of the curve may become poorly defined.

## *MAT_MOHR_COULOMB

This is Material Type 173. It is for solid elements, thick shells, and SPH particles only and is intended to represent cohesive or sandy soils and other granular materials. A simple soil model is obtained by defining Fields 1 through 4 of Card 1 together with PHI and/or CVAL while leaving all other fields blank. Joints (planes of weakness) may be added if required; the material then represents rock. The joint treatment is identical to that of *MAT_JOINTED_ROCK.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | GMOD | RNU | | PHI | CVAL | PSI |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| NOVOID | NPLANES | EXTRA | LCCPDR | LCCPT | LCCJDR | LCCJT | LCSFAC |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| GMODDP | GMODGR | LCGMEP | LCPHIEP | LCPSIEP | LCGMST | CVALGR | ANISO |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if EXTRA > 0.

| LCGMT | LCCVT | LCPHT | EPDAM1 | EPDAM2 | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** Include if NPLANES > 0. Repeat this card for each plane (maximum of 6 planes).

| DIP | DIPANG | CPLANE | FRPLANE | TPLANE | SHRMAX | LOCAL | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | GMOD | RNU | (blank) | PHI | CVAL | PSI |
| Type | A | F | F | F | | F | F | F |
| Default | none | none | none | none | | none | none | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| GMOD | Elastic shear modulus |
| RNU | Poisson's ratio |
| PHI | Angle of friction (radians) |
| CVAL | Cohesion value (shear strength at zero normal stress) |
| PSI | Dilation angle (radians) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NOVOID | NPLANES | EXTRA | LCCPDR | LCCPT | LCCJDR | LCCJT | LCSFAC |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| NOVOID | Voiding behavior flag (see Remarks 8 and 9):<br>EQ.0: Voiding behavior on<br>EQ.1: Voiding behavior off |
| NPLANES | Number of joint planes (maximum of 6) |
| EXTRA | Flag to input further data. If EXTRA > 0, then Card 4 is read. |
| LCCPDR | Load curve for extra cohesion for base material (dynamic relaxation) |
| LCCPT | Load curve for extra cohesion for base material (transient) |
| LCCJDR | Load curve for extra cohesion for joints (dynamic relaxation) |
| LCCJT | Load curve for extra cohesion for joints (transient) |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSFAC | Load curve giving factor on strength as a function of time |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GMODDP | GMODGR | LCGMEP | LCPHIEP | LCPSIEP | LCGMST | CVALGR | ANISO |
| Type | F | F | I | I | I | I | F | F |
| Default | 0.0 | 0.0 | 0 | 0 | 0 | 0 | 0.0 | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| GMODDP | $z$-coordinate at which GMOD and CVAL are correct |
| GMODGR | Gradient of GMOD as a function of $z$-coordinate (usually negative) |
| LCGMEP | Load curve of GMOD as a function of plastic strain (overrides GMODGR) |
| LCPHIEP | Load curve of PHI as a function of plastic strain |
| LCPSIEP | Load curve of PSI as a function of plastic strain |
| LCGMST | (Leave blank) |
| CVALGR | Gradient of CVAL as a function of $z$-coordinate (usually negative) |
| ANISO | Factor applied to elastic shear stiffness in global $XZ$ and $YZ$ planes |

**Card 4.** Define Card 4 only if EXTRA > 0.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCGMT | LCCVT | LCPHT | EPDAM1 | EPDAM2 | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | $10^{20}$ | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCGMT | Load curve of nondimensional factor on GMOD as a function of time |
| LCCVT | Load curve of nondimensional factor on CVAL as a function of time |
| LCPHT | Load curve of nondimensional factor on PHI as a function of time |
| EPDAM1 | Plastic strain or volumetric void strain at which damage begins |
| EPDAM2 | Plastic strain or volumetric void strain at which element is eroded |

**Plane Cards.** Define if NPLANES > 0. Repeat Card 5 for each plane (maximum 6 planes).

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DIP | DIPANG | CPLANE | FRPLANE | TPLANE | SHRMAX | LOCAL | |
| Type | F | F | F | F | F | F | I | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | $10^{20}$ | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| DIP | Angle of the plane in degrees below the horizontal (see Remark 11) |
| DIPANG | Plan view angle (degrees) of downhill vector drawn on the plane |
| CPLANE | Cohesion for shear behavior on plane |
| PHPLANE | Friction angle for shear behavior on plane (degrees) |
| TPLANE | Tensile strength across plane (generally zero or very small) |
| SHRMAX | Max shear stress on plane (upper limit, independent of compression) |
| LOCAL | Axes (see Remark 12):<br>EQ.0: DIP and DIPANG are with respect to the global axes.<br>EQ.1: DIP and DIPANG are with respect to the local element axes. |

**Remarks:**

1. **Mohr-Coulomb yield surface.** This material has a Mohr-Coulomb yield surface, given by

$$\tau_{\max} = C + \sigma_n \tan(\text{PHI}) \ ,$$

where $\tau_{\max}$ is the maximum shear stress on any plane, $\sigma_n$ is the normal stress on that plane (positive in compression), $C$ is the cohesion, and PHI is the friction angle. The plastic potential function is of the form

$$\beta\sigma_k - \sigma_i + \text{constant},$$

where $\sigma_k$ is the maximum principal stress, $\sigma_i$ is the minimum principal stress, and

$$\beta = \frac{1 + \sin(\text{PSI})}{1 - \sin(\text{PSI})} \ .$$

2. **Depth-dependent properties.** If depth-dependent properties are used (see GMODDP, GMODGR, CVALGR), the model must be oriented with the $z$-axis in the upward direction.

3. **Plastic Strain.** Plastic strain is defined as

$$\sqrt{\frac{2}{3}\varepsilon_{pij}\varepsilon_{pij}} \ ,$$

that is, the same way as for other elastoplastic material models.

4. **Plastic strain adjustments.** Friction and dilation angles PHI and PSI may vary with plastic strain (see LCPHIEP and LCPSIEP). To model heavily consolidated materials under large shear strains, as the strain increases, the dilation angle typically reduces to zero, and the friction angle reduces to a lower pre-consolidation value.

   For similar reasons, the shear modulus may reduce with plastic strain (see LCGMEP), but this option may sometimes give unstable results.

5. **Additional cohesion.** The load curves, LCCPDR, LCCPT, LCCJDR, and LCCJT, allow extra cohesion to be specified as a function of time. The cohesion is additional to that specified in the material parameters. This is intended for use during the initial stages of an analysis to allow application of gravity or other loads without cracking or yielding, and for the cracking or yielding then to be introduced in a controlled manner. This is done by specifying extra cohesion that exceeds the expected stresses initially, then declining to zero. If no curves are specified, no extra cohesion is applied.

6. **Time-dependent properties.** LCSFAC, the load curve for factor on strength, applies simultaneously to the cohesion and tan (PHI) of the base material and

all joints. This feature is intended to reduce the strength of the material gradually to explore factors of safety. If no curve is present, a constant factor of 1 is assumed. Values much greater than 1.0 may cause problems with stability. Alternatively, separate functions of time may be defined for each of the properties GMOD, CVAL, and PHI using load curves LCGMT, LCCVT, and LCPHT, respectively.

7. **ANISO.** The anisotropic factor, ANISO, applies to the elastic shear stiffness in the global *XZ* and *YZ* planes. It can be used only in a pure Mohr-Coulomb mode (NPLANES = 0).

8. **Tensile pressure limit.** For a friction angle greater than zero, the Mohr-Coulomb yield surface implies a tensile pressure limit equal to CVAL/tan(PHI). By default, voids develop in the material when this pressure limit is reached, and the pressure will never become more tensile than the tensile pressure limit. The volumetric void strain is tracked and is reversible if the strain is reversed.

9. **NOVOID.** If NOVOID = 1, then the tensile pressure limit is not applied and stress states in which the pressure is more tensile than CVAL/tan(PHI) are permitted but will be purely hydrostatic with no shear stress. NOVOID is recommended in Multi-Material ALE simulations in which the development of voids or air space is already accounted for by the Multi-Material ALE.

10. **Soil or rock.** To model soil, set NPLANES = 0. The joints allow modeling of rock and are treated identically to those of *MAT_JOINTED_ROCK.

11. **Joint plane orientations.** The joint plane orientations are defined by the angle of a "downhill vector" drawn on the plane, meaning the vector is oriented within the plane to obtain the maximum possible downhill angle. DIP is the angle of this line below the horizontal. DIPANG is the plan-view angle of the line (pointing downhill) measured clockwise from the global *Y*-axis about the global *Z*-axis.

12. **Masonry and joint planes.** Joint planes are generally defined in the global axis system if they are taken from survey data, and the material represents rock. For this case, set LOCAL = 0. In other cases, it may be more convenient to define the joint plane angles, DIP and DIPANG, relative to the element local axis system (to do this, set LOCAL = 1). For example, this material model can be used to represent masonry with the weak planes representing the mortar joint. In this situation, these joints may be parallel to the local element axes throughout the mesh.

    The choice of defining the joint angles relative to global versus local coordinates is available only for solid elements. For thick shell elements (*ELEMENT_TSHELL), DIP and DIPANG are always relative to the element's local axis, and the setting of LOCAL is ignored.

13. **Rigid body motion.** The joint planes rotate with the rigid body motion of the elements, irrespective of whether their initial definitions are in the global or local axis system.

14. **Extra history variables.** Extra history variables may be requested (see NEIPH on *DATABASE_EXTENT_BINARY). They are described in the following table:

| History Variable # | Description |
|---|---|
| 1 | Mobilized strength fraction for base material |
| 2 | Volumetric void strain |
| 3 | Maximum stress overshoot during plasticity calculation |
| 4 – 9 | Crack opening strain for planes 1 through 6 |
| 10 – 15 | Crack accumulated engineering shear strain for planes 1 through 6 |
| 16 – 20 | Current shear utilization for planes 1 through 6 |
| 21 – 27 | Maximum shear utilization to date for planes 1 through 6 |
| 33 | Elastic shear modulus (for checking depth-dependent input) |
| 34 | Cohesion (for checking depth-dependent input) |

## *MAT_RC_BEAM

This is Material Type 174. It is for Hughes-Liu beam elements only. The material model can represent plain concrete only, reinforcing steel only, or a smeared combination of concrete and reinforcement. The main emphasis of this material model is the cyclic behavior. It is intended primarily for seismic analysis.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EUNL | PR | FC | EC1 | EC50 | RESID |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | Rem 2 | 0.0 | none | 0.0022 | Rem 2 | 0.2 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FT | UNITC | (blank) | (blank) | (blank) | ESOFT | LCHAR | OUTPUT |
| Type | F | F | F | F | F | F | F | F |
| Default | Rem 2 | 1.0 | none | none | none | Rem 2 | none | 0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FRACR | YMREIN | PRREIN | SYREIN | SUREIN | ESHR | EUR | RREINF |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | none | 0.0 | 0.0 | SYREIN | 0.03 | 0.2 | 4.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| EUNL | Initial unloading elastic modulus (see Remark 2) |
| PR | Poisson's ratio. |
| FC | Cylinder strength (stress units) |
| EC1 | Strain at which stress FC is reached. |
| EC50 | Strain at which the stress has dropped to 50% FC |
| RESID | Residual strength factor |
| FT | Maximum tensile stress |
| UNITC | Factor to convert stress units to MPa (see Remark 2) |
| ESOFT | Slope of stress-strain curve post-cracking in tension |
| LCHAR | Characteristic length for strain-softening behavior |
| OUTPUT | Output flag controlling what is written as "plastic strain" (see Remark 4):<br>EQ.0.0: Curvature<br>EQ.1.0: "High-tide" plastic strain in reinforcement |
| FRACR | Fraction of reinforcement (for example, for 1% reinforcement FRACR = 0.01). See Remark 1. |
| YMREIN | Young's Modulus of reinforcement |
| PRREIN | Poisson's Ratio of reinforcement |
| SYREIN | Yield stress of reinforcement |
| SUREIN | Ultimate stress of reinforcement |
| ESHR | Strain at which reinforcement begins to harden |
| EUR | Strain at which reinforcement reaches ultimate stress |
| R_REINF | Dimensionless Ramberg-Osgood parameter *r*. If zero, a default value of 4.0 will be used. If set to -1, parameters will be calculated from Kent & Park formulae (see Remark 3). |

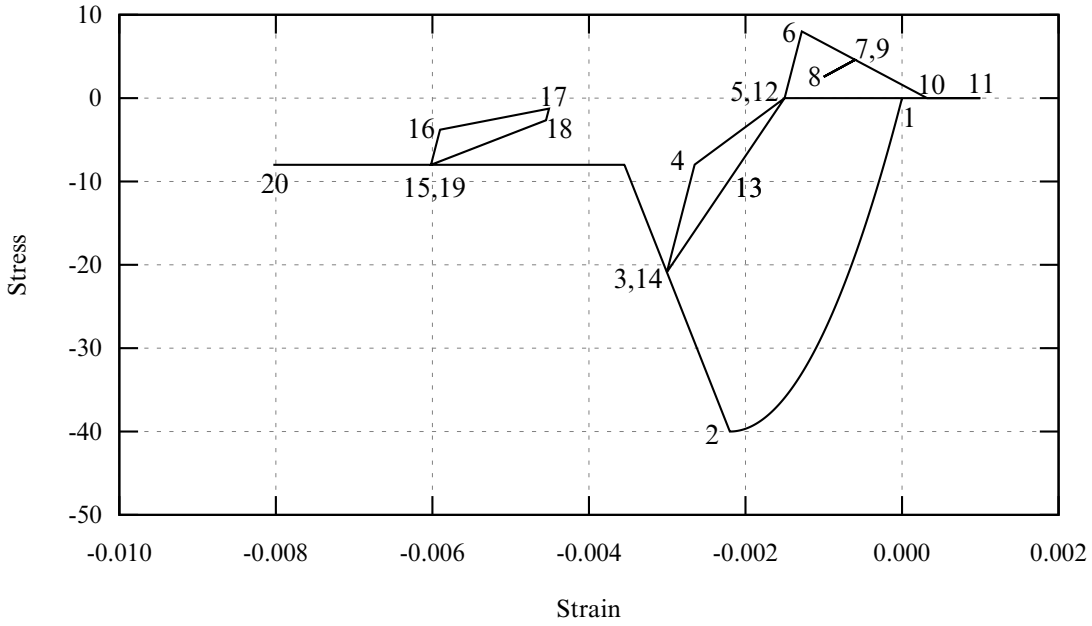



**Figure M174-1.** Example response for Concrete

**Remarks:**

1. **Creating sections for reinforced concrete beams.** This material model can be used to represent unreinforced concrete (FRACR = 0), steel (FRACR = 1), or reinforced concrete with evenly distributed reinforcement (0 < FRACR < 1).

   Alternatively, you can specify the distribution in a section with *INTEGRATION_BEAM. In this case, the PID field for each integration point on *INTEGRATION_BEAM identifies the material for that integration point. You should create one part for concrete and another for steel. These parts should reference two materials of type *MAT_RC_BEAM, but one with FRACR = 0 and the other with FRACR = 1. Then, by assigning one or other of these part IDs to each integration point, the reinforcement can be applied to the correct locations within the section of the beam.

2. **Modeling Concrete.** In monotonic compression, we use the approach of Park and Kent, as described in Park & Paulay [1975]. The material follows a parabolic stress-strain curve up to a maximum stress equal to the cylinder strength FC. Thereafter, the strength decays linearly with strain until the residual strength is reached.

   Default values for some material parameters will be calculated automatically as follows:

$$\mathrm{EC50} = \frac{(3 + 0.29\mathrm{FC})}{145\mathrm{FC} - 1000}$$

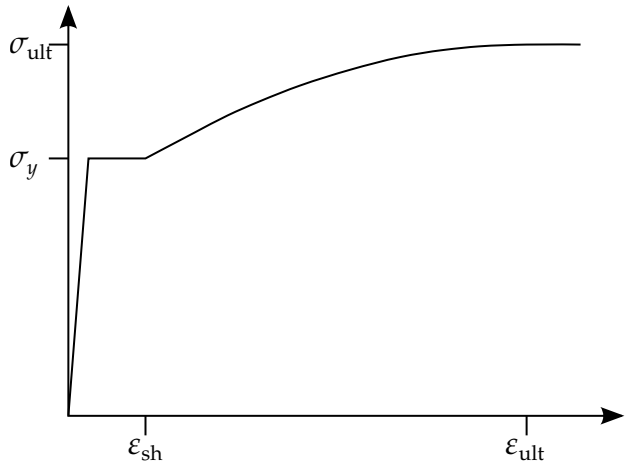



**Figure M174-2.** Monotonic tensile loading of the reinforcement

where FC is in MPa as per Park and Kent test data.

$$\text{EUNL} = \text{initial tangent slope} = \frac{2\text{FC}}{\text{EC1}}$$

Input values for EUNL lower than this are not permitted, but higher values may be defined if desired.

$$\text{FT} = 1.4\left(\frac{\text{FC}}{10}\right)^{\frac{2}{3}}$$

where FC is in MPa as per Park and Kent test data.

$$\text{ESOFT} = \text{EUNL}$$

Input values higher than EUNL are not permitted. UNITC is used only to calculate default values for the above parameters from FC.

Strain-softening behavior tends to lead to deformations being concentrated in one element, and hence the overall force-deflection behavior of the structure can be mesh-size-dependent if the softening is characterized by strain. To avoid this, you may define a characteristic length (LCHAR). This is the length of specimen (or element) that would exhibit the defined monotonic stress-strain relationship. LS-DYNA adjusts the stress-strain relationship after ultimate load for each element, such that all elements irrespective of their length will show the same deflection during strain softening (that is, between ultimate load and residual load). Therefore, although deformation will still be concentrated in one element, the load-deflection behavior should be the same irrespective of element size. For tensile behavior, ESOFT is similarly scaled.

Cyclic behavior is broadly suggested by Blakeley and Park [1973] as described in Park & Paulay [1975]. The stress-strain response lies within the Park-Kent envelope and is characterized by stiff initial unloading response at slope EUNL

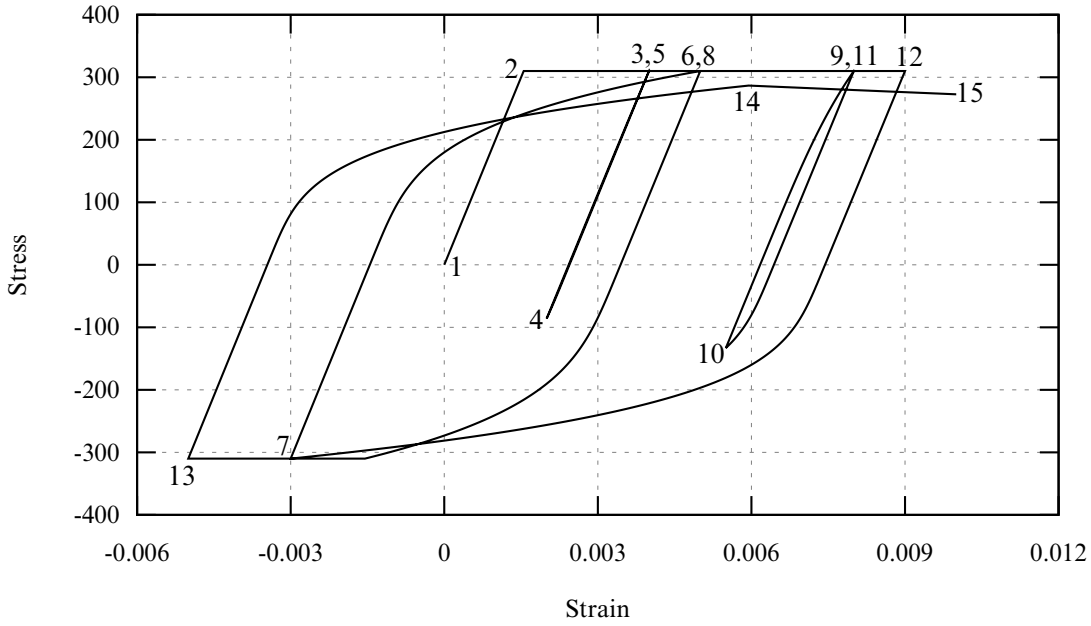



**Figure M174-3.** Stress vs. strain hysteresis plot for the reinforcement with RRE-INF = 4.0

followed by a less stiff response if it unloads to less than half the current strength. Reloading stiffness degrades with increasing strain.

In tension, the stress rises linearly with strain until a tensile limit FT is reached. Thereafter the stiffness and strength decays with increasing strain at a rate ES-OFT. The stiffness also decays such that unloading always returns to strain at which the stress most recently changed to tensile.

3. **Modeling the Reinforcement.** Monotonic loading of the reinforcement results in the stress-strain curve shown in Figure M174-2, which is parabolic between $\varepsilon_{sh}$ and $\varepsilon_{ult}$. The same curve acts as an envelope on the hysteretic behavior when the $x$-axis is cumulative plastic strain.

Unloading from the yielded condition is elastic until the load reverses. Thereafter, the Bauschinger Effect (reduction in stiffness at stresses less than yield during cyclic deformation) is represented by following a Ramberg-Osgood relationship until the yield stress is reached:

$$\varepsilon - \varepsilon_s = \left(\frac{\sigma}{E}\right)\left\{1 + \left(\frac{\sigma}{\sigma_{CH}}\right)^{r-1}\right\}$$

where $\varepsilon$ and $\sigma$ are strain and stress, $\varepsilon_s$ is the strain at zero stress, $E$ is Young's Modulus, and $r$ and $\sigma_{CH}$ are as defined below

We have two options for calculating $r$ and $\sigma_{CH}$, which is performed at each stress reversal:

a) If RREINF is input as -1, $r$ and $\sigma_{CH}$ are calculated internally from formulae given in Kent and Park. Parameter $r$ depends on the number of stress reversals. Parameter $\sigma_{CH}$ depends on the plastic strain that occurred between the previous two stress reversals. The formulae were statistically derived from experiments but may not fit all circumstances. In particular, large differences in behavior may be caused by the presence or absence of small stress reversals such as could be caused by high frequency oscillations. Therefore, results might sometimes be unduly sensitive to small changes in the input data.

b) If RREINF is entered by the user or left blank, $r$ is held constant while $\sigma_{CH}$ is calculated on each reversal such that the Ramberg-Osgood curve meets the monotonic stress-strain curve at the point from which it last unloaded. For example, points 6 and 8 are coincident in Figure M174-3. The default setting of 4.0 for RREINF gives similar hysteresis behavior to that described by Kent & Park but is unlikely to be so sensitive to small changes of input data.

4. **Output.** We recommend setting BEAMIP on *DATABASE_EXTENT_BINARY to request stress and strain output at the individual integration points. Note that for *MAT_RC_BEAM either element curvature or high tide plastic strain for the reinforcement is written to the output files in place of plastic strain depending on the setting of OUTPUT. In the post-processor, select "plastic strain" to display your selection of OUTPUT. For curvature, LS-DYNA compares the absolute values of the curvatures about the local $y$ and $z$ axes and outputs the larger value. In the post-processor, to display the total axial strain (elastic + plastic) at that integration point, select "axial strain." This can be combined with axial stress to create hysteresis plots, such as those shown in Figures M174-1 and M174-3.

## *MAT_VISCOELASTIC_THERMAL

This is Material Type 175. This material model provides a general viscoelastic Maxwell model having up to 12 terms in the prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. Either the coefficients of the prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior. Note that *MAT_GENERAL_VISCOELASTIC (Material Type 76) has all the capability of *MAT_VISCOELASTIC_THERMAL and additionally offers more terms (18) in the prony series expansion and an optional scaling of material properties with moisture content.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | BULK | PCF | EF | TREF | A | B |
|---|---|---|---|---|---|---|---|

**Card 2.** If fitting is done from a relaxation curve, specify fitting parameters on Card 2; *otherwise* if constants are set on Card 3, *LEAVE THIS CARD BLANK*.

| LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
|---|---|---|---|---|---|---|---|

***Card 3.*** These cards are not needed if data is defined using Card 2. This card can be input up to 6 times. The keyword ("*") card terminates this input.

| G*i* | BETA*i* | K*i* | BETAK*i* | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | PCF | EF | TREF | A | B |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Elastic bulk modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| PCF | Tensile pressure elimination flag for solid elements only. If set to unity, tensile pressures are set to zero. |
| EF | Elastic flag:<br>EQ.0: The layer is viscoelastic.<br>EQ.1: The layer is elastic. |
| TREF | Reference temperature for shift function (must be greater than zero) |
| A | Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions |
| B | Coefficient for the Williams-Landel-Ferry shift function |

**Relaxation Curve Card.** If fitting is done from a relaxation curve, specify fitting parameters on Card 2; *otherwise* if constants are set on Viscoelastic Constant Cards, *LEAVE THIS CARD BLANK*.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
| Type | F | I | F | F | F | I | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID | Load curve ID for deviatoric behavior if constants $G_i$ and $\beta_i$ are determined using a least squares fit. This relaxation curve is shown in Figure M175-1. |
| NT | Number of terms in shear fit. If zero, 6 terms are used by default. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is set to 6. |
| BSTART | In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times $\beta_3$, and so on. If zero, BSTART is determined by an iterative trial and error scheme. |
| TRAMP | Optional ramp time for loading |

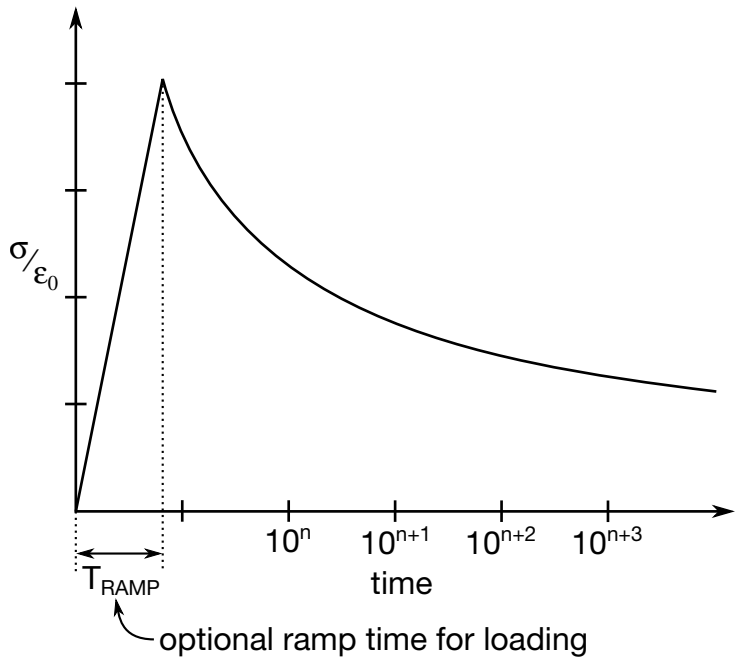



**Figure M175-1.** Relaxation curve. This curve defines stress as a function of time where time is defined on a logarithmic scale. For best results, the points defined in the load curve should be equally spaced on the logarithmic scale. Furthermore, the load curve should be smooth and defined in the positive quadrant. If nonphysical values are determined by least squares fit, LS-DYNA will terminate with an error message after the initialization phase is completed. If the ramp time for loading is included, then the relaxation which occurs during the loading phase is taken into account. This effect may or may not be important.

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDK | Load curve ID for bulk behavior if constants, $K_i$, and $\beta\kappa_i$ are determined using a least squares fit. This relaxation curve is shown in Figure M175-1. |
| NTK | Number of terms desired in bulk fit. If zero 6 terms are used by default. Currently, the maximum number is set to 6. |
| BSTARTK | In the fit, $\beta\kappa_1$ is set to zero, $\beta\kappa_2$ is set to BSTARTK, $\beta\kappa_3$ is 10 times $\beta\kappa_2$, $\beta\kappa_4$ is 10 times $\beta\kappa_3$, and so on. If zero, BSTARTK is determined by an iterative trial and error scheme. |
| TRAMPK | Optional ramp time for bulk loading |

**Viscoelastic Constant Cards.** Up to 6 cards may be input. The next keyword ("*") card terminates this input. These cards are not needed if relaxation data is defined (Card 2). The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included. If an elastic layer is defined, only G*i* and K*i* need to be defined (note in an elastic layer only one card is needed).

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional shear decay constant for the $i^{th}$ term |
| K*i* | Optional bulk relaxation modulus for the $i^{th}$ term |
| BETAK*i* | Optional bulk decay constant for the $i^{th}$ term |

**Remarks:**

Rate effects are taken into accounted through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \ ,$$

where $g_{ijkl(t-\tau)}$ is the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \sum_{m=1}^{N} G_m e^{-\beta_m t} \ .$$

We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. An arbitrary number of terms, up to 6, may be used when applying the viscoelastic model.

For volumetric relaxation, the relaxation function is also represented by the Prony series in terms of bulk moduli:

$$k(t) = \sum_{m=1}^{N} K_m e^{-\beta_{k_m} t}$$

The Arrhenius and Williams-Landel-Ferry (WLF) shift functions account for the effects of the temperature on the stress relaxation. A scaled time, $t'$,

$$t' = \int_0^t \Phi(T) dt \ ,$$

is used in the relaxation function instead of the physical time. The Arrhenius shift function is

$$\Phi(T) = \exp\left[-A\left(\frac{1}{T} - \frac{1}{T_{\text{REF}}}\right)\right] \ ,$$

and the Williams-Landel-Ferry shift function is

$$\Phi(T) = \exp\left(-A\frac{T - T_{\text{REF}}}{B + T - T_{\text{REF}}}\right) .$$

If all three values (TREF, A, and B) are not zero, the WLF function is used; the Arrhenius function is used if B is zero; and no scaling is applied if all three values are zero.

## *MAT_QUASILINEAR_VISCOELASTIC

This is Material Type 176. This is a quasi-linear, isotropic, viscoelastic material based on a one-dimensional model by Fung [1993], which represents biological soft tissues, such as the brain. It is implemented for solid and shell elements. As of LS-DYNA version 971, a second formulation has been implemented that allows for larger strains, but in general, will not give the same results as the previous (default) implementation.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | K | LC1 | LC2 | N | GSTART | M |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SO | E_MIN | E_MAX | GAMA1 | GAMA2 | K | EH | FORM |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is included if and only if LC1 = 0.

| G1 | BETA1 | G2 | BETA2 | G3 | BETA3 | G4 | BETA4 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is included if and only if LC1 = 0.

| G5 | BETA5 | G6 | BETA6 | G7 | BETA7 | G8 | BETA8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is included if and only if LC1 = 0.

| G9 | BETA9 | G10 | BETA10 | G11 | BETA11 | G12 | BETA12 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is included if and only if LC2 = 0.

| C1 | C2 | C3 | C4 | C5 | C6 | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | LC1 | LC2 | N | GSTART | M |
| Type | A | F | F | I | I | F | F | F |
| Default | none | none | none | 0 | 0 | 6 | 1/TMAX | 6 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus |
| LC1 | Load curve ID that defines the relaxation function in shear. This curve is used to fit the coefficients G*i* and BETA*i*. If zero, define the coefficients directly. The latter is recommended. |
| LC2 | Load curve ID that defines the instantaneous elastic response in compression and tension. If zero, define the coefficients directly. *Symmetry is not assumed if only the tension side is defined; therefore, defining the response in tension only, may lead to nonphysical behavior in compression. Also, this curve should give a softening response for increasing strain without any negative or zero slopes. A stiffening curve or one with negative slopes is generally unstable.* |
| N | Number of terms used in the Prony series which must be less than or equal to 6. This number should be equal to the number of decades of time covered by the experimental data. Define this number if LC1 is nonzero. Carefully check the fit in the d3hsp file to ensure that it is valid, since the least square fit is not always reliable. |
| GSTART | Starting value for least square fit. If zero, a default value is set equal to the inverse of the largest time in the experiment. Define this number if LC1 is nonzero. |
| M | Number of terms used to determine the instantaneous elastic response. This variable is ignored with the new formulation but is kept for compatibility with the previous input. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S0 | E_MIN | E_MAX | GAMA1 | GAMA2 | K | EH | FORM |
| Type | F | F | F | F | F | F | F | I |
| Default | 0.0 | -0.9 | 5.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| SO | Strain (logarithmic) output option to control what is written as component 7 to the d3plot database. (LS-PrePost always blindly labels this component as effective plastic strain.) The maximum values are updated for each element each time step:<br>EQ.0.0: Maximum principal strain that occurs during the calculation<br>EQ.1.0: Maximum magnitude of the principal strain values that occurs during the calculation<br>EQ.2.0: Maximum effective strain that occurs during the calculation |
| E_MIN | Minimum strain used to generate the load curve from $C_i$. The default range is -0.9 to 5.1. The computed solution will be more accurate if the user specifies the range used to fit the $C_i$. Linear extrapolation is used outside the specified range. |
| E_MAX | Maximum strain used to generate the load curve from $C_i$. |
| GAMA1 | Material failure parameter (see *MAT_SIMPLIFIED_RUBBER and Figure M181-1) |
| GAMA2 | Material failure parameter (see *MAT_SIMPLIFIED_RUBBER) |
| K | Material failure parameter that controls the volume enclosed by the failure surface (see *MAT_SIMPLIFIED_RUBBER):<br>LE.0.0: Ignore failure criterion<br>GT.0.0: Use actual K value for failure criteria |
| EH | Damage parameter (see *MAT_SIMPLIFIED_RUBBER) |
| FORM | Formulation of model.<br>EQ.0: Original model developed by Fung, which always relaxes to a zero stress state as time approaches infinity<br>EQ.1: Alternative model, which relaxes to the quasi-static elastic response<br>EQ.-1: Improvement on FORM = 0 where the instantaneous elastic response is used in the viscoelastic stress update, not just in the relaxation, as in FORM = 0. Consequently, the constants for the elastic response do not need to be scaled. |

| VARIABLE | DESCRIPTION |
|---|---|
| | In general, formulations 1 and 2 won't give the same responses. |

**Viscoelastic Constants Card 1.** Additional card for LC1 = 0.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G1 | BETA1 | G2 | BETA2 | G3 | BETA3 | G4 | BETA4 |
| Type | F | F | F | F | F | F | F | F |

**Viscoelastic Constants Card 2.** Additional card for LC1 = 0.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G5 | BETA5 | G6 | BETA6 | G7 | BETA7 | G8 | BETA8 |
| Type | F | F | F | F | F | F | F | F |

**Viscoelastic Constants Card 3.** Additional card for LC1 = 0.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G9 | BETA9 | G10 | BETA10 | G11 | BETA11 | G12 | BETA12 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Coefficients of the relaxation function. The number of coefficients is currently limited to 6 although 12 may be read in to maintain compatibility with the previous formulation's input. Define these coefficients if LC1 is set to zero. At least 2 coefficients must be non-zero. |
| BETA*i* | Decay constants of the relaxation function. Define these coefficients if LC1 is set to zero. The number of coefficients is currently limited to 6 although 12 may be read in to maintain compatibility with the previous formulation's input. |

**Instantaneous Elastic Reponses Card.** Additional card for LC2 = 0.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| Ci | Coefficients of the instantaneous elastic response in compression and tension. Define these coefficients only if LC2 is set to zero. |

**Remarks:**

The equations for the original model (FORM = 0) are given as:

$$\sigma_V(t) = \int_0^t G(t-\tau)\frac{\partial \sigma_\varepsilon[\varepsilon(\tau)]}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \tau}d\tau$$

$$G(t) = \sum_{i=1}^{n} G_i e^{-\beta t}$$

$$\sigma_\varepsilon(\varepsilon) = \sum_{i=1}^{k} C_i \varepsilon^i$$

where $G$ is the shear modulus. Effective strain (which can be written to the d3plot database) is calculated as follows:

$$\varepsilon^{\text{eff}} = \sqrt{\frac{2}{3}\varepsilon_{ij}\varepsilon_{ij}}$$

The polynomial for instantaneous elastic response should contain only odd terms if symmetric tension-compression response is desired.

The new model (FORM = 1) is based on the hyperelastic model used in *MAT_SIMPLIFIED_RUBBER assuming incompressibility. The one-dimensional expression for $\sigma_\varepsilon$ generates the uniaxial stress-strain curve and an additional visco-elastic term is added on,

$$\sigma(\varepsilon, t) = \sigma_{SR}(\varepsilon) + \sigma_V(t)$$

$$\sigma_V(t) = \int_0^t G(t-\tau)\frac{\partial \varepsilon}{\partial \tau}d\tau$$

where the first term to the right of the equals sign is the hyperelastic stress and the second is the viscoelastic stress. Unlike the previous formulation, where the stress always relaxed to zero, the current formulation relaxes to the hyperelastic stress.

## *MAT_HILL_FOAM

Purpose: This is Material Type 177. This is a highly compressible foam based on the strain-energy function proposed by Hill [1979]; also see Storakers [1986]. Poisson's ratio effects are taken into account.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | K | N | MU | LCID | FITTYPE | LCSR |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is included if LCID = 0.

| C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is included if LCID = 0.

| B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 |
|---|---|---|---|---|---|---|---|

***Card 4.*** This card is optional.

| R | M | | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | N | MU | LCID | FITTYPE | LCSR |
| Type | A | F | F | F | F | I | I | I |
| Default | none | none | none | 0. | 0. | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus. This modulus is used for determining the contact interface stiffness. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| N | Material constant. Define if LCID = 0 below; otherwise, N is fit from the load curve data. See Remark 2. |
| MU | Damping coefficient. |
| LCID | Load curve ID that defines the force per unit area as a function of the stretch ratio. This curve can be given for either uniaxial or biaxial data depending on FITTYPE. See Remark 1. |
| FITTYPE | Type of fit:<br>EQ.1: Uniaxial data<br>EQ.2: Biaxial data<br>EQ.3: Pure shear data |
| LCSR | Load curve ID that defines the uniaxial or biaxial stretch ratio (see FITTYPE) as a function of the transverse stretch ratio. |

**Material Constant Card 1.** Additional card for LCID = 0.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| C*i* | Material constants. See equations below. Define up to 8 coefficients if LCID = 0. |

**Material Constant Card 2.** Additional card for LCID = 0.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| B*i* | Material constants. See equations below. Define up to 8 coefficients if LCID = 0. |

**Mullins Effect Card.** This card is optional.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | R | M | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| R | Mullins effect model *r* coefficient |
| M | Mullins effect model *m* coefficient |

**Remarks:**

1. **Load Curve Fit.** If load curve data is defined, the fit generated by LS-DYNA must be closely checked in the d3hsp output file. The nonlinear least squares procedure in LS-DYNA, which is used to fit the data, may be inadequate.

2. **Material Model.** The Hill strain energy density function for this highly compressible foam is given by:

$$W = \sum_{j=1}^{m} \frac{C_j}{b_j}\left[\lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n}\left(J^{-nb_j} - 1\right)\right]$$

where $C_j$, $b_j$, and $n$ are material constants. $J = \lambda_1\lambda_2\lambda_3$ and represents the ratio of the deformed to the undeformed state. The constant $m$ is internally set to 4. If the number of points in the curve is less than 8, then $m$ is set to the number of points divided by 2. The principal Cauchy stresses are:

$$t_i = \sum_{j=1}^{m} \frac{C_j}{J}\left[\lambda_i^{b_j} - J^{-nb_j}\right] \quad i = 1,2,3 \ .$$

From the above equations the shear modulus is:

$$\mu = \frac{1}{2}\sum_{j=1}^{m} C_j b_j \ ,$$

and the bulk modulus is:

$$K = 2\mu\left(n + \frac{1}{3}\right) \ .$$

LS-DYNA uses the value for $K$ defined in the input in the calculation of contact forces and for the material time step. Generally, this value should be equal to or greater than the $K$ given in the above equation.

## *MAT_VISCOELASTIC_HILL_FOAM

This is Material Type 178. This material is a highly compressible foam based on the strain-energy function proposed by Hill [1979]; also see Storakers [1986]. The extension to include large strain viscoelasticity is due to Feng and Hallquist [2002].

### Card Summary:

**Card 1.** This card is required.

| MID | RO | K | N | MU | LCID | FITTYPE | LCSR |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCVE | NT | GSTART | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is defined if and only if LCID = 0.

| C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is defined if and only if LCID = 0.

| B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 |
|---|---|---|---|---|---|---|---|

**Card 5.** Include up to 12 of this card. The next keyword ("*") card terminates this input.

| G*i* | BETA*i* | | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | N | MU | LCID | FITTYPE | LCSR |
| Type | A | F | F | F | F | I | I | I |
| Default | none | none | none | 0.0 | 0.05 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| K | Bulk modulus. This modulus is used for determining the contact interface stiffness. |
| N | Material constant. Define if LCID = 0 below; otherwise, N is fit from the load curve data. See remarks below. |
| MU | Damping coefficient (0.05 < recommended value < 0.50) |
| LCID | Load curve ID that defines the force per unit area as a function of the stretch ratio. This curve can be given for either uniaxial or bi-axial data depending on FITTYPE. Load curve LCSR below must also be defined. |
| FITTYPE | Type of fit:<br>EQ.1: Uniaxial data<br>EQ.2: Biaxial data |
| LCSR | Load curve ID that defines the uniaxial or biaxial stress ratio (see FITTYPE) as a function of the transverse stretch ratio |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCVE | NT | GSTART | | | | | |
| Type | I | I | F | | | | | |
| Default | 0 | 6 | 1/TMAX | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCVE | Optional load curve ID that defines the relaxation function in shear. This curve is used to fit the coefficients G*i* and BETA*i* (see Card 5). If zero, define the coefficients directly (recommended). |
| NT | Number of terms used to fit the Prony series, which must be less than or equal to 12. This number should be equal to the number of decades of time covered by the experimental data. Define this number if LCVE is nonzero. Carefully check the fit in the d3hsp file to ensure that it is valid, since the least square fit is not always |

| VARIABLE | DESCRIPTION |
|---|---|
| | reliable. |
| GSTART | Starting value for the least squares fit. If zero, a default value is set equal to the inverse of the largest time in the experiment. Define this number if LCVE is nonzero. See remarks below. |

**Material Constant Card 1.** Additional card for LCID = 0

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

**Material Constant Card 2.** Additional card for LCID = 0

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C*i* | Material constants. See remarks below. Define up to 8 coefficients. |
| B*i* | Material constants. See remarks below. Define up to 8 coefficients. |

**Viscoelastic Constant Cards**. Up to 12 cards may be input. The next keyword ("*") card terminates this input.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA*i* | Optional decay constant if $i^{th}$ term |

**Remarks:**

If load curve data is defined, the fit generated by LS-DYNA must be closely checked in the d3hsp output file. It may occur that the nonlinear least squares procedure in LS-DYNA, which is used to fit the data, is inadequate.

The Hill strain energy density function for this highly compressible foam is given by:

$$W = \sum_{j=1}^{n} \frac{C_j}{b_j}\left[\lambda_1^{b_j} + \lambda_2^{b_j} + \lambda_3^{b_j} - 3 + \frac{1}{n}\left(J^{-nb_j} - 1\right)\right] ,$$

where $C_j$, $b_j$, and $n$ are material constants and $J = \lambda_1\lambda_2\lambda_3$ represents the ratio of the deformed to the undeformed state. The principal Cauchy stresses are

$$\tau_{ii} = \sum_{j=1}^{n} \frac{C_j}{J}\left[\lambda_i^{b_j} - J^{-nb_j}\right] \quad i = 1,2,3$$

From the above equations the shear modulus is:

$$\mu = \frac{1}{2}\sum_{j=1}^{m} C_j b_j$$

and the bulk modulus is:

$$K = 2\mu\left(n + \frac{1}{3}\right)$$

The value for $K$ defined in the input is used in the calculation of contact forces and for the material time step. Generally, this value should be equal to or greater than the $K$ given in the above equation.

Rate effects are taken into account through linear viscoelasticity by a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}\,(t-\tau)\,\frac{\partial \varepsilon_{kl}}{\partial \tau}\,d\tau \ ,$$

or in terms of the second Piola-Kirchhoff stress, $S_{ij}$, and Green's strain tensor, $E_{ij}$,

$$S_{ij} = \int_0^t G_{ijkl}\,(t-\tau)\,\frac{\partial\, E_{kl}}{\partial \tau}\,d\tau \ ,$$

where $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$ are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

If we wish to include only simple rate effects, the relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta_m t}$$

given by,

$$g(t) = \sum_{i=1}^{n} G_i e^{-\beta_i t}$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. The viscoelastic behavior is optional and an arbitrary number of terms may be used.

## *MAT_LOW_DENSITY_SYNTHETIC_FOAM_{OPTION}

This is Material Type 179 (and 180 if the ORTHO option below is active) for modeling rate independent low density foams, which have the property that the hysteresis in the loading-unloading curve is considerably reduced after the first loading cycle. For this material we assume that the loading-unloading curve is identical after the first cycle of loading is completed and that the damage is isotropic, that is, the behavior after the first cycle of loading in the orthogonal directions also follows the second curve. The main application at this time is to model the observed behavior in the compressible synthetic foams that are used in some bumper designs. Tables may be used in place of load curves to account for strain rate effects.

Available options include:

<BLANK>

ORTHO

WITH_FAILURE

ORTHO_WITH_FAILURE

If the foam develops orthotropic behavior, that is, after the first loading and unloading cycle the material in the orthogonal directions are unaffected, then the ORTHO option should be used. If the ORTHO option is active the directionality of the loading is stored. This option requires additional storage for history variables related to the orthogonality and is slightly more expensive.

An optional failure criterion is included. A description of the failure model is provided below for material type 181, *MAT_SIMPLIFIED_RUBBER/FOAM.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | LCID1 | LCID2 | HU | BETA | DAMP |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SHAPE | FAIL | BVFLAG | ED | BETA1 | KCON | REF | TC |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if LCID < 0.

| RFLAG | DTRT | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if using the IF_FAILURE keyword.

| K | GAMA1 | GAMA2 | EH | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | LCID1 | LCID2 | HU | BETA | DAMP |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 1. | none | 0.05 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus. This modulus is used if the elongations are tensile as described for the *MAT_LOW_DENSITY_FOAM. |
| LCID1 | Load curve or table ID describing nominal stress as a function of strain for the undamaged material (see Remark 2):<br>GT.0: Load curve ID (see *DEFINE_CURVE) for nominal stress as a function of strain for the undamaged material.<br>LT.0: -LCID1 is a table ID (see *DEFINE_TABLE) for nominal stress as a function of strain for the undamaged material as a function of strain rate |
| LCID2 | Load curve or table ID. The load curve ID (see *DEFINE_CURVE) defines the nominal stress as a function of strain for the damaged material. The table ID (see *DEFINE_TABLE) defines the nominal stress as a function of strain for the damaged material as a function of strain rate. See Remark 2. Note that if LCID1 refers to a table that LCID2 must also refer to a table. |
| HU | Hysteretic unloading factor between 0.0 and 1.0 (default = 1.0, that is, no energy dissipation); see Figure M179-1 and Remarks 1 and 2. |
| BETA | Decay constant to model creep in unloading, $\beta$ |
| DAMP | Viscous coefficient (.05 < recommended value < .50) to model damping effects.<br>LT.0.0: \|DAMP\| is the load curve ID that defines the damping |

| VARIABLE | DESCRIPTION |
|---|---|
| | constant as a function of the maximum strain in compression defined as: $\varepsilon_{max} = \max(1-\lambda_1, 1-\lambda_2, 1.-\lambda_3)$ . In tension, the damping constant is set to the value corresponding to the strain at 0.0. The abscissa should be defined from 0.0 to 1.0. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SHAPE | FAIL | BVFLAG | ED | BETA1 | KCON | REF | TC |
| Type | F | F | F | F | F | F | F | F |
| Default | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | $10^{20}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| SHAPE | Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor. Values less than one reduce the energy dissipation and greater than one increase dissipation; see also Figure M179-1 and Remarks 1 and 2. |
| FAIL | Failure option after cutoff stress is reached:<br>EQ.0.0: Tensile stress remains at cut-off value,<br>EQ.1.0: Tensile stress is reset to zero. |
| BVFLAG | Bulk viscosity activation flag:<br>EQ.0.0: No bulk viscosity (recommended),<br>EQ.1.0: Bulk viscosity active. |
| ED | Optional Young's relaxation modulus, $E_d$, for rate effects. |
| BETA1 | Optional decay constant, $\beta_1$. |
| KCON | Stiffness coefficient for contact interface stiffness. If undefined, the maximum slope in the stress as a function of strain curve is used. When the maximum slope is used for the contact, the time step size for this material is reduced for stability. In some cases, $\Delta t$ may be significantly smaller, so defining a reasonable stiffness is |

| VARIABLE | DESCRIPTION |
|---|---|
| | recommended. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |
| TC | Tension cut-off stress |

Additional card for LCID1 < 0.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RFLAG | DTRT | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| RFLAG | Rate type for input:<br>EQ.0.0: LCID1 and LCID2 should be input as functions of true strain rate.<br>EQ.1.0: LCID1 and LCID2 should be input as functions of engineering strain rate. |
| DTRT | Strain rate averaging flag:<br>EQ.0.0: Use weighted running average.<br>LT.0.0: Average the last 11 values.<br>GT.0.0: Average over the last DTRT time units. |

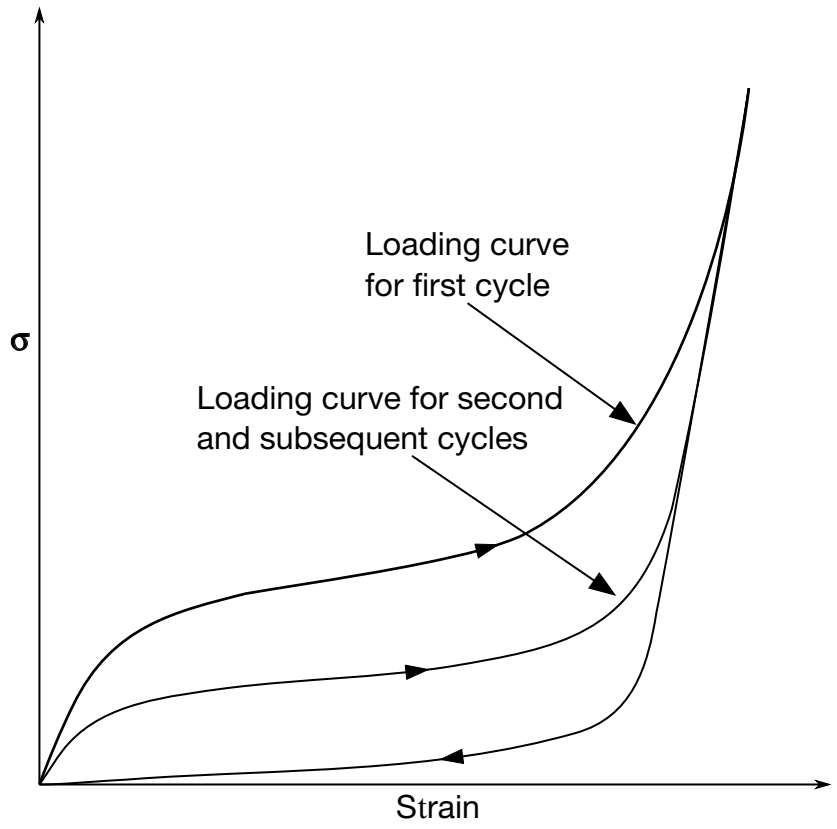



**Figure M179-1.** Loading and reloading curves.

Additional card for WITH_FAILURE keyword option.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | GAMA1 | GAMA2 | EH | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Material failure parameter that controls the volume enclosed by the failure surface.<br>LE.0.0: Ignore failure criterion;<br>GT.0.0: Use actual K value for failure criterions. |
| GAMA1 | Material failure parameter; see Figure M181-1. |
| GAMA2 | Material failure parameter |
| EH | Damage parameter |

**Remarks:**

1. **Uniaxial response.** This model is based on *MAT_LOW_DENSITY_FOAM. The uniaxial response is shown in Figure M179-1 with a large shape factor and small hysteretic factor. If the shape factor is not used, the unloading will occur on the loading curve for the second and subsequent cycles.

2. **Damage and hysteresis.** The damage is defined as the ratio of the current volume strain to the maximum volume strain, and it is used to interpolate between the responses defined by LCID1 and LCID2.

   HU defines a hysteretic scale factor that is applied to the stress interpolated from LCID1 and LCID2,

   $$\sigma = \left[\text{HU} + (1 - \text{HU}) \times \min\left(1, \frac{e_{\text{int}}}{e_{\text{int}}^{\text{max}}}\right)^{S}\right] \sigma(\text{LCID1,LCID2})$$

   where $e_{\text{int}}$ is the internal energy and $S$ is the shape factor. Setting HU to 1 results in a scale factor of 1. Setting HU close to zero scales the stress by the ratio of the internal energy to the maximum internal energy raised to the power $S$, resulting in the stress being reduced when the strain is low.

## *MAT_SIMPLIFIED_RUBBER/FOAM_{OPTION}

This is Material Type 181. This material model provides a rubber and foam model specified with a single uniaxial load curve or a family of uniaxial curves at discrete strain rates. Hysteretic unloading may optionally be modeled through a single uniaxial unloading curve or a two-parameter formulation. Specifying a Poisson's ratio greater than 0.0 and less than 0.49 activates the foam formulation. This material may be used with both shell and solid elements.

Available options include:

<BLANK>

WITH_FAILURE

LOG_LOG_INTERPOLATION

When the WITH_FAILURE keyword option is active, a strain-based failure surface is defined that is suitable for incompressible polymers. It models failure in both tension and compression. With LOG_LOG_INTERPOLATION, LS-DYNA interpolates the strain rate effect in the table TBID using log-log interpolation.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois, LSTC, and Prof. Dave J. Benson, UCSD.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | KM | MU | G | SIGF | REF | PRTEN |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SGL | SW | ST | LC/TBID | TENSION | RTYPE | AVGOPT | PR |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is included if the WITH_FAILURE keyword option is used.

| K | GAMA1 | GAMA2 | EH | | | | |
|---|---|---|---|---|---|---|---|

***Card 4.*** This card is optional. It must be included if Card 5 is included.

| LCUNLD | HU | SHAPE | STOL | VISCO | HISOUT | | |
|---|---|---|---|---|---|---|---|

***Card 5.*** This card is optional. Up to 12 cards in this format may be input. If fewer than 12 cards are input, the next keyword ("*") card terminates this input.

| Gi | BETAi | VFLAG | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | KM | MU | G | SIGF | REF | PRTEN |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| KM | Linear bulk modulus (see PR on Card 2) |
| MU | Damping coefficient (0.05 < recommended value < 0.50; default is 0.10). |
| G | Shear modulus for frequency independent damping. Frequency independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF. See Remark 1. |
| SIGF | Limit stress for frequency independent, frictional damping. See Remark 1. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details).<br>EQ.0.0: Off<br>EQ.1.0: On |
| PRTEN | The tensile Poisson's ratio for shells (optional). If PRTEN is zero, PR will serve as the Poisson's ratio for both tension and |

| VARIABLE | DESCRIPTION |
|---|---|
| | compression in shells. If PRTEN is nonzero, PR will serve only as the compressive Poisson's ratio for shells. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LC/TBID | TENSION | RTYPE | AVGOPT | PR |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| SGL | Specimen gauge length |
| SW | Specimen width |
| ST | Specimen thickness |
| LC/TBID | Load curve ID or table ID (see *DEFINE_TABLE) giving the force as a function of the actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), then curve LC is also engineering stress versus engineering strain. If the table definition is used, a family of curves is defined for discrete strain rates. The curves should cover the complete range of expected response, including both compressive (negative values) and tensile (positive values) regimes. |
| TENSION | Parameter that controls how the rate effects are treated. Applicable to the table definition.<br>EQ.-1.0: Rate effects are considered during tension and compression loading, but not during unloading.<br>EQ.0.0: Rate effects are considered for compressive loading only.<br>EQ.1.0: Rate effects are treated identically in tension and compression. |
| RTYPE | Strain rate type if a table is defined:<br>EQ.0.0: True strain rate<br>EQ.1.0: Engineering strain rate |
| AVGOPT | Averaging option for strain rates to reduce numerical noise: |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: \|AVGOPT\| is a time window/interval over which the strain rates are averaged. This option is recommended because it is time step size independent and generally more stable. |
| | EQ.0.0: Simple average of 12 time steps |
| | EQ.1.0: Running average of last 12 averages |
| PR | Poisson ratio or viscosity coefficient: |
| | LE.0.0: An incompressible rubber material is assumed, using the Ogden strain-energy functional. PR is set to 0.495 internally for computing the time-step only and is not used otherwise. Compressibility is defined using KM. For PR < 0 in solid elements, an incrementally updated mean viscous stress develops according to the following equation with $\beta = \|PR\|$ and $K_m = KM$ (see Card 1): $$p^{n+1} = p^n e^{-\beta\Delta t} + K_m \dot{\varepsilon}_{kk} \left( \frac{1 - e^{-\beta\Delta t}}{\beta} \right) .$$ |
| | GT.0.0.AND.LT.0.49: A foam material is assumed, using the Hill strain-energy function. PR gives the Poisson's ratio. KM on Card 1 should be given a meaningful value as it is used for the critical time step computation, contact penalty stiffness, and hourglass control type 6. Selective-reduced integration is *not* used for fully integrated elements. |
| | GE.0.49.AND.LT.0.5: An incompressible rubber material is assumed, using the Ogden strain-energy functional. PR is used for computing the time-step only. Compressibility is defined using KM on Card 1. Selective-reduced integration is used for fully integrated elements. |

Additional card required for WITH_FAILURE option. Otherwise skip this card.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K | GAMA1 | GAMA2 | EH | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Material failure parameter that controls the volume enclosed by the failure surface.<br>LE.0.0: Ignore failure criterion.<br>GT.0.0: Use actual K value for failure criterion (see Remark 2). |
| GAMA1 | Material failure parameter, $\Gamma_1$; see Remark 2 and Figure M181-1. |
| GAMA2 | Material failure parameter, $\Gamma_2$; see Remark 2. |
| EH | Damage parameter, $h$. See Remark 2. |

**Optional Parameter Card.**

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCUNLD | HU | SHAPE | STOL | VISCO | HISOUT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCUNLD | Optional load curve (see *DEFINE_CURVE) giving the force as a function of actual length during unloading. The unload curve should cover exactly the same range as LC or the load curves of TBID and its end points should have identical values. In other words, the combination of LC and LCUNLD or the first curve of TBID and LCUNLD describes a complete cycle of loading and unloading. See also material *MAT_083. |
| HU | Hysteretic unloading factor between 0 and 1 (default = 1.0, meaning no energy dissipation). See also material *MAT_083 and Figure M57-1. This option is ignored if LCUNLD is used. |

| VARIABLE | DESCRIPTION |
|---|---|
| SHAPE | Shape factor for unloading. Active for nonzero values of the hysteretic unloading factor HU. Values less than one reduces the energy dissipation and greater than one increases dissipation. See also material *MAT_083 and Figure M57-1. |
| STOL | Tolerance in stability check. See Remark 3. |
| VISCO | Flag to invoke viscoelastic formulation. The viscoelastic formulation does not apply to shell elements and will be ignored for shells. See Remark 4.<br>EQ.0.0: Purely elastic<br>EQ.1.0: Viscoelastic formulation (solids only) |
| HISOUT | History output flag.<br>EQ.0.0: Default<br>EQ.1.0: Principal strains are written to history variables 25, 26, and 27. |

**Optional Viscoelastic Constants Cards.** Up to 12 cards in format 5 may be input. A keyword card (with a "*" in column 1) terminates this input if fewer than 12 cards are used.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | VFLAG | | | | | |
| Type | F | F | I | | | | | |
| Default | none | none | 0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term. The G*i* and BETA*i* terms are used only for solid elements when VISCO = 1. See Remark 4. |
| BETA*i* | Optional decay constant if $i^{th}$ term. See Remark 4. |
| VFLAG | Type of viscoelasticity formulation. This appears only on the first line defining G*i*, BETA*i*, and VFLAG. See Remark 4. |

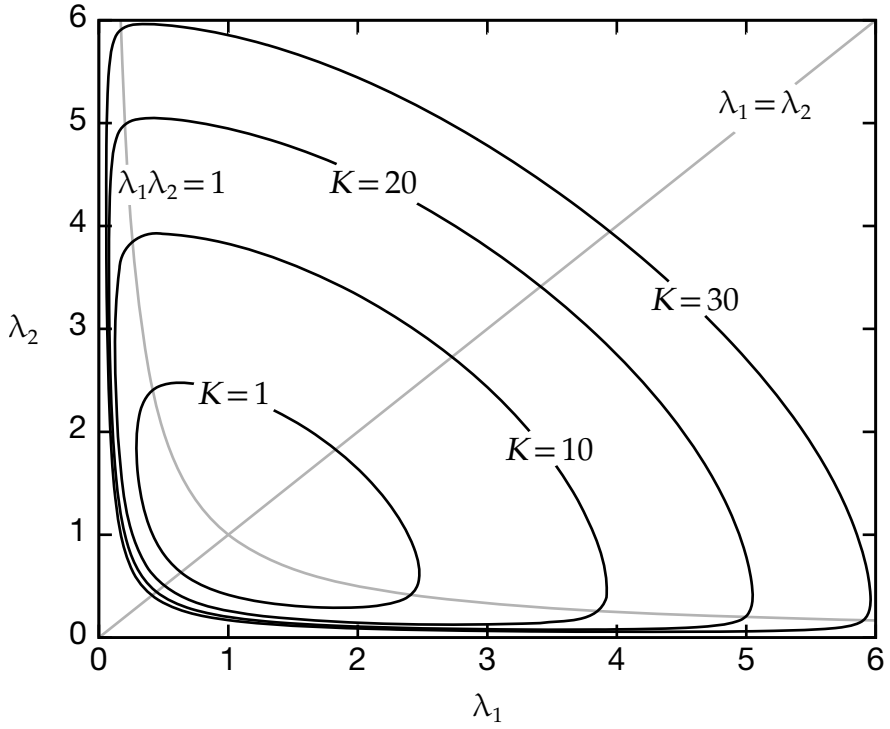
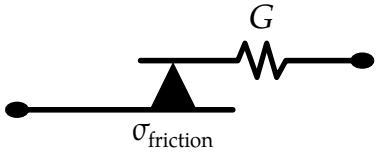



**Figure M181-1.** Failure surface for polymer for $\Gamma_1 = 0$ and $\Gamma_2 = 0.02$.

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0: Standard viscoelasticity formulation (default) |
| | EQ.1: Viscoelasticity formulation using instantaneous elastic stress |

**Remarks:**

1. **Frequency-independent damping.** Frequency-independent damping is obtained by the having a spring and slider in series as shown in the following sketch:




2. **Failure criterion for polymers.** The general failure criterion for polymers is proposed by Feng and Hallquist as

$$f(I_1, I_2, I_3) = (I_1 - 3) + \Gamma_1(I_1 - 3)^2 + \Gamma_2(I_2 - 3) = K$$

where $K$ is a material parameter which controls the size enclosed by the failure surface. $I_1$, $I_2$ and $I_3$ are the three invariants of right Cauchy-Green deformation tensor (**C**):

$$I_1 = C_{ii} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$
$$I_2 = \frac{1}{2}(C_{ii}C_{jj} - C_{ij}C_{ij}) = \lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_3^2$$
$$I_3 = \det(\mathbf{C}) = \lambda_1^2\lambda_2^2\lambda_3^2$$

with $\lambda_i$ as the stretch ratios in the three principal directions.

To avoid sudden failure and numerical difficulty, material failure, which is usually a time point, is modeled as a process of damage growth. In this case, the two threshold values are chosen as $(1 - h)K$ and $K$, where $h$ (also called EH) is a small number chosen based on experimental results reflecting the range between damage initiation and material failure.

The damage is defined as function of $f$:

$$D = \begin{cases} 0 & \text{if} \quad f \le (1-h)K \\ \frac{1}{2}\left[\, 1 + \cos\frac{\pi(f-K)}{hK}\right] & \text{if} \quad (1-h)K < f < K \\ 1 & \text{if} \quad f \ge K \end{cases}$$

With this definition, damage is first-order continuous, and the tangent stiffness matrix will be continuous. The reduced stress considering damage effect is

$$\sigma_{ij} = (1 - D)\sigma_{ij}^o$$

where $\sigma_{ij}^o$ is the undamaged stress. Prior to final failure, material damage is assumed to be recoverable. Once material failure occurs, damage will become permanent.

3. **Stability of the stress-strain response.** Bad choices of curves for the stress-strain response may lead to an unstable model. LS-DYNA can check for stability given a certain tolerance with the field STOL. The check is done by examining the eigenvalues of the tangent modulus at selected stretch points. A warning message is issued if an eigenvalue is less than –STOL × BULK, where BULK indicates the bulk modulus of the material. For STOL < 0, the check is disabled. Otherwise, it should be chosen with care. A too small value may detect instabilities that are insignificant in practice. To avoid significant instabilities, we recommend using smooth curves. At best the curves should be continuously differentiable. For the incompressible case, a sufficient condition for stability is that the stress-stretch curve $S(\lambda)$ can be written as

$$S(\lambda) = H(\lambda) - \frac{H\left(\frac{1}{\sqrt{\lambda}}\right)}{\lambda\sqrt{\lambda}}$$

where $H(\lambda)$ is a function with $H(1) = 0$ and $H'(\lambda) > 0$.

4. **Viscoelasticity.** For solid elements, rate effects may also be taken into account through linear viscoelasticity by setting VISCO = 1.0. For VFLAG = 0, viscoelasticity is modeled through a convolution integral of the form:

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}}{\partial \tau}d\tau \ ,$$

or in terms of the second Piola-Kirchhoff stress, $\mathbf{S}_0$, and Green's strain tensor, $\mathbf{E}_{\mathrm{RT}}$

$$S_{ij} = \int_0^t G_{ijkl}(t-\tau)\frac{\partial E_{kl}}{\partial \tau}d\tau \ .$$

Here $g_{ijkl}(t-\tau)$ and $G_{ijkl}(t-\tau)$ are the relaxation functions for the different stress measures. This stress is added to the stress tensor determined from the strain energy functional.

The relaxation function is represented by six terms from the Prony series:

$$g(t) = \alpha_0 + \sum_{m=1}^{N} \alpha_m e^{-\beta t}$$

given by

$$g(t) = \sum_{i=1}^{n} G_i e^{-\beta_i t} \ .$$

This model is effectively a Maxwell fluid which consists of a dampers and springs in series. We characterize this in the input by shear moduli, $G_i$, and decay constants, $\beta_i$. An arbitrary number of terms may be used.

For VFLAG = 1, the viscoelastic term is

$$\sigma_{ij} = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \sigma_{kl}^E}{\partial \tau}d\tau$$

where $\sigma_{kl}^E$ is the instantaneous stress evaluated from the internal energy functional. The coefficients in the Prony series, therefore, correspond to normalized relaxation moduli instead of elastic moduli.

## *MAT_SIMPLIFIED_RUBBER_WITH_DAMAGE_{OPTION}

Available options include (see Remark 1):

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LOG_LOG_INTERPOLATION

This is Material Type 183. This material model provides an incompressible rubber model defined by a single uniaxial load curve for loading (or a table if rate effects are considered) and a single uniaxial load curve for unloading. This model is similar to *MAT_-181/*MAT_SIMPLIFIED_RUBBER/FOAM. This material may be used with both shell and solid elements.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois, LSTC, and Prof. Dave J. Benson, UCSD.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | MU | G | SIGF | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL | SW | ST | LC / TBID | TENSION | RTYPE | AVGOPT | |
| Type | F | F | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCUNLD | REF | STOL | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| K | Linear bulk modulus |
| MU | Damping coefficient |
| G | Shear modulus for frequency-independent damping. Frequency-independent damping is based on a spring and slider in series. The critical stress for the slider mechanism is SIGF defined below. For the best results, the value of G should be 250-1000 times greater than SIGF. |
| SIGF | Limit stress for frequency-independent, frictional damping. |
| SGL | Specimen gauge length |
| SW | Specimen width |
| ST | Specimen thickness |
| LC/TBID | Load curve ID or table ID (see *DEFINE_TABLE) defining the force as a function of actual change in the gauge length. If SGL, SW, and ST are set to unity (1.0), curve LC is also engineering stress as a function of engineering strain. If the table definition is used, a family of curves is defined for discrete strain rates. The curves should cover the complete range of expected responses, including both compressive (negative values) and tensile (positive values) regimes. See Remark 1. |
| TENSION | Parameter that controls how the rate effects are treated. It is applicable to the table definition.<br>EQ.-1.0: Rate effects are considered during tension and compression loading, but not during unloading.<br>EQ.0.0: Rate effects are considered for compressive loading only.<br>EQ.1.0: Rate effects are treated identically in tension and compression. |
| RTYPE | Strain rate type if a table is defined:<br>EQ.0.0: True strain rate<br>EQ.1.0: Engineering strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| AVGOPT | Averaging option for determining strain rate to reduce numerical noise.<br>EQ.0.0: Simple average of twelve time steps<br>EQ.1.0: Running 12 point average |
| LCUNLD | Load curve (see *DEFINE_CURVE) defining the force as a function of actual change in the gauge length during unloading. The unloading curve should cover exactly the same range as LC (or as the first curve of table TBID) and its endpoints should have identical values, meaning the combination of LC (or the first curve of table TBID) and LCUNLD describes a complete cycle of loading and unloading. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |
| STOL | Tolerance in stability check. See Remark 2. |

**Remarks:**

1. **LOG_LOG_INTERPOLATION.** The LOG_LOG_INTERPOLATION option interpolates the strain rate effect in the table TBID using log-log interpolation.

2. **Stability.** A bad choice of curves for the stress-strain response may lead to an unstable model. STOL enables this check with its value setting the tolerance level. The check is done by examining the eigenvalues of the tangent modulus at selected stretch points and a warning message is issued if an eigenvalue is less than –STOL × BULK, where BULK indicates the bulk modulus of the material. STOL < 0 disables the check. When enabled, the value of STOL should be chosen with care because a too small value may detect instabilities that are insignificant in practice. To avoid significant instabilities it is recommended to use smooth curves. At best the curves should be continuously differentiable. In fact, for the incompressible case, a sufficient condition for stability is that the stress-stretch curve $S(\lambda)$ can be written as

$$S(\lambda) = H(\lambda) - \frac{H(\frac{1}{\sqrt{\lambda}})}{\lambda\sqrt{\lambda}}$$

where $H(\lambda)$ is a function with $H(1) = 0$ and $H'(\lambda) > 0$.

## *MAT_COHESIVE_ELASTIC

This is Material Type 184. It is a simple cohesive elastic model for use with cohesive element formulations; see the field ELFORM in *SECTION_SOLID and *SECTION_-SHELL.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | ET | EN | FN_FAIL | FT_FAIL |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| ROFLG | Flag for whether density is specified per unit area or volume:<br>EQ.0: Specifies the density is per unit volume (default)<br>EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero |
| INTFAIL | The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value.<br>LT.0.0: Employs a Newton-Cotes integration scheme, and the element will be deleted when \|INTFAIL\| integration points have failed.<br>EQ.0.0: Employs a Newton-Cotes integration scheme, and the element will *not* be deleted even if it satisfies the failure criterion.<br>GT.0.0: Employs a Gauss integration scheme, and the element will be deleted when INTFAIL integration points have failed. |
| ET | Stiffness in the plane of the cohesive element |
| EN | Stiffness normal to the plane of the cohesive element |
| FN_FAIL | Traction in the normal direction for tensile failure |

| VARIABLE | DESCRIPTION |
|---|---|
| FT_FAIL | Traction in the tangential direction for shear failure |

**Remarks:**

This material cohesive model outputs three tractions having units of force per unit area into the d3plot database rather than the usual six stress components. The in-plane shear traction along the 1-2 edge replaces the $x$-stress, the orthogonal in plane shear traction replaces the $y$-stress, and the traction in the normal direction replaces the $z$-stress.

## *MAT_COHESIVE_TH

This is Material Type 185. It is a cohesive model by Tvergaard and Hutchinson [1992] for use with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL. The implementation is based on the description of the implementation in the Sandia National Laboratory code, Tahoe [2003].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | SIGMAX | NLS | TLS | TLS2 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LAMDA1 | LAMDA2 | LAMDAF | STFSF | ISW | ALPHA1 | ALPHA2 | |
| Type | F | F | F | F | I | F | F | |

Additional card that may be used for XFEM shells; see *SECTION_SHELL_XFEM.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DR | ALPHA3 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| ROFLG | Flag for whether density is specified per unit area or volume.<br>EQ.0: Specifies density per unit volume (default)<br>EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero |

| VARIABLE | DESCRIPTION |
|---|---|
| INTFAIL | The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 as the recommended value.<br>LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when \|INTFAIL\| integration points have failed.<br>EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion.<br>GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed. |
| SIGMAX | Peak traction |
| NLS | Length scale (maximum separation) in the normal direction |
| TLS | Length scale (maximum separation) in the tangential direction |
| LAMDA1 | Scaled distance to peak traction ($\Lambda_1$) |
| LAMDA2 | Scaled distance to beginning of softening ($\Lambda_2$). |
| LAMDAF | Scaled distance for failure ($\Lambda_{fail}$) |
| STFSF | Penetration stiffness multiplier. The penetration stiffness, *PS*, in terms of input parameters becomes: $$PS = \frac{STFSF \times SIGMAX}{NLS \times \left(\frac{LAMDA1}{LAMDAF}\right)}$$ |
| TLS2 | Length scale (maximum separation) in the tear direction (for XFEM shells only). See Remark 2. |
| ISW | Cohesive law for XFEM shells only (see Remark 2):<br>EQ.-1: Initially rigid cohesive law (type I)<br>EQ.-2: Initially rigid cohesive law (type II) |
| ALPHA1 | Ratio of maximum mode II shear traction to normal traction (for XFEM shells only) |

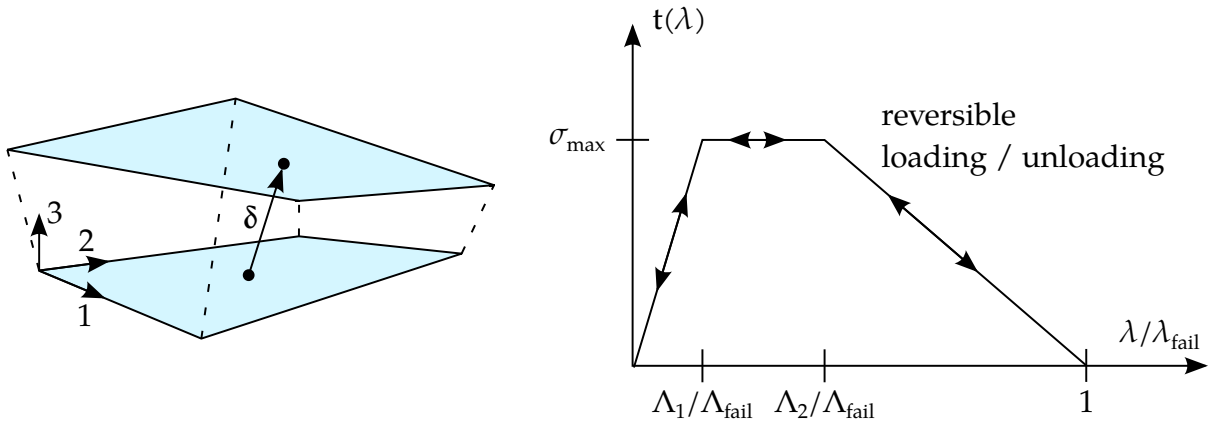



**Figure M185-1.** Relative displacement and trilinear traction-separation law

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA2 | Ratio of maximum mode III shear traction to normal traction (for XFEM shells only) |
| DR | Critical rotation scale (for XFEM shells only) |
| ALPHA3 | Ratio of maximum bending moment to normal traction (for XFEM shells only) |

**Material Model:**

In this cohesive material model, we use a dimensionless separation measure, $\lambda$, for the interaction between relative displacements in the normal ($\delta_3$ - mode I) and tangential ($\delta_1$, $\delta_2$ - mode II) directions (see Figure M185-1 left):

$$\lambda = \sqrt{\left(\frac{\delta_1}{\mathrm{TLS}}\right)^2 + \left(\frac{\delta_2}{\mathrm{TLS}}\right)^2 + \left(\frac{\langle\delta_3\rangle}{\mathrm{NLS}}\right)^2}$$

The Macaulay brackets distinguish between tension ($\delta_3 \geq 0$) and compression ($\delta_3 < 0$). NLS and TLS are critical values, representing the maximum separations in the interface in the normal and tangential directions. For the stress calculation, we use a trilinear traction-separation law, given by (see Figure M185-1 right):

$$t(\lambda) = \begin{cases} \sigma_{\max}\dfrac{\lambda}{\Lambda_1/\Lambda_{\mathrm{fail}}} & \lambda < \Lambda_1/\Lambda_{\mathrm{fail}} \\ \sigma_{\max} & \Lambda_1/\Lambda_{\mathrm{fail}} < \lambda < \Lambda_2/\Lambda_{\mathrm{fail}} \\ \sigma_{\max}\dfrac{1-\lambda}{1-\Lambda_2/\Lambda_{\mathrm{fail}}} & \Lambda_2/\Lambda_{\mathrm{fail}} < \lambda < 1 \end{cases}$$

With this law, the traction drops to zero when $\lambda = 1$. A potential, $\phi$, is defined as:

$$\phi(\delta_1, \delta_2, \delta_3) = \text{NLS} \times \int_0^{\lambda} t(\bar{\lambda})\ d\bar{\lambda}$$

Finally, the tangential components ($t_1$, $t_2$) and normal component ($t_3$) of the traction acting on the interface in the fracture process zone are given by:

$$t_{1,2} = \frac{\partial \phi}{\partial \delta_{1,2}} = \frac{t(\lambda)}{\lambda}\frac{\delta_{1,2}}{\text{TLS}}\frac{\text{NLS}}{\text{TLS}}, \qquad t_3 = \frac{\partial \phi}{\partial \delta_3} = \frac{t(\lambda)}{\lambda}\frac{\delta_3}{\text{NLS}}$$

which in matrix notation is

$$\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \frac{t(\lambda)}{\lambda}\begin{bmatrix} \dfrac{\text{NLS}}{\text{TLS}^2} & 0 & 0 \\ 0 & \dfrac{\text{NLS}}{\text{TLS}^2} & 0 \\ 0 & 0 & \dfrac{1}{\text{NLS}} \end{bmatrix}\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

In the case of compression ($\delta_3 < 0$), penetration is avoided by:

$$t_3 = \frac{\text{STFSF} \times \sigma_{\max}}{\text{NLS} \times \Lambda_1/\Lambda_{\text{fail}}}\delta_3$$

Loading and unloading follows the same path, that is, this model is completely reversible.

**Remarks:**

1. **Traction output to d3plot.** This cohesive material model outputs three tractions having units of force per unit area to the d3plot database rather than the usual six stress components. The in-plane shear traction,$t_1$, along the 1-2 edge replaces the $x$-stress, the orthogonal in-plane shear traction, $t_2$, replaces the $y$-stress, and the traction in the normal direction, $t_3$, replaces the $z$-stress.

2. **XFEM Shells.** For XFEM shells, TLS for $\delta_2$ in the above equation is replaced by TLS2. Since the initially rigid cohesive law is used, an element fails only when the stress level reaches SIGMAX. $\Lambda_1$ is only used to define the penetration stiffness in case of a crack closing (compression).

## *MAT_COHESIVE_GENERAL

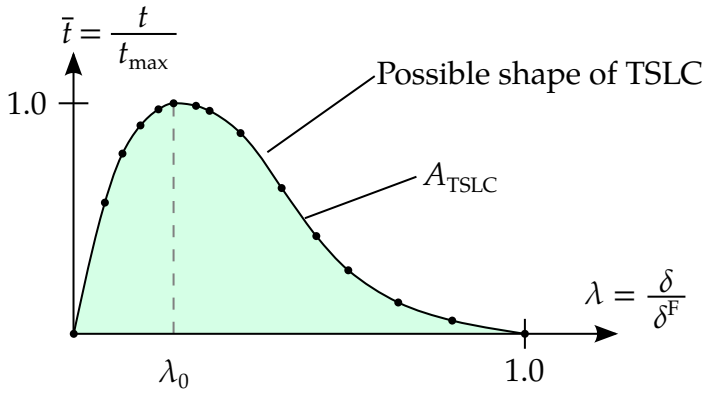
This is Material Type 186. It can be used only with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL. The material model allows you to choose from three general irreversible mixed-mode interaction cohesive formulations. It also includes an arbitrary normalized traction-separation law given by a load curve (TSLC). These three formulations are differentiated through the type of the effective separation parameter (TES). The interaction between fracture modes I and II is considered. Irreversible conditions are enforced with a damage formulation (unloading/reloading path pointing to/from the origin). See remarks for details.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | TES | TSLC | GIC | GIIC |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XMU | T | S | STFSF | TSLC2 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| ROFLG | Flag for whether density is specified per unit area or volume:<br>EQ.0: Specifies density per unit volume (default)<br>EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero |
| INTFAIL | Number of integration points required for a cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value.<br>LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when \|INTFAIL\| integration |

| VARIABLE | DESCRIPTION |
|---|---|
| | points have failed.<br>EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion.<br>GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed. |
| TES | Type of effective separation parameter (ESP).<br>EQ.0.0: A dimensional separation measure is used. For the interaction between modes I and II, a mixed-mode propagation criterion is given by a power law. See Remarks 1 and 2.<br>EQ.1.0: A dimensional separation measure is used. For the interaction between modes I and II, a mixed-mode propagation criterion is given by the Benzeggagh-Kenane law [1996]. See Remarks 1 and 2.<br>EQ.2.0: A dimensionless separation measure is used for the interaction between mode I displacements and mode II displacements (similar to *MAT_185, but with damage and general traction-separation law). See Remarks 1 and 3. |
| TSLC | Normalized traction-separation load curve ID. The curve must be normalized in both coordinates and must contain at least three points: $(0.0, 0.0)$, $(\lambda_0, 1.0)$, and $(1.0, 0.0)$. These points represent the origin, the peak, and the complete failure, respectively (see Figure M186-1). A platform can exist in the curve like the trilinear TSLC (see *MAT_185). See Remark 1. |
| GIC | Fracture toughness / energy release rate $G_I^c$ for mode I |
| GIIC | Fracture toughness / energy release rate $G_{II}^c$ for mode II |
| XMU | Exponent that appears in the power failure criterion (TES = 0.0) or the Benzeggagh-Kenane failure criterion (TES = 1.0). Recommended values for XMU are between 1.0 and 2.0. See Remark 2. |
| T | Peak traction in normal direction (mode I). See Remark 1. |
| S | Peak traction in tangential direction (mode II). See Remark 1. |




| | Mode I | Mode II |
|---|---|---|
| $t_{max}$ | $T$ | $S$ |
| $\delta^F$ | $\frac{G_I^C}{A_{TSLC}T}$ | $\frac{G_{II}^C}{A_{TSLC}S}$ |
| $G^C$ | $G_I^C$ | $G_{II}^C$ |

**Figure M186-1.** Normalized traction-separation law

| VARIABLE | DESCRIPTION |
|---|---|
| STFSF | Penetration stiffness multiplier for compression. Factor = (1.0 + STFSF) is used to scale the compressive stiffness, that is, no scaling is done with STFSF = 0.0 (recommended). |
| TSLC2 | Normalized traction-separation load curve ID for Mode II. The curve must be normalized in both coordinates and must contain at least three points: $(0.0, 0.0)$, $(\lambda_0, 1.0)$, and $(1.0, 0.0)$, which represents the origin, the peak and the complete failure, respectively (see Figure M186-1). If not specified, TSLC is used for Mode II behavior as well. See Remark 1. |

**Remarks:**

1. **Traction-separation behavior.** For all three formulations, the traction-separation behavior of this model is mainly given by $G_I^c$ and T for normal mode I, $G_{II}^c$ and S for tangential mode II, and an arbitrary normalized traction-separation load curve for both modes (see Figure M186-1). The maximum (or failure) separations are then given by:

$$\delta_I^F = \frac{G_I^c}{A_{TSLC} \times \mathrm{T}} \quad , \quad \delta_{II}^F = \frac{G_{II}^c}{A_{TSLC} \times \mathrm{S}}$$

Here $A_{TSLC}$ is the area under the normalized traction-separation curve given with TSLC.

If TSLC2 is defined,

$$\delta_I^F = \frac{G_I^c}{A_{TSLC} \times \mathrm{T}} \quad , \quad \delta_{II}^F = \frac{G_{II}^c}{A_{TSLC2} \times \mathrm{S}}$$

Here $A_{TSLC2}$ is the area under the normalized traction-separation curve given with TSLC2.

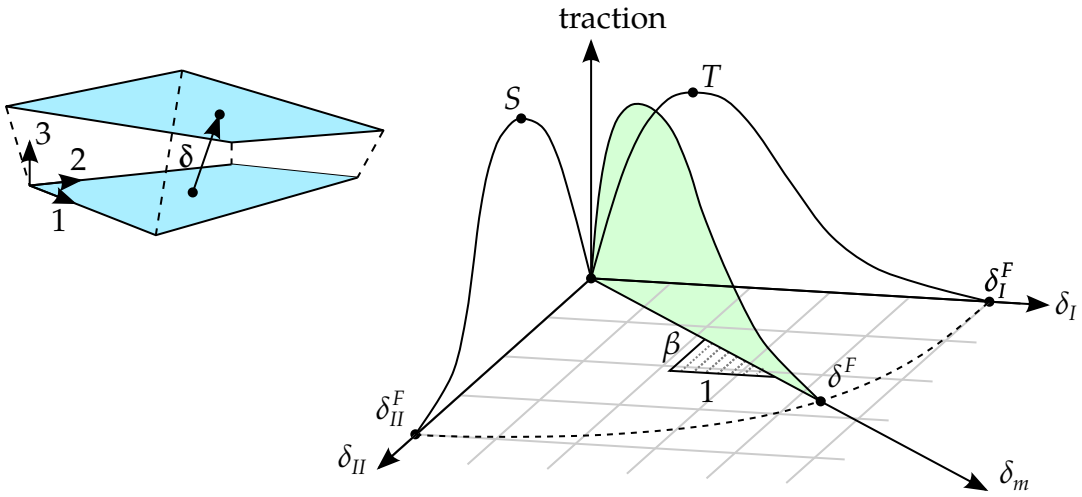



**Figure M186-2**. Mixed mode traction-separation law

2. **First and second mixed-mode interaction cohesive formulations (TES = 0.0 and 1.0).** For mixed-mode behavior, three different formulations are possible. We recommend TES = 0.0 with XMU = 1.0 as a first try. In this remark we will discuss the two formulations with dimensional separation measures.

The total mixed-mode relative displacement $\delta_m$ is defined as $\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2}$, where $\delta_I = \delta_3$ is the separation in normal direction (mode I) and $\delta_{II} = \sqrt{\delta_1^2 + \delta_2^2}$ is the separation in tangential direction (mode II). See Figure M186-2.

The ultimate mixed-mode displacement $\delta^F$ (total failure) for the power law (TES = 0.0) is

$$\delta^F = \frac{1+\beta^2}{A_{\mathrm{TSLC}}}\left[\left(\frac{\mathrm{T}}{G_I^c}\right)^{\mathrm{XMU}} + \left(\frac{\mathrm{S}\times\beta^2}{G_{II}^c}\right)^{\mathrm{XMU}}\right]^{-\frac{1}{\mathrm{XMU}}}$$

If TSLC2 is defined, this changes to:

$$\delta^F = (1+\beta^2)\left[\left(\frac{A_{\mathrm{TSLC}}\times\mathrm{T}}{G_I^c}\right)^{\mathrm{XMU}} + \left(\frac{A_{\mathrm{TSLC2}}\times\mathrm{S}\times\beta^2}{G_{II}^c}\right)^{\mathrm{XMU}}\right]^{-\frac{1}{\mathrm{XMU}}}$$

Alternatively, for the Benzeggagh-Kenane law [1996] (TES = 1.0) $\delta^F$ is given by:

$$\delta^F = \frac{1+\beta^2}{A_{\mathrm{TSLC}}(\mathrm{T}+\mathrm{S}\times\beta^2)}\left[G_I^c + (G_{II}^c - G_I^c)\left(\frac{\mathrm{S}\times\beta^2}{T+\mathrm{S}\times\beta^2}\right)^{\mathrm{XMU}}\right]$$

If TSLC2 is defined, this changes to:

$$\delta^F = \frac{1+\beta^2}{A_{\text{TSLC}} \times \text{T} + A_{\text{TSLC2}} \times \text{S} \times \beta^2} \left[ G_I^c + (G_{II}^c - G_I^c) \left( \frac{A_{\text{TSLC2}} \times \text{S} \times \beta^2}{A_{\text{TSLC}} \times T + A_{\text{TSLC2}} \times \text{S} \times \beta^2} \right)^{\text{XMU}} \right]$$

where $\beta = \delta_{II}/\delta_I$ is the "mode mixity". The larger the chosen exponent, XMU, is, the larger the fracture toughness will be in mixed-mode situations.

In this model, damage of the interface is considered, that is, irreversible conditions are enforced with loading/unloading paths coming from/pointing to the origin. This formulation is similar to *MAT_COHESIVE_MIXED_MODE (*MAT_138), but with the arbitrary traction-separation law TSLC.

3. **Third mixed-mode interaction cohesive formulations (TES = 2.0).** For TES = 2.0, we use a dimensionless effective separation parameter $\lambda$ to model the interaction between relative displacements in normal ($\delta_3$ - mode I) and tangential ($\delta_1$,$\delta_2$ - mode II) directions:

$$\lambda = \sqrt{\left(\frac{\delta_1}{\delta_{II}^F}\right)^2 + \left(\frac{\delta_2}{\delta_{II}^F}\right)^2 + \left\langle\frac{\delta_3}{\delta_I^F}\right\rangle^2}$$

Macaulay brackets distinguish between tension ($\delta_3 \geq 0$) and compression ($\delta_3 < 0$). $\delta_I^F$ and $\delta_{II}^F$ are critical values, representing the maximum separations in the interface in normal and tangential direction. For the stress calculation, the normalized traction-separation load curve TSLC is used:

$$t = t_{\max} \times \bar{t}(\lambda)$$

This formulation is similar to *MAT_COHESIVE_TH (*MAT_185) but with an arbitrary traction-separation law and a damage formulation (that is, irreversible conditions are enforced with loading/unloading paths coming from/pointing to the origin).

### *MAT_SAMP-1

Purpose: This is Material Type 187 (Semi-Analytical Model for Polymers). This material model uses an isotropic C-1 smooth yield surface to describe non-reinforced plastics. [Kolling, Haufe, Feucht, and Du Bois 2005] details the implementation.

This material law has been developed at DaimlerChrysler, Sindelfingen, in collaboration with Paul Du Bois and Dynamore, Stuttgart.

## Card Summary:

**Card 1.** This card is required.

| MID | RO | BULK | GMOD | EMOD | NUE | RBCFAC | NUMINT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCID-T | LCID-C | LCID-S | LCID-B | NUEP | LCID-P | | INCDAM |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| LCID-D | EPFAIL | DEPRPT | LCID_TRI | LCID_LC | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| MITER | MIPS | | INCFAIL | ICONV | ASAF | | NHSV |
|---|---|---|---|---|---|---|---|

***Card 5.*** This card is optional.

| LCEMOD | BETA | FILT | | | VFAST | | |
|---|---|---|---|---|---|---|---|

## Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | GMOD | EMOD | NUE | RBCFAC | NUMINT |
| Type | A | F | F | F | F | F | F | I/F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

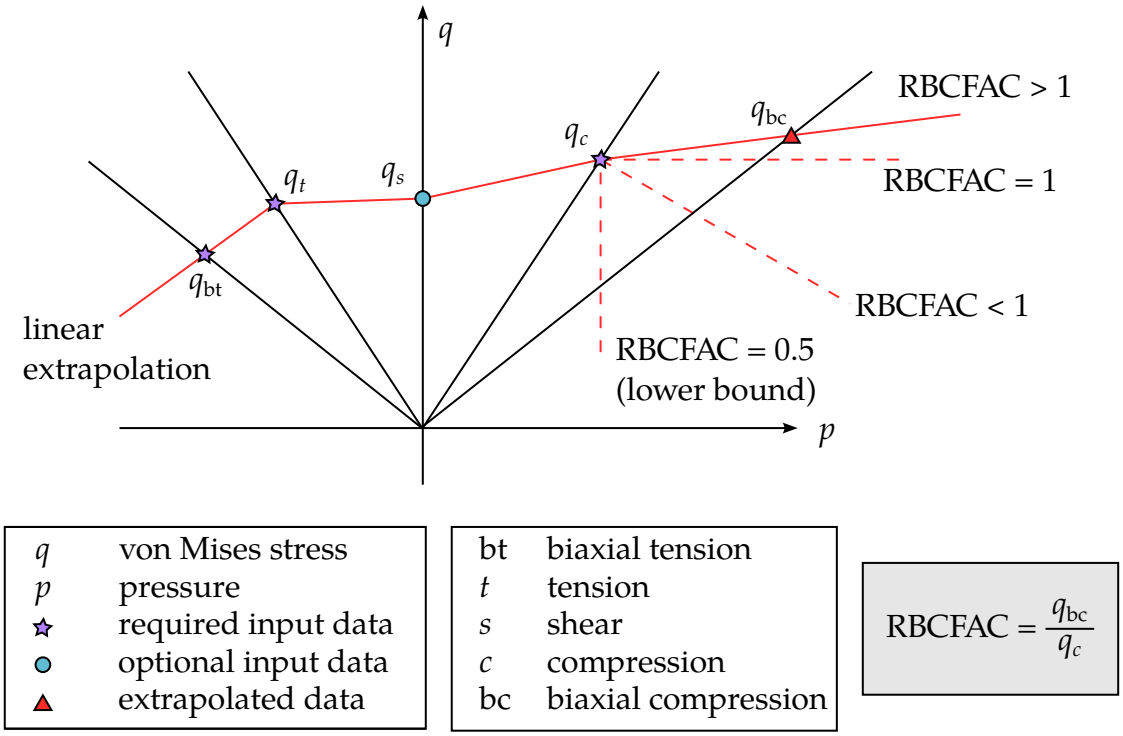



**Figure M187-1.** von Mises stress as a function of pressure

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| BULK | Optional bulk modulus used in the time step calculation for solids only |
| GMOD | Optional shear modulus used in the time step calculation for solids only |
| EMOD | Young's modulus |
| NUE | Poisson ratio (see Remark 6) |
| RBCFAC | Ratio of yield in biaxial compression as a function of yield in uniaxial compression. A nonzero RBCFAC with all four curves LCID-T, LCID-C, LCID-S, and LCID-B defined activates a piecewise-linear yield surface as shown in Figure M187-1. See Remark 3. The default is 0. |
| NUMINT | Number of integration points that must fail before the element is deleted. This option is available for shells and solids.<br>LT.0.0: \|NUMINT\| is the percentage of integration points/layers which must fail before the shell element fails. For fully |

| VARIABLE | DESCRIPTION |
|---|---|
| | integrated shells, a layer fails if one integration point in the layer fails, and then the element fails if the given percentage of layers fails. Only available for shells. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID-T | LCID-C | LCID-S | LCID-B | NUEP | LCID-P | | INCDAM |
| Type | I | I | I | I | F | I | | I |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID-T | Load curve or table ID giving the yield stress as a function of plastic strain. These curves should be obtained from quasi-static and (optionally) dynamic uniaxial tensile tests. This input is mandatory, and the material model will not work unless at least one tensile stress-strain curve is given. If LCID-T is a table ID, the table values are plastic strain rates, and a curve of yield stress versus plastic strain must be given for each of those strain rates. If the first value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of plastic strain rate. When the highest plastic strain rate is several orders of magnitude greater than the lowest, it is recommended that the natural log of plastic strain rate be input in the table. See Remark 4. |
| LCID-C | Optional load curve ID giving the yield stress as a function of plastic strain. This curve should be obtained from a quasi-static uniaxial compression test. |
| LCID-S | Optional load curve ID giving the yield stress as a function of plastic strain. This curve should be obtained from a quasi-static shear test. |
| LCID-B | Optional load curve ID giving the yield stress as a function of plastic strain, this curve should be obtained from a quasi-static biaxial tensile test. |
| NUEP | Plastic Poisson's ratio: an estimated ratio of transversal to longitudinal plastic rate of deformation under uniaxial loading should be given. |
| LCID-P | Load curve ID giving the plastic Poisson's ratio as a function of |

| VARIABLE | DESCRIPTION |
|---|---|
| | plastic strain during uniaxial tensile and uniaxial compressive testing. The plastic strain on the abscissa is negative for compression and positive for tension. It is important to cover both tension and compression. If LCID-P is given, NUEP is ignored. |
| INCDAM | Flag to control the damage evolution as a function of triaxiality:<br>EQ.0: Damage evolution is independent of the triaxiality.<br>EQ.1: An incremental formulation is used to compute the damage. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID-D | EPFAIL | DEPRPT | LCID_TRI | LCID_LC | | | |
| Type | I | F | F | I | I | | | |

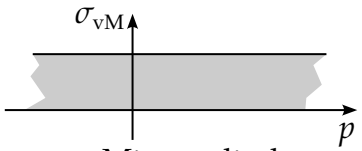
| VARIABLE | DESCRIPTION |
|---|---|
| LCID-D | Load curve ID giving the damage parameter as a function of equivalent plastic strain during uniaxial tensile testing (history variable #2). By default, this option assumes that effective (i.e., undamaged) yield values are used in the load curves LCID-T, LCID-C, LCID-S, and LCID-B. If LCID-D is given a negative value, true (meaning damaged) yield stress values can be used. In this case, an automatic stress-strain recalibration (ASSR) algorithm is activated. The damage value must be defined in the range $0 \leq d < 1$. If EPFAIL and DEPRPT are given, the curve is used only until the effective plastic strain reaches EPFAIL. |
| EPFAIL | This parameter is the equivalent plastic strain at failure under uniaxial tensile loading (history variable #2). If EPFAIL is given as a negative integer, a load curve that defines EPFAIL as a function of the plastic strain rate is expected. The default value is $10^5$. |
| DEPRPT | Increment of equivalent plastic strain under uniaxial tensile loading (history variable #2) between the failure and rupture points. Stresses will fade out to zero between EPFAIL and EPFAIL + DEPRPT. If DEPRPT is given a negative value, a curve definition is expected where DEPRPT is defined as a function of the triaxiality. |
| LCID_TRI | Load curve that specifies a factor that works multiplicatively on the |

| VARIABLE | DESCRIPTION |
|---|---|
| | value of EPFAIL depending on the triaxiality (that is, $p/\sigma_{vM}$). For a triaxiality of -1/3, a value of 1.0 should be specified. |
| LCID_LC | Load curve that specifies a factor that works multiplicatively on the value of EPFAIL, depending on a characteristic element length, defined as the average length of spatial diagonals |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MITER | MIPS | | INCFAIL | ICONV | ASAF | | NHSV |
| Type | I | I | | I | I | F | | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MITER | Maximum number of iterations in the cutting plane algorithm. The default is 400. |
| MIPS | Maximum number of iterations in the secant iteration performed to enforce plane stress (shell elements only). This variable is obsolete. A fixed three-step approach is used by default. |
| INCFAIL | Flag to control the failure evolution as a function of triaxiality:<br>EQ.0: Failure evolution is independent of the triaxiality.<br>EQ.1: Incremental formulation is used to compute the failure value.<br>EQ.-1: The failure model is deactivated. |
| ICONV | Formulation flag:<br>EQ.0: Default<br>EQ.1: The yield surface is internally modified by increasing the shear yield until a convex yield surface is achieved. |
| ASAF | Safety factor used only if ICONV = 1. Values between 1 and 2 can improve convergence; however, the shear yield will be artificially increased if this option is used. The default is 1. |
| NHSV | Number of history variables. The default is 22. Set to 28 if the "instability criterion" should be included in the output (see Remark 5). Note that NEIPS or NEIPH must also be set on *DATA- |

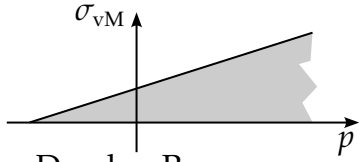
$$\left.\begin{array}{l}\text{LCID-C}=0\\ \text{LCID-S}=0\\ \text{LCID-B}=0\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\sigma_c=\sigma_t\\ \sigma_s=\dfrac{\sigma_t}{\sqrt{3}}\end{array}\right.$$




von Mises cylinder

$$\left.\begin{array}{l}\text{LCID-C}=0\\ \text{LCID-S}\neq 0\\ \text{LCID-B}=0\end{array}\right\} \Rightarrow \sigma_c=\frac{\sqrt{3}\sigma_t\sigma_s}{2\sigma_t-\sqrt{3}\sigma_s}$$

$$\left.\begin{array}{l}\text{LCID-C}\neq 0\\ \text{LCID-S}=0\\ \text{LCID-B}=0\end{array}\right\} \Rightarrow \sigma_s=\frac{2\sigma_c\sigma_t}{\sqrt{3}(\sigma_t+\sigma_c)}$$




Drucker-Prager cone

$$\left.\begin{array}{l}\text{LCID-C}=0\\ \text{LCID-S}=0\\ \text{LCID-B}\neq 0\end{array}\right\} \Rightarrow \left\{\begin{array}{l}\sigma_c=\dfrac{\sigma_t\sigma_b}{3\sigma_b-2\sigma_t}\\ \sigma_s=\dfrac{\sigma_t\sigma_b}{\sqrt{3}(2\sigma_b-\sigma_t)}\end{array}\right.$$

**Figure M187-2.** Fewer than 3 load curves

| VARIABLE | DESCRIPTION |
|---|---|
| | BASE_EXTENT_BINARY for the history variable data to be output. |

**Optional Card.**

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCEMOD | BETA | FILT | | | VFAST | | |
| Type | F | F | F | | | F | | |

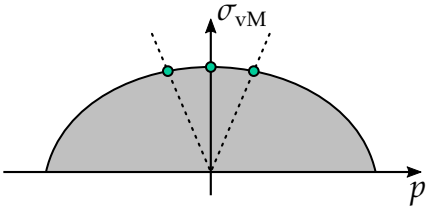
| VARIABLE | DESCRIPTION |
|---|---|
| LCEMOD | Load curve ID defining Young's modulus as a function of effective strain rate. If the first strain rate is negative, all values are expected to represent the natural logarithm of a strain rate. |
| BETA | Decay constant in viscoelastic law:<br>$\dot{\sigma}(t) = -\beta \times \sigma(t) + E(\dot{\varepsilon}(t)) \times \dot{\varepsilon}(t)$<br>If LCEMOD > 0 is used, a non-zero value for BETA is mandatory. |

LCID-C ≠ 0, LCID-S ≠ 0, LCID-B = 0 ⇒ normal SAMP-1 behavior

LCID-C ≠ 0, LCID-S = 0, LCID-B ≠ 0 ⇒ $\sigma_s = \frac{1}{\sqrt{3}} \sqrt{\frac{3\sigma_b^2\sigma_c\sigma_t}{(2\sigma_b+\sigma_c)(2\sigma_b-\sigma_t)}}$

LCID-C = 0, LCID-S ≠ 0, LCID-B ≠ 0 ⇒ $\sigma_c = \frac{6(162\sigma_b^2\sigma_s^2+\sigma_b\sigma_s^2\sigma_t)}{6\sigma_b\sigma_s^2+323\sigma_b^2\sigma_t+3\sigma_s^2\sigma_t}$

LCID-C ≠ 0, LCID-S ≠ 0, LCID-B ≠ 0 ⇒ overspecified, least square




SAMP-1 yield surface defined through load curves

**Figure M187-3.** Three or more load curves

| VARIABLE | DESCRIPTION |
|---|---|
| FILT | Factor for strain rate filtering: $\dot{\varepsilon}_{n+1}^{avg} = (1 - \text{FILT}) \times \dot{\varepsilon}_{n+1}^{\text{cur}} + \text{FILT} \times \dot{\varepsilon}_n^{avg}$ |
| VFAST | Flag to activate a vectorized version of the original implementation, resulting in a reduction of computational effort by up to 30%.<br>EQ.0.0: Original scalar implementation<br>EQ.1.0: Vectorized implementation |

**Load Curves:**

Material SAMP-1 uses three yield curves internally to evaluate a quadratic yield surface. *MAT_SAMP-1 accepts four kinds of yield curves, LCID-T, LCID-C, LCID-S, and LCID-B, where data from tension tests (LCID-T) is always required, but the others are optional. If fewer than three curves are defined, as indicated by setting the missing load curve IDs to 0, the remaining curves are generated internally.

1. **Fewer than three load curves.** In the case of fewer than three load curves, a linear yield surface in the invariant space spanned by the pressure and the von Mises stress is generated using the available data. See Figure M187-2.

2. **Three or more load curves.** See Figure M187-3.

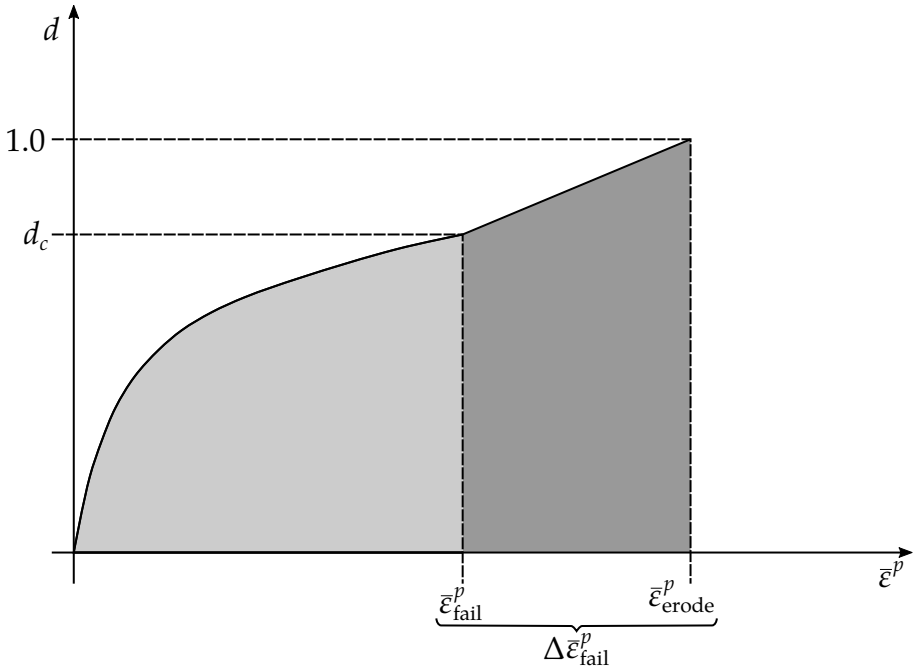



**Figure M187-4.** EPFAIL and DEPRPT defined the failure and fading behavior of a single element.

**Remarks:**

1. **Damage.** If the LCID-D is given, then a damage curve as a function of equivalent plastic strains acting on the stresses is defined as shown in Figure M187-4.

   Since the damaging curve acts on the yield values, the inelastic results are affected by the damage curve. To circumvent this, the load curve LCID-D may be given a negative ID. This will lead to an internal conversion from nominal to effective stresses (ASSR). While this conversion is possible for some combinations of yield curve definitions, plastic Poisson's ratio, and damage curves, the corresponding inverse problem cannot be solved for all combinations. An error message is provided in this case.

2. **Unsolvable yield surface case.** Since the generality of arbitrary curve inputs allows unsolvable yield surfaces, SAMP may modify curves internally. This will always lead to warning messages at the beginning of the simulation run. One modification that is not allowed is negative tangents of the last two data points of any of the yield curves.

3. **RBCFAC.** If RBCFAC is nonzero and curves LCID-T, LCID-C, LCID-S, and LCID-B are specified, the yield surface in $I_1$-$\sigma_{\text{vM}}$ stress space is constructed such that a piecewise-linear yield surface is activated. This option can help promote

the convergence of the plasticity algorithm. Figure M187-1 illustrates the effect of RBCFAC on behavior in biaxial compression.

4. **Dynamic amplification factor for yield stress.** If LCID-T is given as a table specifying strain-rate scaling of the yield stress, then the compressive, shear, and biaxial yield stresses are computed by multiplying their respective static values by the dynamic amplification factor (dynamic/static ratio) of the tensile yield stress.

5. **Instability criterion.** Instability at an integration point is a value between 0 and 1, indicating the integration point's proximity to the damage start. If instability reaches 1, damage starts and grows from 0 to 1. The element then "ruptures". The choice of INCFAIL determines how instability is calculated. For INCFAIL = 0,

$$\text{instability} = (\text{equivalent plastic strain})/\text{EPFAIL}$$

6. **Effective Poisson's ratio.** To avoid numerical difficulties in the plasticity algorithm with a non-associated flow rule, the effective elastic Poisson's ratio $\nu$ is lowered if the plastic Poisson's ratio NUEP (or from LCID-P) falls below the specified value NUE:

$$\nu = \min(\text{NUE}, \text{NUEP})$$

7. **History variables.** This material has the following history variables. NEIPH or NEIPS on *DATABASE_EXTENT_BINARY must be set for the history data to be output.

| History Variable # | Description |
|---|---|
| 2 | Plastic strain in tension/compression |
| 3 | Plastic strain in shear |
| 4 | Biaxial plastic strain |
| 5 | Damage |
| 6 | Volumetric plastic strain |
| 16 | Plastic strain rate in tension/compression |
| 17 | Plastic strain rate in shear |
| 18 | Biaxial plastic strain rate |
| 28 | Instability criterion (set NHSV = 28, see Remark 5) |

## *MAT_SAMP_LIGHT

Purpose: This is a slimmed-down form of Material Type 187. In contrast to the original SAMP-1, the options here are limited to rate-independent or rate-dependent flow in tension and compression, as well as constant or variable plastic Poisson's ratio. Shear and biaxial test data are not incorporated. Damage and failure are not available here. *MAT_ADD_EROSION or *MAT_ADD_DAMAGE can be included for damage and failure. However, a viscoelastic extension can be activated as in the original model.

This model is based on a complete re-coding of the plasticity algorithm. The efficiency was improved, so the computation times should be shorter. As compared to *MAT_SAMP-1, results should differ only slightly. To achieve the most similar results when including rate effects, set RATEOP = 1 because it is equivalent to the viscoplastic formulation in *MAT_SAMP-1.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | | | EMOD | NUE | LCEMOD | BETA |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCID-T | LCID-C | CTFLG | RATEOP | NUEP | LCID-P | RFILTF | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | EMOD | NUE | LCEMOD | BETA |
| Type | A | F | | | F | F | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| EMOD | Young's modulus |
| NUE | Poisson ratio (see Remark 4) |

| VARIABLE | DESCRIPTION |
|---|---|
| LCEMOD | Load curve ID defining Young's modulus as a function of effective strain rate. LCEMOD ≠ 0 activates viscoelasticity (see Remark 3). The parameters BETA and RFILTF must be defined too. If the first strain rate is negative, all values are expected to represent the natural logarithm of a strain rate. |
| BETA | Decay constant in viscoelastic law (see Remark 3). BETA has the unit [1/time]. If LCEMOD > 0 is used, a nonzero value for BETA is mandatory. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID-T | LCID-C | CTFLG | RATEOP | NUEP | LCID-P | RFILTF | |
| Type | I | I | F | I | F | I | F | |
| Default | none | 0 | 0 | 0 | none | 0 | 0.95 | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCID-T | Load curve or table ID giving the yield stress as a function of plastic strain. These curves should be obtained from quasi-static and (optionally) dynamic uniaxial tensile tests. This input is mandatory. If LCID-T is a table ID, the table values are effective strain rates, and a curve of yield stress as a function of plastic strain must be given for each of those strain rates. If the first value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of the effective strain rate. When the highest effective strain rate is several orders of magnitude greater than the lowest, it is recommended that the natural log of strain rate be input in the table. See Remark 1. |
| LCID-C | Optional load curve (or table) ID giving the yield stress as a function of plastic strain (and strain rate). This curve (or table) should be obtained from uniaxial compression tests. If LCID-C is defined as a curve and LCID-T is given as a table, then the rate dependence from the tension table is adopted in compression as well. See Remark 1. |
| CTFLG | Curve treatment flag (for LCID-T, LCID-C, and LCID-P) |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0: Rediscretized curves (default). We recommend this option with an appropriate value of LCINT for accurate resolution of the curves (see *DEFINE_CURVE and *CONTROL_SOLUTION).<br>EQ.1: Original curve values from the input |
| RATEOP | Calculation of effective strain rate option:<br>EQ.0: Original method for calculating the effective *total* strain rate.<br>EQ.1: Viscoplastic formulation, meaning using effective plastic strain rate. Recommended option to achieve the best match with *MAT_SAMP-1.<br>EQ.2: Improved method for calculating the effective total strain rate. This method gives a slightly closer match (compared to RATEOP = 0) to *MAT_SAMP-1. |
| NUEP | Plastic Poisson's ratio: an estimated ratio of transversal to longitudinal plastic rate of deformation under uniaxial loading should be given. Ignored if LCID-P is nonzero. See Remark 1. |
| LCID-P | Load curve ID giving the plastic Poisson's ratio as a function of an equivalent plastic strain measure during uniaxial tensile and uniaxial compressive testing. The plastic strain measure on the abscissa is negative for compression and positive for tension. It is important to cover both tension and compression. If LCID-P is given, NUEP is ignored. See Remark 1. |
| RFILTF | Smoothing factor on the effective strain rate (default is 0.95). The filtered strain rate is used for the rate-dependent plastic flow (LCID-T being a table) as well as for the viscoelasticity (LCEMOD > 0). See Remark 2.<br>$\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$ |

**Remarks:**

1. **Yield surfaces.** In the case of one tensile load curve (or table), LCID-T, the yield surface is von Mises type with associated (NUEP = 0.5) or non-associated (else) plastic flow. A Drucker-Prager type yield surface is used in the case where

LCID-T and LCID-C are both defined. The plastic flow direction again depends on the choice of NUEP/LCID-P.

The yield condition is given by

$$F = \sigma^{\text{eff}} - \sigma^{Y,T} \leq 0$$

with the effective stress

$$\sigma^{\text{eff}} = (1 - \xi) \times \sigma^{\text{vM}} - 3\xi \times p$$

based on the von Mises stress $\sigma^{\text{vM}}$, the pressure $p$ and the Drucker-Prager slope parameter

$$\xi = \frac{\sigma^{Y,C} - \sigma^{Y,T}}{2 \times \sigma^{Y,C}}$$

defined by the tensile and compressive yield stresses $\sigma^{Y,T}$ and $\sigma^{Y,C}$ (input with LCID-T and optionally LCID-C). The plastic potential

$$G = \sqrt{\left(\sigma^{\text{vM}^2} + \alpha \times p^2\right)}$$

with

$$\alpha = \frac{9}{2}\left(\frac{1 - 2\nu^p}{1 + \nu^p}\right)$$

and the plastic Poisson's ratio $\nu^p$ defines the direction of plastic flow. $\nu^p$can be input as a constant with NUEP or as a function of equivalent plastic strain with LCID-P. The elasto-plastic equations are solved with a classical predictor-corrector algorithm for the plastic strain increment $\Delta\varepsilon^p$ and stress $\boldsymbol{\sigma}^{n+1}$.

2. **Effective strain rate.** If tables are used for hardening, the rate dependence is defined using an effective strain rate. An averaged value is used to reduce the noise from the elastic portion of that strain rate. This is governed by the filtering parameter RFILTF as shown above.

3. **Nonlinear viscoelasticity**. For LCEMOD ≠ 0, viscoelasticity is activated. With this model, EMOD becomes the Young's modulus for equilibrium stress ($E_{\text{eq}}$). LCEMOD specifies the viscous Young's modulus, which is a function of the averaged strain rate ($E_v(\dot{\varepsilon}^{\text{avg}})$). BETA gives a constant decay parameter specifying the ratio $\frac{E_v(\dot{\varepsilon}^{\text{avg}})}{\eta(\dot{\varepsilon}^{\text{avg}})}$ where $\eta$ is the viscosity. The viscoelastic stress-strain law is given by:

$$\dot{\sigma}_v = -\text{BETA} \times \sigma(t) + E_v(\dot{\varepsilon}^{\text{avg}}) \times \dot{\varepsilon}^{\text{avg}}$$

The one-dimensional viscoelastic behavior can be visualized by a generalized Maxwell cell as a rheological model, as shown in Figure M187-1.

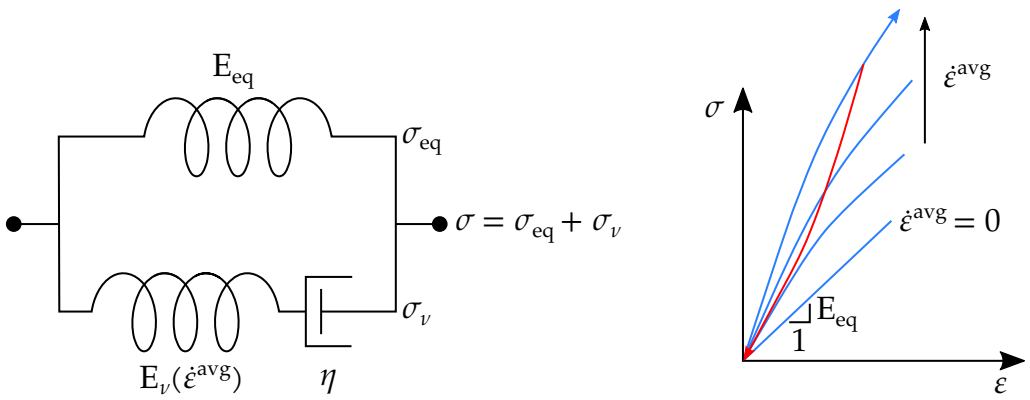



**Figure M187-1.** Viscoelastic model and strain rate dependent stress-strain curves.

We will be using the notation of the viscoelastic-plastic stress update. For simplicity, we will discuss the one-dimensional case. Let $\Delta\varepsilon^{\text{vp}}$ be the viscoplastic strain increment and let $\sigma_{\text{eq}}$ and $\sigma_v$ be the equilibrium and viscous stress portions. The stress update is then:

$$\sigma^{n+1} = \sigma_{\text{eq}}^{n+1} + \sigma_v^{n+1}$$

or with stress increments:

$$\sigma^{n+1} = \sigma^n + \Delta\sigma_{\text{eq}} + \Delta\sigma_v.$$

In the elastic trial step, the equilibrium and viscous stress increments $\Delta\sigma_{\text{eq}}^{\text{tr}}$ and $\Delta\sigma_v^{\text{tr}}$ are calculated as:

$$\Delta\sigma_{\text{eq}}^{\text{tr}} = E_{\text{eq}} \times \Delta\varepsilon$$

and

$$\Delta\sigma_v^{\text{tr}} = E_v(\dot{\varepsilon}^{\text{avg}}) \frac{1 - e^{-\beta\Delta t}}{\beta\Delta t} \left(\dot{\varepsilon}^{\text{avg}}\,\Delta t\right) - \left(1 - e^{-\beta\Delta t}\right)\sigma_v^n$$

The yield condition (see Remark 1) is evaluated:

$$F\left(\sigma^{n+1,\text{tr}}\right) = F(\sigma^n + \Delta\sigma_{\text{eq}}^{\text{tr}} + \Delta\sigma_v^{\text{tr}}).$$

For $F \le 0$, the trial stress state is the current stress, that is, $\sigma^{n+1} = \sigma^n + \Delta\sigma_{\text{eq}}^{\text{tr}} + \Delta\sigma_v^{\text{tr}}$. Otherwise, the plastic strain increment $\Delta\varepsilon^p$ must be evaluated, and the equilibrium and viscous stress increments are updated. For $F\left(\sigma_{\text{eq}}^{\text{tr}}\right) > 0$:

$$\sigma_{\text{eq}}^{n+1} = \sigma_{\text{eq}}^{\text{tr}} - E_{\text{eq}}\Delta\varepsilon^p$$

$$\sigma_v^{n+1} = \sigma_v^{\text{tr}} - E_v(\dot{\varepsilon}^{\text{avg}}) \frac{1 - e^{-\beta\Delta t}}{\beta\Delta t}\Delta\varepsilon^p$$

For $F\left(\sigma_{\text{eq}}^{\text{tr}}\right) \le 0$:

$$\sigma_{\text{eq}}^{n+1} = \sigma_{\text{eq}}^{\text{tr}}$$

4. **Effective Poisson's ratio.** To avoid numerical difficulties in the plasticity algorithm with a non-associated flow rule, the effective elastic Poisson's ratio $\nu$ is lowered if the plastic Poisson's ratio NUEP (or from LCID-P) falls below the specified value NUE:

$$\nu = \min(\text{NUE}, \text{NUEP})$$

5. **History variables.** This material has the following extra history variables:

| History Variable # | Description |
|---|---|
| 1 | Filtered deviatoric strain rate for viscoelasticity (LCEMOD > 0) |
| 2 | Volumetric plastic strain |
| 3 | Number of iterations |
| 4 | Current yield stress |
| 5 | Filtered total strain rate |
| 7 | Deviatoric plastic strain |

## *MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP

This is Material Type 188. In this model, creep is described separately from plasticity using Garafalo's steady-state hyperbolic sine creep law or Norton's power law. Viscous effects of plastic strain rate are considered using the Cowper-Symonds model. Young's modulus, Poisson's ratio, thermal expansion coefficient, yield stress, material parameters of Cowper-Symonds model as well as the isotropic and kinematic hardening parameters are all assumed to be temperature dependent. Application scope includes: simulation of solder joints in electronic packaging, modeling of tube brazing process, creep age forming, etc.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | SIGY | ALPHA | LCSS | REFTEM |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| QR1 | CR1 | QR2 | CR2 | QX1 | CX1 | QX2 | CX2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C | P | LCE | LCPR | LCSIGY | LCQR | LCQX | LCALPH |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| LCC | LCP | LCCR | LCCX | CRPA | CRPB | CRPQ | CRPM |
|---|---|---|---|---|---|---|---|

***Card 5.*** This card is optional.

| CRPLAW | | | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ALPHA | LCSS | REFTEM |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress |
| ALPHA | Thermal expansion coefficient |
| LCSS | Load curve ID or Table ID. The load curve defines effective stress as a function of effective plastic strain. The table defines for each temperature value a load curve ID referencing stress as a function of effective plastic strain for that temperature. The stress as a function of effective plastic strain curve for the lowest value of temperature is used if the temperature falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of temperature is used if the temperature exceeds the maximum value. Card 2 is ignored with this option. |
| REFTEM | Reference temperature that defines thermal expansion coefficient |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QR1 | CR1 | QR2 | CR2 | QX1 | CX1 | QX2 | CX2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| QR1 | Isotropic hardening parameter $Q_{r1}$ |
| CR1 | Isotropic hardening parameter $C_{r1}$ |
| QR2 | Isotropic hardening parameter $Q_{r2}$ |
| CR2 | Isotropic hardening parameter $C_{r2}$ |
| QX1 | Kinematic hardening parameter $Q_{\chi 1}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| CX1 | Kinematic hardening parameter $C_{\chi 1}$ |
| QX2 | Kinematic hardening parameter $Q_{\chi 2}$ |
| CX2 | Kinematic hardening parameter $C_{\chi 2}$ |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCE | LCPR | LCSIGY | LCQR | LCQX | LCALPH |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Viscous material parameter $C$ |
| P | Viscous material parameter $P$ |
| LCE | Load curve for scaling Young's modulus as a function of temperature |
| LCPR | Load curve for scaling Poisson's ratio as a function of temperature |
| LCSIGY | Load curve for scaling initial yield stress as a function of temperature |
| LCQR | Load curve for scaling the isotropic hardening parameters QR1 and QR2 or the stress given by the load curve LCSS as a function of temperature |
| LCQX | Load curve for scaling the kinematic hardening parameters QX1 and QX2 as a function of temperature |
| LCALPH | Load curve for scaling the thermal expansion coefficient as a function of temperature |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCC | LCP | LCCR | LCCX | CRPA | CRPB | CRPQ | CRPM |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LCC | Load curve for scaling the viscous material parameter $C$ as a function of temperature |
| LCP | Load curve for scaling the viscous material parameter $P$ as a function of temperature |
| LCCR | Load curve for scaling the isotropic hardening parameters CR1 and CR2 as a function of temperature |
| LCCX | Load curve for scaling the kinematic hardening parameters CX1 and CX2 as a function of temperature |
| CRPA | Creep law parameter $A$<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-CRPA) which defines $A$ as a function of temperature, $A(T)$ |
| CRPB | Creep law parameter $B$<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-CRPB) which defines $B$ as a function of temperature, $B(T)$ |
| CRPQ | Creep law parameter $Q = E/R$ where $E$ is the activation energy and $R$ is the universal gas constant.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-CRPQ) which defines $Q$ as a function of temperature, $Q(T)$ |
| CRPM | Creep law parameter $m$<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-CRPM) which defines $m$ as a function of temperature, $m(T)$ |

Optional card 5

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CRPLAW | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CRPLAW | Creep law definition (see Remarks):<br>EQ.0.0: Garofalo's hyperbolic sine law (default)<br>EQ.1.0: Norton's power law |

**Remarks:**

If LCSS is not defined, the uniaxial stress-strain curve has the form

$$\sigma(\varepsilon^p_{\text{eff}}) = \sigma_0 + Q_{r1}\left[1 - \exp(-C_{r1}\varepsilon^p_{\text{eff}})\right] + Q_{r2}\left[1 - \exp(-C_{r2}\varepsilon^p_{\text{eff}})\right] + Q_{\chi 1}\left[1 - \exp(-C_{\chi 1}\varepsilon^p_{\text{eff}})\right] + Q_{\chi 2}\left[1 - \exp(-C_{\chi 2}\varepsilon^p_{\text{eff}})\right].$$

Viscous effects are accounted for using the Cowper-Symonds model, which scales the yield stress with the factor:

$$1 + \left(\frac{\dot{\varepsilon}^p_{\text{eff}}}{C}\right)^{1/p} .$$

For CRPLAW = 0, the steady-state creep strain rate of Garafalo's hyperbolic sine equation is given by

$$\dot{\varepsilon}^c = A[\sinh(B\tau^e)]^m \exp\left(-\frac{Q}{T}\right) .$$

For CRPLAW = 1, the steady-state creep strain rate is given by Norton's power law equation:

$$\dot{\varepsilon}^c = A(\tau^e)^B t^m.$$

In the above, $\tau^e$ is the effective elastic stress in the von Mises sense, $T$ is the temperature and $t$ is the time. The following is a schematic overview of the resulting stress update. The multiaxial creep strain increment is given by

$$\Delta\varepsilon^c = \Delta\varepsilon^c \frac{3\boldsymbol{\tau}^e}{2\tau^e} ,$$

where $\boldsymbol{\tau}^e$ is the elastic deviatoric stress tensor. Similarly, the plastic and thermal strain increments are given by

$$\Delta\boldsymbol{\varepsilon}^p = \Delta\varepsilon^p \frac{3\boldsymbol{\tau}^e}{2\tau^e}$$
$$\Delta\boldsymbol{\varepsilon}^T = \alpha_{t+\Delta t}(T - T_{\text{ref}})\mathbf{I} - \boldsymbol{\varepsilon}_t^T$$

where $\alpha$ is the thermal expansion coefficient (note the definition compared to that of other materials). Adding it all together, the stress update is given by

$$\boldsymbol{\sigma}_{t+\Delta t} = \mathbf{C}_{t+\Delta t}\left(\boldsymbol{\varepsilon}_t^e + \Delta\boldsymbol{\varepsilon} - \Delta\boldsymbol{\varepsilon}^p - \Delta\boldsymbol{\varepsilon}^c - \Delta\boldsymbol{\varepsilon}^T\right) \ .$$

The plasticity is isotropic or kinematic but with a von Mises yield criterion, the subscript in the equation above indicates the simulation time of evaluation. Internally, this stress update requires the solution of a nonlinear equation in the effective stress, the viscoelastic strain increment and potentially the plastic strain increment.

*DEFINE_MATERIAL_HISTORIES can be used to output the viscoelastic (creep strain).

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Effective Creep Strain | - - - - | Viscoelastic strain $\varepsilon^c$, see above |
| Plastic Strain Rate | - - - - | Effective plastic strain rate $\dot{\varepsilon}^p_{\text{eff}}$ |

## *MAT_ANISOTROPIC_THERMOELASTIC

This is Material Type 189. This model characterizes elastic materials whose elastic properties are temperature-dependent.

It is available for solid elements, thick shell formulations 3, 5, and 7, and multi-material ALE solid elements. Note that it is not validated for multi-material ALE solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | TA1 | TA2 | TA3 | TA4 | TA5 | TA6 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C11 | C12 | C13 | C14 | C15 | C16 | C22 | C23 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C24 | C25 | C26 | C33 | C34 | C35 | C36 | C44 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C45 | C46 | C55 | C56 | C66 | TGE | TREF | AOPT |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | F | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| TA*i* | Curve IDs defining the coefficients of thermal expansion for the six components of strain tensor as function of temperature. |
| C*ij* | Curve IDs defining the 6 × 6 symmetric constitutive matrix in material coordinate system as function of temperature. Note that 1 corresponds to the *a* material direction, 2 to the *b* material direction, and 3 to the *c* material direction. |
| TGE | Curve ID defining the structural damping coefficient as function of temperature. |
| TREF | Reference temperature for the calculation of thermal loads or the definition of thermal expansion coefficients. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid |

| VARIABLE | DESCRIPTION |
|---|---|
| | elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 6 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4. |
| BETA | Material angle in degrees for AOPT = 3. It may be overwritten on the element card; see *ELEMENT_SOLID_ORTHO. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details).<br>EQ.0.0: Off<br>EQ.1.0: On |

## *MAT_FLD_3-PARAMETER_BARLAT

This is Material Type 190. This model was developed by Barlat and Lian [1989] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. This particular development is due to Barlat and Lian [1989]. It has been modified to include a failure criterion based on the Forming Limit Diagram. The curve can be input as a load curve or calculated based on the n-value and sheet thickness.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | P1 | P2 | ITER |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | R00 | R45 | R90 | LCID | E0 | SPI | P3 |
| Type | F | F | F | F | I | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | C | P | FLDCID | RN | RT | FLDSAFE | FLDNIPF |
| Type | F | F | F | I | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| HR | Hardening rule:<br>EQ.1.0: Linear (default)<br>EQ.2.0: Exponential (Swift)<br>EQ.3.0: Load curve<br>EQ.4.0: Exponential (Voce)<br>EQ.5.0: Exponential (Gosh)<br>EQ.6.0: Exponential (Hocket-Sherby) |
| P1 | Material parameter:<br>HR.EQ.1.0: Tangent modulus<br>HR.EQ.2.0: $k$, strength coefficient for Swift exponential hardening<br>HR.EQ.4.0: $a$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $k$, strength coefficient for Gosh exponential hardening<br>HR.EQ.6.0: $a$, coefficient for Hocket-Sherby exponential hardening |
| P2 | Material parameter:<br>HR.EQ.1.0: Yield stress<br>HR.EQ.2.0: $n$, exponent for Swift exponential hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| | HR.EQ.4.0: $c$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $n$, exponent for Gosh exponential hardening<br>HR.EQ.6.0: $c$, coefficient for Hocket-Sherby exponential hardening |
| ITER | Iteration flag for speed:<br>EQ.0.0: Fully iterative<br>EQ.1.0: Fixed at three iterations<br>Generally, ITER = 0 is recommended. However, ITER = 1 is somewhat faster and may give acceptable results in most problems. |
| M | $m$, exponent in Barlat's yield surface |
| R00 | $R_{00}$, Lankford parameter determined from experiments |
| R45 | $R_{45}$, Lankford parameter determined from experiments |
| R90 | $R_{90}$, Lankford parameter determined from experiments |
| LCID | Load curve ID for the load curve hardening rule (HR = 3.0) |
| E0 | Material parameter<br>HR.EQ.2.0: $\varepsilon_0$ for determining initial yield stress for Swift exponential hardening. The default value is 0.0.<br>HR.EQ.4.0: $b$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $\varepsilon_0$ for determining initial yield stress for Gosh exponential hardening. The default value is 0.0.<br>HR.EQ.6.0: $b$, coefficient for Hocket-Sherby exponential hardening |
| SPI | If E0 is zero above and HR = 2.0:<br>EQ.0.0: $\varepsilon_0 = \left(E/k\right)^{1/(n-1)}$<br>LE.0.2: $\varepsilon_0 = \text{SPI}$<br>GT.0.2: $\varepsilon_0 = \left(\text{SPI}/k\right)^{1/n}$ |
| P3 | Material parameter:<br>HR.EQ.5.0: $p$, parameter for Gosh exponential hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| | HR.EQ.6.0: $n$, exponent for Hocket-Sherby exponential hardening |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| C | $C$ in Cowper-Symonds strain rate model |
| P | $p$ in Cowper-Symonds strain rate model. $p$ = 0.0 for no strain rate effects. |
| FLDCID | Load curve ID defining the Forming Limit Diagram. Minor engineering strains in percent are defined as abscissa values and major engineering strains in percent are defined as ordinate values. The forming limit diagram is shown in Figure M39-1. In defining the curve, list pairs of minor and major strains starting with the leftmost point and ending with the rightmost point. See *DEFINE_CURVE. See Remark 2. |
| RN | Hardening exponent equivalent to the n-value in a power law hardening law. If the parameter FLDCID is not defined, this value in combination with the value RT can be used to calculate a forming limit curve to allow for failure. Otherwise it is ignored. See Remark 2. |
| RT | Sheet thickness used for calculating a forming limit curve. This |

| VARIABLE | DESCRIPTION |
|---|---|
| | value does not override the sheet thickness in any way. It is only used in conjunction with the parameter RN to calculate a forming limit curve if the parameter FLDCID is not defined. See Remark 2. |
| FLDSAFE | A safety offset of the forming limit curve. This value should be input as a percentage (such as 10, not 0.10). This safety margin will be applied to the forming limit curve defined by FLDCID or the curve calculated by RN and RT. |
| FLDNIPF | Numerical integration points failure treatment:<br>GT.0.0: The number of element integration points that must fail before the element is deleted. By default, if one integration point has strains above the forming limit curve, the element is flagged for deletion.<br>LT.0.0: The element is deleted when all integration points within a relative distance of –FLDNIPF from the mid-surface have failed (value between -1.0 and 0.0). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2. |
| V1, V2, V3 | Components of vector **v** for AOPT = 3. |
| D1, D2, D3 | Components of vector **d** for AOPT = 2. |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card. See *ELEMENT_SHELL_BETA. |

**Remarks:**

1. **Theoretical basis.** See *MAT_036 for the theoretical basis.

2. **Forming limit curve.** The forming limit curve can be input directly as a curve by specifying a load curve ID with the parameter FLDCID. When defining such a curve, the major and minor strains must be input as percentages. Alternatively, the parameters RN and RT can be used to calculate a forming limit curve. The use of RN and RT is not recommended for non-ferrous materials. RN and RT are ignored if a nonzero FLDCID is defined.

3. **History variable.** The first history variable is the maximum strain ratio defined by:

$$\frac{\varepsilon_{\text{major}_{\text{workpiece}}}}{\varepsilon_{\text{major}_{\text{fld}}}}$$

corresponding to $\varepsilon_{\text{minor}_{\text{workpiece}}}$. A value between 0 and 1 indicates that the strains lie below the forming limit curve. Values above 1 indicate that the strains are above the forming limit curve.

## *MAT_SEISMIC_BEAM

This is Material Type 191. This material enables lumped plasticity to be developed at the "node 2" end of Belytschko-Schwer beams (resultant formulation). The plastic yield surface allows for interaction between the two moments and the axial force.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | ASFLAG | FTYPE | DEGRAD | IFEMA |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCPMS | SFS | LCPMT | SFT | LCAT | SFAT | LCAC | SFAC |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included if and only if FTYPE = 1.

| ALPHA | BETA | GAMMA | DELTA | A | B | FOFFS | |
|---|---|---|---|---|---|---|---|

**Card 3b.** This card is included if and only if FTYPE = 2.

| SIGY | D | W | TF | TW | | | |
|---|---|---|---|---|---|---|---|

**Card 3c.** This card is included if and only if FTYPE = 4.

| PHI_T | PHI_C | PHI_B | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3d.** This card is included if and only if FTYPE = 5.

| ALPHA | BETA | GAMMA | DELTA | PHI_T | PHI_C | PHI_B | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is included if and only if IFEM > 0.

| PR1 | PR2 | PR3 | PR4 | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is included if and only if IFEM > 0.

| TS1 | TS2 | TS3 | TS4 | CS1 | CS2 | CS3 | CS4 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | ASFLAG | FTYPE | DEGRAD | IFEMA |
| Type | A | F | F | F | F | I | I | I |
| Default | none | none | none | none | 0.0 | 1 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus. |
| PR | Poisson's ratio |
| ASFLAG | Axial strain definition for force-strain curves, degradation and FEMA output:<br>EQ.0.0: True (log) total strain<br>EQ.1.0: Change in length<br>EQ.2.0: Nominal total strain<br>EQ.3.0: FEMA plastic strain ( = nominal total strain minus elastic strain) |
| FTYPE | Formulation type for interaction:<br>EQ.1: Parabolic coefficients, axial load and biaxial bending (default)<br>EQ.2: Japanese code, axial force and major axis bending.<br>EQ.4: AISC utilization calculation but no yielding<br>EQ.5: AS4100 utilization calculation but no yielding |
| DEGRAD | Flag for degrading moment behavior (see Remark 5):<br>EQ.0: Behavior as in previous versions<br>EQ.1: Fatigue-type degrading moment-rotation behavior |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2: FEMA-type degrading moment-rotation behavior |
| IFEMA | Flag for input of FEMA thresholds:<br>EQ.0: No input<br>EQ.1: Input of rotation thresholds only<br>EQ.2: Input of rotation and axial strain thresholds |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCPMS | SFS | LCPMT | SFT | LCAT | SFAT | LCAC | SFAC |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 1.0 | LCMPS | 1.0 | none | 1.0 | LCAT | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCPMS | Load curve ID giving plastic moment as a function of plastic rotation at node 2 about the local *s*-axis. See *DEFINE_CURVE. |
| SFS | Scale factor on *s*-moment at node 2 |
| LCPMT | Load curve ID giving plastic moment as a function of plastic rotation at node 2 about local the *t*-axis. See *DEFINE_CURVE. |
| SFT | Scale factor on *t*-moment at node 2 |
| LCAT | Load curve ID giving axial tensile yield force as a function of total tensile (elastic + plastic) strain or of elongation. See ASFLAG above. All values are positive. See *DEFINE_CURVE. |
| SFAT | Scale factor on axial tensile force |
| LCAC | Load curve ID giving compressive yield force as a function of total compressive (elastic + plastic) strain or of elongation. See ASFLAG above. All values are positive. See *DEFINE_CURVE. |
| SFAC | Scale factor on axial tensile force |

**FTYPE 1 Card.** This card 3 format is used when FTYPE = 1 (default).

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | BETA | GAMMA | DELTA | A | B | FOFFS | |
| Type | F | F | F | F | F | F | F | |
| Default | Rem 1 | Rem 1 | Rem 1 | Rem 1 | Rem 1 | Rem 1 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Parameter to define yield surface |
| BETA | Parameter to define yield surface |
| GAMMA | Parameter to define yield surface |
| DELTA | Parameter to define yield surface |
| A | Parameter to define yield surface |
| B | Parameter to define yield surface |
| FOFFS | Force offset for yield surface (see Remark 2) |

**FTYPE 2 Card.** This card 3 format is used when FTYPE = 2.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGY | D | W | TF | TW | | | |
| Type | F | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGY | Yield stress of material |
| D | Depth of section used to calculate interaction curve |
| W | Width of section used to calculate interaction curve |

| VARIABLE | DESCRIPTION |
|---|---|
| TF | Flange thickness of section used to calculate interaction curve |
| TW | Web thickness used to calculate interaction curve |

**FTYPE 4 Card.** This card 3 format is used when FTYPE = 4.

| Card 3c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PHI_T | PHI_C | PHI_B | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.8 | 0.85 | 0.9 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PHI_T | Factor on tensile capacity, $\phi_t$ |
| PHI_C | Factor on compression capacity, $\phi_c$ |
| PHI_B | Factor on bending capacity, $\phi_b$ |

**FTYPE 5 Card.** This card 3 format is used when FTYPE = 5.

| Card 3d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | BETA | GAMMA | DELTA | PHI_T | PHI_C | PHI_B | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | 1.4 | none | 1.0 | 1.0 | 1.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Parameter to define yield surface |
| BETA | Parameter to define yield surface |
| GAMMA | Parameter to define yield surface |
| DELTA | Parameter to define yield surface |

| VARIABLE | DESCRIPTION |
|---|---|
| PHI_T | Factor on tensile capacity, $\phi_t$ |
| PHI_C | Factor on compression capacity, $\phi_c$ |
| PHI_B | Factor on bending capacity, $\phi_b$ |

**FEMA Limits Card 1.** Additional card for IFEMA > 0.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PR1 | PR2 | PR3 | PR4 | | | | |
| Type | F | F | F | F | | | | |
| Default | 0 | 0 | 0 | 0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PR1 - PR4 | Plastic rotation thresholds 1 to 4 |

**FEMA Limits Card 2.** Additional card for IFEMA = 2.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TS1 | TS2 | TS3 | TS4 | CS1 | CS2 | CS3 | CS4 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0 | 0 | 0 | 0 | TS1 | TS2 | TS3 | TS4 |

| VARIABLE | DESCRIPTION |
|---|---|
| TS1 - TS4 | Tensile axial strain thresholds 1 to 4 |
| CS1 - CS4 | Compressive axial strain thresholds 1 to 4 |

**Remarks:**

1. **FTYPE 1.** Yield surface for formulation type 1 is of the form:

$$\psi = \left(\frac{M_s}{M_{ys}}\right)^{\alpha} + \left(\frac{M_t}{M_{yt}}\right)^{\beta} + A\left(\frac{F}{F_y}\right)^{\gamma} + B\left(\frac{F}{F_y}\right)^{\delta} - 1 \ ,$$

where

$M_s, M_t$ = moments about the local $s$ and $t$ axes
$M_{ys}, M_{yt}$ = current yield moments
$F$ = axial force
$F_y$ = yield force; LCAC in compression or LCAT in tension
$\alpha, \beta, \gamma, \delta$ = input parameters; must be greater than or equal to 1.1
$A, B$ = input parameters

If $\alpha$, $\beta$, $\gamma$, $\delta$, A and B are *all* set to zero, then the following default values are used:

| Field | Default Value |
|---|---|
| ALPHA | 2.0 |
| BETA | 2.0 |
| GAMMA | 2.0 |
| DELTA | 4.0 |
| A | 2.0 |
| B | -1.0 |

2. **FOFFS.** FOFFS offsets the yield surface parallel to the axial force axis. It is the compressive axial force at which the maximum bending moment capacity about the local $s$-axis (determined by LCPMS and SFS) and that about the local $t$-axis (determined by LCPMT and SFT) occur. For steel beams and columns, the value of FOFFS is usually zero. For reinforced concrete beams, columns and shear walls, the maximum bending moment capacity occurs corresponding to a certain compressive axial force, FOFFS. The value of FOFFS can be input as either positive or negative. Internally, LS-DYNA converts FOFFS to, and regards compressive axial force as, negative.

3. **FTYPE 4.** Interaction surface FTYPE 4 calculates a utilisation parameter using the yield force and moment data given on Card 2, but the elements remain elastic even when the forces or moments exceed yield values. This is done for consistency with the design code OBE AISC LRFD (2000). The utilization calculation is as follows:

$$\text{Utilization} = \frac{K_1 F}{\phi F_y} + \frac{K_2}{\phi_b}\left(\frac{M_s}{M_{ys}} + \frac{M_t}{M_{yt}}\right) \ ,$$

where $M_s$, $M_t$, $M_{ys}$, $M_{yt}$, and $F_y$ are as defined in Remark 1. $\phi$ is PHI_T under and PHI_C under compression. $K_1$ and $K_2$ are as follow:

$$K_1 = \begin{cases} 0.5 & \dfrac{F}{\phi F_y} < 0.2 \\ 1.0 & \dfrac{F}{\phi F_y} \geq 0.2 \end{cases}$$

$$K_2 = \begin{cases} 1.0 & \dfrac{F}{\phi F_y} < 0.2 \\ 8/9 & \dfrac{F}{\phi F_y} \geq 0.2 \end{cases}$$

4. **FTYPE 5.** Interaction surface FTYPE 5 is similar to FTYPE 4 (calculates a utilization parameter using the yield data, but the elements do not yield). The equations are taken from Australian code AS4100. The user must select appropriate values of $\alpha$, $\beta$, $\gamma$ and $\delta$ using the various clauses of Section 8 of AS4100. It is assumed that the local $s$-axis is the major axis for bending.

$$\text{Utilization} = \max(U_1, U_2, U_3, U_4, U_5)$$

where

$$U_1 = \frac{F}{\beta\phi_c F_{yc}} \qquad \text{used for members in compression}$$

$$U_2 = \frac{F}{\phi_t F_{yt}} \qquad \text{used for members in tension}$$

$$U_3 = \left[\frac{M_s}{K_2\phi_b M_{ys}}\right]^\gamma + \left[\frac{M_t}{K_1\phi_b M_{yt}}\right]^\gamma \qquad \text{used for members in compression}$$

$$U_4 = \left[\frac{M_s}{K_4\phi_b M_{ys}}\right]^\gamma + \left[\frac{M_t}{K_3\phi_b M_{yt}}\right]^\gamma \qquad \text{used for members in tension}$$

$$U_5 = \frac{F}{\phi_c F_{yc}} + \frac{M_s}{\phi_b M_{ys}} + \frac{M_t}{\phi_b M_{yt}} \qquad \text{used for all members}$$

In the above, $M_s$, $M_t$, $F$, $M_{ys}$, $M_{yt}$, $F_{yt}$ and $F_{yc}$ are as defined in Remark 1. $K_1$, $K_2$, $K_3$, and $K_4$ are subject to a minimum value of $10^{-6}$ and defined as

$$K_1 = 1.0 - \frac{F}{\beta\phi_c F_{yc}}$$

$$K_2 = \min\left[K_1, \alpha\left(1.0 - \frac{F}{\delta\phi_c F_{yc}}\right)\right]$$

$$K_3 = 1.0 - \frac{F}{\phi_t F_{yt}}$$

$$K_4 = \min\left[\mathrm{K}_3, \alpha\left(1.0 + \frac{F}{\phi_t F_{yt}}\right)\right]$$

$\alpha$, $\beta$, $\gamma$, $\delta$, $\phi_t$, $\phi_c$, and $\phi_b$ are input parameters.

5. **DEGRAD.** The option for degrading moment behavior changes the meaning of the plastic moment-rotation curve as follows:

   a) If DEGRAD = 0 (not recommended), the *x*-axis points on the curve represent current plastic rotation (meaning total rotation minus the elastic component of rotation). This quantity can be positive or negative depending on the direction of rotation; during hysteresis the behavior will repeatedly follow backwards and forwards along the same curve. The curve should include negative and positive rotation and moment values. This option is retained so that results from existing models will be unchanged.

   b) If DEGRAD = 1, the *x*-axis points represent cumulative absolute plastic rotation. This quantity is always positive and increases whenever there is plastic rotation in either direction. Thus, during hysteresis, the yield moments are taken from points in the input curve with increasingly positive rotation. If the curve shows a degrading behavior (reducing moment with rotation), then, once degraded by plastic rotation, the yield moment can never recover to its initial value. This option can be thought of as having "fatigue-type" hysteretic damage behavior, where all plastic cycles contribute to the total damage.

   c) If DEGRAD = 2, the *x*-axis points represent the high-tide value (always positive) of the plastic rotation. This quantity increases only when the absolute value of plastic rotation exceeds the previously recorded maximum. If smaller cycles follow a larger cycle, the plastic moment during the small cycles will be constant, since the high-tide plastic rotation is not altered by the small cycles. Degrading moment-rotation behavior is possible. This option can be thought of as showing rotation-controlled damage and follows the FEMA approach for treating fracturing joints.

   DEGRAD applies also to the axial behavior. The same options are available as for rotation: DEGRAD = 0 gives unchanged behavior from previous versions; DEGRAD = 1 gives a fatigue-type behavior using cumulative plastic strain; and DEGRAD = 2 gives FEMA-type behavior, where the axial load capacity depends on the high-tide tensile and compressive strains. The definition of strain for this purpose is according to ASFLAG on Card 1 – it is expected that ASFLAG = 2 will be used with DEGRAD = 2. The "axial strain" variable plotted by post-processors is the variable defined by ASFLAG.

   The output variables plotted as "plastic rotation" have special meanings for this material model– note that hinges form only at Node 2. "Plastic rotation at End 1" is really a high-tide mark of absolute plastic rotation at Node 2, defined as follows:

   d) Current plastic rotation is the total rotation minus the elastic component of rotation.

e) Take the absolute value of the current plastic rotation, and record the maximum achieved up to the current time. This is the high-tide mark of plastic rotation.

If DEGRAD = 0, "Plastic rotation at End 2" is the current plastic rotation at Node 2. If DEGRAD = 1 or 2, "Plastic rotation at End 2" is the current total rotation at Node 2. The total rotation is a more intuitively understood parameter, such as for plotting hysteresis loops. However, with DEGRAD = 0, the previous meaning of that output variable has been retained such that results from existing models are unchanged.

FEMA thresholds are the plastic rotations at which the element is deemed to have passed from one category to the next, e.g. "Elastic", "Immediate Occupancy", "Life Safe", etc. The high-tide plastic rotation (maximum of Y and Z) is checked against the user-defined limits FEMA1, FEMA2, etc. The output flag is then set to 0, 1, 2, 3, or 4: 0 means that the rotation is less than FEMA1; 1 means that the rotation is between FEMA1 and FEMA2, and so on. By contouring this flag, it is possible to see quickly which joints have passed critical thresholds.

6. **Output.** For this material model, special output parameters are written to the d3plot and d3thdt files. The number of output parameters for beam elements is automatically increased to 20 (in addition to the six standard resultants) when parts of this material type are present. Some post-processors may interpret this data as if the elements were integrated beams with 4 integration points. Depending on the post-processor used, the data may be accessed as follows:

| Extra Variable # (Integration Point 4 Description) | Data Description |
|---|---|
| 16 (or Axial Stress) | FEMA rotation flag |
| 17 (or XY Shear Stress) | Current utilization |
| 18 (or ZX Shear Stress) | Maximum utilization to date |
| 20 (or Axial Strain) | FEMA axial flag |

"Utilization" is the yield parameter, where 1.0 is on the yield surface.

## *MAT_SOIL_BRICK

Purpose: This is Material Type 192. It is intended for modeling over-consolidated clay.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | RLAMBDA | RKAPPA | RIOTA | RBETA1 | RBETA2 | RMU |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| RNU | RLCID | TOL | PGCL | SUB-INC | BLK | GRAV | THEORY |
|---|---|---|---|---|---|---|---|

**Card 3a.** This card is included only if THEORY = 4 or 104.

| RVHHH | | | | | | STRSUB | |
|---|---|---|---|---|---|---|---|

**Card 3b.** This card is included only if THEORY = 204 or 304.

| RVHHH | | | | | | STRSUB | CRFLG |
|---|---|---|---|---|---|---|---|

**Card 3c.** This card is included only if THEORY = 7 or 107.

| EHEV | GHHGVH | PRHH | CAP | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is included only if THEORY = 204 or 304 and CRFLG > 0.

| SLRATIO | BETAC | EPSDOT1 | EPSDOT2 | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | RLAMBDA | RKAPPA | RIOTA | RBETA1 | RBETA2 | RMU |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| RLAMBDA | Material coefficient, see Remark 1. |
| RKAPPA | Material coefficient, see Remark 1. |
| RIOTA | Material coefficient, see Remark 1. |
| RBETA1 | Material coefficient, see Remark 1. |
| RBETA2 | Material coefficient, see Remark 1. |
| RMU | Shape factor coefficient. This parameter will modify the shape of the yield surface used. A value of 1.0 implies a von Mises type surface, while 1.1 to 1.25 is more indicative of soils. The default value is 1.0. See Remarks 1 and 9. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RNU | RLCID | TOL | PGCL | SUB-INC | BLK | GRAV | THEORY |
| Type | F | F | F | F | F | F | F | I |
| Default | none | none | 0.0005 | none | none | none | 9.807 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| RNU | Poisson's ratio. See Remarks 1 and 2. |
| RLCID | Load curve ID (see *DEFINE_CURVE) consisting of up to 10 points defining nonlinear response in terms of stiffness degradation. The $x$-axis is strain ("string length"), and the $y$-axis is the ratio of secant stiffness to small-strain stiffness. See Remarks 3 and 12. |
| TOL | User defined tolerance for convergence checking. Default value is set to 0.0005 (recommended). See Remark 11. |
| PGCL | Pre-consolidation ground level. This parameter defines the maximum surface level (relative to $z = 0.0$ in the model) of the soil throughout geological history which is used calculate the |

| VARIABLE | DESCRIPTION |
|---|---|
| | maximum overburden pressure on the soil elements. See Remark 6. |
| SUB-INC | User defined strain increment size. A typical value is 0.005. This is the maximum strain increment permitted in the iteration scheme within the material model. If the value is exceeded, a warning is echoed to the d3hsp file. See Remark 11. |
| BLK | The elastic bulk stiffness of the soil which is used for contact stiffness only. |
| GRAV | The gravitational acceleration which is used to calculate the element stresses due to the overlying soil. Default is set to 9.807 $m/s^2$. |
| THEORY | Version of material subroutines used (see Remarks 7 and 8):<br>EQ.0: 1995 version (default)<br>EQ.4: 2003 version, load/unload initialization<br>EQ.7: 2003 version, load/unload initialization, anisotropy from Ellison et al (2012)<br>EQ.104: 2003 version, load/unload/reload initialization<br>EQ.107: 2003 version, load/unload/reload initialization, anisotropy from Ellison et al (2012)<br>EQ.204: 2015 version, load/unload initialization<br>EQ.304: 2015 version, load/unload/reload initialization |

Define Card 3a only if THEORY = 4 or 104. Omit otherwise.

| Card 3a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RVHHH | | | | | | STRSUB | |
| Type | F | | | | | | F | |
| Default | 0.0 | | | | | | 0.001 | |

| VARIABLE | DESCRIPTION |
|---|---|
| RVHHH | Anisotropy parameter: shear modulus in vertical planes divided by shear modulus in horizontal plane. If this field is blank or zero, |

| VARIABLE | DESCRIPTION |
|---|---|
| | isotropic behavior is assumed. See Remark 10. |
| STRSUB | Strain limit, used to determine whether subcycling within the material model is required (recommended value: 0.001) |

Define Card 3b only if THEORY = 204 or 304. Omit otherwise.

| Card 3b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RVHHH | | | | | | STRSUB | CRFLG |
| Type | F | | | | | | F | F |
| Default | 0.0 | | | | | | 0.001 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| RVHHH | Anisotropy parameter: shear modulus in vertical planes divided by shear modulus in horizontal plane. If this field is blank or zero, isotropic behavior is assumed. See Remark 10. |
| STRSUB | Strain limit, used to determine whether subcycling within the material model is required (recommended value: 0.001) |
| CRFLG | Creep flag:<br>EQ.0: No creep<br>EQ.24: Creep activated; see Card 4. |

Define Card 3c only if THEORY = 7 or 107. Omit otherwise.

| Card 3c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EHEV | GHHGVH | PRHH | CAP | | | | |
| Type | F | F | F | F | | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EHEV | Anisotropy parameter: Young's modulus in horizontal directions divided by Young's modulus in vertical direction. See Remarks 1 and 10. |
| GHHGVH | Anisotropy parameter: shear modulus in horizontal plane divided by shear modulus in vertical planes. See Remarks 1 and 10. |
| PRHH | Anisotropy parameter: Poisson's ratio in horizontal plane. See Remarks 1 and 10. |
| CAP | Anisotropy parameter. See Remarks 1 and 10. |

Define Card 4 only if THEORY = 204 or 304 and CRFLG > 0. Omit otherwise.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SLRATIO | BETAC | EPSDOT1 | EPSDOT2 | | | | |
| Type | F | F | F | F | | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SLRATIO | Creep parameter, see Remark 12 |
| BETAC | Creep parameter, see Remark 12 |
| EPSDOT1 | Creep parameter: reference strain rate for volumetric strains. See Remark 12. |
| EPSDOT2 | Creep parameter: reference strain rate for shear strains. See Remark 12. |

**Remarks:**

1. **Material behavior and references.** The material model consists of up to 10 nested elasto-plastic yield surfaces defined in a transformed stress-strain space, termed "BRICK" space. The sizes of the yield surfaces are given in terms of BRICK strains and are called "string-lengths". Explanation of the input parameters and underlying concepts may be found in Simpson (1992), Lehane &

Simpson (2000), and Ellison et al (2012). The input parameters correspond to those described in Ellison et al as follows:

| LS-DYNA | Ellison et al |
|---|---|
| RLAMBDA | $\lambda$ |
| RKAPPA | $\kappa$ |
| RIOTA | $\iota$ |
| RBETA1 | $\beta^G$ |
| RBETA2 | $\beta^\phi$ |
| RMU | $\mu$ |
| RNU | $\nu$ |
| EHEV | $E_h/E_v$ |
| GHHGVH | $G_{hh}/G_{vh}$ |
| PRHH | $\nu_{hh}$ |
| CAP | $\zeta$ |

See Table 3 in Ellison et al for example input parameter values.

2. **Elastic stiffness.** The elastic bulk modulus is given by $p'/\iota$, where $p'$ is the current mean effective stress (compression positive), and the small-strain elastic shear modulus is calculated from the bulk modulus and Poisson's ratio RNU.

3. **Nonlinear stress-strain response.** The curve RLCID defines the nonlinear behavior in terms of secant stiffness degradation. The same curve is assumed to apply to all six BRICK stress-strain components. For example, shear in the $xz$-plane will follow the curve, such that the $x$-axis points correspond to shear angle $\gamma_{xz}$, while the $y$-axis contains $G/G_{\max}$, where $G$ is the secant stiffness (equal to $\tau_{xz}/\gamma_{xz}$) and $G_{max}$ is the small-strain elastic shear modulus.

4. **Model requirements.** This material type requires that the model be oriented such that the $z$-axis is defined in the upward direction. Compressive initial stress must be defined, using, for example, *INITIAL_STRESS_SOLID or *INITIAL_-STRESS_DEPTH. Stresses must remain compressive throughout the analysis.

5. **Units.** The recommended unit system is kiloNewtons, meters, seconds, tonnes. There are some built-in defaults that assume stress units of kN/m$^2$.

6. **Over-consolidated clays.** Over-consolidated clays have suffered previous loading to higher stress levels than are present at the start of the analysis due to phenomena such as ice sheets during previous ice ages, or the presence of soil

or rock that has subsequently been eroded. The maximum vertical stress during that time is assumed to be:

$$\sigma_{V,MAX} = RO \times GRAV \times (PGCL - Z_{el}) \ ,$$

where

RO, GRAV, and PGCL = input parameters
$Z_{el}$ = z coordinate of center of element

Since that time, the material has been unloaded until the vertical stress equals the user-defined initial vertical stress. The previous load/unload history has a significant effect on the subsequent behavior. For example, the horizontal stress in an over-consolidated clay may be greater than the vertical stress.

7. **Initialization.** This material model initializes each element with a load/unload cycle under uniaxial vertical strain conditions. The element is loaded up to a vertical stress of $\sigma_{V,MAX}$ (defined in Remark 6 above) and then unloaded to the user-defined initial vertical stress $\sigma_{V,USER}$ (see *INITIAL_STRESS_SOLID or *INITIAL_STRESS_DEPTH). During this initialization cycle, the stresses and history variables are updated using the same constitutive behavior as during the main analysis. Therefore, the horizontal stress at the start of the analysis $\sigma_{H,ACTUAL}$ (as seen in the results files at time zero) is an output of the initialization process and will be different from the user-defined initial horizontal stress $\sigma_{H,USER}$; the latter is ignored. Optionally, initialization may be switched to a load/unload/reload cycle (see input settings of THEORY). In this case, the element is loaded up to a vertical stress of $\sigma_{V,MAX}$, unloaded to a stress $\sigma_{V,MIN}$ which is less than $\sigma_{V,USER}$, and then reloaded to $\sigma_{V,USER}$. The value of $\sigma_{V,MIN}$ is calculated automatically to try to minimize the difference between $\sigma_{H,ACTUAL}$ and $\sigma_{H,USER}$.

8. **Material subroutine version.** This material model is developed for a Geotechnical FE program (Oasys Ltd.'s SAFE) written by Arup. The default THEORY = 0 gives a vectorized version ported from SAFE in the 1990's. Since then the material model has been developed further in SAFE, with versions ported to LS-DYNA in 2003 (THEORY = 4 and 104) and 2015 (THEORY = 204 and 304); these are not vectorized and will run more slowly on most computer platforms. Nevertheless, the 2015 version is recommended. THEORY = 0, 4, and 104 are retained only for backward compatibility.

9. **Shape factor.** The shape factor for a typical soil would be 1.25. Do not use values higher than 1.35.

10. **Anisotropy.** Anisotropy is treated by applying stretch factors to the strain axis of the stress-strain curves for certain BRICK shear components. It may be defined by *either* using THEORY = 204 or 304 together with RVHHH on Card 3b *or* using THEORY = 7 or 107 with the parameters on Card 3c. Using THEORY = 4

or 104 with non-zero anisotropy parameters on Card 3a is permitted but not recommended. See Ellison et al (2012) for description of the anisotropy effects modelled by THEORY = 7/107 and the meaning of the parameters on Card 3c. If anisotropy is not required, use THEORY = 204 or 304 and leave Card 3b blank. "Vertical" and "horizontal" are defined in the global coordinate system with "vertical" being the global $z$-axis.

11. **TOL and SUBINC.** These parameters usually have little influence on the result. Smaller values may sometimes improve accuracy, at cost of greater run times.

12. **Creep.** Creep is implemented by scaling the strain "string lengths" (see RLCID) as a function of strain rate:

$$S = \text{SLRATIO} \times S_{\text{RLCID}} \left[ 1 + \text{BETAC} \times ln \left( \frac{|\dot{\varepsilon}|}{\dot{\varepsilon}_{\text{ref}}} + 1 \right) \right]$$

Where $S$ is string length; $S_{RLCID}$ are the string lengths in the curve RLCID; SLRATIO and BETAC are input parameters on Card 4; $|\dot{\varepsilon}|$ is strain rate; and $\dot{\varepsilon}_{\text{ref}}$ is input parameter EPSDOT1 or EPSDOT2 for volumetric and shear strains, respectively. Note that SLRATIO × $S_{\text{RLCID}}$ gives the string lengths for zero strain rate.

**References:**

[1] Simpson, B., "Retaining structures: displacement and design", *Géotechnique*, Vol. 42, No. 4, 539-576, (1992).

[2] Lehane, B. & Simpson, B., "Modelling glacial till conditions using a Brick soil model", *Can. Geotech. J.* Vol. 37, No. 5, 1078–1088 (2000).

[3] Ellison, K., Soga, K., & Simpson, B., "A strain space soil with evolving stiffness memory", *Géotechnique,* Vol. 62, No. 7, 627-641 (2012).

## *MAT_DRUCKER_PRAGER

This is Material Type 193. This material enables modeling soil effectively. The parameters used to define the yield surface are familiar geotechnical parameters (such as the angle of friction). The modified Drucker-Prager yield surface is used in this material model, enabling the shape of the surface to be distorted into a more realistic definition for soils.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | GMOD | RNU | RKF | PHI | CVAL | PSI |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | 1.0 | none | none | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | STR_LIM | | | | | | | |
| Type | F | | | | | | | |
| Default | 0.005 | | | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GMODDP | PHIDP | CVALDP | PSIDP | GMODGR | PHIGR | CVALGR | PSIGR |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| GMOD | Elastic shear modulus |
| RNU | Poisson's ratio |
| RKF | Failure surface shape factor |
| PHI | Angle of friction (radians) |
| CVAL | Cohesion value |
| PSI | Dilation angle (radians) |
| STR_LIM | Minimum shear strength of material is given by STR_LIM × CVAL |
| GMODDP | Depth at which shear modulus (GMOD) is correct |
| PHIDP | Depth at which angle of friction (PHI) is correct |
| CVALDP | Depth at which cohesion value (CVAL) is correct |
| PSIDP | Depth at which dilation angle (PSI) is correct |
| GMODGR | Gradient at which shear modulus (GMOD) increases with depth |
| PHIGR | Gradient at which friction angle (PHI) increases with depth |
| CVALGR | Gradient at which cohesion value (CVAL) increases with depth |
| PSIGR | Gradient at which dilation angle (PSI) increases with depth |

**Remarks:**

1. **Orientation.** This material type requires the model to be oriented such that the *Z*-axis is defined in the upward direction. The key parameters are defined such that they may vary with depth (i.e., the *Z*-axis).

2. **Shape factor.** The shape factor for a typical soil would be 0.8 but should not be pushed further than 0.75.

3. **STR_LIM.** If STR_LIM is set to less than 0.005, the value is reset to 0.005.

4. **Yield function.** The yield function, $F$, and plastic potential, $G$, are defined as

$$F = t - \max(p \times \tan\beta + d, l \times d) = 0$$

and

$$G = t - p \times \tan\psi$$

where:

| Variable | Description |
|---|---|
| $p$ | Hydrostatic pressure, $p = -I_1/3$ |
| $I_1$ | First stress invariant |
| $t$ | $t = q\left(a - b(r/q)^3\right)/2$ |
| $q$ | von Mises stress, $q = \sqrt{3J_2}$ |
| $a$ | $a = 1 + 1/K$ |
| $b$ | $b = 1 - 1/K$ |
| $K$ | Input field RKF |
| $r$ | $r = (27J_3/2)^{1/3}$ |
| $J_2$ | Second deviatoric stress invariant |
| $J_3$ | Third deviatoric stress invariant |
| $\tan\beta$ | $\tan\beta = 6\sin\varphi/(3 - \sin\varphi)$ |
| $d$ | $d = 6C\cos\varphi/(3 - \sin\varphi)$ |
| $\varphi$ | Input field PHI |
| $C$ | Input field CVAL |
| $\psi$ | Input field PSI |
| $l$ | Input field STR_LIM |

5. **Executable precision.** We recommend using this material with a double precision executable.

6. **Output.** This remark applies to versions R14 and onwards. "Plastic Strain" is the deviatoric plastic strain, defined in the same way as for material types 3, 24, etc. Extra history variables may be requested for solid elements (NEIPH on *DATABASE_EXTENT_BINARY). They are described in the following table.

| History Variable # | Description |
|---|---|
| 1 | Volumetric strain |
| 2 | Mobilized fraction (= 1 when on yield surface) |
| 3 | At-rest coefficient (ratio of horizontal stress to vertical stress) |

| History Variable # | Description |
|---|---|
| 4 | Friction angle in radians (differs from input parameter PHI only if PHIDP and PHIGR are used) |
| 5 | Cohesion (differs from input parameter CVAL only if CVALDP and CVALGR are used) |
| 6 | Dilation angle in radians (differs from input parameter PSI only if PSIDP and PSIGR are used) |
| 7 | Shear modulus (differs from input parameter GMOD only if GMODDP and GMODGR are used) |
| 8 | Plastic multiplier |

## *MAT_RC_SHEAR_WALL

Purpose: This is Material Type 194. It is for shell elements only. It uses empirically-derived algorithms to model the effect of cyclic shear loading on reinforced concrete walls. It is primarily intended for modeling squat shear walls but can also be used for slabs. Because the combined effect of concrete and reinforcement is included in the empirical data, crude meshes can be used. The model has been designed such that the minimum amount of input is needed: generally, only the variables on the first card need to be defined.

**NOTE:** This material does not support the specification of a material angle, $\beta_i$, for each through-thickness integration point of a shell.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | TMAX | | | |
| Type | A | F | F | F | F | | | |
| Default | none | none | none | 0.0 | 0.0 | | | |

Include the following data if "Uniform Building Code" formula for maximum shear strength or tensile cracking are required – otherwise leave blank.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FC | PREF | FYIELD | SIGO | UNCONV | ALPHA | FT | ERIENF |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B | C | D | E | F | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.05 | 0.55 | 0.125 | 0.66 | 0.25 | 1.0 | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Y1 | Y2 | Y3 | Y4 | Y5 | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |
| Default | 0.0 | | | | | | | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label not must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| TMAX | Ultimate in-plane shear stress. If set to zero, LS-DYNA calculates TMAX based on the formulae in the Uniform Building Code, using the data on Card 2. See Remark 3. |
| FC | Unconfined compressive strength of concrete. It is used in the calculation of ultimate shear stress. Crushing behavior is not modeled. |
| PREF | Percent reinforcement. For example, if 1.2% of the material is reinforcement, enter 1.2. |
| FYIELD | Yield stress of reinforcement |

| VARIABLE | DESCRIPTION |
|---|---|
| SIG0 | Overburden stress (in-plane compressive stress). It is used in the calculation of ultimate shear stress. Usually, SIG0 is left as zero. |
| UCONV | Unit conversion factor. UCONV is expected to be set such that, $\text{UCONV} = \sqrt{1.0 \text{ PSI in the model's stress units}}$. This factor is used to convert the ultimate tensile stress of concrete which is expressed as $\sqrt{\text{FC}}$ where FC is given in PSI. Therefore a unit conversion factor of $\sqrt{\text{PSI/Stress Unit}}$ is required. Examples: UCONV = 83.3 = $\sqrt{6894}$ if the stress unit is $\text{N/m}^2$ UCONV = 0.083 if the stress unit is $\text{MN/m}^2$ or $\text{N/mm}^2$ |
| ALPHA | Shear span factor. See Remark 3. |
| FT | Cracking stress in direct tension. See Remark 5. The default is 8% of the cylinder strength. |
| ERIENF | Young's modulus of the reinforcement. It is used to calculate the post-cracked stiffness. See Remark 5. |
| A | Hysteresis constants determining the shape of the hysteresis loops |
| B | Hysteresis constants determining the shape of the hysteresis loops |
| C | Hysteresis constants determining the shape of the hysteresis loops |
| D | Hysteresis constants determining the shape of the hysteresis loops |
| E | Hysteresis constants determining the shape of the hysteresis loops |
| F | Strength degradation factor. After the ultimate shear stress has been achieved, F multiplies the maximum shear stress from the curve for subsequent reloading. F = 1.0 implies no strength degradation (default). F = 0.5 implies that the strength is halved for subsequent reloading. |
| Y1, Y2, …, Y5 | Engineering shear strain points on stress-strain curve. By default, these are calculated from the values on Card 1. See Remark 3. |
| T1, T2, …, T5 | Shear stress points on stress-strain curve. By default, these are calculated from the values on Card 1. See Remark 3. |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for more details): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1, and then rotated about the shell element normal by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element (see Figure M2-1). **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

1. **Model limitations.** The element is linear elastic except for in-plane shear and tensile cracking effects. Crushing due to direct compressive stresses is modeled only insofar as there is an in-plane shear stress component. Using this model is not recommended when the nonlinear response to direct compressive or loads is important.

2. **In-plane shear stress.** Note that the in-plane shear stress $t_{xy}$ is defined as the shear stress in the element's local $xy$-plane. This shear stress is not necessarily equal to the maximum shear stress in the plane. For example, if the principal

stresses are at 45 degrees to the local axes, $t_{xy}$ is zero. Therefore, it is important to ensure that the local axes are appropriate. For a shear wall the local axes should be vertical or horizontal. By default, the local $x$-axis points from node 1 to node 2 of the element. It is possible to change the local axes by using AOPT > 0.

3. **TMAX and shear stress as a function of shear strain.** If TMAX is set to zero, the ultimate shear stress is calculated using a formula in the Uniform Building Code 1997, section 1921.6.5:

$$\text{TMAX}_{\text{UBC}} = \text{UCONV} \times \text{ALPHA} \times \sqrt{\text{FC}} + \text{RO} \times \text{FY}$$

where,

UCONV = unit conversion factor, see varriable list

ALPHA = aspect ratio

= 2.0 for $h/l \in (2.0, \infty)$ increases linearly to 3.0 for $h/l \in (2.0,1.5)$

FC = unconfined compressive strength of concrete

RO = fraction of reinforcement

= (percent reinforcement)/100

FY = yield stress of reinforcement

To this we add shear stress due to the overburden to obtain the ultimate shear stress:

$$\text{TMAX}_{\text{UBC}} = \text{TMAX}_{\text{UBC}} + \text{SIG0}$$

where

SIG0 = in-plane compressive stress under static equilibrium conditions

The UBC formula for ultimate shear stress is generally conservative (predicts that the wall is weaker than shown in test), sometimes by 50% or more. A less conservative formula is that of Fukuzawa:

$$\text{TMAX} = \max\left[\left(0.4 + \frac{A_c}{A_w}\right), 1\right] \times 2.7 \times \left(1.9 + \frac{M}{L_v}\right) \times \text{UCONV} + \sqrt{\text{FC}} + 0.5 \times \text{RO} \times \text{FY} + \text{SIG0}$$

where

$A_C$ = Cross-sectional area of stiffening features such as columns or flanges
$A_w$ = Cross-sectional area of wall
$M/L_v$ = Aspect ratio of wall height/length

Other terms are as above. This formula is not included in the material model. TMAX should be calculated by hand and entered on Card 1 if the Fukuzawa formula is required.

Note that none of the available formulae, including Fukuzawa, predict the ultimate shear stress accurately for all situations. Variance from the experimental results can be as great as 50%.

The shear stress as a function of shear strain curve is then constructed automatically as follows, using the algorithm of Fukuzawa extended by Arup:

a) Assume ultimate engineering shear strain, $\gamma_u = 0.0048$

b) First point on curve, corresponding to concrete cracking, is at

$$\left(0.3 \times \frac{\text{TMAX}}{G}, 0.3 \times \text{TMAX}\right),$$

where $G$ is the elastic shear modulus given by

$$G = \frac{E}{2(1+\nu)}.$$

c) Second point, corresponding to the reinforcement yield, is at

$$(0.5 \times \gamma_u, 0.8 \times \text{TMAX}).$$

d) Third point, corresponding to the ultimate strength, is at

$$(\gamma_u, \text{TMAX}).$$

e) Fourth point, corresponding to the onset of strength reduction, is at

$$(2\gamma_u, \text{TMAX}).$$

f) Fifth point, corresponding to failure is at

$$(3\gamma_u, 0.6 \times \text{TMAX}).$$

After failure, the shear stress drops to zero. The curve points can be entered by the user if desired, in which case they override the automatically calculated curve. However, it is anticipated that in most cases the default curve will be preferred due to ease of input.

4. **Hysteresis.** Hysteresis follows the algorithm of Shiga as for the squat shear wall spring (see *MAT_SPRING_SQUAT_SHEARWALL). The hysteresis constants which are defined in fields A, B, C, D, and E can be entered if desired, but it is generally recommended that the default values be used.

5. **Cracking.** Cracking in tension is checked for the local $x$ and $y$ directions only. Cracking is calculated separately from the in-plane shear. A trilinear response is assumed, with turning points at concrete cracking and reinforcement yielding. The three regimes are:

a) *Pre-cracking.* A linear elastic response is assumed using the overall Young's Modulus on Card 1.

b) *Cracking.* Cracking occurs in the local $x$ or $y$ directions when the tensile stress in that direction exceeds the concrete tensile strength FT (if not input on Card 2, this defaults to 8% of the compressive strength FC). Post-cracking, a linear stress-strain response is assumed up to reinforcement yield at a strain defined by reinforcement yield stress divided by reinforcement Young's Modulus.

c) *Post-yield.* A constant stress is assumed (no work hardening).

Unloading returns to the origin of the stress-strain curve. For compressive strains the response is always linear elastic using the overall Young's modulus on Card 1. If insufficient data is entered, no cracking occurs in the model. As a minimum, FC and FY are needed.

6. **History variables.** Extra variables are available for post-processing as follows:

| History Variable # | Description |
|---|---|
| 1 | Current engineering shear strain |
| 2 | Shear status: 0, 1, 2, 3, 4, or 5. The shear status shows how far along the shear stress-strain curve each element has progressed. For instance, status 2 means that the element has passed the second point on the curve. These status levels correspond to performance criteria in building design codes such as FEMA. |
| 3 | Maximum direct strain so far in the local $x$-direction (for tensile cracking) |
| 4 | Maximum direct strain so far in the local $y$-direction (for tensile cracking) |
| 5 | Tensile status:<br>EQ.0: Elastic<br>EQ.1: Cracked<br>EQ.2: Yielded |

## *MAT_CONCRETE_BEAM

This is Material Type 195 for beam elements. This model can define an elasto-plastic material with an arbitrary stress as a function of strain curve and arbitrary strain rate dependency. It supports failure based on a plastic strain or a minimum time step size. See Remarks below.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | ETAN | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | $10^{20}$ | $10^{20}$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | P | LCSS | LCSR | | | | |
| Type | F | F | F | F | | | | |
| Default | 0 | 0 | 0 | 0 | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NOTEN | TENCUT | SDR | | | | | |
| Type | I | F | F | | | | | |
| Default | 0 | $10^{15}$ | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Yield stress |
| ETAN | Tangent modulus; ignored if LCSS > 0 |
| FAIL | Failure flag:<br>LT.0.0: User-defined failure subroutine is called to determine failure.<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic element deletion. |
| C | Strain rate parameter, $C$; see Remarks below. |
| P | Strain rate parameter, $p$; see Remarks below. |
| LCSS | Load curve ID or Table ID. Load curve ID defining effective stress as a function of effective plastic strain. The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate; see Figure M16-1 Stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for the highest value of strain rate is used if the strain rate exceeds the maximum value. The strain rate parameters ($C$ and $p$) and the curve ID, LCSR, are ignored if a Table ID is defined. |
| LCSR | Load curve ID defining strain rate scaling effect on yield stress |
| NOTEN | No-tension flag:<br>EQ.0: Beam takes tension.<br>EQ.1: Beam takes no tension.<br>EQ.2: Beam takes tension up to value given by TENCUT. |
| TENCUT | Tension cutoff value |

| VARIABLE | DESCRIPTION |
|---|---|
| SDR | Stiffness degradation factor |

**Remarks:**

The stress strain behavior may be treated using a bilinear stress strain curve through defining the tangent modulus, ETAN. An effective stress as a function of effective plastic strain curve (LCSS) may be input instead of defining ETAN. The cost is roughly the same for either approach. The most general approach is to use the table definition (LCSS) discussed below.

Three options to account for strain rate effects are possible.

1. Strain rate may be accounted for using the Cowper and Symonds model which scales the yield stress with the factor

$$1 + \left(\frac{\dot{\varepsilon}}{C}\right)^{1/p}$$

where $\dot{\varepsilon}$ is the strain rate. $\dot{\varepsilon} = \sqrt{\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}}$.

2. For complete generality a load curve (LCSR) to scale the yield stress may be input instead. In this curve the scale factor as a function of strain rate is defined.

3. If different stress as a function of strain curves can be provided for various strain rates, the option using the reference to a table (LCSS) can be used.

## *MAT_GENERAL_SPRING_DISCRETE_BEAM

This is Material Type 196. This model permits elastic and elastoplastic springs with damping to be represented with a discrete beam element of type 6 by using six springs, each acting about one of the six local degrees of freedom. For elastic behavior, a load curve defines force or moment as a function of displacement or rotation. For inelastic behavior, a load curve defines yield force or moment as a function of plastic deflection or rotation, which can vary in tension and compression.

The two nodes defining a beam may be coincident to give a zero-length beam or offset to give a finite-length beam. For finite-length discrete beams, the absolute value of the field SCOOR in the *SECTION_BEAM input should be set to a value of 2.0, which causes the local *r*-axis to be aligned along the two nodes of the beam to give physically correct behavior. The distance between the nodes of a beam should not affect the behavior of this material model. A triad is used to orient the beam for the directional springs.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | | | | | MDFAIL | DOSPOT |
|---|---|---|---|---|---|---|---|

**Card 2.** For each active degree of freedom, include a pair of Cards 2 and 3. This data is terminated by the next keyword ("*") card or when all six degrees of freedom have been specified.

| DOF | TYPE | K | D | CDF | TDF | | |
|---|---|---|---|---|---|---|---|

**Card 3.** For each active degree of freedom, include a pair of Cards 2 and 3. This data is terminated by the next keyword ("*") card or when all six degrees of freedom have been specified.

| FLCID | HLCID | C1 | C2 | DLE | GLCID | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | | | MDFAIL | DOSPOT |
| Type | A | F | | | | | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density; see also volume in *SECTION_BEAM definition. |
| MDFAIL | Multidirectional failure calculation method (see Remark 5):<br>EQ.0: Default failure calculation method. It calculates the ratio of actual displacement (or rotation) to its corresponding CDF (or TDF) for each degree of freedom and then uses the maximum of these ratios to evaluate failure.<br>EQ.1: Two separate criteria, one for compression and the other for tension. If any criterion is fulfilled, failure occurs.<br>EQ.2: One criterion that considers the combined effect of all displacements and rotations in tension and compression. |
| DOSPOT | Activate thinning of tied shell elements when SPOTHIN > 0 on *CONTROL_CONTACT.<br>EQ.0: Spot weld thinning is inactive for shells tied to discrete beams that use this material (default).<br>EQ.1: Spot weld thinning is active for shells tied to discrete beams that use this material. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DOF | TYPE | K | D | CDF | TDF | | |
| Type | I | I | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| DOF | Active degree of freedom, a number between 1 and 6 inclusive. Each value of DOF can only be used once. The active degree of freedom is measured in the local coordinate system for the discrete beam element. |
| TYPE | Material behavior:<br>EQ.0: Elastic (default)<br>EQ.1: Inelastic |

| VARIABLE | DESCRIPTION |
|---|---|
| K | Elastic loading/unloading stiffness. This is required input for inelastic behavior. |
| D | Optional viscous damping coefficient |
| CDF | Compressive displacement at failure. Input as a positive number. After failure, no forces are carried. This option does not apply to zero length springs.<br>EQ.0.0: Inactive |
| TDF | Tensile displacement at failure. After failure, no forces are carried.<br>EQ.0.0: Inactive |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FLCID | HLCID | C1 | C2 | DLE | GLCID | | |
| Type | I | I | F | F | F | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FLCID | Load curve ID (see *DEFINE_CURVE):<br>TYPE.EQ.0: This curve defines force or moment as a function of deflection for nonlinear elastic behavior.<br>TYPE.EQ.1: This curve defines the yield force as a function of plastic deflection. If the abscissa of the first point of the curve is 0, the force magnitude is identical in tension and compression, that is, only the sign changes. If not, the yield stress in the compression is used when the spring force is negative. The plastic displacement increases monotonically in this implementation. The load curve is required input. |
| HLCID | Optional load curve ID (see *DEFINE_CURVE) defining force as a function of relative velocity. If the origin of the curve is at (0,0), the force magnitude is identical for a given magnitude of the relative velocity, meaning only the sign changes. |
| C1 | Damping coefficient |

| VARIABLE | DESCRIPTION |
|---|---|
| C2 | Damping coefficient |
| DLE | Factor to scale time units |
| GLCID | Optional load curve ID (see *DEFINE_CURVE) defining a scale factor as a function of deflection for load curve ID, HLCID. If zero, a scale factor of unity is assumed. |

**Remarks:**

1. **Elastic behavior.** If TYPE = 0, elastic behavior is obtained. In this case, if the linear spring stiffness is used, the force, $F$, is given by:

$$F = \mathrm{K} \times \Delta L + \mathrm{D} \times \Delta \dot{L} \ .$$

But if the load curve ID is specified, the force is then given by:

$$F = K f(\Delta L) \left[ 1 + \mathrm{C1} \times \Delta \dot{L} + \mathrm{C2} \times \mathrm{sgn}(\Delta \dot{L}) \ln \left( \max \left\{ 1., \frac{|\Delta \dot{L}|}{\mathrm{DLE}} \right\} \right) \right] + \mathrm{D} \times \Delta \dot{L} + g(\Delta L) h(\Delta \dot{L}) \ .$$

In these equations, $\Delta L$ is the change in length

$$\Delta L = \text{current length} - \text{initial length}$$

For the first three degrees of freedom the fields on Cards 2 and 3 have dimensions as shown below. Being angular in nature, the next three degrees of freedom involve moment instead of force and angle instead of length but are otherwise identical.

$$[\mathrm{K}] = \begin{cases} \dfrac{[\text{force}]}{[\text{length}]} & \mathrm{FLCID} = 0 \\ \text{unitless} & \mathrm{FLCID} > 0 \end{cases}$$

$$[\mathrm{D}] = \frac{[\text{force}]}{[\text{velocity}]} = \frac{[\text{force}][\text{time}]}{[\text{length}]}$$

$$[\mathrm{FLCID}] = [\mathrm{GLCID}] = ([\text{length}], [\text{force}])$$

$$[\mathrm{HLCID}] = ([\text{velocity}], [\text{force}])$$

$$[\mathrm{C1}] = \frac{[\text{time}]}{[\text{length}]}$$

$$[\mathrm{C2}] = \text{unitless}$$

$$[\mathrm{DLE}] = \frac{[\text{length}]}{[\text{time}]}$$

2. **Inelastic behavior.** If TYPE = 1, inelastic behavior is obtained. A trial force is computed as:

$$F^T = F^n + K \times \Delta\dot{L}(\Delta t)$$

and the yield force is taken from the load curve:

$$F^Y = F_y\left(\Delta L^{\text{plastic}}\right) \ ,$$

where $L^{\text{plastic}}$ is the plastic deflection, given by

$$\Delta L^{\text{plastic}} = \frac{F^T - F^Y}{S + K^{\max}} \ .$$

The maximum elastic stiffness is $K^{\max} = \max(K, 2 \times S^{\max})$, where $S$ is the slope of FLCID. The trial force is checked against the yield force to determine $F$:

$$F = \begin{cases} F^Y & \text{if } F^T > F^Y \\ F^T & \text{if } F^T \le F^Y \end{cases}$$

The final force, which includes rate effects and damping, is given by:

$$F^{n+1} = F \times \left[1 + \text{C1} \times \Delta\dot{L} + \text{C2} \times \text{sgn}(\Delta\dot{L}) \ln\left(\max\left\{1., \frac{|\Delta\dot{L}|}{\text{DLE}}\right\}\right)\right] + \text{D} \times \Delta\dot{L} + g(\Delta L)h(\Delta\dot{L}) \ .$$

Unless the origin of the curve starts at (0,0), the negative part of the curve is used when the spring force is negative where the negative of the plastic displacement is used to interpolate, $F_y$. The positive part of the curve is used whenever the force is positive.

3. **Cross-Sectional Area.** The cross-sectional area is defined on the section card for the discrete beam elements. See *SECTION_BEAM. The square root of this area is used as the contact thickness offset if these elements are included in the contact treatment.

4. **Rotational Displacement.** Rotational displacement is measured in radians.

5. **Failure.** The failure calculation depends on the choice of MDFAIL:

| MDFAIL | Damage, $D$ |
|---|---|
| 0 | $\max\left(\max\left(A_i^t\right), \max\left(A_i^c\right)\right)$ |
| 1 | $\max(A^t, A^c)$ |
| 2 | $A^t + A^c$ |

Failure occurs if $D - 1.0 > 0.0$. $A^t$ and $A^c$ are given by:

$$A^t = \left[\frac{\max(0,u_r)}{u_r^{\text{tfail}}}\right]^2 + \left[\frac{\max(0,u_s)}{u_s^{\text{tfail}}}\right]^2 + \left[\frac{\max(0,u_t)}{u_t^{\text{tfail}}}\right]^2 + \left[\frac{\max(0,\theta_r)}{\theta_r^{\text{tfail}}}\right]^2 + \left[\frac{\max(0,\theta_s)}{\theta_s^{\text{tfail}}}\right]^2 + \left[\frac{\max(0,\theta_t)}{\theta_t^{\text{tfail}}}\right]^2$$

$$A^c = \left[\frac{\min(0,u_r)}{u_r^{\text{cfail}}}\right]^2 + \left[\frac{\min(0,u_s)}{u_s^{\text{cfail}}}\right]^2 + \left[\frac{\min(0,u_t)}{u_t^{\text{cfail}}}\right]^2 + \left[\frac{\min(0,\theta_r)}{\theta_r^{\text{cfail}}}\right]^2 + \left[\frac{\min(0,\theta_s)}{\theta_s^{\text{cfail}}}\right]^2 + \left[\frac{\min(0,\theta_t)}{\theta_t^{\text{cfail}}}\right]^2$$

$A_i^t$ and $A_i^c$ are the $i^{\text{th}}$ term ($i$ = 1 to 6) in the equations for $A^t$ and $A^c$, respectively.

6. **Damage.** If *DATABASE_EXTENT_BINARY is used and NEIPB in its Card 4 is set to 12, then the damage is available as history variable #12 in d3plot.

## *MAT_SEISMIC_ISOLATOR

This is Material Type 197 for discrete beam elements. Sliding (pendulum) and elastomeric seismic isolation bearings can be modeled, applying bi-directional coupled plasticity theory. The hysteretic behavior was proposed by Wen [1976] and Park, Wen, and Ang [1986]. The sliding bearing behavior is recommended by Zayas, Low and Mahin [1990]. The algorithm used for implementation was presented by Nagarajaiah, Reinhorn, and Constantinou [1991]. Further options for tension-carrying friction bearings are as recommended by Roussis and Constantinou [2006]. Element formulation type 6 *must* be used. Local axes are defined on *SECTION_BEAM; the default is the global axis system. The local $z$-axis is expected to be vertical. On *SECTION_BEAM SCOOR must be set to zero when using this material model, even if the element has non-zero initial length.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | A | BETA | GAMMA | DISPY | STIFFV | ITYPE |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| PRELOAD | DAMP | MX1 | MX2 | MY1 | MY2 | CDE | IEXTRA |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is used for ITYPE = 0, 2, or 5. Leave this card blank for all other settings of ITYPE.

| FMAX | DELF | AFRIC | RADX | RADY | RADB | STIFFL | STIFFTS |
|---|---|---|---|---|---|---|---|

**Card 4a.** This card is included for ITYPE = 1 or 4.

| FORCEY | ALPHA | STIFFT | DFAIL | | | | |
|---|---|---|---|---|---|---|---|

**Card 4b.** This card is included for ITYPE = 2.

| | | | | FMAXYC | FMAXXT | FMAXYT | YLOCK |
|---|---|---|---|---|---|---|---|

**Card 4c.** This card is included for ITYPE = 3.

| FORCEY | ALPHA | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4d.** This blank card is included for all other settings of ITYPE (0 or5).

| | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

| HTCORE | RCORE | TSHIM | ROLCL | ROSCS | THCST | YLE2 | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

| PCRINI | DIAMB | FCAV0 | CAVK | CAVTR | CAVA | PHIM | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is included for ITYPE = 4 only. Omit for other settings of ITYPE.

| BETA | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is included for ITYPE = 5 only. Omit for other settings of ITYPE.

| FYRIM | DFRIM | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is included if and only if IEXTRA = 1.

| KTHX | KTHY | KTHZ | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | A | BETA | GAMMA | DISPY | STIFFV | ITYPE |
| Type | A | F | F | F | F | F | F | I |
| Default | none | none | 1.0 | 0.5 | 0.5 | none | none | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| A | Nondimensional variable - see below |
| GAMMA | Nondimensional variable - see below |
| BETA | Nondimensional variable - see below |
| DISPY | Yield displacement (length units - must be > 0.0) |

| VARIABLE | DESCRIPTION |
|---|---|
| STIFFV | Vertical stiffness (force/length units) |
| ITYPE | Type:<br>EQ.0: sliding (spherical or cylindrical)<br>EQ.1: elastomeric<br>EQ.2: sliding (two perpendicular curved beams)<br>EQ.3: lead rubber bearing<br>EQ.4: high damping rubber bearing<br>EQ.5: sliding with rim failure |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PRELOAD | DAMP | MX1 | MX2 | MY1 | MY2 | CDE | IEXTRA |
| Type | F | F | F | F | F | F | F | I |
| Default | 0.0 | 1.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| PRELOAD | Vertical preload not explicitly modeled (force units) |
| DAMP | Damping ratio (nondimensional) |
| MX1, MX2 | Moment factor at ends 1 and 2 in local $x$-direction |
| MY1, MY2 | Moment factor at ends 1 and 2 in local $y$-direction |
| CDE | Viscous damping coefficient (ITYPE = 1, 3 or 4) |
| IEXTRA | If IEXTRA = 1, optional Card 9 will be read |

**Sliding Isolator Card.** This card is used for ITYPE = 0, 2, or 5. Leave this card *blank* for all other settings of ITYPE.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FMAX | DELF | AFRIC | RADX | RADY | RADB | STIFFL | STIFFTS |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | $10^{20}$ | $10^{20}$ | $10^{20}$ | STIFFV | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| FMAX | Maximum friction coefficient (dynamic) |
| DELF | Difference between maximum friction and static friction coefficient |
| AFRIC | Velocity multiplier in sliding friction equation (time/length units) |
| RADX | Radius for sliding in local $x$ direction |
| RADY | Radius for sliding in local $y$ direction |
| RADB | Radius of retaining ring |
| STIFFL | Stiffness for lateral contact against the retaining ring |
| STIFFTS | Stiffness for tensile vertical response (default = 0) |

This card is included only for ITYPE = 1 or 4.

| Card 4a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FORCEY | ALPHA | STIFFT | DFAIL | FMAXYC | FMAXXT | FMAXYT | YLOCK |
| Type | F | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.5 × STIFFV | $10^{20}$ | FMAX | FMAX | FMAX | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| FORCEY | Yield force |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Ratio of post-yielding stiffness to pre-yielding stiffness |
| STIFFT | Stiffness for tensile vertical response (elastomeric isolator) |
| DFAIL | Lateral displacement at which the isolator fails |

This card is included only for ITYPE = 2.

| Card 4b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | FMAXYC | FMAXXT | FMAXYT | YLOCK |
| Type | | | | | F | F | F | F |
| Default | | | | | FMAX | FMAX | FMAX | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| FMAXYC | Max friction coefficient (dynamic) for local *y*-axis (compression) |
| FMAXXT | Max friction coefficient (dynamic) for local *x*-axis (tension) |
| FMAXYT | Max friction coefficient (dynamic) for local *y*-axis (tension) |
| YLOCK | Stiffness locking the local *y*-displacement (optional -single-axis sliding) |

This card is included only for ITYPE = 3.

| Card 4c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FORCEY | ALPHA | | | | | | |
| Type | F | F | | | | | | |
| Default | none | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FORCEY | Yield force |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Ratio of post-yielding stiffness to pre-yielding stiffness |

Include this *blank* card for ITYPE = 0 or 5.

| Card 4d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |
| Default | | | | | | | | |

**Lead Rubber Bearing Card.** This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HTCORE | RCORE | TSHIM | ROLCL | ROSCS | THCST | YLE2 | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| HTCORE | Height of lead core (length units) |
| RCORE | Radius of lead core (length units) |
| TSHIM | Total thickness of shim plates (length units) |
| ROLCL | Mass density times specific heat capacity of lead (units: $F.L^{-2}T^{-1}$) |
| ROLCS | Mass density times specific heat capacity of steel (units: $F.L^{-2}T^{-1}$) |
| THCST | Thermal conductivity of steel (units: $F.t^{-1}T^{-1}$) |
| YLE2 | $E_2$ in temperature-dependent yield stress of lead (units: 1/Temperature) |

**Lead Rubber Bearing Card.** This card is included for ITYPE = 3 only. Omit for other settings of ITYPE.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PCRINI | DIAMB | FCAV0 | CAVK | CAVTR | CAVA | PHIM | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| PCRINI | Buckling capacity (force units) |
| DIAMB | External diameter of bearing (length units) |
| FCAV0 | Tensile capacity limited by cavitation (force units) |
| CAVK | Cavitation parameter (units 1/length) |
| TR | Total thickness of rubber (length units) |
| CAVA | Strength degradation parameter (dimensionless) |
| PHIM | Maximum cavitation damage index (dimensionless) |

**High Damping Rubber Bearing Yield Card.** This card is included for ITYPE = 4 only. Omit for other settings of ITYPE.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | BETA | | | | | | | |
| Type | F | | | | | | | |
| Default | 0.0 | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Quadratic factor for yield force |

**Rim Failure Card.** This card is included for ITYPE = 5 only. Omit for other settings of ITYPE.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FYRIM | DFRIM | | | | | | |
| Type | F | F | | | | | | |
| Default | $10^{20}$ | $10^{20}$ | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FYRIM | Radial force at failure of rim |
| DFRIM | Radial displacement of rim to failure after FYRIM is reached |

**Rotational Stiffness Card.** Card 9 for IEXTRA = 1 only. Omit if IEXTRA=0.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | KTHX | KTHY | KTHZ | | | | | |
| Type | F | F | F | | | | | |
| Default | 0 | 0 | 0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| KTHX | Rotational stiffness in local $x$ direction (moment per radian) |
| KTHY | Rotational stiffness in local $y$ direction (moment per radian) |
| KTHZ | Rotational stiffness in local $z$ direction (moment per radian) |

**Remarks:**

1. **Horizontal behavior of the isolator.** The horizontal behavior for all isolator types is governed by plastic history variables Zx and Zy that evolve according to equations given in the reference; A, GAMMA, BETA and DISPYt are the input parameters for this. The intention is to provide smooth build-up, rotation and reversal of forces in response to bidirectional displacement histories in the

horizontal plane. The theoretical model has been correlated to experiments on seismic isolators.

2. **Sliding surface for sliding isolator.** The RADX and RADY inputs for the sliding isolator are optional. If left blank, the sliding surface is assumed to be flat. A cylindrical surface is obtained by defining either RADX or RADY; a spherical surface can be defined by setting RADX = RADY. The effect of the curved surface is to add a restoring force proportional to the horizontal displacement from the center. As seen in elevation, the top of the isolator will follow a curved trajectory, lifting as it displaces away from the center.

3. **Vertical behavior of the isolator.** The vertical behavior for all types is linear elastic, but with different stiffnesses for tension and compression. By default, the tensile stiffness is zero for the sliding types. For the elastomeric type in the case of uplift, the tensile stiffness will be different from the compressive stiffness. For the sliding type, compression is treated as linear elastic, but no tension can be carried.

4. **Vertical preload.** Vertical preload can be modeled either explicitly (for example, by defining gravity), or by using the PRELOAD input. PRELOAD does not lead to any application of vertical force to the model. It is added to the compression in the element before calculating the friction force and tensile/compressive vertical behavior.

5. **Overview of ITYPE.** Various settings of ITYPE are described as follows.

   a) ITYPE = 0 is used to model a single (spherical) pendulum bearing. Triple pendulum bearings can be modelled using three of these elements in series, following the method described by Fenz and Constantinou 2008.

   b) ITYPE = 2 is intended to model uplift-prevention sliding isolators that consist of two perpendicular curved beams joined by a connector that can slide in slots on both beams. The beams are aligned in the local $x$ and $y$ axes, respectively. The vertical displacement is the sum of the displacements induced by the respective curvatures and slider displacements along the two beams. Single-axis sliding is obtained by using YLOCK to lock the local $y$ displacement. To resist uplift, STIFFTS must be defined (recommended value: same as STIFFV). This isolator type allows for different friction coefficients on each beam as well as different values in tension and compression. The total friction, taking into account sliding velocity and the friction history functions, is first calculated using FMAX which applies to the local $x$-axis when in compression, and then scaled as necessary, such as by FMAXXT/FMAX (for the local $x$-axis when in tension) and by FMAXYC/FMAX or FMAXYT/FMAX for the $y$-axis as appropriate. For this reason, FMAX should not be zero.

c) ITYPE = 3 is used to model Lead Rubber Bearings (LRB), made of rubber with a lead core. Phenomenological models following Kumar et al. (2014) are incorporated to simulate the following salient behavior:

i) The properties of the lead core may degrade in the short-term because of substantial internal heat generation from cyclic deformation.

ii) Under larger lateral deformation, the rubber may experience net tension which will affect the compression and tension stiffness, and lead to potential vertical instability.

iii) Cavitation may happen when the bearing is under excessive tension, resulting in permanent damage in the tensile capacity.

d) ITYPE = 4 is used to model higher damping rubber bearings. It differs from elastomeric bearing (ITYPE = 1) in that the time-varying yield force is a function of resultant horizontal displacement, governed by:

$$\text{Yield force} = \text{FORCEY}\left(1 + \text{BETA}\left(\frac{dx^2 + dy^2}{\text{DISPY}^2}\right)\right) .$$

Here $dx$ and $dy$ are the displacements in the local $x$ and $y$ directions.

e) ITYPE = 5 is the same as ITYPE = 0 (spherical sliding bearing), except for the additional capability of yielding and failure of the rim, also called the retaining ring. The rim is intended to prevent the radial displacement of the slider exceeding RADB, but if sufficient radial force is applied, the rim can yield and then fail, leading to the slider falling off the supporting surface. When the rim fails, the isolator element is deleted.

6. **Damping.** DAMP is the fraction of critical damping for free vertical vibration of the isolator, based on the mass of the isolator (including any attached lumped masses) and its vertical stiffness. The viscosity is reduced automatically if it would otherwise infringe numerical stability. Damping is generally recommended:

a) Oscillations in the vertical force have a direct effect on friction forces in sliding isolators.

b) For isolators with curved surfaces, vertical oscillations can be excited as the isolator slides up and down the curved surface.

It may occasionally be necessary to increase DAMP if these oscillations become significant.

7. **Rotational stiffness.** By default, this element has no rotational stiffness - a pin joint is assumed. However, if required, "offset moments" can be generated according to the vertical load multiplied by the lateral displacement of the isolator. This is invoked using MX1, MX2, MY1, MY2. The moment *about* the local $x$-axis (meaning the moment that is dependent on lateral displacement in the local $y$-direction) is reacted on nodes 1 and 2 of the element in the proportions MX1 and MX2, respectively. Similarly, moments about the local $y$-axis are reacted in the proportions MY1 and MY2. These inputs effectively determine the location of the pin joint.

   For example, consider an isolator installed between the top of the foundation of a building (Node 1 of the isolator element) and the base of a column of the superstructure (Node 2 of the isolator element). To model a pin at the base of the column and react the offset moment on the foundation, set MX1 = MY1 = 1.0 and MX2 = MY2 = 0.0. For the same model, MX1 = MY1 = 0.0 and MX2 = MY2 = 1.0 would imply a pin at the top of the foundation - all the moment is transferred to the column. Some isolator designs have the pin at the bottom for moments about one horizontal axis, and at the top for the other axis - these can be modeled by setting MX1 = MY2 = 1.0 and MX2 = MY1 = 0.0. MX1, MX2, MY1 and MY2 are all expected to be greater than or equal to 0 and less than or equal to 1. Also, if MX1 and MX2 are not both zero, then MX1 + MX2 is expected to equal 1.0, and similarly for MY1 and MY2. However, no error checks are performed to ensure this.

   Optionally, rotational stiffnesses that resist rotation of Node 2 relative to Node 1 may be defined on Card 9. These moments are applied equal and opposite on Nodes 1 and 2, irrespective of the settings of MX1, MX2, MY1 and MY2.

8. **Density.** Density should be set to a reasonable value, say 2000 to 8000 kg/m$^3$. The element mass will be calculated as density × volume (volume is entered on *SECTION_BEAM).

9. ***SECTION_BEAM input.** Note on values for *SECTION_BEAM:

   a) Set ELFORM to 6 (discrete beam).

   b) VOL (the element volume) might typically be set to 0.1 m$^3$.

   c) INER always needs to be non-zero. It will influence the solution only when the element has rotational stiffness, that is, when any of MX1, MX2, MY1, MY2, KTHX, KTHY or KTHZ are non-zero. A reasonable value might be 10-20 kg-m$^2$.

   d) CID can be left blank if the isolator is aligned in the global coordinate system, otherwise a coordinate system should be referenced.

e) By default, the isolator will be assumed to rotate with the average rotation of its two nodes. If the base of the column rotates slightly the isolator will no longer be perfectly horizontal: this can cause unexpected vertical displacements coupled with the horizontal motion. To avoid this, rotation of the local axes of the isolator can be eliminated by setting RRCON, SRCON, and TRCON to 1.0. This does not introduce any rotational restraint to the model, it only prevents the orientation of the isolator from changing as the model deforms.

f) SCOOR must be set to zero.

g) All other parameters on *SECTION_BEAM can be left blank.

10. **Post-processing note.** As with other discrete beam material models, the force described in some post-processors as "Axial" is really the force in the local $x$-direction; "Y-Shear" is really the force in the local $y$-direction; and "Z-Shear" is really the force in the local $z$-direction.

## *MAT_JOINTED_ROCK

This is Material Type 198. Joints (planes of weakness) are assumed to exist throughout the material at a spacing small enough to be considered ubiquitous. The planes are assumed to lie at constant orientations defined on this material card. Up to three planes can be defined for each material. The base material is like *MAT_DRUCKER_PRAGER (*MAT_193). Input parameters for the base material are defined on Cards 1 through 3, while the joint planes are defined using Card 4. See *MAT_MOHR_COULOMB (*MAT_173) for a preferred alternative to this material model.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | GMOD | RNU | RKF | PHI | CVAL | PSI |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| STR_LIM | NPLANES | ELASTIC | LCCPDR | LCCPT | LCCJDR | LCCJT | LCSFAC |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| GMODDP | PHIDP | CVALDP | PSIDP | GMODGR | PHIGR | CVALGR | PSIGR |
|---|---|---|---|---|---|---|---|

**Card 4.** Include an instance of this card for each plane. Up to three planes may be defined.

| DIP | STRIKE | CPLANE | FRPLANE | TPLANE | SHRMAX | LOCAL | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | GMOD | RNU | RKF | PHI | CVAL | PSI |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | 1.0 | none | none | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| GMOD | Elastic shear modulus |
| RNU | Poisson's ratio |
| RKF | Failure surface shape parameter, see Remark 5. |
| PHI | Angle of friction (radians) |
| CVAL | Cohesion value (shear strength at zero normal stress) |
| PSI | Dilation angle (radians) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | STR_LIM | NPLANES | ELASTIC | LCCPDR | LCCPT | LCCJDR | LCCJT | LCSFAC |
| Type | F | I | I | I | I | I | I | I |
| Default | 0.005 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| STR_LIM | Minimum shear strength of material is given by STR_LIM × CVAL (see Remark 6) |
| NPLANES | Number of joint planes (maximum of 3) |
| ELASTIC | Behavior of base material (see Remark 3):<br>EQ.0: Nonlinear using all parameters on Cards 1 through 3<br>EQ.1: Linear elastic; only the joint planes are nonlinear |
| LCCPDR | Load curve for extra cohesion for base material (dynamic relaxation) as a function of time. See Remark 8. |
| LCCPT | Load curve for extra cohesion for base material (transient) as a function of time. See Remark 8. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCCJDR | Load curve for extra cohesion for joints (dynamic relaxation) as a function of time. See Remark 8. |
| LCCJT | Load curve for extra cohesion for joints (transient) as a function of time. See Remark 8. |
| LCSFAC | Load curve giving a factor on strength as a function of time (see Remark 9). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GMODDP | PHIDP | CVALDP | PSIDP | GMODGR | PHIGR | CVALGR | PSIGR |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Remark | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |

| VARIABLE | DESCRIPTION |
|---|---|
| GMODDP | Z-coordinate at which GMOD is correct |
| PHIDP | Z-coordinate at which PHI is correct |
| CVALDP | Z-coordinate at which CVAL is correct |
| PSIDP | Z-coordinate at which PSI is correct |
| GMODGR | Gradient of GMOD as a function of Z-coordinate (usually negative) |
| PHIGR | Gradient of PHI as a function of Z-coordinate |
| CVALGR | Gradient of CVAL as a function of Z-coordinate (usually negative) |
| PSIGR | Gradient of PSI as a function of Z-coordinate |

Repeat Card 4 for each plane (maximum of 3 planes):

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DIP | STRIKE | CPLANE | FRPLANE | TPLANE | SHRMAX | LOCAL | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1.e20 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| DIP | Angle of the plane in degrees below the horizontal |
| STRIKE | Plan view angle (degrees) of downhill vector drawn on the plane |
| CPLANE | Cohesion for shear behavior on plane |
| FRPLANE | Friction angle for shear behavior on plane (degrees) |
| TPLANE | Tensile strength across plane (generally zero or very small) |
| SHRMAX | Max shear stress on plane (upper limit, independent of compression) |
| LOCAL | Axes (see Remark 10)<br>EQ.0: DIP and STRIKE are with respect to the global axes.<br>EQ.1: DIP and STRIKE are with respect to the local element axes. |

**Remarks:**

1. **Joint plane orientations.** The joint plane orientations are defined by the angle of a "downhill vector" drawn on the plane, that is, the vector is oriented within the plane to obtain the maximum possible downhill angle. DIP is the angle of this line below the horizontal. STRIKE is the plan-view angle of the line (pointing downhill) measured clockwise from the global *Y*-axis about the global *Z*-axis. Note that DIP and STRIKE can also be with respect to the local element axes. See Remark 10 for details.

2. **Rigid body motion.** The joint planes rotate with the rigid body motion of the elements, irrespective of whether their initial definitions are in the global or local axis system.

3. **Elastic only behavior.** The full facilities of the modified Drucker Prager model for the base material can be used – see description of material type 193. Alternatively, to speed up the calculation, the ELASTIC flag can be set to 1, in which case the yield surface will not be considered, and only RO, GMOD, RNU, GMODDP, GMODGR, and the joint planes will be used.

4. **Model orientation.** This material type requires that the model is oriented such that the Z-axis is defined in the upward direction. The key parameters are defined such that they may vary with depth (i.e. the Z-axis), see Card 3. If Card 3 is left blank, the material properties do not vary with depth.

5. **Shape factor RKF.** The shape factor for a typical soil would be around 0.8. Values less than 0.75 should not be used.

6. **STR_LIM.** If STR_LIM is set to less than 0.005, the value is reset to 0.005.

7. **Correction to Drucker Prager model.** A correction has been introduced into the Drucker Prager model, such that the yield surface never infringes the Mohr-Coulomb criterion. Thus, the model does not give the same results as a "pure" Drucker Prager model.

8. **Load curves giving extra cohesion.** The load curves LCCPDR, LCCPT, LCCJDR, and LCCJT allow additional cohesion to be specified as a function of time. This cohesion is in addition to that specified in the material parameters. This feature is intended for use during the initial stages of an analysis to allow application of gravity or other loads without cracking or yielding, and for the cracking or yielding then to be introduced in a controlled manner. This is done by specifying extra cohesion that exceeds the expected stresses initially, then declining to zero. If no curves are specified, no extra cohesion is applied.

9. **LCSFAC.** The load curve giving a factor on strength applies simultaneously to the cohesion and tan PHI of the base material and all joints. This feature is intended for reducing the strength of the material gradually to explore factors of safety. If no curve is present, a constant factor of 1 is assumed. Values much greater than 1.0 may cause problems with stability.

10. **Masonry and joint planes.** Joint planes are generally defined in the global axis system if they are taken from survey data, and the material represents rock. For this case, set LOCAL = 0. In other cases, it may be more convenient to define the joint plane angles, DIP and STRIKE, relative to the element local axis system (to do this, set LOCAL = 1). For example, this material model can be used to represent masonry with the weak planes representing the mortar joint. In this situation, these joints may be parallel to the local element axes throughout the mesh.

    The choice of defining the joint angles relative to global versus local coordinates is available only for solid elements. For thick shell elements (*ELEMENT_-

TSHELL), DIP and STRIKE are always relative to the element's local axis, and the setting of LOCAL is ignored.

11. **Extra history variables.** Extra history variables may be plotted (see NEIPH on *DATABASE_EXTENT_BINARY). They are described in the following table:

| History Variable # | Description |
|---|---|
| 1 | Mobilized strength fraction for base material |
| 2 | At-rest coefficient (defined as horizontal stress divided by vertical stress, where "horizontal stress" is the average of the stresses in the global *X* and *Y* directions, and "vertical stress" is in the global *Z* direction). |
| 4 – 6 | Crack opening strains for planes 1 through 3 |
| 7 – 9 | Crack accumulated engineering shear strain for planes 1 through 3 |
| 10 – 12 | Current shear utilization for planes 1 through 3 |
| 13 – 15 | Maximum shear utilization to date for planes 1 through 3 |

### *MAT_BARLAT_YLD2004

This is Material Type 199. This model was developed by Aretz and Barlat [2004] and Barlat et al. [2005]. It incorporates a yield criterion called Barlat 2004-18p, where up to 18 material parameters are used to define anisotropy for a full 3D stress state. This model is currently available for solid elements and thick shell formulations 3, 5 and 7.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP12 | CP13 | CP21 | CP23 | CP31 | CP32 | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPP12 | CPP13 | CPP21 | CPP23 | CPP31 | CPP32 | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP44 | CP55 | CP66 | CPP44 | CPP55 | CPP66 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | A | LCSS | | | | | |
| Type | F | F | I | | | | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | I | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus<br>LT.0.0: -E is either a load curve ID for Young's modulus as a function of plastic strain or a table ID for Young's modulus as a function of plastic strain and temperature. |
| PR | Poisson's ratio |
| CP*ij* | 9 coefficients $c'_{ij}$ of the first linear transformation matrix $\mathbf{C}'$ |
| CPP*ij* | 9 coefficients $c''_{ij}$ of the second linear transformation matrix $\mathbf{C}''$ |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| A | Flow potential exponent *a* |
| LCSS | Load curve ID or table ID for (isotropic) hardening: |
| | GT.0: If LCSS is a load curve, then yield stress $\bar{\sigma}$ is a function of plastic strain. If LCSS is a table, then yield stress $\bar{\sigma}$ is a function of plastic strain and plastic strain rate. |
| | LT.0: If -LCSS is a load curve, then yield stress $\bar{\sigma}$ is a function of plastic strain. If -LCSS is a table, then yield stress $\bar{\sigma}$ is a function of plastic strain and temperature. |
| XP YP ZP | Define coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements: |
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation |
| | EQ.-3: Switch material axes *a* and *c* before BETA rotation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO and *ELEMENT_TSHELL_BETA. |

**Remarks:**

The 3D yield condition for this material can be written as (see Barlat et al. [2005])

$$\begin{aligned}\phi &= \phi(\tilde{\mathbf{S}}', \tilde{\mathbf{S}}'') \\ &= |\tilde{S}_1' - \tilde{S}_1''|^a + |\tilde{S}_1' - \tilde{S}_2''|^a + |\tilde{S}_1' - \tilde{S}_3''|^a + |\tilde{S}_2' - \tilde{S}_1''|^a + |\tilde{S}_2' - \tilde{S}_2''|^a \\ &\qquad + |\tilde{S}_2' - \tilde{S}_3''|^a + |\tilde{S}_3' - \tilde{S}_1''|^a + |\tilde{S}_3' - \tilde{S}_2''|^a + |\tilde{S}_3' - \tilde{S}_3''|^a \\ &= 4\bar{\sigma}^a\end{aligned}$$

Here $\tilde{S}_i'$ and $\tilde{S}_i''$ ($i$ = 1,2,3) are the 6 principal values, $a$ is the flow potential exponent, and $\bar{\sigma}$ is the effective uniaxial yield stress (defined with LCSS). The diagonal tensors $\tilde{\mathbf{S}}' = \text{diag}(\tilde{S}_1', \tilde{S}_2', \tilde{S}_3')$ and $\tilde{\mathbf{S}}'' = \text{diag}(\tilde{S}_1'', \tilde{S}_2'', \tilde{S}_3'')$ contain the principal values of $\tilde{\mathbf{s}}'$ and $\tilde{\mathbf{s}}''$. $\tilde{\mathbf{s}}'$ and $\tilde{\mathbf{s}}''$ result from two linear transformations of the deviatoric portion of the Cauchy stress, **s**:

$$\tilde{\mathbf{s}}' = \mathbf{C}'\mathbf{s}$$

$$\tilde{\mathbf{s}}'' = \mathbf{C}''\mathbf{s}$$

$\mathbf{C}'$ and $\mathbf{C}''$ have the following form:

$$\mathbf{C}' = \begin{bmatrix} 0 & -c'_{12} & -c'_{13} & 0 & 0 & 0 \\ -c'_{21} & 0 & -c'_{23} & 0 & 0 & 0 \\ -c'_{31} & -c'_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c'_{66} \end{bmatrix}$$

$$\mathbf{C}'' = \begin{bmatrix} 0 & -c''_{12} & -c''_{13} & 0 & 0 & 0 \\ -c''_{21} & 0 & -c''_{23} & 0 & 0 & 0 \\ -c''_{31} & -c''_{32} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c''_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c''_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c''_{66} \end{bmatrix}$$

Each transformation matrix requires 9 coefficients that must be defined on Cards 2, 3, and 4 of this material model input. For identification of all 18 coefficients, uniaxial tests in several directions, biaxial tests, and crystal plasticity models (for out-of-plane properties) are needed. See Barlat et al. [2005] for more details and examples for parameters sets.

Note that the sequence of stress tensor components in LS-DYNA is as follows

$$\mathbf{s} = \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \\ s_{xy} \\ s_{yz} \\ s_{zx} \end{bmatrix}$$

meaning matrix entry "44" is linked to stress component "$xy$", "55" belongs to "$yz$", and "66" refers to "$zx$". If compared to the paper from Barlat et al. [2005] that means the following relations hold (each equation: LS-DYNA parameters on the left, Barlat coefficients on the right):

$$\text{CP44} = c'_{66} \qquad \text{CP55} = c'_{44} \qquad \text{CP66} = c'_{55}$$
$$\text{CPP44} = c''_{66} \qquad \text{CPP55} = c''_{44} \qquad \text{CPP66} = c''_{55}$$

For example, the following input would correspond to the parameters in Table 2 of that paper for 6111-T4 aluminum alloy:

```
*MAT_BARLAT_YLD2004
...
$      CP12       CP13       CP21       CP23       CP31       CP32
  1.241024   1.078271   1.216463   1.223867   1.093105   0.889161
$     CPP12      CPP13      CPP21      CPP23      CPP31      CPP32
  0.775366   0.922743   0.765487   0.793356   0.918689   1.027625
$      CP44       CP55       CP66      CPP44      CPP55      CPP66
  1.349094   0.501909   0.557173   0.589787   1.115833   1.112273
...
```

## *MAT_BARLAT_YLD2004_27P

This is Material Type 199_27P. This model is a straightforward extension of material type 199. Aretz et al. [2010] developed the extension. It consists of a yield criterion called Barlat 2004-27p, where up to 27 material parameters define anisotropy for a 3D stress state. This model is currently available for solid elements and thick shell formulations 3, 5, and 7.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| CP12 | CP13 | CP21 | CP23 | CP31 | CP32 | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| CPP12 | CPP13 | CPP21 | CPP23 | CPP31 | CPP32 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| CPPP12 | CPPP13 | CPPP21 | CPPP23 | CPPP31 | CPPP32 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| CP44 | CP55 | CP66 | CPP44 | CPP55 | CPP66 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| CPPP44 | CPPP55 | CPPP66 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| AOPT | A | LCSS | | | | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MACF | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus<br>LT.0.0: -E is either a load curve ID for Young's modulus as a function of plastic strain or a table ID for Young's modulus as a function of plastic strain and temperature. |
| PR | Poisson's ratio |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP12 | CP13 | CP21 | CP23 | CP31 | CP32 | | |
| Type | F | F | F | F | F | F | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPP12 | CPP13 | CPP21 | CPP23 | CPP31 | CPP32 | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPPP12 | CPPP13 | CPPP21 | CPPP23 | CPPP31 | CPPP32 | | |
| Type | F | F | F | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP44 | CP55 | CP66 | CPP44 | CPP55 | CPP66 | | |
| Type | F | F | F | F | F | F | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPPP44 | CPPP55 | CPPP66 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CP*ij* | 9 coefficients $c'_{ij}$ of the first linear transformation matrix $\mathbf{C}'$ |
| CPP*ij* | 9 coefficients $c''_{ij}$ of the second linear transformation matrix $\mathbf{C}''$ |
| CPPP*ij* | 9 coefficients $c'''_{ij}$ of the second linear transformation matrix $\mathbf{C}'''$ |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | A | LCSS | | | | | |
| Type | F | F | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the element's keyword input or input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA, depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, $P$, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| A | Flow potential exponent $a$ |
| LCSS | Load curve ID or table ID for (isotropic) hardening:<br>GT.0: If LCSS is a load curve, yield stress, $\bar{\sigma}$, is a function of plastic strain. If LCSS is a table, $\bar{\sigma}$ is a function of plastic strain and plastic strain rate.<br>LT.0: If -LCSS is a load curve, yield stress, $\bar{\sigma}$, is a function of plastic strain. If -LCSS is a table, $\bar{\sigma}$ is a function of plastic strain and temperature. |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP YP ZP | Define coordinates of point, *P*, for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the procedure to obtain the final material axes. If you define BETA on *ELEMENT_SOLID_{OPTION}, LS-DYNA uses that BETA for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 9 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO and *ELEMENT_TSHELL_BETA. |

**Remarks:**

We can write the 3D yield condition for this material as (see Aretz et al. [2010]):

$$\begin{aligned}\phi &= \phi(\tilde{\mathbf{S}}', \tilde{\mathbf{S}}'', \tilde{\mathbf{S}}''') \\ &= \left|\tilde{S}_1' - \tilde{S}_1''\right|^a + \left|\tilde{S}_1' - \tilde{S}_2''\right|^a + \left|\tilde{S}_1' - \tilde{S}_3''\right|^a + \left|\tilde{S}_2' - \tilde{S}_1''\right|^a + \left|\tilde{S}_2' - \tilde{S}_2''\right|^a + \left|\tilde{S}_2' - \tilde{S}_3''\right|^a \\ &\quad + \left|\tilde{S}_3' - \tilde{S}_1''\right|^a + \left|\tilde{S}_3' - \tilde{S}_2''\right|^a + \left|\tilde{S}_3' - \tilde{S}_3''\right|^a + \left|\tilde{S}_1''' - \tilde{S}_2'''\right|^a + \left|\tilde{S}_2''' - \tilde{S}_3'''\right|^a + \left|\tilde{S}_3''' - \tilde{S}_1'''\right|^a \\ &= 6\bar{\sigma}^a\end{aligned}$$

Here $\tilde{S}_i'$, $\tilde{S}_i''$ and $\tilde{S}_i'''$ ($i$ = 1, 2, 3) are the nine principal values, $a$ is the flow potential exponent, and $\bar{\sigma}$ is the effective uniaxial yield stress (defined with LCSS). The diagonal tensors $\tilde{\mathbf{S}}' = \mathrm{diag}(\tilde{S}_1', \tilde{S}_2', \tilde{S}_3')$, $\tilde{\mathbf{S}}'' = \mathrm{diag}(\tilde{S}_1'', \tilde{S}_2'', \tilde{S}_3'')$ and $\tilde{\mathbf{S}}''' = \mathrm{diag}(\tilde{S}_1''', \tilde{S}_2''', \tilde{S}_3''')$ contain the principal values of $\tilde{\mathbf{s}}'$, $\tilde{\mathbf{s}}''$ and $\tilde{\mathbf{s}}'''$. $\tilde{\mathbf{s}}'$, $\tilde{\mathbf{s}}''$ and $\tilde{\mathbf{s}}'''$ result from three linear transformations of the deviatoric portion of the Cauchy stress, $\mathbf{s}$:

$$\tilde{\mathbf{s}}' = \mathbf{C}'\mathbf{s}$$
$$\tilde{\mathbf{s}}'' = \mathbf{C}''\mathbf{s}$$
$$\tilde{\mathbf{s}}''' = \mathbf{C}'''\mathbf{s}$$

$\mathbf{C}'$, $\mathbf{C}''$, and $\mathbf{C}'''$ have the following form:

$$\mathbf{C}' = \begin{bmatrix} 0 & -c_{12}' & -c_{13}' & 0 & 0 & 0 \\ -c_{21}' & 0 & -c_{23}' & 0 & 0 & 0 \\ -c_{31}' & -c_{32}' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}' \end{bmatrix}$$

$$\mathbf{C}'' = \begin{bmatrix} 0 & -c_{12}'' & -c_{13}'' & 0 & 0 & 0 \\ -c_{21}'' & 0 & -c_{23}'' & 0 & 0 & 0 \\ -c_{31}'' & -c_{32}'' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}'' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}'' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}'' \end{bmatrix}$$

$$\mathbf{C}''' = \begin{bmatrix} 0 & -c_{12}''' & -c_{13}''' & 0 & 0 & 0 \\ -c_{21}''' & 0 & -c_{23}''' & 0 & 0 & 0 \\ -c_{31}''' & -c_{32}''' & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44}''' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55}''' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66}''' \end{bmatrix}$$

Each transformation matrix requires nine coefficients input on Cards 2, 3, 4, 5, and 6. You must identify the 27 coefficients from the results of uniaxial tests in several directions, biaxial tests, and crystal plasticity models (for out-of-plane properties). See Barlat et al. [2005] and Aretz et al. [2010] for more details and examples of parameter sets.

Note that the sequence of stress tensor components in LS-DYNA is as follows

$$\mathbf{s} = \begin{bmatrix} s_{xx} \\ s_{yy} \\ s_{zz} \\ s_{xy} \\ s_{yz} \\ s_{zx} \end{bmatrix}$$

meaning matrix entry "44" is linked to stress component "$xy$", "55" belongs to "$yz$", and "66" refers to "$zx$". If compared to the paper from Aretz et al. [2010] that means the following relations hold (each equation: LS-DYNA parameters on the left, Barlat coefficients on the right):

$$\begin{aligned} \text{CP44} &= c'_{66} & \text{CP55} &= c'_{44} & \text{CP66} &= c'_{55} \\ \text{CPP44} &= c''_{66} & \text{CPP55} &= c''_{44} & \text{CPP66} &= c''_{55} \\ \text{CPPP44} &= c'''_{66} & \text{CPPP55} &= c'''_{44} & \text{CPPP66} &= c'''_{55} \end{aligned}$$

For example, the following input corresponds to the anisotropy parameters of the paper from Aretz et al. [2010] for AA3104-H19 aluminum alloy:

```
*MAT_BARLAT_YLD2004_27P
...
$     CP12      CP13      CP21      CP23      CP31      CP32
  0.606220   1.40199  0.367381  0.382048-0.0338334  0.821313
$    CPP12     CPP13     CPP21     CPP23     CPP31     CPP32
   1.47683  0.607440   1.02276  0.529721  0.870353  0.487571
$   CPPP12    CPPP13    CPPP21    CPPP23    CPPP31    CPPP32
 -0.334771 -0.488257  0.559627   1.11301  0.436153  0.808785
$     CP44      CP55      CP66     CPP44     CPP55     CPP66
  0.882890   1.00000   1.00000  0.972130   1.00000   1.00000
$   CPPP44    CPPP55    CPPP66
  0.898776   1.00000   1.00000
...
```

### *MAT_STEEL_EC3

This is Material Type 202. Tables and formulae from Eurocode 3 are used to derive the mechanical properties and their variation with temperature, although these can be overridden by user-defined curves. It is currently available only for Hughes-Liu beam elements. This material model is intended for modelling structural steel in fires.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | | | |
| Type | A | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC_E | LC_PR | LC_AL | TBL_SS | LC_FS | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | | |

Card 3 *must* be included but left blank.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |
| Default | | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus – a reasonable value must be provided even if LC_E is also input. See Remark 2. |
| PR | Poisson's ratio |
| SIGY | Initial yield stress, $\sigma_{y0}$ |
| LC_E | Optional load curve ID giving Young's modulus as a function of temperature (overrides E and factors from EC3) |
| LC_PR | Optional load curve ID giving Poisson's ratio as a function of temperature (overrides PR) |
| LC_AL | Optional load curve ID giving alpha as a function of temperature (over-rides thermal expansion data from EC3) |
| TBL_SS | Optional table ID containing stress-strain curves at different temperatures (overrides curves from EC3) |
| LC_FS | Optional load curve ID giving failure strain as a function of temperature |

**Remarks:**

1. **Eurocode 3 and Required Input.** By default, only E, PR and SIGY must be defined. The Young's Modulus, *E*, will be scaled by a factor that is a function of temperature as specified in EC3. The factor is 1.0 at low temperatures. Eurocode 3 (EC3) Section 3.2 specifies the stress-strain behaviour of carbon steels at temperatures between 20°C and 1200°C. The stress-strain curves given in EC3 are scaled within the material model such that the maximum stress at low temperatures is SIGY (see Figure below). By default, the thermal expansion coefficient as a function of temperature will be as specified in EC3 Section 3.4.1.1.

2. **Optional input.** LC_E, LC_PR and LC_AL are optional; they should have temperature on the *x*-axis and the material property on the *y*-axis, with the points in order of increasing temperature. If defined (that is, nonzero), they override E, PR, and the relationships from EC3. However, a reasonable value for E should always be included, since these values will be used for purposes such as contact stiffness calculation.

   TBL_SS is also optional. It overrides SIGY and the stress-strain relationships from EC3. If present, TBL_SS must be the ID of a *DEFINE_TABLE. The field VALUE on the *DEFINE_TABLE should contain the temperature at which each stress-strain curve is applicable; the temperatures should be in ascending order.

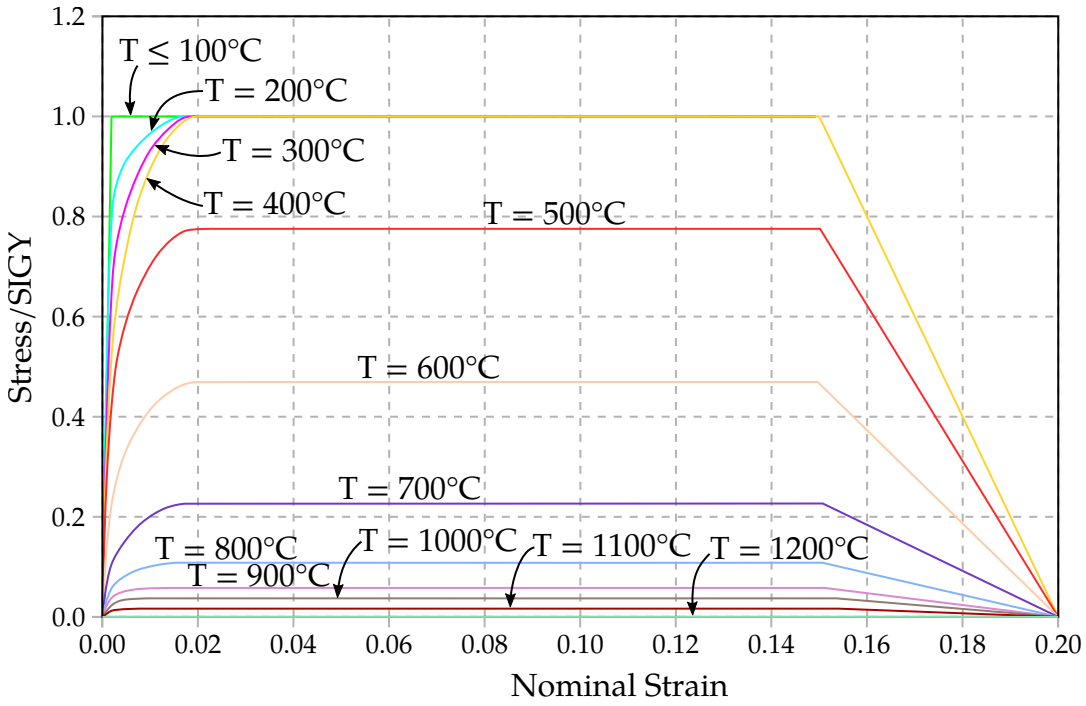



**Figure M202-1.** Stress-strain curves at various temperatures

The curves that follow the temperature values have plastic strain on the $x$-axis and yield stress on the $y$-axis as per other LS-DYNA elasto-plastic material models. As with all instances of *DEFINE TABLE, the curves containing the stress-strain data must immediately follow the *DEFINE_TABLE input data and must be in the correct order (that is, the same order as the temperatures).

3. **Temperature.** Temperature can be defined by any of the *LOAD_THERMAL methods. The temperature does not have to start at zero: the initial temperature will be taken as a reference temperature for each element, so non-zero initial temperatures will not cause thermal shock effects.

4. **Extra history variables.** Temperature is output by this material model as Extra History Variable 1. This can be useful for checking the input in cases where temperature varies across the different integration points, as is the case with *LOAD_THERMAL_VARIABLE_BEAM

## *MAT_HYSTERETIC_REINFORCEMENT

This is Material Type 203. It is intended as an alternative reinforcement model for layered reinforced concrete shell or beam elements, for use in seismic analysis where the nonlinear hysteretic behavior of the reinforcement is important. *PART_COMPOSITE or *INTEGRATION_BEAM should be used to define some integration points as a part made of *MAT_HYSTERETIC_REINFORCEMENT, while other integration points have concrete properties using *MAT_CONCRETE_EC2. When using beam elements, ELFORM = 1 is required.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | YM | PR | SIGY | LAMDA | SBUCK | POWER |
| Type | A | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | SIGY | 0.5 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
|---|---|---|---|---|---|---|---|---|
| Variable | FRACX | FRACY | LCTEN | LCCOMP | AOPT | EBU | DOWNSL | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.1 | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | |
|---|---|---|---|---|---|---|---|---|
| Variable | DBAR | FCDOW | LCHARD | UNITC | UNITL | | | |
| Type | F | F | F | F | F | | | |
| Default | 0.0 | 0.0 | 0.0 | 1.0 | 1.0 | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPDAM1 | EPDAM2 | DRESID | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | 0.0 | 0.0 | | | | | |

Additional Card for AOPT ≠ 0.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |
| Default | | | | 0.0 | 0.0 | 0.0 | | |

Additional Card for AOPT ≠ 0.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| YM | Young's modulus |
| PR | Poisson'srRatio |
| SIGY | Yield stress |

| VARIABLE | DESCRIPTION |
|---|---|
| LAMDA | Slenderness ratio |
| SBUCK | Initial buckling stress (should be positive) |
| POWER | Power law for Bauschinger effect (non-dimensional) |
| FRACX | Fraction of reinforcement at this integration point in local $x$-direction |
| FRACY | Fraction of reinforcement at this integration point in local $y$-direction |
| LCTEN | Optional curve providing the factor on SIGY as a function of plastic strain (tension) |
| LCCOMP | Optional curve providing the factor on SBUCK as a function of plastic strain (compression) |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in Figure M2-1. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. The material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element (see Figure M2-1). **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword.<br>LT.0.0: \|AOPT\| is a coordinate system ID (see *DEFINE_COORDINATE_OPTION). |
| EBU | Optional buckling strain. If defined, it overrides LAMBDA. |

| VARIABLE | DESCRIPTION |
|---|---|
| DOWNSL | Initial downslope of the buckling curve as a fraction of the Young's modulus (dimensionless) |
| DBAR | Reinforcement bar diameter used for dowel action. See Remark 7. |
| FCDOW | Concrete compressive strength used for dowel action. See Remark 7. This field has units of stress. |
| LCHARD | Characteristic length for dowel action (length units) |
| UNITC | Factor to convert model stress units to MPa. For instance, if the model units are Newtons and meters, UNITC = $10^{-6}$. [UNITC] = 1/[STRESS]. |
| UNITL | Factor to convert model length units to millimeters. For example, if the model units are meters, UNITL = 1000. [UNITL] = 1/[LENGTH]. |
| EPDAM1 | Accumulated plastic strain at which hysteretic damage begins |
| EPDAM2 | Accumulated plastic strain at which hysteretic damage is complete |
| DRESID | Residual factor remaining after hysteretic damage |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Angle for AOPT = 0 and 3 |

**Remarks:**

1. **Material directions.** The reinforcement is treated as bars, acting independently in the local material $x$ and $y$ directions. By default, the local material $x$-axis is the element's $x$-axis (parallel to the line from Node 1 to Node 2), but this direction may be overridden using AOPT or the element's BETA angles.

2. **Using this material with shell and beam elements.** For shell elements, the reinforced concrete section should be defined using *PART_COMPOSITE with some integration points being reinforcement (referencing a material ID using this material model) and others being concrete (using, for example, *MAT_CONCRETE_EC2). By default, shear strains in the local $xy$, $yz$, and $zx$ directions are unresisted by this material model, so it should not be used alone (without

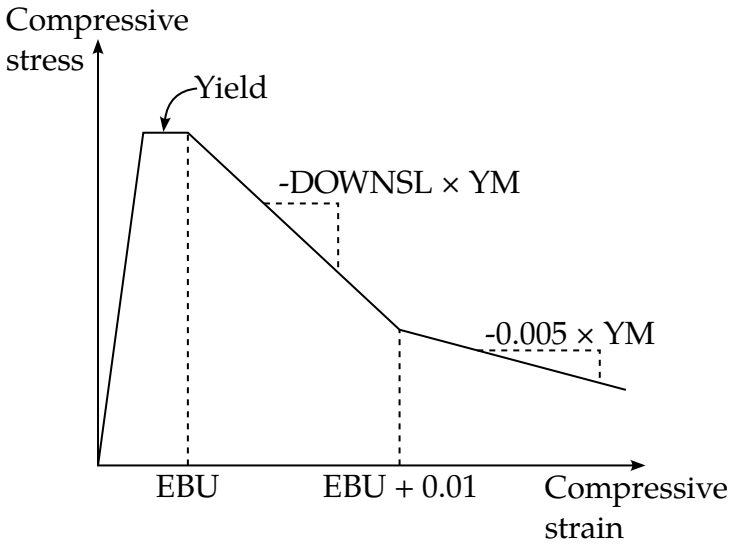



**Figure M203-1.** Buckling behavior using EBU and DOWNSL

concrete). The area fractions of reinforcement in the local $x$ and $y$-directions at each integration point are given by the thickness fraction of the integration point in the *PART_COMPOSITE definition times the fractions FRACX and FRACY on Card 2 above.

For beam elements, *INTEGRATION_BEAM defines the material at the integration point (e.g. reinforcement or concrete). Due to a limitation of *INTEGRATION_BEAM, this material model can be paired only with *MAT_CONCRETE_EC2 within the same *INTEGRATION_BEAM.

3. **Stress-strain response.** The tensile response is elastic-perfectly plastic, using yield stress SIGY. Optionally, load curves may be used to describe the stress-strain response in tension (LCTEN) and compression (LCCOMP). Either, neither, or both curves may be defined. If present, LCTEN overrides the perfectly plastic tensile response, and LCCOMP overrides the buckling curve. The tensile and compressive plastic strains are considered independent of each other.

4. **Bar buckling.** To define bar buckling, set either the slenderness ratio LAMDA or the initial buckling strain EBU with downslope DOWNSL. If bar buckling is not defined, the bars simply yield in compression. If both ways are defined, the buckling behavior defined by EBU and DOWNSL overrides LAMDA.

   The slenderness ratio LAMDA determines buckling behavior and is defined as,

$$\frac{kL}{r},$$

   where $k$ depends on end conditions, and

   $L$ = unsupported length of reinforcement bars
   $r$ = radius of gyration which for round bars is equal to (bar radius)/2.

Users are expected to determine LAMDA accounting for the expected crack spacing.

Figure M203-1 shows the alternative buckling behavior defined by EBU and DOWNSL.

5. **Hysteresis response.** Reloading after a change of load direction follows a Bauschinger-type curve, leading to the hysteresis response shown below:

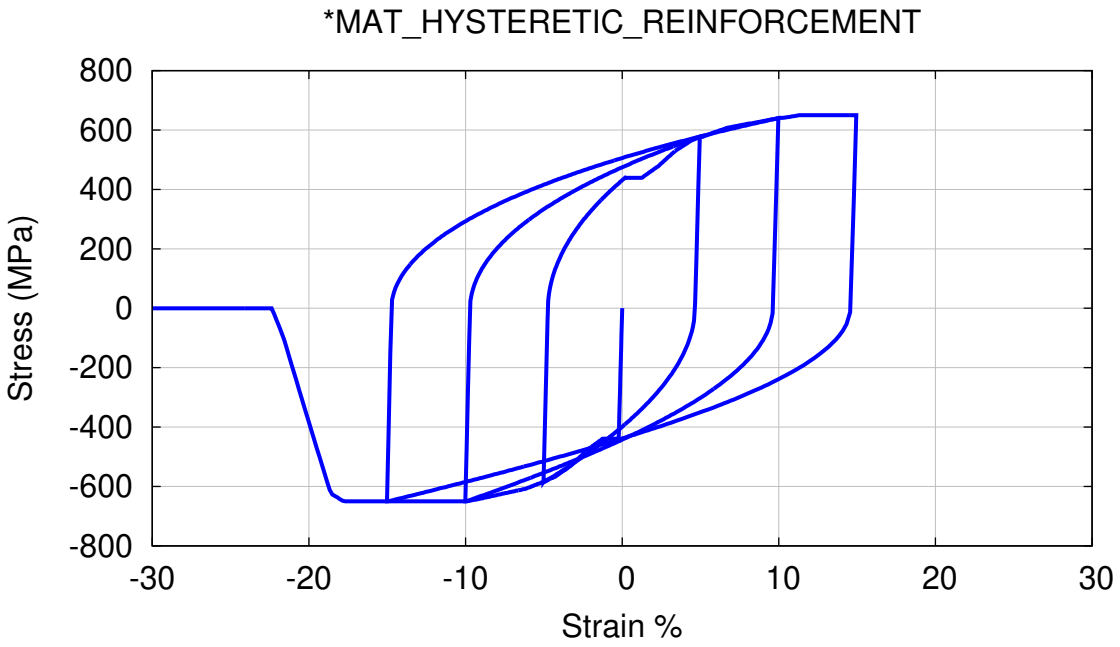



**Figure M203-2.** Example hysteretic response

6. **Damage modeling.** Two types of damage accumulation may be modeled. Damage based on ductility (strain) can be modeled using the curves LCTEN and LCCOMP. At high strain, these curves show reducing stress with increasing strain.

   To model damage based on hysteretic energy accumulation, use the parameters EPDAM1, EPDAM2, and DRESID. The damage is a function of accumulated plastic strain. For this purpose, plastic strain increments are always treated as positive in both tension and compression, and buckling strain also counts towards the accumulated plastic strain. The material has its full stiffness and strength until the accumulated plastic strain reaches EPDAM1. Between plastic strains EPDAM1 and EPDAM2, the stiffness and strength fall linearly with accumulated plastic strain, reaching a factor DRESID at plastic strain EPDAM2.

7. **Dowel action.** The data on Card 3 defines the shear stiffness and strength and is optional. Shear resistance is assumed to occur by dowel action. The bars bend

locally to the crack and crush the concrete. An elastic-perfectly-plastic relation is assumed for all shear components (in-plane and through-thickness). The assumed (smeared) shear modulus and yield stress applicable to the reinforcement bar cross-sectional area are as follows, based on formulae derived from experimental data by El-Ariss, Soroushian, and Dulacska:

$$G[\text{MPa}] = 8.02E^{0.25}F_c^{0.375}L_{\text{char}}D_b^{0.75}$$
$$\tau_y = 1.62\sqrt{F_c S_y}$$

where,

$E$ = steel Young's modulus in MPa
$F_c$ = compressive strength of concrete in MPa
$L_{\text{char}}$ = characteristic length of shear deformation in mm
$D_b$ = bar diameter in mm
$S_y$ = steel yield stress in MPa.

The input parameters should be given in model units. For instance, DBAR and LCHAR are in model length units, and FCDOW is in model stress units. These units are converted internally using UNITL and UNITC.

8. **Element erosion.** By default, no erosion occurs. Elements are deleted only if EPDAM1 and EPDAM2 are nonzero, DRESID is zero, and the accumulated plastic strain reaches EPDAM2. If FRACX and FRACY are both nonzero, i.e., if there is reinforcement in both local directions, elements only erode when the condition above has been reached in both local directions.

9. **Output.** The output stresses, as for all other LS-DYNA material models, are by default in the global coordinate system. They are scaled by the reinforcement fractions FRACX and FRACY. The plastic strain output is the accumulated plastic strain (increments always treated as positive) and is the greater such value of the two local directions. Extra history variables are available as follows:

| History Variable # | Description |
|---|---|
| 1 | Reinforcement stress in the local $x$-direction (not scaled by FRACX) |
| 2 | Reinforcement stress in the local $y$-direction (not scaled by FRACY) |
| 3 | Total strain in the local $x$-direction |
| 4 | Total strain in the local $y$-direction |
| 5 | Accumulated plastic strain in the local $x$-direction |
| 6 | Accumulated plastic strain in the local $y$-direction |
| 7 | Shear stress (dowel action) in local $xy$ |

| History Variable # | Description |
|---|---|
| 8 | Shear stress (dowel action) in local $xz$ |
| 9 | Shear stress (dowel action) in local $yz$ |
| 10 | Maximum (high-tide) total strain in the local $x$-direction |
| 11 | Maximum (high-tide) total strain in the local $y$-direction |

## *MAT_DISCRETE_BEAM_POINT_CONTACT

This is Material Type 205. It is used for discrete beam elements only (ELFORM = 6). It simulates contact forces between a point (Node 2) and an imaginary flat surface (fixed relative to Node 1). The beam elements may have nonzero initial length. On *SECTION_BEAM, SCOOR must be set to -13, the triad rotation option. The triad rotation option ensures that the axis system remains fixed in the imaginary surface.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | STIFF | FRIC | DAMP | DMXPZ | LIMPZ | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| DMXPX | DMXNX | DMXPY | DMXNY | LIMPX | LIMNX | LIMPY | LIMNY |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| KROTX | KROTY | KROTZ | TKROT | FBONDH | FBONDT | DBONDH | DBONDT |
|---|---|---|---|---|---|---|---|

***Card 4.*** This card is optional.

| LCZ | DAMPZ | STIFFH | FRMAX | DAMPH | GAP0 | AFAC | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | STIFF | FRIC | DAMP | DMXPZ | LIMPZ | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | 0.0 | 0.0 | $10^{20}$ | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| STIFF | Stiffness (Force/length units) |
| FRIC | Friction coefficient (dimensionless) |

| VARIABLE | DESCRIPTION |
|---|---|
| DAMP | Damping factor (dimensionless), in the range 0 to 1. We suggest a value of 0.5. |
| DMXPZ | Displacement limit in positive local *z*-direction (uplift) |
| LIMPZ | Action when Node 2 passes DMXPZ:<br>EQ.0: Element is deleted.<br>EQ.1: Further displacement is resisted by stiffness STIFF. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DMXPX | DMXNX | DMXPY | DMXNY | LIMPX | LIMNX | LIMPY | LIMNY |
| Type | F | F | F | F | F | F | F | F |
| Default | $10^{20}$ | DMXPX | DMXPX | DMXPY | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| DMXPX | Displacement limit in positive local *x*-direction |
| DMXNX | Displacement limit in negative local *x*-direction |
| DMXPY | Displacement limit in positive local *y*-direction |
| DMXNY | Displacement limit in negative local *y*-direction |
| LIMPX | Action when Node 2 passes DMXPX:<br>EQ.0: Element is deleted.<br>EQ.1: Further displacement is resisted by stiffness STIFF. |
| LIMNX | Action when Node 2 passes DMXNX:<br>EQ.0: Element is deleted.<br>EQ.1: Further displacement is resisted by stiffness STIFF. |
| LIMPY | Action when Node 2 passes DMXPY:<br>EQ.0: Element is deleted.<br>EQ.1: Further displacement is resisted by stiffness STIFF. |

| VARIABLE | DESCRIPTION |
|---|---|
| LIMNY | Action when Node 2 passes DMXNY:<br>EQ.0: Element is deleted.<br>EQ.1: Further displacement is resisted by stiffness STIFF. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | KROTX | KROTY | KROTZ | TKROT | FBONDH | FBONDT | DBONDH | DBONDT |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | $10^{20}$ | $10^{20}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| KROTX | Rotational stiffness about local $x$ |
| KROTY | Rotational stiffness about local $y$ |
| KROTZ | Rotational stiffness about local $z$ |
| TKROT | Time at which rotational stiffness becomes active |
| FBONDH | Force to break initial bond in plane of contact surface |
| FBONDT | Force to break initial bond in tension, normal to contact surface |
| DBONDH | Displacement over which bond force in the plane of the contact surface reduces from FBONDH to zero |
| DBONDT | Displacement over which bond force normal to the contact surface reduces from FBONDT to zero |

This card is optional

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCZ | DAMPZ | STIFFH | FRMAX | DAMPH | GAP0 | AFAC | |
| Type | I | F | F | F | F | F | F | |
| Default | 0 | 0.0 | STIFF | $\infty$ | 0.0 | 0.0 | 1.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCZ | Optional load curve ID giving force-displacement for compression in local $z$-direction (abscissa: displacement; ordinate: force). The load curve must be defined in the positive quadrant, meaning that the compressive force values should be defined as positive values. |
| DAMPZ | Viscous damping coefficient in local $z$-direction (applied in addition to DAMP) (force/velocity units) |
| STIFFH | Elastic stiffness in local $x$- and $y$-directions |
| FRMAX | Upper limit on friction force |
| DAMPH | Viscous damping coefficient in local $x$- and $y$-directions (applied in addition to DAMP) (force/velocity units). |
| GAP0 | Initial gap in local $z$-direction (length units) |
| AFAC | Scale factor applied to all stiffnesses and forces |

**Remarks:**

1. **Model Description.** This material model simulates contact between a point (Node 2 of the beam element) and an imaginary flat surface which is fixed relative to Node 1. In these remarks we call the imaginary surface the "contact surface" – this does *not* refer to *CONTACT. The local axes are determined by CID on *SECTION_BEAM. The imaginary surface is in the local $xy$-plane passing through the initial coordinates of Node 2. The local $z$-axis points outwards from the surface. The surface translates and rotates with Node 1. SCOOR must be set to -13. The elements may have nonzero length.

   When Node 2 moves in the negative local $z$-direction relative to Node 1 (penetration into the contact surface), the motion is resisted by stiffness STIFF and the force generated is described here as the contact force. By default, uplift (Node 2 moving in the positive local $z$-direction relative to Node 1) is not resisted. If uplift greater than DMXPZ occurs, either the element is deleted (if LIMPZ = 0) or further uplift is resisted by STIFF (if LIMPZ = 1).

   Sliding on the surface (motion of Node 2 in the local $x$- and $y$-directions) is resisted by friction. The maximum friction force is given by FRIC times the contact force, with an upper limit of FRMAX if that parameter is nonzero. When one of the displacement limits, DMXPX, DMXNX, DMXPY, or DMXNY, is reached, the default behavior is for Node 2 to fall off the edge of the contact surface, and the element is deleted (see Figure M205-1). Optionally, the input fields LIMPX, LIMNX, LIMPY, and LIMNY can be used to change the behavior to "hard limits" using stiffness STIFF – these represent contact with a hard surface perpendicular to the surface on which Node 2 slides. In that case, the limit distances, DMXPX, DMXNX, DMXPY, and

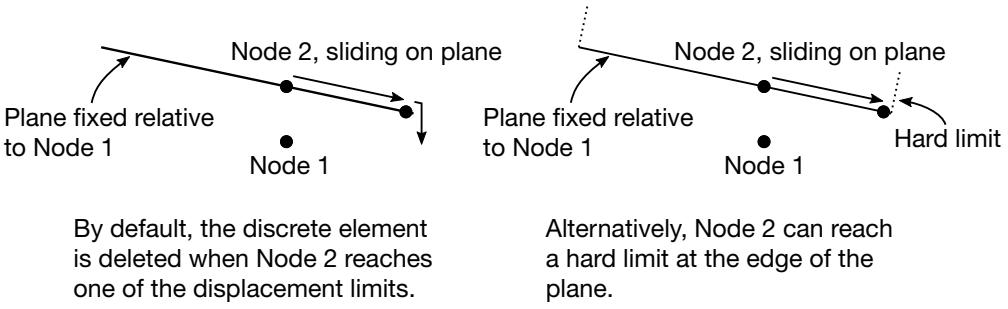



**Figure M205-1.** Illustration of the two different actions that can be applied to the element when Node 2 reaches the edge of the plane. The left schematic depicts the default behavior in which the element is deleted when Node 2 reaches the edge. The right image illustrates Node 2 reaching a hard limit. When it reaches this limit, a contact force is applied.

DMXNY, represent the initial gap between the point (Node 2) and the hard surface.

Optionally, an initial bond strength can be defined. The bond forces are in addition to any contact and friction forces. After breakage of the bond, contact and sliding can continue to occur.

If LCZ is defined, compressive loading in the contact direction follows LCZ while unload/reload is linear with stiffness STIFF. The value of STIFF must not be less than the maximum slope of any segment of the curve LCZ.

You can also define moment stiffnesses through KROTX, KROTY and KROTZ. Optionally, these stiffnesses can be set to become active at a certain time (TKROT). If TKROT is nonzero, the moment stiffness will be zero before that time and during any dynamic relaxation. If TKROT is left zero, the moment stiffness will be active from the start of the analysis including during dynamic relaxation.

Damping is applied to the force normal to the surface, to the bond forces, and to any forces generated by "hard limits" (LIMPX etc.), but not to sliding. Two damping methods are available: DAMP and DAMPZ/DAMPH. DAMP is recommended for general use where the exact amount of damping is unimportant, and the requirement is simply to remove unwanted oscillations. DAMPZ and DAMPH are available for cases where particular values of viscous damping coefficient are required. When LCZ is also defined and the response is following the load curve (meaning not during unloading/reloading), the damping coefficient DAMPZ is scaled to the ratio of the slope of LCZ to STIFF. The slope of LCZ is the gradient at the current point on the load curve.

2. ***SECTION_BEAM Input.** Note on values for *SECTION_BEAM:

   a) Set ELFORM to 6 (discrete beam)

b) CID can be left blank if the contact surface is aligned in the global $XY$-plane, otherwise a coordinate system should be referenced.

c) SCOOR must be set to -13.

3. **Output.** Beam "axial" or "$X$" force is the force in the local $x$-direction. "Shear-$Y$" or "$Y$" force is the force in the local $y$-direction. "Shear-$Z$" or "$Z$" force is the force in the local $z$-direction, normal to the contact surface.

Other output is written to the d3plot and d3thdt files in the places where post-processors expect to find the stress and strain at the first integration point for integrated beams:

| Integration Point | Post-Processing Data Component | Actual Meaning |
|---|---|---|
| 1 | Axial stress | Displacement in the local $x$-direction |
| 1 | $XY$ shear stress | Displacement in the local $y$-direction |
| 1 | $ZX$ shear stress | Displacement in the local $z$-direction |
| 1 | Plastic strain | Minimum overlap, meaning the minimum value so far during the analysis of the remaining displacement before Node 2 falls of the surface in the $x$- or $y$-directions |
| 1 | Axial strain | Bond damage |

## *MAT_SOIL_SANISAND

This is Material Type 207. It is supported for solid elements only. It is intended for modelling sands and sandy soils under monotonic and cyclic loading conditions. Cyclic shear loading leads to dilative and contractive volumetric behavior based on critical state soil mechanics. When used together with LS-DYNA's pore fluid analysis capabilities, liquefaction and related phenomena can be modelled. See Remarks below.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | G0 | K0NU | PREF | RHOC | THETA | X |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| EIN | ALPHAC | E0 | LAMBDA | XI | NB | H0 | CH |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| P0 | CC | ND | A0 | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| ANISO | KH | ZMAX | CZ | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| PATM | M | N | V | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G0 | K0NU | PREF | RHOC | THETA | X |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.37 | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| G0 | Shear-modulus-related dimensionless term $G_0$. See Remark 6. |
| K0NU | Elastic constant (see Remark 6):<br>GT.0.0: Bulk modulus term $K_0$<br>LT.0.0: Absolute value is Poisson's ratio, $\nu$. |
| PREF | Reference pressure in Limiting Compression Curve, associated with unity void ratio. See Remark 10. |
| RHOC | Exponent in Limiting Compression Curve, $\rho_c$. See Remark 10. |
| THETA | Exponent in transitional compression behavior, $\theta$. See Remark 10. |
| X | Material constant $X$. See Remark 10. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EIN | ALPHAC | E0 | LAMBDA | XI | NB | H0 | CH |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | 0.7 | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| EIN | Initial void ratio |
| ALPHAC | Critical surface angle in $q$-$p$ space, $\alpha_c^c$. See Remark 8. |
| E0 | Material constant in Critical State Line, $e_0$. See Remark 8. |
| LAMBDA | Material constant in Critical State Line, $\lambda$. See Remark 8. |
| XI | Material constant in Critical State Line, $\xi$. See Remark 8. |
| NB | Bounding surface parameter, $n^b$. See Remark 8. |

| VARIABLE | DESCRIPTION |
|---|---|
| H0 | Kinematic hardening parameter, $h_0$. See Remark 7. |
| CH | Kinematic hardening parameter, $c_h$. See Remark 7. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P0 | CC | ND | A0 | | | | |
| Type | F | F | F | F | | | | |
| Default | none | 0.778 | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| P0 | Initial value of yield surface parameter, $p_{0.}$ See Remark 7. |
| CC | Ratio of critical surface angle in extension to critical surface angle in compression, $c$. See Remark 8. |
| ND | Dilatancy surface parameter, $n_d$. See Remark 8. |
| A0 | Dilatancy parameter, $A_0$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ANISO | KH | ZMAX | CZ | | | | |
| Type | F | F | F | F | | | | |
| Default | 0.333 | 1.0 | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ANISO | Inherent fabric anisotropy measure. See Remark 12. |
| KH | Material constant, $k_h$, in dependence of hardening parameters on inherent fabric anisotropy. See Remark 12. |
| ZMAX | Material constant for fabric change effect, $z_{max.}$ See Remark 11. |

| VARIABLE | DESCRIPTION |
|---|---|
| CZ | Material constant for fabric change effect, $c_z$. See Remark 11. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PATM | M | N | V | | | | |
| Type | F | F | F | F | | | | |
| Default | none | 0.05 | 20.0 | 1000.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PATM | Atmospheric pressure, $p_{atm}$ |
| M | Yield surface constant, $m$. See Remark 7. |
| N | Yield surface constant, $n$. See Remark 7. |
| V | Flow rule constant, $V$ |

**Remarks:**

1. **References.** SANISAND (an acronym for Simple ANIsotropic SAND) is a family of constitutive models within the frameworks of critical state soil mechanics and bounding surface plasticity. Although the overall principles remain the same, different research teams have developed versions of SANISAND with different details. The material model implemented in LS-DYNA as *MAT_SANISAND is based on Dafalias and Manzari [2004] with high-pressure yield surface cap from Taiebat and Dafalias [2008] and options for inherent fabric anisotropy taken from Dafalias, Papadimitriou, and Li [2004]. The references give the constitutive model equations both in "triaxial formulation" (in which there are only two stress variables $q$ and $p$), and in "multi-axial formulation" (in which the deviatoric stress-related terms are tensors). The LS-DYNA implementation follows the multi-axial formulation, but, in these Remarks, equations are given in triaxial formulation for ease of understanding the principles.

2. **Pore pressure build-up and liquefaction.** Pore water pressure build-up occurs when the "soil skeleton" contracts (for example, under cyclic shearing), leaving a greater proportion of the external load to be supported by the pore water and a lesser proportion acting on the soil skeleton, that is, the effective stress is reduced. Since sandy soils are frictional materials, reduced effective stress leads

to reduced shear stiffness and reduced shear strength. It ultimately leads to liquefaction of the soil. Therefore, accurate modelling of pore pressure build-up and liquefaction effects depend critically on accurate dilation/contraction behavior of the material model, which is a key aspect of the SANISAND material model.

3. **Modeling pore pressure.** Pore pressure is not included in the material model itself. As with other LS-DYNA soil models, the material model represents the effective stress (soil skeleton) behavior. Pore pressure can be modelled with *CONTROL_PORE_FLUID and *BOUNDARY_PORE_FLUID. The analysis type should be set to Undrained or Time Dependent Consolidation (ATYPE = 1 or 3, respectively) in order for pore pressure changes to occur in response to the dilation or contraction of the soil.

4. **Void ratio.** Soils consist of solid particles with voids between them. Many of the equations governing the behavior of this material model are given in terms of the void ratio, $e$. The volume of voids divided by the volume of solids gives the void ratio. It is a measure of how loosely or tightly packed the particles are (the lower the void ratio, the more tightly packed). The void ratio at the start of the analysis is given by input parameter EIN. The void ratio varies during the analysis: volumetric strains are assumed to relate to changes of void volume, while the volume of solids remains unchanged.

5. **Stress ratio.** Most elasto-plastic constitutive models are described in terms of their stress-strain behavior. For frictional materials, such as sandy soils, shear strength is considered proportional to pressure. Thus, it is more appropriate to define the equations in terms of stress ratio rather than stress. Stress ratio in triaxial formulation is a scalar quantity $\eta$ equal to $q/p$ (where $q$ is Von Mises stress and $p$ is pressure). In the multi-axial formulation, the stress ratio is a tensor quantity equal to the deviatoric stress tensor divided by pressure. Stress ratios described in the references include the "dilatancy stress ratio" (also called the "phase transformation line") which marks the boundary between contractive and dilative response to shear strains, the "critical state stress ratio" (see Remark 8), and the "bounding stress ratio" at which shear failure occurs.

6. **Elastic properties.** The elastic shear modulus, $G$, is given by:

$$G = G_0 p_{\text{atm}} \frac{(2.97 - e)^2}{1 + e} \left( \frac{p}{p_{\text{atm}}} \right)^{1/2}$$

where $G_0$and $p_{\text{atm}}$ are the input parameters G0 and PATM, $p$ is the current pressure, and $e$ is the current void ratio. G0 is dimensionless; PATM has stress units.

If input parameter K0NU is positive, then the bulk modulus, $K$, is given by:

$$K = K_0 p_{\text{a}} \frac{1 + e}{e} \left( \frac{p}{p_{\text{atm}}} \right)^{2/3}$$

where $K_0$ is the input parameter K0NU. If input parameter K0NU is negative, then the bulk modulus, $K$, is given by:

$$K = \frac{2(1+\nu)}{3(1-2\nu)}G$$

where $\nu = -$K0NU.

7. **Yield surface and hardening.** The yield surface is defined as a narrow cone of semi-angle $m$ centered on a back stress ratio $\alpha$ which, together with the hardening rule, enables realistic nonlinear unload/reload behavior under cyclic loading. The yield surface is closed at the high pressure end by a cap-like feature that models grain crushing using parameter $p_0$, as described in Taiebat and Dafalias [2008]:

$$f = (q - p\alpha)^2 - m^2 p^2 \left[1 - \left(\frac{p}{p_0}\right)^n\right] = 0$$

where $m$ and $n$ are input parameters M and N (see Card 5), and $p_0$ has an initial value given by input parameter P0. Description of the flow rule and evolution of $\alpha$ and $p_0$ are given in the references. The yield surface as described in Dafalias and Manzari [2004] is the same as above except that it lacks the term in square brackets, which corresponds to a yield surface cap at high pressure. When P0 is set to a high value compared to the expected pressure, the yield surface in this material model becomes the same as that of Dafalias and Manzari [2004].

Hardening is described by a "stress ratio hardening modulus", $H$, which is the rate of change of stress ratio $\eta$ with deviatoric plastic strain:

$$H = \frac{d\eta}{d\varepsilon_p} = \frac{G_0|h_0|(1 - c_h e)(\alpha_b - \eta)}{R}\left(\frac{p}{p_{atm}}\right)^{-1/2}.$$

Here $h_0$ is the input parameter H0, $c_h$ is the input parameter CH, $\alpha_b - \eta$ is the "distance" (in stress ratio terms) between the current stress ratio and its image on the bounding surface for loading in the current loading direction. $R$ is defined as given in Equations 5, 6, and 24 of Dafalias and Manzari [2004] and can be considered as the "distance" (in stress ratio terms) between the current stress ratio and the stress ratio where the current plastic loading cycle began (such as at the most recent loading direction reversal). This term is zero on initial loading and immediately following a stress reversal, giving an instantaneously infinite hardening slope and a smooth transition between elastic and plastic behavior.

If the anisotropy parameter ANISO has a non-default value, anisotropy effects are included as per Equation 15 in Dafalias, Papadimitriou and Li [2004].

8. **Critical state.** The critical state is a combination of values of stress ratio, pressure, and void ratio at which the application of shear strain causes no change to any of those three parameters. It is assumed that soil starting from any initial state, if subjected to sufficient shear strain, will tend towards and eventually

reach a critical state. In MAT_SANISAND, the critical state void ratio, $e_c$, is defined as follows:

$$e_c = e_0 - \lambda(p/p_a)^{\xi} \quad .$$

In the above, $e_0$, $\lambda$, $\xi$, and $p_a$ are input parameters E0, LAMBDA, XI and PATM, respectively. If the anisotropy parameter ANISO has a non-default value, $e_0$ in the above equation is replaced by $e_0 \exp(-A)$ where $A$ is the lode angle-dependent state parameter described in Dafalias, Papadimitriou and Li [2004].

The state parameter $\psi$ is given by $e - e_c$, which may be thought of as the distance from the critical state in terms of void ratio.

The critical state stress ratio, $\alpha_c^c$, is a constant given by the input parameter ALPHAC. If the material is on the critical state line in $e$-$p$ space according to the above equation, that is, when $\psi = 0$, then both the dilatancy stress ratio, $\alpha^d$, and the bounding stress ratio, $\alpha^b$, coincide with the critical state stress ratio. These stress ratios vary with $\psi$ in the following manner:

$$\alpha^d = c\alpha_c^c \exp(n^d \psi)$$
$$\alpha^b = c\alpha_c^c \exp(-n^b \psi)$$

where $n^d$ and $n^b$ are the input parameters ND and NB, respectively, and $c = 1$ for compressive loading. For extension loading, $c$ is equal to the input parameter CC.

9. **Dilation and contraction.** The volumetric plastic strain increment is linked to the deviatoric plastic strain increment through a dilation angle, $D$. $D$ is defined such that the volumetric plastic strain is contractive at stress ratios below $\alpha^d$ and dilative at stress ratios above $\alpha^d$:

$$D = \frac{\dot{\epsilon}_v^p}{\dot{\epsilon}_q^p} = sA_d(\alpha^d - \alpha) \quad .$$

In the above equation, $\dot{\epsilon}_v^p$ and $\dot{\epsilon}_v^p$ are the volumetric and deviatoric plastic strain increments, $s$ takes the value 1 or –1 according to loading direction, and $A_d$ is the input parameter A0. If the fabric change parameters ZMAX and CZ are defined (see Remark 11), $A_d$ is scaled according to Equation 12 of Dafalias and Manzari [2004].

10. **Limiting Compression Curve (LCC).** The maximum pressure that the material can sustain is governed by the LCC, which is analogous to a yield curve in pressure-void ratio space. The LCC consists of points $(p_{\mathrm{LCC}}, e_{\mathrm{LCC}})$ such that:

$$\log e_{\mathrm{LCC}} = -\rho_c \log\left(\frac{p_{\mathrm{LCC}}}{p_{\mathrm{REF}}}\right) \quad .$$

Here $\rho_c$ and $p_{\text{REF}}$ are the input parameters RHOC and PREF. The transition from elastic volumetric response to plastic volumetric deformation along the LCC is a smooth function governed by input parameters THETA, and X. See Taiebat and Dafalias [2008] for further details.

11. **Fabric change.** Input parameters ZMAX and CZ control the fabric change effect described by Dafalias and Manzari [2004]. During cyclic shearing, in the dilatant phase the contact surfaces between the sand particles are re-oriented in a manner that greatly increases the contractive tendency upon subsequent reversal of the shearing direction. If this effect is not modelled, simulations of cyclic shearing may tend to stabilize at a nonzero effective pressure in cases where experiments would show the effective pressure diminishing to zero.

12. **Fabric anisotropy.** Input parameters ANISO and KH control the fabric anisotropy effect described by Dafalias, Papadimitriou and Li [2004]. The default, given by ANISO = 1/3 and KH=1, is isotropic behavior. Non-default values cause the plastic hardening behavior and the critical state line to depend on a lode-angle, leading to a better match of experimental results under a wide variety of stress states. ANISO = 0 corresponds to a fabric formation where particles lie entirely on the global $XY$-plane. ANISO = 1 implies a fabric formation where particles are oriented entirely parallel to the vertical global $Z$-direction. It is expected that the most common cases will be in the range of 0 < ANISO < 1/3, i.e., with a preference toward horizontal orientations.

13. **Output.** The current void ratio, $e$, is output in place of plastic strain.

### References:

[1] Taiebat M. and Dafalias, Y. F. “SANISAND: simple anisotropic sand plasticity model.” *International Journal for Numerical and Analytical Methods in Geomechanics*, 32(8), 915–948 (2008).

[2] Dafalias,Y. F. and Manzari, M.T. “Simple plasticity sand model accounting for fabric change effects.” *Journal of Engineering Mechanics*, 130(6), 622–634 (2004).

[3] Dafalias, Y. F., Papadimitriou, A. G., and Li, X. S. “Sand plasticity model accounting for inherent fabric anisotropy.” *Journal of Engineering Mechanics*,130(11), 1319–1333 (2004).

## *MAT_BOLT_BEAM

This is Material Type 208 for use with beam elements using ELFORM = 6 (discrete beam). The beam elements must have nonzero initial length so that the directions in which tension and compression act can be distinguished. See Remarks 1 and 2.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | KAX | KSHR | blank | blank | FPRE | TRAMP |
| Type | A | F | F | F | | | F | F |
| Default | none | none | 0.0 | 0.0 | | | 0.0 | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCAX | LCSHR | FRIC | CLEAR | DAFAIL | DRFAIL | DAMAG | TOPRE |
| Type | I | I | F | F | F | F | F | F |
| Default | 0 | 0 | 0.0 | 0.0 | $10^{20}$ | $10^{20}$ | 0.1 | 0.0 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DACFAIL | AXSHFL | HOLSHR | IAXIS | | | | |
| Type | F | I | I | I | | | | |
| Default | $10^{20}$ | 0 | 0 | 1 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| KAX | Axial elastic stiffness (Force/Length units) |

| VARIABLE | DESCRIPTION |
|---|---|
| KSHR | Shear elastic stiffness (Force/Length units) |
| FPRE | Preload force |
| TRAMP | Time duration during which preload is ramped up |
| LCAX | Load curve giving axial load as a function of plastic displacement ($x$-axis = displacement (length units), $y$-axis = force). See Remark 4. |
| LCSHR | Load curve ID or table ID giving lateral load as a function of plastic displacement ($x$-axis - displacement (length units), $y$-axis - force). In the table case, each curve in the table represents lateral load as a function of displacement at a given (current) axial load, meaning the values in the table are axial forces. See Remark 4. |
| FRIC | Friction coefficient resisting sliding of bolt head/nut (non-dimensional) |
| CLEAR | Radial clearance (gap between bolt shank and the inner diameter of the hole) (length units). See Remark 5. |
| DAFAIL | Axial tensile displacement at which failure is initiated (length units) |
| DRFAIL | Radial displacement at which failure is initiated (excludes clearance) |
| DAMAG | Failure is completed at<br>(DAFAIL or DRFAIL or DACFAIL) × (1 + DAMAG) |
| T0PRE | Time at which preload application begins |
| DACFAIL | Axial compressive displacement at which failure is initiated (positive value, length units) |
| AXSHFL | Flag to determine effect on axial response of increase of length of element due to shear displacement. In this context, shear displacement excludes sliding within the clearance gap. See Remark 6.<br>EQ.0: Shear-induced length increase treated as axial load.<br>EQ.1: Shear-induced length increase is ignored. |

| VARIABLE | DESCRIPTION |
|---|---|
| HOLSHR | Flag for hole enlargement due to shear (see Remark 7):<br>EQ.0: Hole does not enlarge due to shear deformation.<br>NE.0: Shear deformation after bolt contacts the inner diameter of the hole enlarges the hole. |
| IAXIS | Flag to determine how the bolt axis relates to the beam element local axes. See Remark 2.<br>EQ.1: Each element creates own axes, bolt axis (N1-N2 direction) is local $x$ (default)<br>EQ.2: Each element creates own axes, bolt axis (N1-N2 direction) is local $y$<br>EQ.3: Each element creates own axes, bolt axis (N1-N2 direction) is local $z$<br>EQ.4: Axis system defined by CID on *SECTION_BEAM, bolt axis is local $x$<br>EQ.5: Axis system defined by CID on Section card, bolt axis is local $y$<br>EQ.6: Axis system defined by CID on Section card, bolt axis is local $z$ |

**Remarks:**

1. **Bolted Joint Geometry.** The element represents a bolted joint. The nodes of the beam should be thought of as representing the points at the centers of the holes in the plates that are joined by the bolt.

2. **Local Axes.** There are three options for defining the bolt axis direction and local axis system for elements of this material type. Here, "bolt axis" means the direction in which tension is applied, while shear forces are perpendicular to the bolt axis. "Local axes" means the axis system in which the element's deformations are calculated and its forces are output. By default, the bolt axis coincides with the local $x$-axis, although that can be changed using the input parameter IAXIS.

   a) **Local axes defined by a *DEFINE_COORDINATE_NODES *that has FLAG set to 1.*** The coordinate system is referenced either as CID on *SECTION_BEAM or through PARAM3 on *ELEMENT_BEAM_THICKNESS. The behavior is then as follows:

i) The local axis system is defined by the three nodes of the *DEFINE_COORDINATE_NODES throughout the analysis, as described in the Remarks under *DEFINE_COORDINATE_NODES. The initial direction given by Nodes 1 and 2 of the beam element and the rotation of the local system based on SCOOR from *SECTION_BEAM are both irrelevant in this case.

ii) By default, the axial direction of the bolt coincides with the local $x$-axis of the *DEFINE_COORDINATE_NODES. Tension (positive force) is generated when Node 2 displaces in the positive local $x$ direction relative to Node 1. Care is needed to ensure that the beam element topology is defined with Node 1 and Node 2 the correct way around to generate tension in the expected direction – for example, having Node 2 initially offset in the positive local $x$ direction from Node 1 will mean that tension is generated when the beam element elongates in the local $x$-direction.

iii) The axial direction of the bolt can be oriented along the local $y$- or $z$-axis instead of the local $x$-axis by setting IAXIS to 2 or 3, respectively.

iv) For bolts that are differently oriented, you will need to *either* separate them into different *PARTs (so that the different *DEFINE_COORDINATE_NODES can be referenced by different *SECTION_BEAM cards) *or* have PARAM3 on *ELEMENT_BEAM_THICKNESS reference the needed *DEFINE_COORDINATE_NODES coordinate system.

b) **Bolt axis defined by the initial orientation of the beam element.** To obtain this behavior, set CID = 0 on *SECTION_BEAM (and set PARAM3 = 0 if using *ELEMENT_BEAM_THICKNESS), and position Nodes 1 and 2 of each beam element with a nonzero distance between them, aligned with the axis of the bolt. Note that the behavior described below will also occur if IAXIS < 4 and CID is nonzero, but the referenced coordinate system is a *DEFINE_COORDINATE_SYSTEM or a *DEFINE_COORDINATE_NODES with FLAG = 0 (i.e. it does not fulfill all the conditions for a) above).

i) Each beam element automatically creates its own axis system where the bolt axis is initially in the direction defined by Node 1 to Node 2. Note that this behavior is different than that for other Discrete Beam material types.

ii) During the analysis, the local axis system rotates as defined by SCOOR on *SECTION_BEAM. For example, if SCOOR = -13, the axes rotate with Node 1. We recommend using SCOOR = -13 or +13.

iii) The bolt axis is always initially coincident with the Node 1 to Node 2 direction, but IAXIS = 1, 2 or 3 controls whether the bolt axis is labelled as the local *x*-, *y*- or *z*-axis. This setting has no effect on the analysis, only on the output of results for post-processing. This can be useful when post-processing discrete beams of different material types so that, for example, the beam *z*-force has a similar meaning across the different material types.

c) **Local axes initially defined by a *DEFINE_COORDINATE_*OPTION* with rotation of the axes controlled by SCOOR**. To obtain this behavior, the coordinate system should be specified by either *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_NODES with FLAG = 0. Reference the coordinate system as CID on *SECTION_BEAM (or as PARAM3 on *ELEMENT_BEAM_THICKNESS) and set IAXIS to 4, 5 or 6. The behavior is as follows:

i) The local axis system defined with CID on *SECTION_BEAM (or PARAM3 on *ELEMENT_BEAM_THICKNESS) is used. The initial direction Node 1 to Node 2 has no influence on the local axes or the bolt axis direction.

ii) During the analysis, the local axis system rotates as defined by SCOOR on *SECTION_BEAM. For example, if SCOOR = -13, the axes rotate with Node 1. We recommend using SCOOR = -13 or +13.

iii) The bolt axis is oriented along the local *x*-, *y*- or *z*-axis according to whether IAXIS = 4, 5 or 6, respectively.

iv) Care is needed to ensure that the beam element topology is defined with Node 1 and Node 2 the correct way around to generate tension in the expected direction – for example, if IAXIS = 4, having Node 2 initially offset in the positive local x direction from Node 1 will mean that tension is generated when the beam element elongates in the local *x* direction.

v) For bolts that are differently oriented, you will need to *either* separate them into different *PARTs (so that the different *DEFINE_COORDINATE_NODES can be referenced by different *SECTION_BEAM cards) *or* have PARAM3 on *ELEMENT_BEAM_THICKNESS reference the needed *DEFINE_COORDINATE_NODES coordinate system.

3. **Axial Response.** The axial response is tensile only. If the element shortens in the bolt axis direction, instead of generating a compressive axial load, a gap is assumed to develop between the bolt head (or nut) and the surface of the plate.

Contact between the bolted surfaces must be modelled separately, such as using *CONTACT or another discrete beam element.

4. **Yield Force Curves.** Curves LCAX and LCSHR give yield force as a function of plastic displacement for the axial and shear directions, respectively. The force increments are calculated from the elastic stiffnesses, subject to the yield force limits given by the curves.

5. **Sliding Shear Displacement.** CLEAR allows the bolt to slide in shear, resisted by friction between bolt head/nut and the surfaces of the plates, from the initial position at the center of the hole. CLEAR is the total sliding shear displacement before contact occurs between the bolt shank and the inside surface of the hole. Sliding shear displacement is not included in the displacement used for LCSHR. LCSHR is intended to represent the behavior after the bolt shank contacts the edge of the hole.

6. **Shear Deformation and Axial Tension.** If KSHR and LCSHR represent deformation and rotation of the bolt itself, AXSHFL = 0, the default setting, is recommended. On the other hand, if KSHR and LCSHR represent deformation of the bearing surfaces, AXSHFL = 1 is recommended, and HOLSHR = 1 is likely to be appropriate too (see Remark 7).

   The explanation is as follows. Consider the case where a shear displacement is applied to the bolted joint, while the plates are constrained to remain the same distance apart. Mechanisms by which a bolted joint can displace in the shear direction include:

   a) the bolt sliding within a clearance gap;

   b) rotation, bending and shearing of the bolt itself; and

   c) deformation of the bearing surfaces while the bolt itself remains almost rigid and perpendicular to the plates.

   The tension in the bolt might be expected to increase when mechanism (b) occurs because the length of the bolt itself must increase (in this example, the plates are held the same distance apart). Tension in the bolt would not increase with mechanisms (a) or (c). In the LS-DYNA model, applying shear while holding the plates the same distance apart will cause the element to lengthen. When calculating the axial force in the bolt, *MAT_BOLT_BEAM always ignores any lengthening due to mechanism (a) but is unable to distinguish between (b) and (c) – KSHR and LCSHR could represent either or both types of mechanism. AXSHFL tells *MAT_BOLT_BEAM whether to treat these shear deformations as contributing to changes of length to the bolt itself, and hence whether axial tension will be generated.

7. **Hole Enlargement.** If HOLSHR is nonzero, shear deformation beyond that necessary to close the clearance gap enlarges the hole in the plates and does not deform the bolt itself. The force-deformation relation of this mechanism is still governed by LCSHR, and the deformation (that is, enlargement of the hole) is tracked separately in each of the local *-y*, *+y*, *-z*, and *+z* directions. Thus, for example, enlargement of the hole in the positive *Y* direction has no effect on the position of the edge of the hole in the negative *Y* direction. When HOLSHR is used, AXSHFL should be set to 1.

8. **Output.** Beam "axial" or "X" force is the axial force in the beam. "shear-Y" and "shear-Z" are the shear forces. Other output is written to the d3plot and d3thdt files in the places where post-processors expect to find the stress and strain at the first two integration points for integrated beams.

| Integration Point | Post-Processing Data Component | Actual Meaning |
|---|---|---|
| 1 | Axial Stress | Change of length |
| 1 | *XY* Shear stress | Sliding shear displacement in local *y* |
| 1 | *ZX* Shear stress | Sliding shear displacement in local *Z* |
| 1 | Plastic strain | Resultant shear sliding displacement |
| 1 | Axial strain | Axial plastic displacement |
| 2 | Axial Stress | Shear plastic displacement excluding sliding |
| 2 | XY Shear stress | Not used |
| 2 | ZX Shear stress | Not used |
| 2 | Plastic strain | Not used |
| 2 | Axial strain | Not used |

## *MAT_HYSTERETIC_BEAM

This is Material Type 209. It can be used only with resultant beam elements (ELFORM = 2). It is intended for modelling buildings in seismic analysis and is similar to *MAT_191 but with increased capabilities. Plastic hinges can form at both ends of the element, and plasticity options are available for axial and shear behavior as well as bending. The yield surface incorporates moment-axial interaction. Advanced features implemented for this material include hinge locations and pinching effect (Card 3), asymmetry and shear failure (Card 4), Bauschinger effect (Card 5), stiffness degradation (Cards 6 and 7), and FEMA flags (Cards 8, 9, and 10).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | IAX | ISURF | IHARD | IFEMA |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCPMS | SFS | LCPMT | SFT | LCAT | SFAT | LCAC | SFAC |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| ALPHA | BETA | GAMMA | F0 | PINM | PINS | HLOC1 | HLOC2 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| DELTAS | KAPPAS | DELTAT | KAPPAT | LCSHS | SFSHS | LCSHT | SFSHT |
|---|---|---|---|---|---|---|---|

**Card 4.1.** Include this card if LCSHS < 0.

| LCSHSN | SFSHSN | LCSHTN | SFSHTN | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| HARDMS | GAMMS | HARDMT | GAMMT | HARDAT | GAMAT | HARDAC | GAMAC |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| OMGMS1 | OMGMS2 | OMGMT1 | OMGMT2 | OMGAT1 | OMGAT2 | OMGAC1 | OMGAC2 |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| RUMS | RUMT | DUAT | DUAC | LAM1 | LAM2 | SOFT1 | SOFT2 |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if IFEMA > 0.

| PRS1 | PRS2 | PRS3 | PRS4 | PRT1 | PRT2 | PRT3 | PRT4 |
|---|---|---|---|---|---|---|---|

**Card 9.** Include this card if IFEMA > 1.

| TS1 | TS2 | TS3 | TS4 | CS1 | CS2 | CS3 | CS4 |
|---|---|---|---|---|---|---|---|

**Card 10.** Include this card if IFEMA > 2.

| SS1 | SS2 | SS3 | SS4 | ST1 | ST2 | ST3 | ST4 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | IAX | ISURF | IHARD | IFEMA |
| Type | A | F | F | F | I | I | I | I |
| Default | none | none | none | none | 1 | 1 | 2 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| IAX | Abscissa definition for axial yield force as a function of inelastic deformation/strain curves (LCAT and LCAC on Card 2):<br>EQ.1: Plastic deformation (change in length)<br>EQ.2: Nominal plastic strain, that is, $\frac{\text{plastic deformation}}{\text{initial length}}$ |
| ISURF | Yield surface type for interaction (see Remark 2): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: Simple power law (default)<br>EQ.2: Power law based on resultant moment<br>EQ.3: Skewed yield surface version of ISURF = 2 (see Remark 5) |
| IHARD | Hardening type during cyclic response (see Remark 6):<br>EQ.1: Cumulative absolute deformation<br>EQ.2: Peak deformation<br>EQ.3: Peak deformation, yield-oriented<br>EQ.4: Peak deformation, peak-oriented |
| IFEMA | Flag for input of FEMA thresholds (Cards 8, 9 and 10; see Remarks 9 and 11):<br>EQ.0: No input<br>EQ.1: Input of rotation thresholds only<br>EQ.2: Input of rotation and axial strain thresholds<br>EQ.3: Input of rotation, axial strain and shear strain thresholds |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCPMS | SFS | LCPMT | SFT | LCAT | SFAT | LCAC | SFAC |
| Type | I | F | I | F | I | F | I | F |
| Default | none | 1.0 | LCPMS | SFS | none | 1.0 | LCAT | SFAT |

| VARIABLE | DESCRIPTION |
|---|---|
| LCPMS | Load curve ID (See *DEFINE_CURVE) giving normalized yield moment as a function of plastic rotation at hinges about the local *s*-axis. All values are positive. |
| SFS | Representative yield moment for plastic hinges about local the *s*-axis (scales the normalized moment from LCPMS) |
| LCPMT | Load curve ID (See *DEFINE_CURVE) giving normalized yield moment as a function of plastic rotation at hinges about the local *t*- |

| VARIABLE | DESCRIPTION |
|---|---|
| | axis. All values are positive. |
| SFT | Representative yield moment for plastic hinges about local the *t*-axis (scales the normalized moment from LCPMT) |
| LCAT | Load curve ID (See *DEFINE_CURVE) giving normalized axial tensile yield force as a function of inelastic deformation/strain. See IAX above for definition of deformation/strain. All values are positive. See *DEFINE_CURVE. |
| SFAT | Representative tensile strength (scales the normalized force from LCAT) |
| LCAC | Load curve ID (See *DEFINE_CURVE) giving normalized axial compressive yield force as a function of inelastic deformation/strain. See IAX above for definition of deformation/strain. All values are positive. See *DEFINE_CURVE. |
| SFAC | Representative compressive strength (scales the normalized force from LCAC) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | BETA | GAMMA | F0 | PINM | PINS | HLOC1 | HLOC2 |
| Type | F | F | F | F | F | F | F | F |
| Default | 2.0 | 2.0 | 2.0 | 0.0 | 1.0 | 1.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Parameter to define moment-axial yield surface:<br>GT.0.0: Yield surface parameter ALPHA (must not be < 1.1); see Remark 2.<br>LT.0.0: User-defined yield surface for the local *s*-axis. \|ALPHA\| is the load curve ID giving the yield locus. The abscissa is the moment about the local *s*-axis; the ordinate is the axial force (tensile positive). See Remark 4. |
| BETA | Parameter to define moment-axial yield surface:<br>GT.0.0: Yield surface parameter BETA (must not be < 1.1); see |

| VARIABLE | DESCRIPTION |
|---|---|
| | Remark 2. |
| | LT.0.0: User-defined yield surface for the local *t*-axis. \|BETA\| is the load curve ID giving the yield locus. Abscissa is moment about the local *t*-axis; the ordinate is the axial force (tensile positive). See Remark 4. |
| GAMMA | Parameter to define yield surface which must not be < 1.1 (see Remark 2) |
| F0 | Force at which maximum yield moment is achieved (tensile positive; for reinforced concrete, a negative (compressive) value would be entered). |
| PINM | Pinching factor for flexural hysteresis (for IHARD = 3 or 4 only). See Remark 7. |
| PINS | Pinching factor for shear hysteresis (for IHARD = 3 or 4 only). See Remark 7. |
| HLOC1 | Location of plastic Hinge 1 from Node 1 (see Remark 1): |
| | GE.0.0: HLOC1 is the distance of Hinge 1 to Node 1 divided by element length. |
| | LT.0.0.AND.GT.-1.0: -HLOC1 is the distance of Hinge 1 to Node 1 divided by element length; deactivate shear yielding. |
| | EQ.-1.0: deactivate Hinge 1. |
| | EQ.-10.0: deactivate shear yielding; Hinge 1 is located at Node 1. |
| | EQ.-11.0: deactivate Hinge 1 and shear yielding. |
| HLOC2 | Location of plastic Hinge 2 from Node 2 (see Remark 1): |
| | GE.0.0: HLOC2 is the distance of Hinge 2 to Node 2 divided by element length. |
| | LT.0.0.AND.GT.-1.0: HLOC2 is the distance of Hinge 2 to Node 2 divided by element length; deactivate shear yielding. |
| | EQ.-1.0: deactivate Hinge 2. |
| | EQ.-10.0: deactivate shear yielding; Hinge 2 is located at Node 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-11.0: deactivate Hinge 2 and shear yielding. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DELTAS | KAPPAS | DELTAT | KAPPAT | LCSHS | SFSHS | LCSHT | SFSHT |
| Type | F | F | F | F | I | F | I | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | none | 1.0 | LCSHS | SFSHS |

| VARIABLE | DESCRIPTION |
|---|---|
| DELTAS | Parameter to define the skew for yield surface (ISURF = 3); see Remark 2. |
| KAPPAS | Parameter to define the skew for yield surface (ISURF = 3); see Remark 2. |
| DELTAT | Parameter to define the skew for yield surface (ISURF = 3); see Remark 2. |
| KAPPAT | Parameter to define the skew for yield surface (ISURF = 3); see Remark 2. |
| LCSHS | Load curve ID (see *DEFINE_CURVE) giving yield shear force as a function of inelastic shear strain (shear angle) in the local *s*-direction (see Remark 10).<br>GT.0: Same inelastic response for positive-direction shear force as for negative-direction shear force, controlled by input fields LCSHS through SFSHT.<br>LT.0: Different inelastic response for positive-direction shear force versus negative-direction shear force (invokes Card 4.1). The absolute value of LCSHS and input fields SFSHS through SFSHT control shear in the positive *s*- and *t*-directions. Negative-direction shear behavior is controlled by input fields LCSHSN through SFSHTN on Card 4.1. |
| SFSHS | Scale factor on yield shear force in the local *s*-direction (scales the force from LCSHS): |

| VARIABLE | DESCRIPTION |
|---|---|
| | GT.0.0: Constant scale factor<br>LT.0.0: User-defined interaction with the axial force. \|SFSHS\| is the load curve ID giving the scale factor as a function of normalized axial force (tensile is positive). The normalization uses SFAT for tensile force and SFAC for compressive force. For example, point $(-1.0, 0.5)$ on the curve defines a scale factor of 0.5 for compressive force of -SFAC. |
| LCSHT | Load curve ID (see *DEFINE_CURVE) giving yield shear force as a function of inelastic shear strain (shear angle) in the local *t*-direction (see Remark 10). |
| SFSHT | Scale factor on yield shear force in the local *t*-direction (scales the force from LCSHS).<br>GT.0.0: Constant scale factor<br>LT.0.0: User-defined interaction with the axial force. \|SFSHT\| is the load curve ID giving the scale factor as a function of normalized axial force (tensile is positive). The normalization uses SFAT for tensile force and SFAC for compressive force. For example, point $(-1.0, 0.5)$ on the curve defines a scale factor of 0.5 for compressive force of -SFAC. |

**Negative-Direction Shear Force Card.** Include this card if LCSHS < 0

| Card 4.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSHSN | SFSHSN | LCSHTN | SFSHTN | | | | |
| Type | I | F | I | F | | | | |
| Default | LCSHS | SFSHS | LCSHT | SFSHT | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSHSN | Load curve ID (see *DEFINE_CURVE) giving yield shear force as a function of inelastic shear strain (shear angle) for negative shear in the local *s*-direction (see Remark 10). If left blank, it defaults to LCSHS. |

| VARIABLE | DESCRIPTION |
|---|---|
| SFSHSN | Scale factor on yield shear force for negative shear in the local *s*-direction (scales the force from LCSHSN). If left blank, it defaults to SFSHS.<br>GT.0.0: Constant scale factor<br>LT.0.0: User-defined interaction with the axial force. \|SFSHSN\| is the load curve ID giving the scale factor as a function of normalized axial force (tensile is positive). The normalization uses SFAT for tensile force and SFAC for compressive force. For example, point $(-1.0, 0.5)$ on the curve defines a scale factor of 0.5 for compressive force of -SFAC. |
| LCSHTN | Load curve ID (see *DEFINE_CURVE) giving yield shear force as a function of inelastic shear strain (shear angle) for negative shear in the local *t*-direction (see Remark 10). If left blank, it defaults to LCSHT. |
| SFSHTN | Scale factor on yield shear force for negative shear in the local *t*-direction (scales the force from LCSHTN). If left blank, it defaults to SFSHT.<br>GT.0.0: Constant scale factor<br>LT.0.0: User-defined interaction with the axial force. \|SFSHTN\| is the load curve ID giving the scale factor as a function of normalized axial force (tensile is positive). The normalization uses SFAT for tensile force and SFAC for compressive force. For example, point $(-1.0, 0.5)$ on the curve defines a scale factor of 0.5 for compressive force of -SFAC. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HARDMS | GAMMS | HARDMT | GAMMT | HARDAT | GAMAT | HARDAC | GAMAC |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | HARDMS | GAMMS | 0.0 | 0.0 | HARDAT | GAMAT |

| VARIABLE | DESCRIPTION |
|---|---|
| HARDMS | Kinematic hardening modulus for moment about the local *s*-axis (see Remark 8) |
| GAMMS | Kinematic hardening limit for moment about the local *s*-axis (see Remark 8) |
| HARDMT | Kinematic hardening modulus for moment about the local *t*-axis |
| GAMMT | Kinematic hardening limit for moment about the local *t*-axis |
| HARDAT | Kinematic hardening modulus for tensile axial force |
| GAMAT | Kinematic hardening limit for tensile axial force |
| HARDAC | Kinematic hardening modulus for compressive axial force |
| GAMAC | Kinematic hardening limit for compressive axial force |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OMGMS1 | OMGMS2 | OMGMT1 | OMGMT2 | OMGAT1 | OMGAT2 | OMGAC1 | OMGAC2 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | OMGMS1 | OMGMS2 | 0.0 | 0.0 | OMGAT1 | OMGAT2 |

| VARIABLE | DESCRIPTION |
|---|---|
| OMGMS1 | Damage evolution parameter $\omega_{s1}$ for moment about the local *s*-axis (see Remark 9) |
| OMGMS2 | Damage evolution parameter $\omega_{s2}$ for moment about the local *s*-axis |
| OMGMT1 | Damage evolution parameter $\omega_{t1}$ for moment about the local *t*-axis |
| OMGMT2 | Damage evolution parameter $\omega_{t2}$ for moment about the local *t*-axis |
| OMGAT1 | Damage evolution parameter $\omega_{at1}$ for tensile force |
| OMGAT2 | Damage evolution parameter $\omega_{at2}$ for tensile force |
| OMGAC1 | Damage evolution parameter $\omega_{ac1}$ for compressive force |

| VARIABLE | DESCRIPTION |
|---|---|
| OMGAC2 | Damage evolution parameter $\omega_{ac2}$ for compressive force |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RUMS | RUMT | DUAT | DUAC | LAM1 | LAM2 | SOFT1 | SOFT2 |
| Type | F | F | F | F | F | F | F | F |
| Default | $10^{20}$ | RUMS | $10^{20}$ | DUAT | 0.0 | LAM1 | 3.0 | 4.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| RUMS | Ultimate plastic rotation about *s*-axis for damage calculation (see Remark 9) |
| RUMT | Ultimate plastic rotation about *t*-axis for damage calculation |
| DUAT | Ultimate tensile plastic deformation/strain for damage calculation. See IAX above in Card 1 and Remark 9. |
| DUAC | Ultimate compressive plastic deformation/strain for damage calculation. See IAX above in Card 1. |
| LAM1 | Damage evolution parameter |
| LAM2 | Damage evolution parameter |
| SOFT1 | Threshold index at which softening starts (see Remark 9)<br>LE.4.0: Threshold index for start of softening, see Cards 8 thru 10<br>EQ.5.0: Softening and element deletion are disabled. |
| SOFT2 | Threshold index at which the element is fully softened and to be removed (ignored if SOFT1 = 5) |

**Plastic Rotation Thresholds Card.** Define Card 8 only if IFEMA > 0 (see Remarks 9 and 11).

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PRS1 | PRS2 | PRS3 | PRS4 | PRT1 | PRT2 | PRT3 | PRT4 |
| Type | F | F | F | F | F | F | F | F |
| Default | $10^{20}$ | $2 \times 10^{20}$ | $3 \times 10^{20}$ | $4 \times 10^{20}$ | PRS1 | PRS2 | PRS3 | PRS4 |

| VARIABLE | DESCRIPTION |
|---|---|
| PRS1 – PRS4 | Plastic rotation thresholds 1 to 4 about s-axis |
| PRT1 – PRT4 | Plastic rotation thresholds 1 to 4 about t-axis |

**Plastic Axial Strains Threshold Card.** Define Card 9 only if IFEMA > 1 (see Remarks 9 and 11).

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TS1 | TS2 | TS3 | TS4 | CS1 | CS2 | CS3 | CS4 |
| Type | F | F | F | F | F | F | F | F |
| Default | $10^{20}$ | $2 \times 10^{20}$ | $3 \times 10^{20}$ | $4 \times 10^{20}$ | TS1 | TS2 | TS3 | TS4 |

| VARIABLE | DESCRIPTION |
|---|---|
| TS1 – TS4 | Tensile plastic axial deformation/strain thresholds 1 to 4 |
| CS1 – CS4 | Compressive plastic axial deformation/strain thresholds 1 to 4 |

**Plastic Shear Strains Threshold Card.** Define Card 10 only if IFEMA > 2 (see Remarks 9 and 11).

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SS1 | SS2 | SS3 | SS4 | ST1 | ST2 | ST3 | ST4 |
| Type | F | F | F | F | F | F | F | F |
| Default | $10^{20}$ | $2 \times 10^{20}$ | $3 \times 10^{20}$ | $4 \times 10^{20}$ | SS1 | SS2 | SS3 | SS4 |

| VARIABLE | DESCRIPTION |
|---|---|
| SS1 – SS4 | Plastic shear strain thresholds 1 to 4 in the *s*-direction |
| ST1 – ST4 | Plastic shear strain thresholds 1 to 4 in the *t*-direction |

**Remarks:**

1. **Plastic hinge locations.** Two plastic hinges can be developed at user-specified locations. The default plastic hinge locations are at the ends of the beam element. See Figure M209-1.

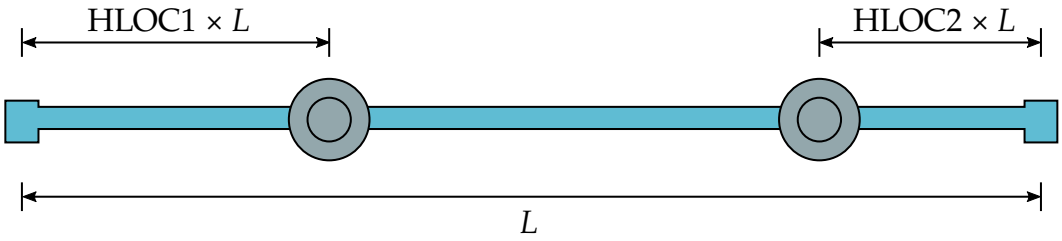



**Figure M209-1.** Plastic hinge locations for HLOC1 > 0.0 and HLOC2 > 0.0

2. **Yield surface**. Axial/moment interaction is defined according to the setting of ISURF on Card 1 (see also Remark 4).

   a) ISURF = 1 (default, simple power law):

$$\psi = \left|\frac{M_s - m_s}{M_{ys}}\right|^{\alpha} + \left|\frac{M_t - m_t}{M_{yt}}\right|^{\beta} + \left|\frac{F - f - F_0}{F_y - F_0}\right|^{\gamma} - 1$$

   b) ISURF = 2 (power law based on resultant moment):

$$\psi = \left[\left(\frac{M_s - m_s}{M_{ys}}\right)^2 + \left(\frac{M_t - m_t}{M_{yt}}\right)^2\right]^{\frac{\alpha}{2}} + \left|\frac{F - f - F_0}{F_y - F_0}\right|^{\gamma} - 1$$

c) ISURF = 3 (skew yield surface version of ISURF = 2; see Remark 5):

$$\psi = \left\{ \left[ \frac{(M_s - m_s) + \delta_s(F - f - F_0)}{(1 - \delta_s\kappa_s)M_{ys}} \right]^2 + \left[ \frac{(M_t - m_t) + \delta_t(F - f - F_0)}{(1 - \delta_t\kappa_t)M_{yt}} \right]^2 \right\}^{\frac{\alpha}{2}} + \left| \frac{(F - f - F_0) + \kappa_s(M_s - m_s) + \kappa_t(M_t - m_t)}{(1 - \delta_s\kappa_s - \delta_t\kappa_t)(F_y - F_0)} \right|^{\gamma} - 1$$

In the above equations,

| Variable | Definition |
|---|---|
| $M_s$ and $M_t$ | Moments about the $s$- and $t$-axes, respectively |
| $F$ | Axial force |
| $M_{ys}$ and $M_{yt}$ | Current yield moments |
| $F_y$ | Current axial yield force, where $F_y = \begin{cases} F_{yt} & \text{for } (F - f) \geq F_0 \\ -F_{yc} & \text{for } (F - f) < F_0 \end{cases}$ |
| $F_{yt}$ and $F_{yc}$ | Current tensile and compressive strengths, respectively |
| $m_s$, $m_t$ and $f$ | Current moments and forces that determine the center of the yield surface. They are closely related to the Bauschinger effect or kinematic hardening discussed below. |
| $\alpha$, $\beta$, and $\gamma$ | Input parameters ALPHA, BETA, and GAMMA on Card 3 which are real numbers ≥ 1.1 unless ALPHA and BETA are < 0 (see Remark 4) |
| $\delta_s$ and $\delta_t$ | Input parameters DELTAS and DELTAT (length units) on Card 4 for skew of yield surface in the local $s$- and $t$-directions, respectively |
| $\kappa_s$ and $\kappa_t$ | Input parameters KAPPAS and KAPPAT (1/length units) on Card 4 for skew of yield surface in the local $s$- and $t$-directions, respectively |
| $F_0$ | Input parameter F0 on Card 3 (see Remark 3) |

3. **Force offset**. The input parameter F0 offsets the yield surface parallel to the axial force axis. It is the axial force at which the maximum bending moment capacity occurs and is treated as tensile if F0 is positive, or compressive if F0 is negative. The same axial force offset F0 is used for both the local axes ($s$ and $t$). For steel components, the value of F0 is usually zero. For reinforced concrete components, F0 should be input as negative, corresponding to the compressive axial force at which the moment capacities are maximum. See Figure M209-2.

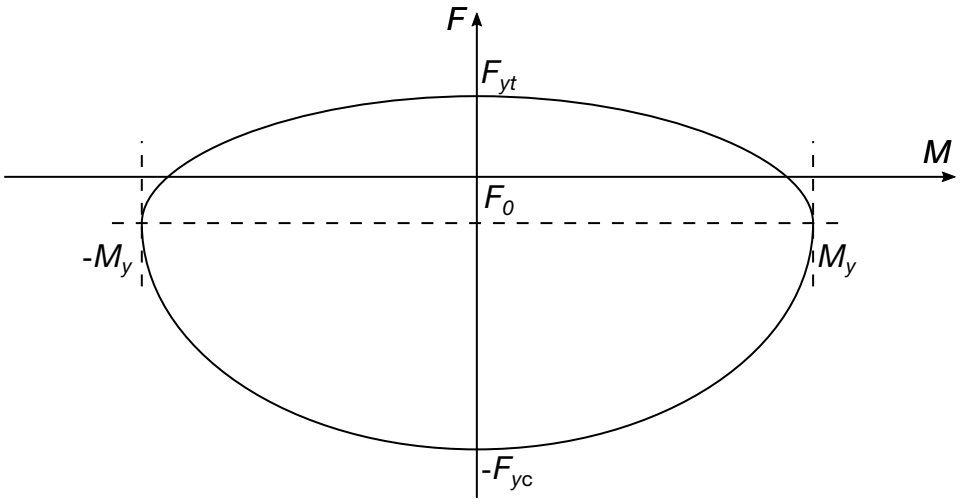



**Figure M209-2.** Effect of force offset on the yield surface

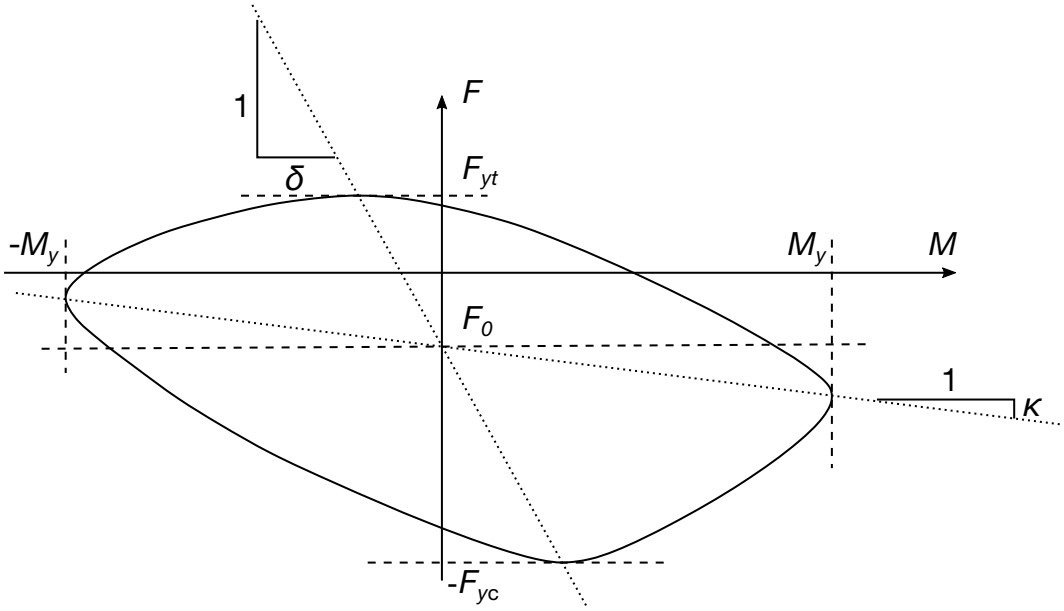



**Figure M209-3.** Example of a skew yield surface

4. **User-defined yield surface shape.** Optionally, you may provide curves defining the shape of the axial-moment yield surface. When ALPHA and BETA are less than 0.0 and GAMMA is equal to 0.0, the absolute values of ALPHA and BETA are the IDs of the load curves (see *DEFINE_CURVE) that define the yield loci in $M_s$- $F$ and $M_t$- $F$ planes. The program will automatically find the set of parameters ALPHA, BETA, GAMMA, SFS, SFT, SFAT, SFAC, F0, DELTAS, KAPPAS, DELTAT and KAPPAT that best fits the yield loci and the yield surface type ISURF.

5. **Skew yield surface.** Reinforced concrete sections with asymmetric reinforcement have a skew yield surface, meaning that the bending moment capacities at

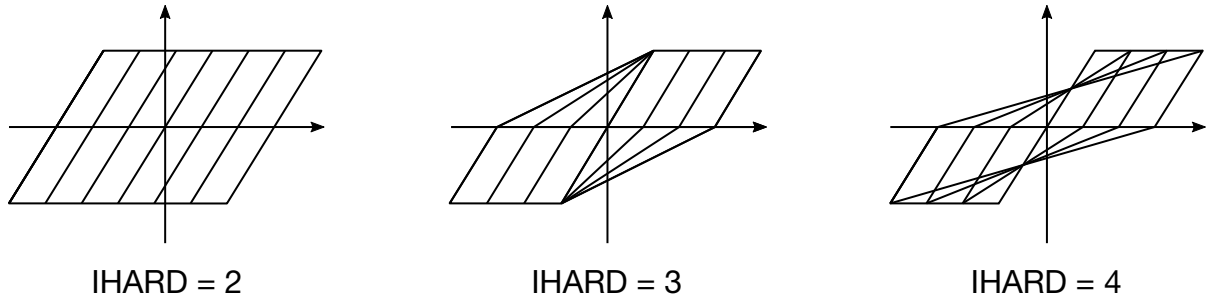



**Figure M209-4.** Example hardening curves

zero axial force are different in positive and negative bending, and the maximum axial capacity occurs at a nonzero bending moment. Furthermore, the axial load at which maximum biaxial bending moment occurs depends on the angle of the bending axis. This can be modelled with ISURF = 3, where DELTAS and DELTAT control the slope of the line connecting peak tensile and compressive strength vertices, and KAPPAS and KAPPAT control the slope of the line connecting peak moment vertices (which lies in the balance plane). See Figure M209-3.

6. **Hardening behavior during cyclic deformation (hysteresis).** The input parameter IHARD determines how the force as a function of deformation and moment as a function of rotation curves on Card 2 are applied during cyclic deformation. In this case, "deformation" includes both axial deformation and rotation at plastic hinges. If IHARD = 1, the abscissa represents cumulative absolute plastic deformation. This quantity is always positive. It increases whenever there is deformation in either direction. Thus, during hysteresis, the yield moments are taken from points in the input curve with increasingly positive deformation. If the curve shows a degrading behavior (reducing strength with deformation), then, once degraded by plastic deformation, the yield force or moment can never recover to its initial value. This option can be described as "fatigue-type" hysteretic behavior, where all plastic cycles contribute to the degradation. In the axial direction, plastic deformation is accumulated separately for tensile and compressive deformations.

   If IHARD = 2, 3 or 4, the abscissa represents the peak absolute value of the plastic deformation. This quantity increases only when the absolute value of plastic deformation exceeds the previously recorded maximum. This option can be described as "high-tide" hardening behavior and follows the FEMA approach. In particular, IHARD of 3 and 4 reproduce the yield-oriented and peak-oriented hysteresis, respectively, as shown in Figure M209-4.

7. **Pinching.** Pinched-shape hysteresis loops are seen in experiments on reinforced concrete members. They are caused by stiffness changes due to cracks opening and closing. This effect on the flexure response may be simulated using input parameter PINM. The default, PINM = 1.0, gives no pinching. The pinch points

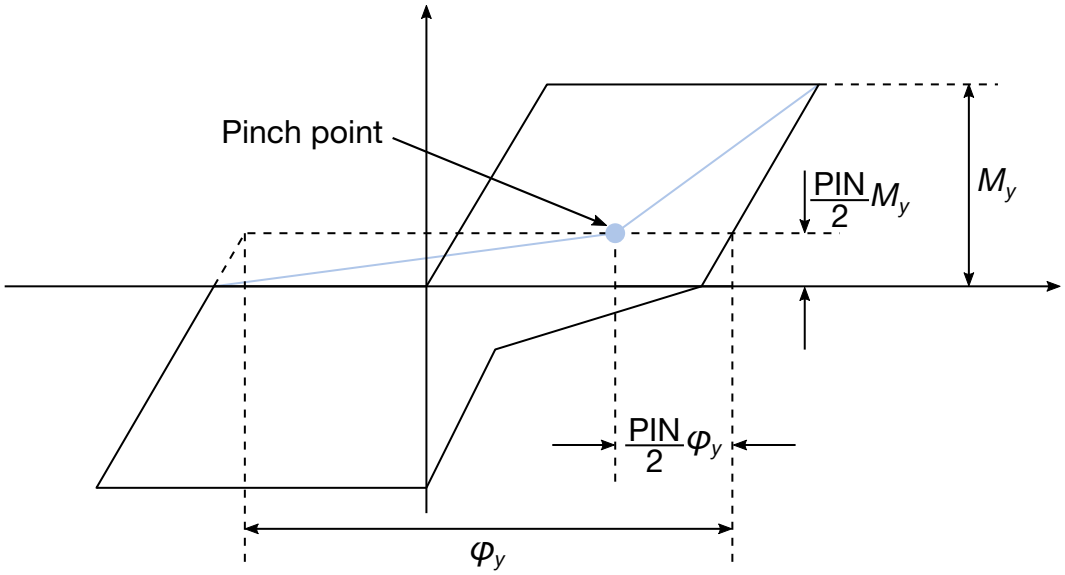



**Figure M209-5.** Example hysteresis curve with pinch point

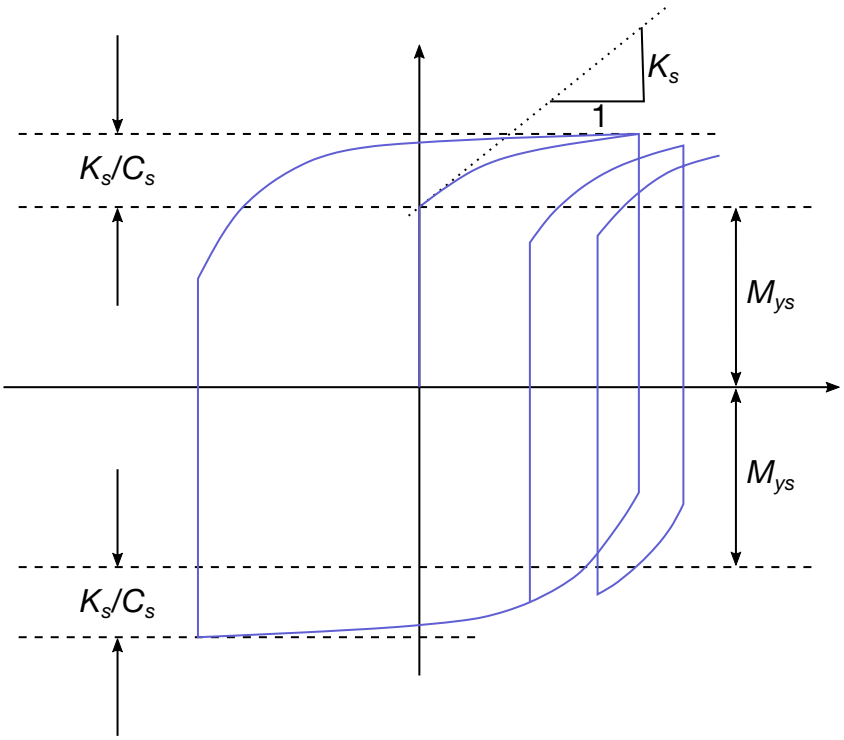



**Figure M209-6.** Example hysteresis curve with kinematic hardening for the moment about the $s$-axis. Here $K_s$ is HARDMS and $C_s$ is GAMMS.

are given by moments and rotations illustrated in the schematic below. Input parameter PINS has the same effect on shear hysteresis as PINM does on flexure hysteresis. See Figure M209-5.

8. **Kinematic hardening.** Kinematic hardening (Bauschinger effect, whereby an increase in tensile yield strength occurs at the expense of compressive yield

strength) is modelled by shift of the yield surface and is controlled by input parameters HARDxx and GAMxx, where xx is MS, MT, AT, and AC for moment about $s$-axis, moment about $t$-axis, axial tension and axial compression, respectively. HARDxx is the rate at which the yield surface shifts, in units of force/displacement or force/strain for axial response (depending on the setting of IAX) and in units of moment/rotation for flexure response. GAMxx is defined such that HARDxx/GAMxx is the maximum force or moment by which the yield surface can shift. See Figure M209-6.

9. **Degradation, damage, and element erosion.** Stiffness and strength degradation are modelled using a damage approach. The damaged fraction of the material does not contribute to the forces, the moments or the stiffness. Damage is calculated in two stages:

   a) a single damage parameter based on passing FEMA thresholds, and

   b) component-specific damage (where "component" means axial tension, axial compression, bending about s-axis and bending about t-axis).

   The force or moment for component $xx$, $F_{xx,\text{actual}}$, is calculated as:

$$F_{xx,\text{actual}} = (1 - D_{xx})(1 - D_{\text{FEMA}})F_{xx,\ \text{nominal}} \ .$$

   $xx$ can be AT for axial tension, AC for axial compression, MS for bending about the $s$-axis, MT for bending about the $t$-axis. Here, $F_{xx,\text{nominal}}$ is the force or moment calculated in the absence of damage for component $xx$. $D_{xx}$ is the component-specific damage fraction for component $xx$, and $D_{\text{FEMA}}$ is the damage calculated from passing FEMA thresholds (see Remark 11).

   The component-specific damage may be defined to be dependent on cumulative plastic deformation, on peak deformation, or a combination of both:

$$D_{xx}(t) = 1 - \left[1 - \omega_{1xx}\frac{\Delta_{xx,peak}}{\Delta_{xx,ult}}\right]^{\lambda_1}\left[1 - \omega_{2xx}\frac{\Delta_{xx,accum}}{\Delta_{xx,ult}}\right]^{\lambda_2} .$$

   Here $\omega_{1xx}$are $\omega_{2xx}$are the input parameters OMGxx1 and OMGxx2. $\Delta_{xx,\text{peak}}$ is the peak deformation (i.e., axial displacement, axial strain or rotation depending on $xx$ and IAX) that has occurred to date; $\Delta_{xx,\text{accum}}$ is the accumulated plastic deformation; and $\Delta_{xx,\text{ult}}$ is the input parameter DUAT, DUAC, RUMS or RUMT. $\lambda_1$ and $\lambda_2$ are the input parameters LAM1 and LAM2. Setting these to zero disables dependence on peak deformation and cumulative plastic deformation, respectively.

   The damage $D_{\text{FEMA}}$ is calculated using input parameters SOFT1 and SOFT2, taking the most damaged component including shear as well as axial and moment components. For example, if SOFT1 = 3 and SOFT2 = 4, and the most damaged

component has reached a FEMA index of 3.25 (meaning one quarter of the way from threshold 3 to threshold 4), then $D_{\text{FEMA}} = 0.25$. When the most damaged component reaches a FEMA index of 4.0, $D_{\text{FEMA}}$ reaches zero and the element will be deleted.

By default, SOFT1 = 3 and SOFT2 = 4. Thus, softening and element removal can occur even if the input parameters SOFT1 and SOFT2 have not been set by the user. This damage mechanism can be switched off by setting SOFT1 = 5. In that case, $D_{\text{FEMA}}$ is always zero irrespective of which thresholds are passed, and elements will not be deleted.

10. **Shear behavior.** Nonlinear shear behavior is controlled using input parameters LCSHS, LCSHT, SFSHS and SFSHT. By default, the shear yield surface is independent of the yield surface for axial and flexure and takes the following form:

$$\psi_s = \left(\frac{V_s}{V_{ys}}\right)^2 + \left(\frac{V_t}{V_{yt}}\right)^2 - 1 \ .$$

Here, $V_s$ and $V_t$ are the current shear forces in the local $s$- and $t$-directions. $V_{ys}$ and $V_{yt}$ are the current yield shear forces in the local $s$- and $t$-directions

The shear yield forces are functions of plastic shear strain (that is, shear angle). The plastic shear strain can be either peak or cumulative, depending on IHARD. Optionally, the shear strengths can be user-defined functions of the axial force; this is obtained by setting SFSHS and SFSHT to negative values.

Optionally, a different nonlinear shear behaviour can be defined for positive-direction and negative-direction shear forces. This is sometimes useful when modeling reinforced concrete beams equipped with inclined shear reinforcement that contribute to the shear capacity only for one direction of shear. This option is invoked by setting LCSHS on Card 4 to a negative value and by defining input fields on Card 4.1 for the negative shear behavior in the local $s$- and $t$-directions. At each time step, the shear yield condition is checked against the current positive or negative shear yield strengths, depending on the sign of the current shear forces in the two local directions. The negative shear strengths can be user-defined functions of the axial force by setting SFSHSN and SFSHTN to negative values.

11. **FEMA thresholds.** FEMA thresholds are used in performance-based earthquake engineering to classify the response according to the level of deformation. The thresholds are the divisions between regimes such as “Elastic”, “Immediate Occupancy”, “Life Safe”, etc. Output parameters indicate the status of each element with respect to these regimes. The thresholds are defined by input parameters PRS$n$, PRT$n$, TS$n$, CS$n$, SS$n$, ST$n$ where $n$ = 1, 2, 3, and 4 for the different regimes. PRS and PRT are plastic rotation about the $s$ and $t$ axes, TS and CS are tensile and compressive strain or deformation according to the setting of IAX,

and SS and ST are shear strains in the *s* and *t* directions. The corresponding output parameters, described as "FEMA flags" (see Remark 12) are "high-tide" values indicating which thresholds have been passed during the analysis, for example 3.25 means that the maximum deformation that has occurred to date exceeds threshold 3 and is one quarter of the way from threshold 3 to threshold 4. See also the influence of SOFT1 and SOFT2 described in Remark 9.

12. **Output.** In addition to the six resultants written for all beam elements, this material model writes a further 50 extra history variables to the d3plot and d3thdt files, given in the table below. The data is written in the same position in these files as where integrated beams write the stresses and strains at integration points requested by BEAMIP on *DATABASE_EXTENT_BINARY. Therefore, some post-processors may interpret this data as if the elements were integrated beams with 10 integration points, and in that case, the data may be accessed by selecting the appropriate integration point and data component:

| Integration point | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Extra (history) variable | 1 | 6 | 11 | 16 | 21 | 26 | 31 | 36 | 41 | 46 | XX(RR) axial stress |
| | 2 | 7 | 12 | 17 | 22 | 27 | 32 | 37 | 42 | 47 | XY(RS) shear stress |
| | 3 | 8 | 13 | 18 | 23 | 28 | 33 | 38 | 43 | 48 | ZX(TR) shear stress |
| | 4 | 9 | 14 | 19 | 24 | 29 | 34 | 39 | 44 | 49 | EPS |
| | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | XX(RR) axial strain |

For example, extra history variable 16 is located at the position that would normally be XX(RR) axial stress for integration point 4.

| Extra Variable | Description |
|---|---|
| 1 | Total axial deformation/strain |
| 2 | Hysteretic bending energy at plastic Hinge 1 |
| 3 | Hysteretic bending energy at plastic Hinge 2 |
| 4 | Plastic rotation about *s*-axis at Hinge 1 |
| 5 | Plastic rotation about *s*-axis at Hinge 2 |
| 6 | Plastic rotation about *t*-axis at Hinge 1 |
| 7 | Plastic rotation about *t*-axis at Hinge 2 |
| 8 | Bending moment about *s*-axis at node 1 |
| 9 | Bending moment about *s*-axis at node 2 |
| 10 | Bending moment about *t*-axis at node 1 |
| 11 | Bending moment about *t*-axis at node 2 |

| Extra Variable | Description |
|---|---|
| 12 | Hysteretic axial deformation energy |
| 13 | Internal energy |
| 14 | N/A |
| 15 | Axial plastic deformation |
| 16 | FEMA rotation flag |
| 17 | Current utilization |
| 18 | Peak utilization |
| 19 | FEMA shear flag |
| 20 | FEMA axial flag |
| 21 | Peak plastic tensile axial deformation/strain |
| 22 | Peak plastic compressive axial deformation/strain |
| 23 | Peak plastic rotation about *s*-axis at Hinge 1 |
| 24 | Peak plastic rotation about *s*-axis at Hinge 2 |
| 25 | Peak plastic rotation about *t*-axis at Hinge 1 |
| 26 | Peak plastic rotation about *t*-axis at Hinge 2 |
| 27 | Cumulative plastic tensile axial deformation/strain |
| 28 | Cumulative plastic compressive axial deformation/strain |
| 29 | Cumulative plastic rotation about *s*-axis at Hinge 1 |
| 30 | Cumulative plastic rotation about *s*-axis at Hinge 2 |
| 31 | Cumulative plastic rotation about *t*-axis at Hinge 1 |
| 32 | Cumulative plastic rotation about *t*-axis at Hinge 2 |
| 33 | Axial tensile damage |
| 34 | Axial compressive damage |
| 35 | Flexural damage about *s*-axis at Hinge 1 |
| 36 | Flexural damage about *s*-axis at Hinge 2 |
| 37 | Flexural damage about *t*-axis at Hinge 1 |
| 38 | Flexural damage about *t*-axis at Hinge 2 |
| 39 | Plastic shear strain in *s*-direction |
| 40 | Plastic shear strain in *t*-direction |
| 41 | Peak plastic shear strain in *s*-direction |
| 42 | Peak plastic shear strain in *t*-direction |

| Extra Variable | Description |
|---|---|
| 43 | Cumulative plastic shear strain in *s*-direction |
| 44 | Cumulative plastic shear strain in *t*-direction |
| 45 | Current axial-flexural utilization at Hinge 1 |
| 46 | Peak axial-flexural utilization at Hinge 1 |
| 47 | Current axial-flexural utilization at Hinge 2 |
| 48 | Peak axial-flexural utilization at Hinge 2 |
| 49 | Current shear utilization |
| 50 | Peak shear utilization |

## *MAT_SPR_JLR

This is Material Type 211. This material model was written for Self-Piercing Rivets (SPR) connecting aluminum sheets. Each SPR should be modeled by a single hexahedral (8-node solid) element, fixed to the sheet either by direct meshing or by tied contact. Pre- and post-processing methods are the same as for solid-element spot welds using *MAT_SPOTWELD. On *SECTION_SOLID, set ELFORM = 1.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HELAS | TELAS | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | 0.0 | 0.0 | | |

Cards 2 and 3 define the input for the "Head" end of the SPR

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCAXH | LCSHH | LCBMH | SFAXH | SFSHH | SFBMH | | |
| Type | I | I | I | F | F | F | | |
| Default | none | none | none | 1.0 | 1.0 | 1.0 | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DFAKH | DFSHH | RFBMH | DMFAXH | DMFSHH | DMFBMH | | |
| Type | F | F | F | F | F | F | | |
| Default | Rem 13 | Rem 13 | Rem 13 | 0.1 | 0.1 | 0.1 | | |

Cards 4 and 5 define the inputs for the "Tail" end of the SPR

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCAXT | LCSHT | LCBMT | SFAXT | SFSHT | SBFMT | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | 1 | 1 | 1 | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DFAXT | DFSHT | RFBMT | DFMAXT | DMFSHT | DMFBMT | | |
| Type | F | F | F | F | F | F | | |
| Default | Rem 13 | Rem 13 | Rem 13 | 0.1 | 0.1 | 0.1 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, used only for contact stiffness calculation. |
| PR | Poisson's ratio, used only for contact stiffness calculation. |
| HELAS | SPR head end behavior flag:<br>EQ.0.0: Nonlinear<br>EQ.1.0: Elastic (use first two points of the load curves). |
| TELAS | SPR tail end behavior flag:<br>EQ.0.0: Nonlinear<br>EQ.1.0: Elastic (use the first two points of the load curves). |
| LCAXH | Load curve ID (see *DEFINE_CURVE) giving axial force as a function of deformation (head) |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSHH | Load curve ID (see *DEFINE_CURVE) giving shear force as a function of deformation (head) |
| LCBMH | Load curve ID, see *DEFINE_CURVE, giving moment as a function of rotation (head) |
| SFAXH | Scale factor on axial force from curve LCAXH |
| SFSHH | Scale factor on shear force from curve LCSHH |
| SFBMH | Scale factor on bending moment from curve LCBMH |
| DFAXH | Optional displacement to start of softening in axial load (head) |
| DFSHH | Optional displacement to start of softening in shear load (head) |
| RFBMH | Optional rotation (radians) to start of bending moment softening (head) |
| DMFAXH | Scale factor on DFAXH |
| DMFSHH | Scale factor on FFSHH |
| DMFBMH | Scale factor on RFBMH |
| LCAXT | Load curve ID (see *DEFINE_CURVE) giving axial force as a function of deformation (tail) |
| LCSHT | Load curve ID (see *DEFINE_CURVE) giving shear force as a function of deformation (tail) |
| LCBMT | Load curve ID (see *DEFINE_CURVE) giving moment as a function of rotation (tail) |
| SFAXT | Scale factor on axial force from curve LCAXT |
| SFSHT | Scale factor on shear force from curve LCSHT |
| SFBMT | Scale factor on bending moment from curve LCBMT |
| DFAXT | Optional displacement to start of softening in axial load (tail) |
| DFSHT | Optional displacement to start of softening in shear load (tail) |
| RFBMT | Optional rotation (radians) to start of bending moment softening (tail) |

| VARIABLE | DESCRIPTION |
|---|---|
| DMFAXT | Scale factor on DFAXT |
| DMFSHT | Scale factor on FFSHT |
| DMFBMT | Scale factor on RFBMT |

**Remarks:**

1. **SPR geometry.** "Head" is the end of the SPR that fully perforates a sheet. "Tail" is the end that is embedded within the thickness of a sheet.

   The sheet planes are defined at the head by the quadrilateral defined by nodes N1-N2-N3-N4 of the solid element; and at the tail by the quadrilateral defined by nodes N5-N6-N7-N8. It is essential that the nodes N1 to N4 are fixed to the head sheet (e.g. by direct meshing or tied contact): the element has no stiffness to resist relative motion of nodes N1 to N4 in the plane of the head sheet. Similarly, nodes N5 to N8 must be fixed to the tail sheet

   The tail of the SPR is defined as a point in the tail sheet plane, initially at the center of the element face. The head of the SPR is initially at the center of the head sheet plane. The SPR axis is defined as the line joining the tail to the head. Thus, the axis of the SPR would typically be coincident with the solid element local $z$-axis if the solid is a cuboid. It is the user's responsibility to ensure that each solid element is oriented correctly.

   During the analysis, the head and tail will always remain in the plane of the sheet but may move away from the centers of the sheet planes if the shear forces in these planes are sufficient.

2. **Young's modulus and Poisson's ratio.** E and PR are used only to calculate contact stiffness. They are not used by the material model.

3. **Axes.** Deformation is in length units and is on the $x$-axis. Force is on the $y$-axis. Rotation is in radians on the $x$-axis. Moment is on the $y$-axis.

4. **Load curve assumptions.** All the load curves are expected to start at (0,0). "Deformation" means the total deformation including both elastic and plastic components, similarly for rotation.

5. **"High tide" algorithm.** A "high tide" algorithm is used to determine the deformation or rotation to be used as the $x$-axis of the load curves when looking up the current yield force or moment. The "high tide" is the greatest displacement or rotation that has occurred so far during the analysis.

6. **Elastic stiffness.** The first two points of the load curve define the elastic stiffness, which is used for unloading.

7. **HELAS and TELAS.** If HELAS > 0, the remainder of the head load curves after the first two points is ignored and no softening or failure occurs. The same applies for TELAS and the tail load curves.

8. **Deformation and rotation of the SPR.** Axial deformation is defined as change of length of the line between the tail and head of the SPR. This line also defines the direction in which the axial force is applied.

   Shear deformation is defined as motion of the tail and head points, in the sheet planes. This deformation is not necessarily perpendicular to axial deformation. Shear forces in these planes are controlled by the load curves LCSHT and LCSHH.

   Rotation at the tail is defined as rotation of the tail-to-head line relative to the normal of the tail sheet plane; and for the head, relative to the normal of the head sheet plane.

9. **Element formulation.** Although ELFORM = 1 is used in the input data, *MAT_SPR_JLR is really a separate unique element formulation. The usual stress/force and hourglass calculations are bypassed, and deformations and nodal forces are calculated by a method unique to *MAT_SPR_JLR; for example, a single *MAT_SPR_JLR element can carry bending loads.

10. **Hourglass.** *HOURGLASS inputs are irrelevant to *MAT_SPR_JLR.

11. **SWFORC file.** Output to the swforc file works in the same way as for spotwelds. Although inside the material model the load curves LCSHT and LCSHH control "shear" forces in the sheet planes, in the swforc file the quoted shear force is the force normal to the axis of the SPR.

12. **Softening.** Before an element fails, it enters a "softening" regime in which the forces, moments and stiffnesses are ramped down as displacement increases (this avoids sudden shocks when the element is deleted). For example, for axial loading at the head, softening begins when the maximum axial displacement exceeds DFAXH. As the displacement increases beyond that point, the load curve will be ignored for that deformation component. The forces, moments and stiffnesses are ramped down linearly with increasing displacement and reach zero at displacement = DFAXH × (1 + DMFAXH) when the element is deleted. The softening factor scales all the force and moment components at both head and tail. Thus, all the force and moment components are reduced when any one displacement component enters the softening regime. For example, if DFAXT = 3.0mm, and DMFAXT = 0.1, then softening begins when axial displacement of the tail reaches 3.0 mm and final failure occurs at 3.3 mm.

13. **Initial softening displacements/rotations.** If the inputs DFAXH, DFSHH, RFBMH, DFAXT, DFSHT, and RFBMT are non-zero, these values must be within the abscissa values of the relevant curve, such that the curve force/moment value is greater than zero at the defined start of softening.

    If the inputs are left blank or zero, they will be calculated internally as follows:

    a) Final failure will occur at the displacement or rotation (DFAIL) at which the load curve reaches zero (determined, if necessary, by extrapolation from the last two points).

    b) Displacement or rotation at which softening begins is then back-calculated. For example, DFAXT = DFAIL/(1 + DMGAXT).

    c) If DMGAXT is left blank or zero, it defaults to 0.1.

    d) If the load curve does not drop to zero, and the final two points have a zero or positive gradient, no failure or softening will be caused by that displacement component.

14. **Output stress.** Output stresses (in the d3plot and time-history output files) are set to zero.

15. **Displacement ratio.** The output variable "displacement ratio" (or rotation ratio for bending), *R*, is defined as follows. See also the Figure M211-1.

    a) *R = 0.0 to 1.0*. The maximum force or moment on the input curve has not yet been reached. *R* is proportional to the maximum force or moment reached so far, with 1.0 being the point of maximum force or moment on the input curve.

    b) *R = 1.0 to 2.0*. The element has passed the point of maximum force but has not yet entered the softening regime. *R* rises linearly with displacement (or rotation) from 1.0 when maximum force occurs to 2.0 when softening begins.

    c) *R = 2.0 to 3.0*. Softening is occurring. *R* rises linearly with displacement from 2.0 at the onset of softening to 3.0 when the element is deleted.

    The displacement (or rotation) ratio is calculated separately for axial, shear, and bending at the tail and head (see Remark 16 below). The output listed by post-processors as "plastic strain" is actually the maximum displacement or rotation ratio of any displacement or rotation component at head or tail. This same variable is also output as "Failure" in the spotweld data in the swforc file (or the swforc section of the binout file).

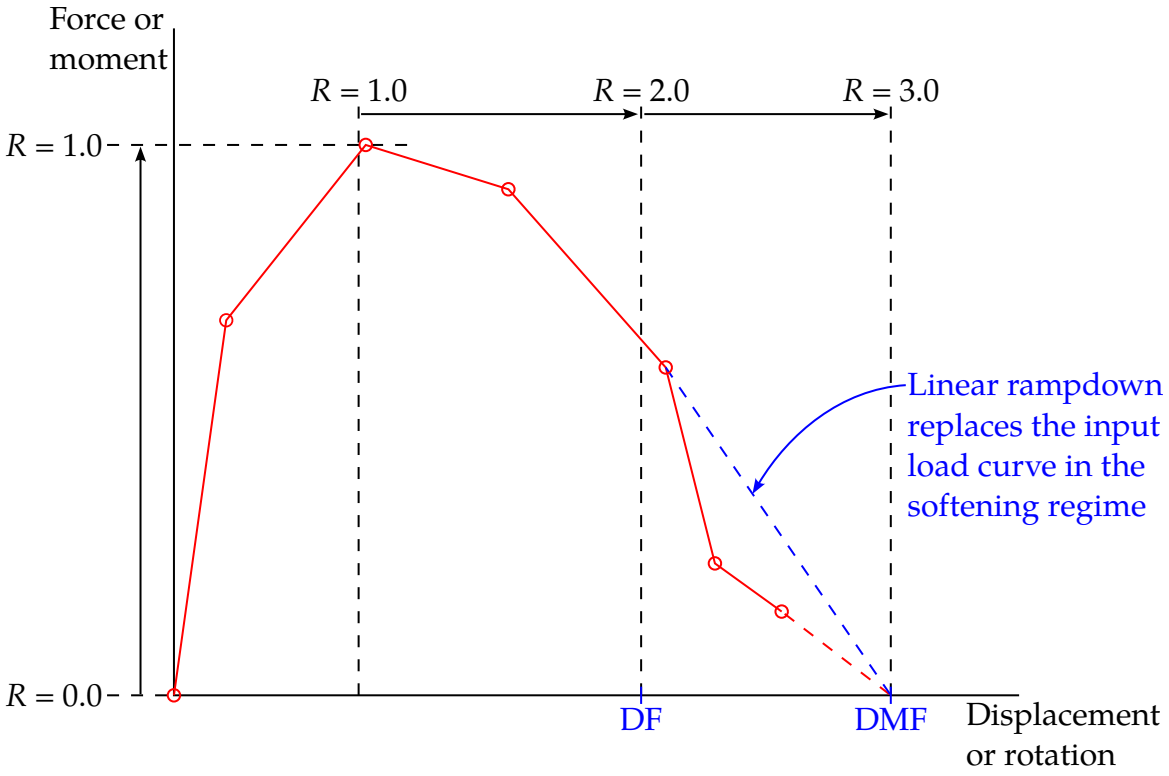



**Figure M211-1.** Output variable "displacement ratio" (or rotation ratio for bending)

16. **Additional history variables.** The additional history variables are listed in the table below.

| History Variable # | Description |
|---|---|
| 1 | Failure time (used for swforc file) |
| 2 | Softening factor used internally to prevent abrupt failure. |
| 3 | Displacement ratio – axial, head |
| 4 | Displacement ratio – axial, tail |
| 5 | Displacement ratio – shear, head |
| 6 | Displacement ratio – shear, tail |
| 7 | Rotation ratio – bending, head |
| 8 | Rotation ratio – bending, tail |
| 9 | (Not meaningful for users) |
| 10 | Shear force in "beam" $x$-axis |
| 11 | Shear force in "beam" $y$-axis |
| 12 | Axial force in "beam" $z$-axis (along "beam") |

| History Variable # | Description |
|---|---|
| 13 | Moment about "beam" $x$-axis at head |
| 14 | Moment about "beam" $y$-axis at head |
| 15 | Moment about "beam" $z$-axis at head (torsion – should be zero) |
| 16 | "Beam" length |
| 17 | Moment about "beam" $x$-axis at tail |
| 18 | Moment about "beam" $y$-axis at tail |
| 19 | Moment about "beam" $z$-axis at tail (torsion – should be zero) |
| 20 | Isoparametric coordinate of head of "beam" ($s$) |
| 21 | Isoparamteric coordinate of head of "beam" ($t$) |
| 22 | Isoparametric coordinate of tail of "beam" ($s$) |
| 23 | Isoparametric coordinate of tail of "beam" ($t$) |
| 24 | Timestep |
| 25 | Plastic displacement – axial, head |
| 26 | Plastic displacement – axial, tail |
| 27 | Plastic rotation – head |
| 28 | Plastic rotation – tail |
| 29 | Plastic displacement – shear in sheet axes, head |
| 30 | Plastic displacement – shear in sheet axes, tail |
| 31 | Global $x$-component of the "beam" $x$-axis |
| 32 | Global $y$-component of the "beam" $x$-axis |
| 33 | Global $z$-component of the "beam" $x$-axis |
| 34 | Shear displacement – local $x$-axis, head |
| 35 | Shear displacement – local $y$-axis, head |
| 36 | Shear displacement – local $x$-axis, tail |
| 37 | Shear displacement – local $y$-axis, tail |
| 38 | Total displacement – axial |
| 39 | Current rotation (radians) – head, local $x$-axis |
| 40 | Current rotation (radians) – head, local $y$-axis |
| 41 | Current rotation (radians) – tail, local $x$-axis |

| History Variable # | Description |
|---|---|
| 42 | Current rotation (radians) – tail, local $y$-axis |

## *MAT_COMPOSITE_TABULATED_PLASTICITY_DAMAGE

(V1.3.8) This is Material Type 213, an orthotropic, visco-elastic-plastic material with temperature and rate dependencies. It has a modular architecture supporting viscoelastic deformations, viscoplastic deformations [1-4, 9, 12], damage [6, 7], failure [8, 15] and probabilistic analysis [5]. It is available for solid, thick shell, and shell elements. Thick shell elements with ELFORM = 1, 2, or 6 follow the shell input format, while those with ELFORM = 3, 5, or 7 follow the solid element input format. For thick shells, we recommend using ELFORM = 1 or 5. In this section, *shell element* refers to both shell elements and ELFORM = 1 thick shell elements. Similarly, *solid element* refers to both solid elements and ELFORM = 5 thick shell elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | PTOL | AOPT | MACF | FILT | VEVP |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | TCSYM |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| H11 | H22 | H33 | H12 | H23 | H13 | H44 | H55 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| H66 | LT1 | LT2 | LT3 | LT4 | LT5 | LT6 | LT7 |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| LT8 | LT9 | LT10 | LT11 | LT12 | YSC | DFLAG | DC |
|---|---|---|---|---|---|---|---|

**Card 8a.1.** Include this card if FTYPE = 0.

| FTYPE | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 8a.2.** Include this card as a blank line if FTYPE = 0.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Card 8b.1.** Include this card if FTYPE = 1 (Puck Failure Criterion).

| FTYPE | FV0 | FV1 | FV2 | FV3 | FV4 | FV5 | FV6 |
|---|---|---|---|---|---|---|---|

**Card 8b.2.** Include this card if FTYPE = 1 (Puck Failure Criterion).

| FV7 | FV8 | FV9 | FV10 | FV11 | FV12 | FV13 | FV14 |
|---|---|---|---|---|---|---|---|

**Card 8c.1.** Include this card if FTYPE = 2 (Tsai-Wu Failure Criterion).

| FTYPE | | FV1 | FV2 | FV3 | FV4 | FV5 | FV6 |
|---|---|---|---|---|---|---|---|

**Card 8c.2.** Include this card if FTYPE = 2. (Tsai-Wu Failure Criterion).

| FV7 | FV8 | FV9 | FV10 | FV11 | FV12 | FV13 | FV14 |
|---|---|---|---|---|---|---|---|

**Card 8d.1.** Include this card if FTYPE = 3 (Generalized Tabulated Failure Criterion).

| FTYPE | | FV1 | FV2 | FV3 | | | |
|---|---|---|---|---|---|---|---|

**Card 8d.2.** Include this card as a blank line if FTYPE = 3 (Generalized Tabulated Failure Criterion).

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Card 8e.1.** Include this card if FTYPE = 4 (Point Cloud Failure Criterion).

| FTYPE | FV0 | FV1 | FV2 | FV3 | FV4 | FV5 | FV6 |
|---|---|---|---|---|---|---|---|

**Card 8e.2.** Include this card if FTYPE = 4 (Point Cloud Failure Criterion).

| FV7 | FV8 | FV9 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 9.** BETA values only need to be specified when VEVP = 1 or 2.

| BETA11 | BETA22 | BETA33 | BETA44 | BETA55 | BETA66 | BETA12 | BETA23 |
|---|---|---|---|---|---|---|---|

**Card 10.** BETA values only need to be specified when VEVP = 1 or 2.

| BETA13 | CP | TQC | TEMP | PMACC | SLM | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in the $a$-direction |
| EB | $E_b$, Young's modulus in the $b$-direction |
| EC | $E_c$, Young's modulus in the $c$-direction |
| PRBA | $\nu_{ba}$, (elastic) Poisson's ratio, $ba$ (see Remark 9) |
| PRCA | $\nu_{ca}$, (elastic) Poisson's ratio, $ca$ (see Remark 9) |
| PRCB | $\nu_{cb}$, (elastic) Poisson's ratio, $cb$ (see Remark 9) |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | PTOL | AOPT | MACF | FILT | VEVP |
| Type | F | F | F | F | F | I | F | I |
| Default | none | none | none | $10^{-6}$ | 0.0 | 0 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus $ab$-plane |
| GBC | $G_{bc}$, shear modulus $bc$-plane |

| VARIABLE | DESCRIPTION |
|---|---|
| GCA | $G_{ca}$, shear modulus *ca*-plane |
| PTOL | Yield function tolerance used during plastic multiplier calculations |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, *P*, defining the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE-_COORDINATE_VECTOR). |
| MACF | Material axes change flag for solid elements: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-4: Switch material axes $b$ and $c$ before BETA rotation<br>EQ.-3: Switch material axes $a$ and $c$ before BETA rotation<br>EQ.-2: Switch material axes $a$ and $b$ before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes $a$ and $b$ after BETA rotation<br>EQ.3: Switch material axes $a$ and $c$ after BETA rotation<br>EQ.4: Switch material axes $b$ and $c$ after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| FILT | Factor for strain rate filtering (optional):<br>$\dot{\varepsilon}_{i+1}^{\text{avg}} = (1 - \text{FILT}) \times \dot{\varepsilon}_{i+1}^{\text{cur}} + \text{FILT} \times \dot{\varepsilon}_{i}^{\text{avg}}$<br>where $i$ is the previous time step. The value of FILT is between 0 and 1. |
| VEVP | Flag to control viscoelastic, viscoplastic behavior:<br>EQ.0: Viscoplastic only with no rate effects in elastic region (default)<br>EQ.1: Viscoelastic, viscoplastic (see Cards 9 and 10)<br>EQ.2: Viscoelastic only (see Cards 9 and 10) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | TCSYM |
| Type | F | F | F | F | F | F | F | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Angle in degrees of a material rotation about the c-axis, available for AOPT = 0 (shells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. |
| TCSYM | Flag for handling tension-compression asymmetry in all three material directions:<br>EQ.0: Do not adjust user-defined data.<br>EQ.1: Compute and use the average tension and compression elastic moduli in adjusting the stress-strain curve. See Remark 7.<br>EQ.2: Use compression modulus as user-defined tension modulus in adjusting the stress-strain curve. See Remark 7.<br>EQ.3: Use tension modulus as user-defined compression modulus in adjusting the stress-strain curve. See Remark 7.<br>EQ.4: Use user-defined tensile curve as the compressive curve overriding the user-defined compressive curve. This implies that the normal stress-strain curves are symmetric including yield values.<br>EQ.5: Use user-defined compressive curve as the tensile curve overriding the user-defined tensile curve. This implies |

| VARIABLE | DESCRIPTION |
|---|---|
| | that the normal stress-strain curves are symmetric including yield values. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | H11 | H22 | H33 | H12 | H23 | H13 | H44 | H55 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | 3.0 | 3.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| H*ij* | Plastic flow rule coefficients. See Remark 1. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | H66 | LT1 | LT2 | LT3 | LT4 | LT5 | LT6 | LT7 |
| Type | F | I | I | I | I | I | I | I |
| Default | 3.0 | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| H*ij* | Plastic flow rule coefficients. See Remark 1. |
| LT1 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *a*-direction tension test. See Remarks 2, 8 and 9. |
| LT2 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *b*-direction tension test. See Remarks 2, 8 and 9. |
| LT3 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *c*-direction tension test. See Remarks 2, 8 and 9. Not required if used with shell elements. |

| VARIABLE | DESCRIPTION |
|---|---|
| LT4 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *a*-direction compression test. See Remarks 2, 8 and 9. |
| LT5 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *b*-direction compression test. See Remarks 2, 8 and 9. |
| LT6 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *c*-direction compression test. See Remarks 2, 8 and 9. Not required if used with shell elements. |
| LT7 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *ab*-plane shear test. See Remarks 2, 8 and 9. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LT8 | LT9 | LT10 | LT11 | LT12 | YSC | DFLAG | DC |
| Type | I | I | I | I | I | I | F | I |
| Default | none | none | none | none | none | none | 0.0 | none |

| VARIABLE | DESCRIPTION |
|---|---|
| LT8 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *bc*-plane shear test. See Remarks 2, 8 and 9. Not required if used with shell elements. |
| LT9 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the *ac*-plane shear test. See Remarks 2, 8 and 9. Not required if used with shell elements. |
| LT10 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the 45° off axis *ab*-plane tension or compression test. See Remarks 2, 8 and 9. Optional for all element types. |

| VARIABLE | DESCRIPTION |
|---|---|
| LT11 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the 45° off axis *bc*-plane tension or compression test. See Remarks 2, 8 and 9. Not required if used with shell elements. Optional for solid elements. |
| LT12 | Table ID for a three-dimensional table (see *DEFINE_TABLE_3D) containing temperature and stress-strain input curves for the 45° off axis *ac*-plane tension or compression test. See Remarks 2, 8 and 9. Not required if used with shell elements. Optional for solid elements. |
| YSC | Load curve ID containing stress-strain curve IDs and associated initial yield strain values. See Remark 3. |
| DFLAG | Damage formulation flag (see Remark 12):<br>EQ.0: Based on effective stress (default)<br>EQ.1: Based on corrected plastic strain |
| DC | Curve ID that specifies which components of the damage model are active. It contains the damage parameter ID and the corresponding damage as a function of total strain curve ID or Table3D ID. Set this value to zero if damage should not be included in the analysis. See Remark 4. |

**No Failure Card.** The following two cards are included if FTYPE = 0. Card 8a.2 must be included as a blank line

| Card 8a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | | | | | | | |
| Type | I | | | | | | | |

| Card 8a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FTYPE | Failure criterion type (see Remarks 5 and 6):<br>EQ.0: No failure considered (default)<br>EQ.1: Puck Failure Criterion (PFC) (solid elements only)<br>EQ.2: Tsai-Wu Failure Criterion (TWFC)<br>EQ.3: Generalized Tabulated Failure Criterion (GTFC)<br>EQ.4: Point Cloud Failure Criterion (PCFC) (shell elements only) |

**PFC Card.** The following two cards are included if FTYPE = 1.

| Card 8b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | FV0 | FV1 | FV2 | FV3 | FV4 | FV5 | FV6 |
| Type | I | F | F | F | F | F | F | F |

| Card 8b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FV7 | FV8 | FV9 | FV10 | FV11 | FV12 | FV13 | FV14 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| FTYPE | Use a value of 1 for PFC (solid elements only). |
| FV0 | $\Gamma_f$, fiber mode fracture energy ($a$-direction) |
| FV1 | Post-peak residual damage in the $a$-direction for tension. Value must be a real number between 0 and 1. |
| FV2 | Post-peak residual damage in the $a$-direction for compression. Value must be a real number between 0 and 1. |
| FV3 | Post-peak residual damage in the $b$ and $c$-directions for tension. Value must be a real number between 0 and 1. |
| FV4 | Post-peak residual damage in the $b$ and $c$-directions for compression. Value must be a real number between 0 and 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| FV5 | Post-peak residual damage in shear. Value must be a real number between 0 and 1. |
| FV6 | magnification factor, $m_f$ |
| FV7 | Slope parameter, $p_{ba}^t$ |
| FV8 | Slope parameter, $p_{ba}^c$ |
| FV9 | Slope parameter, $p_{bb}^t$ |
| FV10 | Slope parameter, $p_{bb}^c$ |
| FV11 | Fiber Poisson's ratio, $\nu_{ab}^f$ |
| FV12 | Fiber Young's modulus, $E_a^f$ |
| FV13 | Inter-fiber mode I fracture energy, $\Gamma_1$ |
| FV14 | Inter-fiber mode II fracture energy, $\Gamma_2$ |

**TWFC Card.** The following two cards are included if FTYPE = 2.

| Card 8c.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | | FV1 | FV2 | FV3 | FV4 | FV5 | FV6 |
| Type | I | | F | F | F | F | F | F |

| Card 8c.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FV7 | FV8 | FV9 | FV10 | FV11 | FV12 | FV13 | FV14 |
| Type | F | F | F | F | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| FTYPE | Use a value of 2 for TWFC. |
| FV1 | $\hat{\sigma}_{aa}^T$, tensile failure stress in the $a$-direction |

| VARIABLE | DESCRIPTION |
|---|---|
| FV2 | $\hat{\sigma}_{aa}^{C}$, compressive failure stress in the *a*-direction (input as a positive value) |
| FV3 | $\hat{\sigma}_{bb}^{T}$, tensile failure stress in the *b*-direction |
| FV4 | $\hat{\sigma}_{bb}^{C}$, compressive failure stress in the *b*-direction (input as a positive value) |
| FV5 | $\hat{\sigma}_{cc}^{T}$, tensile failure stress in the *c*-direction |
| FV6 | $\hat{\sigma}_{cc}^{C}$, compressive failure stress in the *c*-direction (input as a positive value) |
| FV7 | $\hat{\sigma}_{ab}$, shear failure stress in the *ab*-plane |
| FV8 | $\hat{\sigma}_{bc}$, shear failure stress in the *bc*-plane |
| FV9 | $\hat{\sigma}_{ac}$, shear failure stress in the *ac*-plane |
| FV10 | $\hat{\sigma}_{ab}^{45}$, failure stress in the 45° off axis *ab*-plane |
| FV11 | $\hat{\sigma}_{bc}^{45}$, failure stress in the 45° off axis *bc*-plane |
| FV12 | $\hat{\sigma}_{ac}^{45}$, failure stress in the 45° off axis *ac*-plane |
| FV13 | Optional curve ID that defines orientation-dependent erosion strain for all nine stress strain curves (3 tension, 3 compression, and 3 shear). The ordinate values in the load curve define the various erosion strains in the following order: |

| Ordinate Value Descriptions |
|---|
| Tensile erosion strain in the *a*-direction, $\varepsilon_{aaT}$ |
| Compressive erosion strain in the *a*-direction, $\varepsilon_{aaC}$ |
| Tensile erosion strain in the *b*-direction, $\varepsilon_{bbT}$ |
| Compressive erosion strain in the *b*-direction, $\varepsilon_{bbC}$ |
| Tensile erosion strain in the *c*-direction, $\varepsilon_{ccT}$ |
| Compressive erosion strain in the *c*-direction, $\varepsilon_{ccC}$ |
| In-plane shear erosion strain in the *ab*-plane, $\varepsilon_{ab}$ |
| Out-of-plane shear erosion strain in the *bc*-plane, $\varepsilon_{bc}$ |
| Out-of-plane shear erosion strain in the *ac*-plane, $\varepsilon_{bc}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| FV14 | Optional curve ID that defines orientation-dependent post-peak residual strength (PPRD) for all nine stress strain curves (3 tension, 3 compression, and 3 shear). The ordinate values in the load curve define the various residual strengths in the following order: |

| Ordinate Value Descriptions |
|---|
| Tensile post-peak residual strength in the *a*-direction, $PPRD_{aaT}$ |
| Compressive post-peak residual strength in the *a*-direction, $PPRD_{aaC}$ |
| Tensile post-peak residual strength in the *b*-direction, $PPRD_{bbT}$ |
| Compressive post-peak residual strength in the *b*-direction, $PPRD_{bbC}$ |
| Tensile post-peak residual strength in the *c*-direction, $PPRD_{ccT}$ |
| Compressive post-peak residual strength in the *c*-direction, $PPRD_{ddC}$ |
| In-plane shear post-peak residual strength in the *ab*-plane, $PPRD_{ab}$ |
| Out-of-plane shear post-peak residual strength in the *bc*-plane, $PPRD_{bc}$ |
| Out-of-plane shear post-peak residual strength in the *ac*-plane, $PPRD_{ac}$ |

**GTFC Card.** The following two cards are included if FTYPE = 3. Card 8d.2 must be included as a blank line.

| Card 8d.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | | FV1 | FV2 | FV3 | | | |
| Type | I | | F | F | F | | | |

| Card 8d.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FTYPE | Use a value of 3 for GTFC. |
| FV1 | In-plane and out-of-plane interaction term, $n$, used to compute $d$: $$d = \begin{cases} \max(d_1, d_2) & \text{if } n = 0.0 \\ (d_1^n + d_2^n)^{1/n} & \text{otherwise} \end{cases}$$ where, $d_i = \varepsilon^{\text{eq}} / \varepsilon_{\text{fail}}$, $i = 1, 2$. $i = 1$ corresponds to the in-plane mode, while $i = 2$ corresponds to the out-of-plane mode. Here, $$\varepsilon^{\text{eq}} = \begin{cases} \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + 2\varepsilon_{12}^2} & \text{for in-plane} \\ \sqrt{\varepsilon_{33}^2 + 2\varepsilon_{13}^2 + 2\varepsilon_{23}^2} & \text{for out-of-plane} \end{cases}$$ An element is eroded if $d$ reaches a value of 1.0 for solid elements. For shell elements, an element is eroded if $d_1$ reaches a value of 1.0 since only the in-plane mode of failure is considered. $n$ is not required for shell elements. |
| FV2 | Table ID for the table containing in-plane $(\theta_{\text{IP}}, \varepsilon_{\text{fail}}^{\text{eq}})$ values with respect to the specified $a$-direction stress. Note, $$\theta_{\text{IP}} = \cos^{-1}\left(\frac{\sigma_{22}}{\sqrt{\sigma_{22}^2 + \sigma_{12}^2}}\right)$$ |
| FV3 | Table ID for the table containing out-of-plane $(\theta_{\text{OOP}}, \varepsilon_{\text{fail}}^{\text{eq}})$ values with respect to the specified normal $c$-direction stress. Note, $$\theta_{OOP} = \cos^{-1}\left(\frac{\sigma_{13}}{\sqrt{\sigma_{13}^2 + \sigma_{23}^2}}\right)$$ |

**PCFC Card.** The following two cards are included if FTYPE = 4. Card 8e.2 must be included as a blank line if the field FV7, FV8, and FV9 are unused.

| Card 8e.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | FV0 | FV1 | FV2 | FV3 | FV4 | FV5 | FV6 |
| Type | I | I | I | F | F | I | I | I |

| Card 8e.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FV7 | FV8 | FV9 | | | | | |
| Type | I | I | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FTYPE | Use a value of 4 for PCFC (shell elements only). |
| FV0 | Flag to define PCFC methods:<br>EQ.0: Simplified Approximate Nearest Neighbor (SANN) (default)<br>EQ.1: Neural network (NN) |
| FV1 | Number of neighborhood points ($k$) for the SANN method (FV0 = 0) |
| FV2 | Meaning depends on FV0:<br>FV0.EQ.0: $\alpha$ (see Remark 5)<br>FV0.EQ.1: $\beta$ (see Remark 5) |
| FV3 | Equivalent erosion strain, $\varepsilon_{\text{IP}}^{f}$. An element is eroded if $\varepsilon^{\text{eq}} = \sqrt{\varepsilon_{11}^2 + \varepsilon_{22}^2 + 2\varepsilon_{12}^2} > \varepsilon_{\text{IP}}^{f}$. |
| FV4 | Curve ID that defines orientation-dependent strain for 5 stress-strain curves (2 tension, 2 compression, and 1 shear). The ordinate values in the load curve define the various strains in the following order: |

| Ordinate Value Descriptions |
|---|
| Tensile erosion strain in the $a$-direction, $\varepsilon_{aaT}$ |
| Compressive erosion strain in the $a$-direction, $\varepsilon_{aaC}$ |
| Tensile erosion strain in the $b$-direction, $\varepsilon_{bbT}$ |
| Compressive erosion strain in the $b$-direction, $\varepsilon_{bbC}$ |
| In-plane shear erosion strain in the $ab$-plane, $\varepsilon_{ab}$ |

| | |
|---|---|
| FV5 | Curve ID that defines orientation-dependent post-peak residual strength (PPRD) for all 5 stress-strain curves (2 tension, 2 compression, and 1 shear). The ordinate values in the load curve define the |

| VARIABLE | DESCRIPTION |
|---|---|

various residual strengths in the following order:

| Ordinate Value Descriptions |
|---|
| Tensile post-peak residual strength in the *a*-direction, $PPRD_{aaT}$ |
| Compressive post-peak residual strength in the *a*-direction, $PPRD_{aaC}$ |
| Tensile post-peak residual strength in the *b*-direction, $PPRD_{bbT}$ |
| Compressive post-peak residual strength in the *b*-direction, $PPRD_{bbC}$ |
| In-plane shear post-peak residual strength in the *ab*-plane, $PPRD_{ab}$ |

FV6 **SANN.** Table ID that contains the point cloud data as a function of the index number of each point in the point cloud for each component of stress.

| Table Value | Curve Description |
|---|---|
| 1 | $\sigma_{11}$ component of point cloud data as a function of point index number |
| 2 | $\sigma_{22}$ component of point cloud data as a function of point index number |
| 3 | $\sigma_{12}$ component of point cloud data as a function of point index number |

**NN.** Table ID for a table that provides the weight matrix $(W_{m\times n})$ of the neural network. The values of the table index curves that provide the columns of the weight matrix. For each curve, the abscissa of the curve is the row number, with the ordinate being the component of the weight matrix for that row and column.

| Table Value | Curve Descriptions |
|---|---|
| 1 | $W_{i1}$ first column of the weight matrix $(W_{m\times n})$ |
| 2 | $W_{i2}$ second column of the weight matrix $(W_{m\times n})$ |
| ⋮ | ⋮ |
| *n* | $W_{in}$ $n^{\text{th}}$ column of the weight matrix $(W_{m\times n})$ |

FV7 **SANN.** Flag to select the weighting method to compute the average distance using data from neighborhood points:

| VARIABLE | DESCRIPTION |
|---|---|

EQ.0: Assign equal weight to all neighbors.

EQ.N: Use an inverse distance weight method with weights proportional to $1/d^N$, where $d$ is the distance and $N$ is the specified power.

**NN.** Table ID that provides for each layer of the neural network a bias vector for that layer. The values of the table are the layer. Each value indexes a curve providing the bias vector for that layer. The abscissa of the curve is the index of the bias vector and ordinate is the value of the component of the bias vector for that index.

| Curve Descriptions |
|---|
| $b_{i1}$ bias vector for the first layer |
| $b_{i2}$ bias vector for the second layer |
| ⋮ |
| $b_{iN}$ bias vector for the N$^{th}$ layer |

FV8 This field only applies for NN (FV0 = 1). Curve ID that specifes the number of neurons in each layer of the neural network. The abscissa is the layer number, and the ordinate is the number of neurons in the layer.

| Ordinate Value Descriptions |
|---|
| Number of neurons in the first layer |
| Number of neurons in the second layer |
| ⋮ |
| Number of neurons in the N$^{th}$ layer |

FV9 This field only applies for NN (FV0 = 1). Curve ID that specifies the activation function used in each layer of the neural network. The abscissa is the layer number. Three activation functions are considered: ReLU, TanH, and Linear, which are defined as numeric values on the ordinate as follows: 1 for ReLU, 2 for TanH, and 3 for Linear.

| Ordinate Value Descriptions |
|---|
| Activation function for the first layer |
| Activation function for the second layer |

| VARIABLE | DESCRIPTION |
|---|---|
| | ⋮ |
| | Activation function for the $N^{th}$ layer |

**Viscoelasticity Card.** BETA values only need to be specified when VEVP = 1 or 2.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | BETA11 | BETA22 | BETA33 | BETA44 | BETA55 | BETA66 | BETA12 | BETA23 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | ↓ | ↓ |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA11 | Decay constant for the relaxation matrix of the viscoelastic law in 1-direction (default = 0.001). It must be greater than or equal to zero. |
| BETA22 | Decay constant for the relaxation matrix of the viscoelastic law in 2-direction (default = 0.001). It must be greater than or equal to zero. |
| BETA33 | Decay constant for the relaxation matrix of the viscoelastic law in 3-direction (default = 0.001). This field is *not* required for shell elements. It must be greater than or equal to zero. |
| BETA44 | Decay constant for the relaxation matrix of the viscoelastic law in 12-shear (default = 0.001). It must be greater than or equal to zero. |
| BETA55 | Decay constant for the relaxation matrix of the viscoelastic law in 23-shear (default = 0.001). This field is *not* required for shell elements. It must be greater than or equal to zero. |
| BETA66 | Decay constant for the relaxation matrix of the viscoelastic law in 13-shear (default = 0.001). This field is *not* required for shell elements. It must be greater than or equal to zero. |
| BETA12 | Decay constant for the relaxation matrix of the viscoelastic law 12-coupling (default = (BETA11 + BETA22)/2). It must be greater than or equal to zero. |
| BETA23 | Decay constant for the relaxation matrix of the viscoelastic law 23-coupling (default = (BETA22 + BETA33)/2). This field is *not* |

| VARIABLE | DESCRIPTION |
|---|---|
| | required for shell elements. It must be greater than or equal to zero. |

**Viscoelasticity Card.** BETA values only need to be specified when VEVP = 1 or 2.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | BETA13 | CP | TQC | TEMP | PMACC | SLM | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA13 | Decay constant for the relaxation matrix of the viscoelastic law 13-coupling (default = (BETA11 + BETA33)/2). This field is *not* required for shell elements. It must be greater than or equal to zero. |
| CP | Specific heat capacity (per unit mass) |
| TQC | Taylor-Quinney Coefficient (see Remark 14) |
| TEMP | This is the reference (or initial) temperature used to obtain the corresponding stress-strain curves. |
| PMACC | Plastic multiplier computational accuracy (see Remark 10):<br>EQ.0: Use up to a maximum of 1000 increments (default)<br>EQ.N: Specify a positive value, N, greater than 1 as the maximum number of increments. An error message is issued if a converged solution cannot be found. |
| SLM | Step length multiplier used in bounding the plastic multiplier calculations. The default value is 1.1. |

**Remarks:**

1. **Flow rule coefficients.** Flow rule coefficients (FRCs) are determined using the plastic Poisson's ratios. H33, H55 and H66 are not required for shell elements. Details on how to compute FRCs can be found in [10]. Setting all the FRCs to zero invokes a simplified *MAT_213 material model. See Remark 13.

2. **Temperature-strain rate test result tables.** A minimum of two sets of (strain rate-temperature) curves are needed. If the material is not temperature and rate-

sensitive, make the two sets of table data identical (note that MAT_213 does not support DEFINE_TABLE_2D, meaning *DEFINE_TABLE must be used instead). If the material is rate and temperature sensitive, the curve corresponding to the smallest total strain rate for the given reference temperature (TEMP in Card 10) is assumed to be the quasi-static, room temperature (QS-RT) curve and influences the viscoelastic-plastic computations.

An example TABLE_3D (LT*i*) structure for 3 total strain rates and 3 temperatures for a tension in the *b*-direction test is shown in [10] as Example 7.1.

3. **YSC.** Curve of initial yield strain values (YSC) must list curves in ascending order as abscissa values with the corresponding yield strains given as the ordinate values. An example YSC data is shown in [10] as Example 7.2.

4. **Damage curve.** Include in this curve only the active damage parameter ID and the corresponding curve or Table3D ID. Note that damage data can be rate and temperature dependent and are used with all relevant input stress-strain curves in MAT_213 V1.3.6 and later versions. The damage parameter ID definitions are shown in the following table. For shell elements, only in-plane damage is considered and only parameters 1, 2, 4, 5, 7, 13, 15, 16, 18, 21, 23, 24, 26, 37, 38, 40, 42, 45, 46, 48, 50, 61, 62, 64, 65 are active.

| Damage Parameter ID | Damage Parameter | Damage Parameter ID | Damage Parameter | Damage Parameter ID | Damage Parameter |
|---|---|---|---|---|---|
| 1 | $d_{aa_T}^{aa_T}(\varepsilon_{aa_T})$ | 29 | $d_{bb_T}^{aa_T}(\varepsilon_{cc_T})$ | 57 | $d_{cc_C}^{bb_C}(\varepsilon_{cc_C})$ |
| 2 | $d_{bb_T}^{bb_T}(\varepsilon_{bb_T})$ | 30 | $d_{cc_T}^{bb_T}(\varepsilon_{cc_T})$ | 58 | $d_{cc_C}^{ab}(\varepsilon_{cc_C})$ |
| 3 | $d_{cc_T}^{cc_T}(\varepsilon_{cc_T})$ | 31 | $d_{cc_T}^{aa_C}(\varepsilon_{cc_T})$ | 59 | $d_{cc_C}^{bc}(\varepsilon_{cc_C})$ |
| 4 | $d_{aa_C}^{aa_C}(\varepsilon_{aa_C})$ | 32 | $d_{cc_T}^{bb_C}(\varepsilon_{cc_T})$ | 60 | $d_{cc_C}^{ac}(\varepsilon_{cc_C})$ |
| 5 | $d_{bb_C}^{bb_C}(\varepsilon_{bb_C})$ | 33 | $d_{cc_T}^{cc_C}(\varepsilon_{cc_T})$ | 61 | $d_{ab}^{aa_T}(\varepsilon_{ab})$ |
| 6 | $d_{cc_C}^{cc_C}(\varepsilon_{cc_C})$ | 34 | $d_{cc_T}^{ab}(\varepsilon_{cc_T})$ | 62 | $d_{ab}^{bb_T}(\varepsilon_{ab})$ |
| 7 | $d_{ab}^{ab}(\varepsilon_{ab})$ | 35 | $d_{cc_T}^{bc}(\varepsilon_{cc_T})$ | 63 | $d_{ab}^{cc_T}(\varepsilon_{ab})$ |
| 8 | $d_{bc}^{bc}(\varepsilon_{bc})$ | 36 | $d_{cc_T}^{ac}(\varepsilon_{cc_T})$ | 64 | $d_{ab}^{aa_C}(\varepsilon_{ab})$ |
| 9 | $d_{ac}^{ac}(\varepsilon_{ac})$ | 37 | $d_{aa_C}^{aa_T}(\varepsilon_{aa_T})$ | 65 | $d_{ab}^{bb_C}(\varepsilon_{ab})$ |
| 10 | $d_{oab}^{oab}(\varepsilon_{oab})$ | 38 | $d_{aa_C}^{bb_T}(\varepsilon_{aa_C})$ | 66 | $d_{ab}^{cc_C}(\varepsilon_{ab})$ |
| 11 | $d_{obc}^{obc}(\varepsilon_{obc})$ | 39 | $d_{aa_C}^{cc_T}(\varepsilon_{aa_C})$ | 67 | $d_{ab}^{bc}(\varepsilon_{ab})$ |
| 12 | $d_{oac}^{oac}(\varepsilon_{oac})$ | 40 | $d_{aa_C}^{bb_C}(\varepsilon_{aa_C})$ | 68 | $d_{ab}^{ac}(\varepsilon_{ab})$ |

| Damage Parameter ID | Damage Parameter | Damage Parameter ID | Damage Parameter | Damage Parameter ID | Damage Parameter |
|---|---|---|---|---|---|
| 13 | $d_{aa_T}^{bb_T}(\varepsilon_{aa_T})$ | 41 | $d_{aa_C}^{cc_C}(\varepsilon_{aa_C})$ | 69 | $d_{bc}^{aa_T}(\varepsilon_{bc})$ |
| 14 | $d_{aa_T}^{cc_T}(\varepsilon_{aa_T})$ | 42 | $d_{aa_C}^{ab}(\varepsilon_{aa_C})$ | 70 | $d_{bc}^{bb_T}(\varepsilon_{bc})$ |
| 15 | $d_{aa_T}^{aa_C}(\varepsilon_{aa_T})$ | 43 | $d_{aa_C}^{bc}(\varepsilon_{aa_C})$ | 71 | $d_{bc}^{cc_T}(\varepsilon_{bc})$ |
| 16 | $d_{aa_T}^{bb_C}(\varepsilon_{aa_T})$ | 44 | $d_{aa_C}^{ac}(\varepsilon_{aa_C})$ | 72 | $d_{bc}^{aa_c}(\varepsilon_{bc})$ |
| 17 | $d_{aa_T}^{cc_C}(\varepsilon_{aa_T})$ | 45 | $d_{bb_C}^{aa_T}(\varepsilon_{bb_C})$ | 73 | $d_{bc}^{bb_c}(\varepsilon_{bc})$ |
| 18 | $d_{aa_T}^{ab}(\varepsilon_{aa_T})$ | 46 | $d_{bb_C}^{bb_T}(\varepsilon_{bb_C})$ | 74 | $d_{bc}^{cc_c}(\varepsilon_{bc})$ |
| 19 | $d_{aa_T}^{bc}(\varepsilon_{aa_T})$ | 47 | $d_{bb_C}^{cc_T}(\varepsilon_{bb_C})$ | 75 | $d_{bc}^{ab}(\varepsilon_{bc})$ |
| 20 | $d_{aa_T}^{ac}(\varepsilon_{aa_T})$ | 48 | $d_{bb_C}^{aa_C}(\varepsilon_{bb_C})$ | 76 | $d_{bc}^{ac}(\varepsilon_{bc})$ |
| 21 | $d_{bb_T}^{aa_T}(\varepsilon_{bb_T})$ | 49 | $d_{bb_C}^{cc_C}(\varepsilon_{bb_C})$ | 77 | $d_{ac}^{aa_T}(\varepsilon_{ac})$ |
| 22 | $d_{bb_T}^{cc_T}(\varepsilon_{bb_T})$ | 50 | $d_{bb_C}^{ab}(\varepsilon_{bb_C})$ | 78 | $d_{ac}^{bb_T}(\varepsilon_{ac})$ |
| 23 | $d_{bb_T}^{aa_C}(\varepsilon_{bb_T})$ | 51 | $d_{bb_C}^{bc}(\varepsilon_{bb_C})$ | 79 | $d_{ac}^{cc_T}(\varepsilon_{ac})$ |
| 24 | $d_{bb_T}^{bb_C}(\varepsilon_{bb_T})$ | 52 | $d_{bb_C}^{ac}(\varepsilon_{bb_C})$ | 80 | $d_{ac}^{aa_C}(\varepsilon_{ac})$ |
| 25 | $d_{bb_T}^{cc_C}(\varepsilon_{bb_T})$ | 53 | $d_{cc_C}^{aa_T}(\varepsilon_{cc_C})$ | 81 | $d_{ac}^{bb_C}(\varepsilon_{ac})$ |
| 26 | $d_{bb_T}^{ab}(\varepsilon_{bb_T})$ | 54 | $d_{cc_C}^{bb_T}(\varepsilon_{cc_C})$ | 82 | $d_{ac}^{cc_C}(\varepsilon_{ac})$ |
| 27 | $d_{bb_T}^{bc}(\varepsilon_{bb_T})$ | 55 | $d_{cc_C}^{cc_T}(\varepsilon_{cc_C})$ | 83 | $d_{ac}^{ab}(\varepsilon_{ac})$ |
| 28 | $d_{bb_T}^{ac_C}(\varepsilon_{bb_T})$ | 56 | $d_{cc_C}^{aa_C}(\varepsilon_{cc_C})$ | 84 | $d_{ac}^{bc}(\varepsilon_{ac})$ |

A typical damage curve has the total strain in the loading direction as abscissa values with the corresponding damage value given as the ordinate values. The final strain value in the curve must correspond to the final strain in the corresponding QS-RT input stress-strain curve.

Examples 7.3 and 7.4 in [10] illustrate providing damage data that is rate and temperature independent and rate and temperature dependent, respectively.

5. **Failure criterion.** Use Cards 8*n*.1 and 8*n*.2 for the failure criterion to be included in the failure model. The failure criterion and associated values are discussed below (see [8, 15] for details).

   a) *Puck Failure Criterion (PFC).* For this criterion, FTYPE = 1 and FV0, …, FV14 are the magnitudes of fracture energy, damage, magnification factor, slope

parameters, and material parameters for the fiber. For an example showing how to use PFC data for solid elements, refer to Example 7.5 in [10].

b) *Tsai-Wu Failure Criterion (TWFC).* For this criterion, FTYPE = 2 and FV1, ..., FV12 are the magnitudes of the failure stresses $\hat{\sigma}_{aa}^{T}$, $\hat{\sigma}_{aa}^{C}$, $\hat{\sigma}_{bb}^{T}$, $\hat{\sigma}_{bb}^{C}$, $\hat{\sigma}_{cc}^{T}$, $\hat{\sigma}_{cc}^{C}$, $\hat{\sigma}_{ab}$, $\hat{\sigma}_{bc}$, $\hat{\sigma}_{ac}$, $\hat{\sigma}_{ab}^{45}$, $\hat{\sigma}_{bc}^{45}$, and $\hat{\sigma}_{ac}^{45}$. FV13 and FV14 are erosion-related values and residual strength values. For an example showing how to use TWFC data for shell elements, refer to Example 7.6 in [10].

c) *Generalized Tabulated Failure Criterion (GTFC).* For solid elements with FTYPE = 3, FV1 is $n$, the in-plane and out-of-plane interaction term. FV2 and FV3 are the table IDs of the two tables for the in-plane and the out-of-plane $(\theta, \varepsilon_{fail})$ values that define the in-plane and out-of-plane failure surfaces, respectively. For the in-plane failure surface, the table contains the $a$-direction stress (S11) value-curve ID pairs. For the out-of-plane failure surface, the table contains the normal $c$-direction stress (S33) value-curve ID pairs. For shell elements, only FV2 is used with the preceding definition, and the element fails when the damage is 1. There is no data in card 8d.2. For an example showing how to use GTFC data for solid elements, refer to Example 7.7 in [10].

d) *Point Cloud Failure Criterion (PCFC) using SANN.* For this criterion FTYPE = 4 and FV0 = 0. Failure is detected by using the distances of the FEA-generated stress/strain Gauss point $L_2^{\text{FEA}}$ and the average distance of $k$ neighborhood points on the failure surface generated data $\left(L_2^{\text{SGD}}\right)_{\text{avg}}$ from the origin. Distances are calculated with the general Minkowski norm of order $r$:

$$L_r = \left( \sqrt{\sum_{i=1}^{d} \left| v_i^j \right|^r} \right)$$

where $d$ is the dimension the point cloud space, and $v_i^j$ is a component of the position vector in the $d$-dimensional space for point $j$. Here, $d$ is 3 for the failure space of $\{\sigma_{11}, \sigma_{22}, \sigma_{12}\}$ and $r$ is 2.

To detect failure, $L_2^{\text{FEA}}$ is compared with $\left(L_2^{\text{SGD}}\right)_{\text{avg}}$ with a safety factor, $\alpha\sigma_{\text{range}}$. Here, $\sigma_{\text{range}} = \max\left(\sigma_{11}^{\max} - \sigma_{11}^{\min},\ \sigma_{22}^{\max} - \sigma_{22}^{\min},\ \sigma_{12}^{\max} - \sigma_{12}^{\min}\right)$ and $0 < \alpha < 1$. An element is detected as failed if the failure parameter, FP, is greater than or equal to 0:

$$\text{FP} = L_2^{\text{FEA}} - \left( \left(L_2^{\text{SGD}}\right)_{\text{avg}} - \alpha\sigma_{\text{range}} \right)$$

For an example that shows how to define PCFC material data card using SANN, refer to Example 7.8 in [10].

e) *Point Cloud Failure Criterion (PCFC) using NN.* For this criterion FTYPE = 4 and FV0 = 1. Failure for the $i^{th}$ Gauss point of element $k$ is detected by comparing $(\sigma_{12})_i$ with neural-network-predicted $\left(\hat{\sigma}_{12,f}\right)_i$ . If $(\sigma_{12})_i \leq \beta\left(\hat{\sigma}_{12,f}\right)_i$, then there is no onset of failure. If $(\sigma_{12})_i > \beta\left(\hat{\sigma}_{12,f}\right)_i$, then failure begins.

For an example that shows how to define PCFC material data card using the NN method, refer to Example 7.9 in [10].

6. **Element erosion.** Element is eroded if failure occurs at any one Gauss point. Note that *DEFINE_ELEMENT_EROSION_SHELL is required for shell element erosion; the number of integration points needed to fail to erode the element is defined there.

7. **Adjusting stress-strain curves.** The user-defined stress-strain curves are adjusted when using TCSYM = 1, 2, and 3 as follows. If $E_a^{T_0}$ and $E_a^{C_0}$ represent the original $a$-direction (1-direction) elastic tensile and compressive moduli, respectively, then the modified elastic moduli are computed as

a) $TCSYM = 1.\ E_a^T = E_a^C = 0.5\left(E_a^{T_0} + E_a^{C_0}\right)$

b) $TCSYM = 2.\ E_a^T = E_a^{T_0}$ and $E_a^C = E_a^{T_0}$

c) $TCSYM = 3.\ E_a^T = E_a^{C_0}$ and $E_a^C = E_a^{C_0}$.

Let $R_a^T = E_a^T / E_a^{T_0}$ and $R_a^C = E_a^C / E_a^{C_0}$. The adjusted tensile strain is then computed as the original tensile strain divided by $R_a^T$, the adjusted compressive strain is computed as the original compressive strain divided by $R_a^C$, the adjusted tensile yield strain is computed as the original tensile yield strain divided by $R_a^T$, and the adjusted compressive yield strain is computed as the original compressive yield strain divided by $R_a^C$. The same process is then applied to the other two normal directions.

8. **Curve discretization.** For this material, only LCINT on *CONTROL_SOLUTION (not with *DEFINE_CURVE) can be used to specify the number of discretized points for the input curves. The default value is 100.

9. **Poisson's ratios.** If necessary, the input Poisson's ratios are adjusted internally in LS-DYNA to satisfy the criteria described in [11]. However, the user may enter all Poisson's Ratios to be zero in which case Poisson's ratio checks are not carried out.

10. **Plastic multiplier.** The plastic multiplier computations involve finding the root of the yield function. The root is computed numerically, not analytically. The

first step is to find the interval bounding the root. The value of N set in field PMACC on Card 10 controls the discretization of the interval to find the bound. The larger the value of N, the more accurate the bound. However, computational time is likely to increase with larger values of N.

11. **Stochastic variation.** A stochastic variation can be added using keyword *DEFINE_STOCHASTIC_VARIATION_PROPERTIES. There are six quantities (see table below) which can be varied, so Card 2 in *DEFINE_STOCHASTIC_VARIATION_PROPERTIES must be defined six times and assumes that the quantities being varied are in the order specified.

| Material Properties to be Varied in Predefined Order |
|---|
| $E_a$ |
| $G_{ab}$ |
| $G_{bc}$ |
| $G_{ca}$ |
| In-plane failure radius |
| Out-of-plane failure radius |

12. **Damage formulation.** By default, damage calculations are carried out using effective stress as the internal state variable for tracking growth of damage parameters $d_{cd}^{ab}$. An alternate formulation is available where the damage parameters are taken as functions of directional plastic strains to track damage growth. Details of both the formulations are available in [7].

13. **Simplified material model**. Inputting all the FRCs (see Remark 1) as zero invokes a simplified material model, resulting essentially in a linear analysis. Rate and temperature dependencies are not supported. If needed, damage and failure models can be turned on with additional input. This simplified material model is available for solid and shell elements. For example input data that uses SMM, refer to Example 7.10 in [10].

14. **Taylor-Quinney Coefficient (TQC) Data.** How the TQC data is used to compute the rise in temperature is explained in [8, 10].

### External Files Generated by MAT_213:

Two sets of external files are generated, which contain information connected with the input stress-strain curves. The first set of files has a naming convention of "MAT_213-__input_curve__stress-strain__curve_id_*i*.plt" Here, $i$ is the $i^{\text{th}}$ load curve input into

*MAT_213. Each of these files contains LCINT (see Remark 8) stress-strain curve data points. An example of the set of files generated is shown below.

| File Name | Load Curve |
|---|---|
| MAT_213__input_curve__stress-strain__curve_id_1.plt | LC1 |
| MAT_213__input_curve__stress-strain__curve_id_2.plt | LC2 |
| MAT_213__input_curve__stress-strain__curve_id_3.plt | LC3 |
| MAT_213__input_curve__stress-strain__curve_id_4.plt | LC4 |
| MAT_213__input_curve__stress-strain__curve_id_5.plt | LC5 |
| MAT_213__input_curve__stress-strain__curve_id_6.plt | LC6 |
| MAT_213__input_curve__stress-strain__curve_id_7.plt | LC7 |
| MAT_213__input_curve__stress-strain__curve_id_8.plt | LC8 |
| MAT_213__input_curve__stress-strain__curve_id_9.plt | LC9 |
| MAT_213__input_curve__stress-strain__curve_id_10.plt | LC10 |
| MAT_213__input_curve__stress-strain__curve_id_11.plt | LC11 |
| MAT_213__input_curve__stress-strain__curve_id_12.plt | LC12 |
| MAT_213__input_curve__stress-strain__curve_id_13.plt | LC13 |
| MAT_213__input_curve__stress-strain__curve_id_14.plt | LC14 |
| MAT_213__input_curve__stress-strain__curve_id_15.plt | LC15 |
| MAT_213__input_curve__stress-strain__curve_id_16.plt | LC16 |
| MAT_213__input_curve__stress-strain__curve_id_17.plt | LC17 |
| MAT_213__input_curve__stress-strain__curve_id_18.plt | LC18 |
| MAT_213__input_curve__stress-strain__curve_id_19.plt | LC19 |
| MAT_213__input_curve__stress-strain__curve_id_20.plt | LC20 |
| MAT_213__input_curve__stress-strain__curve_id_21.plt | LC21 |
| MAT_213__input_curve__stress-strain__curve_id_22.plt | LC22 |
| MAT_213__input_curve__stress-strain__curve_id_23.plt | LC23 |
| MAT_213__input_curve__stress-strain__curve_id_24.plt | LC24 |

The second set of files have a naming convention of "MAT_213__modified_curve__-stress-pl_strain__curve_id_*i*.plt". As above, *i* is the $i^{th}$ load curve input into *MAT_213. Each of these files contains LCINT stress-effective plastic strain curve data points. We urge you to use these plot files to check if the stress-effective plastic strain curves for each of the 12 components intersect or not when rate and temperature sensitive data are input.

Intersecting curves or intersecting extrapolated curves for a component are likely to lead to inconsistent results. An example of the set of files generated is shown below.

| File Name | Load Curve |
|---|---|
| MAT_213__modified_curve__stress-pl_strain__curve_id_1.plt | LC1 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_2.plt | LC2 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_3.plt | LC3 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_4.plt | LC4 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_5.plt | LC5 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_6.plt | LC6 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_7.plt | LC7 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_8.plt | LC8 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_9.plt | LC9 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_10.plt | LC10 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_11.plt | LC11 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_12.plt | LC12 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_13.plt | LC13 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_14.plt | LC14 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_15.plt | LC15 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_16.plt | LC16 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_17.plt | LC17 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_18.plt | LC18 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_19.plt | LC19 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_20.plt | LC20 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_21.plt | LC21 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_22.plt | LC22 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_23.plt | LC23 |
| MAT_213__modified_curve__stress-pl_strain__curve_id_24.plt | LC24 |

Each of the aforementioned "MAT_213__modified_curve__stress-pl_strain__curve_-id_*i*.plt" files have similar headings

```
Curveplot
MAT213 mod crv i (EPSR =       xxxx.xxx)
effective plastic strain
```

```
stress
stress curve
stress #pts=     "LCINT"
```

**List of History Variables for Solid Elements (LS-PrePost):**

| History Variable # | Symbols | Description |
|---|---|---|
| 15 | $c_1^d$ | Damage in *a*-direction, tension |
| 16 | $c_2^d$ | Damage in *b*-direction, tension |
| 17 | $c_3^d$ | Damage in *c*-direction, tension |
| 18 | $c_4^d$ | Damage in *a*-direction, compression |
| 19 | $c_5^d$ | Damage in *b*-direction, compression |
| 20 | $c_6^d$ | Damage in *c*-direction, compression |
| 21 | $c_7^d$ | Damage in *ab*-plane, shear |
| 22 | $c_8^d$ | Damage in *bc*-plane, shear |
| 23 | $c_9^d$ | Damage in *a*c-plane, shear |
| 24 | $\varepsilon_{aa}^0$, F, or $d$ | FTYPE.EQ.1: Strain at failure onset in *a*-direction<br>FTYPE.EQ.2: Flag for failure of the integration point, which is 1 if $d \geq 1$<br>FTYPE.EQ.3: Failure term |
| 25 | $\varepsilon_{aa}^f$ or $\varepsilon_{aaT}^p$ | FTYPE.EQ.1: Strain for erosion in *a*-direction<br>FTYPE.NE.1: Tensile plastic strain in *a*-direction |
| 26 | $\varepsilon_{bb}^0$ or $\varepsilon_{bbT}^p$ | FTYPE.EQ.1: Strain at failure onset in *b*-direction<br>FTYPE.NE.1: Tensile plastic strain in *b*-direction |
| 27 | $\varepsilon_{bb}^f$, $d$, or $r_{\text{IP}}^f$ | FTYPE.EQ.1: Strain for erosion in *b*-direction<br>FTYPE.EQ.2: Failure term<br>FTYPE.EQ.3: Equivalent failure strain for in-plane mode |
| 28 | $\varepsilon_{cc}^0$ or $r_{\text{IP}}$ | FTYPE.EQ.1: Strain at failure onset in *c*-direction<br>FTYPE.EQ.3: Equivalent strain for in-plane mode |
| 29 | $\varepsilon_{cc}^f$ or $\theta_{\text{IP}}$ | FTYPE.EQ.1: Strain for erosion in *c*-direction<br>FTYPE.EQ.3: Failure angle for in-plane mode |
| 30 | $\varepsilon_{ab}^0$ or $r_{\text{OOP}}^f$ | FTYPE.EQ.1: Tensorial shear strain at failure onset in *ab*-plane |

| History Variable # | Symbols | Description |
|---|---|---|
| | | FTYPE.EQ.3: Equivalent failure strain for out-of-plane mode |
| 31 | $\varepsilon_{ab}^{f}$ or $r_{\mathrm{OOP}}$ | FTYPE.EQ.1: Tensorial shear strain for erosion in *ab*-plane<br>FTYPE.EQ.3: Equivalent strain for out-of-plane mode |
| 32 | $\varepsilon_{bc}^{0}$ or $\theta_{\mathrm{OOP}}$ | FTYPE.EQ.1: Tensorial shear strain at failure onset in *bc*-plane<br>FTYPE.EQ.3: Failure angle for out-of-plane mode |
| 33 | $\varepsilon_{bc}^{f}$ or $\varepsilon_{ccT}^{p}$ | FTYPE.EQ.1: Tensorial shear strain for erosion in *bc*-plane<br>FTYPE.NE.1: Tensile plastic strain in *c*-direction |
| 34 | $\varepsilon_{ac}^{0}$ or $\varepsilon_{aaC}^{p}$ | FTYPE.EQ.1: Tensorial shear strain at failure onset in *ac*-plane<br>FTYPE.NE.1: Compressive plastic strain in *a*-direction |
| 35 | $\varepsilon_{ac}^{f}$ or $\varepsilon_{bbC}^{p}$ | FTYPE.EQ.1: Tensorial shear strain for erosion in *ac*-plane<br>FTYPE.NE.1: Compressive plastic strain in *b*-direction |
| 36 | FF or $\varepsilon_{ccC}^{p}$ | FTYPE.EQ.1: Flag for fiber-fracture<br>FTYPE.NE.1: Compressive plastic strain in *c*-direction |
| 37 | IFF or $\varepsilon_{ab}^{p}$ | FTYPE.EQ.1: Flag for inter-fiber-fracture<br>FTYPE.NE.1: Plastic tensorial strain in *ab*-plane |
| 38 | $T$ | Temperature |
| 39 | $\dot{\varepsilon}_{aa_T}$ | Tensile strain rate in *a*-direction |
| 40 | $\dot{\varepsilon}_{bb_T}$ | Tensile strain rate in *b*-direction |
| 41 | $\dot{\varepsilon}_{cc_T}$ | Tensile strain rate in *c*-direction |
| 42 | $\dot{\varepsilon}_{aa_C}$ | Compressive strain rate in *a*-direction |
| 43 | $\dot{\varepsilon}_{bb_C}$ | Compressive strain rate in *b*-direction |
| 44 | $\dot{\varepsilon}_{cc_C}$ | Compressive strain rate in *c*-direction |
| 45 | $\dot{\varepsilon}_{ab}$ | Tensorial shear strain rate in *ab*-plane |
| 46 | $\dot{\varepsilon}_{bc}$ | Tensorial shear strain rate in *bc*-plane |

| History Variable # | Symbols | Description |
|---|---|---|
| 47 | $\dot{\varepsilon}_{ac}$ | Tensorial shear strain rate in *ac*-plane |
| 48 | $\sigma^{\text{eff}}_{aa_T}$ or $\varepsilon^{\text{cp}}_{aa_T}$ | DFLAG.EQ.0: Effective tensile stress in the *a*-direction<br>DFLAG.EQ.1: Corrected tensile plastic strain the *a*-direction |
| 49 | $\sigma^{\text{eff}}_{bb_T}$ or $\varepsilon^{\text{cp}}_{bb_T}$ | DFLAG.EQ.0: Effective tensile stress in the *b*-direction<br>DFLAG.EQ.1: Corrected tensile plastic strain the *b*-direction |
| 50 | $\sigma^{\text{eff}}_{cc_T}$ or $\varepsilon^{\text{cp}}_{cc_T}$ | DFLAG.EQ.0: Effective tensile stress in the *c*-direction<br>DFLAG.EQ.1: Corrected tensile plastic strain the *c*-direction |
| 51 | $\sigma^{\text{eff}}_{aa_C}$ or $\varepsilon^{\text{cp}}_{aa_C}$ | DFLAG.EQ.0: Effective compressive stress in the *a*-direction<br>DFLAG.EQ.1: Corrected compressive plastic strain the *a*-direction |
| 52 | $\sigma^{\text{eff}}_{bb_C}$ or $\varepsilon^{\text{cp}}_{bb_C}$ | DFLAG.EQ.0: Effective compressive stress in the *b*-direction<br>DFLAG.EQ.1: Corrected compressive plastic strain the *b*-direction |
| 53 | $\sigma^{\text{eff}}_{cc_C}$ or $\varepsilon^{\text{cp}}_{cc_C}$ | DFLAG.EQ.0: Effective compressive stress in the *c*-direction<br>DFLAG.EQ.1: Corrected compressive plastic strain the *c*-direction |
| 54 | $\sigma^{\text{eff}}_{ab}$ or $\varepsilon^{\text{cp}}_{ab}$ | DFLAG.EQ.0: Effective shear stress in the *ab*-plane<br>DFLAG.EQ.1: Corrected plastic tensorial strain the *ab*-plane |
| 55 | $\sigma^{\text{eff}}_{bc}$ or $\varepsilon^{\text{cp}}_{bc}$ | DFLAG.EQ.0: Effective shear stress in the *bc*-plane<br>DFLAG.EQ.1: Corrected plastic tensorial strain the *bc*-plane |
| 56 | $\sigma^{\text{eff}}_{ac}$ or $\varepsilon^{\text{cp}}_{ac}$ | DFLAG.EQ.0: Effective shear stress in the *ac*-plane<br>DFLAG.EQ.1: Corrected plastic tensorial strain the *ac*-plane |
| 57 | $\lambda$ | Effective plastic strain |
| 58 | $\varepsilon_{aa}$ | Strain in *a*-direction |

| History Variable # | Symbols | Description |
|---|---|---|
| 59 | $\varepsilon_{bb}$ | Strain in *b*-direction |
| 60 | $\varepsilon_{cc}$ | Strain in *c*-direction |
| 61 | $\varepsilon_{ab}$ | Tensorial shear strain in *ab*-plane |
| 62 | $\varepsilon_{bc}$ | Tensorial shear strain in *bc*-plane |
| 63 | $\varepsilon_{ac}$ | Tensorial shear strain in *ac*-plane |
| 64 | $\sigma^{y}_{aa_T}$ | Yield stress in tension *a*-direction |
| 65 | $\sigma^{y}_{bb_T}$ | Yield stress in tension *b*-direction |
| 66 | $\sigma^{y}_{cc_T}$ | Yield stress in tension *c*-direction |
| 67 | $\sigma^{y}_{aa_C}$ | Yield stress in compression *a*-direction |
| 68 | $\sigma^{y}_{bb_C}$ | Yield stress in compression *b*-direction |
| 69 | $\sigma^{y}_{cc_C}$ | Yield stress in compression *c*-direction |
| 70 | $\sigma^{y}_{ab}$ | Yield stress in shear *ab*-plane |
| 71 | $\sigma^{y}_{bc}$ | Yield stress in shear *bc*-plane |
| 72 | $\sigma^{y}_{ac}$ | Yield stress in shear *ac*-plane |
| 73 | $T$ | Current cycle |
| 74 | $\sigma^{e}_{aa}$ | Equilibrium stress in *a*-direction |
| 75 | $\sigma^{e}_{bb}$ | Equilibrium stress in *b*-direction |
| 76 | $\sigma^{e}_{cc}$ | Equilibrium stress in *c*-direction |
| 77 | $\sigma^{e}_{ab}$ | Equilibrium shear stress in *ab*-plane |
| 78 | $\sigma^{e}_{bc}$ | Equilibrium shear stress in *bc*-plane |
| 79 | $\sigma^{e}_{ac}$ | Equilibrium shear stress in *ac*-plane |
| 80 | $\sigma^{v}_{aa}$ | Viscous stress in *a*-direction |
| 81 | $\sigma^{v}_{bb}$ | Viscous stress in *b*-direction |
| 82 | $\sigma^{v}_{cc}$ | Viscous stress in *c*-direction |
| 83 | $\sigma^{v}_{ab}$ | Viscous shear stress in *ab*-plane |
| 90 | $\sigma^{v}_{bc}$ | Viscous shear stress in *bc*-plane |
| 91 | $\sigma^{v}_{ac}$ | Viscous shear stress in *ac*-plane |
| 92 | $\dot{\lambda}$ | Effective plastic strain rate |
| 93 | $\varepsilon^{p}_{bc}$ | Plastic tensorial strain in *bc*-plane (FTYPE ≠ 1) |

| History Variable # | Symbols | Description |
|---|---|---|
| 94 | $\varepsilon_{ac}^{p}$ | Plastic tensorial strain in *ac*-plane (FTYPE ≠ 1) |

**List of History Variables for Shell Elements (LS-PrePost):**

| History Variable # | Symbols | Description |
|---|---|---|
| 13 | $d_{\max}$ | Maximum damage parameter |
| 14 | $c_1^d$ | Damage in *a*-direction, tension |
| 15 | $c_2^d$ | Damage in *b*-direction, tension |
| 16 | $c_4^d$ | Damage in *a*-direction, compression |
| 17 | $c_5^d$ | Damage in *b*-direction, compression |
| 18 | $c_7^d$ | Damage in *ab*-plane, shear |
| 19 | $d$ | Failure term (FTYPE = 3 and FTYPE = 4 (SANN)) |
| 20 | $d$ or $r_{\text{IP}}$ | FTYPE.EQ.2: Failure term<br>FTYPE.EQ.3: Equivalent strain for in-plane mode |
| 21 | $\theta_{\text{IP}}$ | Failure angle for in-plane mode (FTYPE = 3) |
| 22 | F | Flag for failure of the integration point. "1" if $d \geq 1$ |
| 23 | $T$ | Temperature |
| 24 | $\dot{\lambda}$ | Effective plastic strain rate |
| 25 | $\dot{\varepsilon}_{aa_T}$ | Tensile strain rate in the *a*-direction |
| 26 | $\dot{\varepsilon}_{bb_T}$ | Tensile strain rate in the *b*-direction |
| 27 | $\dot{\varepsilon}_{aa_C}$ | Compressive strain rate in the *a*-direction |
| 28 | $\dot{\varepsilon}_{bb_C}$ | Compressive strain rate in the *b*-direction |
| 29 | $\dot{\varepsilon}_{ab}$ | Tensorial shear strain rate in the *ab*-plane |
| 30 | $\varepsilon_{aa_T}^{p}$ | Tensile plastic strain in the *a*-direction |
| 31 | $\varepsilon_{bb_T}^{p}$ | Tensile plastic strain in the *b*-direction |
| 32 | $\varepsilon_{cc_T}^{p}$ | Tensile plastic strain in the *c*-direction |
| 33 | $\varepsilon_{aa_C}^{p}$ | Compressive plastic strain in the *a*-direction |
| 34 | $\varepsilon_{bb_C}^{p}$ | Compressive plastic strain in the *b*-direction |
| 35 | $\varepsilon_{cc_C}^{p}$ | Compressive plastic strain in the *c*-direction |
| 36 | $\varepsilon_{ab}^{p}$ | Plastic tensorial shear strain in the *ab*-plane |

| History Variable # | Symbols | Description |
|---|---|---|
| 37 | $\lambda$ | Effective plastic strain |
| 38 | $\varepsilon_{aa}$ | Strain in *a*-direction |
| 39 | $\varepsilon_{bb}$ | Strain in *b*-direction |
| 40 | $\varepsilon_{cc}$ | Strain in *c*-direction |
| 41 | $\varepsilon_{ab}$ | Tensorial shear strain in *ab*-plane |
| 42 | $\varepsilon_{bc}$ | Tensorial shear strain in *bc*-plane |
| 43 | $\varepsilon_{ac}$ | Tensorial shear strain in *ac*-plane |
| 44 | $\sigma_{aa}$ | Stress in *a*-direction |
| 45 | $\sigma_{bb}$ | Stress in *b*-direction |
| 46 | $\sigma_{ab}$ | Shear stress in *ab*-plane |
| 47 | $\sigma_{bc}$ | Shear stress in *bc*-plane |
| 48 | $\sigma_{ac}$ | Shear stress in *ac*-plane |
| 49 | $\sigma_{aa}^{e}$ | Equilibrium stress in *a*-direction |
| 50 | $\sigma_{bb}^{e}$ | Equilibrium stress in *b*-direction |
| 51 | $\sigma_{ab}^{e}$ | Equilibrium shear stress in *ab*-plane |
| 52 | $\sigma_{aa}^{v}$ | Viscous stress in *a*-direction |
| 53 | $\sigma_{bb}^{v}$ | Viscous stress in *b*-direction |
| 54 | $\sigma_{ab}^{v}$ | Viscous shear stress in *ab*-plane |
| 55 | $\varepsilon_{aa_T}^{\mathrm{cp}}$ | Corrected plastic strain in *a*-direction, tension |
| 56 | $\varepsilon_{bb_T}^{\mathrm{cp}}$ | Corrected plastic strain in *b*-direction, tension |
| 57 | $\varepsilon_{aa_C}^{\mathrm{cp}}$ | Corrected plastic strain in *a*-direction, compression |
| 58 | $\varepsilon_{bb_C}^{\mathrm{cp}}$ | Corrected plastic strain in *b*-direction, compression |
| 59 | $\sigma_{aa_T}^{y}$ | Yield stress in tension *a*-direction |
| 60 | $\sigma_{bb_T}^{y}$ | Yield stress in tension *b*-direction |
| 61 | $\sigma_{aa_C}^{y}$ | Yield stress in compression *a*-direction |
| 62 | $\sigma_{bb_C}^{y}$ | Yield stress in compression *b*-direction |
| 63 | $\sigma_{ab}^{y}$ | Yield stress in shear *ab*-plane |
| 64 | $\sigma_{ab}^{y,45}$ | Yield stress in 45° off-axis *ab*-plane |
| 65 | $E_p$ | Dissipated plastic energy |

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## *MAT_DRY_FABRIC

This is Material Type 214. This material model can be used to model high strength woven fabrics, such as Kevlar® 49, with transverse orthotropic behavior for use in structural systems where high energy absorption is required (Bansal et al., Naik et al., Stahlecker et al.). The major applications of the model are for the materials used in propulsion engine containment system, body armor and personal protections.

Woven dry fabrics are described in terms of two principal material directions, longitudinal warp and transverse fill yarns. The primary failure mode in these materials is the breaking of either transverse or longitudinal yarn. An equivalent continuum formulation is used and an element is designated as having failed when it reaches some critical value for strain in either directions. A linearized approximation to a typical stress-strain curve is shown in Figure M214-1 and to a typical engineering shear stress-strain curve is shown in the figure corresponding to the GAB*i* field in the variable list. Note that the principal directions are labeled *a* for the warp and *b* for the fill, and the direction *c* is orthogonal to *a* and *b*.

The material model is available for membrane elements and it is recommended to use a double precision version of LS-DYNA.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | GAB1 | GAB2 | GAB3 | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GBC | GCA | GAMAB1 | GAMAB2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | | | | | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| EACRF | EBCRF | EACRP | EBCRP | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| EASF | EBSF | EUNLF | ECOMF | EAMAX | EBMAX | SIGPOST | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

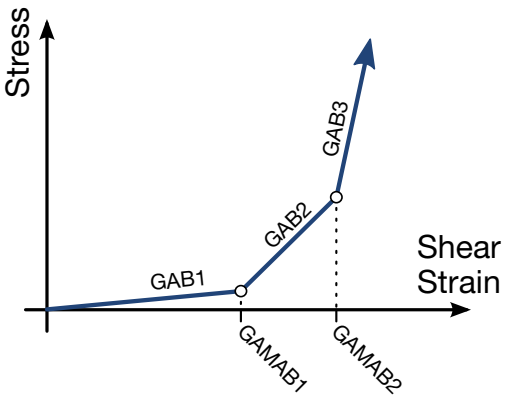
| CCE | PCE | CSE | PSE | DFAC | EMAX | EAFAIL | EBFAIL |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | GAB1 | GAB2 | GAB3 | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GBC | GCA | GAMAB1 | GAMAB2 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Continuum equivalent mass density |
| EA | Modulus of elasticity in the longitudinal (warp) direction, which corresponds to the slope of segment AB in Figure M214-1 |
| EB | Modulus of elasticity in the transverse (fill) direction, which corresponds to the slope of segment of AB in Figure M214-1 |
| GAB*i* / GAMAB*i* | Shear stress-strain behavior is modeled as piecewise linear in three segments. *See the figure to the right*. The shear moduli GAB*i* correspond to the slope of the $i^{th}$ segment. The start and end points for the segments are specified in the GAMAB[1-2] fields. |



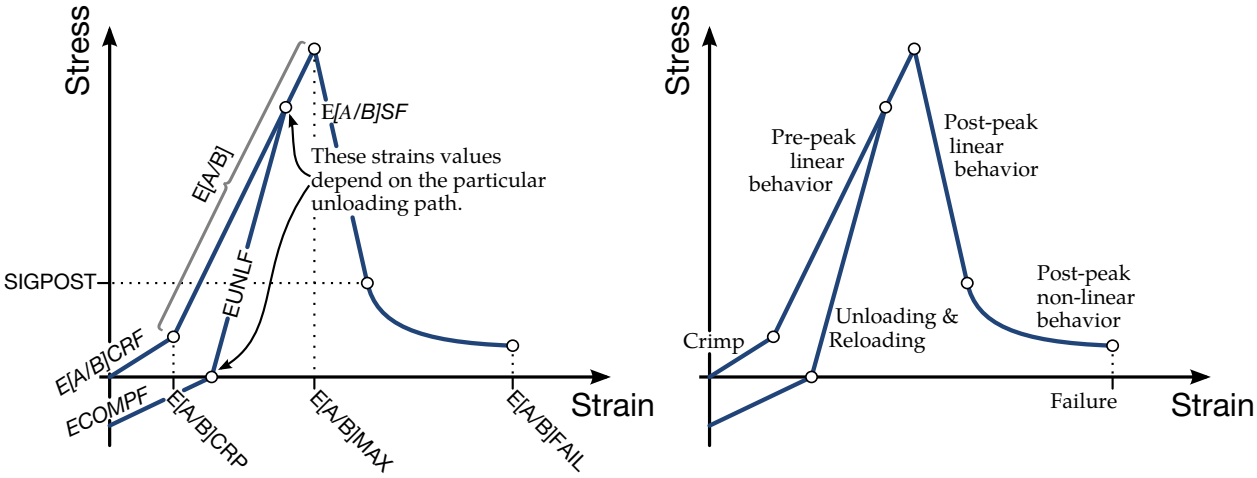



**Figure M214-1.** Stress – Strain curve for *MAT_DRY_FABRIC. This curve models the force-response in the longitudinal and transverse directions.

| VARIABLE | DESCRIPTION |
|---|---|
| GBC | $G_{bc}$, shear modulus in *bc* direction |
| GCA | $G_{ca}$, shear modulus in *ca* direction |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | XP | YP | ZP | A1 | A2 | A3 |
| Type | F | | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option. See *MAT_*OPTION*TROPIC_ELASTIC for a more complete description: |

EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the element normal by an angle BETA

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR

EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by

| VARIABLE | DESCRIPTION |
|---|---|
| | the cross product of the vector **v** with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EACRF | EBCRF | EACRP | EBCRP | | | | |
| Type | F | F | F | F | | | | |
| Remarks | 2 | 2 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EACRF | Factor for crimp region modulus of elasticity in longitudinal direction (see Figure M214-1): $E_{a,\text{crimp}} = E_{a,\text{crimpfac}}E, \quad E_{a,\text{crimpfac}} = \text{EACRF}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| EBCRF | Factor for crimp region modulus of elasticity in transverse direction (see Figure M214-1): $E_{b,\text{crimp}} = E_{b,\text{crimpfac}}E, \quad E_{b,\text{crimpfac}} = \text{EBCRF}$ |
| EACRP | Crimp strain in longitudinal direction (see Figure M214-1), $\varepsilon_{a,\text{crimp}}$ |
| EBCRP | Crimp strain in transverse direction (see Figure M214-1), $\varepsilon_{b,\text{crimp}}$ |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EASF | EBSF | EUNLF | ECOMF | EAMAX | EBMAX | SIGPOST | |
| Type | F | F | F | F | F | F | F | |
| Remarks | 2 | 2 | 2 | 2 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EASF | Factor for post-peak region modulus of elasticity in longitudinal direction (see Figure M214-1): $E_{a,\text{soft}} = E_{a,\text{softfac}}E, \quad E_{a,\text{softfac}} = \text{EASF}$ |
| EBSF | Factor for post-peak region modulus of elasticity in transverse direction (see Figure M214-1): $E_{b,\text{soft}} = E_{b,\text{softfac}}E, \quad E_{b,\text{softfac}} = \text{EBSF}$ |
| EUNLF | Factor for unloading modulus of elasticity (see Figure M214-1): $E_{\text{unload}} = E_{\text{unloadfac}}E, \quad E_{\text{unloadfac}} = \text{EUNLF}$ |
| ECOMPF | Factor for compression zone modulus of elasticity (see Figure M214-1): $E_{\text{comp}} = E_{\text{compfac}}E, \quad E_{\text{compfac}} = \text{ECOMPF}$ |
| EAMAX | Strain at peak stress in longitudinal direction (see Figure M214-1), $\varepsilon_{a,\text{max}}$ |
| EBMAX | Strain at peak stress in transverse direction (see Figure M214-1), $\varepsilon_{b,\text{max}}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGPOST | Stress value in post-peak region at which nonlinear behavior begins (see Figure M214-1), $\sigma_{\text{post}}$ |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CCE | PCE | CSE | PSE | DFAC | EMAX | EAFAIL | EBFAIL |
| Type | F | F | F | F | F | F | F | F |
| Remarks | 1 | 1 | 1 | 1 | 2 | 3 | 2, 3 | 2, 3 |

| VARIABLE | DESCRIPTION |
|---|---|
| CCE | Strain rate parameter $C$, Cowper-Symonds factor for modulus. If zero, rate effects are not considered. |
| PCE | Strain rate parameter $P$, Cowper-Symonds factor for modulus. If zero, rate effects are not considered. |
| CSE | Strain rate parameter $C$, Cowper-Symonds factor for stress to peak / failure. If zero, rate effects are not considered. |
| PSE | Strain rate parameter $P$, Cowper-Symonds factor for stress to peak / failure. If zero, rate effects are not considered. |
| DFAC | Damage factor, $d_{\text{fac}}$ |
| EMAX | Erosion strain of element, $\varepsilon_{\max}$ |
| EAFAIL | Erosion strain in longitudinal direction (see Figure M214-1), $\varepsilon_{a,\text{fail}}$ |
| EBFAIL | Erosion strain in transverse direction (see Figure M214-1), $\varepsilon_{b,\text{fail}}$ |

**Remarks:**

1. **Strain rate effects.** Strain rate effects are accounted for using a Cowper-Symonds model which scales the stress according to the strain rate:

$$\sigma^{\text{adj}} = \sigma\left(1 + \frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}} .$$

In the above equation $\sigma$ is the quasi-static stress, $\sigma^{\text{adj}}$ is the adjusted stress accounting for strain rate $\dot{\varepsilon}$, and $C$ (CCE) and $P$ (PCE) are the Cowper-Symonds factors which must be determined experimentally for each material.

The model captures the non-linear strain rate effects in many materials. With its less than unity exponent, $1/p$, this model captures the rapid increase in material properties at low strain rate, while increasing less rapidly at very high strain rates. Because stress is a function of strain rate the elastic stiffness also is:

$$\mathbf{E}^{\text{adj}} = \mathbf{E}\left(1 + \frac{\dot{\varepsilon}}{C}\right)^{\frac{1}{P}}$$

where $\mathbf{E}^{\text{adj}}$ is the adjusted elastic stiffness. Additionally, the strains to peak and strains to failure are assumed to follow a Cowper-Symonds model with, *possibly different*, constants

$$\varepsilon^{\text{adj}} = \varepsilon\left(1 + \frac{\dot{\varepsilon}}{C_s}\right)^{\frac{1}{P_s}} ,$$

where, $\varepsilon^{\text{adj}}$ is the adjusted effective strain to peak stress or strain to failure, and $C_s$ and $P_s$ are CSE and PSE respectively.

2. **Stress-strain beyond peak stress.** When strained beyond the peak stress, the stress decreases linearly until it attains a value equal to SIGPOST, at which point the stress-strain relation becomes nonlinear. In the non-linear region the stress is given by

$$\sigma = \sigma_{\text{post}}\left[1 - \left(\frac{\varepsilon - \varepsilon_{[a/b],\text{post}}}{\varepsilon_{[a/b],\text{fail}} - \varepsilon_{[a/b],\text{post}}}\right)^{d_{\text{fac}}}\right] ,$$

where $\sigma_{\text{post}}$ and $\varepsilon_{\text{post}}$ are, respectively, the stress and strain demarcating the onset of nonlinear behavior. The value of SIGPOST is the same in both the transverse and longitudinal directions, whereas $\varepsilon_{\text{a,post}}$ and $\varepsilon_{\text{b,post}}$ depend on direction and are derived internally from EASF, EBSF, and SIGPOST. The failure strain, $\varepsilon_{[a/b],\text{fail}}$, specifies the onset of failure and differs in the longitudinal and transverse directions. Lastly the exponent, $d_{\text{fac}}$, determines the shape of nonlinear stress-strain curve between $\varepsilon_{\text{post}}$ and $\varepsilon_{[a/b],\text{fail}}$.

3. **Element erosion.** The element is eroded if either (a) or (b) is satisfied:

   a) $\varepsilon_a > \varepsilon_{a,\text{fail}}$ and $\varepsilon_b > \varepsilon_{b,\text{fail}}$

   b) $\varepsilon_a > \varepsilon_{\max}$ and $\varepsilon_b > \varepsilon_{\max}$.

## *MAT_4A_MICROMEC

This is Material Type 215. A micromechanical material that distinguishes between a fiber/inclusion and a matrix material, developed by 4a engineering GmbH. It is available for shell, thick shell, and solid elements. Useful hints and an input example can be found in [1]. More theory and application notes are provided in [2].

This material is intended for anisotropic composite materials, especially for short (SFRT) and long fiber thermoplastics (LFRT). The matrix behavior is modeled with an isotropic elasto-viscoplastic von Mises model. The fiber/inclusion behavior is transverse isotropic elastic. This material model can be used for classical endless fiber composites.

The inelastic homogenization for describing the composite deformation behavior is based on:

- Mori Tanaka Meanfield Theory [3,4]
- ellipsoidal inclusions using Eshelby's solution [5,6]
- orientation averaging [7]
- a linear fitted closure approximation to determine the $4^{th}$ order fiber orientation tensor out of the user provided $2^{nd}$ order fiber orientation tensor.

The software product 4a micromec can calculate and export the thermo-elastic composite properties [8].

Failure/damage of the composite can be considered with:

- a ductile damage initiation and evolution model for the matrix (DIEM)
- fiber failure with a maximum stress criterion

References [9] and [10] provide more details on the material characterization.

The (fiber) orientation can be defined either for the whole material using Cards 2 and 3 or elementwise using *ELEMENT_(T)SHELL_BETA or *ELEMENT_SOLID_ORTHO. The manufacturing process highly influences the mechanical properties of SFRT and LFRT in injection molded parts. By mapping the fiber orientation from the process simulation to the structural analysis the local anisotropy can be considered [11,12]. The fiber orientation, length and volume fraction can therefore as well be defined for each integration point by using *INITIAL_STRESS_(T)SHELL(SOLID) [2]. Details on the history variables that can be initialized (extra history variables 9-18) can be found in the output section.

**Card Summary:**

Cards 2 through 4 specify fiber orientation. They may be overwritten with may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID. Cards 5 and 6 are for specifying parameters for the fiber/inclusion material. Cards 7 through 9 give properties associated with the matrix material.

**Card 1.** This card is required.

| MID | MMOPT | BUPD | | | FAILM | FAILF | NUMINT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| FVF | | FL | FD | | A11 | A22 | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| ROF | EL | ET | GLT | PRTL | PRTT | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XT | | | | | | SLIMXT | NCYRED |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| ROM | E | PR | | | | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| SIGYT | ETANT | | | EPS0 | C | | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| LCIDT | | | | LCDI | UPF | | NCYRED2 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | MMOPT | BUPD | | | FAILM | FAILF | NUMINT |
| Type | A | F | F | | | F | F | F |
| Default | none | 0.0 | 0.01 | | | 0.0 | 0.0 | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| MMOPT | Option to define micromechanical material behavior:<br>EQ.0.0: Elastic<br>EQ.1.0: Elastic-plastic |
| BUPD | Tolerance for update of Strain-Concentration Tensor |
| FAILM | Option for matrix failure using a ductile DIEM model. See sections Damage Initiation and Damage Evolution in the manual page for *MAT_ADD_DAMAGE_DIEM for a description of ductile damage initialization (DITYP = 0) based on stress triaxiality and a linear damage evolution (DETYP = 0) type. Also see fields LCDI and UPF on Card 9.<br>LT.0.0: \|FAILM\| is effective plastic matrix strain at failure. When the matrix plastic strain reaches this value, the element is deleted from the calculation.<br>EQ.0.0: Only visualization (triaxiality of matrix stresses)<br>EQ.1.0: Active DIEM (triaxiality of matrix stresses)<br>EQ.10.0: Only visualization (triaxiality of composite stresses)<br>EQ.11.0: Active DIEM (triaxiality of composite stresses) |
| FAILF | Option for fiber failure:<br>EQ.0.0: Only visualization (equivalent fiber stresses)<br>EQ.1.0: Active (equivalent fiber stresses) |

| VARIABLE | DESCRIPTION |
|---|---|
| NUMINT | Number or percentage of failed integration points prior to element deletion (default value is 1):<br>GT.0.0: Number of integration points which must fail before element is deleted.<br>LT.0.0: Applies only to shells. \|NUMINT\| is the percentage of layers which must fail before an element fails. For shell formulations with 4 integration points per layer, the layer is considered failed if any of the integration points in the layer fails. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is |

| VARIABLE | DESCRIPTION |
|---|---|
| | determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector, **v**, and an originating point, *P*, defining the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE-_COORDINATE_VECTOR). |
| | The fiber orientation information may be overwritten using *INITIAL_STRESS_(T)SHELL/SOLID |
| MACF | Material axes change flag for solid elements: |
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation |
| | EQ.-3: Switch material axes *a* and *c* before BETA rotation |
| | EQ.-2: Switch material axes *a* and *b* before BETA rotation |
| | EQ.1: No change, default |
| | EQ.2: Switch material axes *a* and *b* after BETA rotation |
| | EQ.3: Switch material axes *a* and *c* after BETA rotation |
| | EQ.4: Switch material axes *b* and *c* after BETA rotation |
| | Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 3 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Define components of vector **v** for AOPT = 3 and 4. |
| D1, D2, D3 | Define components of vector **d** for AOPT = 2. |
| BETA | Angle in degrees of a material rotation about the *c*-axis, available for AOPT = 0 (shells and tshells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVF | | FL | FD | | A11 | A22 | |
| Type | F | | F | F | | F | F | |
| Default | 0.0 | | 0.0 | 1.0 | | 1.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVF | Fiber volume or mass fraction:<br>GT.0.0: Fiber volume fraction<br>LT.0.0: \|FVF\| is the fiber mass fraction. |
| FL | Fiber length unless FD = 1. If FD = 1, then it is the aspect ratio (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID) |
| FD | Fiber diameter |

| VARIABLE | DESCRIPTION |
|---|---|
| A11 | Value of first principal fiber orientation (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID) |
| A22 | Value of second principal fiber orientation (may be overwritten by *INITIAL_STRESS_(T)SHELL/SOLID) |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ROF | EL | ET | GLT | PRTL | PRTT | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ROF | Mass density of fiber |
| EL | $E_L$, Young's modulus of fiber in the longitudinal direction |
| ET | $E_T$, Young's modulus of fiber in the transverse direction |
| GLT | $G_{LT}$, shear modulus LT |
| PRTL | $\nu_{TL}$, Poisson's ratio TL |
| PRTT | $\nu_{TT}$, Poisson's ratio TT |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XT | | | | | | SLIMXT | NCYRED |
| Type | F | | | | | | F | F |
| Default | 0.0 | | | | | | 0.0 | 10 |

| VARIABLE | DESCRIPTION |
|---|---|
| XT | Fiber tensile strength in the longitudinal direction |

| VARIABLE | DESCRIPTION |
|---|---|
| SLIMXT | Factor to determine the minimum stress limit in the fiber after stress maximum (fiber tension) |
| NCYRED | Number of cycles for stress reduction from maximum to minimum (fiber tension) |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ROM | E | PR | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | 0.0 | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ROM | Mass density of matrix |
| E | Young's modulus of matrix |
| PR | Poisson's ratio of matrix |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGYT | ETANT | | | EPS0 | C | | |
| Type | F | F | | | F | F | | |
| Default | 0.0 | 0.0 | | | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGYT | Yield stress of matrix in tension |
| ETANT | Tangent modulus of matrix in tension, ignore if LCIDT > 0 is defined. |
| EPS0 | Quasi-static threshold strain rate (Johnson-Cook model) for bilinear hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Johnson-Cook constant for bilinear hardening |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDT | | | | LCDI | UPF | | NCYRED2 |
| Type | F | | | | F | F | | F |
| Default | 0 | | | | 0 | 0.0 | | 1 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDT | Load curve ID or table ID for defining effective stress as a function of effective plastic strain in tension of matrix material (Table to include strain-rate effects, viscoplastic formulation.) |
| LCDI | Curve/table for ductile damage initiation parameter. The definitions depend on if the element is a shell or solid.<br>**Shell elements.** LCDI can be a load curve or table ID. A load curve represents plastic strain at onset of damage as function of stress triaxiality. A table represents plastic strain at onset of damage as function of stress triaxiality and plastic strain rate.<br>**Solid elements.** LCDI can be a load curve, table, or 3D table ID. A load curve represents plastic strain at onset of damage as function of stress triaxiality. A table represents plastic strain at onset of damage as function of stress triaxiality and lode angle. A 3D table represents plastic strain at onset of damage as function of stress triaxiality, lode angle and plastic strain rate. |
| UPF | Damage evolution parameter<br>GT.0.0: Plastic displacement at failure, $u_f^p$<br>LT.0.0: \|UPF\| is a table ID for $u_f^p$ as a function of triaxiality and damage |
| NCYRED2 | In case of matrix failure (IFAILM = 1 or 11), number of cycles for stress reduction of fiber stresses until the integration point will be marked as failed. |

**Output:**

For this material, "Plastic Strain" is the equivalent plastic strain in the matrix. Extra history variables may be requested for (t)shell (NEIPS) and solid (NEIPH) elements with *DATABASE_EXTENT_BINARY. Extra history variables 1 through 8 are intended for post-processing while 9 through18 are intended for initialization with *INITIAL_-STRESS_(T)SHELL/SOLID. They have the following meaning:

| History Variable # | Description |
|---|---|
| 1 | Equivalent plastic strain rate of matrix |
| 2 | Triaxiality of matrix, $\eta = -p/q$ |
| 3 | Lode parameter of matrix, $\xi = -\frac{27J_3}{2q}$ |
| 4 | Damage initiation, $d$, of matrix (Ductile Criteria) |
| 5 | Damage evolution, $D$, of matrix |
| 6 | Fiber reserve factor |
| 7 | Fiber damage variable |
| 8 | Fiber stress reduction variable (NCYRED2) |
| 9 | Value of first principal fiber orientation, A11 |
| 10 | Value of second principal fiber orientation, A22 |
| 11 | For shells, $\cos\alpha$ where $\alpha$ is the in-plane angle between the material coordinate system and the element coordinate system. For solids, $q_{11}$ where $q_{11}$ is the $x$-direction component of the first orientation direction in the element coordinate system. |
| 12 | For shells, -sin $\alpha$ where $\alpha$ is the in-plane angle between the material coordinate system and the element coordinate system. For solids, $q_{12}$ where $q_{12}$ is the $y$-direction component of the first orientation direction in the element coordinate system. |
| 13 | For shells, unused. For solids, $q_{13}$ where $q_{13}$ is the $z$-direction component of the first orientation direction in the element coordinate system. |
| 14 | For shells, unused. For solids, $q_{31}$ where $q_{31}$ is the $x$-direction component of the third orientation direction in the element coordinate system. |
| 15 | For shells, unused. For solids, $q_{32}$ where $q_{32}$ is the $y$-direction component of the third orientation direction in the element coordinate system. |

| History Variable # | Description |
|---|---|
| 16 | For shells, unused. For solids, $q_{33}$ where $q_{33}$ is the $z$-direction component of the third orientation direction in the element coordinate system. |
| 17 | Fiber volume fraction, FVF |
| 18 | Fiber length, FL |

**Material Orientation:**

Figure 4 of Reference 13 shows the 2nd order orientation tensor for which there are eigenvectors and corresponding eigenvalues. The coordinate system based on the eigenvectors is the material coordinate system. The values $q_{11}$,…, $q_{33}$ for solids (history variables 11 through 16) and the values $\cos(\alpha)$ and -sin $\alpha$ for shells (history variables 11 and 12) specify this material coordinate system with respect to the element coordinate system. The values $a_1$ and $a_2$ (A11 and A22 of history variables 9 and 10) shown in the figure represent the eigenvalues, or in other words, the lengths of the ellipsoid. Thus. history variables 9 and 10 give the shape of the ellipsoid while history variables 11 through 16 give the orientation.

**References:**


[1] Reithofer, P., et. al, *MAT_4A_MICROMEC – micro mechanic based material model, 14th German LS-DYNA Conference (2016), Bamberg.

[2] Reithofer, P., et. al, *MAT_4A_MICROMEC – Theory and application notes, 11th European LS-DYNA Conference (2017), Salzburg.

[3] Mori, T., Tanaka, K., Average Stress in Matrix and Average elastic Energy of Materials with misfitting Inclusions, Acta Metallurgica, Vol.21, pp.571-574, (1973).

[4] Tucker Ch. L. III, Liang Erwin: Stiffness Predictions for Unidirectional Short-Fibre Composites: Review and Evaluation, Composites Science and Technology, 59, (1999).

[5] Maewal A., Dandekar D.P., Effective Thermoelastic Properties of Short-Fibre Composites, Acta Mechanica, 66, (1987).

[6] Eshelby, J. D., The determination of the elastic field of an ellipsoidal inclusion, and related problems, Proceedings of the Royal Society, London, Vol.A, No241, pp.376-396, (1957).

[7] Mlekusch, B., Kurzfaserverstärkte Thermoplaste, Dissertation, Montanuniversität Leoben (1997).

[8] http://micromec.4a.co.at

[9] Reithofer et al., *Material characterization of composites using micro mechanic models as key enabler*, NAFEMS DACH, Bamberg 2016.

[10] http://impetus.4a.co.at

[11] Reithofer et al., *Short and long fiber reinforced thermoplastics material models in LS-DYNA*, 10th European LS-DYNA Conference, Würzburg 2015.

[12] http://fibermap.4a.co.at

[13] Liebold et al., *The Significance of the Production Process of FRP Parts for the Performance in Crashworthiness*, 14th International LS-DYNA Users Conference, 2016, http://www.dyna-look.com/14th-international-ls-dyna-conference/simulation/the-significance-of-the-production-process-of-frp-parts-for-the-performance-in-crashworthiness

## *MAT_ELASTIC_PHASE_CHANGE

This is Material Type 216, a generalization of Material Type 1, for which material properties change on an element-by-element basis upon crossing a plane in space. This is an isotropic hypoelastic material and is available *only* for shell element types.

**Phase 1 Properties.**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO1 | E1 | PR1 | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | 0.0 | | | | |

**Phase 2 Properties.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | RO2 | E2 | PR2 | | | | |
| Type | | F | F | F | | | | |
| Default | | none | none | 0.0 | | | | |

**Transformation Plane Card**.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | X1 | Y1 | Z1 | X2 | Y2 | Z2 | THKFAC | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | 1.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO*i* | Mass density for phase *i* |
| E*i* | Young's modulus for phase *i* |
| PR*i* | Poisson's ratio for phase *i* |
| X1, Y1, Z1 | Coordinates of a point on the phase transition plane |
| X2, Y2, Z2 | Coordinates of a point that defines the exterior normal with the first point |
| THKFAC | Scale factor applied to the shell thickness after the phase transformation |

**Phases:**

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, coordinates X1, Y1, and Z1, lies on the plane. The second point, coordinates X2, Y2, and Z2, defines the exterior normal as a unit vector in the direction from the first point to the second point.

**Remarks:**

This hypoelastic material model may not be stable for finite (large) strains. If large strains are expected, a hyperelastic material model, such as *MAT_002 or *MAT_217, would be more appropriate.

## *MAT_ORTHOTROPIC_ELASTIC_PHASE_CHANGE

This is Material Type 217. It is a generalization of the orthotropic version of Material Type 2 for which material properties change on an element-by-element basis upon crossing a plane in space.

This material is valid *only* for shells. The stress update is incremental. The elastic constants are formulated in terms of Cauchy stress and true strain.

**Phase 1 Material Parameters Card 1.**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO1 | EA1 | EB1 | EC1 | PRBA1 | PRCA1 | PRCB1 |
| Type | A | F | F | F | F | F | F | F |

**Phase 1 Material Parameters Card 2.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB1 | GBC1 | GCA1 | AOPT1 | | | | |
| Type | F | F | F | F | | | | |

**Local Coordinate System Card 1 (phase 1).**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A11 | A21 | A31 | | |
| Type | | | | F | F | F | | |

**Local Coordinate System Card 2 (phase 1).**

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V11 | V21 | V31 | D11 | D21 | D31 | BETA1 | |
| Type | F | F | F | F | F | F | F | |

**Phase 2 Material Parameters Card 1.**

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | R02 | EA2 | EB2 | EC2 | PRBA2 | PRCA2 | PRCB2 |
| Type | | F | F | F | F | F | F | F |

**Phase 2 Material Parameters Card 2.**

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB2 | GBC2 | GCA2 | | | | | |
| Type | F | F | F | | | | | |

**Local Coordinate System Card 1 (phase 2).**

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A12 | A22 | A32 | | |
| Type | | | | F | F | F | | |

**Local Coordinate System Card 2 (phase 2).**

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V12 | V22 | V32 | D12 | D22 | D32 | BETA2 | |
| Type | F | F | F | F | F | F | F | |

**Transformation Plane Card.**

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | X1 | Y1 | Z1 | X2 | Y2 | Z2 | THKFAC | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | 1.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO*i* | Mass density for phase *i* |
| EA*i* | $E_a$, Young's modulus in *a*-direction for phase *i* |
| EB*i* | $E_b$, Young's modulus in *b*-direction for phase *i* |
| EC*i* | $E_c$, Young's modulus in *c*-direction phase *i* (nonzero value required but not used for shells) |
| PRBA*i* | $\nu_{ba}$, Poisson's ratio in the *ba* direction for phase *i* |
| PRCA*i* | $\nu_{ca}$, Poisson's ratio in the *ca* direction for phase *i* |
| PRCB*i* | $\nu_{cb}$, Poisson's ratio in the *cb* direction for phase *i* |
| GAB*i* | $G_{ab}$, shear modulus in the *ab* direction for phase *i* |
| GBC*i* | $G_{bc}$, shear modulus in the *bc* direction for phase *i* |
| GCA*i* | $G_{ca}$, shear modulus in the *ca* direction for phase *i* |
| AOPT*i* | Material axes option for phase *i* (see *MAT_*OPTION*TROPIC for more details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes as shown in part (a) of Figure M2-1. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. The material axes are then rotated about the normal vector to the surface of the shell by the angle BETA. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then, **a** and **b** are rotated about **c** by an angle BETA.<br>LT.0.0: \|AOPT\| is a coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| A1*i*, A2*i*, A3*i* | Define components of the $i^{th}$ phase's vector **a** for AOPT = 2 |
| V1*i*, V2*i*, V3*i* | Define components of the $i^{th}$ phase's vector **v** for AOPT = 3 |
| D1*i*, D2*i*, D3*i* | Define components of the $i^{th}$ phase's vector **d** for AOPT = 2 |
| BETA*i* | Material angle of $i^{th}$ phase in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |
| X1, Y1, Z1 | Coordinates of a point on the phase transition page |
| X2, Y2, Z2 | Coordinates of a point that defines the exterior normal with the first point |
| THKFAC | Scale factor applied to the shell thickness after the phase transformation. |

**Phases:**

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates X1, Y1, and Z1, lies on the plane. The second point, defined by the coordinates X2, Y2, and Z2, define the exterior normal as a unit vector in the direction from the first point to the second point.

**Material Formulation:**

The material law that relates stresses to strains is defined as:

$$\mathbf{C} = \mathbf{T}^{\mathrm{T}}\mathbf{C}_L\mathbf{T}$$

where $\mathbf{T}$ is a transformation matrix, and $\mathbf{C}_L$ is the constitutive matrix defined in terms of the material constants of the orthogonal material axes, $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$. The inverse of $\mathbf{C}_L$ is defined as:

$$\mathbf{C}_L^{-1} = \begin{bmatrix} \frac{1}{E_a} & -\frac{v_{ba}}{E_b} & -\frac{v_{ca}}{E_c} & 0 & 0 & 0 \\ -\frac{v_{ab}}{E_a} & \frac{1}{E_b} & -\frac{v_{cb}}{E_c} & 0 & 0 & 0 \\ -\frac{v_{ac}}{E_a} & -\frac{v_{bc}}{E_b} & \frac{1}{E_c} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{ab}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{bc}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}} \end{bmatrix}$$

where,

$$\frac{v_{ab}}{E_a} = \frac{v_{ba}}{E_b}, \frac{v_{ca}}{E_c} = \frac{v_{ac}}{E_a}, \frac{v_{cb}}{E_c} = \frac{v_{bc}}{E_b}.$$

## *MAT_MOONEY-RIVLIN_PHASE_CHANGE

This is Material Type 218. It is a generalization of Material Type 27, for which material properties change on an element-by-element basis upon crossing a plane in space.

**Phase 1 Card 1.**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO1 | PR1 | A1 | B1 | REF | | |
| Type | A | F | F | F | F | F | | |

**Phase 1 Card 2.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL1 | SW1 | ST1 | LCID1 | | | | |
| Type | F | F | F | F | | | | |

**Phase 2 Card 1.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | RO2 | PR2 | A2 | B2 | | | |
| Type | | F | F | F | F | | | |

**Phase 2 Card 2.**

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SGL2 | SW2 | ST2 | LCID2 | | | | |
| Type | F | F | F | F | | | | |

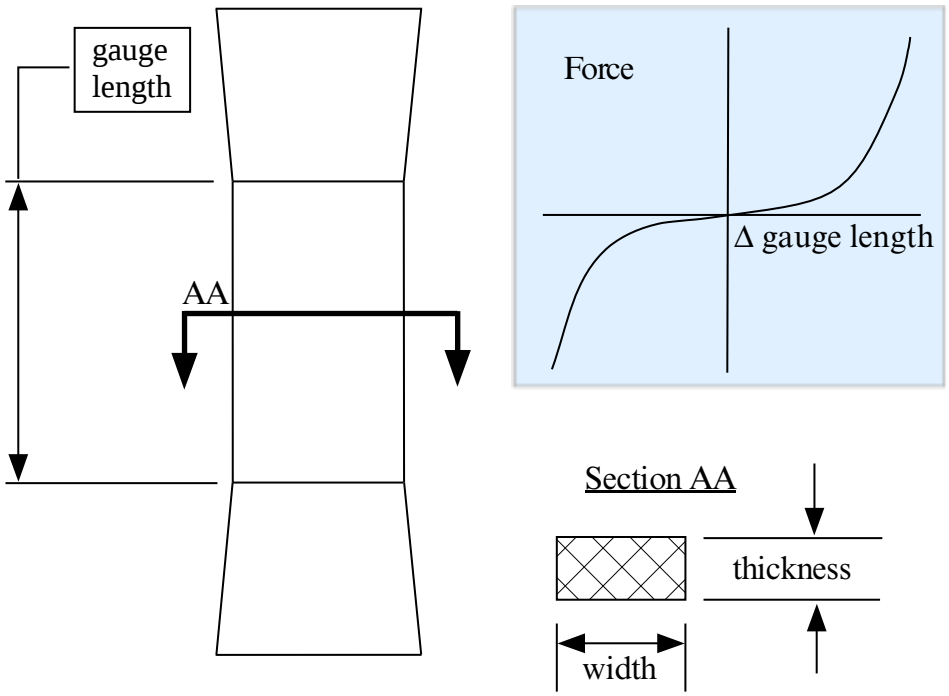



**Figure M218-1.** Uniaxial specimen for experimental data

**Transformation Plane Card.**

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | X1 | Y1 | Z1 | X2 | Y2 | Z2 | THKFAC | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | 1.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO*i* | Mass density for phase *i*. |
| PR*i* | Poisson's ratio where *i* indicates the phase. A value between 0.49 and 0.5 is recommended. Smaller values may not work. |
| A*i* | Constant for the $i^{th}$ phase. See the literature and the equations defined in Material Formulation. |

| VARIABLE | DESCRIPTION |
|---|---|
| B*i* | Constant for the $i^{th}$ phase. See the literature and the equations defined in Material Formulation. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY (see there for more details).<br>EQ.0.0: Off,<br>EQ.1.0: On. |

If A = B = 0.0, then a least squares fit is computed from tabulated uniaxial data via a load curve. The following information should be defined:

| VARIABLE | DESCRIPTION |
|---|---|
| SGL*i* | Specimen gauge length $l_0$ for the $i^{th}$ phase, see Figure M218-1. |
| SW*i* | Specimen width for the $i^{th}$ phase, see Figure M218-1. |
| ST*i* | Specimen thickness for the $i^{th}$ phase, see Figure M218-1. |
| LCID*i* | Curve ID for the $i^{th}$ phase (see *DEFINE_CURVE) giving the force as a function of actual change, $\Delta L$, in the gauge length. See also Figure M218-2 for an alternative definition. |
| X1, Y1, Z1 | Coordinates of a point on the phase transition plane. |
| X2, Y2, Z2 | Coordinates of a point that defines the exterior normal with the first point. |
| THKFAC | Scale factor applied to the shell thickness after the phase transformation. |

**Phases:**

The material properties for each element are initialized using the data for the first phase. After the center of the element passes through the transition plane defined by the two points, the material properties are irreversibly changed to the second phase.

The plane is defined by two points. The first point, defined by the coordinates X1, Y1, and Z1, lies on the plane. The second point, defined by the coordinates X2, Y2, and Z2, define the exterior normal as a unit vector in the direction from the first point to the second point.

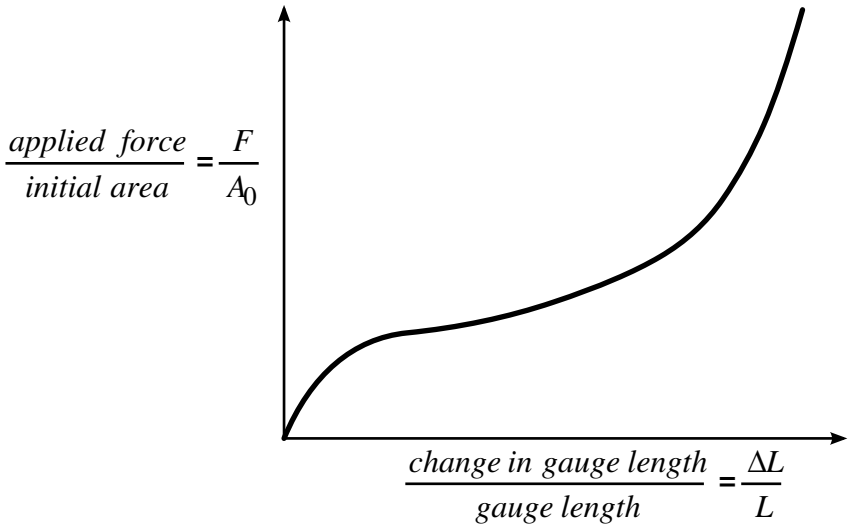



**Figure M218-2** The stress as a function strain curve can be used instead of the force as a function of the change in the gauge length by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force.

**Material Formulation:**

The strain energy density function is defined as:

$$W = A(I-3) + B(II-3) + C\left(III^{-2}-1\right) + D(III-1)^2$$

where

$$\begin{aligned} C &= 0.5\,A + B \\ D &= \frac{A(5v-2)+B(11v-5)}{2(1-2v)} \\ \nu &= \text{Poisson's ratio} \\ 2(A+B) &= \text{shear modulus of linear elasticity} \\ I, II, III &= \text{invariants of right Cauchy-Green Tensor C.} \end{aligned}$$

The load curve definition that provides the uniaxial data should give the change in gauge length, $\Delta L$, as a function of the corresponding force. In compression, both the force and the change in gauge length must be specified as negative values. In tension the force and change in gauge length should be input as positive values. The principal stretch ratio in the uniaxial direction, $\lambda_1$, is then given by

$$\lambda_1 = \frac{L_0 + \Delta L}{L_0}$$

with $L_0$ being the initial length and $L$ being the actual length.

Alternatively, the stress as a function strain curve can also be input by setting the gauge length, thickness, and width to unity (1.0) and defining the engineering strain in place of the change in gauge length and the nominal (engineering) stress in place of the force. See Figure M218-1.

The initialization phase performs a least squares fit to the experimental data. The d3hsp file provides a comparison between the fit and the actual input. It is a good idea to visually check to make sure it is acceptable. The coefficients $A$ and $B$ are also printed in the output file. It is also advised to use the material driver (see Appendix K) for checking out the material model.

## *MAT_CODAM2

This is Material Type 219. This material model is the second generation of the UBC Composite Damage Model (CODAM2) for solid, shell, and thick shell elements developed at The University of British Columbia. The model is a sub-laminate-based continuum damage mechanics model for fiber reinforced composite laminates made up of transversely isotropic layers. The material model includes an optional non-local averaging and element erosion.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | | PRBA | | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | | | NLAYER | R1 | R2 | NFREQ | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | AOPT | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | MACF |
|---|---|---|---|---|---|---|---|

**Card 5.** For each of the NLAYER layers specify on angle. Include as many cards as needed to set NLAYER values.

| ANGLE1 | ANGLE2 | ANGLE3 | ANGLE4 | ANGLE5 | ANGLE6 | ANGLE7 | ANGLE8 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| IMATT | IFIBT | ILOCT | IDELT | SMATT | SFIBT | SLOCT | SDELT |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| IMATC | IFIBC | ILOCC | IDELC | SMATC | SFIBC | SLOCC | SDELC |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| ERODE | ERPAR1 | ERPAR2 | RESIDS | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | | PRBA | | PRCB |
| Type | A | F | F | F | | F | | F |
| Default | none | none | none | none | | none | | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in $a$-direction. This is the modulus along the direction of fibers. |
| EB | $E_b$, Young's modulus in $b$-direction. This is the modulus transverse to fibers. |
| PRBA | $\nu_{ba}$, Poisson's ratio, $ba$ (minor in-plane Poisson's ratio). |
| PRCB | $\nu_{cb}$, Poisson's ratio, $cb$ (Poisson's ratio in the plane of isotropy). |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | | | NLAYER | R1 | R2 | NFREQ | |
| Type | F | | | I | F | F | I | |
| Default | none | | | 0 | 0.0 | 0.0 | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ba}$, Shear modulus, $ab$ (in-plane shear modulus). |
| NLAYER | Number of layers in the sub-laminate excluding symmetry. As an example, in a $[0/45/-45/90]_{3s}$, NLAYER = 4. |

| VARIABLE | DESCRIPTION |
|---|---|
| R1 | Non-local averaging radius |
| R2 | Currently not used |
| NFREQ | Number of time steps between update of neighbor list for nonlocal smoothing.<br>EQ.0: do only one search at the start of the calculation. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | AOPT | |
| Type | F | F | F | F | F | F | I | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| AOPT | Material axes option (see *MAT_OPTIONTROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center. This is the **a**-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d**, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between |

| VARIABLE | DESCRIPTION |
|---|---|
| | the inner surface and outher surface defined by the first four nodes and the last four nodes of the connectivity, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then, **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: \|AOPT\| is a coordinate system ID (see *DEFINE_COORDINATE_OPTION). |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | MACF |
| Type | F | F | F | F | F | F | F | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 1 |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. BETA may be overriden on the element card. See *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO. |
| MACF | Material axes change flag for solid elements: |
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-3: Switch material axes *a* and *c* before BETA rotation |
| | EQ.-2: Switch material axes *a* and *b* before BETA rotation |
| | EQ.1: No change, default |
| | EQ.2: Switch material axes *a* and *b* after BETA rotation |
| | EQ.3: Switch material axes *a* and *c* after BETA rotation |
| | EQ.4: Switch material axes *b* and *c* after BETA rotation |

Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes switch as specified by MACF, but no BETA rotation is performed.

**Angle Cards.** For each of the NLAYER layers specify on angle. Include as many cards as needed to set NLAYER values.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ANGLE1 | ANGLE2 | ANGLE3 | ANGLE4 | ANGLE5 | ANGLE6 | ANGLE7 | ANGLE8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ANGLE*i* | Rotation angle in degrees of the layers with respect to the material axes. Input one for each layer. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IMATT | IFIBT | ILOCT | IDELT | SMATT | SFIBT | SLOCT | SDELT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| IMATT | Initiation strain for damage in the matrix (transverse) under tensile conditions |
| IFIBT | Initiation strain for damage in the fiber (longitudinal) under tensile conditions |
| ILOCT | Initiation strain for the anti-locking mechanism. This parameter should be equal to the saturation strain for the fiber damage mechanism under tensile conditions. |
| IDELT | Not working in the current version. It can be used for visualization purposes only. |
| SMATT | Saturation strain for damage in the matrix (transverse) under tensile conditions |
| SFIBT | Saturation strain for damage in the fiber (longitudinal) under tensile conditions |
| SLOCT | Stuation strain for the anti-locking mechanism under tensile conditions. The recommended value for this parameter is ILOCT + 0.02. |
| SDELT | Not working for the current version. It can be used for visualization purposes only. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IMATC | IFIBC | ILOCC | IDELC | SMATC | SFIBC | SLOCC | SDELC |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| IMATC | Initiation strain for damage in matrix (transverse) under compressive condition |
| IFIBC | Initiation strain for damage in the fiber (longitudinal) under compressive condition |
| ILOCC | Initiation strain for the anti-locking mechanism. This parameter should be equal to the saturation strain for the fiber damage mechanism under compressive condition. |

| VARIABLE | DESCRIPTION |
|---|---|
| IDELC | Initiation strain for delamination. Not working in the current version. Can be used for visualization purpose only. |
| SMATC | Saturation strain for damage in matrix (transverse) under compressive condition |
| SFIBC | Saturation strain for damage in the fiber (longitudinal) under compressive condition |
| SLOCC | Saturation strain for the anti-locking mechanism under compressive condition. The recommended value for this parameter is ILOCC + 0.02. |
| SDELC | Delamination strain. Not working in the current version. Can be used for visualization purpose only. |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ERODE | ERPAR1 | ERPAR2 | RESIDS | | | | |
| Type | I | F | F | F | | | | |
| Default | 0 | none | none | 0.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ERODE | Erosion Flag (see Element Erosion in the remarks)<br>EQ.0: erosion is turned off.<br>EQ.1: non-local strain based erosion criterion<br>EQ.2: local strain based erosion criterion<br>EQ.3: use both ERODE = 1 and ERODE = 2 criteria. |
| ERPAR1 | The erosion parameter #1 used in ERODE types 1 and 3. ERPAR1 ≥ 1.0. The recommended value of ERPAR1 is 1.2. |
| ERPAR2 | The erosion parameter #2 used in ERODE types 2 and 3. The recommended value is five times SLOCC defined in Card 7. |
| RESIDS | Residual strength for layer damage |

### Model Description:

CODAM2 is developed for modeling the nonlinear, progressive damage behavior of laminated fiber-reinforced plastic materials. The model is based on the work by (Forghani, 2011; Forghani et al. 2011a; Forghani et al. 2011b) and is an extension of the original model, CODAM (Williams et al. 2003).

Briefly, the model uses a continuum damage mechanics approach and the following assumptions have been made in its development:

1. The material is an orthotropic medium consisting of a number of repeating units through the thickness of the laminate, called sub-laminates. For example, $[0/\pm45/90]$ is in a $[0/\pm45/90]_{8S}$ laminate.

2. The nonlinear behavior of the composite sub-laminate is only caused by damage evolution. Nonlinear elastic or plastic deformations are not considered.

### Formulation:

The in-plane secant stiffness of the damaged laminate is represented as the summation of the effective contributions of the layers in the laminate as shown.

$$\mathbf{A}^d = \sum t_k \mathbf{T}_k^{\mathrm{T}} \mathbf{Q}_k^d \mathbf{T}_k$$

where $\mathbf{T}_k$ is the transformation matrix for the strain vector, and $\mathbf{Q}_k^d$ is the in-plane secant stiffness of $k^{\text{th}}$ layer in the principal orthotropic plane, and $t_k$ is the thickness of the $k^{\text{th}}$ layer of an $n$-layered laminate.

A physically-based and yet simple approach has been employed here to derive the damaged stiffness matrix. Two reduction coefficients, $R_f$ and $R_m$, that represent the reduction of stiffness in the longitudinal (fiber) and transverse (matrix) directions have been employed. The shear modulus has also been reduced by the matrix reduction parameter. The major and minor Poisson's ratios have been reduced by $R_f$and $R_m$respectively. A sub-laminate-level reduction, $R_L$, is incorporated to avoid spurious stress locking in the damaged zone. This would lead to an effective reduced stiffness matrix $\mathbf{Q}_k^d$. The reduction coefficients are equal to 1 in the undamaged condition and gradually decrease to 0 for a saturated damage condition.

$$\mathbf{Q}_k^d = R_L \begin{bmatrix} \dfrac{(R_f)_k E_1}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & \dfrac{(R_f)_k (R_m)_k \nu_{12} E_2}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & 0 \\ \dfrac{(R_f)_k (R_m)_k \nu_{12} E_2}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & \dfrac{(R_m)_k E_2}{1-(R_f)_k (R_m)_k \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & (R_m)_k G_{12} \end{bmatrix} = \mathbf{Q}_k^{d^{\mathrm{T}}} ,$$

where $E_1$, $E_2$, $\nu_{12}$, $\nu_{21}$, and $G_{12}$ are the elastic constants of the lamina.

**Damage Evolution:**

In CODAM2, the evolution of damage mechanisms is expressed in terms of equivalent strain parameters. The equivalent strain function that governs the fiber stiffness reduction parameter is written in terms of the longitudinal normal strains by

$$\varepsilon_{f,k}^{\text{eq}} = \varepsilon_{11,k}, \qquad k = 1, \ldots, n$$

The equivalent strain function that governs the matrix stiffness reduction parameter is written in an interactive form in terms of the transverse and shear components of the local strain:

$$\varepsilon_{m,k}^{\text{eq}} = \text{sign}(\varepsilon_{22,k})\sqrt{(\varepsilon_{22,k})^2 + \left(\frac{\gamma_{12,k}}{2}\right)^2}, \qquad k = 1, \ldots, n$$

The sign of the transverse normal strain plays a very important role in the initiation and growth of damage since it indicates the compressive or tensile nature of the transverse stress. Therefore, the equivalent strain for the matrix damage carries the sign of the transverse normal strain.

Evolution of the overall damage mechanism (anti-locking) is written in terms of the maximum principal strains:

$$\varepsilon_L^{\text{eq}} = \max[\text{prn}(\varepsilon)] \ .$$

Within the framework of non-local strain-softening formulations adopted here, all damage modes, be it intra-laminar (i.e. fiber and matrix damage) or overall sub-laminate modes are considered to be a function of the non-local (averaged) equivalent strain defined as:

$$\bar{\varepsilon}_\alpha^{\text{eq}} = \int_{\Omega_{\mathbf{X}}} \varepsilon_\alpha^{\text{eq}}(\mathbf{x}) w_\alpha(\mathbf{X} - \mathbf{x}) d\Omega \ ,$$

where the subscript $\alpha$ denotes the mode of damage: fiber ($\alpha = f$) and matrix ($\alpha = m$) damage in each layer, $k$, within the sub-laminate or associated with the overall sub-laminate, namely, locking ($\alpha = L$). Thus, for a given sub-laminate with $n$ layers, $\varepsilon_\alpha^{\text{eq}}$ and $\bar{\varepsilon}_\alpha^{\text{eq}}$ are vectors of size $2n + 1$. $\mathbf{X}$ represents the position vector of the original point of interest and $\mathbf{x}$ denotes the position vector of all other points (Gauss points) in the averaging zone denoted by $\Omega$. In classical isotropic non-local averaging approach, this zone is taken to be spherical (or circular in 2D) with a radius of $r$ (named R1 in the material input card). The parameter, $r$, which affects the size of the averaging zone, introduces a length scale into the model that is linked directly to the predicted size of the damage zone. Averaging is done with a bell-shaped weight function, $w_\alpha$, evaluated by

$$w_\alpha = \left[1 - \left(\frac{d}{r}\right)^2\right]^2 \ ,$$

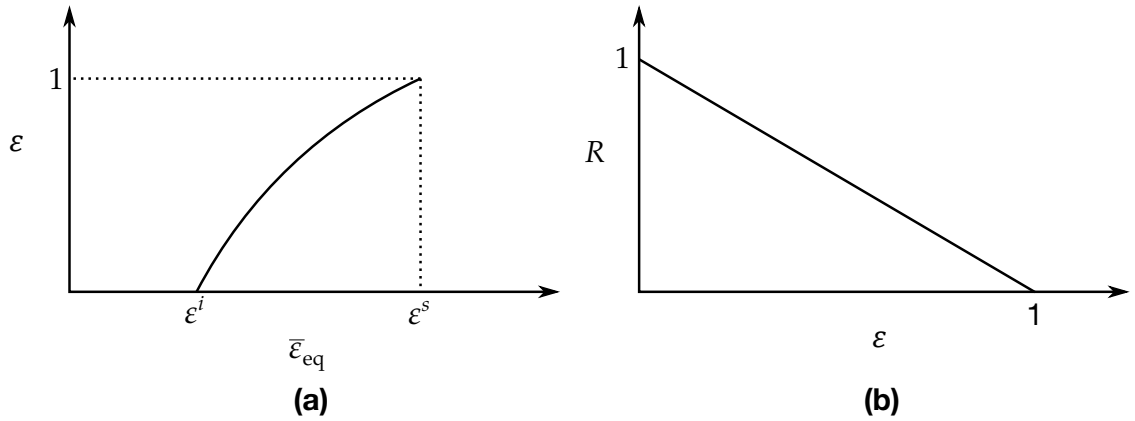



**Figure M219-1.** Illustrations of (a) damage parameter and (b) reduction parameter.

where $d$ is the distance from the integration point of interest to another integration point within the averaging zone.

The damage parameters, $\omega_\alpha$, are calculated as a function of the corresponding averaged equivalent strains. In CODAM2 the damage parameters are assumed to grow as a hyperbolic function of the damage potential (non-local equivalent strains) such that when used in conjunction with stiffness reduction factors that vary linearly with the damage parameters they result in a linear strain-softening response (or a bilinear stress-strain curve) for each mode of damage

$$\omega_\alpha = \frac{\left(|\bar{\varepsilon}_\alpha^{\mathrm{eq}}| - \varepsilon_\alpha^i\right)}{\left(\varepsilon_\alpha^s - \varepsilon_\alpha^i\right)} \frac{\varepsilon_\alpha^s}{|\bar{\varepsilon}_\alpha^{eq}|}, \qquad |\bar{\varepsilon}_\alpha^{\mathrm{eq}}| - \varepsilon_\alpha^i > 0$$

where superscripts $i$ and $s$ denote, respectively, the damage initiation and saturation values of the strain quantities to which they are assigned. The initiation and saturation parameters are defined in Cards 6 and 7. Damage is considered to be a monotonically increasing function of time, $t$, such that

$$\omega_\alpha = \max_{\tau<t}(\omega_\alpha^\tau) \ ,$$

where $\omega_\alpha^t$ is the value of $\omega_\alpha$ for the current time (load state), and $\omega_\alpha^\tau$ represents the state of damage at previous times $\tau \le t$.

Damage is applied by scaling the layer stress by reduction parameters

$$R_\alpha = 1 - \omega_\alpha$$

where $\alpha = f$ and $\alpha = m$. The layer stresses are summed and then then scaled by reduction parameter

$$R_L = 1 - \omega_L \ .$$

Figures M219-1 (a) and (b) show the relationship between the damage and reduction parameters

If the parameter RESIDS > 0, damage in the layers is limited such that

$$R_f = \max(\text{RESIDS}, 1 - \omega_f)$$
$$R_m = \max(\text{RESIDS}, 1 - \omega_m)$$

**Element Erosion:**

When ERODE > 0, an erosion criterion is checked at each integration point. Shell elements and thick shell elements will be deleted when the erosion criterion has been met at all integration points. Brick elements will be deleted when the erosion criterion is met at any of the integration points. For ERODE = 1, the erosion criterion is met when maximum principal strain exceeds either SLOCT × ERPAR1 for elements in tension, or SLOCC × ERPAR1 for elements in compression. Elements are in tension when the magnitude of the first principal strain is greater than the magnitude of the third principal strain and in compression when the third principal strain is larger. When R1 > 0, the ERODE = 1 criterion is checked using the non-local (averaged) principal strain. For ERODE = 2, the erosion criterion is met when the local (non-averaged) maximum principal strain exceeds ERPAR2. For ERODE = 3, both of these erosion criteria are checked. For visualization purposes, the ratio of the maximum principal strain over the limit is stored in the location of plastic strain which is written by default to the elout and d3plot files.

**History Variables:**

History variables for CODAM2 are enumerated in the following tables. To include them in the d3plot database, use NEIPH (solids) or NEIPS (shells) on *DATABASE_EXTENT_BINARY. For solid elements, add 4 to the variable numbers in the table because the first 6 history variables are reserved.

***Damage parameters***

| History Variable # | Description |
|---|---|
| 3 | Overall (anti-locking) Damage. |
| 4 | Delamination Damage (for visualization only) |
| 5 | Fiber damage in the first layer |
| 6 | Matrix damage in the first layer |
| 7 | Fiber damage in the second layer |
| 8 | Matrix damage in the second layer |
| ⋮ | ⋮ |
| 3 + 2 × NLAYER | Fiber damage in the last layer |

| History Variable # | Description |
|---|---|
| $4 + 2 \times$ NLAYER | Matrix damage in the last layer |

***Equivalent Strains used to evaluate damage (averaged if R1 > 0)***

| History Variable # | Description |
|---|---|
| $5 + 2 \times$ NLAYER | $\varepsilon_R^{\text{eq}}$ |
| $6 + 2 \times$ NLAYER | $\varepsilon_{f,1}^{\text{eq}}$ |
| $7 + 2 \times$ NLAYER | $\varepsilon_{m,1}^{\text{eq}}$ |
| $8 + 2 \times$ NLAYER | $\varepsilon_{f,2}^{\text{eq}}$ |
| $9 + 2 \times$ NLAYER | $\varepsilon_{m,2}^{\text{eq}}$ |
| ⋮ | ⋮ |
| $4 + 4 \times$ NLAYER | $\varepsilon_{f,n}^{\text{eq}}$ |
| $5 + 4 \times$ NLAYER | $\varepsilon_{f,n}^{\text{eq}}$ |

***Total Strain***

| History Variable # | Description |
|---|---|
| $6 + 4 \times$ NLAYER | $\varepsilon_x$ |
| $7 + 4 \times$ NLAYER | $\varepsilon_y$ |
| $8 + 4 \times$ NLAYER | $\varepsilon_z$ |
| $9 + 4 \times$ NLAYER | $\gamma_{xy}$ |
| $10 + 4 \times$ NLAYER | $\gamma_{yz}$ |
| $11 + 4 \times$ NLAYER | $\gamma_{zx}$ |

## *MAT_RIGID_DISCRETE

This is Material Type 220. It is a rigid material for shells or solids. Unlike *MAT_020, a *MAT_220 part can be discretized into multiple disjoint pieces and have each piece behave as an independent rigid body. The inertia properties for the disjoint pieces are determined directly from the finite element discretization.

Nodes of a *MAT_220 part cannot be shared by any other rigid part. A *MAT_220 part may share nodes with deformable structural and solid elements.

This material option can be used to model granular material where the grains interact through an automatic single surface contact definition. Another possible use includes modeling bolts as rigid bodies where the bolts belong to the same part ID. This model eliminates the need to represent each rigid piece with a unique part ID.

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |

## *MAT_ORTHOTROPIC_SIMPLIFIED_DAMAGE

This is Material Type 221. It is an orthotropic material with optional simplified damage and optional failure for composites. This model is valid for 3D solid elements, for thick shell formulations 3, 5, and 7, and for SPH elements. The elastic behavior is the same as *MAT_022. Nine damage variables are defined such that damage is different in tension and compression. These damage variables are applicable to $E_a$, $E_b$, $E_c$, $G_{ab}$, $G_{bc}$ and $G_{ca}$. In addition, nine failure criteria on strains are available. When failure occurs, elements are deleted (erosion). Failure depends on the number of integration points failed through the element. See the material description below.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | | AOPT | MACF | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| NERODE | NDAM | EPS1TF | EPS2TF | EPS3TF | EPS1CF | EPS2CF | EPS3CF |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| EPS12F | EPS23F | EPS13F | EPSD1T | EPSC1T | CDAM1T | EPSD2T | EPSC2T |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| CDAM2T | EPSD3T | EPSC3T | CDAM3T | EPSD1C | EPSC1C | CDAM1C | EPSD2C |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| EPSC2C | CDAM2C | EPSD3C | EPSC3C | CDAM3C | EPSD12 | EPSC12 | CDAM12 |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| EPSD23 | EPSC23 | CDAM23 | EPSD31 | EPSC31 | CDAM31 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in $a$-direction |
| EB | $E_b$, Young's modulus in $b$-direction |
| EC | $E_c$, Young's modulus in $c$-direction |
| PRBA | $\nu_{ba}$, Poisson ratio |
| PRCA | $\nu_{ca}$, Poisson ratio |
| PRCB | $\nu_{cb}$, Poisson ratio |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | | AOPT | MACF | | |
| Type | F | F | F | | F | I | | |
| Default | none | none | none | | 0.0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, Shear modulus |
| GBC | $G_{bc}$, Shear modulu |
| GCA | $G_{ca}$, Shear modulus |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, $P$, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| VARIABLE | DESCRIPTION |
|---|---|
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 4 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card, see *ELEMENT_SOLID_ORTHO. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NERODE | NDAM | EPS1TF | EPS2TF | EPS3TF | EPS1CF | EPS2CF | EPS3CF |
| Type | I | I | F | F | F | F | F | F |
| Default | 0 | 0 | $10^{20}$ | $10^{20}$ | $10^{20}$ | $-10^{20}$ | $-10^{20}$ | $-10^{20}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| NERODE | Element erosion flag. For multi-integration point elements, each of the failure strains mentioned below for NERODE ≥ 2 need only occur in one integration point to trigger element erosion. For NERODE values 6 to 11, which require more than one failure strain be reached, those failure strains need not occur in the same integration point.<br>EQ.0: No erosion (default)<br>EQ.1: Erosion occurs when one failure strain is reached in all integration points.<br>EQ.2: Erosion occurs when one failure strain is reached.<br>EQ.3: Erosion occurs when a tension or compression failure strain in the *a*-direction is reached. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4: Erosion occurs when as a tension or compression failure strain in the *b*-direction is reached. |
| | EQ.5: Erosion occurs when a tension or compression failure strain in the *c*-direction is reached. |
| | EQ.6: Erosion occurs when tension or compression failure strain in both the *a*- and *b*-directions are reached. |
| | EQ.7: Erosion occurs when tension or compression failure strain in both the *b*- and *c*-directions are reached. |
| | EQ.8: Erosion occurs when tension or compression failure strain in both the *a*- and *c*-directions are reached. |
| | EQ.9: Erosion occurs when tension or compression failure strain in all 3 directions are reached. |
| | EQ.10: Erosion occurs when tension or compression failure strain in both the *a*- and *b*-directions is reached and either of the out-of-plane failure shear strains (*bc* or *ac*) is reached. |
| | EQ.11: Erosion occurs when tension failure strain in either the *a*- or *b*-directions is reached and either of the out-of-plane failure shear strains (*bc* or *ac*) is reached. |
| NDAM | Damage flag:<br>EQ.0: No damage (default)<br>EQ.1: Damage in tension only (null for compression)<br>EQ.2: Damage in tension and compression |
| EPS1TF | Failure strain in tension along the *a*-direction |
| EPS2TF | Failure strain in tension along the *b*-direction |
| EPS3TF | Failure strain in tension along the *c*-direction |
| EPS1CF | Failure strain in compression along the *a*-direction |
| EPS2CF | Failure strain in compression along the *b*-direction |
| EPS3CF | Failure strain in compression along the *c*-direction |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS12F | EPS23F | EPS13F | EPSD1T | EPSC1T | CDAM1T | EPSD2T | EPSC2T |
| Type | F | F | F | F | F | F | F | F |
| Default | $10^{20}$ | $10^{20}$ | $10^{20}$ | 0. | 0. | 0. | 0. | 0. |

| VARIABLE | DESCRIPTION |
|---|---|
| EPS12F | Failure shear strain in the *ab*-plane |
| EPS23F | Failure shear strain in the *bc*-plane |
| EPS13F | Failure shear strain in the *ac*-plane |
| EPSD1T | Damage threshold in tension along the *a*-direction, $\varepsilon_{1t}^{s}$ |
| EPSC1T | Critical damage threshold in tension along the *a*-direction, $\varepsilon_{1t}^{c}$ |
| CDAM1T | Critical damage in tension along the *a*-direction, $D_{1t}^{c}$ |
| EPSD2T | Damage threshold in tension along the *b*-direction, $\varepsilon_{2t}^{s}$ |
| EPSC2T | Critical damage threshold in tension along the *b*-direction, $\varepsilon_{2t}^{c}$ |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CDAM2T | EPSD3T | EPSC3T | CDAM3T | EPSD1C | EPSC1C | CDAM1C | EPSD2C |
| Type | I | I | F | F | F | F | F | F |
| Default | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

| VARIABLE | DESCRIPTION |
|---|---|
| CDAM2T | Critical damage in tension along the *b*-direction, $D_{2t}^{c}$ |
| EPSD3T | Damage threshold in tension along the *c*-direction, $\varepsilon_{3t}^{s}$ |
| EPSC3T | Critical damage threshold in tension along the *c*-direction, $\varepsilon_{3t}^{c}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| CDAM3T | Critical damage in tension along the *c*-direction, $D_{3t}^c$ |
| EPSD1C | Damage threshold in compression along the *a*-direction, $\varepsilon_{1c}^s$ |
| EPSC1C | Critical damage threshold in compression along the *a*-direction, $\varepsilon_{1c}^c$ |
| CDAM1C | Critical damage in compression along the *a*-direction, $D_{1c}^c$ |
| EPSD2C | Damage threshold in compression along the *b*-direction, $\varepsilon_{2c}^s$ |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSC2C | CDAM2C | EPSD3C | EPSC3C | CDAM3C | EPSD12 | EPSC12 | CDAM12 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

| VARIABLE | DESCRIPTION |
|---|---|
| EPSC2C | Critical damage threshold in compression along the *b*-direction, $\varepsilon_{2c}^c$ |
| CDAM2C | Critical damage in compression along the *b*-direction, $D_{2c}^c$ |
| EPSD3C | Damage threshold in compression along the *c*-direction, $\varepsilon_{3c}^s$ |
| EPSC3C | Critical damage threshold in compression along the *c*-direction, $\varepsilon_{3c}^c$ |
| CDAM3C | Critical damage in compression along the *c*-direction, $D_{3c}^c$ |
| EPSD12 | Damage threshold for shear in the *ab*-plane, $\varepsilon_{12}^s$ |
| EPSC12 | Critical damage threshold for shear in the *ab*-plane, $\varepsilon_{12}^c$ |
| CDAM12 | Critical damage for shear in the *ab*-plane, $D_{12}^c$ |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSD23 | EPSC23 | CDAM23 | EPSD31 | EPSC31 | CDAM31 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0. | 0. | 0. | 0. | 0. | 0. | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EPSD23 | Damage threshold for shear in the *bc*-plane, $\varepsilon_{23}^{s}$ |
| EPSC23 | Critical damage threshold for shear in the *bc*-plane, $\varepsilon_{23}^{c}$ |
| CDAM23 | Critical damage for shear in the *bc*-plane, $D_{23}^{c}$ |
| EPSD31 | Damage threshold for shear in the *ac*-plane, $\varepsilon_{31}^{s}$ |
| EPSC31 | Critical damage threshold for shear in the *ac*-plane, $\varepsilon_{31}^{c}$ |
| CDAM31 | Critical damage for shear in the *ac*-plane, $D_{31}^{c}$ |

**Remarks:**

If $\varepsilon_k^c < \varepsilon_k^s$ , no damage is considered. Failure occurs only when failure strain is reached.

Failure can occur along the 3 orthotropic directions, in tension, in compression and for shear behavior. Nine failure strains drive the failure. When failure occurs, elements are deleted (erosion). Under the control of the NERODE flag, failure may occur either when only one integration point has failed, when several integration points have failed or when all integrations points have failed.

Damage applies to the 3 Young's moduli and the 3 shear moduli. Damage is different for tension and compression. Nine damage variables are used: $d_{1t}$, $d_{2t}$, $d_{3t}$, $d_{1c}$, $d_{2c}$, $d_{3c}$, $d_{12}$, $d_{23}$, $d_{13}$. The damaged flexibility matrix is:

$$
-S^{\text{dam}} = \begin{pmatrix}
\frac{1}{E_a(1-d_{1[t,c]})} & \frac{-v_{ba}}{E_b} & \frac{-v_{ca}}{E_c} & 0 & 0 & 0 \\
\frac{-v_{ba}}{E_b} & \frac{1}{E_b(1-d_{2[t,c]})} & \frac{-v_{cb}}{E_c} & 0 & 0 & 0 \\
\frac{-v_{ca}}{E_c} & \frac{-v_{cb}}{E_c} & \frac{1}{E_c(1-d_{3[t,c]})} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{ab}(1-d_{12})} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{bc}(1-d_{23})} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{ca}(1-d_{31})}
\end{pmatrix}
$$

The nine damage variables are calculated as follows:

$$
d_k = \max\left(d_k, D_k^c \left\langle \frac{\varepsilon_k - \varepsilon_k^s}{\varepsilon_k^c - \varepsilon_k^s} \right\rangle_+ \right)
$$

with $k = 1t, 2t, 3t, 1c, 2c, 3c, 12, 23, 31$.

$$
\langle \quad \rangle_+ \text{ is the positive part: } \langle x \rangle_+ = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}.
$$

Damage in compression may be deactivated with the NDAM flag. In this case, damage in compression is null, and only damage in tension and for shear behavior are taken into account.

The nine damage variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input by the optional *DATABASE_EXTENT_BINARY card as variable NEIPH. These additional variables are tabulated below:

| History Variable | Description | Value | LS-PrePost History Variable |
|---|---|---|---|
| $d_{1t}$ | damage in traction along $a$ | | plastic strain |
| $d_{2t}$ | damage in traction along $b$ | | 1 |
| $d_{3t}$ | damage in traction along $c$ | | 2 |
| $d_{1c}$ | damage in compression along $a$ | 0 - no damage | 3 |
| $d_{2c}$ | damage in compression along $b$ | | 4 |
| $d_{3c}$ | damage in compression along $c$ | $0 < d_k < D_k^c$ - damage | 5 |
| $d_{12}$ | shear damage in $ab$-plane | | 6 |
| $d_{23}$ | shear damage in $bc$-plane | | 7 |
| $d_{13}$ | shear damage in $ac$-plane | | 8 |

The first damage variable is stored in the place of effective plastic strain. The eight other damage variables may be plotted in LS-PrePost as element history variables.

## *MAT_TABULATED_JOHNSON_COOK_{OPTION}

This is Material Type 224. This type models an elasto-viscoplastic material with arbitrary stress versus strain curve(s) and arbitrary strain rate dependency. Plastic heating causes the temperature to increase adiabatically and material softening. Optional plastic failure strain can be a function of triaxiality, strain rate, temperature, and/or element size. Please take careful note of the sign convention for triaxiality, as illustrated in Figure M224-1. This material model resembles the original Johnson-Cook material (see *MAT_015) but with the possibility of general tabulated input parameters.

An equation of state (*EOS) is optional for solid elements, tshell formulations 3 and 5, and 2D continuum elements. It is invoked by setting EOSID to a nonzero value in *PART. If an equation of state is used, the material model gives only the deviatoric stresses, and the equation of state provides the pressure.

Available options include:

<BLANK>

LOG_INTERPOLATION

With LOG_INTERPOLATION, the strain rate effect in table LCK1 is interpolated with logarithmic interpolation.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | CP | TR | BETA | NUMINT |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 1.0 | 1.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCK1 | LCKT | LCF | LCG | LCH | LCI | BFLG | |
| Type | I | I | I | I | I | I | I | |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

This card is optional.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FAILOPT | NUMAVG | NCYFAIL | ERODE | LCPS | | | |
| Type | I | I | I | I | I | | | |
| Default | 0 | 1 | 1 | 0 | 0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus:<br>GT.0.0: Constant value is used.<br>LT.0.0: -E gives curve ID for temperature dependence. |
| PR | Poisson's ratio |
| CP | Specific heat (superseded by heat capacity in *MAT_THERMAL_*OPTION* if a coupled thermal/structural analysis) |
| TR | Room temperature |
| BETA | Fraction of plastic work converted into heat (supersedes FWORK in *CONTROL_THERMAL_SOLVER if a coupled thermal/structural analysis):<br>EQ.0.0: Defaults to 1.0.<br>GT.0.0: Constant value is used.<br>LT.0.0: -BETA gives a curve ID for strain rate dependence, a table ID for strain rate and temperature dependence, a 3-dimensional table ID for temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence, or a 4-dimensional table ID for triaxiality (TABLE_4D), temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence. Please see the description of BFLG below for an alternative interpretation of TABLE_3D arguments. |

| VARIABLE | DESCRIPTION |
|---|---|
| NUMINT | GT.0.0: Number of integration points that must fail before the element is deleted. Available for shells and solids.<br>LT.0.0: -NUMINT is the percentage of integration points/layers that must fail before the shell element fails. For fully integrated shells, a layer fails if one integration point fails. Then, the given percentage of layers must fail before the element fails. It is only available for shells.<br>EQ.-200: Turns off erosion for shells and solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE. |
| LCK1 | Load curve ID, table ID, or 3D table ID. The load curve gives effective stress as a function of effective plastic strain. The table gives for each plastic strain rate value a load curve ID specifying the (isothermal) effective stress as a function of effective plastic strain for that rate. As in *MAT_024, natural logarithmic strain rates can be used by setting the *first* strain rate to a negative value. See Remark 1.<br>If referring to a three-dimensional table ID, the yield stress can be a function of temperature (TABLE_3D), plastic strain rate (TABLE), and plastic strain (CURVE). LCKT is ignored in that case. |
| LCKT | Table ID defining for each temperature value a load curve ID giving the (quasi-static) effective stress versus effective plastic strain for that temperature. See Remark 1. |
| LCF | Load curve ID or table ID. The load curve ID defines plastic failure strain (or scale factor – see Remark 2) as a function of triaxiality. The table ID defines for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI. |
| LCG | Load curve ID defining plastic failure strain (or scale factor – see Remark 2) as a function of plastic strain rate (*The curve should not extrapolate to zero or failure may occur at low strain*). If the *first* abscissa value in the curve corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for all abscissa values. See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCH | Load curve ID defining plastic failure strain (or scale factor – see Remark 2) as a function of temperature. See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI. |
| LCI | Load curve ID, table ID, or table_3D ID. The load curve ID defines plastic failure strain (or scale factor – see Remark 2) as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality. If referring to a three-dimensional table ID, plastic failure strain can be a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2 for a description of the combination of LCF, LCG, LCH, and LCI. |
| BFLG | Flag for treatment of case BETA < 0 with TABLE_3D (available for solid elements only):<br>EQ.0: Dissipation factor $\beta$ is a function of temperature, strain rate, and plastic strain (as described above).<br>EQ.1: Dissipation factor $\beta$ is a function of maximum shear strain (TABLE_3D), strain rate (TABLE), and element size (CURVE). |
| FAILOPT | Flag for additional failure criterion $F_2$ (see Remark 3).<br>EQ.0: Off (default)<br>EQ.1: On |
| NUMAVG | Number of time steps for the running average of the plastic failure strain in the additional failure criterion. The default is 1 (no averaging). See Remark 3. |
| NCYFAIL | Number of time steps that the additional failure criterion must be met before element deletion. The default is 1. See Remark 3. |
| ERODE | Erosion flag (only for solid elements):<br>EQ.0: Default, element erosion is allowed.<br>EQ.1: Element does not erode; deviatoric stresses set to zero when element fails.<br>EQ.2: Element does not erode. The stress response is uncoupled from material damage. We intend this option for forging simulations with 3D *r*-adaptivity. |
| LCPS | Table ID with first principal stress limit as a function of plastic |

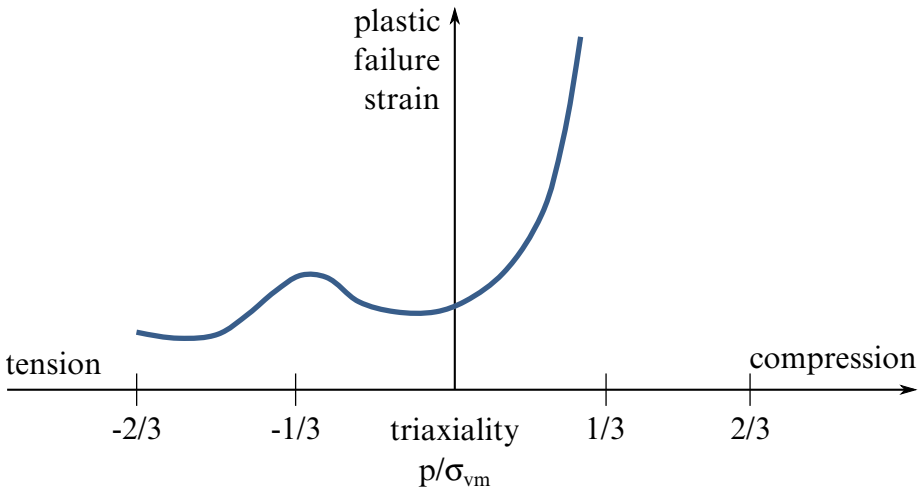



**Figure M224-1.** Typical failure curve for metal sheet, modeled with shell elements.

| VARIABLE | DESCRIPTION |
|---|---|
| | strain (curves) and plastic strain rate (table). This option is for post-processing purposes only and gives an indication of areas in the structure where failure is likely to occur. History variable #17 is 1.0 for integration points that have exceeded the limit; otherwise, it has a value of 0.0. |

**Remarks:**

1. **Flow stress.** The flow stress $\sigma_y$ is expressed as a function of plastic strain $\varepsilon_p$, plastic strain rate $\dot{\varepsilon}_p$ and temperature $T$ through the following formula (using load curves/tables LCK1 and LCKT):

$$s_y = k_1(\varepsilon_p, \dot{\varepsilon}_p)\frac{k_t(\varepsilon_p, T)}{k_t(\varepsilon_p, T_R)}$$

   Note that $T_R$ is a material parameter and should correspond to the temperature used when performing the room temperature tensile tests. If simulations are to be performed with an initial temperature $T_I$ deviating from $T_R$, then this temperature should be set using *INITIAL_STRESS_SOLID/SHELL by setting history variable #14 for solid elements or history variable #10 for shell elements.

2. **Plastic failure strain.** Optional plastic failure strain is defined as a function of triaxiality $p/\sigma_{\text{vm}}$, Lode parameter, plastic strain rate $\dot{\varepsilon}_p$, temperature $T$ and initial element size $l_c$ (square root of element area for shells and volume over maximum area for solids) by

$$\varepsilon_{\text{pf}} = f\left(\frac{p}{\sigma_{\text{vm}}}, \frac{27J_3}{2\sigma_{\text{vm}}^3}\right) g(\dot{\varepsilon}_p) h(T) i\left(l_c, \frac{p}{\sigma_{\text{vm}}}\right)$$

using load curves/tables LCF, LCG, LCH, and LCI. If more than one of these four variables, LCF, LCG, LCH, and LCI, are defined, be aware that the net plastic failure strain is essentially the product of multiple functions, as shown in the above equation. This means that one and only one of the variables LCF, LCG, LCH, and LCI can point to a curve(s) that has plastic strain along the curve ordinate. The remaining nonzero variable(s) LCF, LCG, LCH, and LCI should point to a curve(s) that has a unitless scaling factor along the curve ordinate.

A typical failure curve LCF for a metal sheet, modeled with shell elements is shown in Figure M224-1. Triaxiality should be monotonically increasing in this curve. A reasonable range for triaxiality is -2/3 to 2/3 if shell elements are used (plane stress). For 3-dimensional stress states (solid elements), the possible range of triaxiality goes from -∞ to +∞, but to get a good resolution in the internal load curve discretization (depending on parameter LCINT of *CONTROL_SOLUTION) you should define lower limits, e.g. -1 to 1 if LCINT = 100 (default).

3. **Failure criterion.** The default failure criterion of this material model depends on plastic strain evolution $\dot{\varepsilon}_p$ and on plastic failure strain $\varepsilon_{\text{pf}}$ and is obtained by accumulation over time:

$$F = \int \frac{\dot{\varepsilon}_p}{\varepsilon_{pf}} dt$$

where element erosion takes place when $F \geq 1$. This accumulation provides load-path-dependent treatment of failure. The value of $F$ is stored as history variable #8 for shells and #12 for solids.

An additional, load-path independent, failure criterion can be invoked by setting FAILOPT = 1, where the current state of plastic strain is used:

$$F_2 = \frac{\varepsilon_p}{\varepsilon_{pf}}$$

Two additional parameters can be used as countermeasures against stress oscillations for this failure criterion. With NUMAVG active, plastic failure strain is averaged over NUMAVG time steps for the $F_2$ criterion. The value of $F_2$, taking into account any averaging per NUMAVG, is stored as history variable #14 for shells and #16 for solids. NUMAVG cannot exceed 30. NCYFAIL defines the number of time steps that $F_2 \geq 1$ must be met before element deletion takes place. The number of time steps that $F_2 \geq 1$ is stored as history variable #15 for shells and #19 for solids.

4. **Change in temperature.** Temperature increase is caused by plastic work

$$T = T_R + \frac{\beta}{C_p \rho} \int \sigma_y \dot{\varepsilon}_p dt$$

with room temperature $T_R$, dissipation factor $\beta$, specific heat $C_p$, and density $\rho$. If a coupled thermal/structural analysis is performed, temperatures from the thermal solver are used. In that case, the thermal solver receives the plastic work that is to be converted to heat as an additional volumetric heat source. If no dissipation factor is defined, the value of FWORK in *CONTROL_THERMAL_-SOLVER is used.

5. **Failure when used with *CONSTRAINED_TIED_NODES_WITH_FAILURE.** For *CONSTRAINED_TIED_NODES_WITH_FAILURE, the failure is based on the damage instead to the plastic strain.

6. **History variables.** History variables may be post-processed through additional variables. The number of additional variables for shells/solids written to the d3plot and d3thdt databases is input by the optional *DATABASE_EXTENT_-BINARY card as variable NEIPS/NEIPH. Specifically, when used with shell element type 14 or 15, history variable output will be as a solid element, not a shell element. The relevant additional variables of this material model are tabulated below:

| History Variable # | Description for Shell Elements |
|---|---|
| 1 | Plastic strain rate |
| 7 | Plastic work |
| 8 | Ratio of plastic strain to plastic failure strain |
| 9 | Element size |
| 10 | Temperature |
| 11 | Plastic failure strain |
| 12 | Triaxiality |
| 16 | Fraction of plastic work to heat |
| 17 | LCPS: critical value |

| History Variable # | Description for Solid Elements |
|---|---|
| 5 | Plastic strain rate |
| 8 | Plastic failure strain |
| 9 | Triaxiality |
| 10 | Lode parameter |
| 11 | Plastic work |
| 12 | Ratio of plastic strain to plastic failure strain |

| History Variable # | Description for Solid Elements |
|---|---|
| 13 | Element size |
| 14 | Temperature |
| 17 | LCPS: critical value |
| 18 | Fraction of plastic work to heat |

## *MAT_TABULATED_JOHNSON_COOK_GYS_{OPTION}

This is Material Type 224_GYS. It is an isotropic, elastic-plastic material law with a J3-dependent yield surface. This material considers tensile/compressive asymmetry in the material response, which is essential for HCP metals. It is available for solid elements.

Available options include:

<BLANK>

LOG_INTERPOLATION

With LOG_INTERPOLATION, the strain rate effect in table LCK1 (Card 2) is interpolated with logarithmic interpolation.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | CP | TR | BETA | NUMINT |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 1.0 | 1.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCK1 | LCKT | LCF | LCG | LCH | LCI | | |
| Type | I | I | I | I | I | I | | |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCCR | LCCT | LCSR | LCST | IFLAG | SFIEPM | NITER | |
| Type | I | I | I | I | I | F | I | |
| Default | 0 | 0 | 0 | 0 | 0 | 1 | 100 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus:<br>GT.0.0: Constant value is used.<br>LT.0.0: Temperature-dependent Young's modulus given by load curve ID = -E |
| PR | Poisson's ratio |
| CP | Specific heat |
| TR | Room temperature |
| BETA | Fraction of plastic work converted into heat (superseded by FWORK in *CONTROL_THERMAL_SOLVER if a coupled thermal/structural analysis):<br>GT.0.0: Constant value is used.<br>LT.0.0: -BETA gives a load curve ID for strain rate dependence, a table ID for strain rate and temperature dependence, a 3-dimensional table ID for temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence, or a 4-dimensional table ID for triaxiality (TABLE_4D), temperature (TABLE_3D), strain rate (TABLE), and plastic strain (CURVE) dependence. |
| NUMINT | Number of integration points which must fail before the element is deleted.<br>EQ.-200: Turns off erosion for solids. Not recommended unless used in conjunction with *CONSTRAINED_TIED_NODES_FAILURE. |
| LCK1 | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) effective stress as a function of effective plastic strain for that rate. |
| LCKT | Table ID defining for each temperature value a load curve ID giving the (qu asi-static) effective stress as a function of effective plastic strain for that temperature. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCF | Load curve ID or table ID. The load curve specifies plastic failure strain as a function of triaxiality. The table specifies for each Lode parameter a load curve ID giving the plastic failure strain versus triaxiality for that Lode parameter. (Table option not yet generally supported). |
| LCG | Load curve ID for specifying plastic failure strain as a function of plastic strain rate. |
| LCH | Load curve ID for specifying plastic failure strain as a function of temperature |
| LCI | Load curve ID, table ID, or 3D table ID. The load curve ID defines plastic failure strain as a function of element size. The table ID defines for each triaxiality a load curve ID giving the plastic failure strain versus element size for that triaxiality. If referring to a three-dimensional table ID, plastic failure strain can be a function of Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). |
| LCCR | Table ID. The curves in this table define compressive yield stress as a function of plastic strain or effective plastic strain (see IFLAG). The table ID defines for each plastic strain rate value or effective plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain or effective plastic strain for that rate. |
| LCCT | Table ID defining for each temperature value a load curve ID giving the (quasi-static) compressive yield stress as a function of strain for that temperature. The curves in this table define compressive yield stress as a function of plastic strain or effective plastic strain (see IFLAG). |
| LCSR | Table ID. The load curves define shear yield stress in function of plastic strain or effective plastic strain (see IFLAG). The table ID defines for each plastic strain rate value or effective plastic strain rate value a load curve ID giving the (isothermal) shear yield stress as a function of plastic strain or effective plastic strain for that rate. |
| LCST | Table ID defining for each temperature value a load curve ID giving the (quasi-static) shear yield stress as a function of strain for that temperature. The load curves define shear yield stress as a function of plastic strain or effective plastic strain (see IFLAG). |

| VARIABLE | DESCRIPTION |
|---|---|
| IFLAG | Flag to specify abscissa for LCCR, LCCT, LCSR, LCST:<br>EQ.0: Compressive and shear yields are given as functions of plastic strain as defined in Remark 1 (default).<br>EQ.1: Compressive and shear yields are given as functions of effective plastic strain. |
| SFIEPM | Scale factor on the initial estimate of the plastic multiplier |
| NITER | Number of secant iterations to be performed |

**Remarks:**

1. **IFLAG.** If IFLAG = 0, the compressive and shear curves are defined as follows:

$$\sigma_c(\varepsilon_{pc},\dot{\varepsilon}_{pc}), \quad \varepsilon_{pc} = \varepsilon_c - \frac{\sigma_c}{E}, \quad \dot{\varepsilon}_{pc} = \frac{\partial \varepsilon_{pc}}{\partial t}$$
$$\sigma_s(\gamma_{ps},\dot{\gamma}_{ps}), \quad \gamma_{ps} = \gamma_s - \frac{\sigma_s}{G}, \quad \dot{\gamma}_{ps} = \frac{\partial \gamma_{ps}}{\partial t}$$

   Two history variables (#16 plastic strain in compression and #17 plastic strain in shear) are stored in addition to those history variables already stored for *MAT_224.

   If IFLAG = 1, the compressive and shear curves are defined as follows:

$$\sigma_c(\dot{\lambda},\lambda), \quad \sigma_s(\dot{\lambda},\lambda), \quad \dot{W}_p = \sigma_{\text{eff}}\dot{\lambda}$$

2. **History variables.** History variables may be post-processed through additional variables. The number of additional variables for solids written to the d3plot and d3thdt databases is input through NEIPH on optional *DATABASE_EXTENT_BINARY. The relevant additional history variables for this material model are listed below:

| History Variable # | Description |
|---|---|
| 5 | Plastic strain rate |
| 8 | Plastic failure strain |
| 9 | Triaxiality |
| 10 | Lode parameter |
| 11 | Plastic work |
| 12 | Damage |
| 13 | Element size |

| History Variable # | Description |
|---|---|
| 14 | Temperature |
| 16 | Plastic strain in compression |
| 17 | Plastic strain in shear |

## *MAT_VISCOPLASTIC_MIXED_HARDENING

This is Material Type 225. An elasto-viscoplastic material with an arbitrary stress as a function of strain curve and arbitrary strain rate dependency can be defined. Kinematic, isotropic, or a combination of kinematic and isotropic hardening can be specified. Also, failure based on plastic strain can be defined.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | LCSS | BETA | | |
| Type | A | F | F | F | I | F | | |
| Default | none | none | none | none | none | 0.0 | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FAIL | | | | | | | |
| Type | F | | | | | | | |
| Default | $10^{20}$ | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (*PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| LCSS | Load curve ID or Table ID. Load curve ID defining effective stress as a function of effective plastic strain The table ID defines for each strain rate value a load curve ID giving the stress as a function of effective plastic strain for that rate, See Figure M24-1. The stress as a function of effective plastic strain curve for the lowest value of strain rate is used if the strain rate falls below the minimum value. Likewise, the stress as a function of effective plastic strain curve for |

| VARIABLE | DESCRIPTION |
|---|---|
| | the highest value of strain rate is used if the strain rate exceeds the maximum value. NOTE: The strain rate values defined in the table may be given as the natural logarithm of the strain rate. If the *first* stress-strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used. Since the tables are internally discretized to equally space the points, natural logarithms are necessary, for example, if the curves correspond to rates from $10^{-4}$ to $10^{4}$. |
| BETA | Hardening parameter, 0.0 < BETA < 1.0:<br>EQ.0.0: Pure kinematic hardening<br>EQ.1.0: Pure isotropic hardening<br>0.0 < BETA < 1.0: Mixed hardening |
| FAIL | Failure flag:<br>LT.0.0: User-defined failure subroutine is called to determine failure<br>EQ.0.0: Failure is not considered. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |

## *MAT_KINEMATIC_HARDENING_BARLAT89_*{OPTION}*

This is Material Type 226. This model combines the Yoshida & Uemori non-linear kinematic hardening rule (*MAT_125) with the 3-parameter material model of Barlat and Lian [1989] (*MAT_36) to model metal sheets under cyclic plasticity loading with anisotropy in plane stress condition. Lankford parameters are used for the definition of the anisotropy. Yoshida's theory describes the hardening rule with a "two surfaces" method: the yield surface and the bounding surface. In the forming process, the yield surface does not change in size, but its center moves with deformation; the bounding surface changes both in size and location.

Available options include:

<BLANK>

NLP

The NLP option estimates necking failure using the Formability Index (F.I.), which accounts for the non-linear strain paths seen in metal forming applications (see Remark 4). When using this option, specify IFLD in Card 3. Since the NLP option also works with a linear strain path, it is recommended to be used as the default failure criterion in metal forming. The NLP option is also available for *MAT_036, *MAT_037, and *MAT_125.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | M | R00 | R45 | R90 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| CB | Y | SC | K | RSAT | SB | H | HLCID |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | IOPT | C1 | C2 | IFLD | EA | COE | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | M | R00 | R45 | R90 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$. Optionally, the Young's modulus can be a function of effective plastic strain. See Remark 5. In that case this is the initial Young's modulus. |
| PR | Poisson's ratio, $\nu$ |
| M | the exponent in Barlat's yield criterion, $m$ |
| R00 | $R_{00}$, Lankford parameter in 0° direction |
| R45 | $R_{45}$, Lankford parameter in 45° direction |
| R90 | $R_{90}$, Lankford parameter in 90° direction |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CB | Y | SC | K | RSAT | SB | H | HLCID |
| Type | F | F | F | F | F | F | F | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | none |

| VARIABLE | DESCRIPTION |
|---|---|
| CB | The uppercase $B$ defined in Yoshida's equations |

| VARIABLE | DESCRIPTION |
|---|---|
| Y | Hardening parameter as defined in Yoshida's equations |
| SC | The lowercase *c* defined in the Yoshida & Uemori's equations |
| K | Hardening parameter as defined in the Yoshida & Uemori's equations |
| RSAT | Hardening parameter as defined in the Yoshida and Uemori's equations |
| SB | The lowercase *b* as defined in the Yoshida & Uemori's equations |
| H | Anisotropic parameter associated with work-hardening stagnation, defined in the Yoshida and Uemori's equations |
| HLCID | Load curve ID (see *DEFINE_CURVE) giving true strain as a function of true stress. The load curve is optional and is used for error calculation only. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | IOPT | C1 | C2 | IFLD | EA | COE | |
| Type | F | I | F | F | I | F | F | |
| Default | none | none | 0.0 | 0.0 | none | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an |

| VARIABLE | DESCRIPTION |
|---|---|
| | angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE__COORDINATE_VECTOR).. |
| IOPT | Kinematic hardening rule flag:<br>EQ.0: Original Yoshida & Uemori formulation,<br>EQ.1: Modified formulation; define C1, C2 as below. |
| C1, C2 | Constants used to modify $R$:<br>$R = \text{RSAT} \times [(C_1 + \bar{\varepsilon}^p)^{c_2} - {C_1}^{c_2}]$ |
| IFLD | ID of a load curve of the traditional Forming Limit Diagram (FLD) for the linear strain paths. In the load curve, abscissas represent minor strains while ordinates represent major strains. Define only when the NLP option is used. |
| EA | Variable controlling the change of Young's modulus, $E^A$. See Remark 5. |
| COE | Variable controlling the change of Young's modulus, $\zeta$. See Remark 5. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

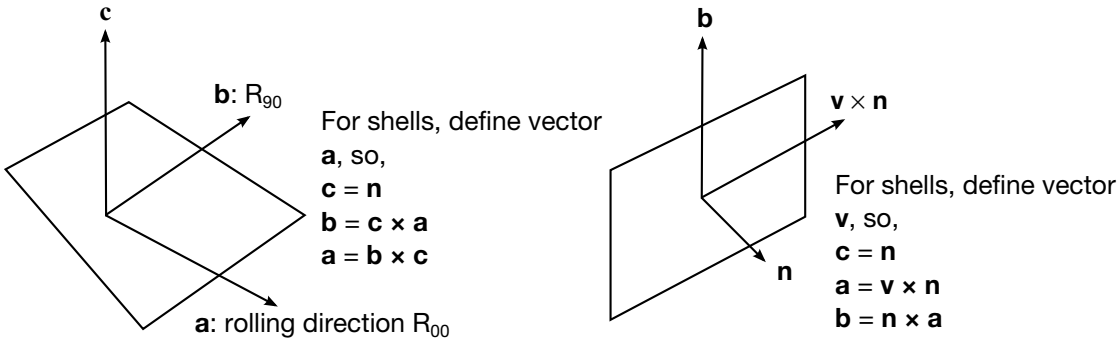



**Figure M226-1.** Defining sheet metal rolling direction.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3, may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

**Remarks:**

1. **Barlat and Lian's yield vriterion.** The *R*-values are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width *W* and thickness *T* are measured as functions of strain, then the corresponding *R*-value is given by:

$$R = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T} .$$

Input R00, R45 and R90 to define sheet anisotropy in the rolling, 45° and 90° direction.

Barlat and Lian's [1989] anisotropic yield criterion Φ for plane stress is defined as:

$$\Phi = a|K_1 + K_2|^m + a|K_1 - K_2|^m + c|2K_2|^m = 2\sigma_Y^m$$

for face centered cubic (FCC) materials exponent $m = 8$ is recommended and for body centered cubic (BCC) materials $m = 6$ may be used. Detailed description on the criterion can be found in the *MAT_036 manual pages.

2. **Yoshida & Uemori nonliner kinematic hardening model.** See manual pages for *MAT_125 for more details.

3. **Rolling direction of sheet metal.** The variable AOPT is used to define the rolling direction of the sheet metals. When AOPT = 2, define vector components of **a** in the direction of the rolling ($R_{00}$); when AOPT = 3, define vector components of **v** perpendicular to the rolling direction, as shown in Figure M226-1.

4. **A failure criterion for nonlinear strain paths (NLP).** The NLP failure criterion and corresponding post processing procedures are described in the entries for *MAT_036 and *MAT_037. The history variables for every element stored in d3plot files include:

   a) Formability Index (F.I.): #1

   b) Strain ratio $\beta$ (in-plane minor strain increment/major strain increment): #2

   c) Effective strain from the planar isotropic assumption: #3

   To enable the output of these history variables to the d3plot files, NEIPS on the *DATABASE_EXTENT_BINARY card must be set to at least 3.

5. **Change in Young's modulus.** The optional change in Young's modulus is defined as a function of effective plastic strain,

$$E = E_0 - (E_0 - E_A)[1 - \exp(-\zeta\bar{\varepsilon}^p)] \ .$$

## *MAT_PML_ELASTIC

This is Material Type 230. This is a perfectly-matched layer (PML) material — an absorbing layer material used to simulate wave propagation in an unbounded isotropic elastic medium — and is available only for solid 8-node bricks (element type 2). This material implements the three-dimensional version of the Basu-Chopra PML [Basu and Chopra (2003,2004), Basu (2009)].

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |

**Remarks:**

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.

2. **Bounded Domain Material Properties.** It is assumed the material in the bounded domain near the layer is, or behaves like, an isotropic linear elastic material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces

of this box may be open, as required by the geometry of the problem. For example, for a half-space problem, the "top" of the box should be open.

Internally, LS-DYNA will partition the entire PML into regions which form the "faces," "edges" and "corners" of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.

4. **Number of Elements in Layer.** The layer should have 5-10 elements through its depth. Typically, 5-6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8-10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints, such as *CONSTRAINED_GLOBAL or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve, will not be recognized.

   Note that Ansys Mechanical uses *BOUNDARY_PRESCRIBED_MOTION with a zero-value load curve for constraining nodal degrees-of-freedom. Because of this limitation, you will have to edit the input file with a text editor.

6. **Stress and Strain.** The stress and strain values reported by this material do not have any physical significance.

## *MAT_PML_ELASTIC_FLUID

This is Material Type 230_FLUID. This model is a perfectly-matched layer (PML) material with a pressure fluid constitutive law, to be used in a wave-absorbing layer adjacent to a fluid material (*MAT_ELASTIC_FLUID) in order to simulate wave propagation in an unbounded fluid medium. See the Remarks sections of *MAT_PML_ELASTIC (*MAT_230) and *MAT_ELASTIC_FLUID (*MAT_001_FLUID) for further details.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | VC | | | | |
| Type | A | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus |
| VC | Tensor viscosity coefficient |

## *MAT_PML_ACOUSTIC

This is Material Type 231. This is a perfectly-matched layer (PML) material — an absorbing layer material used to simulate wave propagation in an unbounded acoustic medium — and can be used only with the acoustic pressure element formulation (element type 14). This material implements the three-dimensional version of the Basu-Chopra PML for anti-plane motion [Basu and Chopra (2003,2004), Basu (2009)].

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | C | | | | | |
| Type | A | F | F | | | | | |
| Default | none | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| C | Sound speed |

**Remarks:**

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any hydrostatic pressure.

2. **Material in Bounded Domain.** It is assumed the material in the bounded domain near the layer is an acoustic material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem, e.g., for a half-space problem, the "top" of the box should be open.

Internally, LS-DYNA will partition the entire PML into regions which form the "faces," "edges" and "corners" of the above cuboid box, and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.

4. **Number of Elements in Layer.** The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints, such as *CONSTRAINED_GLOBAL or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve, will not be recognized.

   Note that Ansys Mechanical uses *BOUNDARY_PRESCRIBED_MOTION with a zero-value load curve for constraining nodal degrees-of-freedom. Because of this limitation, you will have to edit the input file with a text editor.

6. **Pressure Values.** The pressure values reported by this material do not have any physical significance.

## *MAT_BIOT_HYSTERETIC

This is Material Type 232. This is a Biot linear hysteretic material, to be used for modeling the nearly-frequency-independent viscoelastic behavior of soils subjected to cyclic loading, such as in soil-structure interaction analysis [Spanos and Tsavachidis (2001), Makris and Zhang (2000), Muscolini, Palmeri and Ricciardelli (2005)]. The hysteretic damping coefficient for the model is computed from a prescribed damping ratio by calibrating with an equivalent viscous damping model for a single-degree-of-freedom system. The damping increases the stiffness of the model and thus reduces the computed time-step size.

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | ZT | FD | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | 0.0 | 3.25 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| ZT | Damping ratio |
| FD | Dominant excitation frequency in Hz |

**Remarks:**

1. **Stress.** The stress is computed as a function of the strain rate as

$$\sigma(t) = \int_0^t C_R(t-\tau)\dot{\varepsilon}(\tau)\,d\tau$$

where

$$C_R(t) = C\left[1 + \frac{2\eta}{\pi}E_1(\beta t)\right].$$

In the above, $C$ is the elastic isotropic constitutive tensor, $\eta$ is the hysteretic damping factor, and $\beta = 2\pi f_d/10$, where $f_d$ is the dominant excitation frequency in Hz. The function $E_1$ is given by

$$E_1(s) = \int_s^\infty \frac{e^{-\xi}}{\xi} d\xi$$

For efficient implementation, this function is approximated by a 5-term Prony series as

$$E_1(s) \approx \sum_{k=1}^{5} b_k e^{a_k s} ,$$

such that $b_k > 0$.

2. **Hysteretic damping factor.** The hysteretic damping factor $\eta$ is obtained from the prescribed damping ratio $\varsigma$ as

$$\eta = \pi\varsigma/\text{atan}(10) = 2.14\varsigma$$

by assuming that, for a single degree-of-freedom system, the energy dissipated per cycle by the hysteretic material is the same as that by a viscous damper, if the excitation frequency matches the natural frequency of the system.

3. **Young's modulus.** The consistent Young's modulus for this model is given by

$$E_c = E\left[1 + \frac{2\eta}{\pi} g\right] ,$$

where

$$g = \sum_{k=1}^{5} b_k \frac{1}{a_k \beta \Delta t_n} [\exp(a_k \beta \Delta t_n) - 1] .$$

Because $g > 0$, the computed element time-step size is smaller than that for the corresponding elastic element. Furthermore, the time-step size computed at any time depends on the previous time-step size. It can be demonstrated that the new computed time-step size stays within a narrow range of the previous time-step size and for a uniform mesh, converges to a constant value. For $f_d = 3.25$ Hz and $\varsigma = 0.05$, the percentage decrease in time-step size can be expected to be about 12 - 15% for initial time-step sizes of less than 0.02 secs, and about 7 - 10% for initial time-step sizes larger than 0.02 secs.

4. **Dominant frequency.** The default value of the dominant frequency is chosen to be valid for earthquake excitation.

## *MAT_CAZACU_BARLAT

This is Material Type 233. This material model is for Hexagonal Closed Packet (HCP) metals and is based on the work by Cazacu et al. (2006). This model is capable of describing the yielding asymmetry between tension and compression for such materials. Moreover, a parameter fit is optional and can be used to find the material parameters that describe the experimental yield stresses. The experimental data that you should supply consists of yield stresses for tension and compression in the 00 direction, tension in the 45 and the 90 directions, and a biaxial tension test.

Available options include:

<BLANK>

MAGNESIUM

Including MAGNESIUM invokes a material model developed by the USAMP consortium to simulate cast Magnesium under impact loading. The model includes rate effects having a tabulated failure model including equivalent plastic strain to failure as a function of stress triaxiality and effective plastic strain rate. Element erosion will occur when the number of integration points where the damage variable has reached unity reaches some specified threshold (NUMINT). Alternatively, a Gurson type failure model can be activated, which requires less experimental data.

You must provide the evolution of the Cazacu-Barlat effective stress as a function of the energy conjugate plastic strain in the input for the hardening curve for MAT_233. With the MAGNESIUM option an alternative option for the hardening curve is available: von Mises effective stress as a function of equivalent plastic strain, which is energy conjugate to the von Mises stress.

Finally, the MAGNESIUM option allows for distortional hardening by providing hardening curves as measured in tension and compression tests. This option is however incompatible with the activation of rate effects (visco-plasticity).

With the MAGNESIUM option this material model is also available for solid elements.

> **NOTE:** Activating the MAGNESIUM options *requires* setting HR = 3 and FIT = 0.0 (also see below).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | HR | P1 | P2 | ITER |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| A | C11 | C22 | C33 | LCID | E0 | K | P3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| AOPT | | | | C12 | C13 | C23 | C44 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | FIT |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is included if and only if the MAGNESIUM keyword option is used.

| LC1ID | LC2ID | NUMINT | LCCID | ICFLAG | IDFLAG | LC3ID | EPSFG |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | P1 | P2 | ITER |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be used (see *PART). |
| RO | Constant mass density |
| E | Young's modulus:<br>GT.0.0: Constant value<br>LT.0.0: \|E\| is a load curve ID that defines the Young's modulus as a function of plastic strain. |
| PR | Poisson's ratio |
| HR | Hardening rules: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Linear hardening (default)<br>EQ.2.0: Exponential hardening (Swift)<br>EQ.3.0: Load curve<br>EQ.4.0: Exponential hardening (Voce)<br>EQ.5.0: Exponential hardening (Gosh)<br>EQ.6.0: Exponential hardening (Hocken-Sherby)<br>HR must be set to 3 if the MAGNESIUM keyword option is active. |
| P1 | Material parameter:<br>HR.EQ.1.0: Tangent modulus<br>HR.EQ.2.0: $q$, coefficient for exponential hardening law (Swift)<br>HR.EQ.4.0: $a$, coefficient for exponential hardening law (Voce)<br>HR.EQ.5.0: $q$, coefficient for exponential hardening law (Gosh)<br>HR.EQ.6.0: $a$, coefficient for exponential hardening law (Hocket-Sherby) |
| P2 | Material parameter:<br>HR.EQ.1.0: Yield stress for the linear hardening law<br>HR.EQ.2.0: $n$, coefficient for (Swift) exponential hardening<br>HR.EQ.4.0: $c$, coefficient for exponential hardening law (Voce)<br>HR.EQ.5.0: $n$, coefficient for exponential hardening law (Gosh)<br>HR.EQ.6.0: $c$, coefficient for exponential hardening law (Hocket-Sherby) |
| ITER | Iteration flag for speed:<br>EQ.0.0: Fully iterative<br>EQ.1.0: Fixed at three iterations. Generally, ITER = 0.0 is recommended. However, ITER = 1.0 is faster and may give acceptable results in most problems. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | C11 | C22 | C33 | LCID | E0 | K | P3 |
| Type | F | F | F | F | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A | Exponent in Cazacu-Barlat's orthotropic yield surface (A > 1) |
| C11 | Material parameter (see Card 5 pos. 8):<br>FIT.EQ.1.0.OR.EQ.2.0: Yield stress for tension in the 00 direction<br>FIT.EQ.0.0: Material parameter $c_{11}$ |
| C22 | Material parameter (see Card 5 pos.8):<br>FIT.EQ.1.0.OR.EQ.2.0: Yield stress for tension in the 45 direction<br>FIT.EQ.0.0: Material parameter $c_{22}$ |
| C33 | Material parameter (see Card 5 pos.8):<br>FIT.EQ.1.0.OR.EQ.2.0: Yield stress for tension in the 90 direction<br>FIT.EQ.0.0: Material parameter $c_{33}$ |
| LCID | Load curve ID for the hardening law (HR = 3.0), 2D Table ID for rate dependent hardening or 3D Table ID for rate-and-temperature-dependent hardening if the MAGNESIUM option is active. For the 3D table case, *MAT_ADD_THERMAL_EXPANSION could be used for thermal stress/strain effects. Note that the 3D table option is only valid for shell elements. |
| E0 | Material parameter:<br>HR.EQ.2.0: $\varepsilon_0$, initial yield strain for exponential hardening law (Swift) (default = 0.0)<br>HR.EQ.4.0: $b$, coefficient for exponential hardening (Voce)<br>HR.EQ.5.0: $\varepsilon_0$, initial yield strain for exponential hardening (Gosh); default = 0.0<br>HR.EQ.6.0: $b$, coefficient for exponential hardening law (Hocket- |

| VARIABLE | DESCRIPTION |
|---|---|
| | Sherby) |
| K | Material parameter (see Card 5 pos.8):<br>FIT.EQ.1.0.OR.EQ.2.0: Yield stress for compression in the 00 direction<br>FIT.EQ.0.0: Material parameter ($-1 < k < 1$) |
| P3 | Material parameter:<br>HR.EQ.5.0: $p$, coefficient for exponential hardening (Gosh)<br>HR.EQ.6.0: $n$, exponent for exponential hardening law (Hocket-Sherby) |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | C12 | C13 | C23 | C44 |
| Type | F | | | | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see*MAT_*OPTION*TROPIC_ELASTIC for a more complete description).<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the $a$-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle BETA, from a line in the plane of the element defined by the cross product of the vector, **v**, with the element |

| VARIABLE | DESCRIPTION |
|---|---|
| | normal<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *p*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR). |
| C12 | Material parameter, $c_{12}$. If parameter identification (FIT = 1.0) is turned on, C12 is not used. |
| C13 | Material parameter, $c_{13}$. If parameter identification (FIT = 1.0) is turned on, C13 = 0.0 |
| C23 | Material parameter. If parameter identification (FIT = 1.0) is turned on, C23 = 0.0 |
| C44 | Material parameter (see Card 5 pos.8)<br>FIT.EQ.1.0.OR.EQ.2.0: Yield stress for the balanced biaxial tension test.<br>FIT.EQ.0.0: Material parameter, $c_{44}$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP - ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1 - A3 | Components of vector **a** for AOPT = 2.0 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | FIT |
| Type | F | F | F | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| V1 - V3 | Components of vector **v** for AOPT = 3.0 |
| D1 - D3 | Components of vector **d** for AOPT = 2.0 |
| BETA | Material angle in degrees for AOPT = 0 and 3. Note that BETA may be overridden on the element card; see *ELEMENT_SHELL_BETA |
| FIT | Flag for parameter identification algorithm:<br>EQ.0.0: No parameter identification routine is used. The variables K, C11, C22, C33, C44, C12, C13 and C23 are interpreted as material parameters. FIT MUST be set to zero if MAGNESIUM option is active<br>EQ.1.0: Parameter fit is used. The variables C11, C22, C33, C44 and K are interpreted as yield stresses in the 00 degree direction, the 45 degree direction, the 90 degree direction, the balanced biaxial tension, and the 00 degree compression, respectively. It is recommended to always check the d3hsp file to see the fitted parameters before complex jobs are submitted.<br>EQ.2.0: Same as EQ.1.0 but also produce contour plots of the yield surface. For each material three xy-data files are created: Contour1_*n*, Contour2_*n* and Contour3_*n* where *n* equals the material number. |

**Magnesium Card.** Additional card for MAGNESIUM keyword option.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC1ID | LC2ID | NUMINT | LCCID | ICFLAG | IDFLAG | LC3ID | EPSFG |
| Type | I | I | F | I | I | I | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| LC1ID | Load curve ID giving equivalent plastic strain to failure as a function of stress triaxiality or a table ID giving plastic strain to failure as a function of Lode parameter and stress triaxiality (solids) |
| LC2ID | Load curve ID giving equivalent plastic strain to failure as a function of equivalent plastic strain rate. The failure strain will be computed as the product of the values on LC1ID and LC2ID. |
| NUMINT | Number of through thickness integration points which must fail before the element is deleted (inactive for solid elements) |
| LCCID | Load curve ID giving effective stress as a function of plastic strain obtained from a compression stress. This load curve will activate distortional hardening. It is *not* compatible with the use of strain rate effects. |
| ICFLAG | Automated input conversion flag:<br>EQ.0: Load curves provided under LCID and LCCID contain Cazacu-Barlat effective stress as a function of energy conjugate plastic strain.<br>EQ.1: Both load curves are given in terms of von Mises stress as a function of equivalent plastic strain |
| IDFLAG | Damage flag:<br>EQ.0: Failure model is of the Johnson Cook type and requires LC1ID and LC2ID as additional input.<br>EQ.1: Failure model is of the Gurson type and requires LC3ID and EPSFG as additional input. |
| LC3ID | Load curve giving the critical void fraction of the Gurson model as a function of the plastic strain to failure measured in the uniaxial tensile test |
| EPSFG | Plastic strain to failure measured in the uniaxial tensile test. This value is used by the Gurson type failure model only. |

**Remarks:**

This material model (*MAT_CAZACU_BARLAT) aims to model materials with strength differential and orthotropic behavior under plane stress. The yield condition includes a parameter $k$ that describes the asymmetry between yield in tension and compression. Moreover, to include the anisotropic behavior the stress deviator, **S**, undergoes a linear

transformation. The principal values of the Cauchy stress deviator are substituted with the principal values of the transformed tensor, **Z**, which is represented as a vector field, defined as:

$$\mathbf{Z} = \mathbf{CS} \ . \tag{233.1}$$

Here **S** is the field comprised of the four stresses deviator components, $S_I = (s_{11}, s_{22}, s_{33}, s_{12})$,

$$\mathbf{s} = \sigma - \frac{1}{3}\mathrm{tr}(\sigma)\delta \ .$$

In the above equation, $\mathrm{tr}(\sigma)$ is the trace of the Cauchy stress tensor and $\delta$ is the Kronecker delta. For the 2D plane stress condition, the orthotropic condition gives 7 independent coefficients. The tensor **C** is represented by the $4 \times 4$ matrix

$$C_{IJ} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & \\ c_{12} & c_{22} & c_{23} & \\ c_{13} & c_{23} & c_{33} & \\ & & & c_{44} \end{pmatrix}.$$

The principal values of **Z** are denoted $\Sigma_1$, $\Sigma_2$, and $\Sigma_3$ and are given as the eigenvalues to the matrix composed by the components $\Sigma_{xx}$, $\Sigma_{yy}$, $\Sigma_{zz}$, and $\Sigma_{xy}$through

$$\begin{aligned} \Sigma_1 &= \frac{1}{2}\left(\Sigma_{xx} + \Sigma_{yy} + \sqrt{\left(\Sigma_{xx} - \Sigma_{yy}\right)^2 + 4\Sigma_{xy}^2}\right), \\ \Sigma_2 &= \frac{1}{2}\left(\Sigma_{xx} + \Sigma_{yy} - \sqrt{\left(\Sigma_{xx} - \Sigma_{yy}\right)^2 + 4\Sigma_{xy}^2}\right), \\ \Sigma_3 &= \Sigma_{zz} \end{aligned}$$

where

$$\begin{aligned} 3\Sigma_{xx} &= (2c_{11} - c_{12} - c_{13})\sigma_{xx} + (-c_{11} + 2c_{12} - c_{13})\sigma_{yy}, \\ 3\Sigma_{yy} &= (2c_{12} - c_{22} - c_{23})\sigma_{xx} + (-c_{12} + 2c_{22} - c_{23})\sigma_{yy}, \\ 3\Sigma_{zz} &= (2c_{13} - c_{23} - c_{33})\sigma_{xx} + (-c_{13} + 2c_{23} - c_{33})\sigma_{yy}, \\ \Sigma_{xy} &= c_{44}\sigma_{12} \end{aligned}$$

Note that the symmetry of $\Sigma_{xy}$ follows from the symmetry of the Cauchy stress tensor.

The yield condition is written in the following form:

$$f\left(\Sigma, k, \varepsilon_{\mathrm{ep}}\right) = \sigma_{\mathrm{eff}}(\Sigma_1, \Sigma_2, \Sigma_3, k) - \sigma_y\left(\varepsilon_{\mathrm{ep}}\right) \le 0 \ , \tag{233.2}$$

where $\sigma_y\left(\varepsilon_{\mathrm{ep}}\right)$ is a function representing the current yield stress dependent on current effective plastic strain and $k$ is the asymmetric parameter for yield in compression and tension. The effective stress $\sigma_{\mathrm{eff}}$ is given by

$$\sigma_{\mathrm{eff}} = \left[(|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a\right]^{1/a} \ , \tag{233.3}$$

where $k \in [-1,1]$ and $a \ge 1$. Now, let $\sigma_{00}^T$ and $\sigma_{00}^C$ represent the yield stress along the rolling (00 degree) direction in tension and compression, respectively. Furthermore let $\sigma_{45}^T$ and $\sigma_{90}^T$ represent the yield stresses in the 45 and the 90 degree directions, and last let

$\sigma_B^T$ be the balanced biaxial yield stress in tension. Following Cazacu et al. (2006) the yield stresses can easily be derived.

To simplify the equations it is preferable to make the following definitions:

$$\Phi_1 = \frac{1}{3}(2c_{11} - c_{12} - c_{13}) \qquad \Psi_1 = \frac{1}{3}(-c_{11} + 2c_{12} - c_{13})$$
$$\Phi_2 = \frac{1}{3}(2c_{12} - c_{22} - c_{23}) \quad \text{and} \quad \Psi_2 = \frac{1}{3}(-c_{12} + 2c_{22} - c_{23})$$
$$\Phi_3 = \frac{1}{3}(2c_{13} - c_{23} - c_{33}) \qquad \Psi_3 = \frac{1}{3}(-c_{13} + 2c_{23} - c_{33})$$

The yield stresses can now be written as:

1. In the 00 degree direction:

$$\sigma_{00}^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Phi_1| - k\Phi_1)^a + (|\Phi_2| - k\Phi_2)^a + (|\Phi_3| - k\Phi_3)^a}\right]^{1/a},$$
$$\sigma_{00}^C = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Phi_1| + k\Phi_1)^a + (|\Phi_2| + k\Phi_2)^a + (|\Phi_3| + k\Phi_3)^a}\right]^{1/a} \tag{233.4}$$

2. In the 45 degree direction:

$$\sigma_{45}^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Lambda_1| - k\Lambda_1)^a + (|\Lambda_2| - k\Lambda_2)^a + (|\Lambda_3| - k\Lambda_3)^a}\right]^{1/a} \tag{233.5}$$

where

$$\Lambda_1 = \frac{1}{4}\left[\Phi_1 + \Phi_2 + \Psi_1 + \Psi_2 + \sqrt{(\Phi_1 + \Psi_1 - \Phi_2 - \Psi_2)^2 + 4c_{44}^2}\right],$$
$$\Lambda_2 = \frac{1}{4}\left[\Phi_1 + \Phi_2 + \Psi_1 + \Psi_2 - \sqrt{(\Phi_1 + \Psi_1 - \Phi_2 - \Psi_2)^2 + 4c_{44}^2}\right],$$
$$\Lambda_3 = \frac{1}{2}[\Phi_3 + \Psi_3].$$

3. In the 90 degree direction:

$$\sigma_{90}^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Psi_1| - k\Psi_1)^a + (|\Psi_2| - k\Psi_2)^a + (|\Psi_3| - k\Psi_3)^a}\right]^{1/a} \tag{233.6}$$

4. In the balanced biaxial yield occurs when both $\sigma_{xx}$ and $\sigma_{yy}$are equal to:

$$\sigma_B^T = \left[\frac{(\sigma_{\text{eff}})^a}{(|\Omega_1| - k\Omega_1)^a + (|\Omega_2| - k\Omega_2)^a + (|\Omega_3| - k\Omega_3)^a}\right]^{1/a} \tag{233.7}$$

where

$$\Omega_1 = \frac{1}{3}(c_{11} + c_{12} - 2c_{13})$$
$$\Omega_2 = \frac{1}{3}(c_{12} + c_{22} - 2c_{23})$$
$$\Omega_3 = \frac{1}{3}(c_{13} + c_{23} - 2c_{33})$$

**Hardening laws:**

The following hardening laws are implemented:

1. Swift hardening law
2. Voce hardening law
3. Gosh hardening law
4. Hocket-Sherby hardening law
5. Loading curve, where the yield stress is given as a function of the effective plastic strain

The Swift's hardening law can be written as

$$\sigma_y(\varepsilon_{\text{ep}}) = q(\varepsilon_0 + \varepsilon_{\text{ep}})^n$$

where $q$ and $n$ are material parameters.

The Voce's equation says that the yield stress can be written in the following form

$$\sigma_y(\varepsilon_{\text{ep}}) = a - be^{-c\varepsilon_{\text{ep}}}$$

where $a$, $b$, and $c$ are material parameters. The Gosh's equation is similar to Swift's equation. They only differ by a constant

$$\sigma_y(\varepsilon_{\text{ep}}) = q(\varepsilon_0 + \varepsilon_{\text{ep}})^n - p \ .$$

Here $q$, $\varepsilon_0$, $n$ and $p$ are material constants. The Hocket-Sherby equation resembles the Voce's equation, but with an additional parameter added

$$\sigma_y(\varepsilon_{\text{ep}}) = a - be^{-c\varepsilon_{\text{ep}}^n} \ ,$$

where $a$, $b$, $c$ and $n$ are material parameters.

**Constitutive relation and material stiffness:**

The classical elastic constitutive equation for linear deformations is the well-known Hooke's law. This relation written in a rate formulation is given by

$$\dot{\sigma} = \mathbf{D}\dot{\varepsilon}_e \ , \tag{233.8}$$

where $\varepsilon_e$ is the elastic strain and $\mathbf{D}$ is the constitutive matrix. An over imposed dot indicates differentiation respect to time. Introducing the total strain, $\varepsilon$, and the plastic strain, $\varepsilon_p$, Eq. (233.8) is classically rewritten as

$$\dot{\sigma} = \mathbf{D}\left(\dot{\varepsilon} - \dot{\varepsilon}_p\right) \ , \tag{233.9}$$

where

$$\mathbf{D} = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & v & & & \\ v & 1 & & & \\ & & \frac{1-v}{2} & & \\ & & & \frac{1-v}{2} & \\ & & & & \frac{1-v}{2} \end{pmatrix} \text{ and } \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p\right) = \begin{pmatrix} \dot{\varepsilon}_{11} - \left(\dot{\varepsilon}_p\right)_{11} \\ \dot{\varepsilon}_{22} - \left(\dot{\varepsilon}_p\right)_{22} \\ 2\left[\dot{\varepsilon}_{12} - \left(\dot{\varepsilon}_p\right)_{12}\right] \\ 2\left[\dot{\varepsilon}_{13} - \left(\dot{\varepsilon}_p\right)_{13}\right] \\ 2\left[\dot{\varepsilon}_{23} - \left(\dot{\varepsilon}_p\right)_{23}\right] \end{pmatrix} .$$

The parameters $E$and $v$ are the Young's modulus and Poisson's ratio, respectively.

The material stiffness $\mathbf{D}_p$ that is needed for implicit analysis can be calculated from Equation (233.9) as

$$\mathbf{D}_p = \frac{\partial \dot{\sigma}}{\partial \dot{\boldsymbol{\varepsilon}}} \ .$$

The associative flow rule for the plastic strain is usually written as

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda}\frac{\partial f}{\partial \sigma} \ , \tag{233.10}$$

and the consistency condition reads as

$$\frac{\mathrm{d}f}{\mathrm{d}\sigma}\dot{\sigma} + \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{\mathrm{ep}}}\dot{\boldsymbol{\varepsilon}}_{\mathrm{ep}} = 0 \ . \tag{233.11}$$

Note that the centralized "dot" means scalar product between two vectors. Using standard calculus one easily derives from Equations (233.9), (233.10) and (233.11) an expression for the stress rate

$$\dot{\sigma} = \left[\mathbf{D} - \frac{\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)}{\frac{df}{d\sigma}\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right) - \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{ep}}}\right]\dot{\boldsymbol{\varepsilon}} \tag{233.12}$$

That means that the material stiffness used for implicit analysis is given by

$$\mathbf{D}_p = \mathbf{D} - \frac{\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right)}{\frac{\mathrm{d}f}{\mathrm{d}\sigma}\cdot\left(\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\right) - \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{\mathrm{ep}}}}. \tag{233.13}$$

To be able to do a stress update we need to calculate the tangent stiffness and the derivative with respect to the corresponding hardening law.

When a suitable hardening law has been chosen the corresponding derivative is simple and will be left out from this document. However, the stress gradient of the yield surface is more complicated and will be outlined here.

$$\begin{aligned}\frac{\partial f}{\partial \sigma_{11}} = \frac{\partial f}{\partial \Sigma_3}\frac{1}{2}\frac{\partial f}{\partial \Sigma_1}\left[\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_1+\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_2\right] \\ +\frac{1}{2}\frac{\partial f}{\partial \Sigma_2}\left[\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_1+\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Phi_2\right]+\Phi_3\end{aligned} \tag{233.14}$$

$$\begin{aligned}\frac{\partial f}{\partial \sigma_{22}} = \frac{1}{2}\frac{\partial f}{\partial \Sigma_1}\left[\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_1+\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_2\right] \\ +\frac{1}{2}\frac{\partial f}{\partial \Sigma_2}\left[\left(1-\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_1+\left(1+\frac{\Sigma_{xx}-\Sigma_{yy}}{\sqrt{\Sigma_T}}\right)\Psi_2\right]+\frac{\partial f}{\partial \Sigma_3}\Psi_3\end{aligned} \tag{233.15}$$

and the derivative with respect to the shear stress component is

$$\frac{\partial f}{\partial \sigma_{12}} = c_{44}\frac{2\Sigma_{xy}}{\sqrt{\Sigma_T}}\left(\frac{\partial f}{\partial \Sigma_1}-\frac{\partial f}{\partial \Sigma_2}\right) \tag{233.16}$$

where

$$\Sigma_T = \left(\Sigma_{xx}-\Sigma_{yy}\right)^2+4\Sigma_{xy}^2 \tag{233.17}$$

and

$$\frac{\partial f}{\partial \Sigma_i} = f\left(\Sigma,k,\varepsilon_{ep}\right)^{\frac{1}{a}-1}\left(|\Sigma_i|-k\Sigma_i\right)^{a-1}\left(\mathrm{sgn}(\Sigma_i)-k\right)\ \text{ for }\ i=1,2,3 \tag{233.18}$$

**Implementation:**

Assume that the stress and strain is known at time $t^n$. A trial stress $\tilde{\sigma}^{n+1}$ at time $t^{n+1}$ is calculated by assuming a pure elastic deformation, that is,

$$\tilde{\boldsymbol{\sigma}}^{n+1} = \boldsymbol{\sigma}^n + \mathbf{D}\left(\boldsymbol{\varepsilon}^{n+1}-\boldsymbol{\varepsilon}^n\right) \tag{233.19}$$

Now, if $f(\Sigma,k,\varepsilon_{ep}) \le 0$, the deformation is purely elastic, and the new stress and plastic strain are determined as

$$\begin{aligned}\boldsymbol{\sigma}^{n+1} &= \tilde{\boldsymbol{\sigma}}^{n+1} \\ \varepsilon_{\mathrm{ep}}^{n+1} &= \varepsilon_{\mathrm{ep}}^{n}\end{aligned} \tag{233.20}$$

The thickness strain increment is given by

$$\Delta\varepsilon_{33} = \varepsilon_{33}^{n+1}-\varepsilon_{33}^{n} = -\frac{v}{1-v}\left(\Delta\varepsilon_{11}+\Delta\varepsilon_{22}\right) \tag{233.21}$$

If the deformation is not purely elastic, the stress is not inside the yield surface and a plastic iterative procedure must take place as described in the following:

1. Set $m = 0$, $\boldsymbol{\sigma}_{(0)}^{n+1} = \widetilde{\boldsymbol{\sigma}}^{n+1}$, $\varepsilon_{\mathrm{ep}(0)}^{n+1} = \varepsilon_{\mathrm{ep}}^{n}$ and $\Delta\varepsilon_{11}^{p(0)} = \Delta\varepsilon_{22}^{p(0)} = 0$

2. Determine the plastic multiplier as

$$\Delta\lambda = \frac{f\left(\boldsymbol{\sigma}_{(m)}^{n+1}, \varepsilon_{\mathrm{ep}(m)}^{n+1}\right)}{\frac{\mathrm{d}f}{\mathrm{d}\sigma}\left(\boldsymbol{\sigma}_{(m)}^{n+1}\right) \cdot \mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\left(\sigma_{(m)}^{n+1}\right) - \frac{\mathrm{d}f}{\mathrm{d}\varepsilon_{\mathrm{ep}}}\left(\varepsilon_{\mathrm{ep}(m)}^{n+1}\right)} \tag{233.22}$$

3. Perform a plastic corrector step: $\boldsymbol{\sigma}_{(m+1)}^{n+1} = \boldsymbol{\sigma}_{(m)}^{n+1} - \Delta\lambda\mathbf{D}\frac{\mathrm{d}f}{\mathrm{d}\sigma}\left(\boldsymbol{\sigma}_{(m)}^{n+1}\right)$ and find the increments in plastic strain according to

$$\begin{aligned}
\varepsilon_{\mathrm{ep}(m+1)}^{n+1} &= \varepsilon_{\mathrm{ep}(m)}^{n+1} + \Delta\lambda \\
\Delta\varepsilon_{11}^{p(n+1)} &= \Delta\varepsilon_{11}^{p(n)} + \Delta\lambda\frac{\partial f}{\partial\sigma_{11}}\left(\sigma_{(m)}^{n+1}\right) \\
\Delta\varepsilon_{22}^{p(n+1)} &= \Delta\varepsilon_{22}^{p(n)} + \Delta\lambda\frac{\partial f}{\partial\sigma_{22}}\left(\sigma_{(m)}^{n+1}\right)
\end{aligned} \tag{233.23}$$

4. If $\left|f\left(\boldsymbol{\sigma}_{(m+1)}^{n+1}, \varepsilon_{\mathrm{ep}}^{n}\right)\right| < \mathrm{tol}$ or $m = m_{\max}$; stop and set

$$\begin{aligned}
\boldsymbol{\sigma}^{n+1} &= \boldsymbol{\sigma}_{(m+1)}^{n+1}, \\
\varepsilon_{\mathrm{ep}}^{n+1} &= \varepsilon_{\mathrm{ep}(m+1)}^{n+1}, \\
\Delta\varepsilon_{11}^{p} &= \Delta\varepsilon_{11}^{p(m+1)}, \\
\Delta\varepsilon_{22}^{p} &= \Delta\varepsilon_{22}^{p(m+1)}.
\end{aligned} \tag{233.24}$$

Otherwise set $m = m + 1$and return to 2.

The thickness strain increment for plastic yield is calculated as

$$\Delta\varepsilon_{33} = -\frac{1}{1-v}\left(\Delta\varepsilon_{11} + \Delta\varepsilon_{22}\right) - \left(1 - \frac{v}{1-v}\right)\left(\Delta\varepsilon_{11}^{p} + \Delta\varepsilon_{22}^{p}\right) \tag{233.25}$$

**History Variables for the MAGNESIUM keyword option:**

The following history variables will be stored for the MAGNESIUM option:

| History Variable # | Description |
|---|---|
| 10 | Gurson damage |
| 11 | Void fraction |
| 12 | Void fraction star |

| History Variable # | Description |
|---|---|
| 14 | Damage |
| 15 | Plastic strain to failure |
| 17 | Equivalent plastic strain (energy conjugate to von Mises stress) |
| 19 | Effective stress (Cazacu-Barlat) |

## *MAT_VISCOELASTIC_LOOSE_FABRIC

This is Material Type 234 developed and implemented by Tabiei et al [2004]. The model is a mechanism incorporating the crimping of the fibers as well as the trellising with re-orientation of the yarns and the locking phenomenon observed in loose fabric. The equilibrium of the mechanism allows the straightening of the fibers depending on the fiber tension. The contact force at the fiber cross over point determines the rotational friction that dissipates a part of the impact energy. The stress-strain relationship is viscoelastic based on a three-element model. The failure of the fibers is strain rate dependent. *DAMPING_PART_MASS is recommended to be used in conjunction with this material model. This material is valid for modeling the elastic and viscoelastic response of loose fabric used in body armor, blade containments, and airbags.

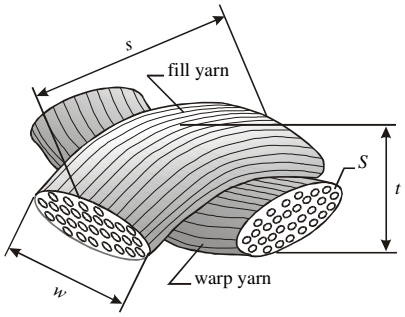



**Figure M234-1.** Representative Volume Cell (RVC) of the model

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | E2 | G12 | EU | THL | THI |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TA | W | s | T | H | S | EKA | EUA |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VMB | C | G23 | EKB | AOPT | | | |
| Type | F | F | F | F | F | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E1 | $E_1$, Young's modulus in the yarn axial direction |
| E2 | $E_2$, Young's modulus in the yarn transverse-direction |
| G12 | $G_{12}$, shear modulus of the yarns |
| EU | Ultimate strain at failure |
| THL | Yarn locking angle |
| THI | Initial braid angle |
| TA | Transition angle to locking |
| W | Fiber width |

| VARIABLE | DESCRIPTION |
|---|---|
| s | Span between the fibers |
| T | Real fiber thickness |
| H | Effective fiber thickness |
| S | Fiber cross-sectional area |
| EKA | Elastic constant of element "a" |
| EUA | Ultimate strain of element "a" |
| VMB | Damping coefficient of element "b" |
| C | Coefficient of friction between the fibers |
| G23 | Transverse shear modulus |
| EKB | Elastic constant of element "b" |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description).<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR). |
| A1 - A3 | Components of vector **a** for AOPT = 2.0 |
| V1 - V3 | Components of vector **v** for AOPT = 3.0 |
| D1 - D3 | Components of vector **d** for AOPT = 2.0 |

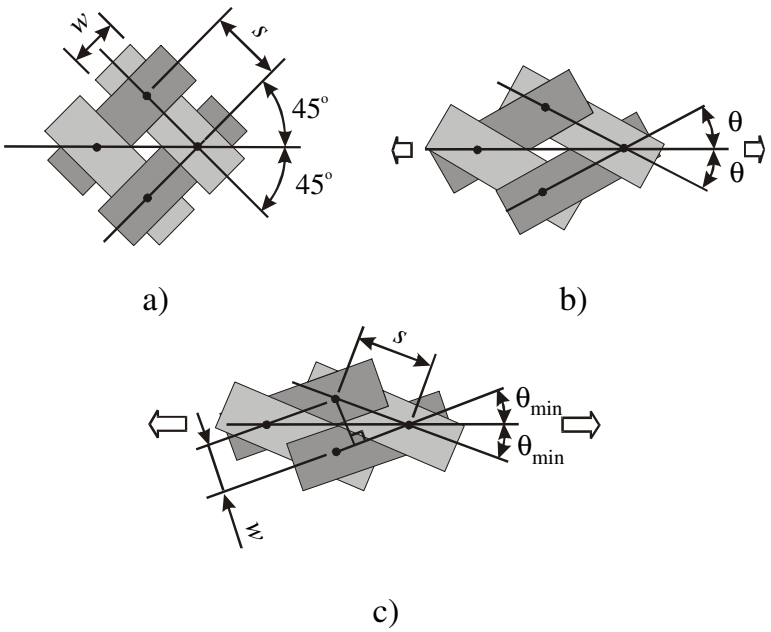



**Figure M234-2.** Plain woven fabric as trellis mechanism: a) initial state; b) slightly stretched in bias direction; c) stretched to locking.

**Remarks:**

The parameters of the Representative Volume Cell (RVC) are: the yarn span, $s$, the fabric thickness, $t$, the yarn width, $w$, and the yarn cross-sectional area, $A$. The initially orthogonal yarns (see Figure M234-2a) are free to rotate (see Figure M234-2b) up to some angle and after that the lateral contact between the yarns causes the locking of the trellis mechanism and the packing of the yarns (see Figure M234-2c).The minimum braid angle, $\theta_{\min}$, can be calculated from the geometry and the architecture of the fabric material having the yarn width, $w$, and the span between the yarns, $s$:

$$\sin(2\theta_{\min}) = \frac{w}{s} \ .$$

The range angle, $\theta_{\text{lock}}$, and the maximum braid angle, $\theta_{\max}$, are then easily determined as:

$$\theta_{\text{lock}} = 45° - \theta_{\min} \ , \qquad \theta_{\max} = 45° + \theta_{\text{lock}}$$

The material behavior of the yarn can be simply described by a combination of one Maxwell element without the dashpot and one Kelvin-Voigt element. The 1-D model of viscoelasticity is shown in Figure M234-3. The differential equation of viscoelasticity of the yarns can be derived from the model equilibrium as in the following equation:

$$(K_a + K_b)\sigma + \mu_b\dot{\sigma} = K_a K_b \varepsilon + \mu_b K_a \dot{\varepsilon}$$

The input parameters for the viscoelasticity model of the material are only the static Young's modulus $E_1$, the Hookian spring coefficient (EKA) $K_a$, the viscosity coefficient

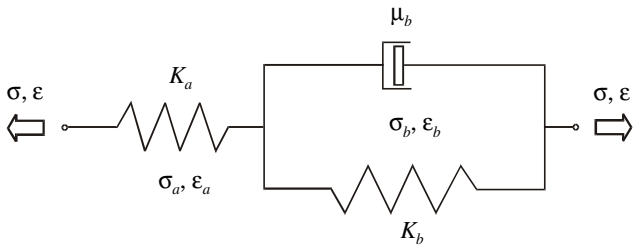



**Figure M234-3.** Three-element viscoelasticity model

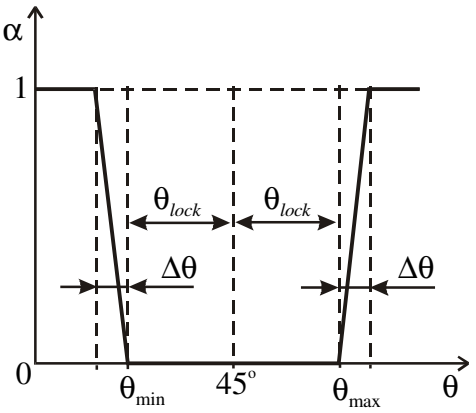



**Figure M234-4.** The lateral contact factor as a function of average braid angle, $\theta$.

(VMB) $\mu_b$, the static ultimate strain (EU) $\varepsilon_{\max}$, and the Hookian spring ultimate strain (EUA)$\varepsilon_{a,\max}$. The other parameters can be obtained as follows:

$$K_b = \frac{K_a E_1}{K_a - E_1}$$
$$\varepsilon_{b,\max} = \frac{K_a - E_1}{K_a}\varepsilon_{\max}$$

The stress in the yarns for the fill and warp is updated for the next time step as:

$$\sigma_f^{(n+1)} = \sigma_f^{(n)} + \Delta\sigma_f^{(n)}, \qquad \sigma_w^{(n+1)} = \sigma_w^{(n)} + \Delta\sigma_w^{(n)}$$

where $\Delta\sigma_f$ and $\Delta\sigma_w$ are the stress increments in the yarns. We can imagine that the RVC is smeared to the parallelepiped in order to transform the stress acting on the yarn cross-section to the stress acting on the element wall. The thickness of the membrane shell element used should be equal to the effective thickness, $t_e$, that can be found by dividing the areal density of the fabric by its mass density. The in-plane stress components acting on the RVC walls in the material direction of the yarns are calculated as follows for the fill and warp directions:

$$\sigma_{f11}^{(n+1)} = \frac{2\sigma_f^{(n+1)}S}{st_e} \qquad \sigma_{w11}^{(n+1)} = \frac{2\sigma_w^{(n+1)}S}{st_e}$$

$$\sigma_{f22}^{(n+1)} = \sigma_{f22}^{(n)} + \alpha E_2 \Delta\varepsilon_{f22}^{(n)} \qquad \sigma_{w22}^{(n+1)} = \sigma_{w22}^{(n)} + \alpha E_2 \Delta\varepsilon_{w22}^{(n)}$$

$$\sigma_{f12}^{(n+1)} = \sigma_{f12}^{(n)} + \alpha G_{12} \Delta\varepsilon_{f12}^{(n)} \qquad \sigma_{w12}^{(n+1)} = \sigma_{w12}^{(n)} + \alpha G_{12} \Delta\varepsilon_{w12}^{(n)}$$

where $E_2$ is the transverse Young's modulus of the yarns, $G_{12}$ is the longitudinal shear modulus, and $\alpha$ is the lateral contact factor. The lateral contact factor is zero when the trellis mechanism is open and unity if the mechanism is locked with full lateral contact between the yarns. There is a transition range, $\Delta\theta \times \mathrm{TA}$, of the average braid angle, $\theta$, in which the lateral contact factor, $\alpha$, is a linear function of the average braid angle. The graph of the function $\alpha(\theta)$ is shown in Figure M234-4.

## *MAT_MICROMECHANICS_DRY_FABRIC

This is Material Type 235 developed and implemented by Tabiei et al [2001]. The material model derivation includes the micro-mechanical approach and the homogenization technique usually used in composite material models. The model accounts for reorientation of the yarns and the fabric architecture. The behavior of the flexible fabric material is achieved by discounting the shear moduli of the material in free state which allows the simulation of the trellis mechanism before packing the yarns. This material is valid for modeling the elastic response of loose fabric used in inflatable structures, parachutes, body armor, blade containments, and airbags.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | E2 | G12 | G23 | V12 | V23 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XT | THI | THL | BFI | BWI | DSCF | CNST | ATLR |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VME | VMS | TRS | FFLG | AOPT | | | |
| Type | F | F | F | F | F | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E1 | $E_1$, Young's modulus of the yarn in the axial-direction |
| E2 | $E_2$, Young's modulus of the yarn in the transverse-direction |
| G12 | $G_{12}$, shear modulus of the yarns. |
| G23 | $G_{23}$, transverse shear modulus of the yarns. |
| V12 | Poisson's ratio |
| V23 | Transverse Poisson's ratio |
| XT | Stress or strain to failure (see FFLG) |
| THI | Initial braid angle |
| THL | Yarn locking angle |
| BFI | Initial undulation angle in fill direction |
| BWI | Initial undulation angle in warp direction |
| DSCF | Discount factor |
| CNST | Reorientation damping constant |
| ATLR | Angle tolerance for locking |
| VME | Viscous modulus for normal strain rate |
| VMS | Viscous modulus for shear strain rate |
| TRS | Transverse shear modulus of the fabric layer |

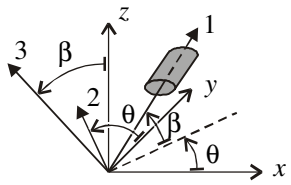



**Figure M235-1.** Yarn orientation schematic.

| VARIABLE | DESCRIPTION |
|---|---|
| FFLG | Flag for stress-based or strain-based failure:<br>EQ.0: XT is a stress to failure.<br>NE.0: XT is a strain to failure. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for more complete description).<br>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2 and 4, as with *DEFINE_COORDINATE_NODES<br>EQ.2.0: globally orthotropic with material axes determined by vectors defined below, as with *DEFINED_COORDINATE_VECTOR<br>EQ.3.0: locally orthotropic material axes defined by the cross product of the vector **v**with the element normal<br>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR). |
| A1 - A3 | Components of vector **a** for AOPT = 2.0 |
| V1 - V3 | Components of vector **v** for AOPT = 3.0 |
| D1 - D3 | Components of vector **d** for AOPT = 2.0 |

**Remarks:**

The Representative Volume Cell (RVC) approach is used in the micro-mechanical model development. The direction of the yarn in each sub-cell is determined by two angles – the braid angle, $\theta$ *(the initial braid angle is 45 degrees)*, and the undulation angle of the yarn, which is different for the fill and warp-yarns, $\beta_f$ and $\beta_w$ (the initial undulations are normally a few degrees), respectively. The starting point for the homogenization of the material properties is the determination of the yarn stiffness matrices.

The elasticity tensor is given by

$$[C'] = [S']^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{12}}{E_1} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{23}}{E_2} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\mu G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\mu G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\mu G_{12}} \end{bmatrix}^{-1}$$

where $E_1$, $E_2$, $\nu_{12}$, $\nu_{23}$, $G_{12}$ and $G_{23}$ are Young's moduli, Poisson's ratios, and the shear moduli of the yarn material, respectively. $\mu$ is a discount factor, which is function of the braid angle, $\theta$, and has value between $\mu_0$ and 1 as shown in the next figure. Initially, in a free stress state, the discount factor is a small value (DSCF = $\mu_0 \ll 1$) and the material has very small resistance to shear deformation if any.

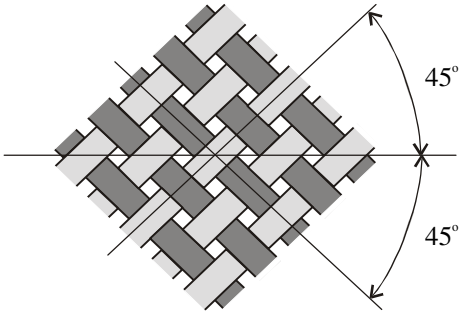



**Figure M235-2.** Free state of the plain woven fabric

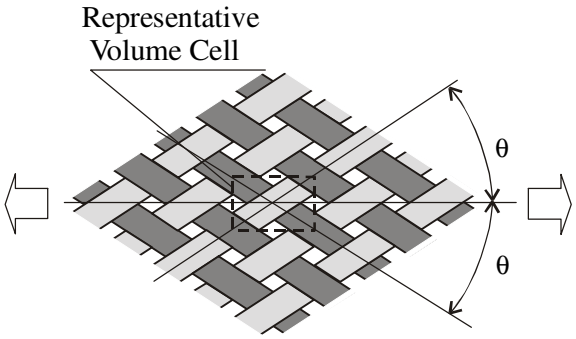



**Figure M235-3.** Stretched state of the plain woven fabric

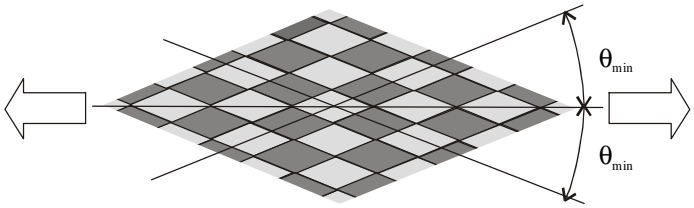



**Figure M235-4.** Compacted state of the plain woven fabric

When locking occurs, the fabric yarns are packed and behave like elastic media. The discount factor is unity as shown in the next figure. The micro-mechanical model is developed to account for the reorientation of the yarns up to the locking angle. The locking angle, $\theta_{\text{lock}}$, can be obtained from the yarn width and the spacing parameter of the fabric using simple geometrical relationship. The transition range, $\Delta\theta$ (angle tolerance for locking), can be chosen to be as small as possible, but big enough to prevent high frequency oscillations during the transition to the compacted state which depends on the range to the locking angle and the dynamics of the simulated problem. The reorientation damping constant damps some of the high frequency oscillations. A simple rate effect is added by defining the viscous modulus for normal or shear strain rate (VME × $\dot{\varepsilon}_{11 \text{ or } 22}$ for normal components and VMS × $\dot{\varepsilon}_{12}$ for the shear components).

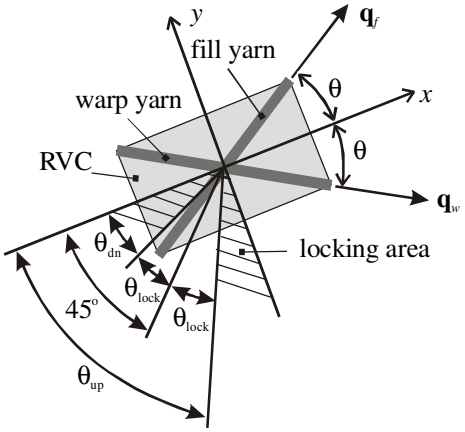



**Figure M235-5.** Locking angles

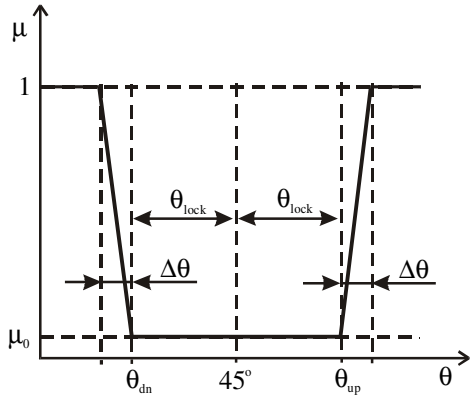



**Figure M235-6.** Discount factor as a function of braid angle, $\theta$

## *MAT_SCC_ON_RCC

This is Material Type 236 developed by Carney, Lee, Goldberg, and Santhanam [2007]. This model simulates silicon carbide coating on Reinforced Carbon-Carbon (RCC), a ceramic matrix. It is based upon a quasi-orthotropic, linear-elastic, plane-stress model. Additional constitutive model attributes include a simple (meaning non-damage model based) option that can model the tension crack requirement: a "stress-cutoff" in tension. This option satisfies the tension crack requirements by limiting the stress in tension but not compression and having the tensile "yielding" (that is, the stress-cutoff) be fully recoverable – not plasticity or damage based.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E0 | E1 | E2 | E3 | E4 | E5 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PR | G | G_SCL | TSL | EPS_TAN | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E0 | $E_0$; see Remarks below. |
| E1 | $E_1$; see Remarks below. |
| E2 | $E_2$; see Remarks below. |
| E3 | $E_3$; see Remarks below. |
| E4 | $E_4$; see Remarks below. |
| E5 | $E_5$, Young's modulus of the yarn in transverse-direction |
| PR | Poisson's ratio |

| VARIABLE | DESCRIPTION |
|---|---|
| G | Shear modulus |
| G_SCL | Shear modulus multiplier (default = 1.0) |
| TSL | Tensile limit stress |
| EPS_TAN | Strain at which E = tangent to the polynomial curve |

**Remarks:**

This model for the silicon carbide coating on RCC is based upon a quasi-orthotropic, linear-elastic, plane-stress model, given by:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

Additional constitutive model requirements include a simple (meaning non-damage model based) option that can model the tension crack requirement: a "stress-cutoff" in tension. This option satisfies the tension crack requirements by limiting the stress in tension but not compression and having the tensile "yielding" (that is, the stress-cutoff) be fully recoverable – not plasticity or damage based.

The tension stress-cutoff separately resets the stress to a limit value when it is exceeded in each of the two principal directions. There is also a strain-based memory criterion that ensures unloading follows the same path as loading: the "memory criterion" is the tension stress assuming that no stress cutoffs were in effect. In this way, when the memory criterion exceeds the user-specified cutoff stress, the actual stress will be set to that value. When the element unloads and the memory criterion falls back below the stress cutoff, normal behavior resumes. Using this criterion is a simple way to ensure that unloading does not result in any hysteresis. The cutoff criterion cannot be based on an effective stress value because effective stress does not discriminate between tension and compression while also including shear. This means that the in plane, 1- and 2- directions must be modeled as independent to use the stress cutoff. Because the Poisson's ratio is not zero, this assumption is not true for cracks that may arbitrarily lie along any direction. However, careful examination of damaged RCC shows that the surface cracks do, generally, tend to lie in the fabric directions, meaning that cracks tend to open in the 1- or the 2- direction independently. So the assumption of directional independence for tension cracks may be appropriate for the coating because of this observed orthotropy.

The quasi-orthotropic, linear-elastic, plane-stress model with tension stress cutoff (to simulate tension cracks) can model the as-fabricated coating properties, which do not show

nonlinearities, but not the non-linear response of the flight-degraded material. Explicit finite element analysis (FEA) lends itself to *nonlinear-elastic* stress-strain relation instead of linear-elastic. Thus, instead of $\sigma = \mathbf{E}\varepsilon$, the modulus will be defined as a function of some effective strain quantity, or $\sigma = \mathbf{E}(\varepsilon_{\text{eff}})\varepsilon$, even though it is uncertain, from the available data, whether the coating response is completely nonlinear-elastic and does not include some damage mechanism.

This nonlinear-elastic model cannot be implemented into a closed form solution or into an implicit solver; however, for explicit FEA such as is used for LS-DYNA impact analysis, the modulus can be adjusted at each time step to a higher or lower value as desired. In order to model the desired S-shape response curve of flight-degraded RCC coating, a function of strain that replicates the desired response must be found. The nonlinearities in the material are assumed recoverable (elastic) and the modulus is assumed to be communicative between the 1- and 2- directions (going against the tension-crack assumption that the two directions do not interact). Sometimes stability can be a problem for this type of nonlinearity modeling; however, stability was not found to be a problem with the material constants used for the coating.

The von Mises strain is selected for the effective strain definition as it couples the 3-dimensional loading but reduces to uniaxial data, so that the desired uniaxial compressive response can be reproduced. So,

$$\varepsilon_{\text{eff}} = \frac{1}{\sqrt{2}}\frac{1}{1+\nu}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_1-\varepsilon_3)^2+3\gamma_{12}^2}\ ,$$

where for a two-dimensional, isotropic shell element case, the $z$-direction strain is given by:

$$\varepsilon_3 = \frac{-\nu}{1-\nu}(\varepsilon_1+\varepsilon_2)\ .$$

The function for modulus is implemented as an arbitrary 5th order polynomial:

$$E(\varepsilon_{\text{eff}}) = A_0\varepsilon_{\text{eff}}^0 + A_1\varepsilon_{\text{eff}}^1 + \cdots + A_5\varepsilon_{\text{eff}}^5\ .$$

In the case of as-fabricated material the first coefficient, $A_0$, is simply the modulus $E$, and the other coefficients, $A_{n>0}$, are zero, reducing to a 0th order polynomial, or linear. To match the degraded stress-strain compression curve, a higher order polynomial is needed. Six conditions on stress were used (stress and its derivative at beginning, middle, and end of the curve) to obtain a 5th order polynomial, and then the derivative of that equation was taken to obtain modulus as a function of strain, yielding a 4th order polynomial that represents the degraded coating modulus as strain curve.

For values of strain which exceed the failure strain observed in the laminate compression tests, the higher order polynomial will no longer match the test data. Therefore, after a specified effective-strain, representing failure, the modulus is defined to be the tangent of the polynomial curve. As a result, the stress/strain response has a continuous derivative, which aids in avoiding numerical instabilities. The test data does not clearly define

the failure strain of the coating, but in the impact test it appears that the coating has a higher compressive failure strain in bending than the laminate failure strain.

The two dominant modes of loading which cause coating loss on the impact side of the RCC (the front-side) are in-plane compression and transverse shear. The in-plane compression is measured by the peak out of plane tensile strain, $\varepsilon_3$. As there is no direct loading of a shell element in this direction, $\varepsilon_3$ is computed through Poisson's relation:

$$\varepsilon_3 = \frac{-v}{1-v}(\varepsilon_1 + \varepsilon_2) \ .$$

When $\varepsilon_3$ is tensile, it implies that the average of $\varepsilon_1$ and $\varepsilon_2$ is compressive. This failure mode will likely dominate when the RCC undergoes large bending, putting the front-side coating in high compressive strains. A transverse shear failure mode is expected to dominate when the debris source is very hard or very fast. By definition, the shell element cannot give a precise account of the transverse shear throughout the RCC's thickness. However, the Belytschko-Tsay shell element formulation in LS-DYNA has a first-order approximation of transverse shear that is based on the out-of-plane nodal displacements and rotations that should suffice to give a qualitative evaluation of the transverse shear. By this formulation, the transverse shear is constant through the entire shell thickness and thus violates surface-traction conditions. The constitutive model implementation records the peak value of the tensile out-of-plane strain ($\varepsilon_3$) and peak root-mean-sum transverse-shear: $\sqrt{\varepsilon_{13}^2 + \varepsilon_{23}^2}$.

## *MAT_PML_HYSTERETIC

This is Material Type 237. This is a perfectly-matched layer (PML) material with a Biot linear hysteretic constitutive law, to be used in a wave-absorbing layer adjacent to a Biot hysteretic material (*MAT_BIOT_HYSTERETIC) in order to simulate wave propagation in an unbounded medium with material damping. This material is the visco-elastic counterpart of the elastic PML material (*MAT_PML_ELASTIC). See the Remarks sections of *MAT_PML_ELASTIC (*MAT_230) and *MAT_BIOT_HYSTERETIC (*MAT_232) for further details.

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | ZT | FD | | |
| Type | A | F | F | F | F | F | | |
| Default | none | none | none | none | 0.0 | 3.25 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| ZT | Damping ratio |
| FD | Dominant excitation frequency in Hz |

## *MAT_PERT_PIECEWISE_LINEAR_PLASTICITY

This is Material Type 238. It is a duplicate of Material Type 24 (*MAT_PIECEWISE_LINEAR_PLASTICITY) modified for use with *PERTURBATION_MATERIAL and solid elements in an explicit analysis. It should give exactly the same values as the original material, if used exactly the same. It exists as a separate material type because of the speed penalty (an approximately 10% increase in the overall execution time) associated with the use of a material perturbation.

See Material Type 24 (*MAT_PIECEWISE_LINEAR_PLASTICITY) for a description of the material parameters. All of the documentation for Material Type 24 applies. First creating the input deck using Material Type 24 is recommended. Additionally, the CMP variable in the *PERTURBATION_MATERIAL must be set to affect a specific variables in the MAT_238 definition as defined in the following table; for example, CMP = 5 will perturb the yield stress.

| CMP Value | Material Variable |
|---|---|
| 3 | E |
| 5 | SIGY |
| 6 | ETAN |
| 7 | FAIL |

## *MAT_COHESIVE_MIXED_MODE_ELASTOPLASTIC_RATE_{OPTION}

Available options include:

<BLANK>

THERMAL

3MODES

FUNCTIONS

This is Material Type 240. This model is a rate-dependent, elastic-ideally plastic cohesive zone model. It includes a tri-linear traction-separation law with a quadratic yield and damage initiation criterion in mixed-mode loading (mode I – mode II), while the damage evolution is governed by a power-law formulation. It can be used only with cohesive element formulations; see ELFORM in *SECTION_SOLID and *SECTION_SHELL.

With the THERMAL option, some properties are defined as functions of temperature, meaning fields EMOD, GMOD, G1C_0, G2C_0, T0, S0, FG1, and FG2 must be defined as curve IDs instead of scalar values.

With the FUNCTIONS option, some properties are defined as functions of connection partner properties, meaning fields EMOD, GMOD, G1C_0, G2C_0, T0, S0, FG1, and FG2 must be defined as function IDs instead of scalar values. See Remark 2 for details.

The keyword option 3MODES activates the possibility to include deformation/fracture mode III which could be useful for cohesive shells. Corresponding fields can be defined on optional Cards 4 and 5.

Note that 3MODES is compatible with THERMAL and FUNCTIONS, but THERMAL and FUNCTIONS cannot be used together. In other words, THERMAL_3MODES and FUNCTIONS_3MODES are allowed as keyword options, but THERMAL_FUNCTIONS is not allowed.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | ROFLG | INTFAIL | EMOD | GMOD | THICK | INICRT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| G1C_0 | G1C_INF | EDOT_G1 | T0 | T1 | EDOT_T | FG1 | LCG1C |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| G2C_0 | G2C_INF | EDOT_G2 | S0 | S1 | EDOT_S | FG2 | LCG2C |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if the 3MODES keyword option is used.

| G3C_0 | G3C_INF | EDOT_G3 | R0 | R1 | EDOT_R | FG3 | LCG3C |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card if the 3MODES keyword option is used.

| GMOD3 | | | | | | | |
|---|---|---|---|---|---|---|---|

***Card 6.*** This card is optional.

| RFILTF | COMPY | SMOLIM | XMU | | USRFAIL | | |
|---|---|---|---|---|---|---|---|

**Card 7.** Include this card if USRFAIL = 1.

| USRV1 | USRV2 | USRV3 | USRV4 | USRV5 | USRV6 | USRV7 | USRV8 |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if USRFAIL = 1.

| USRV9 | USRV10 | USRV11 | USRV12 | USRV13 | USRV14 | USRV15 | USRV16 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | EMOD | GMOD | THICK | INICRT |
| Type | A | F | I | I | F/I | F/I | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| ROFLG | Flag for whether density is specified per unit area or volume:<br>EQ.0: Specified density per unit volume (default)<br>EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero |

| VARIABLE | DESCRIPTION |
|---|---|
| INTFAIL | Number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value.<br>LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when \|INTFAIL\| integration points have failed.<br>EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion.<br>GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed. |
| EMOD | Young's modulus of the material (Mode I).<br>GT.0.0: Constant value. Curve ID for the THERMAL keyword option. Function ID for the FUNCTIONS keyword option.<br>LT.0.0: Load curve ID = \|EMOD\|, which defines Young's modulus as a function of strain rate. This feature is only available if the keyword option is unset (<BLANK>). If the first abscissa value is negative, then logarithmic interpolation is used. |
| GMOD | The shear modulus of the material (Mode II).<br>GT.0.0: Constant value. Curve ID for the THERMAL keyword option. Function ID for the FUNCTIONS keyword option.<br>LT.0.0: Load curve ID = \|GMOD\|, which defines the shear modulus as a function of strain rate. This feature is only available if the keyword option is unset (<BLANK>). If the first abscissa value is negative, then logarithmic interpolation is used. |
| THICK | Initial thickness:<br>GT.0.0: Cohesive thickness<br>LE.0.0: Initial thickness is calculated from nodal coordinates. |
| INICRT | Yield and damage initiation criterion:<br>EQ.0.0: Quadratic nominal stress (default) |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Maximum nominal stress |
| | EQ.2.0: Maximum nominal stress (same as INICRT = 1.0). Additionally, it flags outputting the maximum strain as history variable #15. |
| | LT.0.0: Mixed mode with flexible exponent \|INICRT\| |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G1C_0 | G1C_INF | EDOT_G1 | T0 | T1 | EDOT_T | FG1 | LCG1C |
| Type | F/I | F | F | F/I | F | F | F/I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| G1C_0 | Energy release rate $G_{IC}$ in Mode I or the lower bound of the rate-dependent $G_{IC}$: |
| | GT.0.0: Energy release rate $G_{IC}$ in Mode I. G1C_0 is a curve ID if the THERMAL keyword option is used. G1C_0 is a function ID if the FUNCTIONS keyword option is used. |
| | LE.0.0: Lower bound value of rate-dependent $G_{IC}$ |
| G1C_INF | Upper bound value of rate-dependent $G_{IC}$ (only considered if G1C_0 < 0) |
| EDOT_G1 | Equivalent strain rate at yield initiation to describe the rate dependency of $G_{IC}$ (only considered if G1C_0 < 0) |
| T0 | Yield stress in Mode I or a component of the rate-dependent model for determining the yield stress: |
| | GT.0.0: Yield stress in Mode I. T0 is a curve ID if the THERMAL keyword option is used. T0 is a function ID if the FUNCTIONS keyword option is used. |
| | LT.0.0: Rate-dependency is considered; see T1 and EDOT_T. |
| T1 | Field T1, only considered if T0 < 0: |
| | GT.0.0: Quadratic logarithmic model |
| | LT.0.0: Linear logarithmic model |

| VARIABLE | DESCRIPTION |
|---|---|
| EDOT_T | Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode I (only considered if T0 < 0) |
| FG1 | $f_{G1}$, describes the tri-linear shape of the traction-separation law in Mode I. See Remark 1. It is a curve ID if the THERMAL keyword option is used. It is a function ID if the FUNCTIONS keyword option is used.<br>GT.0.0: FG1 is the ratio of fracture energies, $G_{I,P}/G_{IC}$.<br>LT.0.0: \|FG1\| is ratio of displacements, $(\delta_{n2} - \delta_{n1})/(\delta_{nf} - \delta_{n1})$. |
| LCG1C | Load curve ID which defines fracture energy GIC as a function of cohesive element thickness. G1C_0 and G1C_INF are ignored in this case. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G2C_0 | G2C_INF | EDOT_G2 | S0 | S1 | EDOT_S | FG2 | LCG2C |
| Type | F/I | F | F | F/I | F | F | F/I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| G2C_0 | Energy release rate $G_{IIC}$ in Mode II or the lower bound of the rate-dependent $G_{IIC}$:<br>GT.0.0: Energy release rate $G_{IIC}$ in Mode II. If the THERMAL keyword option is used, it is a load curve ID. For the FUNCTIONS keyword option, it is a function ID.<br>LE.0.0: Lower bound value of rate-dependent $G_{IIC}$ |
| G2C_INF | Upper bound value of $G_{IIC}$ (only considered if G2C_0 < 0) |
| EDOT_G2 | Equivalent strain rate at yield initiation to describe the rate dependency of $G_{IIC}$ (only considered if G2C_0 < 0) |
| S0 | Yield stress in Mode II or a component of the rate-dependent model for determining the yield stress:<br>GT.0.0: Yield stress in Mode II. It is a load curve ID for the THERMAL keyword option. It is a function ID for the FUNCTIONS keyword option. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: Rate-dependency is considered; see S1 and EDOT_S |
| S1 | Parameter S1, only considered if S0 < 0:<br>GT.0.0: Quadratic logarithmic model is applied.<br>LT.0.0: Linear logarithmic model is applied. |
| EDOT_S | Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode II (only considered if S0 < 0) |
| FG2 | $f_{G2}$, describes the trilinear shape of the traction-separation law in Mode II (see Remark 1). It is a load curve ID for the THERMAL keyword option. It is a function ID for the FUNCTIONS keyword option.<br>GT.0.0: FG2 is the ratio of fracture energies, $G_{II,P}/G_{IIC}$.<br>LT.0.0: \|FG2\| is the ratio of displacements, $(\delta_{t2} - \delta_{t1})/(\delta_{tf} - \delta_{t1})$. |
| LCG2C | Load curve ID for the load curve that defines fracture energy GIIC as a function of cohesive element thickness. G2C_0 and G2C_INF are ignored in that case. |

**Additional Cards 4 and 5 for 3MODES keyword option.** Properties for Mode III (out-of-plane mode in cohesive shell elements).

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G3C_0 | G3C_INF | EDOT_G3 | R0 | R1 | EDOT_R | FG3 | LCG3C |
| Type | F/I | F | F | F/I | F | F | F/I | I |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GMOD3 | | | | | | | |
| Type | F/I | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G3C_0 | Energy release rate $G_{IIIC}$ in Mode II or the lower bound of the rate-dependent $G_{IIIC}$:<br>GT.0.0: Energy release rate $G_{IIIC}$ in Mode III. G3C_0 is a load curve ID for the THERMAL keyword option. G3C_0 is a function ID for the FUNCTIONS keyword option.<br>LE.0.0: Lower bound value of rate-dependent $G_{IIIC}$ |
| G3C_INF | Upper bound value of rate-dependent $G_{IIIC}$ (only considered if G3C_0 < 0) |
| EDOT_G3 | Equivalent strain rate at yield initiation to describe the rate dependency of $G_{IIIC}$ (only considered if G1C_0 < 0) |
| R0 | Yield stress in Mode III or a component of the rate-dependent model for determining the yield stress:<br>GT.0.0: Yield stress in Mode III. R0 is a load curve ID for the THERMAL keyword option. R0 is a function ID for the FUNCTIONS keyword option.<br>LT.0.0: Rate-dependency is considered |
| R1 | Parameter R1, only considered if R0 < 0:<br>GT.0.0: Quadratic logarithmic model<br>LT.0.0: Linear logarithmic model |
| EDOT_R | Equivalent strain rate at yield initiation to describe the rate dependency of the yield stress in Mode III (only considered if R0 < 0) |
| FG3 | $f_{G3}$, describes the tri-linear shape of the traction-separation law in Mode III; see Remark 1. It is a load curve ID if the THERMAL keyword option is used. It is a function ID if the FUNCTIONS keyword option is used.<br>GT.0.0: FG3 is ratio of fracture energies, $G_{III,P}/G_{IIIC}$.<br>LT.0.0: \|FG3\| is ratio of displacements, $(\delta_{s2} - \delta_{s1})/(\delta_{sf} - \delta_{s1})$. |
| LCG3C | Load curve ID which defines fracture energy GIIIC as a function of cohesive element thickness. G3C_0 and G3C_INF are ignored in that case. |
| GMOD3 | Shear modulus for Mode III. GMOD3 is a load curve ID for the THERMAL keyword option. GMOD3 is a function ID for the |

| VARIABLE | DESCRIPTION |
|---|---|
| | FUNCTIONS keyword option. |

This card is optional.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RFILTF | COMPY | SMOLIM | XMU | | USRFAIL | | |
| Type | F | F | F | F | | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| RFILTF | Smoothing factor on the equivalent strain rate using an exponential moving average method: $\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$ This option invokes a modified handling of strain rates: GT.0.0: RFILTF applied on the equivalent plastic strain rate. LT.0.0: \|RFILTF\| applied on the equivalent total strain rate. |
| COMPY | Yield under compression flag: EQ.0: Off (default) EQ.1: On |
| SMOLIM | Smooth treatment of asymptotic limits (such as pure shear). EQ.0: Off (default) EQ.1: On |
| XMU | Exponent of the mixed mode failure criterion. Default is 1.0. |
| USRFAIL | User-defined failure flag: EQ.0: No user subroutine is called (default). EQ.1: User subroutine `ucohfail` in dyn21umatc.f is called. |

**User-Subroutine-Based Failure Card 1.** Include this card if USRFAIL = 1.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | USRV1 | USRV2 | USRV3 | USRV4 | USRV5 | USRV6 | USRV7 | USRV8 |
| Type | F | F | F | F | F | F | F | F |

**User-Subroutine-Based Failure Card 2.** Include this card if USRFAIL = 1.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | USRV9 | USRV10 | USRV11 | USRV12 | USRV13 | USRV14 | USRV15 | USRV16 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| USRV$n$ | Failure constants for the user-defined failure subroutine |

**Remarks:**

1. **Material model.** The model is a tri-linear elastic-ideally plastic Cohesive Zone Model, developed by Marzi et al. [2009]. It looks similar to *MAT_185 but considers effects of plasticity and rate-dependency. Since the entire separation at failure is plastic, no brittle fracture behavior can be modeled with this material type.

   The following description of the model is for two deformation/fracture modes (I and II) only, meaning without the option 3MODES. This is a natural choice for cohesive solid elements, where no specific distinction between in-plane and out-of-plane shear can be made. On the other hand, if this material model is used with the cohesive shell element type ±29, shear deformation can clearly be separated into in-plane shear (Mode II) and out-of-plane shear (Mode III). This can be taken into account by adding 3MODES to the keyword and defining additional Cards 4 and 5. Corresponding equations including Mode III are not explicitly given here (for the sake of brevity), but derivation of them is straightforward.

   The separations, $\Delta_n$ and $\Delta_t$, in the normal (peel) and tangential (shear) directions, respectively, are calculated from the element's separations in the integration points,

$$\Delta_n = \max(u_n, 0)$$

and

$$\Delta_t = \sqrt{u_{t1}^2 + u_{t2}^2} \ .$$

$u_n$ is the separation in the normal direction while $u_{t1}$ and $u_{t2}$ is the separation in both tangential directions of the element coordinate system. The total (mixed-mode) separation $\Delta_m$ is determined by

$$\Delta_m = \sqrt{\Delta_n^2 + \Delta_t^2} \ .$$

The initial stiffnesses in both modes are calculated from the elastic Young's and shear moduli and are, respectively,

$$E_n = \frac{\text{EMOD}}{\text{THICK}}$$
$$E_t = \frac{\text{GMOD}}{\text{THICK}}$$

where THICK, the element's thickness, is an input parameter. If THICK ≤ 0, it is calculated from the distance between the initial positions of the element's corner nodes (Nodes 1-5, 2-6, 3-7 and 4-8, respectively).

While the total energy under the traction-separation law is given by $G_C$, one further parameter is needed to describe the exact shape of the tri-linear material model. If the area (energy) under the constant stress (plateau) region is denoted $G_P$ (see Figure M240-1), a parameter $f_G$ defines the shape of the traction-separation law,

$$0 \le f_{G1} = \frac{G_{I,P}}{G_{IC}} < 1 - \frac{T^2}{2G_{IC}E_n} < 1 \quad \text{for mode I loading}$$
$$0 \le f_{G2} = \frac{G_{II,P}}{G_{IIC}} < 1 - \frac{S^2}{2G_{IIC}E_t} < 1 \quad \text{for mode II loading}$$

As a recommended alternative, the shape of the tri-linear model can be described by the following displacement ratios (triggered by negative input values for $f_G$):

$$0 < |f_{G1}| = \left|\frac{\delta_{n2} - \delta_{n1}}{\delta_{nf} - \delta_{n1}}\right| < 1 \quad \text{for mode I loading}$$
$$0 < |f_{G2}| = \left|\frac{\delta_{t2} - \delta_{t1}}{\delta_{tf} - \delta_{t1}}\right| < 1 \quad \text{for mode II loading}$$

While $f_{G1}$ and $f_{G2}$ are always constant values, $T$, $S$, $G_{IC}$, and $G_{IIC}$ may be chosen as functions of an equivalent strain rate $\dot{\varepsilon}_{\text{eq}}$, which is evaluated by

$$\dot{\varepsilon}_{\text{eq}} = \frac{\sqrt{\dot{u}_n^2 + \dot{u}_{t1}^2 + \dot{u}_{t2}^2}}{\text{THICK}} \ .$$

Here $\dot{u}_n$, $\dot{u}_{t1}$, and $\dot{u}_{t2}$ are the velocities corresponding to the separations $u_n$, $u_{t1}$, and $u_{t2}$, respectively.

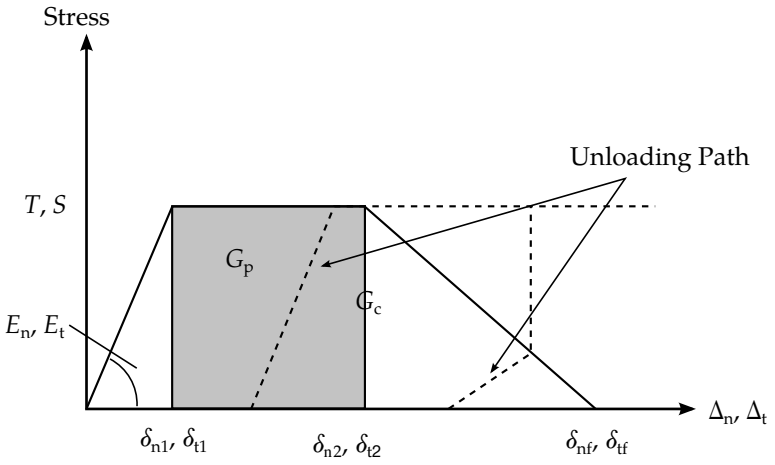



**Figure M240-1.** Trilinear traction-separation law

For the yield stresses, two rate-dependent formulations are implemented:

a) A quadratic logarithmic function:

$$T(\dot{\varepsilon}_{\text{eq}}) = |\text{T0}| + |\text{T1}|\left[\max\left(0, \ln\frac{\dot{\varepsilon}_{\text{eq}}}{\text{EDOT_T}}\right)\right]^2 \quad \text{for mode I if T0} < 0 \text{ and T1} > 0$$

$$S(\dot{\varepsilon}_{\text{eq}}) = |\text{S0}| + |\text{S1}|\left[\max\left(0, \ln\frac{\dot{\varepsilon}_{\text{eq}}}{\text{EDOT_S}}\right)\right]^2 \quad \text{for mode II if S0} < 0 \text{ and S1} > 0$$

b) A linear logarithmic function:

$$T(\dot{\varepsilon}_{\text{eq}}) = |\text{T0}| + |\text{T1}|\max\left(0, \ln\frac{\dot{\varepsilon}_{\text{eq}}}{\text{EDOT_T}}\right) \quad \text{for mode I if T0} < 0 \text{ and T1} < 0$$

$$S(\dot{\varepsilon}_{\text{eq}}) = |\text{S0}| + |\text{S1}|\max\left(0, \ln\frac{\dot{\varepsilon}_{\text{eq}}}{\text{EDOT_S}}\right) \quad \text{for mode II if S0} < 0 \text{ and S1} < 0$$

Alternatively, $T$ and $S$ can be set to constant values:

$$T(\dot{\varepsilon}_{\text{eq}}) = \text{T0} \quad \text{for mode I if T0} > 0$$
$$S(\dot{\varepsilon}_{\text{eq}}) = \text{S0} \quad \text{for mode II if S0} > 0$$

The rate-dependent fracture energies are given by:

$$G_{IC}(\dot{\varepsilon}_{\text{eq}}) = |\text{G1C_0}| + (\text{G1C_INF} - |\text{G1C_0}|)\exp\left(-\frac{\text{EDOT_G1}}{\dot{\varepsilon}_{\text{eq}}}\right) \quad \text{if G1C_0} < 0$$

$$G_{IIC}(\dot{\varepsilon}_{\text{eq}}) = |\text{G2C_0}| + (\text{G2C_INF} - |\text{G2C_0}|)\exp\left(-\frac{\text{EDOT_G2}}{\dot{\varepsilon}_{\text{eq}}}\right) \quad \text{if G2C_0} < 0$$

If positive values are chosen for G1C_0 or G2C_0, no rate-dependency is considered for this parameter, and its value remains constant as specified by the user.

As an alternative, fracture energies GIC and GIIC can be defined as functions of cohesive element thickness by using load curves LCG1C and LCG2C, respectively. In that case, parameters G1C_0, G1C_INF, G2C_0, and G2C_INF will be ignored, and no rate dependence is considered.

Note that the equivalent strain rate $\dot{\varepsilon}_{\text{eq}}$ is updated until $\Delta_m > \delta_{m1}$. Then, the model behavior depends on the equivalent strain rate at yield initiation. A modified handling of strain rates is invoked by RFILTF ≠ 0 with which filtered strain rates are updated throughout the whole process.

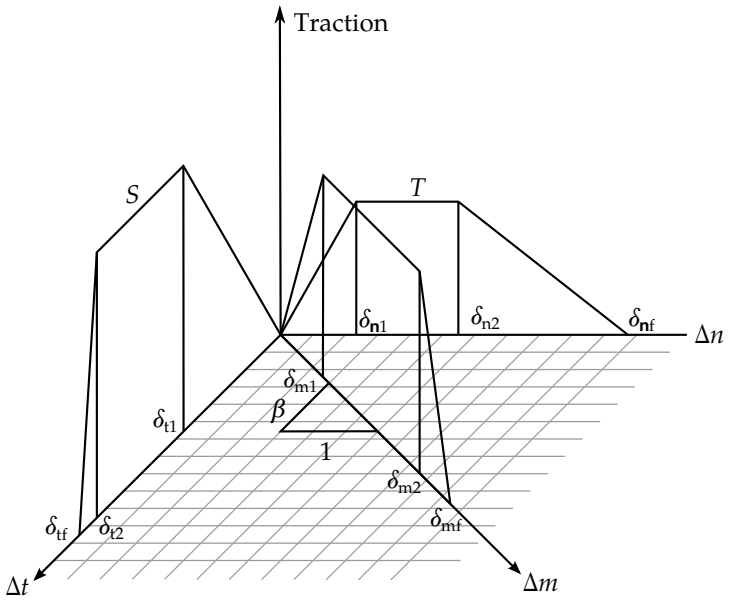



**Figure M240-2.** Trilinear mixed-mode traction-separation law

Having defined the parameters describing the single modes, the mixed-mode behavior is formulated by quadratic initiation criteria for both yield stress and damage initiation, while the damage evolution follows a power law. Due to reasons of readability, the following simplifications are made,

$$T = T(\dot{\varepsilon}_{\text{eq}})$$
$$S = S(\dot{\varepsilon}_{\text{eq}})$$
$$G_{IC} = G_{IC}(\dot{\varepsilon}_{\text{eq}})$$
$$G_{IIC} = G_{IIC}(\dot{\varepsilon}_{\text{eq}})$$

If the quadratic nominal stress criterion is used (INICRT = 0), the mixed-mode yield initiation displacement $\delta_{m1}$ is defined as

$$\delta_{m1} = \delta_{n1}\delta_{t1}\sqrt{\frac{1+\beta^2}{\delta_{t1}^2 + (\beta\delta_{n1})^2}}\ ,$$

where $\delta_{n1} = T/E_n$ and $\delta_{t1} = S/E_t$ are the single-mode yield initiation displacements and $\beta = \Delta_t/\Delta_n$ is the mixed-mode ratio. As an analog to the yield initiation, the damage initiation displacement $\delta_{m2}$ is defined as:

$$\delta_{m2} = \delta_{n2}\delta_{t2}\sqrt{\frac{1+\beta^2}{{\delta_{t2}}^2 + (\beta\delta_{n2})^2}},$$

where

$$\delta_{n2} = \delta_{n1} + \frac{f_{G1}G_{IC}}{T}$$
$$\delta_{t2} = \delta_{t1} + \frac{f_{G2}G_{IIC}}{S}$$

As an alternative, a maximum nominal stress criterion could be used (INICRT = 1) which results in the following expressions for yield and damage initiation displacements:

$$\delta_{m1} = \begin{cases} \delta_{n1}\sqrt{1+\beta^2} & \text{if } \beta \le \dfrac{\delta_{t1}}{\delta_{n1}} \\ \dfrac{\delta_{t1}}{\beta}\sqrt{1+\beta^2} & \text{if } \beta > \dfrac{\delta_{t1}}{\delta_{n1}} \end{cases}$$

$$\delta_{m2} = \begin{cases} \delta_{n2}\sqrt{1+\beta^2} & \text{if } \beta \le \dfrac{\delta_{t2}}{\delta_{n2}} \\ \dfrac{\delta_{t2}}{\beta}\sqrt{1+\beta^2} & \text{if } \beta > \dfrac{\delta_{t2}}{\delta_{n2}} \end{cases}$$

A third possibility is to choose INICRT < 0, which invokes a nominal stress criterion with flexible exponent:

$$\delta_{m1} = \delta_{n1}\delta_{t1}\sqrt{1+\beta^2}\left(\delta_{t1}^{|\text{INICRT}|} + (\beta\delta_{n1})^{|\text{INICRT}|}\right)^{-1/|\text{INICRT}|}$$
$$\delta_{m2} = \delta_{n2}\delta_{t2}\sqrt{1+\beta^2}\left(\delta_{t2}^{|\text{INICRT}|} + (\beta\delta_{n2})^{|\text{INICRT}|}\right)^{-1/|\text{INICRT}|}$$

Obviously, the special case of INICRT = -2 would lead to the same result as the quadratic criterion, INICRT = 0.

With $\gamma = \arccos\left(\frac{\langle u_n \rangle}{\Delta_m}\right)$, the ultimate (failure) displacement $\delta_{mf}$ can be written,

$$\delta_{mf} = \frac{\delta_{m1}(\delta_{m1}-\delta_{m2})E_nG_{IIC}\cos^2\gamma + G_{IC}\left(2G_{IIC} + \delta_{m1}(\delta_{m1}-\delta_{m2})E_t\sin^2\gamma\right)}{\delta_{m1}(E_nG_{IIC}\cos^2\gamma + E_tG_{IC}\sin^2\gamma)}.$$

This formulation describes a power-law damage evolution with an exponent $\eta = 1.0$ (see *MAT_138). This is the case for XMU = 0.0 or 1.0.

With the definition of an arbitrary value for XMU, the failure displacement is given by

$$\delta_{mf} = \max\left(\delta_{m2}, \delta_{m1} - \delta_{m2} + \frac{2}{\delta_{m1}}\left[\left(\frac{E_n \cos^2\gamma}{G_{IC}}\right)^{\mathrm{XMU}} + \left(\frac{E_t \sin^2\gamma}{G_{IIC}}\right)^{\mathrm{XMU}}\right]^{-1/\mathrm{XMU}}\right)$$

After the shape of the mixed-mode traction-separation law has been determined by $\delta_{m1}$, $\delta_{m2}$, and $\delta_{mf}$, the plastic separation in each element direction, $u_{n,P}$, $u_{t1,P}$, and $u_{t2,P}$ can be calculated. The plastic separation in peel direction is given by

$$u_{n,P} = \max(u_{n,P,\Delta t-1}, u_n - \delta_{m1}\cos(\gamma)\,, 0)\ .$$

In the shear direction, a shear yield separation $\delta_{t,y}$,

$$\delta_{t,y} = \sqrt{(u_{t1} - u_{t1,P,\Delta t-1})^2 + (u_{t2} - u_{t2,P,\Delta t-1})^2},$$

is defined. If $\delta_{t,y} > \delta_{m1}\sin\gamma$, the plastic shear separations in the element coordinate system are updated,

$$u_{t1,P} = u_{t1,P,\Delta t-1} + u_{t1} - u_{t1,\Delta t-1}$$
$$u_{t2,P} = u_{t2,P,\Delta t-1} + u_{t2} - u_{t2,\Delta t-1}$$

In the formulas above, $\Delta t - 1$ indicates the individual value from the last time increment. In case $\Delta_m > \delta_{m2}$, the damage initiation criterion is satisfied and a damage variable $D$ increases monotonically,

$$D = \max\left(\frac{\Delta_m - \delta_{m2}}{\delta_{mf} - \delta_{m2}}, D_{\Delta t-1}, 0\right)\ .$$

When $\Delta_m > \delta_{mf}$, complete damage ($D = 1$) is reached and the element fails in the corresponding integration point.

Finally, the peel and the shear stresses in element directions are calculated,

$$\sigma_{t1} = E_t(1 - D)(u_{t1} - u_{t1,P})$$
$$\sigma_{t2} = E_t(1 - D)(u_{t2} - u_{t2P})$$

In the peel direction, no damage under pressure loads is considered if $u_n - u_{n,P} > 0$

$$\sigma_n = E_n(u_n - u_{n,P})\ .$$

Otherwise,

$$\sigma_n = E_n(1 - D)(u_n - u_{n,P})\ .$$

2. **Arguments for *DEFINE_FUNCTION.** If the FUNCTIONS keyword option is used, parameters EMOD, GMOD, G1C_0, G2C_0, T0, S0, FG1, and FG2 (as well as GMOD3, G3C_0, R0, and FG3 if combined with 3MODES) should refer to *DEFINE_FUNCTION IDs. The arguments of those functions include several properties of both connection partners if corresponding solid elements are in a tied contact with shell elements.

These functions depend on:

(t1, t2) = thicknesses of both bond partners
(sy1, sy2) = initial yield stresses at plastic strain of 0.002
(sm1, sm2) = maximum engineering yield stresses (necking points)
r = strain rate
a = element area
(e1, e2) = Young's moduli
(ut1,ut2,un) = tangential/normal separations
(utp1,utp2,unp) = plastic tangential/normal separations
thk = cohesive thickness

For T0 = -100 such a function could look like:

```
*DEFINE_FUNCTION
           100
    func(t1,t2,sy1,sy2,sm1,sm2,r,a,e1,e2,ut1,ut2,un,utp1,ut
p2,unp,thk)=0.5*(sy1+sy2)
```

Since material parameters must be identified from both bond partners during initialization, this feature is only available for a subset of material models at the moment, namely material models 24, 36, 120, 123, 124, 251, and 258.

3. **History variables.** The following history variables are available for this material (see NEIPH on *DATABASE_EXTENT_BINARY):

| History Variable # | Description |
|---|---|
| eqp | Damage |
| 1 | Failure flag |
| 2 | Reference strain rate |
| 3 | Thickness |
| 4 | $G_{IIC}$ |
| 5 | Strength $S$ (Mode II) |
| 6 | $G_{IC}$ |
| 7 | Strength $T$ (Mode I) |
| 8 | Mixed-mode separation at plateau start |
| 9 | Plastic separation in $t1$ direction |
| 10 | Plastic separation in $t2$ direction |
| 11 | Plastic separation in peel direction |
| 12 | Separation $u_{t1}$ for increment |
| 13 | Separation $u_{t2}$ for increment |

| History Variable # | Description |
|---|---|
| 14 | Separation $u_n$for increment |
| 15 | For all keyword options except THERMAL, state variable that means elastic for values between 0 and 1, plastic for values between 1 and 2, and softening/damage for values between 2 and 3. For the THERMAL keyword option, this history variable is temperature. |
| 16 | Curve shape factor $f_{G1}$ |
| 17 | Curve shape factor $f_{G2}$ |
| 25 | For the THERMAL keyword option only, state variable that means elastic for values between 0 and 1, plastic for values between 1 and 2, and softening/damage for values between 2 and 3 |

**Reference:**

S. Marzi, O. Hesebeck, M. Brede and F. Kleiner (2009), A Rate-Dependent, Elasto-Plastic Cohesive Zone Mixed-Mode Model for Crash Analysis of Adhesively Bonded Joints, In Proceeding: *7th European LS-DYNA Conference, Salzburg*

## *MAT_JOHNSON_HOLMQUIST_JH1

This is Material Type 241. This Johnson-Holmquist Plasticity Damage Model is useful for modeling ceramics, glass and other brittle materials. This version corresponds to the original version of the model, JH1, and Material Type 110 corresponds to JH2, the updated model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | G | P1 | S1 | P2 | S2 | C |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS0 | T | | ALPHA | SFMAX | BETA | DP1 | |
| Type | F | F | | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPFMIN | EPFMAX | K1 | K2 | K3 | FS | FDAM | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Density |
| G | Shear modulus |
| P1 | Pressure point 1 for intact material |
| S1 | Effective stress at P1 |
| P2 | Pressure point 2 for intact material |
| S2 | Effective stress at P2 |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Strain rate sensitivity factor |
| EPS0 | Quasi-static threshold strain rate. See *MAT_015. |
| T | Maximum tensile pressure strength. This value is positive in tension. |
| ALPHA | Initial slope of the fractured material strength curve. See Figure M241-1. |
| SFMAX | Maximum strength of the fractured material |
| BETA | Fraction of elastic energy loss converted to hydrostatic energy (affects bulking pressure (history variable 1) that accompanies damage). |
| DP1 | Maximum compressive pressure strength. This value is positive in compression. |
| EPFMIN | Plastic strain for fracture at tensile pressure $T$. See Figure M241-2. |
| EPFMAX | Plastic strain for fracture at compressive pressure DP1. See Figure M241-1. |
| K1 | First pressure coefficient (equivalent to the bulk modulus) |
| K2 | Second pressure coefficient |
| K3 | Third pressure coefficient |
| FS | Element deletion due to hydrostatic pressure or equivalent plastic strain:<br>LT.0.0: Delete if $P$ < FS (tensile failure)<br>EQ.0.0: No element deletion (default)<br>GT.0.0: Delete element if the $\bar{\varepsilon}^p$ > FS |
| FDAM | Failure damage value. If this damage value is reached, the element is deleted. A meaningful value would be FDAM = 1.0, for instance.<br>EQ.0.0: No element deletion due to damage (default) |

**Remarks:**

The equivalent stress for both intact and fractured ceramic-type materials is given by:

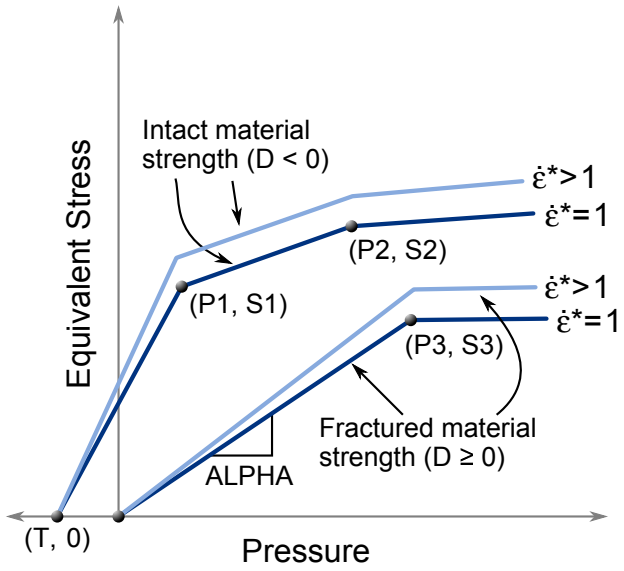



**Figure M241-1.** Strength: equivalent stress versus pressure.

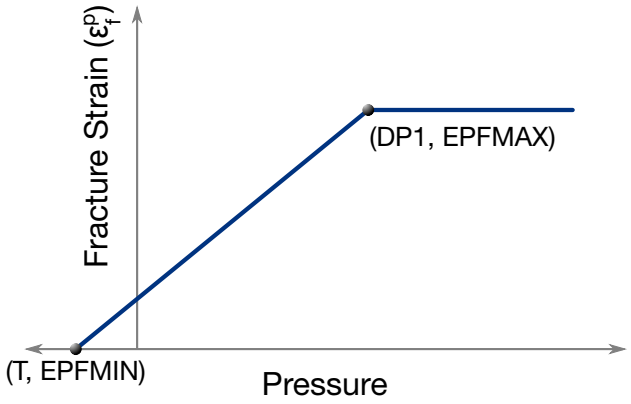



**Figure M241-2.** Fracture strain versus pressure.

$$\sigma_y = (1 + c \ln \dot{\varepsilon}^*)\sigma(P)$$

where $\sigma(P)$ is evaluated according to Figure M241-1.

$$D = \sum \Delta\varepsilon^p / \varepsilon_f^p \, (P)$$

represents the accumulated damage (history variable 2) based upon the increase in plastic strain per computational cycle and the plastic strain to fracture is evaluated according to Figure M241-2.

In undamaged material, the hydrostatic pressure is given by

$$P = k_1\mu + k_2\mu^2 + k_3\mu^3 + \Delta P$$

in compression and by

$$P = k_1\mu + \Delta P$$

in tension, where $\mu = \rho/\rho_0 - 1$. A fraction, between 0 and 1, of the elastic energy loss, $\beta$, is converted into hydrostatic potential energy (pressure). The pressure increment, $\Delta P$, associated with the increment in the hydrostatic potential energy is calculated at fracture, where $\sigma_y$ and $\sigma_y^f$ are the intact and failed yield stresses, respectively. This pressure increment is applied in both compression and tension, which is not true for JH2 where the increment is added only in compression.

$$\Delta P = -k_1\mu_f + \sqrt{(k_1\mu_f)^2 + 2\beta k_1 \Delta U}$$

$$\Delta U = \frac{\sigma_y - \sigma_y^f}{6G}$$

## *MAT_KINEMATIC_HARDENING_BARLAT2000

This is Material Type 242. This model combines the Yoshida non-linear kinematic hardening rule (*MAT_125) with the 8-parameter material model (*MAT_133) of Barlat et al. (2003) to model metal sheets under cyclic plasticity loading with anisotropy in plane stress conditions (see also *MAT_226). This material is available only for shell elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | EA | COE | M | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card must be included as a blank card.

| | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card must be included as a blank card.

| | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| CB | Y | SC1 | K | RSAT | SB | H | SC2 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| AOPT | | IOPT | C1 | C2 | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | EA | COE | M | |
| Type | A | F | F | F | F | F | F | |
| Default | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| EA | $E^A$, parameter controlling the change of Young's modulus; see the remarks of *MAT_125.<br>LT.0.0: \|EA\| is a curve ID giving the change of Young's modulus as a function of effective plastic strain. |
| COE | $\zeta$, parameter controlling the change of Young's modulus; see the remarks of *MAT_125. |
| M | Flow potential exponent. For face centered cubic (FCC) materials $m = 8$ is recommended and for body centered cubic (BCC) materials $m = 6$ may be used.<br>LT.0.0: \|M\| is a load curve ID specifying the flow potential exponent as a function of effective plastic strain. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA*i* | $\alpha_i$, material constants in Barlat's yield equation<br>LT.0.0: \|ALPHA*i*\| is a load curve ID specifying $\alpha_i$ as a function of effective plastic strain. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |
| Default | | | | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | | | | | |
| Type | | | | | | | | |
| Default | | | | | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CB | Y | SC1 | K | RSAT | SB | H | SC2 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| CB | The uppercase $B$ defined in Yoshida's equations. |
| Y | Anisotropic parameter associated with work-hardening stagnation, defined in Yoshida's equations |
| SC1 | The lowercase $c_2$ defined in Yoshida & Uemori's equations. Note the equation below from the paper: $$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$$ See more details in mat_125_SC1_SC2 in the remarks section of *MAT_125. |
| K | Hardening parameter as defined in Yoshida's equations |
| RSAT | Hardening parameter as defined in Yoshida's equations |
| SB | The lowercase $b$ as defined in Yoshida's equations |
| H | Anisotropic parameter associated with work-hardening stagnation, defined in Yoshida's equations |
| SC2 | The lowercase $c_1$ defined in the Yoshida and Uemori's equations. Note the equation below from the paper: $$c = \begin{cases} c_1 & \max(\bar{\alpha}_*) < B - Y \\ c_2 & \text{otherwise} \end{cases}$$ See more details in mat_125_SC1_SC2 in the remarks section of *MAT_125. If SC2 equals 0.0, is left blank, or equals SC1, then it turns into the basic model (the one $c$ model). |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | IOPT | C1 | C2 | | | |
| Type | I | | I | F | F | | | |
| Default | none | | none | 0.0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description). Note AOPT may need to set to 0.0 for a simulation using the dynain file from a previous simulation.<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| IOPT | Kinematic hardening rule flag:<br>EQ.0: Original Yoshida formulation<br>EQ.1: Modified formulation. Define C1, C2 below. |
| C1, C2 | Constants used to modify $R$:<br>$R = \text{RSAT} \times [(C_1 + \bar{\varepsilon}^p)^{c_2} - {C_1}^{c_2}]$ |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |
| Default | | | | none | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

**Remarks:**

1. **Yield Surface.** A total of eight parameters ($\alpha_1$ to $\alpha_8$) are needed to describe the yield surface. The parameters can be determined with tensile tests in three directions and one equal biaxial tension test. For detailed theoretical background and material parameters of some typical FCC materials, see Remarks in *MAT_133 and Barlat et al. (2003) paper.

2. **Yoshida Model.** For a more detailed description on the Yoshida model and parameters, see Remarks in *MAT_226 and *MAT_125.

3. **AOPT.** For information on the variable AOPT, see Remarks in *MAT_226.

4. **Convergence and Springback.** To improve convergence, it is recommended that *CONTROL_IMPLICIT_FORMING type '1' be used when conducting a springback simulation.

**Revisions:**

1. This material model is available starting in LS-DYNA R5 Revision 58432.

2. The variables EA, COE, SC1, and SC2 are available starting in Revision 133318.

## *MAT_HILL_90

This is Material Type 243. This model was developed by Hill [1990] for modeling sheets with anisotropic materials under plane stress conditions. This material allows the use of the Lankford parameters for the definition of the anisotropy. All features of this model are the same as in *MAT_036, only the yield condition and associated flow rules are replaced by the Hill90 equations.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | HR | P1 | P2 | ITER |
|---|---|---|---|---|---|---|---|

**Card 2a.** This card is included if FLAG = 0 (see Card 4).

| M | R00 | R45 | R90 | LCID | E0 | SPI | P3 |
|---|---|---|---|---|---|---|---|

**Card 2b.** This card is included if FLAG = 1 (see Card 4).

| M | AH | BH | CH | LCID | E0 | SPI | P3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is included if M < 0 on Card 2a/2b.

| CRC1 | CRA1 | CRC2 | CRA2 | CRC3 | CRA3 | CRC4 | CRA4 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | C | P | VLCID | | FLAG | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

***Card 7.*** This card is optional.

| USRFAIL | | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | P1 | P2 | ITER |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus, $E$<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-E) which defines Young's modulus as a function of plastic strain. See Remark 1. |
| PR | Poisson's ratio, $\nu$ |
| HR | Hardening rule:<br>EQ.1.0: Linear (default)<br>EQ.2.0: Exponential (Swift; see Remark 3)<br>EQ.3.0: Load curve or table with strain rate effects (see Remark 1)<br>EQ.4.0: Exponential (Voce; see Remark 3)<br>EQ.5.0: Exponential (Gosh; see Remark 3)<br>EQ.6.0: Exponential (Hocket-Sherby; see Remark 3)<br>EQ.7.0: Load curves in three directions (see Remark 1)<br>EQ.8.0: Table with temperature dependence (see Remark 1)<br>EQ.9.0: 3D table with temperature and strain rate dependence (see Remark 1) |
| P1 | Material parameter:<br>HR.EQ.1.0: Tangent modulus, |

| VARIABLE | DESCRIPTION |
|---|---|
| | HR.EQ.2.0: $k$, strength coefficient for Swift exponential hardening<br>HR.EQ.4.0: $a$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $k$, strength coefficient for Gosh exponential hardening<br>HR.EQ.6.0: $a$, coefficient for Hocket-Sherby exponential hardening<br>HR.EQ.7.0: Load curve ID for hardening in 45 degree direction. See Remark 1. |
| P2 | Material parameter:<br>HR.EQ.1.0: Yield stress<br>HR.EQ.2.0: $n$, exponent for Swift exponential hardening<br>HR.EQ.4.0: $c$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $n$, exponent for Gosh exponential hardening<br>HR.EQ.6.0: $c$, coefficient for Hocket-Sherby exponential hardening<br>HR.EQ.7.0: Load curve ID for hardening in 90 degree direction. See Remark 1. |
| ITER | Iteration flag for speed:<br>EQ.0.0: Fully iterative<br>EQ.1.0: Fixed at three iterations<br>Generally, we recommend ITER = 0.0. However, ITER = 1.0 is somewhat faster and may give acceptable results in most problems. |

**Lankford Parameters Card.** This card is included if FLAG = 0 on Card 4.

| Card 2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | R00 | R45 | R90 | LCID | E0 | SPI | P3 |
| Type | F | F | F | F | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| M | $m$, exponent in Hill's yield surface. If negative, the absolute value is used. Typically, $m$ ranges between 1 and 2 for low-$r$ materials, such as aluminum (AA6111: $m \approx 1.5$) and is greater than 2 for high $r$-materials, as in steel (DP600: $m \approx 4$). See Remark 3. |
| R00 | Lankford parameter in 0 degree direction, $R_{00}$ (see Remark 3):<br>GT.0.0: Constant value<br>LT.0.0: Load curve or table ID = (-R00) which defines $R_{00}$ as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1 and 2. |
| R45 | Lankford parameter in 45 degree direction, $R_{45}$ (see Remark 3):<br>GT.0.0: Constant value<br>LT.0.0: Load curve or table ID = (-R45) which defines $R_{45}$ as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1 and 2. |
| R90 | Lankford parameter in 90 degree direction, $R_{90}$ (see Remark 3):<br>GT.0.0: Constant value<br>LT.0.0: Load curve or table ID = (-R90) which defines $R_{90}$ as a function of plastic strain (curve) or as a function of temperature and plastic strain (table). See Remarks 1 and 2. |
| LCID | Load curve/table ID for hardening in the 0 degree direction (applies for HR = 3, 7, 8, and 9). See Remark 1. |
| E0 | Material parameter (see Remark 3):<br>HR.EQ.2.0: $\varepsilon_0$ for determining initial yield stress for Swift exponential hardening (default = 0.0)<br>HR.EQ.4.0: $b$, coefficient for Voce exponential hardening<br>HR.EQ.5.0: $\varepsilon_0$ for determining initial yield stress for Gosh exponential hardening (default = 0.0)<br>HR.EQ.6.0: $b$, coefficient for Hocket-Sherby exponential hardening |
| SPI | Case I: If $\varepsilon_0$ is zero above and HR = 2.0 (see Remark 3). (Default = 0.0)<br>EQ.0.0: $\varepsilon_0 = \left(E/k\right)^{1/(n-1)}$ |

| VARIABLE | DESCRIPTION |
|---|---|
| | LE.0.02: $\varepsilon_0$ = SPI<br>GT.0.02: $\varepsilon_0 = \left(\text{SPI}/k\right)^{1/n}$<br>Case II: If HR = 5.0 the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR = 2.0 (see Remark 3). |
| P3 | Material parameter (see Remark 3):<br>HR.EQ.5.0: $p$, parameter for Gosh exponential hardening<br>HR.EQ.6.0: $n$, exponent for Hocket-Sherby exponential hardening |

**Hill90 Parameters Card.** This card is included for FLAG = 1.

| Card 2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | AH | BH | CH | LCID | E0 | SPI | P3 |
| Type | F | F | F | F | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| M | $m$, exponent in Hill's yield surface. If negative, the absolute value is used. Typically, $m$ ranges between 1 and 2 for low-$R$ materials, such as aluminum (AA6111: $m \approx 1.5$) and is greater than 2 for high $R$-materials, as in steel (DP600: $m \approx 4$). See Remark 3. |
| AH | $a$, Hill90 parameter (see Remark 3) |
| BH | $b$, Hill90 parameter (see Remark 3) |
| CH | $c$, Hill90 parameter (see Remark 3) |
| LCID | Load curve/table ID for hardening in the 0 degree direction (applies for HR = 3, 7, 8, and 9). See Remark 1. |
| E0 | Material parameter (see Remark 3):<br>HR.EQ.2.0: $\varepsilon_0$ for determining initial yield stress for Swift exponential hardening (default = 0.0)<br>HR.EQ.4.0: $b$, coefficient for Voce exponential hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| | HR.EQ.5.0: $\varepsilon_0$ for determining initial yield stress for Gosh exponential hardening (default = 0.0) |
| | HR.EQ.6.0: $b$, coefficient for Hocket-Sherby exponential hardening |
| SPI | Case I: If $\varepsilon_0$ is zero above and HR = 2.0 (see Remark 3). (default = 0.0) |
| | EQ.0.0: $\varepsilon_0 = \left(E/k\right)^{1/(n-1)}$ |
| | LE.0.02: $\varepsilon_0$ = SPI |
| | GT.0.02: $\varepsilon_0 = \left(\text{SPI}/k\right)^{1/n}$ |
| | Case II: If HR = 5.0 the strain at plastic yield is determined by an iterative procedure based on the same principles as for HR = 2.0 (see Remark 3). |
| P3 | Material parameter (see Remark 3): |
| | HR.EQ.5.0: $p$, parameter for Gosh exponential hardening |
| | HR.EQ.6.0: $n$, exponent for Hocket-Sherby exponential hardening |

**Hardening Card.** Additional Card for M < 0.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CRC1 | CRA1 | CRC2 | CRA2 | CRC3 | CRA3 | CRC4 | CRA4 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CRC$n$ | Chaboche-Rousselier hardening parameters. See Remark 4 |
| CRA$n$ | Chaboche-Rousselier hardening parameters. See Remark 4. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | C | P | VLCID | | FLAG | | |
| Type | F | F | F | I | | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. The material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| C | *C* in Cowper-Symonds strain rate model (see Remark 3) |
| P | *p* in Cowper-Symonds strain rate model (see Remark 3). $p = 0.0$ for no strain rate effects. |
| VLCID | Volume correction curve ID defining the relative volume change (change in volume relative to the initial volume) as a function of the effective plastic strain. This is only used when nonzero. See Remark 1. |

| VARIABLE | DESCRIPTION |
|---|---|
| FLAG | Flag for interpretation of parameters. If FLAG = 1, parameters AH, BH, and CH are read instead of R00, R45, and R90. See Remark 3. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 and 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA. |

This card is optional.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | USRFAIL | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| USRFAIL | User defined failure flag: |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0: No user subroutine is called |
| | EQ.1: User subroutine `matusr_24` in dyn21.f is called. |

**Remarks:**

1. **Plastic Strain in Curve Definitions.** The effective plastic strain used in this model is defined to be plastic work equivalent. A consequence of this is that for parameters defined as functions of effective plastic strain, the rolling (00) direction should be used as reference direction. For instance, the hardening curve for HR = 3 is the stress as function of strain for uniaxial tension in the rolling direction, the curve VLCID should give the relative volume change as function of strain for uniaxial tension in the rolling direction, and the load curve with ID -E should give the Young's modulus as function of strain for uniaxial tension in the rolling direction. Optionally the curve can be substituted for a table defining hardening as function of plastic strain rate (HR = 3), temperature (HR = 8), or both (HR = 9).

   Exceptions from this rule are curves defined as functions of plastic strain in the 45 and 90 directions, such as P1 and P2 for HR = 7 and negative R45 or R90. The hardening curves are here defined as measured stress as function of measured plastic strain for uniaxial tension in the direction of interest, meaning as determined from experimental testing using a standard procedure. Moreover, the curves defining the *R*-values are functions of the measured plastic strain for uniaxial tension in the direction of interest. These curves are transformed internally to be used with the effective stress and strain properties in the actual model. The effective plastic strain does not coincide with the plastic strain components in directions other than the rolling direction and may be somewhat confusing. Therefore, the von Mises work equivalent plastic strain is output as history variable #2 if HR = 7 or if any *R*-value is defined as function of the plastic strain.

2. **Determining *R*-Values from Curves.** The *R*-values in curves are defined as the ratio of instantaneous width change to instantaneous thickness change. That is, assume that the width $W$ and thickness $T$ are measured as a function of strain. Then the corresponding *R*-value is given by:

$$R = \frac{\frac{dW}{d\varepsilon}/W}{\frac{dT}{d\varepsilon}/T}$$

3. **Yield Criterion, Hill90 Parameters, and Hardening Models.** The anisotropic yield criterion $\Phi$ for plane stress is defined as:

$$\Phi = K_1^m + K_3 K_2^{(m/2)-1} + c^m K_4^{m/2} = (1 + c^m - 2a + b)\sigma_Y^m$$

where $\sigma_Y$ is the yield stress. $K_i, i = 1, \ldots, 4$ are given by:

$$K_1 = |\sigma_x + \sigma_y|$$
$$K_2 = |\sigma_x^2 + \sigma_y^2 + 2\sigma_{xy}^2|$$
$$K_3 = -2a(\sigma_x^2 - \sigma_y^2) + b(\sigma_x - \sigma_y)^2$$
$$K_4 = \left|(\sigma_x - \sigma_y)^2 + 4\sigma_{xy}^2\right|$$

If FLAG = 0, the anisotropic material constants $a$, $b$, and $c$ are obtained through $R_{00}$, $R_{45}$, and $R_{90}$ using these 3 equations:

$$1 + 2R_{00} = \frac{c^m - a + \{(m+2)/2m\}b}{1 - a + \{(m-2)/2m\}b}$$
$$1 + 2R_{45} = c^m$$
$$1 + 2R_{90} = \frac{c^m + a + \{(m+2)/2m\}b}{1 + a + \{(m-2)/2m\}b}$$

If FLAG = 1, material parameters $a$ (AH), $b$ (BH), and $c$ (CH) are used directly.

For material parameters $a$, $b$, $c$, and $m$, the following condition must be fulfilled, otherwise an error termination occurs:

$$1 + c^m - 2a + b \quad > \quad 0$$

Two even more strict conditions should be satisfied to ensure convexity of the yield surface according to Hill (1990). A warning message will be output if at least one of them is violated:

$$b > -2^{\left(\frac{m}{2}\right)-1} c^m$$
$$b > a^2 - c^m$$

For the Swift hardening law (HR = 2), the yield strength of the material can be expressed in terms of $k$ and $n$:

$$\sigma_Y = k\varepsilon^n = k(\varepsilon_0 + \bar{\varepsilon}^p)^n$$

where $\varepsilon_0$ is the elastic strain to yield and $\bar{\varepsilon}^p$ is the effective plastic strain (logarithmic). $\varepsilon_0$ can be given in the input with E0 or determined using SPI. If E0 and SPI are both set to zero, the strain to yield is found by solving for the intersection of the linearly elastic loading equation with the strain hardening equation:

$$\sigma = E\varepsilon$$
$$\sigma = k\varepsilon^n$$

which gives the elastic strain at yield as:

$$\varepsilon_0 = \left(\frac{E}{k}\right)^{\left[\frac{1}{n-1}\right]}$$

If E0 is zero and SPI is nonzero and greater than 0.02 then:

$$\varepsilon_0 = \left(\frac{\sigma_Y}{k}\right)^{\left[\frac{1}{n}\right]}$$

The other available hardening models include the Voce equation (HR = 4) given by

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p},$$

the Gosh equation (HR = 5) given by

$$\sigma_Y(\varepsilon_p) = k(\varepsilon_0 + \varepsilon_p)^n - p,$$

and finally, the Hocket-Sherby equation (HR = 6) given by

$$\sigma_Y(\varepsilon_p) = a - be^{-c\varepsilon_p^n}.$$

For the Gosh hardening law, the interpretation of the variable SPI is the same as for the Swift hardening law, meaning if set to zero (along with E0), the strain at yield is determined implicitly from the intersection of the strain hardening equation with the linear elastic equation.

To include strain rate effects in the model, we multiply the yield stress by a factor depending on the effective plastic strain rate. We use the Cowper-Symonds model, hence the yield stress can be written

$$\sigma_Y(\varepsilon_p, \dot{\varepsilon}_p) = \sigma_Y^s(\varepsilon_p)\left[1 + \left(\frac{\dot{\varepsilon}_p}{C}\right)^{1/p}\right].$$

Here $\sigma_Y^s$ denotes the static yield stress, $C$ and $p$ are material parameters, and $\dot{\varepsilon}_p$ is the effective plastic strain rate.

4. **Kinematic Hardening Model.** A kinematic hardening model is implemented following the works of Chaboche and Roussilier. A back stress $\boldsymbol{\alpha}$ is introduced such that the effective stress is computed as

$$\sigma_{\text{eff}} = \sigma_{\text{eff}}(\sigma_{11} - 2\alpha_{11} - \alpha_{22}, \sigma_{22} - 2\alpha_{22} - \alpha_{11}, \sigma_{12} - \alpha_{12})$$

The back stress is the sum of up to four terms according to

$$\alpha_{ij} = \sum_{k=1}^{4} \alpha_{ij}^k$$

and the evolution of each back stress component is as follows

$$\delta\alpha_{ij}^k = C_k\left(a_k \frac{s_{ij} - \alpha_{ij}}{\sigma_{\text{eff}}} - \alpha_{ij}^k\right)\delta\varepsilon_p$$

where $C_k$ and $a_k$ are material parameters,$s_{ij}$ is the deviatoric stress tensor, $\sigma_{\text{eff}}$ is the effective stress, and $\varepsilon_p$ is the effective plastic strain.

## *MAT_UHS_STEEL

This is Material Type 244. This material model is developed for both shell and solid models. It is mainly suited for hot stamping processes where phase transformations are crucial. It has five phases, and it is assumed that the blank is fully austenitized before cooling. The model also includes optional algorithms for switching between heating and cooling. The basic constitutive model is based on the work done by P. Akerstrom [2, 7].

**NOTE 1:** For this material "weight%" means "ppm × $10^{-4}$".

**NOTE 2:** We include baseline values in the variable tables as possible starting values that lead to reasonable results for an alloy called 22MnB5. The values are taken from the literature.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | TUNIT | CRSH | PHASE | HEAT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCY1 | LCY2 | LCY3 | LCY4 | LCY5 | KFER | KPER | B |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C | Co | Mo | Cr | Ni | Mn | Si | V |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| W | Cu | P | Al | As | Ti | CWM | LCTRE |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| THEXP1 | THEXP5 | LCTH1 | LCTH5 | TREF | LAT1 | LAT5 | TABTH |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| QR2 | QR3 | QR4 | ALPHA | GRAIN | TOFFE | TOFPE | TOFBA |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| PLMEM2 | PLMEM3 | PLMEM4 | PLMEM5 | STRC | STRP | REACT | TEMPER |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is included if HEAT = 1.

| AUST | FERR | PEAR | BAIN | MART | GRK | GRQR | TAU1 |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is included if HEAT = 1.

| GRA | GRB | EXPA | EXPB | GRCC | GRCM | HEATN | TAU2 |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is included if REACT = 1.

| FS | PS | BS | MS | MSIG | LCEPS23 | LCEPS4 | LCEPS5 |
|---|---|---|---|---|---|---|---|

**Card 11.** This card is included if TEMPER = 1.

| LCH4 | LCH5 | DTCRIT | TSAMP | | | | |
|---|---|---|---|---|---|---|---|

**Card 12.** This card is included if CWM = 1.

| TASTART | TAEND | TLSTART | TLEND | EGHOST | PGHOST | AGHOST | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | TUNIT | CRSH | PHASE | HEAT |
| Type | A | F | F | F | F | I | I | I |
| Defaults | none | none | none | none | 3600 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). | |
| RO | Material density | 7830 Kg/m$^3$ |
| E | Young's modulus:<br>GT.0.0: Constant value<br>LT.0.0: Temperature dependent Young's modulus given by | 100 GPa [1] |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | load curve or table ID = -E. See Remark 9 for more information about using a table to specify the Young's modulus. | |
| PR | Poisson's ratio:<br>GT.0.0: Constant value<br>LT.0.0: Temperature dependent Poisson ratio given by load curve or table ID = -PR. The table input is described in Remark 9. | 0.30 [1] |
| TUNIT | Number of time units per hour. Default is seconds, that is, 3600 time units per hour. TUNIT is used only for hardness calculations. | 3600. |
| CRSH | Switch to use a simple and fast material model but with the actual phases active.<br>EQ.0: The original model where phase transitions are active and trip is used.<br>EQ.1: A simpler and faster version. This option is mainly used when transferring the quenched blank into a crash analysis where all properties from the cooling are maintained. This option must be used with a *INTERFACE_SPRINGBACK keyword and should be used after a quenching analysis.<br>EQ.2: Same as 0 but trip effect is not used. | 0 |
| PHASE | Switch to include or exclude middle phases from the simulation.<br>EQ.0: All phases active (default)<br>EQ.1: Pearlite and bainite excluded | 0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | EQ.2: Bainite excluded<br>EQ.3: Ferrite and pearlite excluded<br>EQ.4: Ferrite and bainite excluded<br>EQ.5: Exclude middle phases (only austenite → martensite) | |
| HEAT | Switch to activate the heating algorithms (see Remarks 7 and 8):<br>EQ.0: Heating is not activated which means that no transformation to austenite is possible.<br>EQ.1: Heating is activated which means that only transformation to austenite is possible.<br>EQ.2: Automatic switching between cooling and heating. LS-DYNA checks the temperature gradient and calls the appropriate algorithms. For example, this can be used to simulate the heat affected zone during welding.<br>LT.0: The switch between cooling and heating is defined by a time dependent load curve with ID = \|HEAT\|. The ordinate should be 1.0 when heating is applied and 0.0 if cooling is preferable. | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCY1 | LCY2 | LCY3 | LCY4 | LCY5 | KFER | KPER | B |
| Type | I | I | I | I | I | F | F | F |
| Defaults | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| LCY1 | Load curve or table ID for austenite hardening.<br>**Load Curve.** When LCY1 is a load curve ID, it defines input yield stress as a function of effective plastic strain.<br>**Tabular Data (LCY1 > 0).** When LCY1 is greater than 0 and references a table ID, a 2D table references for each temperature value a hardening curve.<br>**Tabular Data (LCY1 < 0).** When LCY1 is less than 0, \|LCY1\| is a 3D table ID. Each input temperature value gives a table ID which defines for each a strain rate a hardening curve. | |
| LCY2 | Load curve ID for ferrite hardening (stress as a function of effective plastic strain) | |
| LCY3 | Load curve or table ID for pearlite. See LCY1 for description. | |
| LCY4 | Load curve or table ID for bainite. See LCY1 for description. | |
| LCY5 | Load curve or table ID for martensite. See LCY1 for description. | |
| KFERR | Correction factor for boron in the ferrite reaction. | $1.9 \times 10^5$ [2] |
| KPEAR | Correction factor for boron in the pearlite reaction. | $3.1 \times 10^3$ [2] |
| B | Boron [weight %] | 0.003 [2] |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | Co | Mo | Cr | Ni | Mn | Si | V |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| C | Carbon [weight %] | 0.23 [2] |
| Co | Cobolt [weight %] | 0.0 [2] |
| Mo | Molybdenum [weight %] | 0.0 [2] |
| Cr | Chromium [weight %] | 0.21 [2] |
| Ni | Nickel [weight %] | 0.0 [2] |
| Mn | Manganese [weight %] | 1.25 [2] |
| Si | Silicon [weight %] | 0.29 [2] |
| V | Vanadium [weight %] | 0.0 [2] |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | W | Cu | P | Al | As | Ti | CWM | LCTRE |
| Type | F | F | F | F | F | F | I | I |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 | none |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| W | Tungsten [weight %] | 0.0 [2] |
| Cu | Copper [weight %] | 0.0 [2] |
| P | Phosphorous [weight %] | 0.013 [2] |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| Al | Aluminum [weight %] | 0.0 [2] |
| As | Arsenic [weight %] | 0.0 [2] |
| Ti | Titanium [weight %] | 0.0 [2] |
| CWM | Flag for computational welding mechanics input. One additional input card is read.<br>EQ.1.0: Active<br>EQ.0.0: Inactive | |
| LCTRE | Load curve for transformation induced strains. See Remark 14. | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | THEXP1 | THEXP5 | LCTH1 | LCTH5 | TREF | LAT1 | LAT5 | TABTH |
| Type | F | F | I | I | F | F | F | I |
| Defaults | 0.0 | 0.0 | none | none | 273.15 | 0.0 | 0.0 | none |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| THEXP1 | Coefficient of thermal expansion in austenite | $25.1 \times 10^{-6}$ 1/K [7] |
| THEXP5 | Coefficient of thermal expansion in martensite | $11.1 \times 10^{-6}$ 1/K [7] |
| LCTH1 | Load curve for the thermal expansion coefficient for austenite:<br>LT.0: Curve ID = -LCTH1 and TREF is used as reference temperature<br>GT.0: Curve ID = LCTH1 | 0 |
| LCTH5 | Load curve for the thermal expansion coefficient for martensite: | 0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | LT.0: Curve ID = -LCTH5 and TREF is used as reference temperature<br>GT.0: Curve ID = LCTH5 | |
| TREF | Reference temperature for thermal expansion. Used if LCTH1 < 0.0, LCTH5 < 0.0, or TABTH < 0. | 293.15 |
| LAT1 | Latent heat for the decomposition of austenite into ferrite, pearlite and bainite.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID or table ID. See Remark 10 for more information. | $590 \times 10^6$ J/m$^3$ [2] |
| LAT5 | Latent heat for the decomposition of austenite into martensite.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID giving latent heat as a function of temperature<br>Note that LAT5 is ignored if a table ID is used in LAT1. | $640 \times 10^6$ J/m$^3$ [2] |
| TABTH | Table ID for thermal expansion coefficient. With this option active THEXP1, THEXP2, LCTH1 and LCTH5 are ignored. See Remark 11.<br>GT.0: A table for instantaneous thermal expansion (TREF is ignored).<br>LT.0: A table with thermal expansion with reference to TREF. | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QR2 | QR3 | QR4 | ALPHA | GRAIN | TOFFE | TOFPE | TOFBA |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| QR2 | Activation energy divided by the universal gas constant for the diffusion reaction of the austenite-ferrite reaction: Q2/$R$. $R$ = 8.314472 [J/mol K]. | 10324 K [3] = (23000 cal/mole) × (4.184 J/cal) / (8.314 J/mole/K) |
| QR3 | Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-pearlite reaction: Q3/$R$. $R$ = 8.314472 [J/mol K]. | 13432. K [3] |
| QR4 | Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-bainite reaction: Q4/$R$. R = 8.314472 [J/mol K]. | 15068. K [3] |
| ALPHA | Material constant for the martensite phase. A value of 0.011 means that 90% of the available austenite is transformed into martensite at 210 degrees below the martensite start temperature (see messag file for information), whereas a value of 0.033 means a 99.9% transformation. | 0.011 |
| GRAIN | ASTM grain size number for austenite, usually a number between 7 and 11. | 6.8 |
| TOFFE | Number of degrees that the ferrite is bleeding over into the pearlite reaction | 0.0 |
| TOFPE | Number of degrees that the pearlite is bleeding over into the bainite reaction | 0.0 |
| TOFBA | Number of degrees that the bainite is | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | bleeding over into the martensite reaction | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PLMEM2 | PLMEM3 | PLMEM4 | PLMEM5 | STRC | STRP | REACT | TEMPER |
| Type | I | F | F | F | F | F | I | I |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0 | 0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| PLMEM2 | Memory coefficient for the plastic strain that is carried over from the austenite. A value of 1 means that all plastic strains from austenite is transferred to the *ferrite* phase and a value of 0 means that nothing is transferred. | 0.0 |
| PLMEM3 | Same as PLMEM2 but between austenite and pearlite | 0.0 |
| PLMEM4 | Same as PLMEM2 but between austenite and bainite | 0.0 |
| PLMEM5 | Same as PLMEM3 but between austenite and martensite | 0.0 |
| STRC | Effective strain rate parameter *C*.<br>LT.0.0: load curve ID = -STRC<br>GT.0.0: constant value<br>EQ.0.0: strain rate NOT active | 0.0 |
| STRP | Effective strain rate parameter *P*.<br>LT.0.0: load curve ID = -STRP<br>GT.0.0: constant value<br>EQ.0.0: strain rate NOT active | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| REACT | Flag for advanced reaction kinetics input. One additional input card is read.<br>EQ.1.0: active<br>EQ.0.0: inactive | 0.0 |
| TEMPER | Flag for tempering input. One additional input card is read.<br>EQ.1.0: active<br>EQ.0.0: inactive | 0.0 |

**Heat Card 1.** Additional Card for HEAT = 1.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AUST | FERR | PEAR | BAIN | MART | GRK | GRQR | TAU1 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.08E8 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| AUST | If a heating process is initiated at t = 0, this field sets the initial amount of austenite in the blank. If heating is activated at t > 0 during a simulation, this value is ignored. Note that,<br>$AUST + FERR + PEAR + BAIN + MART = 1.0$ | 0.0 |
| FERR | See AUST for description | 0.0 |
| PEAR | See AUST for description | 0.0 |
| BAIN | See AUST for description | 0.0 |
| MART | See AUST for description | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| GRK | Growth parameter $k$ ($\mu m^2$/sec) | $10^{11}$ [9] |
| GRQR | Grain growth activation energy (J/mol) divided by the universal gas constant: $Q/R$ where $R$ = 8.314472 (J/mol K) | $3 \times 10^4$ [9] |
| TAU1 | Empirical grain growth parameter $c_1$ describing the function $\tau(T)$ | $2.08 \times 10^8$ [9] |

**Heat Card 2.** Additional Card for HEAT =1.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GRA | GRB | EXPA | EXPB | GRCC | GRCM | HEATN | TAU2 |
| Type | F | F | F | F | F | F | F | F |
| Default | 3.11 | 7520. | 1.0 | 1.0 | none | none | 1.0 | 4.806 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| GRA | Grain growth parameter $A$ | [9] |
| GRB | Grain growth parameter $B$. A table of recommended values of GRA and GRB is included in Remark 8. | [9] |
| EXPA | Grain growth parameter $a$ | 1.0 [9] |
| EXPB | Grain growth parameter $b$ | 1.0 [9] |
| GRCC | Grain growth parameter with the concentration of non-metals in the blank, weight% of C or N | [9] |
| GRCM | Grain growth parameter with the concentration of metals in the blank, lowest weight% of Cr, V, Nb, Ti, Al | [9] |
| HEATN | Grain growth parameter $n$ for the austenite formation | 1.0 [9] |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| TAU2 | Empirical grain growth parameter $c_2$ describing the function $\tau(T)$ | 4.806 [9] |

**Reaction Card.** Addition card for REACT = 1.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FS | PS | BS | MS | MSIG | LCEPS23 | LCEPS4 | LCEPS5 |
| Type | F | F | F | F | F | I | I | I |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | none | none | none | none |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| FS | Manual start temperature Ferrite<br>GT.0.0: Same temperature is used for heating and cooling.<br>LT.0.0: Different start temperatures for cooling and heating given by load curve ID = -FS. First ordinate value is used for cooling, last ordinate value for heating. | |
| PS | Manual start temperature Pearlite. See FS for description. | |
| BS | Manual start temperature Bainite. See FS for description. | |
| MS | Manual start temperature Martensite. See FS for description. | |
| MSIG | Describes the increase of martensite start temperature for cooling due to applied stress.<br>LT.0: Load curve ID describes MSIG as a function of triaxiality (pressure | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | / effective stress).<br>$MS^* = MS + MSIG \times \sigma_{eff}$ | |
| LCEPS23 | Load curve ID dependent on plastic strain that scales the activation energy QR2 and QR3.<br>$QRx = Qx \times LCEPS23(\varepsilon_{pl})/R$ | |
| LCEPS4 | Load Curve ID dependent on plastic strain that scales the activation energy QR4.<br>$QR4 = Q4 \times LCEPS4(\varepsilon_{pl})/R$ | |
| LCEPS5 | Load curve ID which describe the increase of the martensite start temperature for cooling as a function of plastic strain.<br>$MS^* = MS + MSIG \times \sigma_{eff} + LCEPS5(\varepsilon_{pl})$ | |

**Tempering Card.** Additional card for TEMPR = 1.

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCH4 | LCH5 | DTCRIT | TSAMP | | | | |
| Type | I | I | F | F | | | | |
| Default | 0 | 0 | 0.0 | 0.0 | | | | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| LCH4 | Load curve ID of Vicker hardness as a function of temperature for Bainite hardness calculation | |
| LCH5 | Load curve ID of Vicker hardness as a function of temperature for Martensite hardness calculation | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| DTCRIT | Critical cooling rate to detect holding phase | |
| TSAMP | Sampling interval for temperature rate monitoring to detect the holding phase | |

**Computational Welding Mechanics Card.** Additional card for CWM = 1.

| Card 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TASTART | TAEND | TLSTART | TLEND | EGHOST | PGHOST | AGHOST | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| TASTART | Annealing temperature start | |
| TAEND | Annealing temperature end | |
| TLSTART | Birth temperature start | |
| EGHOST | Young's modulus for ghost (quiet) material | |
| PGHOST | Poisson's ratio for ghost (quiet) material | |
| AGHOST | Thermal expansion coefficient for ghost (quiet) material | |

**Discussion:**

The phase distribution during cooling is calculated by solving the following rate equation for each phase transition

$$\dot{X}_k = g_k(G, C, T_k, Q_k) f_k(X_k) \ , \qquad k = 2,3,4$$

where $g_k$ is a function, taken from Li et al., dependent on the grain number $G$, the chemical composition $C$, the temperature $T$, and the activation energy $Q$. Moreover, the function $f_k$ is dependent on the actual phase $X_k = x_k / x_{\text{eq}}$

$$f_k(X_k) = X_k^{0.4(X_k-1)}(1-X_k)^{0.4X_k} \quad , \qquad k = 2,3,4$$

The true amount of martensite, that is, $k = 5$, is modelled by using the true amount of the austenite left after the bainite phase:

$$x_5 = x_1\left[1 - e^{-\alpha(\mathrm{MS}-T)}\right] \ ,$$

where $x_1$ is the true amount of austenite left for the reaction, $\alpha$ is a material dependent constant and MS is the start temperature of the martensite reaction.

The start temperatures are automatically calculated based on the composition:

1. Ferrite,

$$\mathrm{FS} = 1185 - 203 \times \sqrt{\mathrm{C}} - 15.2 \times \mathrm{Ni} + 44.7 \times \mathrm{Si} + 104 \times \mathrm{V} + 31.5 \times \mathrm{Mo} + 13.1 \times \mathrm{W} - 30 \times \mathrm{Mn} - 11 \times \mathrm{Cr} - 20 \times \mathrm{Cu} + 700 \times \mathrm{P} + 400 \times \mathrm{Al} + 120 \times \mathrm{As} + 400 \times \mathrm{Ti}$$

2. Pearlite,

$$\mathrm{PS} = 996 - 10.7 \times \mathrm{Mn} - 16.9 \times \mathrm{Ni} + 29 \times \mathrm{Si} + 16.9 \times \mathrm{Cr} + 290 \times \mathrm{As} + 6.4 \times \mathrm{W}$$

3. Bainite,

$$\mathrm{BS} = 910 - 58 \times \mathrm{C} - 35 \times \mathrm{Mn} - 15 \times \mathrm{Ni} - 34 \times \mathrm{Cr} - 41 \times \mathrm{Mo}$$

4. Martensite,

$$\mathrm{MS} = 812 - 423 \times \mathrm{C} - 30.4 \times \mathrm{Mn} - 17.7 \times \mathrm{Ni} - 12.1 \times \mathrm{Cr} - 7.5 \times \mathrm{Mo} + 10 \times \mathrm{Co} - 7.5 \times \mathrm{Si}$$

where the element weight values are input on Cards 2 through 4.

The automatic start temperatures are printed to the messag file and if they are not accurate enough you can manually set them in the input deck (must be set in absolute temperature, Kelvin). If HEAT > 0, the temperature FSnc (ferrite without C) is also echoed. If the specimen exceeds that temperature, all remaining ferrite is instantaneously transformed to austenite.

**Remarks:**

1. **History Variables.** History variables 1 through 8 include the different phases, the Vickers hardness, the yield stress and the ASTM grain size number. Set NEIPS = 8 (shells) or NEIPH = 8 (solids) on *DATABASE_EXTENT_BINARY.

| History Variable | Description |
|---|---|
| 1 | Amount austenite |
| 2 | Amount ferrite |
| 3 | Amount pearlite |

| History Variable | Description |
|---|---|
| 4 | Amount bainite |
| 5 | Amount martensite |
| 6 | Vickers hardness |
| 7 | Yield stress |
| 8 | ASTM grain size number (a low value means large grains and vice versa) |

2. **Excluding Phases.** To exclude a phase from the simulation, set the PHASE parameter accordingly.

3. **STRC and STRP.** Note that both strain rate parameters must be set to include the effect. It is possible to use a temperature dependent load curve for both parameters simultaneously or for one parameter keeping the other constant.

4. **TUNIT.** TUNIT is time units per hour and is only used for calculating the Vicker Hardness. By default, it is assumed that the time unit is seconds. If another time unit is used, for example milliseconds, then TUNIT must be changed to TUNIT $= 3.6 \times 10^6$

5. **TSF.** The thermal speedup factor TSF of *CONTROL_THERMAL_SOLVER is used to scale reaction kinetics and hardness calculations in this material model. Strain rate dependent properties (see LCY1 to LCY5 or STRC/STRP), however, are not scaled by TSF.

6. **CRSH.** With the CRSH = 1 option it is now possible to transfer the material properties from a hot stamping simulation (CRSH = 0) into another simulation. The CRSH = 1 option reads a dynain file from a simulation with CRSH = 0 and keeps all the history variables (austenite, ferrite, pearlite, bainite, martensite, etc.) constant. This will allow steels with inhomogeneous strength to be analyzed in, for example, a crash simulation. The speed with the CRSH = 1 option is comparable with *MAT_024. Note that for keeping the speed the temperature used in the CRSH simulation should be constant and the thermal solver should be inactive.

7. **Heating and Cooling and Transformation Temperatures.** To activate the heating algorithm, set HEAT = 1 or 2. HEAT = 0 (default) activates only the cooling algorithm. Note that for HEAT = 0 you *must* check that the initial temperature of this material is above the start temperature for the ferrite transformation. The transformation temperatures are echoed in the messag and d3hsp files.

If HEAT > 0 the temperature that instantaneously transforms all ferrite back to austenite is also echoed in the messag file. If you want to heat up to 100% austenite, you must let the specimen's temperature exceed that temperature.

8. **HEAT, Grain Growth, and Re-austenization.** When HEAT is activated the re-austenitization and grain growth algorithms are also activated. The grain growth is activated when the temperature exceeds a threshold value that is given by

$$T = \frac{B}{A - \log_{10}[(\mathrm{GRCM})^a (\mathrm{GRCC})^b]} ,$$

and the rate equation for the grain growth is,

$$\dot{g} = \frac{k}{2g} e^{-\frac{Q}{RT}} .$$

The rate equation for the phase re-austenitization is given in Oddy (1996) and is here mirrored

$$\dot{x}_a = n \left[ \ln \left( \frac{x_{eu}}{x_{eu} - x_a} \right) \right]^{\frac{n-1}{n}} \left[ \frac{x_{eu} - x_a}{\tau(T)} \right] ,$$

where $n$ is the parameter HEATN. The temperature dependent function $\tau(T)$ is given from Oddy as $\tau(T) = c_1 (T - T_s)^{c_2}$. The empirical parameters $c_1$ and $c_2$ are calibrated in Oddy to $2.06 \times 10^8$ and 4.806 respectively. Note that $\tau$ above given in *seconds*.

Recommended values for GRA and GRB are given in the following table.

| Compound | Metal | Non-metal | GRA | GRB |
|---|---|---|---|---|
| $Cr_{23}C_6$ | Cr | C | 5.90 | 7375 |
| $V_4C_3$ | V | $C_{0.75}$ | 5.36 | 8000 |
| TiC | Ti | C | 2.75 | 7000 |
| NbC | Nb | $C_{0.7}$ | 3.11 | 7520 |
| $Mo_2C$ | Mo | C | 5.0 | 7375 |
| Nb(CN) | Nb | (CN) | 2.26 | 6770 |
| VN | V | N | 3.46 + 0.12%Mn | 8330 |
| AlN | Al | N | 1.03 | 6770 |
| NbN | Nb | N | 4.04 | 10230 |
| TiN | Ti | N | 0.32 | 8000 |

9. **Using the Table Capability for Temperature Dependence of Young's Modulus.** Use *DEFINE_TABLE_2D and set the abscissa value equal to 1 for the austenite YM-curve, equal to 2 for the ferrite YM-curve, equal to 3 for the pearlite YM curve, equal to 4 for the bainite YM-curve and finally equal to 5 for the martensite YM-curve. If you use the PHASE option you only need to define the curves for the included phases, but you can define all five. LS-DYNA uses the number 1 - 5 to get the right curve for the right phase. The total YM is calculated by a linear mixture law: $YM = YM1 \times PHASE1 + \cdots + YM5 \times PHASE5$. For example:

```
*DEFINE_TABLE_2D
$ The number before curve id:s define which phase the curve
$ will be applied to.  1 = Austenite, 2 = Ferrite, 3 = Pearlite,
$ 4 = Bainite and 5 = Martensite.
      1000       0.0       0.0
                 1.0                 100
                 2.0                 200
                 3.0                 300
                 4.0                 400
                 5.0                 500
$
$ Define curves 100 - 500
*DEFINE_CURVE
$ Austenite Temp (K) - YM-Curve (MPa)
       100         0       1.0       1.0
              1300.0              50.E+3
               223.0             210.E+3
```

10. **Using the Table Capability for Latent Heat.** When using a table ID for the latent heat (LAT1) you can describe all phase transition individually. Use *DEFINE_TABLE_2D and set the abscissa values to the corresponding phase transition number, that is, 2 for austenite to ferrite, 3 for austenite to pearlite, 4 for austenite to bainite and 5 for austenite to martensite. Remark 9 demonstrates the form of a correct table definition. If a curve is missing, the corresponding latent heat for that transition will be set to zero. Also, when a table is used the LAT5 is ignored. If HEAT > 0, you also have the option to include latent heat for the transition back to Austenite. This latent heat curve is marked as 1 in the table definition of LAT1.

11. **Using the Table Capability for Thermal Expansion.** When using a table ID for the thermal expansion you can specify the expansion characteristics for each phase. That is, you can have a curve for each of the 5 phases (austenite, ferrite, pearlite, bainite, and martensite). The input is identical to the above table definitions. The table must have the abscissa values between 1 and 5 where the number correspond to phase 1 to 5. To exclude one phase from influencing the thermal expansion you simply input a curve that is zero for that phase or even easier, exclude that phase number in the table definition. For example, to exclude the bainite phase you only define the table with curves for the indices 1, 2, 3 and 5.

12. **TEMPER.** Tempering is activated by setting TEMPER to 1. When active, the default hardness calculation for bainite and martensite is altered to use an incremental update formula. The total hardness is given by $\sum_{i=1}^{5} \mathrm{HV}_i \times x_i$. When holding phases are detected, the hardness for Bainite and Martensite is updated according to

$$\mathrm{HV}_4^{n+1} = \frac{x_4^n}{x_4^{n+1}}\mathrm{HV}_4^n + \frac{\Delta x_4}{x_4^{n+1}}h_4(T), \qquad \Delta x_4 = x_4^{n+1} - x_4^n$$

$$\mathrm{HV}_5^{n+1} = \frac{x_5^n}{x_5^{n+1}}\mathrm{HV}_5^n + \frac{\Delta x_5}{x_5^{n+1}}h_5(T), \qquad \Delta x_5 = x_5^{n+1} - x_5^n$$

We detect the holding phase for Bainite and Martensite when the temperature is in the appropriate range and if average temperature rate is below DTCRIT. The average temperature rate is calculated as $T_{\mathrm{tresh}}/t_{\mathrm{tresh}}$, where $T_{\mathrm{tresh}}^{n+1} = T_{\mathrm{tresh}}^n + |\dot{T}|\Delta t$ and $t_{\mathrm{tresh}}^{n+1} = t_{\mathrm{tresh}}^n + \Delta t$. The average temperature and time are updated until $t_{\mathrm{tresh}} \geq t_{\mathrm{samp}}$.

13. **CWM (Welding).** When computational welding mechanics is activated with CWM = 1, the material can be defined to be initially in a quiet state. In this state the material (often referred to as ghost material) has thermo-mechanical properties defined by an additional card. The material is activated when the temperature reaches the birth temperature. See MAT_CWM (MAT_270) for a detailed description.

14. **LCTRE (Transformation Induced Strains).** Transformation induced strains can be included with a load curve LCTRE as a function of temperature. The load curve represents the difference between the hard phases and the austenite phase in the dilatometer curves. Therefore, positive curve values result in a negative transformation strain for austenitization and a positive transformation strain for the phase transformation from austenite to one of the hard phases.

**Boron steel composition from the literature:**

| Element | HAZ code | Akerstrom (2) | Naderi (8) | ThyssenKrupp (5) (max amount) |
|---|---|---|---|---|
| B | | 0.003 | 0.003 | 0.005 |
| C | 0.168 | 0.23 | 0.230 | 0.250 |
| Co | | | | |
| Mo | 0.036 | | | 0.250 |
| Cr | 0.255 | 0.211 | 0.160 | 0.250 |
| Ni | 0.015 | | | |
| Mn | 1.497 | 1.25 | 1.18 | 1.40 |
| Si | 0.473 | 0.29 | 0.220 | 0.400 |

| Element | HAZ code | Akerstrom (2) | Naderi (8) | ThyssenKrupp (5) (max amount) |
|---|---|---|---|---|
| V | 0.026 | | | |
| W | | | | |
| Cu | 0.025 | | | |
| P | 0.012 | 0.013 | 0.015 | 0.025 |
| Al | 0.020 | | | |
| As | | | | |
| Ti | | | 0.040 | 0.05 |
| S | | 0.003 | 0.001 | 0.010 |

## References:


[1] Numisheet 2008 Proceedings, The Numisheet 2008 Benchmark Study, Chapter 3, Benchmark 3, Continuous Press Hardening, Interlaken, Switzerland, Sept. 2008.

[2] P. Akerstrom and M. Oldenburg, “Austenite Decomposition During Press hardening of a Boron Steel – Computer Simulation and Test”, Journal of Material processing technology, 174 (2006), pp399-406.

[3] M.V Li, D.V Niebuhr, L.L Meekisho and D.G Atteridge, “A Computatinal model for the prediction of steel hardenability”, Metallurgical and materials transactions B, 29B, 661-672, 1998.

[4] D.F. Watt, “An Algorithm for Modelling Microstructural Development in Weld Heat-Affected Zones (Part A) Reaction Kinetics”, Acta metal. Vol. 36., No. 11, pp. 3029-3035, 1988.

[5] ThyssenKrupp Steel, “Hot Press hardening Manganese-boron Steels MBW”, product information Manganese-boron Steels, Sept. 2008.

[6] Malek Naderi, “Hot Stamping of Ultra High Strength Steels”, Doctor of Engineering Dissertation, Technical University Aachen, Germany, 2007.

[7] P. Akerstrom, “Numerical Implementation of a Constitutive model for Simulation of Hot Stamping”, Division of Solid Mechanics, Lulea University of technology, Sweden.

[8] Malek Naderi, “A numerical and Experimental Investigation into Hot Stamping of Boron Alloyed Heat treated Steels”, Steel research Int. 79 (2008) No. 2.

[9] A.S. Oddy, J.M.J. McDill and L. Karlsson, “Microstructural predictions including arbitrary thermal histories, reaustenitization and carbon segregation effects” (1996).

## *MAT_PML_*OPTION*TROPIC_ELASTIC

This is Material Type 245. This is a perfectly-matched layer (PML) material for orthotropic or anisotropic media. It is to be used in a wave-absorbing layer adjacent to an orthotropic/anisotropic material (*MAT_*OPTION*TROPIC_ELASTIC) in order to simulate wave propagation in an unbounded ortho/anisotropic medium.

This material is a variant of *MAT_PML_ELASTIC (*MAT_230). It is available only for solid 8-node bricks (element type 2). The input cards exactly follow *MAT_*OPTION*-TROPIC_ELASTIC as shown below. See the variable descriptions and Remarks section of *MAT_*OPTION*TROPIC_ELASTIC (*MAT_002) for further details.

Available options include:

ORTHO

ANISO

such that the keyword cards appear:

*MAT_PML_ORTHOTROPIC_ELASTIC or MAT_245 (4 cards follow)

*MAT_PML_ANISOTROPIC_ELASTIC or MAT_245_ANISO (5 cards follow)

### Card Summary:

**Card 1a.1.** This card is required for the ORTHO keyword option.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 1a.2.** This card is required for the ORTHO keyword option.

| GAB | GBC | GCA | AOPT | G | SIGF | | |
|---|---|---|---|---|---|---|---|

**Card 1b.1.** This card is required for the ANISO keyword option.

| MID | RO | C11 | C12 | C22 | C13 | C23 | C33 |
|---|---|---|---|---|---|---|---|

**Card 1b.2.** This card is required for the ANISO keyword option.

| C14 | C24 | C34 | C44 | C15 | C25 | C35 | C45 |
|---|---|---|---|---|---|---|---|

**Card 1b.3.** This card is required for the ANISO keyword option.

| C55 | C16 | C26 | C36 | C46 | C56 | C66 | AOPT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MACF | |
|---|---|---|---|---|---|---|---|

**Card 3.**

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

**Orthotropic Card 1.** Included for the ORTHO keyword option.

| Card 1a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

**Orthotropic Card 2.** Included for the ORTHO keyword option.

| Card 1a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | G | SIGF | | |
| Type | F | F | F | F | F | F | | |

**Anisotropic Card 1.** Included for the ANISO keyword option.

| Card 1b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | C11 | C12 | C22 | C13 | C23 | C33 |
| Type | A | F | F | F | F | F | F | F |

**Anisotropic Card 2.** Included for the ANISO keyword option.

| Card 1b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C14 | C24 | C34 | C44 | C15 | C25 | C35 | C45 |
| Type | F | F | F | F | F | F | F | F |

**Anisotropic Card 3.** Included for the ANISO keyword option.

| Card 1b.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C55 | C16 | C26 | C36 | C46 | C56 | C66 | AOPT |
| Type | F | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | I | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | F |

**Remarks:**

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary. The layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.

2. **Material Properties of Bounded Domain.** The material in the bounded domain near the layer is assumed to be, or behaves like, a linear ortho/anisotropic

material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem. For instance, for a half-space problem, the "top" of the box should be open.

   Internally, LS-DYNA partitions the entire PML into regions which form the "faces", "edges" and "corners" of the above cuboid box. LS-DYNA generates a new material for each region. This partitioning will be visible in the d3plot file. You may safely ignore this partitioning.

4. **Number of Elements in the Layer.** The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints — such as *CONSTRAINED_GLOBAL, or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve — will not be recognized. (Ansys Workbench uses the latter approach).

6. **Stress and Strain.** The stress and strain values reported by this material do not have any physical significance.

## *MAT_PML_NULL

This is Material Type 246. This is a perfectly-matched layer (PML) material with a pressure fluid constitutive law computed using an equation of state, to be used in a wave-absorbing layer adjacent to a fluid material (*MAT_NULL with an EOS) in order to simulate wave propagation in an unbounded fluid medium. Only *EOS_LINEAR_POLYNOMIAL and *EOS_GRUNEISEN are allowed with this material. See the Remarks section of *MAT_NULL (*MAT_009) for further details. Accurate results are to be expected only for the case where the EOS presents a linear relationship between the pressure and volumetric strain.

This material is a variant of *MAT_PML_ELASTIC (*MAT_230) and is available only for solid 8-node bricks (element type 2).

| Card | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | MU | | | | | |
| Type | A | F | F | | | | | |
| Default | none | none | 0.0 | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| MU | Dynamic viscosity coefficient |

**Remarks:**

1. **Unboundedness.** A layer of this material may be placed at a boundary of a bounded domain to simulate unboundedness of the domain at that boundary: the layer absorbs and attenuates waves propagating outward from the domain, without any significant reflection of the waves back into the bounded domain. The layer cannot support any static displacement.

2. **Material Properties of Bounded Domain.** It is assumed the material in the bounded domain near the layer is, or behaves like, a linear fluid material. The material properties of the layer should be set to the corresponding properties of this material.

3. **Layer Geometry.** The layer should form a cuboid box around the bounded domain, with the axes of the box aligned with the coordinate axes. Various faces of this box may be open, as required by the geometry of the problem. For example, for a half-space problem, the "top" of the box should be open.

   Internally, LS-DYNA will partition the entire PML into regions which form the "faces," "edges" and "corners" of the above cuboid box and generate a new material for each region. This partitioning will be visible in the d3plot file. The user may safely ignore this partitioning.

4. **Number of Elements in the Layer.** The layer should have 5 - 10 elements through its depth. Typically, 5 - 6 elements are sufficient if the excitation source is reasonably distant from the layer, and 8 - 10 elements if it is close. The size of the elements should be similar to that of elements in the bounded domain near the layer, and should be small enough to sufficiently discretize all significant wavelengths in the problem.

5. **Nodal Constraints.** The nodes on the outer boundary of the layer should be fully constrained using either *BOUNDARY_SPC or TC on *NODE. Other constraints, such as *CONSTRAINED_GLOBAL or *BOUNDARY_PRESCRIBED_MOTION with a zero-valued load curve, will not be recognized.

   Note that Ansys Mechanical uses *BOUNDARY_PRESCRIBED_MOTION with a zero-value load curve for fully constraining nodal degrees-of-freedom. Because of this limitation, you will have to edit the input file with a text editor.

6. **Stress and Strain.** The stress and strain values reported by this material do not have any physical significance.

## *MAT_PHS_BMW

This is Material Type 248. This model is intended for hot stamping processes with phase transformation effects. It is available for shell elements *only* and is based on Material Type 244 (*MAT_UHS_STEEL). As compared with Material Type 244, Material Type 248 features:

1. a more flexible choice of evolution parameters,
2. an approach for transformation induced strains,
3. and a more accurate density calculation of individual phases.

Thus, the physical effects of the metal can be taken into account calculating the volume fractions of ferrite, pearlite, bainite and martensite for fast supercooling as well as for slow cooling conditions. Furthermore, this material model features cooling-rate dependence for several of its more crucial material parameters in order to accurately calculate the Time-Temperature-Transformation diagram dynamically. A detailed description can be found in Hippchen et al. [2013] and Hippchen [2014].

**NOTE 1:** For this material "weight%" means "ppm × $10^{-4}$".

**NOTE 2:** For this material the phase fractions are calculated in volume percent (vol%).

**NOTE 3:** The baseline values for this material are mainly taken from those for *MAT_244. They are provided to give reasonable starting results. These values may not reproduce the results from the papers by Hippchen.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | TUNIT | TRIP | PHASE | HEAT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCY1 | LCY2 | LCY3 | LCY4 | LCY5 | C_F | C_P | C_B |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C | Co | Mo | Cr | Ni | Mn | Si | V |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| W | Cu | P | Al | As | Ti | B | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| | | TABRHO | | TREF | LAT1 | LAT5 | TABTH |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| QR2 | QR3 | QR4 | ALPHA | GRAIN | TOFFE | TOFPE | TOFBA |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| PLMEM2 | PLMEM3 | PLMEM4 | PLMEM5 | STRC | STRP | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| FS | PS | BS | MS | MSIG | LCEPS23 | LCEPS4 | LCEPS5 |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| LCH4 | LCH5 | DTCRIT | TSAMP | ISLC | IEXTRA | | |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is required.

| ALPH_M | N_M | PHI_M | PSI_M | OMG_F | PHI_F | PSI_F | CR_F |
|---|---|---|---|---|---|---|---|

**Card 11.** This card is required.

| OMG_P | PHI_P | PSI_P | CR_P | OMG_B | PHI_B | PSI_B | CR_B |
|---|---|---|---|---|---|---|---|

**Card 12.** This card is included if HEAT ≠ 0.

| AUST | FERR | PEAR | BAIN | MART | GRK | GRQR | TAU1 |
|---|---|---|---|---|---|---|---|

**Card 13.** This card is included if HEAT ≠ 0.

| GRA | GRB | EXPA | EXPB | GRCC | GRCM | HEATN | TAU2 |
|---|---|---|---|---|---|---|---|

**Card 14.** This card is included if IEXTRA ≥ 1.

| FUNCA | FUNCB | FUNCM | TCVUP | TCVLO | CVCRIT | TCVSL | |
|---|---|---|---|---|---|---|---|

**Card 15.** This card is included if IEXTRA ≥ 2.

| EPSP | EXPON | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | TUNIT | TRIP | PHASE | HEAT |
| Type | A | F | F | F | F | I | I | I |
| Defaults | none | none | none | none | 3600 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). | |
| RO | Material density at room temperature (necessary for calculating transformation induced strains) | 7830 $Kg/m^3$ |
| E | Youngs' modulus:<br>GT.0.0: Constant value is used.<br>LT.0.0: Temperature dependent Young's modulus given by load curve or table ID = -E. The table input is described in Remark 10. | 100.e+09 Pa [1] |
| PR | Poisson's ratio:<br>GT.0.0: Constant value is used.<br>LT.0.0: Temperature dependent Poisson's ratio given by load curve or table ID = -PR. The table input is described in Remark 10. | 0.30 [1] |
| TUNIT | Number of time units per hour. Default is seconds, that is, 3600 time units per hour. It is used only for hardness calculations. | 3600. |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| TRIP | Flag to activate (0) or deactivate (1) trip effect calculation | 0 |
| PHASE | Switch to exclude middle phases from the simulation:<br>EQ.0: All phases active (default)<br>EQ.1: Pearlite and bainite active<br>EQ.2: Bainite active<br>EQ.3: Ferrite and pearlite active<br>EQ.4: Ferrite and bainite active<br>EQ.5: No active middle phases (only austenite → martensite) | 0 |
| HEAT | Heat flag as in MAT_244:<br>EQ.0: Heating is not activated.<br>EQ.1: Heating is activated.<br>EQ.2: Automatic switching between cooling and heating<br>LT.0: Switch between cooling and heating is defined by a time dependent load curve with ID \|HEAT\|. | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCY1 | LCY2 | LCY3 | LCY4 | LCY5 | C_F | C_P | C_B |
| Type | I | I | I | I | I | F | F | F |
| Defaults | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| LCY1 | Load curve or table ID for austenite hardening. | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | **Load Curve.** When LCY1 is a load curve ID, it defines input yield stress as a function of effective plastic strain.<br>**Tabular Data (LCY1 > 0).** When LCY1 is greater than 0 and references a table ID, a 2D table references for each temperature value a hardening curve.<br>**Tabular Data (LCY1 < 0).** When LCY1 is less than 0, \|LCY1\| is a 3D table ID. Each input temperature value gives a table ID which defines for each a strain rate a hardening curve. | |
| LCY2 | Load curve or table ID for ferrite. See LCY1 for description. | |
| LCY3 | Load curve or table ID for pearlite. See LCY1 for description. | |
| LCY4 | Load curve or table ID for bainite. See LCY1 for description. | |
| LCY5 | Load curve or table ID for martensite. See LCY1 for description. | |
| C_F | Alloy dependent factor $C_f$ for ferrite (controls the alloying effects beside of Boron on the time-temperature-transformation start line of ferrite) | |
| C_P | Alloy dependent factor $C_p$ for pearlite (see C_F for description) | |
| C_B | Alloy dependent factor $C_b$ for bainite (see C_F for description) | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | Co | Mo | Cr | Ni | Mn | Si | V |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| C | Carbon [weight %] | 0.23 [2] |
| Co | Cobolt [weight %] | 0.0 [2] |
| Mo | Molybdenum [weight %] | 0.0 [2] |
| Cr | Chromium [weight %] | 0.21 [2] |
| Ni | Nickel [weight %] | 0.0 [2] |
| Mn | Manganese [weight %] | 1.25 [2] |
| Si | Silicon [weight %] | 0.29 [2] |
| V | Vanadium [weight %] | 0.0 [2] |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | W | Cu | P | Al | As | Ti | B | |
| Type | F | F | F | F | F | F | F | |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| W | Tungsten [weight %] | 0.0 [2] |
| Cu | Copper [weight %] | 0.0 [2] |
| P | Phosphorous [weight %] | 0.013 [2] |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| Al | Aluminum [weight %] | 0.0 [2] |
| As | Arsenic [weight %] | 0.0 [2] |
| Ti | Titanium [weight %] | 0.0 [2] |
| B | Boron [weight %] | 0.0 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | TABRH0 | | TREF | LAT1 | LAT5 | TABTH |
| Type | | | I | | F | F | F | I |
| Defaults | | | none | | none | 0.0 | 0.0 | none |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| TABRHO | Table definition for phase and temperature dependent densities. Needed for calculation of transformation induced strains. | |
| TREF | Reference temperature for thermal expansion (only necessary for thermal expansion calculation with the secant method). | 293.15 |
| LAT1 | Latent heat for the decomposition of austenite into ferrite, pearlite and bainite.<br>GT.0.0: Constant value<br>LT.0.0: Curve ID or table ID. See Remark 11 for more information. | $590 \times 10^6$ J/m$^3$ [2] |
| LAT5 | Latent heat for the decomposition of austenite into martensite.<br>GT.0.0: Constant value<br>LT.0.0: Curve ID. Note that LAT 5 is ignored if a table ID is used in LAT1. | $640 \times 10^6$ J/m$^3$ [2] |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| TABTH | Table definition for thermal expansion coefficient. See Remark 12.<br>GT.0: A table for instantaneous thermal expansion (TREF is ignored)<br>LT.0: A table with thermal expansion with reference to TREF | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | QR2 | QR3 | QR4 | ALPHA | GRAIN | TOFFE | TOFPE | TOFBA |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| QR2 | Activation energy divided by the universal gas constant for the diffusion reaction of the austenite-ferrite reaction: Q2/$R$. $R$ = 8.314472 [J/mol K]. Load curve ID if ISLC = 2 on Card 9 (function of cooling rate). | 10324 K [3] = (23000 cal/mole) × (4.184 J/cal) / (8.314 J/mole/K) |
| QR3 | Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-pearlite reaction: Q3/$R$. $R$ = 8.314472 [J/mol K]. Load curve ID if ISLC = 2 on Card 9 (function of cooling rate). | 13432. K [3] |
| QR4 | Activation energy divided by the universal gas constant for the diffusion reaction for the austenite-bainite reaction: Q4/$R$. $R$ = 8.314472 [J/mol K]. Load curve ID if ISLC = 2 on Card 9 (function of cooling rate). | 15068. K [3] |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| ALPHA | Material constant for the martensite phase. A value of 0.011 means that 90% of the available austenite is transformed into martensite at 210 degrees below the martensite start temperature (see messag file for information), whereas a value of 0.033 means a 99.9% transformation. | 0.011 |
| GRAIN | ASTM grain size number $G$ for austenite, usually a number between 7 and 11. | 6.8 |
| TOFFE | Number of degrees that the ferrite is bleeding over into the pearlite reaction: $T_{\text{off},f}$ | 0.0 |
| TOFPE | Number of degrees that the pearlite is bleeding over into the bainite reaction: $T_{\text{off},p}$ | 0.0 |
| TOFBA | Number of degrees that the bainite is bleeding over into the martensite reaction: $T_{\text{off},b}$ | 0.0 |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PLMEM2 | PLMEM3 | PLMEM4 | PLMEM5 | STRC | STRP | | |
| Type | F | F | F | F | F | F | | |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| PLMEM2 | Memory coefficient for the plastic strain that is carried over from the austenite. A value of 1 means that all plastic strains from austenite is transferred to the ferrite phase and a value of 0 means that nothing is transferred. | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| PLMEM3 | Same as PLMEM2 but between austenite and pearlite | 0.0 |
| PLMEM4 | Same as PLMEM2 but between austenite and bainite | 0.0 |
| PLMEM5 | Same as PLMEM3 but between austenite and martensite | 0.0 |
| STRC | Cowper and Symonds strain rate parameter *C*<br>LT.0.0: Load curve ID = -STRC<br>GT.0.0: Constant value<br>EQ.0.0: Strain rate *not* active | 0.0 |
| STRP | Cowper and Symonds strain rate parameter *P*<br>LT.0.0: Load curve ID = -STRP<br>GT.0.0: Constant value<br>EQ.0.0: Strain rate *not* active | 0.0 |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FS | PS | BS | MS | MSIG | LCEPS23 | LCEPS4 | LCEPS5 |
| Type | F | F | F | F | F | I | I | I |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | none | none | none | none |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| FS | Manual start temperature ferrite, $F_S$.<br>GT.0.0: Same temperature is used for heating and cooling.<br>LT.0.0: Different start temperatures for cooling and heating given by load curve ID = -FS. First | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | ordinate value is used for cooling, last ordinate value for heating. | |
| PS | Manual start temperature pearlite, $P_S$. See FS for description. | |
| BS | Manual start temperature bainite, $B_S$. See FS for description. | |
| MS | Manual start temperature martensite, $M_S$. See FS for description. | |
| MSIG | Describes the increase of martensite start temperature for cooling due to applied stress.<br>LT.0: Load curve ID describes MSIG as a function of triaxiality (pressure / effective stress).<br>$\mathrm{MS}^* = \mathrm{MS} + \mathrm{MSIG} \times \sigma_{\mathrm{eff}}$ | |
| LCEPS23 | Load curve ID dependent on plastic strain that scales the activation energy QR2 and QR3.<br>$\mathrm{QRn} = \mathrm{Qn} \times \mathrm{LCEPS23}(\varepsilon_{\mathrm{pl}})/R$ | |
| LCEPS4 | Load curve ID dependent on plastic strain that scales the activation energy QR4.<br>$\mathrm{QR4} = \mathrm{Q4} \times \mathrm{LCEPS4}(\varepsilon_{\mathrm{pl}})/R$ | |
| LCEPS5 | Load Curve ID which describe the increase of the martensite start temperature for cooling as a function of plastic strain.<br>$\mathrm{MS}^* = \mathrm{MS} + \mathrm{MSIG} \times \sigma_{\mathrm{eff}} + \mathrm{LCEPS5}(\varepsilon_{\mathrm{pl}})$ | |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCH4 | LCH5 | DTCRIT | TSAMP | ISLC | IEXTRA | | |
| Type | I | I | F | F | I | I | | |
| Defaults | 0 | 0 | 0.0 | 0.0 | 0 | 0 | | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| LCH4 | Load curve ID of Vickers hardness as a function of temperature for bainite hardness calculation | |
| LCH5 | Load curve ID of Vickers hardness as a function of temperature for martensite hardness calculation | |
| DTCRIT | Critical cooling rate to detect holding phase | |
| TSAMP | Sampling interval for temperature rate monitoring to detect the holding phase | |
| ISLC | Flag for definition of evolution parameters on Cards 10 and 11.<br>EQ.0.0: All 16 fields on Cards 10 and 11 are constant values.<br>EQ.1.0: PHI_F, CR_F, PHI_P, CR_P, PHI_B, and CR_B are load curves defining values as functions of cooling rate. The remaining 10 fields on Cards 10 and 11 are constant values.<br>EQ.2.0: QR2, QR3, and QR4 from Card 6 and all 16 fields on Cards 10 and 11 are load curves defining values as functions of cooling rate. | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| IEXTRA | Flag to read extra cards (see Cards 14 and 15) | |

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPH_M | N_M | PHI_M | PSI_M | OMG_F | PHI_F | PSI_F | CR_F |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0428 | 0.191 | 0.382 | 2.421 | 0.41 | 0.4 | 0.4 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| ALPH_M | Martensite evolution parameter $\alpha_m$ | 0.0428 |
| N_M | Martensite evolution parameter $n_m$ | 0.191 |
| PHI_M | Martensite evolution parameter $\varphi_m$ | 0.382 |
| PSI_M | Martensite evolution exponent $\psi_m$. If $\psi_m < 0$, then $\psi_m = \lvert\psi_m\rvert(2 - \varsigma_a)$. | 2.421 |
| OMG_F | Ferrite grain size factor $\omega_f$ (mainly controls the alloying effect of Boron on the time-temperature-transformation start line of ferrite) | 0.41 |
| PHI_F | Ferrite evolution parameter $\varphi_f$ (controls the incubation time till 1 vol% of ferrite is built) | 0.4 |
| PSI_F | Ferrite evolution parameter $\psi_f$ (controls the time till 99 vol% of ferrite is built without effect on the incubation time) | 0.4 |
| CR_F | Ferrite evolution parameter $C_{r,f}$ (retardation coefficient to influence the kinetics of phase transformation of ferrite, should be determined at slow cooling conditions, can also be defined in dependency to the | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| | cooling rate) | |

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | OMG_P | PHI_P | PSI_P | CR_P | OMG_B | PHI_B | PSI_B | CR_B |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.32 | 0.4 | 0.4 | 0.0 | 0.29 | 0.4 | 0.4 | 0.0 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| OMG_P | Pearlite grain size factor $\omega_p$ (see OMG_F for description) | 0.32 |
| PHI_P | Pearlite evolution parameter $\varphi_p$ (see PHI_F for description) | 0.4 |
| PSI_P | Pearlite evolution parameter $\psi_p$ (see PSI_F for description) | 0.4 |
| CR_P | Pearlite evolution parameter $C_{r,p}$ (see CR_F for description) | 0.0 |
| OMG_B | Bainite grain size factor $\omega_b$ (see OMG_F for description) | 0.32 |
| PHI_B | Bainite evolution parameter $\varphi_b$ (see PHI_F for description) | 0.4 |
| PSI_B | Bainite evolution parameter $\psi_b$ (see PSI_-F for description) | 0.4 |
| CR_B | Bainite evolution parameter $C_{r,b}$ (see CR_F for description) | 0.0 |

**Heat Card 1.** Additional Card for HEAT ≠ 0.

| Card 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AUST | FERR | PEAR | BAIN | MART | GRK | GRQR | TAU1 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 2.08E+8 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| AUST | If a heating process is initiated at $t = 0$, this field sets the initial amount of austenite in the blank. If heating is activated at $t > 0$ during a simulation, this value is ignored. Note that, $\text{AUST} + \text{FERR} + \text{PEAR} + \text{BAIN} + \text{MART} = 1.0$ | 0.0 |
| FERR | See AUST for description | 0.0 |
| PEAR | See AUST for description | 0.0 |
| BAIN | See AUST for description | 0.0 |
| MART | See AUST for description | 0.0 |
| GRK | Growth parameter $k$ ($\mu m^2$/sec) | $10^{11}$[9] |
| GRQR | Grain growth activation energy (J/mol) divided by the universal gas constant: $Q/R$. $R = 8.314472$ (J/mol K) | $3.0 \times 10^4$[9] |
| TAU1 | Empirical grain growth parameter $c_1$ describing the function $\tau(T)$ | $2.08 \times 10^8$[9] |

**Heat Card 2.** Additional Card for HEAT ≠ 0.

| Card 13 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GRA | GRB | EXPA | EXPB | GRCC | GRCM | HEATN | TAU2 |
| Type | F | F | F | F | F | F | F | F |
| Default | 3.11 | 7520. | 1.0 | 1.0 | none | none | 1.0 | 4.806 |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| GRA | Grain growth parameter $A$ | [9] |
| GRB | Grain growth parameter $B$. A table of recommended values of GRA and GRB is included in Remark 8 of *MAT_244. | [9] |
| EXPA | Grain growth parameter $a$ | 1.0 [9] |
| EXPB | Grain growth parameter $b$ | 1.0 [9] |
| GRCC | Grain growth parameter with the concentration of nonmetals in the blank, weight% of C or N | [9] |
| GRCM | Grain growth parameter with the concentration of metals in the blank, lowest weight% of Cr, V, Nb, Ti, Al | [9] |
| HEATN | Grain growth parameter $n$ for the austenite formation | 1.0 [9] |
| TAU2 | Empirical grain growth parameter $c_2$ describing the function $\tau(T)$ | 4.806 [9] |

**Extra Card 1.** Additional Card for IEXTRA ≥ 1

| Card 14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FUNCA | FUNCB | FUNCM | TCVUP | TCVLO | CVCRIT | TCVSL | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| FUNCA | ID of a *DEFINE_FUNCTION for saturation stress $A$ (Hockett-Sherby approach) | |
| FUNCB | ID of a *DEFINE_FUNCTION for initial yield stress $B$ (Hockett-Sherby approach) | |
| FUNCM | ID of a *DEFINE_FUNCTION for saturation rate $M$ (Hockett-Sherby approach) | |
| TCVUP | Upper temperature for determination of average cooling velocity | |
| TCVLO | Lower temperature for determination of average cooling velocity | |
| CVCRIT | Critical cooling velocity. If the average cooling velocity is less than or equal to CVCRIT, the cooling rate at temperature TCVSL is used. | |
| TCVSL | Temperature for determination of cooling velocity for small cooling velocities. | |

**Extra Card 2.** Additional Card for IEXTRA ≥ 2

| Card 15 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSP | EXPON | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION | BASELINE VALUE |
|---|---|---|
| EPSP | Plastic strain in Hockett-Sherby approach | |
| EXPON | Exponent in Hockett-Sherby approach | |

**Remarks:**

1. **Start temperatures.** Start temperatures for ferrite, pearlite, bainite, and martensite can be defined manually using FS, PS, BS, and MS. Or they are initially defined using the following composition equations:

$$F_S = 273.15 + 912 - 203 \times \sqrt{\mathrm{C}} - 15.2 \times \mathrm{Ni} + 44.7 \times \mathrm{Si} + 104 \times \mathrm{V} + 31.5 \times \mathrm{Mo} + 13.1 \times \mathrm{W} - 30 \times \mathrm{Mn} - 11 \times \mathrm{Cr} - 20 \times \mathrm{Cu} + 700 \times \mathrm{P} + 400 \times \mathrm{Al} + 120 \times \mathrm{As} + 400$$

$$P_S = 273.15 + 723 - 10.7 \times \mathrm{Mn} - 16.9 \times \mathrm{Ni} + 29 \times \mathrm{Si} + 16.9 \times \mathrm{Cr} + 290 \times \mathrm{As} + 6.4 \times \mathrm{W}$$

$$B_S = 273.15 + 637 - 58 \times \mathrm{C} - 35 \times \mathrm{Mn} - 15 \times \mathrm{Ni} - 34 \times \mathrm{Cr} - 41 \times \mathrm{Mo}$$

$$M_S = 273.15 + 539 - 423 \times \mathrm{C} - 30.4 \times \mathrm{Mn} - 17.7 \times \mathrm{Ni} - 12.1 \times \mathrm{Cr} - 7.5 \times \mathrm{Mo} + 10 \times \mathrm{Co} - 7.5 \times \mathrm{Si}$$

2. **Martensite phase evolution.** Martensite phase evolution according to Lee et al. [2008, 2010] if PSI_M > 0:

$$\frac{\mathrm{d}\xi_m}{\mathrm{d}T} = \alpha_m (M_S - T)^n \xi_m^{\varphi_m} (1 - \xi_m)^{\psi_m}$$

Martensite phase evolution according to Lee et al. [2008, 2010] with extension by Hippchen et al. [2013] if PSI_M < 0:

$$\frac{\mathrm{d}\xi_m}{\mathrm{d}T} = \alpha_m (M_S - T)^n \xi_m^{\varphi_m} (1 - \xi_m)^{\psi_m (2 - \zeta_a)}$$

3. **Phase change kinetics for ferrite, pearlite and bainite.**

$$\frac{d\xi_f}{dt} = 2^{\omega_f G}\frac{\exp\left(-\frac{Q_f}{RT}\right)}{C_f}(F_S - T)^3\frac{\xi^{\varphi_f(1-\xi_f)}(1-\xi_f)^{\psi_f\xi_f}}{\exp\left(C_{r,f}\xi_f^2\right)}$$

$$\text{for} \quad F_S \geq T \geq (P_S - T_{\text{off},f})$$

$$\frac{d\xi_p}{dt} = 2^{\omega_p G}\frac{\exp\left(-\frac{Q_p}{RT}\right)}{C_p}(P_S - T)^3\frac{\xi^{\varphi_p(1-\xi_p)}(1-\xi_p)^{\psi_p\xi_p}}{\exp\left(C_{r,p}\xi_p^2\right)}$$

$$\text{for} \quad P_S \geq T \geq (B_S - T_{\text{off},p})$$

$$\frac{d\xi_b}{dt} = 2^{\omega_b G}\frac{\exp\left(-\frac{Q_b}{RT}\right)}{C_b}(B_S - T)^2\frac{\xi^{\varphi_b(1-\xi_b)}(1-\xi_b)^{\psi_b\xi_b}}{\exp\left(C_{r,b}\xi_b^2\right)}$$

$$\text{for} \quad M_S \geq T \geq (M_S - T_{\text{off},b})$$

4. **History variables.** History variables of this material model are listed in the following table. To be able to post-process that data, parameters NEIPS (shells) or NEIPH (solids) must be defined on *DATABASE_EXTENT_BINARY.

| History Variable # | Description |
|---|---|
| 1 | Amount austenite |
| 2 | Amount ferrite |
| 3 | Amount pearlite |
| 4 | Amount bainite |
| 5 | Amount martensite |
| 6 | Vickers hardness |
| 7 | Yield stress |
| 8 | ASTM grain size number |
| 9 | Young's modulus |
| 10 | Saturation stress A (H-S approach) |
| 11 | Initial yield stress B (H-S approach) |
| 12 | Saturation rate M (H-S approach) |
| 13 | Yield stress of H-S approach $\sigma_y = A - (A - B) \times e^{-M \times \text{EPSP}^{\text{EXPON}}}$ |
| 17 | Temperature rate |
| 19 | Current temperature |

| History Variable # | Description |
|---|---|
| 25 | Plastic strain rate |
| 26 | Effective thermal expansion coefficient |

5. **Choosing/excluding phases.** To exclude a phase from the simulation, set the PHASE parameter accordingly.

6. **Strain rate effects.** Note that both strain rate parameters (STRC and STRP) must be set to include the effect. It is possible to use a temperature dependent load curve for both parameters simultaneously or for one parameter keeping the other constant.

7. **Time units.** TUNIT is time units per hour and is only used for calculating the Vicker Hardness. By default, it is assumed that the time unit is seconds. If another time unit is used, for example milliseconds, then TUNIT must be changed to TUNIT = $3.6 \times 10^6$.

8. **Thermal speedup factor.** The thermal speedup factor TSF of *CONTROL_THERMAL_SOLVER is used to scale reaction kinetics and hardness calculations in this material model. Strain rate dependent properties (see LCY1 to LCY5 or STRC/STRP), however, are not scaled by TSF.

9. **Re-austenization and grain growth with the HEAT field.** When HEAT is activated, the re-austenitization and grain growth algorithms are also activated. See Remark 8 of MAT_244 for details.

10. **Phase indexed tables.** When using a table ID to describe the Young's modulus as dependent on the temperature, use *DEFINE_TABLE_2D. Set the abscissa value equal to 1 for the austenite YM-curve, equal to 2 for the ferrite YM-curve, equal to 3 for the pearlite YM curve, equal to 4 for the bainite YM-curve and finally equal to 5 for the martensite YM-curve. When using the PHASE option only the curves for the included phases are required, but all five phases may be included. The total YM is calculated by a linear mixture law:

$$YM = YM1 \times PHASE1 + \cdots + YM5 \times PHASE5$$

For example:

```
*DEFINE_TABLE_2D
$ The number before curve id:s define which phase the curve
$ will be applied to.  1 = Austenite, 2 = Ferrite, 3 = Pearlite,
$ 4 = Bainite and 5 = Martensite.
      1000       0.0       0.0
                 1.0                 100
                 2.0                 200
                 3.0                 300
                 4.0                 400
                 5.0                 500
$
```

```
$ Define curves 100 - 500
*DEFINE_CURVE
$ Austenite Temp (K) - YM-Curve (MPa)
       100          0       1.0       1.0
              1300.0              50.E+3
               223.0             210.E+3
```

11. **Phase-indexed latent heat table.** A table ID may be specified for the Latent heat (LAT1) to describe each phase change individually. Use *DEFINE_TABLE_2D and set the abscissa values to the corresponding phase transition number. That is, 2 for the Austenite – Ferrite, 3 for the Austenite – Pearlite, 4 for the Austenite – Bainite and 5 for the Austenite – Martensite. See Remark 10 for an example of a correct table definition. If a curve is missing, the corresponding latent heat for that transition will be set to zero. Also, when a table is used, LAT2 is ignored. If HEAT > 0, the latent for the transition back to Austenite can also be included. This latent heat curve is marked as 1 in the table definition of LAT1.

12. **Phase-indexed thermal expansion table.** Tables are supported for defining different thermal expansion properties for each phase. The input is identical to the above table definitions. The table must have the abscissa values between 1 and 5 where the number correspond to phase 1 to 5. To exclude one phase from influencing the thermal expansion you simply input a curve that is zero for that phase or even easier, exclude that phase number in the table definition. For example, to exclude the bainite phase you only define the table with curves for the indices 1, 2, 3 and 5.

13. **Phase-indexed transformation induced strain properties.** Transformation induced strains can be define with a table TABRHO, where densities are defined as functions of phase (table abscissas) and temperature (load curves).

## *MAT_REINFORCED_THERMOPLASTIC

This is Material Type 249. This material model describes a reinforced thermoplastic composite material. The reinforcement is defined as an anisotropic hyper-elastic material with up to three distinct fiber directions. It can be used to model unidirectional layers as well as woven and non-crimped fabrics. The matrix is modeled with a simple thermal elasto-plastic material formulation. For a composite, the overall stress is found by adding the fiber and matrix stresses.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EM | LCEM | PRM | LCPRM | LCSIGY | BETA |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| NFIB | AOPT | | | | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | MANGL | THICK |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| IDF1 | ALPH1 | EF1 | LCEF1 | G23_1 | G31_1 | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| G12 | LCG12 | ALOC12 | GLOC12 | METH12 | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| IDF2 | ALPH2 | EF2 | LCEF2 | G23_2 | G31_2 | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| G23 | LCG23 | ALOC23 | GLOC23 | METH23 | | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| IDF3 | ALPH3 | EF3 | LCEF3 | G23_3 | G31_3 | | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is optional.

| POSTV | IHIS | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EM | LCEM | PRM | LCPRM | LCSIGY | BETA |
| Type | A | F | F | I | F | I | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Density |
| EM | Young's modulus of matrix material |
| LCEM | Curve ID for Young's modulus of matrix material as a function of temperature. With this option active, EM is ignored. |
| PR | Poisson's ratio for matrix material |
| LCPR | Curve ID for Poisson's ratio of matrix material versus temperature. With this option active, PR is ignored. |
| LCSIGY | Load curve or table ID for strain hardening of the matrix. If a curve, then it specifies yield stress as a function of effective plastic strain. If a table, then temperatures are the table values indexing curves giving yield stress as a function of effective plastic strain (see *DEFINE_TABLE). |
| BETA | Parameter for mixed hardening, $0.0 \le \beta \le 1.0$. Set $\beta = 0.0$ for pure kinematic hardening and $\beta = 1.0$ for pure isotropic hardening. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NFIB | AOPT | | | | A1 | A2 | A3 |
| Type | I | F | | | | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| NFIB | Number of fiber families to be considered |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDI-NATE_-NODES, and then rotated about the shell element normal by the angle MANGL.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDI-NATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. The angle may be set in the keyword input for the element or in the input for this keyword with MANGL.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_CO-ORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | MANGL | THICK |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| MANGL | Material angle in degrees for AOPT = 0 and 3. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |

| VARIABLE | DESCRIPTION |
|---|---|
| THICK | Balance thickness changes of the material due to the matrix description by scaling fiber stresses<br>EQ.0: No scaling<br>EQ.1: Scaling |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF1 | ALPH1 | EF1 | LCEF1 | G23_1 | G31_1 | | |
| Type | I | F | F | I | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF1 | ID for 1st fiber family for post-processing |
| ALPH1 | Orientation angle $\alpha_1$ for 1st fiber with respect to overall material direction |
| EF1 | Young's modulus for 1st fiber family |
| LCEF1 | Load curve for stress as a function of fiber strain of 1st fiber. With this option active, EF1 is ignored. If a curve, then it specifies input stress as a function of fiber strain. If a table, then temperatures are the table values indexing curves giving stress as function of fiber strain. The table data will be extrapolated for both strains and temperatures where necessary. |
| G23_1 | Transverse shear modulus orthogonal to direction of 1st fiber |
| G31_1 | Transverse shear modulus in direction of 1st fiber |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G12 | LCG12 | ALOC12 | GLOC12 | METH12 | | | |
| Type | F | I | F | F | I | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G12 | Linear shear modulus for shearing between fiber families 1 and 2 |
| LCG12 | Curve ID for shear stress as a function of shearing type as specified with METH12 between the 1st and 2nd fibers. See Remark 1. |
| ALOC12 | Locking angle (in radians) for shear between fiber families 1 and 2 |
| GLOC12 | Linear shear modulus for shear angles larger than ALOC12 |
| METH12 | Option for shear response between fiber 1 and 2 (see Remark 1):<br>EQ.0: Elastic shear response. Curve LCG12 specifies shear stress as a function of the scalar product of the fiber directions.<br>EQ.1: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of the normalized scalar product of the fiber directions.<br>EQ.2: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers.<br>EQ.3: Elasto-plastic shear response. Curve LCG12 defines yield shear stress as a function of normalized shear angle between the fibers.<br>EQ.4: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching.<br>EQ.5: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching.<br>EQ.10: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. This option is tailored for woven fabrics and guarantees a pure shear stress response.<br>EQ.11: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle. This option is tailored for woven fabrics and guarantees a pure shear stress response. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF2 | ALPH2 | EF2 | LCEF2 | G23_2 | G31_2 | | |
| Type | I | F | F | I | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF2 | ID for 2nd fiber family for post-processing |
| ALPH2 | Orientation angle $\alpha_2$ for 2nd fiber with respect to overall material direction |
| EF2 | Young's modulus for 2nd fiber family |
| LCEF2 | Load curve for stress as a function of fiber strain of 2nd fiber. With this option active, EF2 is ignored. If a curve, then it specifies input stress as a function of fiber strain. If a table, then temperatures are the table values indexing curves giving stress as function of fiber strain. The table data will be extrapolated for both strains and temperatures where necessary. |
| G23_2 | Transverse shear modulus orthogonal to direction of 2nd fiber |
| G31_2 | Transverse shear modulus in direction of 2nd fiber |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G23 | LCG23 | ALOC23 | GLOC23 | METH23 | | | |
| Type | F | I | F | F | I | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G23 | Linear shear modulus for shearing between fiber families 2 and 3 |
| LCG23 | Curve ID for shear stress as a function of shearing type as specifies with METH23 between the 2nd and 3rd fibers. See Remark 1. |
| ALOC23 | Locking angle (in radians) for shear between fiber families 2 and 3 |
| GLOC23 | Linear shear modulus for shear angles larger than ALOC23 |

| VARIABLE | DESCRIPTION |
|---|---|
| METH23 | Option for shear response between fibers 2 and 3 (see METH12 for input options and Remark 1). |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF3 | ALPH3 | EF3 | LCEF3 | G23_3 | G31_3 | | |
| Type | I | F | F | I | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF3 | ID for 3rd fiber family for post-processing |
| ALPH3 | Orientation angle $\alpha_3$ for 3rd fiber with respect to overall material direction |
| EF3 | Young's modulus for 3rd fiber family |
| LCEF3 | Load curve for stress versus fiber strain of 3rd fiber. With this option active, EF3 is ignored. If a curve, then it specifies input stress as a function of fiber strain. If a table, then temperatures are the table values indexing curves giving stress as function of fiber strain. The table data will be extrapolated for both strains and temperatures where necessary. |
| G23_3 | Transverse shear modulus orthogonal to direction of 3rd fiber |
| G31_3 | Transverse shear modulus in direction of 3rd fiber |

This card is optional.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | POSTV | IHIS | | | | | | |
| Type | I | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| POSTV | Defines additional history variables that might be useful for post-processing. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| IHIS | Flag for material properties initialization:<br>EQ.0: Material properties defined in Cards 1 - 8 are used.<br>GE.1: Use *INITIAL_STRESS_SHELL to initialize some material properties on an element-by-element basis (see Remark 3 below). |

**Remarks:**

1. **Stress Calculation.** This material features an additive split of the matrix and reinforcement contributions to the stress. Therefore, the combined stress response, $\sigma$, equals the sum $\sigma^m + \sigma^f$.

   The matrix model uses an elastic-plastic material formulation with a von-Mises yield criterion. Material parameters, such as Young's modulus, Poisson's ratio and yield stress, can be given as functions of temperature. This material supports a mixed hardening approach.

   We formulated the contribution of the fiber reinforcement as a hyperelastic material. Based on the orientation angle, $\alpha_i$, of the $i^{\text{th}}$ fiber family, LS-DYNA computes an initial fiber direction in the element coordinate system $\mathbf{m}_i^0$. By using the deformation gradient, $\mathbf{F}$, the current fiber configuration is given as $\mathbf{m}_\text{i} = \mathbf{F}\,\mathbf{m}_i^0$, containing all necessary information on fiber strain and reorientation. Here, this vector is always orthogonal to the shell normal and can, thus, be represented by the two in-plane vector components.

   Following standard textbook mechanics for anisotropic and hyperelastic materials, the elastic stresses within the fibers due to tension or compression are given as

$$\sigma_T^f = \sum_{i=1}^{n} \sigma_{T,i}^f(\lambda_i) = \sum_{i=1}^{n} \frac{1}{J} f_i(\lambda_i)(\mathbf{m}_\text{i} \otimes \mathbf{m}_\text{i}) \quad ,$$

   where the function $f_i$ of the fiber strain $\lambda_i$ corresponds to the load curve LCEF*i*. $n$ is the number of fiber families.

   The shear behavior of the reinforcement can be controlled by METH*ij*. For values less than 10, the behavior is again standard textbook mechanics:

$$\sigma_S^f = \sum_{i=1}^{n-1} \sigma_{S,i,i+1}^f = \sum_{i=1}^{n-1} \frac{1}{J} g_{i,i+1}(\kappa_{i,i+1})(\mathbf{m}_\text{i} \otimes \mathbf{m}_\text{i+1}) \quad .$$

   Here $\kappa_{i,i+1}$ represents the employed shear measure (scalar product or shear angle in radians). In general, the dyadic product $\mathbf{m}_\text{i} \otimes \mathbf{m}_\text{i+1}$ does not define a shear

stress tensor. This formulation might result in unphysical shear behavior in the case of woven fabrics. Therefore, we devised METH*ij* = 10 and 11 to always give a pure shear stress tensor, $\boldsymbol{\sigma}_S^f$.

For even values of METH*ij*, an elastic shear response is assumed. If defined, the load curve LCG*ij* corresponds to function $g_{i,j}$. In this case the values of G*ij*, ALOC*ij* and GLOC*ij* are ignored.

For odd values of METH*ij* on the other hand, an elasto-plastic shear behavior is assumed and the load curve LCG*ij* defines the yield stress value as function of a normalized shear parameter. This implies that the load curve needs to be defined for abscissa values between 0.0 and 1.0. A first elastic regime, which is controlled by the linear shear stiffness G*ij*, is assumed until the yield stress given in the load curve for normalized shear value 0.0 is reached. A second linear elastic regime is defined for shear angles ($\xi_{ij}$)/ fiber angles ($\eta_{ij}$) larger than the locking angle ALOC*ij*. The corresponding stiffness in that regime is GLOC*ij*. At the transition point to the second elastic regime, the shear stress corresponds to the load curve value for a normalized shear of 1.0.

2. **History Data.** This material formulation outputs to d3plot additional data for post-processing to the set of history variables if requested. The parameter POSTV specifies the data to be written. Its value is calculated as

$$\text{POSTV} = a_1 + 2\,a_2 + 4\,a_3 + 8\,a_4 + 16\,a_5 + 32\,a_6 + 64\,a_7.$$

Each flag $a_i$ is a binary number (can be either 1 or 0) and corresponds to one particular type of post-processing variable according to the following table.

| Flag | Description | Variables | # History Var |
|---|---|---|---|
| $a_1$ | Fiber angle | $\eta_{12}$, $\eta_{23}$ | 2 |
| $a_2$ | Fiber ID | IDF1, IDF2, IDF3 | 3 |
| $a_3$ | Fiber strain | $\lambda_1, \lambda_2, \lambda_3$ | 3 |
| $a_4$ | Fiber direction (in global coordinates) | $\mathbf{m}_1^{\mathrm{g}}$, $\mathbf{m}_2^{\mathrm{g}}$, $\mathbf{m}_3^{\mathrm{g}}$ | 9 |
| $a_5$ | Individual fiber stresses | $f_1(\lambda_1), f_2(\lambda_2), f_3(\lambda_3)$ | 3 |
| $a_6$ | Fiber stress tensor | $\sigma_{11}^f$, $\sigma_{22}^f$, $\sigma_{33}^f$, $\sigma_{12}^f$, $\sigma_{23}^f$, $\sigma_{31}^f$ | 6 |
| $a_7$ | Fiber direction (in material coordinates) | $\mathbf{m}_1^{\mathrm{m}}$, $\mathbf{m}_2^{\mathrm{m}}$, $\mathbf{m}_3^{\mathrm{m}}$ | 6 |

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 32 for POSTV = 127.

As mentioned in Remark 1 fiber orientation is represented in the material subroutine as vector $\mathbf{m}_i$ defined in the element coordinate system. Prior to writing to the list of histories the vector is transformed into the global coordinate system with three vector components for $a_4 = 1$ and/or into the overall material coordinate system with two vector components for $a_7 = 1$.

A more complete list of potentially helpful history variables is given in the following table. The variable NEIPS in *DATABASE_EXTENT_BINARY must be set to output these history variables.

| History Variable # | Description |
|---|---|
| 3 | Number of fibers |
| 4 | NXH |
| 5 → NXH + 4 | Variables as described in preceding table |
| NXH + 5 | POSTV |
| NXH + 6, NXH + 7 | Shear angles $\xi_{12}$and $\xi_{23}$ |
| NXH + 8 | Matrix damage parameter $d^m$ |
| NXH + 9 → NXH + 11 | Fiber tensile damage parameter $d_i^{f,t}$ |
| NXH + 12 → NXH + 14 | Fiber compressive damage param. $d_i^{f,c}$ |
| NXH + 15 → NXH + 20 | Matrix stress tensor in element coordinate system |
| NXH+21 → NXH + 26 | Deformation gradient |

3. **Description of IHIS.** Some material data can be initialized on an element-by-element basis through history variables defined with *INITIAL_STRESS_SHELL starting at position HISV5.

How the data is interpreted depends on the parameter IHIS. Following the same concept as for parameter POSTV, the value of IHIS is computed by the following expression:

$$\text{IHIS} = a_1 + 2\,a_2$$

Each flag $a_i$ is a binary number (can be either 1 or 0) and corresponds to one particular type of material variable. So far, the only material variables implemented are fiber orientation in two different coordinate systems, global and material. Thus, at most one of the flags $a_1$ and $a_2$ should be set to 1.

| Flag | Description | Variables | # History Var |
|---|---|---|---|
| $a_1$ | Fiber direction (in global coordinates) | $\mathbf{m}_1^g$, $\mathbf{m}_2^g$, $\mathbf{m}_3^g$ | 9 |
| $a_2$ | Fiber direction (in material coordinates) | $\mathbf{m}_1^m$, $\mathbf{m}_2^m$, $\mathbf{m}_3^m$ | 6 |

## *MAT_REINFORCED_THERMOPLASTIC_CRASH

This is Material Type 249. This material model describes a reinforced thermoplastic composite material with its damage and failure behavior. The reinforcement is modeled as an anisotropic hyper-elastic material with up to three distinguished fiber directions. It can be used to model unidirectional layers as well as woven and non-crimped fabrics. The matrix is modeled with a simple elastic plastic material formulation. For a composite, the overall stress is found by adding the fiber and matrix stresses.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EM | PRM | LCSIGY | BETA | PFL | VISC |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| NFIB | AOPT | | | | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | MANGL | THICK |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is included if VISC > 0.

| VG1 | VB1 | VG2 | VB2 | VG3 | VB3 | VG4 | VB4 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| IDF1 | ALPH1 | EF1 | LCEF1 | G23_1 | G31_1 | DAF1 | DAM1 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| G12 | LCG12 | ALOC12 | GLOC12 | METH12 | DAM12 | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| IDF2 | ALPH2 | EF2 | LCEF2 | G23_2 | G31_2 | DAF2 | DAM2 |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| G23 | LCG23 | ALOC23 | GLOC23 | METH23 | DAM23 | | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| IDF3 | ALPH3 | EF3 | LCEF3 | G23_3 | G31_3 | DAF3 | DAM3 |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is optional.

| POSTV | VISCS | IHIS | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EM | PRM | LCSIGY | BETA | PFL | VISC |
| Type | A | F | F | F | I | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, $\rho$ |
| N | Number of phases |
| EM | Young's modulus of matrix material |
| PRM | Poisson's ratio for matrix material |
| LCSIGY | Load curve or table ID for strain hardening of the matrix. If a curve, then it specifies yield stress as a function of effective plastic strain. If a table, then temperatures are the table values indexing curves giving yield stress as a function of effective plastic strain (see *DEFINE_TABLE). |
| BETA | Parameter for mixed hardening, $0.0 \le \beta \le 1.0$. Set $\beta = 0.0$ for pure kinematic hardening and $\beta = 1.0$ for pure isotropic hardening. |
| PFL | Percentage of layers that must fail to initiate failure of the element (default is 100) |
| VISC | Viscous formulation for fibers:<br>EQ.0: Elastic behavior<br>GE.1: Viscoelastic behavior modeled with a Prony series. See Remark 3. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NFIB | AOPT | | | | A1 | A2 | A3 |
| Type | I | F | | | | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| NFIB | Number of fiber families to be considered (up to 3) |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDI-NATE_-NODES, and then rotated about the shell element normal by the angle MANGL.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDI-NATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotatingthe material axes about the element normal by an angle, MANGL, from a line in the plane of the element defined by the cross product of the vector v with the element normal<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_CO-ORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | MANGL | THICK |
| Type | F | F | F | F | F | F | F | I |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| MANGL | Material angle in degrees for AOPT = 0 and 3. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |
| THICK | Balance thickness changes of the material due to the matrix response when calculating the fiber stresses. Stresses can be scaled to account for the fact that fiber cross-sectional usually does not change.<br>EQ.0: No scaling<br>EQ.1: Scaling |

**Fiber Viscosity Card.** Additional card for VISC > 0 only. See Remark 3.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VG1 | VB1 | VG2 | VB2 | VG3 | VB3 | VG4 | VB4 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| VG*k* | Relaxation modulus $G_k$ for the $k^{th}$ term of the Prony series for viscoelastic fibers |
| VB*k* | Decay constant $\beta_k$ for the $k^{th}$ term of the Prony series for viscoelastic fibers |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF1 | ALPH1 | EF1 | LCEF1 | G23_1 | G31_1 | DAF1 | DAM1 |
| Type | I | F | F | I | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF1 | ID for 1st fiber family for post-processing |
| ALPH1 | Orientation angle $\alpha_1$ for 1st fiber with respect to overall material direction |

| VARIABLE | DESCRIPTION |
|---|---|
| EF1 | Young's modulus for 1st fiber family |
| LCEF1 | Load curve for stress as a function of fiber strain of 1st fiber. With this option active, EF1 is ignored. |
| G23_1 | Transverse shear modulus orthogonal to direction of 1st fiber |
| G31_1 | Transverse shear modulus in direction of 1st fiber |
| DAF1 | Load curve or table ID for damage parameter $d_1^f$ for 1st fiber (see Remark 2). If a curve, DAF1 specifies damage as a function of fiber strain (for compression and elongation). If DAF1 refers to a table, then two different damage functions for tensile and compressive stresses are input. The values in the table are arbitrary and exist only to index the two curves. The first indexed curve is assumed to specify tensile damage as a function of fiber strains while second curve specifies compressive damage as a function of fiber strains. input different damage functions for tensile and compressive stresses. Any other curves input with the table definition are ignored.<br>The damager parameter $d_1^f$ ranges from 0.0 for an undamaged fiber to 1.0 for a failed fiber family. If all families have failed, material failure at the integration point is initiated. |
| DAM1 | Load curve or table ID for damage parameter $d_1^m$ for matrix material based on the current deformation status of the 1st fiber (see Remark 2). If a curve, it specifies damage as a function of fiber strain (for compression and elongation). If a table, then the values are fiber strain rates which index damage as a function of fiber strain curves.<br>The damager parameter $d_1^m$ ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage $d_1^m$ of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G12 | LCG12 | ALOC12 | GLOC12 | METH12 | DAM12 | | |
| Type | F | I | F | F | I | I | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G12 | Linear shear modulus for shearing between fiber families 1 and 2 |
| LCG12 | Curve ID for shear stress as a function of shearing type as specified with METH12 between the 1st and 2nd fibers. See Remark 1. |
| ALOC12 | Locking angle (in radians) for shear between fiber families 1 and 2 |
| GLOC12 | Linear shear modulus for shear angles larger than ALOC12 |
| METH12 | Option for shear response between fiber 1 and 2 (see Remark 1):<br>EQ.0: Elastic shear response. Curve LCG12 specifies shear stress as a function of the scalar product of the fiber directions.<br>EQ.1: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of the normalized scalar product of the fiber directions.<br>EQ.2: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers.<br>EQ.3: Elasto-plastic shear response. Curve LCG12 defines yield shear stress as a function of normalized shear angle between the fibers.<br>EQ.4: Elastic shear response. Curve LCG12 specifies shear stress as a function of shear angle (radians) between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching.<br>EQ.5: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle between the fibers. This option is a special implementation for non-crimped fabrics, where one of the fiber families corresponds to a stitching.<br>EQ.10: Elastic shear response. Curve LCG12 specifies shear |

| VARIABLE | DESCRIPTION |
|---|---|
| | stress as a function of shear angle (radians) between the fibers. This option is tailored for woven fabrics and guarantees a pure shear stress response. |
| | EQ.11: Elasto-plastic shear response. Curve LCG12 specifies yield shear stress as a function of normalized shear angle. This option is tailored for woven fabrics and guarantees a pure shear stress response |
| DAM12 | Load curve ID defining the damage parameter $d_{12}^m$ for the matrix as function of shear angle (radians) between the 1st and 2nd fiber (see Remark 2). The damage parameter $d_{12}^m$ ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage $d_{12}^m$ of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF2 | ALPH2 | EF2 | LCEF2 | G23_2 | G31_2 | DAF2 | DAM2 |
| Type | I | F | F | I | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF2 | ID for 2nd fiber family for post-processing |
| ALPH2 | Orientation angle $\alpha_2$ for 2nd fiber with respect to overall material direction |
| EF2 | Young's modulus for 2nd fiber family |
| LCEF2 | Load curve for stress as a function of fiber strain of 2nd fiber. With this option active, EF2 is ignored. |
| G23_2 | Transverse shear modulus orthogonal to direction of 2nd fiber |
| G31_2 | Transverse shear modulus in direction of 2nd fiber |
| DAF2 | Load curve or table ID for damage parameter $d_2^f$ for 2nd fiber (see Remark 2). If a curve, DAF2 specifies damage as a function of fiber strain (for compression and elongation). If DAF2 refers to a table, |

| VARIABLE | DESCRIPTION |
|---|---|
| | then two different damage functions for tensile and compressive stresses are input. The values in the table are arbitrary and exist only to index the two curves. The first indexed curve is assumed to specify tensile damage as a function of fiber strains while second curve specifies compressive damage as a function of fiber strains. input different damage functions for tensile and compressive stresses. Any other curves input with the table definition are ignored.<br>The damager parameter $d_2^f$ ranges from 0.0 for an undamaged fiber to 1.0 for a failed fiber family. If all families have failed, material failure at the integration point is initiated. |
| DAM2 | Load curve or table ID for damage parameter $d_2^m$ for matrix material based on the current deformation status of the 2nd fiber (see Remark 2). If a curve, it specifies damage as a function of fiber strain (for compression and elongation). If a table, then the values are fiber strain rates which index damage as a function of fiber strain curves.<br>The damager parameter $d_2^m$ ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage $d_2^m$ of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5. |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G23 | LCG23 | ALOC23 | GLOC23 | METH23 | DAM23 | | |
| Type | F | I | F | F | I | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G23 | Linear shear modulus for shearing between fiber families 2 and 3 |
| LCG23 | Curve ID for shear stress as a function of shearing type as specifies with METH23 between the 2nd and 3rd fibers. See Remark 1. |
| ALOC23 | Locking angle (in radians) for shear between fiber families 2 and 3 |
| GLOC23 | Linear shear modulus for shear angles larger than ALOC23 |

| VARIABLE | DESCRIPTION |
|---|---|
| METH23 | Option for shear response between fibers 2 and 3 (see METH12 for input options and Remark 1). |
| DAM23 | Load curve ID defining the damage parameter $d_{23}^m$ for the matrix as function of shear angle (in radians) between the 1st and 2nd fiber (see Remark 2). The damager parameter $d_{23}^m$ ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage $d_{23}^m$ of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5. |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF3 | ALPH3 | EF3 | LCEF3 | G23_3 | G31_3 | DAF3 | DAM3 |
| Type | I | F | F | I | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF3 | ID for 3rd fiber family for post-processing |
| ALPH3 | Orientation angle $\alpha_3$ for 3rd fiber with respect to overall material direction |
| EF3 | Young's modulus for 3rd fiber family |
| LCEF3 | Load curve for stress versus fiber strain of 3rd fiber. With this option active, EF3 is ignored. |
| G23_3 | Transverse shear modulus orthogonal to direction of 3rd fiber |
| G31_3 | Transverse shear modulus in direction of 3rd fiber |
| DAF3 | Load curve or table ID for damage parameter $d_3^f$ for 3rd fiber (see Remark 2). If a curve, DAF3 specifies damage as a function of fiber strain (for compression and elongation). If DAF3 refers to a table, then two different damage functions for tensile and compressive stresses are input. The values in the table are arbitrary and exist only to index the two curves. The first indexed curve is assumed to specify tensile damage as a function of fiber strains while second curve specifies compressive damage as a function of fiber strains. |

| VARIABLE | DESCRIPTION |
|---|---|
| | input different damage functions for tensile and compressive stresses. Any other curves input with the table definition are ignored.<br>The damager parameter $d_3^f$ ranges from 0.0 for an undamaged fiber to 1.0 for a failed fiber family. If all families have failed, material failure at the integration point is initiated. |
| DAM3 | Load curve or table ID for damage parameter $d_3^m$ for matrix material based on the current deformation status of the 3rd fiber (see Remark 2). If a curve, it specifies damage as a function of fiber strain (for compression and elongation). If a table, then the values are fiber strain rates which index damage as a function of fiber strain curves.<br>The damager parameter $d_3^m$ ranges from 0.0 to 1.5. A value of 0.0 indicates an undamaged matrix, whereas 1.0 refers to a completely damaged matrix. To initiate failure of the composite at the integration point, a matrix damage $d_3^m$ of 1.5 must be reached. Naturally, the mechanical behavior of the matrix does not change for damage values between 1.0 and 1.5. |

The following card is optional.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | POSTV | VISCS | IHIS | | | | | |
| Type | I | F | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| POSTV | Parameter for outputting additional history variables that might be useful for post-processing. See Remark 4. |
| VISCS | Portion of viscous relaxation moduli VG*k* that is accounted for in the time step size calculation |
| IHIS | Flag for material properties initialization:<br>EQ.0: Material properties defined in Cards 1 - 9 are used<br>GE.1: Use *INITIAL_STRESS_SHELL to initialize some material properties on an element-by-element basis (see Remark 5 below). |

**Remarks:**

1. **Stress calculation.** This material features an additive split of the matrix and reinforcement contributions to the stress. Therefore, the combined stress response, $\sigma$, equals the sum $\sigma^m + \sigma^f$.

   The matrix uses an elastic-plastic material formulation with a von-Mises yield criterion. This material supports a mixed hardening approach.

   We formulated the contribution of the fiber reinforcement as a hyperelastic material. Based on the orientation angle, $\alpha_i$, of the $i^{\text{th}}$ fiber family, LS-DYNA computes an initial fiber direction in the element coordinate system $\mathbf{m}_i^0$. By using the deformation gradient, $\mathbf{F}$, the current fiber configuration is given as $\mathbf{m}_\text{i} = \mathbf{F}\,\mathbf{m}_i^0$, containing all necessary information on fiber strain and reorientation. Here, this vector is always orthogonal to the shell normal and can, thus, be represented by the two in-plane vector components.

   Following standard textbook mechanics for anisotropic and hyperelastic materials, the elastic stresses within the fibers due to tension or compression are given as

$$\sigma_T^f = \sum_{i=1}^{n} \sigma_{T,i}^f(\lambda_i) = \sum_{i=1}^{n} \frac{1}{J} f_i(\lambda_i)(\mathbf{m}_\text{i} \otimes \mathbf{m}_\text{i}) \quad ,$$

   where the function $f_i$ of the fiber strain $\lambda_i$ corresponds to the load curve LCEF*i*. $n$ is the number of fiber families.

   The shear behavior of the reinforcement can be controlled by METH*ij*. For values less than 10, the behavior is again standard textbook mechanics:

$$\sigma_S^f = \sum_{i=1}^{n-1} \sigma_{S,i,i+1}^f = \sum_{i=1}^{n-1} \frac{1}{J} g_{i,i+1}(\kappa_{i,i+1})(\mathbf{m}_\text{i} \otimes \mathbf{m}_\text{i+1}) \quad .$$

   Here $\kappa_{i,i+1}$ represents the employed shear measure (scalar product or shear angle in radians). In general, the dyadic product $\mathbf{m}_\text{i} \otimes \mathbf{m}_\text{i+1}$ does not define a shear stress tensor. This formulation might result in unphysical shear behavior in the case of woven fabrics. Therefore, we devised METH*ij* = 10 and 11 to always give a pure shear stress tensor, $\sigma_S^f$.

   For even values of METH*ij*, an elastic shear response is assumed. If defined, the load curve LCG*ij* corresponds to function $g_{i,j}$. In this case the values of G*ij*, ALOC*ij* and GLOC*ij* are ignored.

   For odd values of METH*ij* on the other hand, an elasto-plastic shear behavior is assumed and the load curve LCG*ij* defines the yield stress value as function of a normalized shear parameter. This implies that the load curve needs to be defined for abscissa values between 0.0 and 1.0. A first elastic regime, which is

controlled by the linear shear stiffness G*ij*, is assumed until the yield stress given in the load curve for normalized shear value 0.0 is reached. A second linear elastic regime is defined for shear angles ($\xi_{ij}$)/ fiber angles ($\eta_{ij}$) larger than the locking angle ALOC*ij*. The corresponding stiffness in that regime is GLOC*ij*. At the transition point to the second elastic regime, the shear stress corresponds to the load curve value for a normalized shear of 1.0.

2. **Damage and failure.** This material features a phenomenological description of damage and failure. User-defined load curves specify several damage variables as functions of the fiber strain values $\lambda_i$ or shear $\kappa_{i,i+1}$. Here, damage parameters are always accumulated and cannot decrease during the simulation.

   If input parameter DAF*i* refers to a load curve, it specifies the damage parameter $d_i^f$ as function of the fiber strain $\lambda_i$. If DAF*i* refers to a table, the material distinguishes between tensile and compressive damage. In that case, two parameters $d_i^{f,t}$ and $d_i^{f,c}$ are introduced as functions of the fiber strain $\lambda_i$ (given by two load curves referred to by the table definition) and are both evaluated in every time step. The effective damager parameter $d_i^f$ is then defined as

$$d_i^f(\lambda_i) = \begin{cases} d_i^{f,c}(\lambda_i), & \lambda_i < 0 \\ d_i^{f,t}(\lambda_i), & \lambda_i \geq 0 \end{cases}$$

   The damage parameter $d_i^f$ degrades the fiber stress contribution $\sigma_{T,i}^f$:

$$\widehat{\sigma}_T^f = \sum_{i=1}^{n} \left( \mathbf{1} - d_i^f(\lambda_i) \right) \sigma_{T,i}^f(\lambda_i) \quad .$$

   Failure of the composite material at the integration point is initiated as soon as all fibers have failed: $\min_i d_i^f = 1.0$.

   We assume matrix damage to result from the fiber straining and reorientating. Consequently, the input includes load curves DAM*i* and DAM*ij* to specify damage parameters $d_i^m(\lambda_i)$ and $d_{i,i+1}^m(\kappa_{i,i+1})$, respectively. The overall matrix damage parameter $d^m$ is given by

$$d^m = \max\left( \max_{i \leq n} d_i^m(\lambda_i), \max_{i<n} d_{i,i+1}^m(\kappa_{i,i+1}) \right) \quad .$$

   Matrix failure ($d^m = 1.0$) does not necessarily initiate failure of the composite material. In this implementation, matrix damage parameters $d_i^m$ and $d_{i,i+1}^m$ that exceed a value of 1.0 are admissible. Failure of the composite is initiated as soon as the damage parameter reaches 1.5. To account for this delayed failure, the degradation of the matrix stresses is given by:

$$\widehat{\sigma}^m = \left(1 - \min(1.0, d^m)\right)\sigma^m \quad .$$

3. **Fiber viscosity.** Input parameter VISC activates fiber viscosity. This feature adds numerical damping to the post-damage behavior of the material. Damping

might be necessary since brittle fiber failure tends to induce shockwaves through the material, resulting in oscillations or even unphysical damage propagation.

If activated, an additional viscous stress term is added to the fiber contribution:

$$\boldsymbol{\sigma}^{f}_{T,v} = \sum_{i=1}^{n} \frac{1}{J} \left( \int_0^t f_v(t-\tau) \frac{\partial \lambda_i(\tau)}{\partial \tau} d\tau \right) (\mathbf{m}_i \otimes \mathbf{m}_i) \quad .$$

The relaxation function, $f_v$, is represented by up to four terms of the Prony series expansions and thus reads

$$f_v(t) = \sum_k G_k e^{-\beta_k t}$$

with relaxation moduli $G_k$ and decay constants $\beta_k$.

4. **History data.** This material formulation outputs to d3plot additional data for post-processing to the set of history variables if requested. The parameter POSTV specifies the data to be written. Its value is calculated as

$$\text{POSTV} = a_1 + 2\,a_2 + 4\,a_3 + 8\,a_4 + 16\,a_5 + 32\,a_6 + 64\,a_7.$$

Each flag $a_i$ is a binary number (can be either 1 or 0) and corresponds to one particular type of post-processing variable according to the following table.

| Flag | Description | Variables | # of History Variables |
|---|---|---|---|
| $a_1$ | Fiber angle | $\eta_{12}$, $\eta_{23}$ | 2 |
| $a_2$ | Fiber ID | IDF1, IDF2, IDF3 | 3 |
| $a_3$ | Fiber strain | $\lambda_1, \lambda_2, \lambda_3$ | 3 |
| $a_4$ | Fiber direction (in global coordinates) | $\mathbf{m}_1^{\mathrm{g}}$, $\mathbf{m}_2^{\mathrm{g}}$, $\mathbf{m}_3^{\mathrm{g}}$ | 9 |
| $a_5$ | Individual fiber stresses | $f_1(\lambda_1), f_2(\lambda_2), f_3(\lambda_3)$ | 3 |
| $a_6$ | Fiber stress tensor | $\sigma^f_{11}$, $\sigma^f_{22}$, $\sigma^f_{33}$, $\sigma^f_{12}$, $\sigma^f_{23}$, $\sigma^f_{31}$ | 6 |
| $a_7$ | Fiber direction (in material coordinates) | $\mathbf{m}_1^{\mathrm{m}}$, $\mathbf{m}_2^{\mathrm{m}}$, $\mathbf{m}_3^{\mathrm{m}}$ | 6 |

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 32 for POSTV = 127.

As mentioned in Remark 1 fiber orientation is represented in the material subroutine as vector $\mathbf{m}_i$ defined in the element coordinate system. Prior to writing to the list of histories the vector is transformed into the global coordinate system

with three vector components for $a_4 = 1$ and/or into the overall material coordinate system with two vector components for $a_7 = 1$.

A more complete list of potentially helpful history variables is given in the following table. The variable NEIPS in *DATABASE_EXTENT_BINARY must be set to output these history variables.

| History Variable # | Description |
|---|---|
| 3 | Number of fibers |
| 4 | NXH |
| 5 → NXH + 4 | Variables as described in preceding table |
| NXH + 5 | POSTV |
| NXH + 6, NXH + 7 | Shear angles $\xi_{12}$and $\xi_{23}$ |
| NXH + 8 | Matrix damage parameter $d^m$ |
| NXH + 9 → NXH + 11 | Fiber tensile damage parameter $d_i^{f,t}$ |
| NXH + 12 → NXH + 14 | Fiber compressive damage param. $d_i^{f,c}$ |
| NXH + 15 → NXH + 20 | Matrix stress tensor in element coordinate system |
| NXH+21 → NXH + 26 | Deformation gradient |

5. **Description of IHIS.** Some material data can be initialized on an element-by-element basis through history variables defined with *INITIAL_STRESS_SHELL starting at position HISV5.

How the data is interpreted depends on the parameter IHIS. Following the same concept as for parameter POSTV, the value of IHIS is computed by the following expression:

$$\text{IHIS} = a_1 + 2\,a_2$$

Each flag $a_i$ is a binary number (can be either 1 or 0) and corresponds to one particular type of material variable. So far, the only material variables implemented are fiber orientation in two different coordinate systems, global and material. Thus, at most one of the flags $a_1$ and $a_2$ should be set to 1.

| Flag | Description | Variables | # of History Variables |
|---|---|---|---|
| $a_1$ | Fiber direction (in global coordinates) | $\mathbf{m}_1^g$, $\mathbf{m}_2^g$, $\mathbf{m}_3^g$ | 9 |
| $a_2$ | Fiber direction (in material coordinates) | $\mathbf{m}_1^m$, $\mathbf{m}_2^m$, $\mathbf{m}_3^m$ | 6 |

## *MAT_REINFORCED_THERMOPLASTIC_UDFIBER

This is Material Type 249. It describes a material with unidirectional fiber reinforcements and considers up to three distinct fiber directions. Each fiber family is described by a spatially transversely isotropic neo-Hookean constitutive law. The implementation is based on an adapted version of the material described by Bonet and Burton (1998). The material is only available for thin shell elements and in explicit simulations.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EM | PRM | G | EZDEF | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| NFIB | AOPT | XP | YP | ZP | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | MANGL | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| IDF1 | ALPH1 | EF1 | KAP1 | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| IDF2 | ALPH2 | EF2 | KAP2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| IDF3 | ALPH3 | EF3 | KAP3 | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EM | PRM | G | EZDEF | | |
| Type | A | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label be specified (see *PART). |
| RO | Density |
| EM | Isotropic Young's modulus, $E_{iso}$ |
| PR | Poisson's ratio, $v$ |
| G | Linear shear modulus, $G_{fib}$ |
| EZDEF | Algorithmic parameter. If set to 1, last row of deformation gradient is not updated during the calculation. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NFIB | AOPT | XP | YP | ZP | A1 | A2 | A3 |
| Type | I | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| NFIB | Number of fiber families to be considered (maximum of 3) |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle MANGL. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR. |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then, **a** and **b** are rotated about **c** by an angle MANGL. MANGL may be set in the keyword input for the |

| VARIABLE | DESCRIPTION |
|---|---|
| | element or in the input for this keyword. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | MANGL | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| MANGL | Material angle in degrees for AOPT = 0 and. It may be overwritten on the element card; see *ELEMENT_SHELL_BETA. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF1 | ALPH1 | EF1 | KAP1 | | | | |
| Type | I | F | F | F | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF2 | ALPH2 | EF2 | KAP2 | | | | |
| Type | I | F | F | F | | | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IDF3 | ALPH3 | EF3 | KAP3 | | | | |
| Type | I | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| IDF*i* | ID for $i^{\text{th}}$ fiber family for post-processing |
| ALPH*i* | Orientation angle $\alpha_i$ for $i^{\text{th}}$ fiber with respect to overall material direction |
| EF*i* | Young's modulus $E_i$ for $i^{\text{th}}$ fiber family |
| KAP*i* | Fiber volume ratio $\kappa_i$ of $i^{\text{th}}$ fiber family |

**Stress Calculation:**

In this model up to three distinct fiber families are considered. We assume that there is no interaction between the families. Thus, the resulting stress tensor is the sum of the single fiber responses. Each fiber response is the sum of an isotropic and a spatially transversely isotropic neo-Hookean stress contribution, $\sigma_i^{\text{iso}}$ and $\sigma_i^{\text{tr}}$ , respectively. The implementation is based on the work of Bonet and Burton (1998), adapted by BMW for simulation of unidirectional fabrics (see references below).

The isotropic stress tensor, $\sigma_i^{\text{iso}}$, depends on an isotropic shear modulus, $\mu$, and an isotropic bulk modulus, $\lambda_i$ where:

$$\mu = \frac{E_{\text{iso}}}{2(1+\nu)} \text{ and } \lambda_i = \frac{E_{\text{iso}}(\nu + n_i\nu^2)}{2(1+\nu)}.$$

Here, the variable $n_i$ denotes the ratio between stiffness orthogonally to the fibers and in fiber direction, that is, $n_i = E_{\text{iso}}/E_i$. $E_{\text{iso}}$, $\nu$, and $E_i$ are input parameters. Using the left Cauchy-Green tensor, **b,** the isotropic neo-Hookean model reads:

$$\sigma_i^{\text{iso}} = \frac{\mu}{J}(\mathbf{b} - \mathbf{I}) + \lambda_i(J-1)\mathbf{I}\,.$$

Based on the orientation angel $\alpha_i$ of the $i^{\text{th}}$ fiber family, an initial fiber direction $\mathbf{m}_i^0$ is computed. The deformation gradient, **F**, transforms the initial fiber configuration to the current fiber configuration as $\mathbf{m}_i = \mathbf{F}\mathbf{m}_i^0$. This vector contains all necessary information on fiber elongation and reorientation.

The spatially transversely isotropic neo-Hookean formulation is given by:

$$J\boldsymbol{\sigma}_i^{\mathrm{tr}} = 2\beta_i(I_4-1)\mathbf{I} + 2\big(\alpha + 2\beta_i \ln J + 2\gamma_i(I_4-1)\big)\mathbf{m}_i \otimes \mathbf{m}_i - \alpha(\mathbf{b}\mathbf{m}_i \otimes \mathbf{m}_i + \mathbf{m}_i \otimes \mathbf{b}\mathbf{m}_i)$$

with material parameters

$$\alpha = \mu - G_{\mathrm{fib}}, \qquad \beta_i = \frac{E_{\mathrm{iso}}\nu^2(1-n_i)}{4m_i(1+\nu)}, \qquad m_i = 1-\nu-2n_i\nu^2,$$

$$\gamma_i = \frac{E_i\kappa_i(1-\nu)}{8m} - \frac{\lambda_i+2\mu}{8} + \frac{\alpha}{2} - \beta_i.$$

The parameter EZDEF activates a modification of the model. Instead of the standard deformation gradient, $\mathbf{F}$, a modified tensor $\tilde{\mathbf{F}}$ is employed to calculate current fiber directions $\mathbf{m}_i$ and left Cauchy-Green tensor $\mathbf{b}$. For tensor $\tilde{\mathbf{F}}$ only the first two rows of the deformation gradient are updated based on the deformation of the element. This simplification can in some cases increase the stability of the model, especially if the structure undergoes large deformations.

### References:


[1] Bonet, J., and A. J. Burton. "A simple orthotropic, transversely isotropic hyperelastic constitutive equation for large strain computations." *Computer methods in applied mechanics and engineering* 162.1 (1998): 151-164.

[2] Senner, T., et al. "A modular modeling approach for describing the in-plane forming behavior of unidirectional non-crimp-fabrics." *Production Engineering* 8.5 (2014): 635-643.

[3] Senner, T., et al. "Bending of unidirectional non-crimp-fabrics: experimental characterization, constitutive modeling and application in finite element simulation." *Production Engineering* 9.1 (2015): 1-10.


### History Data:

| History Variable # | Description |
|---|---|
| 3 | ID of 1st fiber |
| 4 | ID of 2nd fiber |
| 5 | ID of 3rd fiber |
| 6 → 8 | Current direction of 1st fiber |
| 9 → 11 | Current direction of 2nd fiber |
| 12 → 14 | Current direction of 3rd fiber |

| History Variable # | Description |
|---|---|
| 15 | Number of fibers |
| 16 | Projected orthogonal fiber strain (1st fiber) |
| 17 | Projected parallel fiber strain (1st fiber) |
| 18 | Shear angle (1st fiber) in rad |
| 19 | Euler-Almansi strain (1st fiber) |
| 20 | Porosity (1st fiber) |
| 21 | Fiber volume ratio (1st fiber) |
| 22 | Projected orthogonal fiber strain (2nd fiber) |
| 23 | Projected parallel fiber strain (2nd fiber) |
| 24 | Shear angle (2nd fiber) in rad |
| 25 | Euler-Almansi strain (2nd fiber) |
| 26 | Porosity (2nd fiber) |
| 27 | Fiber volume ratio (2nd fiber) |
| 28 | Projected orthogonal fiber strain (3rd fiber) |
| 29 | Projected parallel fiber strain (3rd fiber) |
| 30 | Shear angle (3rd fiber) in rad |
| 31 | Euler-Almansi strain (3rd fiber) |
| 32 | Porosity (3rd fiber) |
| 33 | Fiber volume ratio (3rd fiber) |

## *MAT_TAILORED_PROPERTIES_{OPTION}

Available options include:

<BLANK>

LOG_INTERPOLATION

STOCHASTIC

This is Material Type 251. It is similar to *MAT_PIECEWISE_LINEAR_PLASTICITY or (*MAT_024). Unlike *MAT_024, it has a 3D table option that uses a history variable (that could be hardness, pre-strain, or some other quantity) from a previous calculation to evaluate the plastic behavior as a function of 1) history variable, 2) strain rate, and 3) plastic strain. Starting with release R12, it is also possible to use a 4D table option with two history variables, that is, the plastic behavior would be a function of 1) history variable HISVN + 1, 2) history variable HISVN, 3) strain rate, and 4) plastic strain. Starting with release R15, the Young's modulus can be scaled with a factor given on history variable #8. Beginning with R16, external variables (see *LOAD_EXTERNAL_VARIABLE and Remark 5) can be used instead of history variables for evaluating the plastic and scaling the Young's modulus. This material is available for shell and solid elements.

The LOG_INTERPOLATION keyword option interpolates the strain rate effect in table LCSS with logarithmic interpolation.

The STOCHASTIC keyword option allows spatially varying yield behavior. The scaling factor for the yield stress is on history variable #9. The current yield stress can be found on history variable #5. See *DEFINE_STOCHASTIC_VARIATION for additional information.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | FAIL | TDEL |
| Type | A | F | F | F | | | F | F |
| Default | none | none | none | none | | | $10^{20}$ | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | LCSS | | VP | HISVN | PHASE | |
| Type | | | I | | F | I | F | |
| Default | | | 0 | | 0.0 | 0 | 0.0 | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPS1 | EPS2 | EPS3 | EPS4 | EPS5 | EPS6 | EPS7 | EPS8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ES1 | ES2 | ES3 | ES4 | ES5 | ES6 | ES7 | ES8 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus. Spatial variation is possible using history variable #8 or an external variable. See Remarks 1 and 5. |
| PR | Poisson's ratio |
| FAIL | Failure flag:<br>LT.0.0: Call user-defined failure subroutine, `matusr_24` in |

| VARIABLE | DESCRIPTION |
|---|---|
| | dyn21.F, to determine failure<br>EQ.0.0: Do not consider failure. This option is recommended if failure is not of interest since many calculations will be saved.<br>GT.0.0: Effective plastic strain to failure. When the plastic strain reaches this value, the element is deleted from the calculation. |
| TDEL | Minimum time step size for automatic element deletion |
| LCSS | Load curve ID or table ID<br>**Load Curve.** When LCSS is a load curve ID, it is taken as defining stress as a function of effective plastic strain. If defined, EPS1 - EPS8 and ES1 - ES8 are ignored.<br>**Tabular Data.** The table ID defines for each strain rate value a load curve ID giving the stress as a function effective plastic strain for that rate; see Figure M24-1. When the strain rate falls below the minimum value, the stress as a function of effective plastic strain curve for the lowest value of strain rate is used. Likewise, when the strain rate exceeds the maximum value the stress as a function of effective plastic strain curve for the highest value of strain rate is used. EPS1 - EPS8 and ES1 - ES8 are ignored if a table ID is defined. Linear interpolation between the discrete strain rates is used by default; logarithmic interpolation is used when the LOG_INTERPOLATION option is invoked.<br>**Logarithmically Defined Tables.** Logarithmic interpolation between discrete strain rates is assumed if the *first* value in the table is negative, in which case LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated with invoking logarithmic interpolation.<br>**Multi-Dimensional Tables.** Stress values can also depend on history variables (or external variables). The 3D table gives stress versus plastic strain as a function of strain rates as a function of one history variable (see HISVN) or one external variable. The 4D table gives stress versus plastic strain as a function of strain rates as a |

| VARIABLE | DESCRIPTION |
|---|---|
| | function of two history variable values (see HISVN) or two external variables. See Remarks 2 and 5. |
| VP | Formulation for rate effects:<br>EQ.0.0: Scale yield stress (default)<br>EQ.1.0: Viscoplastic formulation |
| HISVN | Location of the history variable in the history array of *INITIAL_-STRESS_SHELL/SOLID that is used to evaluate the 3D table LCSS. If a 4D table is used, then HISVN is the location of the history variable for the *TABLE_3D value, and HISVN + 1 is the location of the history variable for the *TABLE_4D values. See Remark 4. |
| PHASE | Constant value to evaluate the 3D table LCSS. PHASE is only used if HISVN = 0. |
| EPS1 - EPS8 | Effective plastic strain values (optional). At least 2 points should be defined. The first point must be zero corresponding to the initial yield stress. |
| ES1 - ES8 | Corresponding yield stress values to EPS1 - EPS8 |

**Remarks:**

1. **Scaling the Young's modulus.** The Young's modulus can be scaled by a factor given on history variable HISV8 of *INITIAL_STRESS_SHELL/SOLID. A value of 1.0 means no scaling (default). Alternatively, it can be scaled by providing an external variable and *LOAD_EXTERNAL_VARIABLE. See Remark 5.

2. **LCSS as multi-dimensional table.** If using a 3D or 4D for LCSS, interpolation is used to find the corresponding stress value for the current plastic strain, strain rate, and history variable(s). In addition, extrapolation is used for the history variable evaluation, which means that some upper and lower "limit curves" have to be used, if extrapolation is not desired.

3. **Location of material history variables in dynain.** If using *INTERFACE_-SPRINGBACK_LSDYNA to write material history to the dynain file, the history variables of *MAT_251 (for example, hardness and temperature) are written to positions HISV6 and HISV7 of *INITIAL_STRESS_SHELL/SOLID.

4. **HISVN.** We recommend setting HISVN = 6 and putting the history variables on position HISV6 (and HISV7 if TABLE_4D is used) if using *MAT_251 in combination with *MAT_ADD_...

5. **Effect of external variables.** Instead of using history variables, it is also possible to define the spatial variation of material properties with external variables. Depending on the input in *LOAD_EXTERNAL_VARIABLE (see IMP), the Young's modulus can be scaled by the current value of an external variable (material property index IMP = 1) and/or the external variable can be used when evaluating the multi-dimensional table LCSS (indices IMP = 2 and IMP = 3 replace the first and second history variables, respectively). Note that history variables and external variables *cannot* be used at the same time in a single *MAT_251 material. For instance, if an external variable is used for a 4D LCSS, both history variables must be replaced by external variables, and either an external variable scales the Young's modulus, or the Young's is not scaled. The following table summarizes the meaning of the indices that can be set in IMP of *LOAD_EXTERNAL_VARIABLE:

| Property index | Property name | Table |
|---|---|---|
| 1 | Young's modulus, *E* | - |
| 2 | Yield stress | LCSS (3D and 4D) |
| 3 | Yield stress | LCSS (4D) |

## *MAT_TOUGHENED_ADHESIVE_POLYMER

This is Material Type 252, the Toughened Adhesive Polymer model (TAPO). It is based on non-associated $I_1$ - $J_2$ plasticity constitutive equations and was specifically developed to represent the mechanical behaviour of crash optimized high-strength adhesives under combined shear and tensile loading. This model includes material softening due to damage, rate-dependency, and a constitutive description for the mechanical behaviour of bonded connections under compression.

A detailed description of this material can be found in Matzenmiller and Burbulla [2013]. This material model can be used with solid elements or with cohesive elements in combination with *MAT_ADD_COHESIVE.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | FLG | JCFL | DOPT | |
| Type | A | F | F | F | I | I | I | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCSS | TAU0 | Q | B | H | C | GAM0 | GAMM |
| Type | I | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A10 | A20 | A1H | A2H | A2S | POW | | SRFILT |
| Type | F | F | F | F | F | F | | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | IHIS | | D1 | D2 | D3 | D4 | D1C | D2C |
| Type | F | | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, $\rho$ |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| FLG | Flag to choose between yield functions $f$ and $\hat{f}$ (see Material Model):<br>EQ.0: Cap in tension and nonlinear Drucker & Prager in compression<br>EQ.2: Cap in tension and von Mises in compression |
| JCFL | Johnson & Cook constitutive failure criterion flag (see Material Model):<br>EQ.0: use triaxiality factor only in tension,<br>EQ.1: use triaxiality factor in tension and compression. |
| DOPT | Damage criterion flag $\widehat{D}$ or $\breve{D}$ (see Material Model):<br>EQ.0: damage model uses damage plastic strain $r$,<br>EQ.1: damage model uses plastic arc length $\gamma_v$. |
| LCSS | Curve ID or Table ID<br>**Load Curve.** The curve specifies yield stress $\tau_Y$ as a function of plastic strain $r$.<br>**Table Data.** If a 2D table is defined, for each strain rate value the table specifies a curve ID giving the yield stress as a function of plastic strain for that strain rate (see *DEFINE_TABLE). If a 3D table is defined, for each temperature value, a table ID is specified which, in turn, maps strain rates to curves giving the yield stress as a function of plastic strain (see DEFINE_TABLE_3D).<br>The yield stress as a function of plastic strain curve for the lowest value of strain rate or temperature is used when the strain rate or temperature falls below the minimum value. Likewise, maximum values cannot be exceeded. Hardening variables are ignored with this option (TAU0, Q, B, H, C, GAM0, and GAMM). |
| TAU0 | Initial shear yield stress, $\tau_0$ |

| VARIABLE | DESCRIPTION |
|---|---|
| Q | Isotropic nonlinear hardening modulus, $q$ |
| B | Isotropic exponential decay parameter, $b$ |
| H | Isotropic linear hardening modulus, $H$ |
| C | Strain rate coefficient $C$. |
| GAM0 | Quasi-static threshold strain rate, $\gamma_0$ |
| GAMM | Maximum threshold strain rate, $\gamma_{\mathrm{m}}$ |
| A10 | Yield function parameter: initial value $a_{10}$ of $a_1 = \hat{a}_1(r)$ |
| A20 | Yield function parameter: initial value $a_{20}$ of $a_2 = \hat{a}_2(r)$ |
| A1H | Yield function parameter $a_1^{\mathrm{H}}$ for formative hardening (ignored if FLG = 2) |
| A2H | Yield function parameter $a_2^{\mathrm{H}}$ for formative hardening (ignored if FLG = 2) |
| A2S | Plastic potential parameter $a_2^*$ for hydrostatic stress term |
| POW | Exponent $n$ of the phenomenological damage model |
| SRFILT | Strain rate filtering parameter in exponential moving average with admissible values ranging from 0 to 1: $\dot{\varepsilon}_n^{\mathrm{avg}} = \mathrm{SRFILT} \times \dot{\varepsilon}_{n-1}^{\mathrm{avg}} + (1 - \mathrm{SRFILT}) \times \dot{\varepsilon}_n$ |
| IHIS | Flag for additional material properties initialization based on a prior process simulation:<br>EQ.0: No additional initialization<br>GE.1: Use *INITIAL_STRESS_SOLID to initialize additional material properties on an element-by-element basis (see Remark 1). |
| D1 | Johnson & Cook failure parameter $d_1$ |
| D2 | Johnson & Cook failure parameter $d_2$ |
| D3 | Johnson & Cook failure parameter $d_3$ |
| D4 | Johnson & Cook rate dependent failure parameter $d_4$ |

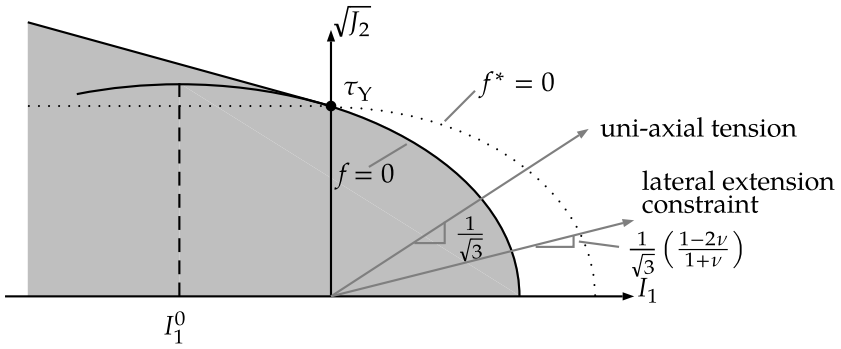



**Figure M252-1.** Yield function $f$ and plastic flow potential $f^*$

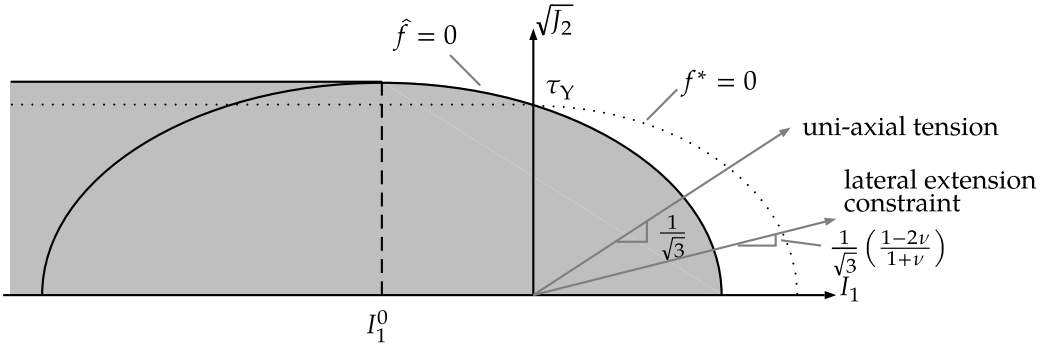



**Figure M252-2.** Yield function $\hat{f}$ and plastic flow potential $f^*$

| VARIABLE | DESCRIPTION |
|---|---|
| D1C | Johnson & Cook damage threshold parameter $d_{1c}$ |
| D2C | Johnson & Cook damage threshold parameter $d_{2c}$ |

**Material Model:**

Two different $I_1$-$J_2$ yield criteria for isotropic plasticity can be defined by parameter FLG:

1. FLG = 0 is used for the yield criterion $f$ which is changed at the case of hydrostatic pressure $I_1 = 0$ into a nonlinear *Drucker & Prager* model (DP)

$$f := \frac{J_2}{(1-D)^2} + \frac{1}{\sqrt{3}} a_1 \tau_0 \frac{I_1}{1-D} + \frac{a_2}{3}\left\langle \frac{I_1}{1-D} \right\rangle^2 - \tau_Y^2 = 0$$

with the *Macauley* bracket $\langle \bullet \rangle$, the first invariant of the stress tensor $I_1 = \text{tr}\,\boldsymbol{\sigma}$, and the second invariant of the stress deviator $J_2 = (1/2)\text{tr}(\mathbf{s})^2$ (see Figure M252-1).

2. FLG = 2 is used for the yield criterion $\hat{f}$ which is changed at the vertex into the deviatoric *von Mises* yield function (see Figure M252-2) and is used for

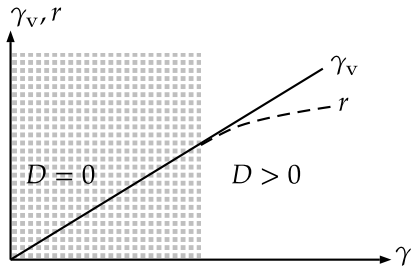



**Figure M252-3.** Accumulated plastic strain $\gamma_v$ and damage plastic strain $r$ as a function of strain $\gamma$

conservative calculation in case of missing uniaxial compression or combined compression and shear experiments:

$$\hat{f} := \frac{J_2}{(1-D)^2} + \frac{a_2}{3}\left\langle \frac{I_1}{1-D} + \frac{\sqrt{3}a_1\tau_0}{2a_2}\right\rangle^2 - \left(\tau_Y^2 + \frac{a_1^2\tau_0^2}{4a_2}\right) = 0 \ .$$

The yield functions $f$ and $\hat{f}$ are formulated in terms of the effective stress tensor

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}/(1-D)$$

and the isotropic material damage $D$ according to the continuum damage mechanics in Lemaitre [1992]. The stress tensor $\boldsymbol{\sigma}$ is defined in terms of the elastic strain $\boldsymbol{\varepsilon}^e$ and the isotropic damage $D$:

$$\boldsymbol{\sigma} = (1-D)\mathbb{C}\boldsymbol{\varepsilon}^e \ .$$

The continuity $(1-D)$ in the elastic constitutive equation above degrades the fourth order elastic stiffness tensor $\mathbb{C}$,

$$\mathbb{C} = 2G\left(\mathbb{I} - \frac{1}{3}\mathbf{1}\otimes\mathbf{1}\right) + K\,\mathbf{1}\otimes\mathbf{1}$$

with shear modulus $G$, bulk modulus $K$, fourth order identity tensor $\mathbb{I}$, and second order identity tensor $\mathbf{1}$. The plastic strain rate $\dot{\boldsymbol{\varepsilon}}^p$ is given by the non-associated flow rule

$$\dot{\boldsymbol{\varepsilon}}^p = \lambda\frac{\partial f^*}{\partial\boldsymbol{\sigma}} = \frac{\lambda}{(1-D)^2}\left(\mathbf{s} + \frac{2}{3}a_2^*\langle I_1\rangle\mathbf{1}\right)$$

with the potential $f^*$ and an additional parameter $a_2^* < a_2$ to reduce plastic dilatancy.

$$f^* := \frac{J_2}{(1-D)^2} + \frac{a_2^*}{3}\left\langle\frac{I_1}{1-D}\right\rangle^2 - \tau_Y^2$$

The plastic arc length $\dot{\gamma}_v$ characterizes the inelastic response of the material and is defined by the Euclidean norm:

$$\dot{\gamma}_v := \sqrt{2\,\mathrm{tr}(\dot{\boldsymbol{\varepsilon}}^p)^2} = \frac{2\lambda}{(1-D)^2}\sqrt{J_2 + \frac{2}{3}(a_2^*\langle I_1\rangle)^2} \ .$$

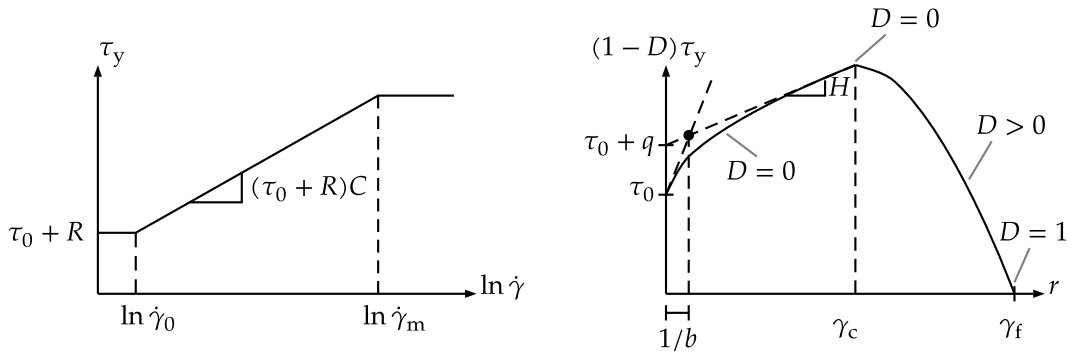



**Figure M252-4.** Rate-dependent tensile strength $\tau_Y$ as a function of effective strain rate $\dot{\gamma}$ (left) and effective damage plastic strain $r$ (right)

In addition, the arc length of the damage plastic strain rate $\dot{r}$ is introduced by means of the arc length $\dot{\gamma}_v$ and the continuity $(1-D)$ as in Lemaitre [1992], where $\tilde{I}_1 = I_1/(1-D)$ and $\tilde{J}_2 = J_2/(1-D)^2$ are the effective stress invariants (see Figure M252-3).

$$\dot{r} := (1-\mathrm{D})\dot{\gamma}_v = 2\lambda\sqrt{\tilde{J}_2 + \frac{2}{3}\left(a_2^*\langle\tilde{I}_1\rangle\right)^2}$$

The rate-dependent yield strength for shear $\tau_Y$ can be defined by two alternative expressions. The first representation is an analytic expression for $\tau_Y$:

$$\tau_Y = (\tau_0 + R)\left[1 + C\left(\left\langle \ln\frac{\dot{\gamma}}{\dot{\gamma}_0}\right\rangle - \left\langle \ln\frac{\dot{\gamma}}{\dot{\gamma}_m}\right\rangle\right)\right], \text{ with } \dot{\gamma} = \sqrt{2\,\mathrm{tr}(\dot{\varepsilon})^2} \ ,$$

where the first factor $(\tau_0 + R)$ in $\tau_Y$ is given by the static yield strength with the initial yield $\tau_0$ and the non-linear hardening contribution

$$R = q[1 - \exp(-br)] + Hr \ .$$

The second factor $[\ldots]$ in $\tau_Y$ describes the rate dependency of the yield strength by a modified Johnson & Cook approach with the reference strain rates $\dot{\gamma}_0$ and $\dot{\gamma}_m$ which limit the shear strength $\tau_Y$ (see Figure M252-4).

The second representation of the yield strength $\tau_Y$ is the table definition LCSS, where hardening can be defined as a function of plastic strain, strain rate, and temperature.

Toughened structural adhesives show distortional hardening under plastic flow, that is, the yield surface changes its shape. This formative hardening can be phenomenological described by simple evolution equations of parameters $a_1 = \hat{a}_1(r) \ \wedge \ a_2 = \hat{a}_2(r)$ in the yield criterions $f$ with the initial values $a_{10}$ and $a_{20}$:

$$a_1 = \hat{a}_1(r) \wedge \dot{a}_1 = a_1^H\dot{r}$$
$$a_2 = \hat{a}_2(r) \wedge a_2 \geq 0 \wedge \dot{a}_2 = a_2^H\dot{r}$$

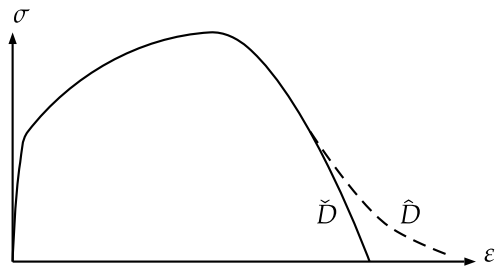



**Figure M252-5.** Influence of DOPT on damage softening

The parameters $a_1^H$ and $a_2^H$ can take positive or negative values as long as the inequality $a_2 \geq 0$ is satisfied. The criterion $a_2 \geq 0$ ensures an elliptic yield surface. The yield criterion $\hat{f}$ uses only the initial values $a_1 = a_{10}$ and $a_2 = a_{20}$ without the distortional hardening.

The empirical isotropic damage model $D$ is based on the approach in Lemaitre [1985]. Two different evolution equations $\hat{\dot{D}}(r, \dot{r})$ and $\check{\dot{D}}(\gamma_v, \dot{\gamma}_v)$ are available (see Figure M252-5). The damage variable $D$ is formulated in terms of the damage plastic strain rate $\dot{r}$ (DOPT = 0)

$$\dot{D} = \hat{\dot{D}}(r, \dot{r}) = n \left\langle \frac{r - \gamma_c}{\gamma_f - \gamma_c} \right\rangle^{n-1} \frac{\dot{r}}{\gamma_f - \gamma_c}$$

or of the plastic arc length $\dot{\gamma}_v$ (DOPT = 1)

$$\dot{D} = \check{\dot{D}}(\gamma_v, \dot{\gamma}_v) = n \left\langle \frac{\gamma_v - \gamma_c}{\gamma_f - \gamma_c} \right\rangle^{n-1} \frac{\dot{\gamma}_v}{\gamma_f - \gamma_c} \ ,$$

where $r$ in contrast to $\gamma_v$ increases non-proportionally slowly (see Figure M252-5). The strains at the thresholds $\gamma_c$ and $\gamma_f$ for damage initiation and rupture are functions of the triaxiality $T = \sigma_m / \sigma_{eq}$ with the hydrostatic stress $\sigma_m = I_1/3$ and the von Mises equivalent stress $\sigma_{eq} = \sqrt{3J_2}$ as in Johnson and Cook [1985].

$$\gamma_c = [d_{1c} + d_{2c} \exp(-d_3 \langle T \rangle)] \left( 1 + d_4 \left( \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle - \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_m} \right\rangle \right) \right)$$

$$\gamma_f = [d_1 + d_2 \exp(-d_3 \langle T \rangle)] \left( 1 + d_4 \left( \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_0} \right\rangle - \left\langle \ln \frac{\dot{\gamma}}{\dot{\gamma}_m} \right\rangle \right) \right)$$

The option JCFL controls the influence of triaxiality $T = \sigma_m / \sigma_{eq}$ in the pressure range for the thresholds $\gamma_c$ and $\gamma_f$. JCFL = 0 makes use of the Macauley bracket $\langle T \rangle$ for the triaxiality $T = \sigma_m / \sigma_{eq}$ while JCFL = 1 omits the Macauley bracket $\langle T \rangle$.

**Remarks:**

1. **Description of IHIS.** To account for results from *a prior process simulation*, it is possible to define additional material parameters on an element-by-element basis. The parameters influence stiffness, plasticity and damage behavior of the

material. LS-DYNA reads the data from the *INITIAL_STRESS_SOLID keyword beginning with history position HISV4. IHIS governs the number of read history values and their interpretation. It is defined as:

$$\text{IHIS} = a_0 + 2\,a_1 + 4\,a_2 + 8\,a_3.$$

Here, each $a_i$ is a binary flag set to either 1 or 0, activating or deactivating the input of particular properties as summarized in the following table, which also indicates the order in which the additional data is read.

| Flag | Description | Variables | # |
|---|---|---|---|
| $a_0$ | Scaling factors for elastic properties | $\alpha_E, \alpha_\nu$ | 2 |
| $a_1$ | Scaling factors for initial yield stress and hardening | $\chi_c, \phi_c$ | 2 |
| $a_2$ | Scaling factors for damage strain thresholds | $\beta, \delta$ | 2 |
| $a_3$ | Structural pre-damage | $D_2$ | 1 |

If defined by an appropriate value of IHIS ($a_0 = 1$), $\alpha_E$ and $\alpha_\nu$ are scaling factors for Young's modulus $E$ and Poissons's ratio $\nu$, respectively. If $a_1 = 1$, then the plastic behavior is changed: the initial shear yield stress $\tau_0$ is multiplied by factor $\chi_c$, and the hardening modulus $R$ is scaled by $\phi_c$. Choosing $a_2 = 1$ allows locally modifying the strain thresholds $\gamma_c$ and $\gamma_f$ by multiplying them by $\beta$ and $\delta$, respectively. Finally, setting $a_3 = 1$ causing accounting for a pre-damaged $D_2$. The two damage mechanisms, represented by $D$ and $D_2$, are applied multiplicatively, such that the effective stress is given by

$$\tilde{\sigma} = \sigma / \big((1 - D)(1 - D_2)\big).$$

Note that parameter NHISV of *INITIAL_STRESS_SOLID has to be consistent with the choice of IHIS:

$$\text{NHISV} = 3 + 2\,a_0 + 2\,a_1 + 2\,a_2 + a_3$$

2. **History Variables.** The following additional history variables are available for this keyword:

| History Variable # | Description |
|---|---|
| 1 | Damage variable, $D$ |
| 2 | Plastic arc length, $\gamma_v$ |
| 3 | Effective strain rate |
| 4 | Temperature |
| 5 | Yield stress |

| History Variable # | Description |
|---|---|
| 6 | Damaged yield stress |
| 7 | Triaxiality |
| 8 | threshold, $\gamma_c$ |
| 9 | threshold, $\gamma_f$ |

## *MAT_GENERALIZED_PHASE_CHANGE

This is Material Type 254. It is designed to model phase transformations in materials and the implied changes in the material properties. It is applicable to hot stamping, heat treatment, and welding processes for a wide range of materials. It accounts for up to 24 phases and provides a list of generic phase change mechanisms for each possible phase change. The parameters for the phase transformation laws are to be given in tabulated form.

Given the current microstructure composition, the formulation implements a temperature and strain-rate-dependent elastic-plastic material with non-linear hardening behavior. Above a certain temperature, the model shows an ideal elastic-plastic behavior with no evolution of plastic strains.

The material has been implemented for solid and shell elements and is suitable for explicit and implicit analysis.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | N | E | PR | MIX | MIXR | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| TASTART | TAEND | CTE | | | EPSINI | DTEMP | |
|---|---|---|---|---|---|---|---|

**Card 2.1.** Include this card if TASTART > 0 and TAEND = 0.

| XASTR | XAEND | XAIPA1 | XAIPA2 | XAIPA3 | XAFPA | CTEANN | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| PTLAW | PTSTR | PTEND | PTX1 | PTX2 | PTX3 | PTX4 | PTX5 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| PTTAB1 | PTTAB2 | PTTAB3 | PTTAB4 | PTTAB5 | PTTAB6 | PTTAB7 | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| PTEPS | PTRIP | PTLAT | POSTV | NUSHIS | GRAIN | T1PHAS | T2PHAS |
|---|---|---|---|---|---|---|---|

**Card 5.1.** Include this card if NUSHIS > 0.

| FUSHI1 | FUSHI2 | FUSHI3 | FUSHI4 | FUSHI5 | FUSHI6 | FUSHI7 | FUSHI8 |
|---|---|---|---|---|---|---|---|

**Card 6.** For each of the N phases, one parameter SIGY*i* must be specified. A maximum of 10 instantiations of this card may be included. The next keyword ("*") card terminates this input.

| SIGY1 | SIGY2 | SIGY3 | SIGY4 | SIGY5 | SIGY6 | SIGY7 | SIGY8 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | N | E | PR | MIX | MIXR | |
| Type | A | F | I | F | F | I | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, $\rho$ |
| N | Number of phases |
| E | Young's modulus:<br>GT.0.0: Constant value is used.<br>LT.0.0: Temperature dependent Young's modulus given by load curve or table ID = -E. Tables are used to describe a temperature dependent modulus for each phase individually. |
| PR | Poisson's ratio:<br>GT.0.0: Constant value is used.<br>LT.0.0: Temperature dependent Poisson's ratio given by load curve or table ID = -E. Tables are used to describe a temperature dependent Poisson's ratio for each phase individually. |
| MIX | Load curve ID with initial phase concentrations |
| MIXR | Load curve or table ID for mixture rule. Tables are used to define temperature dependency. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TASTART | TAEND | CTE | | | EPSINI | DTEMP | |
| Type | F | F | I | | | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| TASTART | Temperature start for simple linear annealing (see Remark 5). If TASTART > 0 and TAEND = 0, an enhanced annealing algorithm is used (see Remark 6). In that case, TASTART is interpreted as an anneal option, and Card 2.1 is required. Possible values for the extended anneal option are:<br>EQ.1: Linear annealing<br>EQ.2: JMAK |
| TAEND | Temperature end for simple linear annealing. See Remark 5. If TASTART > 0 and TAEND = 0, an enhanced annealing algorithm is used. See Remark 6. |
| CTE | Coefficient of thermal expansion:<br>GT.0.0: Constant value is used.<br>LT.0.0: Temperature dependent CTE given by load curve or table ID = -CTE. Tables give CTE as a function of temperature for each phase individually. |
| EPSINI | Initial plastic strains, uniformly distributed within the part |
| DTEMP | Maximum temperature variation within a time step. If exceeded during the analysis, a local sub-cycling is used for the calculation of the phase transformations. |

**Enhanced Annealing Card.** Additional card for TASTART > 0 and TAEND = 0 only. See Remark 6 for details.

| Card 2.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XASTR | XAEND | XAIPA1 | XAIPA2 | XAIPA3 | XAFPA | CTEANN | |
| Type | F | F | I | I | I | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XASTR | Annealing start temperature |
| XAEND | Annealing end temperature |
| XAIPA*i* | Load curve or table ID defining the $i^{th}$ parameter of the enhanced annealing option. Interpretation of the parameter depends on TASTART. |
| XAFPA | Scalar parameter of the enhanced annealing option if applicable. Interpretation of the parameter depends on TASTART. |
| CTEAN | Annealing option for thermal expansion:<br>LT.0: \|CTEAN\| defines the upper temperature limit (cut-off temperature) for evaluation of thermal strains.<br>EQ.0: No modification of thermal strains<br>EQ.1: XAEND defines the upper temperature limit (cut-off temperature) for evaluation of thermal strains. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PTLAW | PTSTR | PTEND | PTX1 | PTX2 | PTX3 | PTX4 | PTX5 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| PTLAW | Table ID to define the phase transformation model as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify transformation model (ordinate) as a function of phase number after transformation.<br>LT.0: transformation model used in heating<br>EQ.0: no transformation<br>GT.0: transformation model is used in cooling<br>A variety of transformation models can be specified as ordinate values of the curves:<br>EQ.1: Koinstinen-Marburger |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2: Johnson-Mehl-Avrami-Kolmogorov (JMAK)<br>EQ.3: Akerstrom (only for cooling)<br>EQ.4: Oddy (only for heating)<br>EQ.5: Phase Recovery I (only for heating)<br>EQ.6: Phase Recovery II (only for heating)<br>EQ.7: Parabolic Dissolution I (only for heating)<br>EQ.8: Parabolic Dissolution II (only for heating)<br>EQ.9: extended Koinstinen-Marburger (only for cooling)<br>EQ.12: JMAK for both cooling and heating<br>See Remarks 1 and 2 for further details. |
| PTSTR | Table ID to define start temperatures for the transformations as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify start temperature (ordinate) as a function of phase number after transformation (abscissa). |
| PTEND | Table ID to define end temperatures for the transformations as a function of source phase and target phase. The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify end temperature (ordinate) as a function of phase number after transformation (abscissa). |
| PTX*i* | Table ID defining the $i^{th}$ scalar-valued phase transformation parameter as function of source phase and target phase (see Remark 2 and Table M254-1 to determine which parameters apply). The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify scalar parameter (ordinate) as a function of phase number after transformation (abscissa). |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PTTAB1 | PTTAB2 | PTTAB3 | PTTAB4 | PTTAB5 | PTTAB6 | PTTAB7 | |
| Type | I | I | I | I | I | I | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| PTTAB*i* | Table ID for a 3D table defining the $i^{th}$ tabulated phase transformation parameter as a function of source phase and target phase (see Remark 2 and Table M254-1 to determine which parameters apply).<br><br>The values in *DEFINE_TABLE_3D are the phase numbers before transformation. The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase numbers after transformation. The curves referenced by the 2D tables specify a tabulated parameter (ordinate) as a function of either temperature or temperature rate (abscissa). |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PTEPS | PTRIP | PTLAT | POSTV | NUSHIS | GRAIN | T1PHAS | T2PHAS |
| Type | I | F | I | I | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| PTEPS | Table ID defining transformation induced strains.<br><br>If ID of 2D table<br><br>The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify strains (ordinate) as a function of phase number after transformation (abscissa).<br><br>If ID of 3D table<br><br>The values in *DEFINE_TABLE_3D are the phase numbers before transformation. The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase number after transformation. The curves referenced by the 2D tables specify induced strains as a function of temperature. |
| PTRIP | Flag for transformation induced plasticity (TRIP). Algorithm active for positive values of PTRIP. |
| PTLAT | Table ID defining transformation induced heat generation (latent heat).<br><br>If ID of 2D table |

| VARIABLE | DESCRIPTION |
|---|---|
| | The values in *DEFINE_TABLE are the phase numbers before transformation. The curves referenced by the table specify heat values (ordinate) versus phase number after transformation (abscissa).<br>If ID of 3D table<br>The values in *DEFINE_TABLE_3D are the phase numbers before transformation. The values in the 2D tables referenced by *DEFINE_TABLE_3D are the phase number after transformation. The curves referenced by the 2D tables specify induced heat as function of temperature. |
| POSTV | Define additional pre-defined history variables that might be useful for post-processing. See Remark 4. |
| NUHIS | Number of additional user defined history variables. For details see Remarks 3 and 4. |
| GRAIN | Initial grain size |
| T1PHAS | Lower temperature limit for cooling rate evaluation. Cooling rate can be used as input for user defined variables. |
| T2PHAS | Upper temperature limit for cooling rate evaluation. Cooling rate can be used as input for user defined variables. |

**User History Card.** Additional card for NUSHIS > 0 only.

| Card 5.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FUSHI1 | FUSHI2 | FUSHI3 | FUSHI4 | FUSHI5 | FUSHI6 | FUSHI7 | FUSHI8 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| FUSHI*i* | Function ID for user defined history variables. See Remarks 3 and 4. |

**Phase Yield Stress Cards.** For each of the N phases, one parameter SIGY*i* must be specified. A maximum of 10 of this card may be included. The next keyword ("*") card terminates this input.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIGY1 | SIGY2 | SIGY3 | SIGY4 | SIGY5 | SIGY6 | SIGY7 | SIGY8 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| SIGY*i* | Load curve or table ID for hardening of phase *i*. |

If load curve ID

Input yield stress as a function of effective plastic strain.

If table ID of 2D table

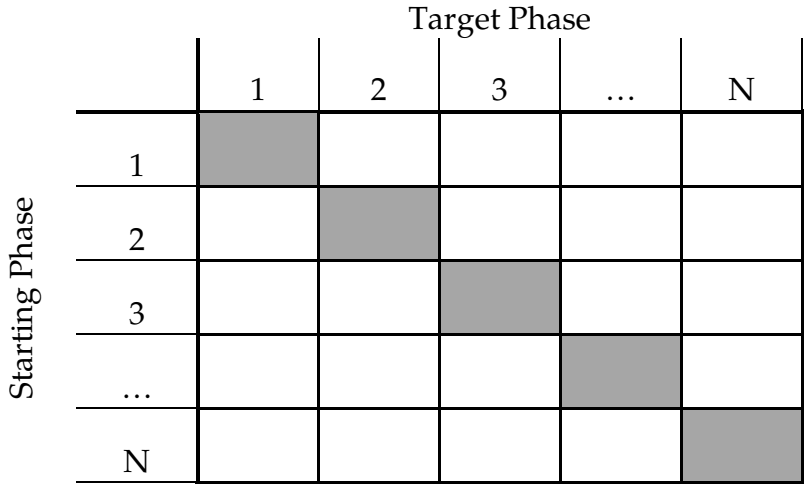
Input temperatures as table values and hardening curves (yield stress as a function of effective plastic strain) as targets for those temperatures.

If table ID of 3D table

Input temperatures as main table values and strain rates as values for the sub-tables. Hardening curves (yield stress as a function of effective plastic strain) are targets for those strain rates.

**Remarks:**

1. **Phase transformation matrix.** All data defining the microstructure evolution is expected to be given in a tabular form. The input is structured as a two-dimensional matrix containing one row for any starting phase and one row for any target phase. The basic structure is depicted in the following table:




For the input in Card 3, the entry at position *ij* of this matrix is interpreted as scalar data used for the transformation from phase *i* to phase *j*. This could, for example, be the transformation law or the start time. In LS-DYNA, such a matrix is defined by the keyword *DEFINE_TABLE(_2D). The abscissa values are the starting phase IDs. Each load curve (*DEFINE_CURVE) that is referenced consequently defines one row of the matrix.

Some of the implemented transformation models require input data that is a function of temperature, temperature rate, equivalent plastic strain, or other values. The input of this data has the same basic input structure as the scalar values, but the matrix entries are now load curve IDs. Therefore, the input is a three-dimensional table (*DEFINE_TABLE_3D), and each row of the matrix is represented by a two-dimensional table itself defined by *DEFINE_TABLE(_2D).

2. **Phase transformation models.** This material features temperature and phase-composition-dependent elastic-plastic behavior. The phase composition is determined using a list of generic phase transformation mechanisms you can choose from for each of the possible phase transformations. So far, eight different transformation models have been implemented to describe the transition from source phase concentration, $x_a$, to target phase concentration, $x_b$. Table M254-1 at the end of this remark summarizes the input parameters necessary for the individual models.

a) *Koistinen-Marburger (KM), law 1.*

The KM formulation is tailored for non-diffusive transformations. In the most basic and commonly used version, the temperature-dependent amount of the target phase is computed as

$$x_b = (x_a + x_b)\left(1.0 - e^{-\alpha_{\mathrm{KM}}(T_{\mathrm{start}} - T)}\right) .$$

PTX1 defines the so-called Koistinen-Marburger factor, $\alpha_{\mathrm{KM}}$.

b) *Generalized Johnson-Mehl-Avrami-Kolmogorov (JMAK), law 2/12.*

This is a widely used model for diffusive phase transformation. In literature, often the incremental form of the JMAK equation is given for an isothermal, incomplete transformation:

$$x_b = x_{\mathrm{eq}}(T)(x_a + x_b)\left(1 - e^{-\left(\frac{t}{\tau(T)}\right)^{n(T)}}\right) .$$

In the previous equation, exponent, $n$; equilibrium concentration, $x_{\mathrm{eq}}$; and relaxation time, $\tau$, are functions of the temperature.

In this material model, the differential form of the JMAK equation is employed which makes the model readily applicable for non-isothermal processes:

$$\frac{dx_b}{dt} = n(T)(k_{ab}x_a - k'_{ab}x_b)\left(\ln\left(\frac{k_{ab}(x_a + x_b)}{k_{ab}x_a - k'_{ab}x_b}\right)\right)^{\frac{n(T)-1.0}{n(T)}} ,$$

In this evolution equation, the following factors are defined:

$$k_{ab} = \frac{x_{eq}(T)}{\tau(T) \times \alpha(\varepsilon^p)} f(\dot{T})$$
$$k'_{ab} = \frac{1.0 - x_{eq}(T)}{\tau(T) \times \alpha(\varepsilon^p)} f'(\dot{T})$$

As user input, load curve data for the exponent, $n(T)$, is defined in PTTAB1, the equilibrium concentration, $x_{\mathrm{eq}}(T)$, in PTTAB2, and the relaxation time, $\tau(T)$, in PTTAB3. This model is a generalized JMAK approach that features additional parameters, such as the temperature rate correction factors, $f(\dot{T})$ and $f'(\dot{T})$, given in PTTAB4 and PTTAB5, respectively. As an optional parameter an accelerator $\alpha(\varepsilon^p)$ for the transformation can be defined as a function of equivalent plastic strain or of an external variable (see Remark 7) in PTTAB6. If not defined, a constant value of 1.0 is assumed.

Like the Koistinen-Marburger case, a temperature-dependent equilibrium concentration, $x_{\mathrm{eq},a}$, of the source phase can optionally be defined. If

defined in PTTAB7, the transformation is only active if the source phase fraction exceeds the equilibrium, meaning $x_a > x_{\mathrm{eq},a}$.

Note that the JMAK evolution can not only be activated by a choice of -2 (heating) and 2 (cooling), but also by choosing the law to be 12. In that case, the sign of the temperature rate is not checked, and the model is always active if the temperature is in the temperature window defined by the start and end temperature of the transformation.

c) *Kirkaldy, law 3.*

d) Similar to the implementation of *MAT_244, the transformation for cooling phases can be computed by the evolution equation:

$$\frac{dX_b}{dt} = 2^{0.5(G-1)} f(C)(T_{\mathrm{start}} - T)^{n_T} D(T) \frac{X_b^{n_1(1.0-X_b)}(1.0-X_b)^{n_2 X_b}}{\mathrm{Y}(X_b)} \ ,$$

formulated in the normalized phase concentration

$$X_b = \frac{x_b}{x_{\mathrm{eq}}(T)} \ .$$

In contrast to *MAT_244, the parameters for the evolution equation are not determined from the chemical composition of the material but defined directly as user input. The scalar data in PTX1 to PTX4 are interpreted as $f(C)$, $n_T$, $n_1$, and $n_2$. Tabulated data for $D(T)$, $Y(X_b)$, and $x_{\mathrm{eq}}(T)$ are given in PTTAB1 to PTTAB3.

e) *Oddy, law 4.*

For phase transformation in heating, the equation of Oddy can be used, which can be interpreted as a simplified JMAK relation and reads as

$$\frac{dx_b}{dt} = n \frac{x_a}{c_1(T - T_{\mathrm{start}})^{-c_2}} \left( \ln\left( \frac{(x_a + x_b)}{x_a} \right) \right)^{\frac{n-1.0}{n}} \ .$$

Its application requires the input of three scalar parameters, $n$, $c_1$, and $c_2$, that are read from the respective positions in the tables in PTX1 to PTX3.

f) *Phase recovery I, law 5.*

This phase transformation law is motivated by the recovery of the $\beta$-phase and $\alpha$-phase from martensitic $\alpha$-phase in titanium alloys and is a generalization of the algorithms described in the literature for this process.

The transformation takes place if and only if the amount of the target phase is below a user-defined, temperature dependent threshold, $x_b^{\mathrm{tre}}$. This threshold can be defined in PTTAB3.

For $x_b < x_b^{\text{tre}}$, the transformation scheme comprises three steps. First, a temperature dependent equilibrium fraction for the starting phase is calculated based on an incomplete KM equation:

$$x_{\text{eq},a} = (x_a + x_b - x_{\text{inc}})\left(1.0 - e^{-\alpha_{\text{KM}}(T_{\text{KM},s}-T)}\right) .$$

The KM-parameter $\alpha_{\text{KM}}$ and the start temperature $T_{\text{KM},s}$ must be given in PTX1 and PTX2, respectively. The incompleteness parameter $x_{\text{inc}}$ is a function of temperature defined in PTTAB4.

Second, if the current fraction of the starting phase $x_a$ exceeds the calculated equilibrium concentration $x_{\text{eq},a}$, a diffusional process follows. It is described by a JMAK approach. Its incremental form for an isothermal process is given by

$$x_a = x_{\text{eq},a} + \left(x_a + x_b - x_{\text{eq},a}\right)e^{-\left(\frac{t}{\tau(T)}\right)^{n(T)}} .$$

Naturally, a differential form of this equation is used in the model in order to be applicable to non-isothermal situations. The final calculated change $\Delta x_a$ in identified with the formation of a recovery phase $x_a^r = -\Delta x_a$. The parameters for the JMAK equation are given in PTTAB1 ($n$) and PTTAB2 ($\tau$).

Third, some of the recovery phase is partially transformed into the target phase:

$$\Delta x_b = \gamma(T) x_a^r .$$

The quotient $\gamma(T)$ can be defined in PTTAB5.

g) *Phase recovery II, law 6.*

This is the second part of the recovery and can only be defined if the previous transformation law (law 5) has also been defined with the same starting phase. This second step aims to transform the remaining fraction of the virtual, recovery phase $x_a^r$ into the physical phases defined in the material.

In order to allow for the definition of more than two target phases for one recovery process, an optional parameter $\eta(T)$ can be defined as the only input of this transformation in PTTAB1. It is used to control the transformation by

$$\Delta x_b = \eta(T) x_a^r .$$

Note that $x_a^r$ here refers to the complete fraction of the recovery phase as calculated by the JMAK approach. If the parameter is not defined, then the remainder of the virtual phase fraction is completely transformed.

h) *Parabolic growth I, law 7.*

The transformation laws 7 and 8 model the subsequent dissolution of a group of phases into one common target phase. The remaining fraction of the group after dissolution within a time step is denoted by $x_{\text{diss}}$. The groups are identified by a group ID that is here defined in PTX1.

You can define a dissolution function, $f_{\text{diss}}$, and a critical time, $t_{\text{crit}}$. These values are expected to be functions of temperature and are defined in PTTAB3 and PTTAB4, respectively. Based on those and the temperature dependent equilibrium concentration $x_{\text{eq},b}$ (PTTAB2), a characteristic dissolution time, $t_{\text{diss}}$, can be calculated as

$$t_{\text{diss}} = \left( \frac{x_b(T)}{x_b^{\text{eq}}(T)} \right)^2 t_{\text{crit}}(T) \quad .$$

Depending on the relative size of the step increment, $\Delta t$, with respect to the critical and characteristic dissolution time, the remaining group fraction $x_{\text{diss}}$ is calculated as

$$x_{\text{diss}} = \begin{cases} 1 - x_b^{\text{eq}}(T) f(T) \sqrt{\Delta t + t_{\text{diss}}(T)}, & \text{for } \Delta t + t_{\text{diss}} < t_{\text{crit}} \\ 1 - x_b^{\text{eq}}(T), & \text{otherwise} \end{cases}$$

Now, the fraction $x_a$ (the transformation of which is defined by law 7) is always assumed to be the first member of the group to by dissolved. It is algorithmically assured that there cannot be an increase in fraction $x_a$.

i) *Parabolic growth II, law 8.*

This law cannot be defined separately, but simulates the dissolution of the further members of the group already defined for a transformation with law 7. Naturally, the group ID must also be referenced here, and it is again given in PTX1. Furthermore, in the case of three or more members within a group the order in which the fractions are to be dissolved must be defined. For that purpose, the position in the group is defined in PTX2.

j) *Extenden Koistinen-Marburger, law 9.*

This extension to the standard Koistinen-Marburger (law 1) is motivated by the application of the material formulation to titanium and is only available in cooling.

An equilibrium concentration $x_{\text{eq},a}$ of the source phase can be defined as function of the current temperature in parameter PTTAB1. The transformation is only active if the source phase fraction exceeds the equilibrium, meaning $x_a > x_{\text{eq},a}$.

Furthermore, an incomplete transformation is possible in case of relatively slow cooling rates. For this purpose, you can define two rate limits $\dot{T}_{\text{lim},1}$ and $\dot{T}_{\text{lim},2}$ in PTX2 and PTX3, respectively, and an incompleteness parameter $x_{\text{inc}}(T)$ as a function of temperature in PTTAB2. The corresponding equation for the transformation then is given by:

$$x_b = \begin{cases} (x_a + x_b)\left(1.0 - e^{-\alpha_{\text{KM}}(T_{\text{start}} - T)}\right) & \text{, for } \dot{T} < \dot{T}_{\text{lim},1} \\ (x_a + x_b - x_{\text{inc}})\left(1.0 - e^{-\alpha_{\text{KM}}(T_{\text{start}} - T)}\right) & \text{, for } \dot{T}_{\text{lim},1} < \dot{T} < \dot{T}_{\text{lim},2} \end{cases}$$

A summary of input parameters for the different material laws is given in the following table. If not stated otherwise, the parameters in the tabular data PTTAB*i* are expected to be functions of the current temperature, $T$.

**Table M254-1.** Summary of input parameters for the various laws

| | PTLAW # | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Parameters | 1 | 2/12 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| PTX1 | $\alpha_{\text{KM}}$ | | $f(C)$ | $n$ | $\alpha_{\text{KM}}$ | | GID | GID | $\alpha_{\text{KM}}$ |
| PTX2 | | | $n_T$ | $c_1$ | $T_{\text{KM},s}$ | | | POS | $\dot{T}_{\text{lim},1}$ |
| PTX3 | | | $n_1$ | $c_2$ | | | | | $\dot{T}_{\text{lim},2}$ |
| PTX4 | | | $n_2$ | | | | | | |
| PTTAB1 | | $n$ | $D$ | | $n$ | $\eta$ | $x_b^{\text{tre}}$ | | $x_{\text{eq},a}$ |
| PTTAB2 | | $x_{\text{eq}}$ | $Y(X_b)$ | | $\tau$ | | $x_{\text{eq},b}$ | | $x_{\text{inc}}$ |
| PTTAB3 | | $\tau$ | $x_{\text{eq}}$ | | $x_b^{\text{tre}}$ | | $f_{\text{diss}}$ | | |
| PTTAB4 | | $f(\dot{T})$ | | | $x_{\text{inc}}$ | | $t_{\text{crit}}$ | | |
| PTTAB5 | | $f'(\dot{T})$ | | | $\gamma$ | | | | |
| PTTAB6 | | $\alpha(\varepsilon^{\text{pl}})$ | | | | | | | |
| PTTAB7 | | $x_{\text{eq},a}$ | | | | | | | |

3. **User-defined history data.** You can define up to eight additional history variables that are added to the list of history variables starting at position 31 (see Remark 4). These values can, for example, be used to evaluate the hardness of the material based on different formulas given in the literature.

   The additional variables are to be given by respective *DEFINE_FUNCTION keywords in the input as functions of the current time, the user-defined histories themselves, the current phase concentrations, the current temperature, the peak temperature, the average temperature rate between T2PHASE and T1PHASE, the current yield stress, the stress tensor, and the current values for the equivalent plastic strain of the individual phases.

For example, if all 24 phases are used (N = 24) and eight additional history variables (NUSHIS = 8) are defined, a function declaration could look as follows:

```
*DEFINE_FUNCTION
1,user defined history 1
uhist(time,usrhst1,usrhst2,…,usrhst8,phase1,
phase2,…,phase24,T,Tpeak,Trate,sigy,
sigma1,sigma2,…,sigma6,
epspl1,epspl2,…,epspl24)= ...
```

In contrast, for four considered phases (N = 4) and two additional histories (NUSHIS = 2) the keyword input could be

```
*DEFINE_FUNCTION
2,user defined history 1
uhist(time,usrhst1,usrhst2,phase1,phase2,phase3,phase4,
T,Tpeak,Trate,sigy,sigma1,sigma2,…,sigma6,epspl1,epspl2,
epspl3,epspl4)= ...
```

4. **History values.** To be able to post-process values of history variables, fields NEIPS (shells) or NEIPH (solids) must be defined on *DATABASE_EXTENT_BINARY.

Aside from the user-defined history variables discussed in Remark 3, this material formulation can output additional pre-defined history values for post-processing. The input value of field POSTV defines the data to be written. Its value is calculated as

$$\text{POSTV} = \text{a}_1 + 2\,a_2 + 4\,a_3 + 8\,a_4$$

Each flag $a_i$ is a binary number (can be either 1 or 0) and corresponds to one particular post-processing variable according to the following table. This table also shows the order of output as well as the number of extra history variables associated with the particular flag. The values of these user-defined histories are reset when the temperature is in the annealing range.

| Flag | Description | Variables | # Hist |
|---|---|---|---|
| $a_1$ | Accumulated thermal strain | $\varepsilon_T$ | 1 |
| $a_2$ | Accumulated strain tensor | $\boldsymbol{\varepsilon}$ | 6 |
| $a_3$ | Plastic strain tensor | $\boldsymbol{\varepsilon}_p$ | 6 |
| $a_4$ | Equivalent strain | $\varepsilon_{\text{VM}}$ | 1 |

In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 14 for POSTV = 15.

A complete list of history variables for the material is given in the following table. "Position" refers to the history variable number as listed by LS-PrePost when post-processing the d3plot database. The value of N indicates the number of phases accounted for in the model.

| History Variable # | Description |
|---|---|
| 1 → N | Phase concentration |
| N + 1 | Maximum temperature |
| N + 2 | Cooling rate between T2PHAS and T1PHAS |
| N + 3 | Yield stress |
| N + 4 | Young's modulus |
| N + 5 | Indicator of plastic behavior |
| N + 6 → N + 5 + NUSHIS | User-defined history variables |
| N + 6 +NUSHIS | Current temperature |
| N + 7 + NUHIS → N + 6 + NUHIS + NXH | Post-process history data as described in the previous table |
| N + 7 + NUHIS + NXH → 2 × N + 6 + NUHIS + NXH | Effective plastic strain for each phase in the microstructure |

5. **Simple annealing.** When the temperature reaches the start annealing temperature TASTART, the material starts assuming its virgin properties. Beyond the start annealing temperature, it behaves as an ideal elastic-plastic material but with no evolution of plastic strains.

   For non-zero values of both TASTART and TAEND a simple annealing strategy is used. The resetting of effective plastic properties in the annealing temperature interval is done by modifying the effective plastic strain for each phase as

   $$\varepsilon_p^n = \varepsilon_{p,\text{start}}^n \frac{T_a^{\text{end}} - T}{T_a^{\text{end}} - T_a^{\text{start}}} ,$$

   where $\varepsilon_{p,\text{start}}^n$ is the plastic strain for phase $n$ at the beginning of the annealing process.

6. **Enhanced annealing.** For a positive value of TASTART and TAEND = 0, an enhanced annealing strategy is employed. It requires the input of an additional keyword card.

   Above the annealing start temperature $T_a^{\text{start}}$, defined by XASTR, the material behaves as an ideal-plastic material, but instead of an evolution of the plastic strains, the equivalent plastic strain $\varepsilon^p$is reduced by a scale factor $\alpha(T,t)$ within the annealing temperature window

$$\varepsilon_p^n = \varepsilon_{p,\text{start}}^n(\alpha(T,t)) \quad .$$

The base value $\varepsilon_{p,\text{start}}^n$ refers to the equivalent plastic strain found in the phase, $n$, when the temperature reaches the annealing start temperature for the first time. The algorithm used to determine the value of $\alpha$ depends on the annealing option TASTART.

a) *Linear annealing.* For TASTART = 1 a linear relation between temperature and the annealing effect is assumed, similar to the simple annealing option discussed above. But in this case an incomplete reset of the equivalent plastic strain data is possible. The scale factor, $\alpha$, is a function of temperature and is given by

$$\alpha = \frac{T_a^{\text{end}} - T}{T_a^{\text{end}} - T_a^{\text{start}}} + \alpha_{\text{eq}} \frac{T - T_a^{\text{start}}}{T_a^{\text{end}} - T_a^{\text{start}}}$$

Here, the end temperature, $T_a^{\text{end}}$, is defined by XAEND and the newly introduced incompleteness factor, $\alpha_{\text{eq}}$, as scalar input data in XAFPA.

b) *Johnson-Mehl-Avrami-Kolmogorov (JMAK).* For TASTART = 2, the evolution of the scale factor follows a JMAK-type approach. For isothermal situations and assuming a start time for the process of 0.0, an incremental form can be explicitly stated a

$$\alpha = \alpha_{\text{eq}}(T) + \Big(1 - \alpha_{\text{eq}}(T)\Big) e^{-\left(\frac{t}{\tau(T)}\right)^{n(T)}} \quad .$$

In the last equation, $n(T)$ denotes the exponent for the differential equation, $\tau(T)$ the relaxation time and $\alpha_{eq}(T)$ denotes the limit value for the scale factor for infinitely long processes. All of those are functions of temperature and, thus, require the input of load curve IDs in XAIPA1 ($n(T)$), XAIPA2 ($\alpha_{\text{eq}}(T)$) and XAIPA3 ($\tau(T)$).

In the material implementation a differential form of the JMAK approach is invoked, which makes the formulation applicable to non-isothermal processes as well as independent of the start time of annealing.

7. **Effect of external variables on phase transformation.** As discussed in some detail in Remark 2b, the JMAK transformation model offers the possibility to modify the transformation speed by means of an accelerator $\alpha$. $\alpha$ is a function of the equivalent plastic strain or an external variable input as a load curve in PTTAB6. By default, the load curve is assumed to depend on equivalent plastic strain. If IMP on *LOAD_EXTERNAL_VARIABLE references the accelerator property index, which is 1, the load curve depends on the external variable instead.

## *MAT_PIECEWISE_LINEAR_PLASTIC_THERMAL

This is Material Type 255, an isotropic elastoplastic material with thermal properties. It can be used for both explicit and implicit analyses. Young's modulus and Poisson's ratio can depend on the temperature by defining two load curves. Moreover, the yield stress in tension and compression are given as load curves for different temperatures by using two tables. The thermal coefficient of expansion can be given as a constant ALPHA or as a load curve LALPHA. A positive curve ID for LALPHA models the instantaneous thermal coefficient, whereas a negatives curve ID models the thermal coefficient relative to a reference temperature, TREF. The strain rate effects are modelled with the Cowper-Symonds rate model with the parameters C and P on Card 1. Failure can be based on effective plastic strain or using the *MAT_ADD_EROSION keyword.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | C | P | FAIL | TDEL |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TABIDC | TABIDT | LALPHA | | VP | | | |
| Type | I | I | I | | F | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| E | Young's modulus:<br>LT.0.0: \|E\| is a load curve ID where $E$ is given as a function of temperature, $T$. The curve consists of $(T, E)$ data pairs.<br>GT.0.0: Constant |
| PR | Poisson's ratio.<br>LT.0.0: \|PR\| is a load curve ID for Poisson's ratio as a function of temperature.<br>GT.0.0: Constant |
| C | Strain rate parameter. See Remark 1. |
| P | Strain rate parameter. See Remark 1. |
| FAIL | Effective plastic strain when the material fails. User defined failure subroutine, matusr_24 in dyn21.F, is called to determine failure when FAIL < 0. Note that for solids the *MAT_ADD_EROSION can be used for additional failure criteria. |
| TDEL | A time step less than TDEL is not allowed. A step size less than TDEL will trigger automatic element deletion. This option is ignored for implicit analyses. |
| TABIDC | Table ID for yield stress in compression; see Remark 2. |
| TABIDT | Table ID for yield stress in tension; see Remark 2. |
| LALPHA | Load curve ID for thermal expansion coefficient as a function of temperature:<br>GT.0: The instantaneous thermal expansion coefficient based on the following formula: $$d\varepsilon_{ij}^{\text{thermal}} = \alpha(T) dT \delta_{ij}$$<br>LT.0: The thermal coefficient is defined relative a reference temperature TREF, such that the total thermal strain is given by: $$\varepsilon_{ij}^{\text{thermal}} = \alpha(T)(T - T_{\text{ref}})\delta_{ij}$$<br>With this option active, ALPHA is ignored. |
| VP | Formulation for rate effects; see Remarks 1 and 2.<br>EQ.0.0: effective total strain rate (default) |

| VARIABLE | DESCRIPTION |
|---|---|
| | NE.0.0: effective plastic strain rate |
| ALPHA | Coefficient of thermal expansion |
| TREF | Reference temperature, which is required if and only if LALPHA is given with a negative load curve ID |

**Remarks:**

1. **Strain rate effects.** The strain rate effect is modelled by using the Cowper and Symonds model which scales the yield stress according to the factor

$$1 + \left(\frac{\dot{\varepsilon}_{\text{eff}}}{C}\right)^{1/P}$$

where $\dot{\varepsilon}_{\text{eff}} = \sqrt{\text{tr}(\dot{\varepsilon}\dot{\varepsilon}^{\text{T}})}$ is the Euclidean norm of the total strain rate tensor if VP = 0 (default), otherwise $\dot{\varepsilon}_{\text{eff}} = \dot{\varepsilon}^{p}_{\text{eff}}$.

2. **Yield stress tables.** The dependence of the yield stresses on the effective plastic strains is given in two tables.

   a) TABIDC gives the behaviour of the yield stresses in compression

   b) TABIDT gives the behaviour of the yield stresses in tension.

   The table indices consist of temperatures, and at each temperature a yield stress curve must be defined.

   Both TABIDC and TABIDT can be 3D tables, in which temperatures indexes the main table and strain rates are defined as values for the sub tables with hardening curves as targets for those strain rates. If the same yield stress should be used in both tension and compression, only one table needs to be defined and the same TABID is put in position 1 and 2 on Card 2. If VP = 0, effective total strain rates are used in the 3D tables, otherwise plastic strain rates.

3. **History variables.** Two history variables are added to the d3plot file, the Young's modulus and the Poisson's ratio, respectively. They can be requested through the *DATABASE_EXTENT_BINARY keyword.

4. **Nodal temperatures.** Nodal temperatures must be defined by using a coupled analysis or some other way to define the temperatures, such as *LOAD_THERMAL_VARIABLE or *LOAD_THERMAL_LOAD_CURVE.

## *MAT_AMORPHOUS_SOLIDS_FINITE_STRAIN

This is Material Type 256, an isotropic elastic-viscoplastic material model intended to describe the behaviour of amorphous solids such as polymeric glasses. The model accurately captures the hardening-softening-hardening sequence and the Bauschinger effect experimentally observed at tensile loading and unloading respectively. The formulation is based on hyperelasticity and uses the multiplicative split of the deformation gradient $F$ which makes it naturally suitable for both large rotations and large strains. Stress computations are performed in an intermediate configuration and are therefore preceded by a pull-back and followed by a push-forward. The model was originally developed by Anand and Gurtin [2003] and implemented for solid elements by Bonnaud and Faleskog [2019].

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | G | MR | LL | NU0 | M |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | H0 | SCV | B | ECV | G0 | S0 | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus |
| G | Shear modulus |
| MR | Kinematic hardening parameter, $\mu_R$ (see Remark 1) |
| LL | Kinematic hardening parameter, $\lambda_L$ (see Remark 1) |
| NU0 | Creep parameter, $\nu_0$ (see Remark 2) |
| M | Creep parameter, $m$ (see Remark 2) |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA | Creep parameter, $\alpha$ (see Remark 2) |
| H0 | Isotropic hardening parameter, $h_0$ (see Remark 3) |
| SCV | Isotropic hardening parameter, $s_{cv}$ (see Remark 3) |
| B | Isotropic hardening parameter, $b$ (see Remark 3) |
| ECV | Isotropic hardening parameter, $\eta_{cv}$ (see Remark 3) |
| G0 | Isotropic hardening parameter, $g_0$ (see Remark 3) |
| S0 | Isotropic hardening parameter, $s_0$ (see Remark 3) |

**Remarks:**

1. **Kinematic Hardening.** Kinematic hardening gives rise to the second hardening occurrence in the hardening-softening-hardening sequence. The constants $\mu_R$ and $\lambda_L$ enter the back stress, $\mu B$ (where $B$ is the left Cauchy-Green deformation tensor), through the function $\mu$ according to:

$$\mu = \mu_R \left(\frac{\lambda_L}{3\lambda^p}\right) L^{-1}\left(\frac{\lambda^p}{\lambda_L}\right) \ , \tag{256.1}$$

where $\lambda^p = \frac{1}{\sqrt{3}}\sqrt{tr(B^p)}$ and $B^p$ is the plastic part of the left Cauchy-Green deformation tensor and $L$ is the Langevin function defined by:

$$L(X) = \coth(X) - X^{-1} \ .$$

2. **Creep.** This material model assumes plastic incompressibility. Nevertheless in order to account for the different behaviours in tension and compression a Drucker-Prager law is included in the creep law according to:

$$\nu^p = \nu_0 \left(\frac{\bar{\tau}}{s + \alpha\pi}\right)^{1/m} \ , \tag{256.2}$$

where $\nu^p$ is the equivalent plastic shear strain rate, $\bar{\tau}$ is the equivalent shear stress, $s$ is the internal variable defined below and $-\pi$ is the hydrostatic stress.

3. **Isotropic Hardening.** Isotropic hardening gives rise to the first hardening occurrence in the hardening-softening-hardening sequence. Two coupled internal variables are defined: the resistance to plastic flow, $s$, and the local free volume, $\eta$. Their evolution equations are:

$$\dot{s} = h_0 \left[1 - \frac{s}{\tilde{s}(\eta)}\right] \nu^p \tag{256.3}$$

$$\dot{\eta} = g_0 \left( \frac{s}{s_{cv}} - 1 \right) \nu^p \tag{256.4}$$

$$\tilde{s}(\eta) = s_{cv}[1 + b(\eta_{cv} - \eta)] \tag{256.5}$$

4. **Typical Material Parameters.** Typical material parameters values are given in [1] for Polycarbonate:

| Variable | Value |
|---|---|
| K | 2.24 GPa |
| G | 0.857 GPA |
| MR | 11.0 MPa |
| LL | 1.45 |
| NUO | 0.0017 $s^{-1}$ |
| M | 0.011 |
| ALPHA | 0.08 |
| H0 | 2.75 GPa |
| SCV | 24.0 MPa |
| B | 825 |
| ECV | 0.001 |
| G0 | 0.006 |
| S0 | 20.0 MPa |

**References:**

[1] Anand, L., Gurtin, M.E., 2003, "A theory of amorphous solids undergoing large deformations, with application to polymeric glasses," *International Journal of Solids and Structures*, 40, pp. 1465-1487.

[2] Bonnaud, E.L., Faleskog, J., 2019, "Explicit, fully implicit and forward gradient numerical integration of a hyperelasto-viscoplastic constitutive model for amorphous polymers undergoing finite deformation," *Computational Mechanics*, 64, pp.1389–1401.

## *MAT_NON_QUADRATIC_FAILURE

This is Material Type 258. This is an elastic-(visco)plastic material with a non-quadratic yield surface where isotropic work hardening is included. A ductile failure model is included in the form of a damage indicator model. The extended Cockcroft-Latham criterion is used to represent the dependence of the failure strain on stress state; see Gruben et. al. [2012]. Mesh dependency of the failure strain is damped out using a regularization scheme based on the deformation mode of the shell element. A more detailed description of this model can be found in the paper by Costas et al. [2018]. The material is available for shell elements only.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | SIGY | A | KSI | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | THETA1 | Q1 | THETA2 | Q2 | THETA3 | Q3 | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CS | PDOTS | | | | | | |
| Type | F | F | | | | | | |
| Default | none | none | | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DCRIT | WCB | WCL | WCS | CC | PHI | GAMMA | THICK |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| SIGY | Initial yield stress |
| A | Exponent of Hershey yield criterion |
| KSI | Coefficient governing critical strain increment for substepping |
| THETA*i* | Initial hardening modulus of $R_i$ |
| Q*i* | Saturation value of $R_i$ |
| CS | Rate sensitivity of flow stress |
| PDOTS | Reference strain rate |
| DCRIT | Critical damage |
| WCB | Constant defining the damage evolution |
| WCL | Constant defining the damage evolution |
| WCS | Constant defining the damage evolution |
| CC | Constant defining the damage evolution |
| PHI | Constant defining the damage evolution |

| VARIABLE | DESCRIPTION |
|---|---|
| GAMMA | Constant defining the damage evolution |
| THICK | Element thickness if using shell formulation 16. Since releases R12.1 and R13.0, setting THICK to zero causes the thickness to be taken from *SECTION_SHELL or *ELEMENT_SHELL_THICKNESS. |

**Remarks:**

The yield function is defined on the form

$$f = \varphi(\boldsymbol{\sigma}) - (\sigma_0 + R)$$

where $\sigma_{eq} \equiv \varphi(\boldsymbol{\sigma})$ is the equivalent stress, $\sigma_0$ is the initial yield stress, and $R$ is the isotropic hardening variable, which is a function of the equivalent plastic strain $p$. The equivalent stress is defined as

$$\varphi(\boldsymbol{\sigma}) = \left[\frac{1}{2}\{|\sigma_1 - \sigma_2|^a + |\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a\}\right]^{\frac{1}{a}}$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the ordered principal stresses. The isotropic hardening variable is expressed as

$$R(p) = \sum_{i=1}^{3} R_i(p) = \sum_{i=1}^{3} Q_i \left(1 - \exp\left(-\frac{\theta_i}{Q_i} p\right)\right)$$

where $Q_i$ and $\theta_i$ are in turn the saturation value and initial hardening modulus of the hardening variable $R_i$. As $p \to \infty$, $R$ attains its saturation value $R_{\text{sat}}$, given by

$$R_{\text{sat}} = \sum_{i=1}^{3} Q_i \ .$$

Note that you can provide nonzero initial values o fplastic strain with *INITIAL_STRESS_SHELL which initializes the simulation with nonzero $R_i(p)$ at $t = 0$.

Rate-dependent plasticity is described by a fully viscoplastic formulation. If rate dependence is invoked, the equivalent stress $\sigma_{\text{eq}}$ is constrained by the viscoplastic relation

$$\sigma_{\text{eq}} = (\sigma_0 + R(p)) \left(1 + \frac{\dot{p}}{\dot{p}_\sigma}\right)^{C_\sigma} \quad \text{for } f > 0$$

in the plastic domain. The parameters $C_\sigma$ and $\dot{p}_\sigma$ govern the rate dependence of the material, where $\dot{p}_\sigma$ is a reference strain rate.

The uncoupled version of the Extended Cockcroft-Latham (ECL) criterion is applied here to define damage evolution

$$\dot{D} = \frac{\varphi(\sigma)}{W_c}\left\langle \phi\frac{\sigma_1}{\varphi(\sigma)} + (1-\phi)\frac{\sigma_1-\sigma_3}{\varphi(\sigma)}\right\rangle^{\gamma}\dot{p}$$

where $D$ is the damage variable and $W_c$, $\phi$, and $\gamma$ are parameters governing the damage evolution and its dependence of the stress triaxiality and the Lode parameter. By setting $\phi = \gamma = 1$, we get the Cockcroft-Latham criterion:

$$\dot{D} = \frac{\langle\sigma_1\rangle}{W_c}\dot{p}$$

where $W_c$ is the Cockcroft-Latham (CL) fracture parameter.

In simulations with shell elements, the CL fracture parameter $W_c$ is defined by

$$W_c = \Omega W_c^b + (1-\Omega)W_c^m$$

where $W_c^b$ is the CL parameter in pure bending, $W_c^m$ is a mesh-dependent CL parameter in membrane loading, and $\Omega$ is a bending indicator given as

$$\Omega = \frac{1}{2}\frac{\left|\dot{\varepsilon}_{3p}^{+} - \dot{\varepsilon}_{3p}^{-}\right|}{\max\left(\left|\dot{\varepsilon}_{3p}^{+}\right|, \left|\dot{\varepsilon}_{3p}^{-}\right|\right)}$$

where $\dot{\varepsilon}_{3p}^{+}$ and $\dot{\varepsilon}_{3p}^{-}$ are the plastic thickness strain rates on the two sides of the shell element. Thus, the bending indicator is $\Omega = 1$ for pure bending ($\dot{\varepsilon}_{3p}^{-} = -\dot{\varepsilon}_{3p}^{+}$) and $\Omega = 0$ ($\dot{\varepsilon}_{3p}^{-} = \dot{\varepsilon}_{3p}^{+}$) for pure membrane loading. The mesh-dependent CL parameter for membrane loading is defined by

$$W_c^m = W_c^l + \left(W_c^s - W_c^l\right)\exp\left(-c\left(\frac{l_e}{t_e} - 1\right)\right)$$

where $W_c^l$, $W_c^s$, and $c$ are parameters, $l_e$ is the characteristic size of the shell element, and $t_e$ is the thickness of the shell element.

## *MAT_STOUGHTON_NON_ASSOCIATED_FLOW_{OPTION}

This is Material Type 260A. This material model is implemented based on non-associated flow rule models (Stoughton 2002 and 2004). Strain rate sensitivity can be included using a load curve. This model applies to both shell and solid elements. It is available for explicit in both MPP and SMP.

Available options include:

<BLANK>

XUE

The option XUE is available for solid elements only.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | R00 | R45 | R90 | SIG00 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| SIG45 | SIG90 | SIG_B | LCIDS | LCIDV | SCALE | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is included for the XUE keyword option.

| EF0 | PLIM | Q | GAMA | M | BETA | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | R00 | R45 | R90 | SIG00 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | 1.0 | R00 | R00 | Rem 1 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's Modulus |
| PR | Poisson's ratio |
| R00, R45, R90 | Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions, respectively; determined from experiments. Note if R00, R45, and R90 are not defined or are set to 0.0, then R00 = R45 = R90 = 1.0, which degenerates to the Von-Mises yield. |
| SIG00 | Initial yield stress from uniaxial tension tests in rolling (0°) direction |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SIG45 | SIG90 | SIG_B | LCIDS | LCIDV | SCALE | | |
| Type | F | F | F | I | I | F | | |
| Default | Rem 1 | Rem 1 | Rem 1 | none | none | 1.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SIG45 | Initial yield stress from uniaxial tension tests in diagonal (45°) direction |

| VARIABLE | DESCRIPTION |
|---|---|
| SIG90 | Initial yield stress from uniaxial tension tests in transverse (90°) directions |
| SIG_B | Initial yield stress from equi-biaxial stretching tests |
| LCIDS | ID of load curve giving stress as a function of strain hardening behavior from a uniaxial tension test along the rolling direction |
| LCIDV | ID of a load curve defining stress scale factors as a function strain rates, determined from experiments. An example of the curve can be found in Figure M260A-2. To know which values are used, the strain rates and strain rate scale factors are stored in the d3plot file as history variables #5 and #6, respectively. |
| SCALE | Parameter for speeding up the simulation while equalizing the strain rate effect. It is useful in cases where the pulling speed or punch speed is slow. See Remark 2. |

**XUE Card.** This card is included for the XUE keyword option.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EF0 | PLIM | Q | GAMA | M | BETA | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EF0, PLIM, Q, GAMA, M, BETA | Material parameters for the option XUE. The parameter $k$ in the original paper is assumed to be 1.0. Note the default BETA value of 0.0 means no progressive weakening damage. For details, refer to Xue, L., Wierzbicki, T.'s 2009 paper "*Numerical simulation of fracture mode transition in ductile plates*" in the *International Journal of Solids and Structures*. See Remark 3. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | I | | | | | | | |
| Default | none | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

### The Stoughton Non-Associated Flow Rule:

In a non-associated flow rule, the material yield function is not equal to the plastic flow potential. According to Thomas B. Stoughton's paper titled "*A non-associated flow rule for sheet metal forming*" in 2002 International Journal of Plasticity 18, 687-714, and "*A pressure-sensitive yield criterion under a non-associated flow rule for sheet metal forming*" in 2004 International Journal of Plasticity 20, 705-731, the plastic potential is given by:

$$\overline{\sigma}_p = \sqrt{\sigma_{11}^2 + \lambda_p \sigma_{22}^2 - 2\nu_p \sigma_{11}\sigma_{22} + 2\rho_p \sigma_{12}^2}$$

where $\sigma_{ij}$ is the stress tensor component. Here,

$$\lambda_p = \frac{1+\frac{1}{r_{90}}}{1+\frac{1}{r_0}} \ ,$$
$$\nu_p = \frac{r_0}{1+r_0} \ ,$$
$$\rho_p = \frac{\frac{1}{r_0}+\frac{1}{r_{90}}}{1+\frac{1}{r_0}}\left(\frac{1}{2}+r_{45}\right) \ .$$

$r_0$, $r_{45}$, and $r_{90}$ are Lankford parameters in the rolling (0°), the diagonal (45°) and the transverse (90°) directions, respectively.

The yield function is given by:

$$\overline{\sigma}_y = \sqrt{\sigma_{11}^2 + \lambda_y \sigma_{22}^2 - 2\nu_y \sigma_{11}\sigma_{22} + 2\rho_y \sigma_{12}^2} \ ,$$

where

$$\lambda_y = \left(\frac{\sigma_0}{\sigma_{90}}\right)^2,$$
$$\nu_y = \frac{1}{2}\left[1+\lambda_y - \left(\frac{\sigma_0}{\sigma_b}\right)^2\right] \ ,$$
$$\rho_y = \frac{1}{2}\left[\left(\frac{2\sigma_0}{\sigma_{45}}\right)^2 - \left(\frac{\sigma_0}{\sigma_b}\right)^2\right] \ .$$

Here $\sigma_0$, $\sigma_{45}$, $\sigma_{90}$ are the initial yield stresses from uniaxial tension tests in the rolling, diagonal, and transverse directions, respectively. $\sigma_b$ is the initial yield stress from an equi-biaxial stretching test.

**Remarks:**

1. **Defaults for SIG00, SIG45, SIG90, and SIG_B.** If not specified, SIG00, SIG45, SIG90, and SIG_B default to the first stress value in LCIDS. Note that if all four values are not specified, the non-associated flow rule degenerates to the associated flow rule.

2. **SCALE.** The variable SCALE is very useful in speeding up the simulation while equalizing the strain rate effect. For example, if the pulling speed is 15 mm/s but running the simulation at this speed will take a long time, you can increase the pulling speed to 500 mm/s while setting SCALE to 0.03. The latter settings will give the same results with the benefit of greatly reduced computational time (see Figures M260A-3 and M260A-4). Note that the increased absolute value (within a reasonable range) of mass scaling, $-1.0 \times$ dt2ms, frequently used in forming simulation does not affect the strain rates, as shown in the Figure M260A-5. See examples in Verification.

3. **XUE Parameters.** The following table lists variable names used in this material model with the corresponding symbols in Xue et al [2009] for the option XUE:

| EF0 | PLIM | Q | GAMA | M |
|---|---|---|---|---|
| $\varepsilon_{f0}$ | $P_{lim}$ | $q$ | $\gamma$ | $m$ |

4. **History Variables.** The history variables output to d3plot for this material depend on whether the XUE keyword option is used and whether this material is used with an EOS. When the XUE option is used, damage accumulation is output to d3plot. It is history variable #1 without an EOS and history variable #5 with an EOS. The value ranges from 0.0 to 1.0. When XUE is not used, history variable #5 is strain rates and history variable #6 is strain rate scale factors.

**Verification:**

Uniaxial tension tests were done on a single shell element as shown in Figure M260A-1. Strain rate effect LCIDV is input as shown in Figure M260A-2. In Figure M260A-3, pulling stress as a function of strain from various test conditions are compared with input stress-strain curve A. In summary, using the parameter SCALE, the element can be pulled much faster (500 mm/s vs. 15 mm/s) but achieve the same stress vs. strain results, the same strain rates (history variable #5), and the same strain rate scale factor (history variable #6 in Figure M260A-4). Simulation speed can be improved further with increased mass scaling ($-1.0 \times$ dt2ms) without affecting the results; see Figure M260A-5.

A partial keyword input is provided below, for the case with pulling speed of 500 mm/s, strain hardening curve ID of 100, LCIDV curve ID of 105, and strain rate scale factor of 0.03.

```
*KEYWORD
*parameter_expression
R endtime     0.012
R v           500.0
*CONTROL_TERMINATION
$    ENDTIM    ENDCYC     DTMIN    ENDNEG    ENDMAS
&endtime
*MAT_STOUGHTON_NON_ASSOCIATED_FLOW
$#     mid        Ro         E        PR       R00       R45       R90     SIG00
         1 7.8000E-9   2.10E05  0.300000       1.1       1.2       1.3     311.0
$    SIG45     SIG90     SIG_B     LCIDS     LCIDV     SCALE
     305.4     321.1     290.3       100       105      0.03
$     AOPT
         3
$       XP        YP        ZP        A1        A2        A3

$       V1        V2        V3        D1        D2        D3      BETA
                 1.0
*DEFINE_CURVE
       100
         0.00000E+00         0.30130E+03
         0.10000E-01         0.42295E+03
         0.20000E-01         0.47991E+03
```

```
         0.30000E-01         0.52022E+03
         0.40000E-01         0.55126E+03
         0.50000E-01         0.57615E+03
                                        ⋮
*DEFINE_CURVE
       105
         0.00000E+00         0.10000E+01
         0.10000E+00         0.10608E+01
         0.50000E+00         0.10828E+01
         0.10000E+01         0.10923E+01
                                        ⋮
*END
```

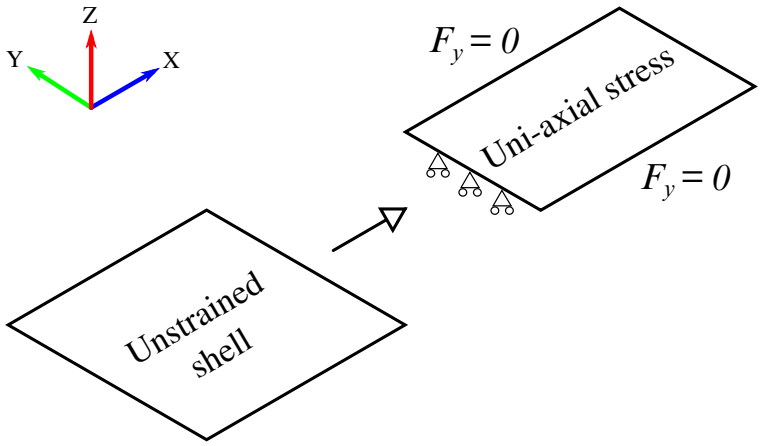



**Figure M260A-1.** Uniaxial tension tests on a single shell element.

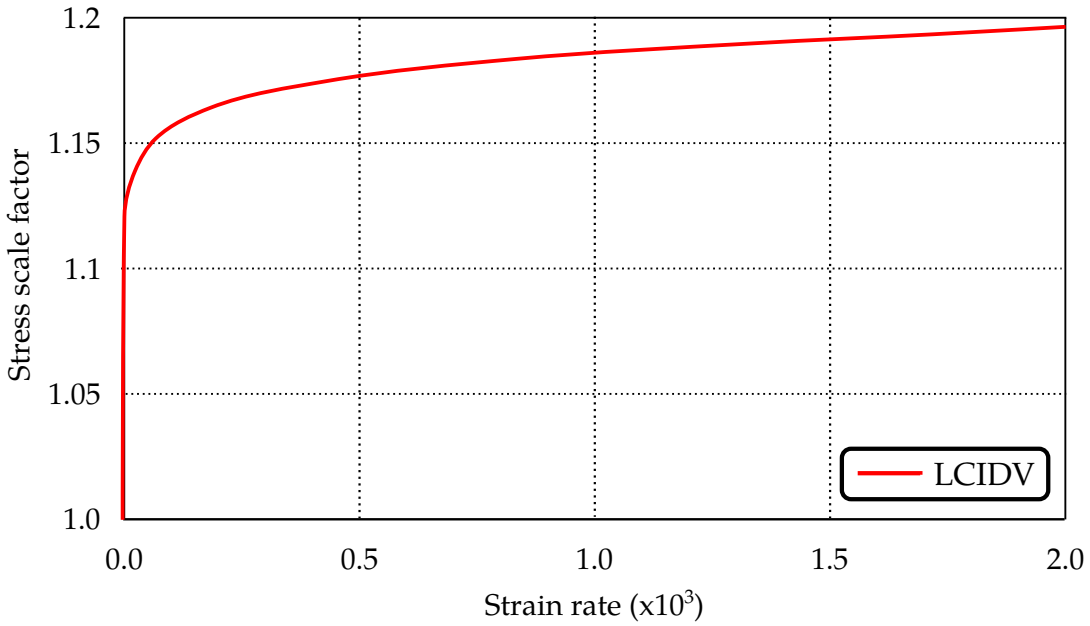



**Figure M260A-2.** Input LCIDV

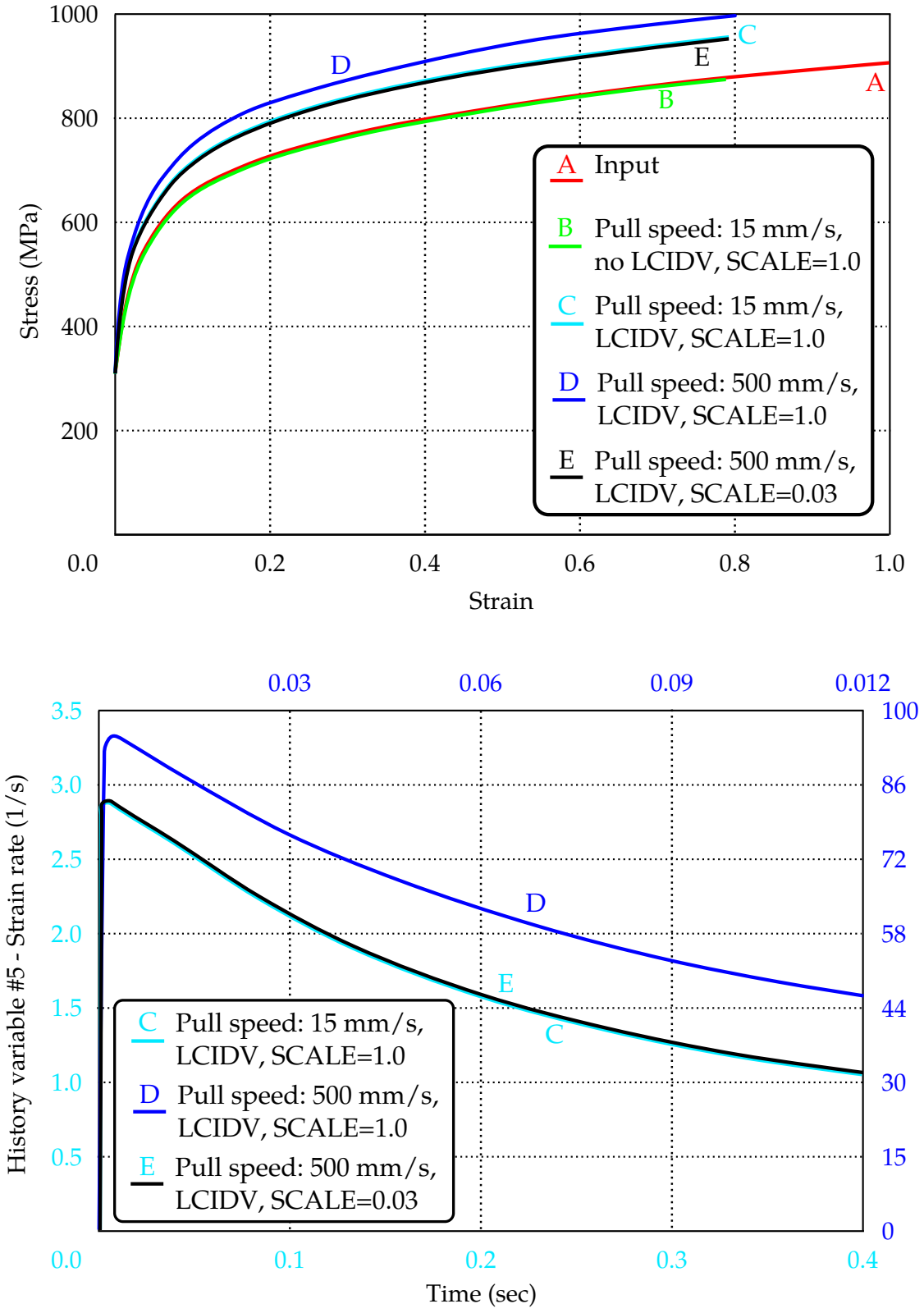



**Figure M260A-3.** Recovered stress-strain curve (top) and strain rates (bottom) under various conditions shown.

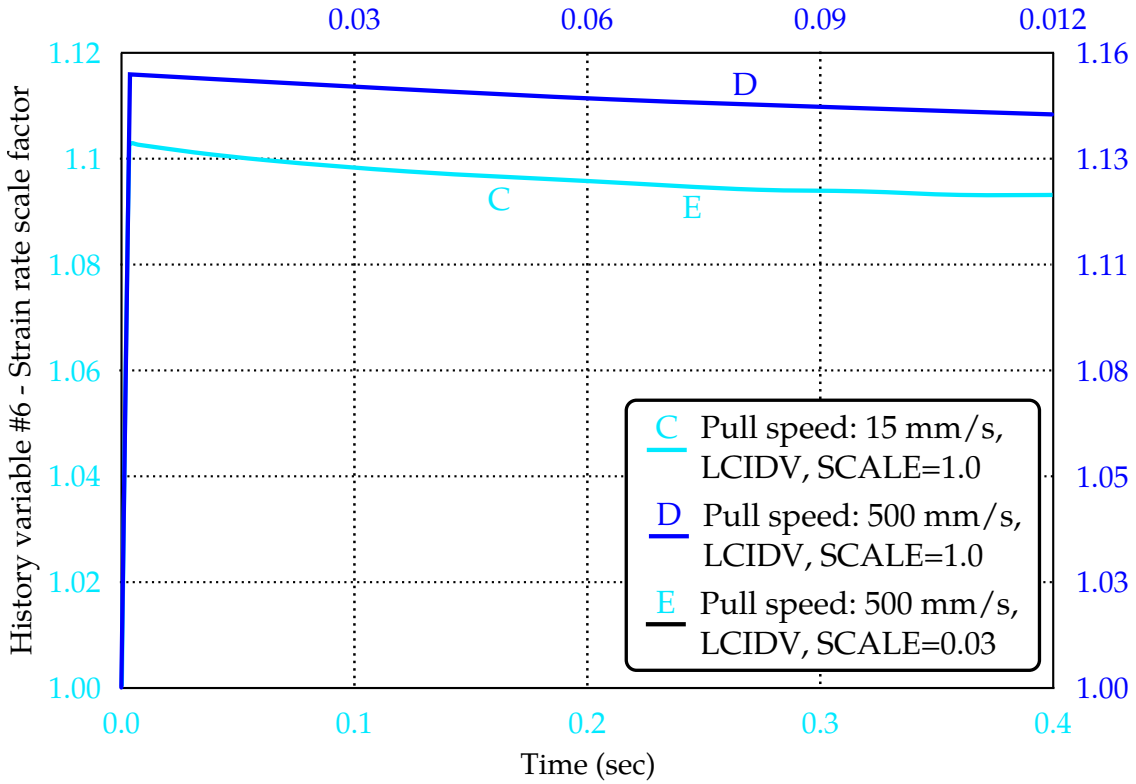



**Figure M260A-4.** Recovered strain rate scale factors under various conditions shown.

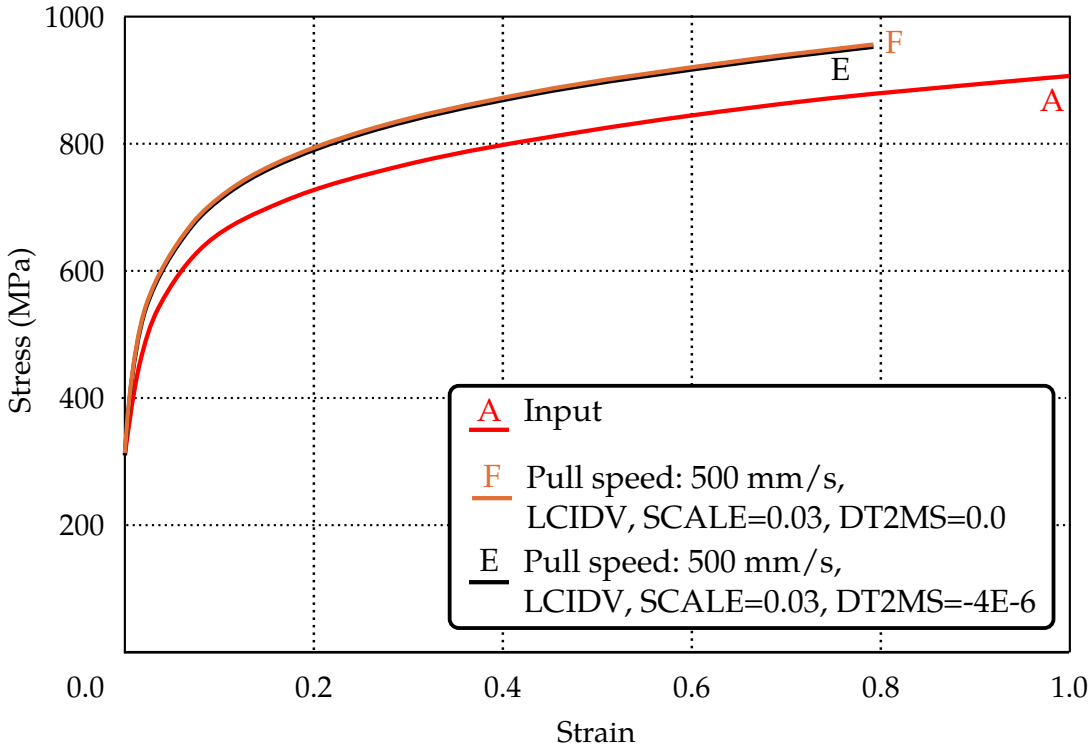



**Figure M260A-5.** Effect of mass scaling (-1.0*dt2ms).

## *MAT_MOHR_NON_ASSOCIATED_FLOW_{OPTION}

This is Material Type 260B. This material model is implemented based on the papers by Mohr, D., et al. (2010) and Roth, C.C. and Mohr, D. (2014) [1, 2]. The Johnson-Cook plasticity model which includes strain hardening, strain rate hardening, and temperature softening is modified with a mixed Swift-Voce strain hardening function coupled with a non-associated flow rule. For certain Advanced High Strength Steels (AHSS), the non-associated flow rule accounts for the difference between directional dependency of the *r*-values (planar anisotropic) and the planar isotropic material response. A ductile fracture model is included based on Hosford-Coulomb fracture initiation model. This model applies to shell elements only.

Available options include:

<BLANK>

XUE

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | P12 | P22 | P33 | G12 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| G22 | G33 | LCIDS | LCIDV | LCIDT | LFLD | LFRAC | W0 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| A | B0 | GAMMA | C | N | SCALE | SIZE0 | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| TREF | TMELT | M | ETA | CP | TINI | DEPS0 | DEPSAD |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is included if the XUE keyword option is used.

| EF0 | PLIM | Q | GAMA | M | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| AOPT | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| | | | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| V1 | V2 | V3 | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | P12 | P22 | P33 | G12 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | -0.5 | 1.0 | 3.0 | -0.5 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's Modulus |
| PR | Poisson's ratio |
| P12, P22, P33 | Yield function parameters, defined by Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions, respectively; see Remark 1. |
| G12 | Plastic flow potential parameters, defined by Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions; see Remark 1. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G22 | G33 | LCIDS | LCIDV | LCIDT | LFLD | LFRAC | W0 |
| Type | F | F | I | I | I | I | I | F |
| Default | 1.0 | 3.0 | none | none | none | 0 | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| G22, G33 | Plastic flow potential parameters, defined by Lankford parameters in rolling (0°), diagonal (45°) and transverse (90°) directions; see Remark 1. |
| LCIDS | Load curve ID defining stress as a function of strain hardening from a uniaxial tension test; it must be along the rolling direction. Also see Remark 2. |
| LCIDV | Load curve ID defining stress scale factors as a function of strain rates (see Figure M260B-1 middle) as determined from experiments. Strain rates are stored in history variable #5. Strain rate scale factors are stored in history variable #6. To output these history variables to d3plot, set NEIPS to at least "6" in *DATABASE_EXTENT_BINARY. Also see Remark 2 |
| LCIDT | Load curve ID defining stress scale factors as a function of temperature in Kelvin (see Figure M260B-1 bottom) as determined from experiments. Temperatures are stored in history variable #4. Temperature scale factors are stored in history variable #7. To output these history variables to d3plot, set NEIPS to at least "7" in *DATABASE_EXTENT_BINARY. Also see Remark 2. |
| LFLD | Load curve ID defining a traditional Forming Limit Diagram for linear strain paths |
| LFRAC | Load curve ID defining a fracture limit curve. Leave this field empty if fields A, B0, GAMMA, C, and N are defined. However, if this field is defined, fields A, B0, GAMMA, C, and N will be ignored even if they are defined. |
| W0 | Neck (FLD failure) width which typically is the blank thickness |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | B0 | GAMMA | C | N | SCALE | SIZE0 | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | 1.0 | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| A, B0, GAMMA, C, N | Material parameters ($a$, $b_0$, $\gamma$, $c$, $n$) for the rate-dependent Hosford-Coulomb fracture initiation model; see Remark 3. Ignored if LFRAC is defined. |
| SCALE | This field can be used to speed up the simulation while equalizing the strain rate effect, which is useful especially in cases where the pulling speed or punch speed is slow. For example, if the pulling speed is 15 mm/s but running the simulation at this speed will take a long time, the pulling speed can be increased to 500 mm/s while "SCALE" can be set to 0.03, giving the same results as those from 15 mm/s with greatly reduced computational time; see examples and Figures in *MAT_260A for details. Furthermore, the increased absolute value (within a reasonable range) of mass scaling -1.0 × dt2ms frequently used in forming simulation does not affect the strain rates, as shown in the examples and Figures in *MAT_260A. |
| SIZE0 | Fracture gauge length used in an experimental measurement, typically between 0.2~0.5 mm |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TREF | TMELT | M | ETA | CP | TINI | DEPSO | DEPSAD |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| TREF | Material parameters for strain softening effect due to temperature. TINI is the initial temperature. See Remark 2 for other parameters' definitions. |
| TMELT | Reference temperature for modified Johnson-Cook Plasticity Model; see Remark 2. |
| M | Exponent coefficient, $m$, for modified Johnson-Cook Plasticity Model; see Remark 2. |
| ETA | Taylor-Quinney coefficient, $\eta_k$; see Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| CP | Heat capacity, $C_p$; see Remark 2. |
| TINI | Initial temperature; see Remark 2. |
| DEPS0 | $\dot{\varepsilon}_{it}/\dot{\varepsilon}_0$; see Remark 2. |
| DEPSAD | $\dot{\varepsilon}_a$; see Remark 2. |

**XUE Card.** This card is included if the XUE keyword option is used.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EF0 | PLIM | Q | GAMA | M | | | |
| Type | F | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EF0, PLIM, Q, GAMA, M | Material parameters for the option XUE. The parameter $k$ in the original paper is assumed to be 1.0. For details, refer to Xue, L. and Wierzbicki, T.'s 2009 paper "Numerical simulation of fracture mode transition in ductile plates" in the *International Journal of Solids and Structures* [4]. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | | | | | | | |
| Type | F | | | | | | | |
| Default | none | | | | | | | |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |
| Default | | | | none | none | none | | |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | | | | | |
| Type | F | F | F | | | | | |
| Default | none | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see *MAT_*OPTION* TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then rotated about the shell element normal by the angle BETA.<br>EQ.2.0: globally orthotropic with material axes determined by the vector **a** for shells, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: locally orthotropic material axes determined by a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>LT.0.0: the absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE__COORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |

**Remarks:**

1. **Non-associated Flow Rule.** Referring to [1] and [2], Hill's 1948 quadratic yield function is written as:

$$f(\boldsymbol{\sigma}, k) = \overline{\sigma} - k = 0 \ ,$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor and $\overline{\sigma}$ is the equivalent stress, defined by:

$$\overline{\sigma} = \sqrt{(\mathbf{P}\boldsymbol{\sigma}) \bullet \boldsymbol{\sigma}} \ .$$

**P** is a symmetric positive-definite matrix defined through three independent parameters, $P_{12}$, $P_{22}$, and $P_{33}$:

$$\mathbf{P} = \begin{bmatrix} 1 & \mathrm{P}_{12} & 0 \\ \mathrm{P}_{12} & \mathrm{P}_{22} & 0 \\ 0 & 0 & \mathrm{P}_{33} \end{bmatrix} .$$

The flow rule, which defines the incremental plastic strain tensor, is written as follows:

$$d\boldsymbol{\varepsilon}_p = d\delta \frac{\partial g(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} \ ,$$

where $d\delta$ is a scalar plastic multiplier. The plastic potential function $g(\sigma)$ can be defined as a quadratic function in stress space:

$$g(\boldsymbol{\sigma}) = \sqrt{(\mathbf{G}\boldsymbol{\sigma}) \bullet \boldsymbol{\sigma}}$$

with,

$$\mathbf{G} = \begin{bmatrix} 1 & G_{12} & 0 \\ \mathrm{G}_{12} & \mathrm{G}_{22} & 0 \\ 0 & 0 & \mathrm{G}_{33} \end{bmatrix} .$$

When **P≠G,** the flow rule is non-associated. The associated flow rule is recovered if $\mathbf{P} = \mathbf{G}$. For example, **P** can represent an isotropic von-Mises yield surface by setting $P_{11} = P_{22} = 1.0$, $P_{12} = -0.5$, and $P_{33} = 3.0$. **G** can represent an orthotropic plastic flow potential by setting:

$$\begin{aligned} G_{12} &= -\frac{r_0}{1 + r_0} \ , \\ G_{22} &= \frac{r_0(1 + r_{90})}{r_{90}(1 + r_0)} \ , \\ G_{33} &= \frac{(1 + 2r_{45})(r_0 + r_{90})}{r_{90}(1 + r_0)} \ . \end{aligned}$$

Here $r_0$, $r_{45}$, and $r_{90}$ are the Lankford coefficients in the rolling, diagonal and transverse directions, respectively. Experiments have shown on the stress level, some AHSS (Advanced High Strength Steel), e.g., DP590, and TRIP780 show strong directional dependency for the $r$-values, while nearly the same stress-strain curves have been measured in all directions. The directional dependency of $r$-values suggests planar anisotropy while the material response for the stress

is planar isotropic, which is the main reason to employ the non-associated flow rule.

2. **A Modified Johnson-Cook Plasticity Model with Mixed Swift-Voce Hardening.** The Johnson-Cook plasticity model (1983) multiplicatively decomposes the deformation resistance into three functions representing the effects of strain hardening, strain rate, and temperature. The Johnson-Cook model is modified to include hardening saturation with a mixed Swift-Voce hardening law (Sung et al, 2010 [3]), which gives a better description of the hardening at large strain levels, thus improving the prediction of the necking and post-necking response of metal sheet:

$$\sigma_y = \left(\alpha\left(A\left(\bar{\varepsilon}_{pl}+\varepsilon_0\right)^n\right)+(1-\alpha)\left(k_0+Q\left(1-e^{-\beta\bar{\varepsilon}_{pl}}\right)\right)\right)\left(1+C\ln\left(\frac{\dot{\bar{\varepsilon}}_{pl}}{\dot{\varepsilon}_0}\right)\right)\left(1-\left(\frac{T-T_r}{T_m-T_r}\right)^m\right)$$

where $\bar{\varepsilon}_{pl}$ and $\dot{\bar{\varepsilon}}_{pl}$ are effective plastic strain and strain rate, respectively; $T_m$ (TMELT), $T_r$ (TREF) and $T$ are the melting temperature, reference temperature (ambient temperature 293 K) and current temperature, respectively; and $m$ (M) is an exponent coefficient. For other symbols' definitions refer to the aforementioned paper.

To make this material model more general and flexible, three load curves are used to define the three components of the deformation resistance. A load curve (LCIDS) is used to describe the strain hardening:

$$\alpha\left(A\left(\bar{\varepsilon}_{pl}+\varepsilon_0\right)^n\right)+(1-\alpha)\left(k_0+Q\left(1-e^{-\beta\bar{\varepsilon}_{pl}}\right)\right)$$

Strain rate is described by a load curve LCIDV (stress scale factor vs. strain rates, Figure M260B-1 middle), which scales the stresses based on the strain rates during a simulation:

$$1+Cln\left(\frac{\dot{\bar{\varepsilon}}_{pl}}{\dot{\varepsilon}_0}\right)$$

The temperature softening effect is defined by another load curve LCIDT (stress scale factor as a function of temperature, Figure M260B-1 bottom), which scales the stresses based on the temperatures during the simulation:

$$1-\left(\frac{T-T_r}{T_m-T_r}\right)^m$$

The temperature effect is a self-contained model, meaning it does not require thermal exchange with the environment. It calculates temperatures based on plastic strain and strain rate.

The temperature evolution is determined with:

$$dT = \omega\left[\dot{\bar{\varepsilon}}_{pl}\right]\frac{\eta_k}{\rho C_p}\bar{\sigma}d\bar{\varepsilon}_{pl} \ .$$

where $\eta_k$ (ETA) is the Taylor-Quinney coefficient; $\rho$ (R0) is the mass density; $C_p$ (CP) is the heat capacity; and

$$\omega\left[\dot{\bar{\varepsilon}}_{pl}\right] = \begin{cases} 0 & for\ \dot{\bar{\varepsilon}}_{pl} < \dot{\varepsilon}_{it} \\ \dfrac{\left(\dot{\bar{\varepsilon}}_{pl} - \dot{\varepsilon}_{it}\right)^2\left(3\dot{\varepsilon}_a - 2\dot{\bar{\varepsilon}}_{pl} - \dot{\varepsilon}_{it}\right)}{\left(\dot{\varepsilon}_a - \dot{\varepsilon}_{it}\right)^3} & for\ \dot{\varepsilon}_{it} \le \dot{\bar{\varepsilon}}_{pl} \le \dot{\varepsilon}_a \\ 1 & for\ \dot{\varepsilon}_a < \dot{\bar{\varepsilon}}_{pl} \end{cases}$$

Here $\dot{\varepsilon}_{it} > 0$ and $\dot{\varepsilon}_a > \dot{\varepsilon}_{it}$ define the limits of the respective domains of isothermal and adiabatic conditions ($\dot{\varepsilon}_a$ = DEPSAD). For simplicity, $\dot{\varepsilon}_{it} = \dot{\varepsilon}_0 \times$ DEPS0.

As shown in a single shell element undergoing uniaxial stretching (see Figure M260B-1), the general effect of LCIDV is to elevate the strain hardening behavior as the strain rate increases (curve "D" in Figure M260B-2 top), while the effect of LCIDT is strain softening as the temperature rises (curve "C" in Figure M260B-2 top). Initially, due to the combined effect of both LCIDV and LCIDT strain hardening may occur before the temperature rises enough to cause strain softening in the model (curve "E" in Figure M260B-2 top). The temperature and strain rates calculated for each element can be viewed with history variables #4 and #5 (curves "C" and "D" in Figure M260B-2 bottom), respectively, while the strain rate scale factors and temperature scale factors can be viewed with history variable #6 and #7, respectively.

3. **Rate-dependent Hosford-Coulomb Fracture Initiation Model.** An extension of the Hosford-Coulomb fracture initiation model is used to account for the effect of strain rate on ductile fracture. The damage accumulation is calculated through history variable #3. When this history variable reaches 1.0, fracture occurs at an equivalent plastic strain, $\bar{\varepsilon}_f$, that is,

$$\int_0^{\varepsilon_f} \frac{d\bar{\varepsilon}_{pl}}{\bar{\varepsilon}_f^{pr}[\eta, \bar{\theta}]} = 1 \ .$$

Here $\bar{\varepsilon}_f^{pr}$, $\eta$, and $\bar{\theta}$ are strain to fracture, stress triaxiality, and the Lode parameter, respectively.

The fracture parameters, A, B0, GAMMA, C, and N, are used in the following equations as ($a$, $b_0$, $\gamma$, $c$, $n$), respectively. Strain to fracture for a proportional load is given as:

$$\bar{\varepsilon}_f^{pr}[\eta,\bar{\theta}] = b(1+c)^{\frac{1}{n}}\left(\left\{\frac{1}{2}\left((f_1-f_2)^a+(f_2-f_3)^a+(f_1-f_3)^a\right)\right\}^{\frac{1}{a}} + c(2\eta+f_1+f_3)\right)^{-\frac{1}{n}}$$

where $a$ is the Hosford exponent, $c$ is the friction coefficient controlling the effect of triaxiality, and $n$ is the stress state sensitivity. The Lode angle parameter dependent trigonometric functions are given as:

$$f_1[\bar{\theta}] = \frac{2}{3}\cos\left[\frac{\pi}{6}(1-\bar{\theta})\right]$$

$$f_2[\bar{\theta}] = \frac{2}{3}\cos\left[\frac{\pi}{6}(3+\bar{\theta})\right]$$

$$f_3[\bar{\theta}] = -\frac{2}{3}\cos\left[\frac{\pi}{6}(1+\bar{\theta})\right]$$

The coefficient $b$ (strain to fracture for uniaxial or equi-biaxial stretching) is:

$$b = \begin{cases} b_0 & for\ \dot{\varepsilon}_p < \dot{\varepsilon}_0 \\ b_0\left(1+\gamma ln\left[\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0}\right]\right) & for\ \dot{\varepsilon}_p > \dot{\varepsilon}_0 \end{cases}$$

where $\gamma$ is the strain rate sensitivity.

4. **Corresponding Parameters Summary.** The following table lists variable names used in this material model and corresponding symbols employed in [1], [2], and [3]:

| Variable | P12 | P22 | P33 | G12 | G22 | G33 | A | B0 |
|---|---|---|---|---|---|---|---|---|
| Symbol | $P_{12}$ | $P_{22}$ | $P_{33}$ | $G_{12}$ | $G_{22}$ | $G_{33}$ | $a$ | $b_0$ |
| Variable | GAMMA | C | N | TREF | TMELT | M | ETA | CP |
| Symbol | $\gamma$ | $c$ | $N$ | $T_r$ | $T_m$ | $m$ | $\eta_k$ | $C_p$ |
| Variable | DEPS0 | DEPSAD | R0 | | | | | |
| Symbol | $\dot{\varepsilon}_{it}/\dot{\varepsilon}_0$ | $\dot{\varepsilon}_a$ | $\rho$ | | | | | |

The following table lists variable names used in this material model and corresponding symbols in Xue's 2009 paper [4], for the option XUE:

| Variable | EF0 | P22 | P33 | G12 | G22 | | | |
|---|---|---|---|---|---|---|---|---|
| Symbol | $\varepsilon_{f0}$ | $P_{\lim}$ | $q$ | $\gamma$ | $m$ | | | |

5. **Additional History Variables.** The table below lists the extra history variables associated with this material. See NEIPS on the manual page for *DATABASE_EXTENT_BINARY.

| History Variable # | Description |
|---|---|
| 3 | Damage accumulation. Elements will be deleted if this variable reaches 1.0 for more than half of the through-thickness integration points. |
| 4 | Temperatures |
| 5 | Strain rates |
| 6 | Strain rate scale factors |
| 7 | Temperature scale factor |

**Keyword Example Input:**

A sample material input card can be found below, with parameters from Mohr, D., et al. (2010) and Roth, C.C. and Mohr, D. (2014).

```
*MAT_MOHR_NON_ASSOCIATED_FLOW
$#     mid        R0         E        PR       P12       P22       P33       G12
         1 7.8000E-9  2.10E05  0.300000      -0.5       1.0       3.0   -0.4946
$      G22       G33     LCIDS     LCIDV     LCIDT      LFLD     LFRAC        W0
    0.9318    2.4653       100       105       102
$        A         B0     GAMMA         C         N     SCALE
      1.97      0.82     0.025      0.00     0.199  3.132E-3
$     TREF     TMELT         M       ETA        CP      TINI     DEPSO    DEPSAD
     293.0   1673.70     0.921       0.9     420.0     293.0  0.001164     1.379
$     AOPT
         3
$       XP        YP        ZP        A1        A2        A3

$       V1        V2        V3        D1        D2        D3      BETA
       1.0
*DEFINE_CURVE
100
         0.00000E+00         0.30130E+03
```

```
         0.10000E-01         0.42295E+03
         0.20000E-01         0.47991E+03
         0.30000E-01         0.52022E+03
         0.40000E-01         0.55126E+03
                                          ⋮
*DEFINE_CURVE
105
         0.00000E+00         0.10000E+01
         0.10000E+00         0.10608E+01
         0.50000E+00         0.10828E+01
         0.10000E+01         0.10923E+01
                                          ⋮
*DEFINE_CURVE
102
         0.29300E+03         0.10000E+01
         0.33300E+03         0.96168E+00
         0.37300E+03         0.92744E+00
         0.41300E+03         0.89459E+00
         0.45300E+03         0.86261E+00
                                          ⋮
```

## References:


[1] D. Mohr, M. Dunand, K. Kim, "Evaluation of associated and non-associated quadratic plasticity models for advanced high strength steel sheets under multi-axial loading," *International Journal of Plasticity*, Vol 26, Issue 7, https://doi.org/10.1016/j.ijplas.2009.11.006, July 2010.

[2] C.C. Roth and D. Mohr, "Effect of strain rate on ductile fracture initiation in advanced high strength steel sheets: Experiments and modeling," *International Journal of Plasticity*, Vol 56, https://doi.org/10.1016/j.ijplas.2014.01.003, May 2014.

[3] J.H. Sung, J.H. Kim, R.H. Wagoner, "A plastic constitutive equation incorporating strain, strain-rate, and temperature," *International Journal of Plasticity*, Vol 26, Issue 12, https://doi.org/10.1016/j.ijplas.2010.02.005, December 2010.

[4] L. Xue and T. Wierzbicki, "Numerical simulation of fracture mode transition in ductile plates," *International Journal of Solids and Structures*, Vol 46, Issue 6, https://doi.org/10.1016/j.ijsolstr.2008.11.009, March 2009.


## Revision Information::

This material model is available in SMP starting in Revision 102375. Revision history is listed below:

- Element deletion feature based on damage accumulation: Revision 109792.
- The option XUE is available starting on Revision 111531.
- Set default values for P12, P22, P33, G12, G22 and G33: Revision 116262.

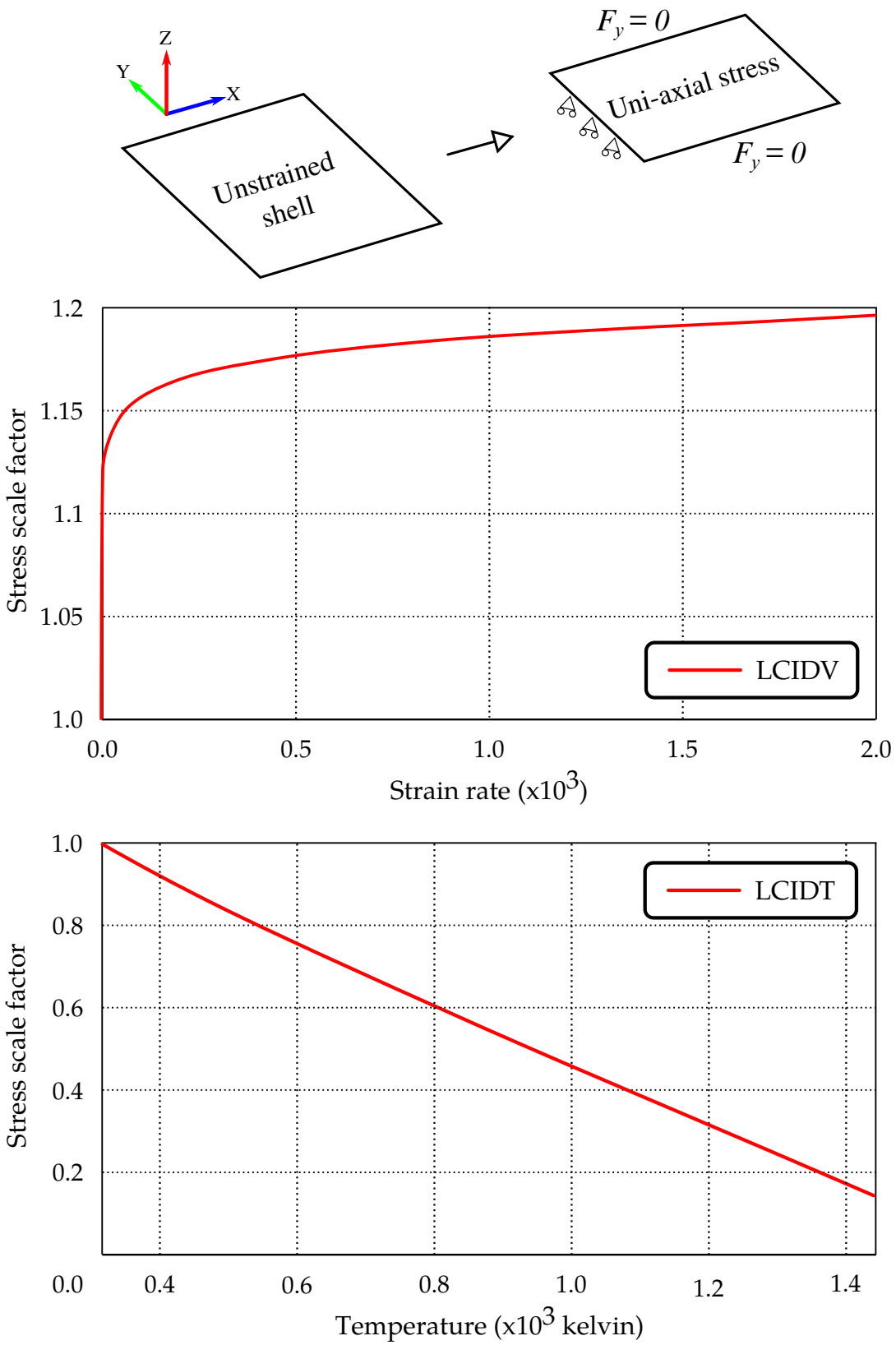



**Figure M260B-1.** Uniaxial stretching on a single shell element; Input curves LCIDV and LCIDT.

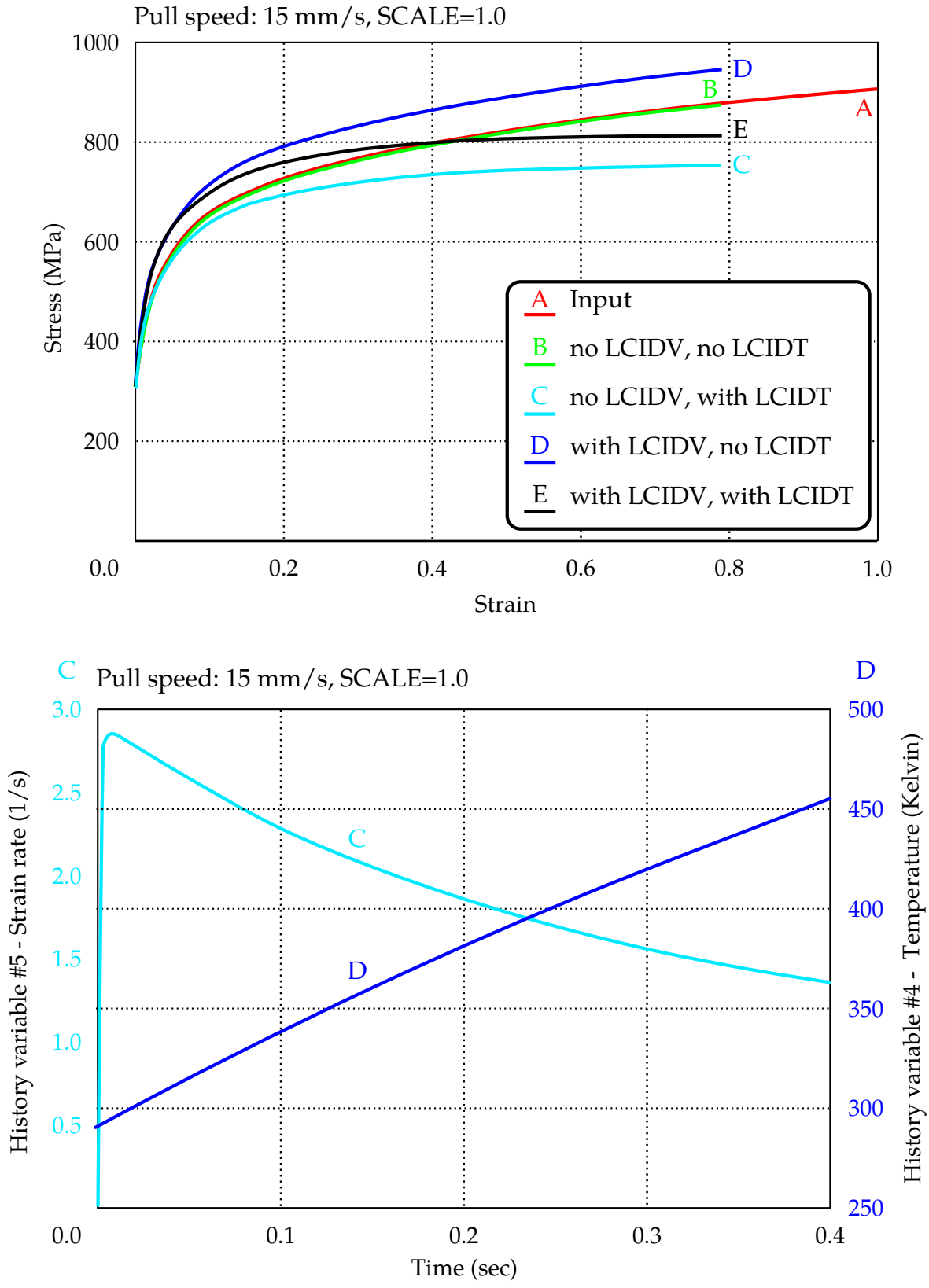



**Figure M260B-2.** Results of a single element uniaxial stretching - stress-strain curves (top), strain rates and temperature history under various conditions.

## *MAT_LAMINATED_FRACTURE_DAIMLER_PINHO

This is Material Type 261 which is an orthotropic continuum damage model for laminated fiber-reinforced composites. See Pinho, Iannucci and Robinson [2006]. It is based on a physical model for each failure mode and considers non-linear in-plane shear behavior.

This model is implemented for shell, thick shell and solid elements.

> **NOTE:** To apply laminated shell theory, set LAMSHT $\geq 3$ in *CONTROL_SHELL.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | AOPT | DAF | DKF | DMF | EFS |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | MANGLE | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| ENKINK | ENA | ENB | ENT | ENL | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XC | XT | YC | YT | SL | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| FIO | SIGY | LCSS | BETA | PFL | PUCK | SOFT | DT |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus in $a$-direction (longitudinal) |
| EB | $E_b$, Young's modulus in $b$-direction (transverse) |
| EC | $E_c$, Young's modulus in $c$-direction |
| PRBA | $\nu_{ba}$, Poisson's ratio $ba$ |
| PRCA | $\nu_{ca}$, Poisson's ratio $ca$ |
| PRCB | $\nu_{cb}$, Poisson's ratio $cb$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | DAF | DKF | DMF | EFS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus $ab$ |
| GBC | $G_{bc}$, shear modulus $bc$ |
| GCA | $G_{ca}$, shear modulus $ca$ |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by an angle (see MANGLE on Card 4). |
| | EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. This angle may be set in the keyword input for the element or in the input for this keyword (see MANGLE on Card 4). |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| DAF | Flag to control failure of an integration point based on longitudinal (fiber) tensile failure (see Remarks 1 and 2): |
| | LT.-0.01: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. \|DAF\| limits the stress reduction due to damage from longitudinal tensile failure. |
| | EQ.0.00: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, |

| VARIABLE | DESCRIPTION |
|---|---|
| | reaches 1.0. DAF does not limit the stress reduction due to damage from longitudinal tensile failure. |
| | GT.0.01: No failure of integration point due to fiber tensile failure. This condition corresponds to history variable "da(i)" reaching 1.0. The value of DAF limits the stress reduction due to damage from longitudinal tensile failure. |
| DKF | Flag to control failure of an integration point based on longitudinal (fiber) compressive failure (see Remarks 1 and 2): |
| | LT.-0.01: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. \|DKF\| limits the stress reduction due to damage from longitudinal compressive failure. |
| | EQ.0.00: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. DKF does not limit the stress reduction due to damage from longitudinal tensile failure. |
| | GT.0.01: No failure of integration point due to fiber compressive failure. This condition corresponds to history variable "dkink(i)" reaching 1.0. The value of DKF limits the stress reduction due to damage from longitudinal compressive failure. |
| DMF | Flag to control failure of an integration point based on transverse (matrix) failure (see Remarks 1 and 2): |
| | LT.-0.01: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. \|DMF\| limits the stress reduction due to the damage from the matrix failure. |
| | EQ.0.00: Integration point fails if any damage variable, that has a corresponding flag (DAF, DKF, or DMF) set ≤ 0.0, reaches 1.0. DMF does not limit the stress reduction due to damage from longitudinal tensile failure. |
| | GT.0.01: No failure of integration point due to matrix failure. This condition corresponds to history variable "dmat(i)" reaching 1.0. The value of DMF limits the stress reduction due to damage from matrix failure. |
| EFS | Maximum effective strain for element layer failure. A value of |

| VARIABLE | DESCRIPTION |
|---|---|
| | unity would equal 100% strain. |
| | GT.0.0: Fails when effective strain calculated assuming material is volume preserving exceeds EFS |
| | LT.0.0: Fails when effective strain calculated from the full strain tensor exceeds \|EFS\| |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | MANGLE | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| MANGLE | Material angle in degrees for AOPT = 0 (shells and tshells only) and AOPT = 3. MANGLE may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ENKINK | ENA | ENB | ENT | ENL | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ENKINK | Fracture toughness for longitudinal (fiber) compressive failure mode.<br>GT.0.0: The given value will be regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = (-ENKINK). The load curve defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. Neither case includes further regularization. The table must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5. |
| ENA | Fracture toughness for longitudinal (fiber) tensile failure mode.<br>GT.0.0: The given value will be regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = (-ENA) The load curve defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. Neither case includes further regularization. The table definition must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5. |
| ENB | Fracture toughness for intralaminar matrix tensile failure.<br>GT.0.0: The given value will be regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = (-ENB). The load curve defines |

| VARIABLE | DESCRIPTION |
|---|---|
| | the fracture toughness for intralaminar matrix tensile failure as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for intralaminar matrix tensile failure as a function of characteristic element length. Neither case includes further regularization. The table definition must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5. |
| ENT | Fracture toughness for intralaminar matrix transverse shear failure.<br>GT.0.0: The given value will be regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = (-ENT). The load curve defines the fracture toughness for intralaminar matrix transverse shear failure as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for intralaminar matrix transverse shear failure as a function of characteristic element length.Neither case includes further regularization. The table must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5 |
| ENL | Fracture toughness for intralaminar matrix longitudinal shear failure.<br>GT.0.0: The given value will be regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = (-ENL). The load curve defines the fracture toughness for intralaminar matrix longitudinal shear failure as a function of characteristic element length. The table defines for each strain rate value a load curve that defines the fracture toughness for intralaminar matrix longitudinal shear failure as a function of characteristic element length. Neither case includes further regularization. The table must use the absolute value of the strain rate unless providing a logarithmically defined table. See Remark 5. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SL | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength, *a*-axis (positive value).<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength as a function of the absolute value of the longitudinal strain rate ($\dot{\epsilon}_{aa}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves. |
| XT | Longitudinal tensile strength, *a*-axis.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength as a function of the absolute value of the longitudinal strain rate ($\dot{\epsilon}_{aa}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves. |
| YC | Transverse compressive strength, *b*-axis (positive value).<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength as a function of the absolute value of the transverse strain rate ($\dot{\epsilon}_{bb}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves. |
| YT | Transverse tensile strength, *b*-axis.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-YT) which defines the transverse tensile strength as a function of the absolute value of the transverse strain rate ($\dot{\epsilon}_{bb}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves. |

| VARIABLE | DESCRIPTION |
|---|---|
| SL | Longitudinal shear strength.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SL) which defines the longitudinal shear strength as a function of the absolute value of the in-plane shear strain rate ($\dot{\epsilon}_{ab}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FIO | SIGY | LCSS | BETA | PFL | PUCK | SOFT | DT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| FIO | Fracture angle in pure transverse compression (default = 53.0°) |
| SIGY | In-plane shear yield stress (only used when BETA < 1.0) |
| LCSS | Load curve ID or Table ID.<br>**Load Curve.** When LCSS is a load curve ID, it defines the nonlinear in-plane shear stress as a function of the absolute value of in-plane shear strain ($\gamma_{ab}$), except in the case of logarithmically defined curves. See Remark 5 for a discussion of logarithmically defined curves.<br>**Tabular Data.** The table maps in-plane strain rate values ($\dot{\gamma}_{ab}$) to a load curve giving the in-plane shear stress as a function of the absolute value of in-plane shear strain. Note that except for in the case of logarithmically definied tables, the strain rates and shear strains must be the absolute values (see Remark 5). For strain rates below the minimum value, the curve for the lowest defined value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the curve for the highest defined value of strain rate is used. |
| BETA | Hardening parameter for in-plane shear plasticity ($0.0 \le$ BETA $\le 1.0$).<br>EQ.0.0: Pure kinematic hardening |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Pure isotropic hardening<br>0.0 < BETA < 1.0: Mixed hardening |
| PFL | Percentage of shell or tshell layers which must fail until crashfront is initiated. For example, if \|PFL\| = 80.0, then 80% of layers must fail until strengths are reduced in neighboring elements. Default: all layers must fail. A single layer fails if 1 in-plane integration point fails (PFL > 0) or if 4 in-plane integration points fail (PFL < 0). |
| PUCK | Flag for evaluation and post-processing of Puck's inter-fiber-failure criterion (IFF); see Puck, Kopp and Knops [2002].<br>EQ.0.0: No evaluation of Puck's IFF-criterion.<br>EQ.1.0: Puck's IFF-criterion will be evaluated. |
| SOFT | Softening reduction factor for material strength in crashfront elements (default = 1.0) |
| DT | Strain rate averaging option:<br>EQ.0.0: Strain rate is evaluated using a running average.<br>LT.0.0: Strain rate is evaluated using an average of the last 11 time steps.<br>GT.0.0: Strain rate is averaged over the last DT time units. |

**Remarks:**

1. **Failure surfaces.** We assemble *four* sub-surfaces, representing different failure mechanisms, to obtain the failure surface to limit the elastic domain. See Figure M261-1 for a definition of angles. They are defined as follows:

   a) longitudinal (fiber) tension,

   $$f_a = \frac{\sigma_a}{X_T} = 1$$

   b) longitudinal (fiber) compression (3D-kinking model) – (transformation to fracture plane),

   $$f_{\text{kink}} = \begin{cases} \left(\dfrac{\tau_T}{S_T - \mu_T \sigma_n}\right)^2 + \left(\dfrac{\tau_L}{S_L - \mu_L \sigma_n}\right)^2 = 1 & \text{if} \quad \sigma_{b^m} \le 0 \\ \left(\dfrac{\sigma_n}{Y_T}\right)^2 + \left(\dfrac{\tau_T}{S_T}\right)^2 + \left(\dfrac{\tau_L}{S_L}\right)^2 = 1 & \text{if} \quad \sigma_{b^m} > 0 \end{cases}$$

   with

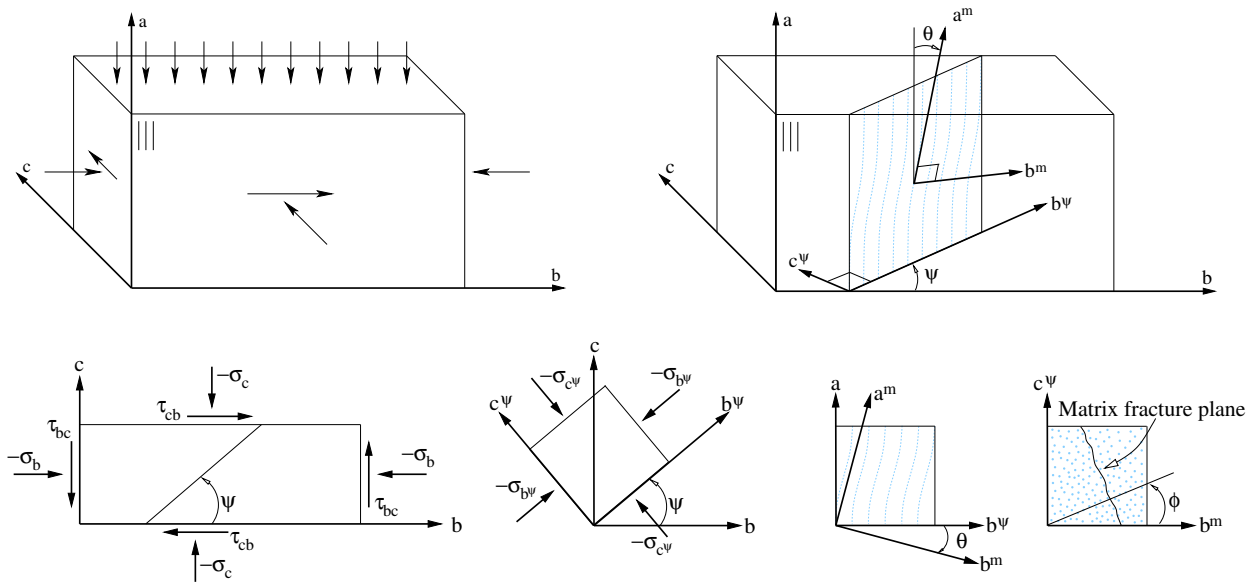



**Figure M261-1.** Definition of angles and stresses in fracture plane

$$S_T = \frac{Y_C}{2\tan(\phi_0)}$$

$$\mu_T = -\frac{1}{\tan(2\phi_0)}$$

$$\mu_L = S_L \frac{\mu_T}{S_T}$$

$$\sigma_n = \frac{\sigma_{b^m} + \sigma_{c^\psi}}{2} + \frac{\sigma_{b^m} - \sigma_{c^\psi}}{2}\cos(2\phi) + \tau_{b^m c^\psi}\sin(2\phi)$$

$$\tau_T = -\frac{\sigma_{b^m} - \sigma_{c^\psi}}{2}\sin(2\phi) + \tau_{b^m c^\psi}\cos(2\phi)$$

$$\tau_L = \tau_{a^m b^m}\cos(\phi) + \tau_{c^\psi a^m}\sin(\phi)$$

c) transverse (matrix) failure: transverse tension,

$$f_{\text{mat}} = \left(\frac{\sigma_n}{Y_T}\right)^2 + \left(\frac{\tau_T}{S_T}\right)^2 + \left(\frac{\tau_L}{S_L}\right)^2 = 1 \quad \text{if} \quad \sigma_n \geq 0$$

with

$$\sigma_n = \frac{\sigma_b + \sigma_c}{2} + \frac{\sigma_b - \sigma_c}{2}\cos(2\phi) + \tau_{bc}\sin(2\phi)$$

$$\tau_T = -\frac{\sigma_b - \sigma_c}{2}\sin(2\phi) + \tau_{bc}\cos(2\phi)$$

$$\tau_L = \tau_{ab}\cos\phi + \tau_{ca}\sin\phi$$

d) transverse (matrix) failure: transverse compression/shear,

$$f_{\text{mat}} = \left(\frac{\tau_T}{S_T - \mu_T\sigma_n}\right)^2 + \left(\frac{\tau_L}{S_L - \mu_L\sigma_n}\right)^2 = 1 \quad \text{if} \quad \sigma_n < 0$$

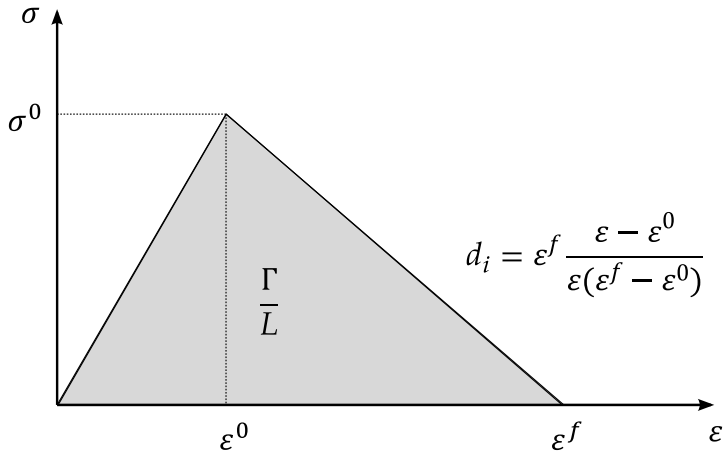



**Figure M261-2.** Damage evolution law

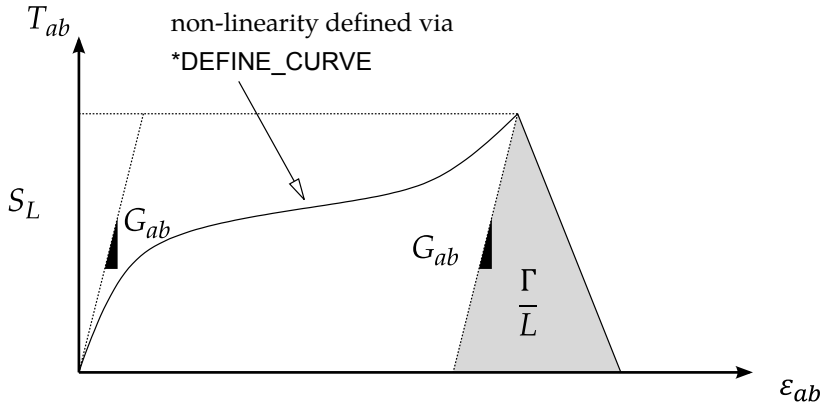



**Figure M261-3.** Definition of nonlinear in-plane shear behavior

2. **Damage evolution.** As long as the stress state is located within the failure surface, the model behaves in an orthotropic elastic manner. When reaching the failure criteria, the effective (undamaged) stresses will be reduced by a factor of $(1-d)$. Here, the damage variable $d$ represents one of the damage variables defined for the different failure mechanisms or a limit to the stress reduction set with DAF, DKF, and DMF due to the damage variable. In other words, $d$ corresponds to one of the following values, depending on the failure mechanism: $\min(d_{\text{da}}, |\text{DAF}|)$ for logitudinal tension damage, $\min(d_{\text{kink}}, |\text{DKF}|)$ for logitudinal compression damage, or $\min(d_{\text{mat}}, |\text{DMF}|)$ for matrix damage. Note that D*i*F = 0, where *i* is A, K, or M, means no limit to the stress reduction due to the corresponding damage variable. Note that once the integration point fails that the stress goes to zero. A linear damage evolution law based on fracture toughnesses ($\Gamma \rightarrow$ ENKINK, ENA, ENB, ENT, ENL) and a characteristic internal element length, $L$, to account for objectivity drive the growth of the damage variables ($d_{\text{da}}$, $d_{\text{kink}}$, and $d_{\text{mat}}$). See Figure M261-2.

3. **Nonlinear in-plane shear.** To account for the characteristic nonlinear in-plane shear behavior of laminated fiber-reinforced composites, we couple a 1D elasto-

plastic formulation to a linear damage behavior upon reaching the maximum allowable stress state for shear failure. To introduce nonlinearity in the shear behavior, use *DEFINE_CURVE to define an explicit shear stress as a function of engineering shear strain curve (LCSS). See Figure M261-3 (in which $\epsilon_{ab}$ designates engineering shear strain rather than tensorial shear strain).

4. **Element deletion.** When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. Elements that share nodes with the deleted element become "crashfront" elements and can have their strengths reduced by using the SOFT parameter. An earlier initiation of crashfront elements is possible by using the parameter PFL.

5. **Logarithmically defined tables and curves.** An alternative way to invoke logarithmic interpolation between discrete strain rates or strains is described as follows. If the *first* value in the table or curve is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate or all the curve abscissa values represent the natural logarithm of a strain rate or strain. Since the tables and curves are internally discretized to equally space the table and curve values, it makes good sense from an accuracy standpoint that the table and curve values represent the natural log of strain rate or strain when the lowest strain rate or strain and highest strain rate or strain differ by several orders of magnitude. Invoking logarithmic interpolation incurs some additional computational cost.

6. **History variables.** The number of additional integration point variables written to the LS-DYNA database is input by the *DATABASE_EXTENT_BINARY definition with the variable NEIPS (shells) and NEIPH (solids). These additional variables are tabulated below ($i$ = integration point).

   When intending to initialize the stress state using *INTIAL_STRESS_*OPTION*, the stress values SIGXX, SIGYY, etc. in *INITIAL_STRESS_*OPTION* are not used, rather stresses are determined from the total strain history variables 31 to 36.

| History Variable | Description | Value | History Variable # |
|---|---|---|---|
| fa($i$) | fiber tensile mode | $0 \rightarrow 1$: elastic<br>1: failure criterion reached | 1 |
| fkink($i$) | fiber compressive mode | $0 \rightarrow 1$: elastic<br>1: failure criterion reached | 2 |
| fmat($i$) | matrix mode | $0 \rightarrow 1$: elastic<br>1: failure criterion reached | 3 |

| History Variable | Description | Value | History Variable # |
|---|---|---|---|
| da(*i*) | damage fiber tension | 0: elastic<br>1: fully damaged | 5 |
| dkink(*i*) | damage fiber compression | 0: elastic<br>1: fully damaged | 6 |
| dmat(*i*) | damage transverse | 0: elastic<br>1: fully damaged | 7 |
| dam(*i*) | crashfront | -1: element intact<br>$10^{-8}$: element in crashfront<br>+1: element failed | 9 |
| fmt_p(*i*) | tensile matrix mode (Puck criteria) | 0 → 1: elastic<br>1: failure criterion reached | 10 |
| fmc_p(*i*) | compressive matrix mode (Puck criteria) | 0 → 1: elastic<br>1: failure criterion reached | 11 |
| theta_p(*i*) | angle of fracture plane (radians, Puck criteria) | | 12 |
| d1tot(*i*) | Total strain in material 11-direction | | 31 |
| d2tot(*i*) | Total strain in material 22-direction | | 32 |
| d3tot(*i*) | Total strain in material 33-direction | | 33 |
| d4tot(*i*) | Total strain in material 12-direction | | 34 |
| d5tot(*i*) | Total strain in material 23-direction | | 35 |
| d6tot(*i*) | Total strain in material 31-direction | | 36 |
| theta | Angle $\theta$ in Figure M261-1 | | 49 |
| psi | Angle ψ in Figure M261-1 | | 50 |
| e1dot(*i*) | Averaged strain rate in longitudinal direction | | 54 |
| e2dot(*i*) | Averaged strain rate in transverse direction | | 55 |

| History Variable | Description | Value | History Variable # |
|---|---|---|---|
| e4dot(*i*) | Averaged engineering shear strain rate in in-plane direction | | 56 |

**References:**

More detailed information about this material model can be found in Pinho, Iannucci and Robinson [2006].

## *MAT_LAMINATED_FRACTURE_DAIMLER_CAMANHO

This is Material Type 262 which is an orthotropic continuum damage model for laminated fiber-reinforced composites. See Maimí, Camanho, Mayugo and Dávila [2007]. It is based on a physical model for each failure mode and considers a simplified non-linear in-plane shear behavior. This model is implemented for shell, thick shell and solid elements.

Applus+ IDIADA and Toyota Motor Corporation/TOYOTA GAZOO Racing Europe developed enhancements to improve the accuracy of in-plane shear mechanisms and the predictability of material behavior under crushing-bending loads. See Alameda, Dominguez, Martin-Santos and Miura [2024]. These features have been implemented in material 262 in collaboration with DYNAmore and are specified in Cards 9 and 10.

> **NOTE:** Laminated shell theory can be applied by setting LAMSHT ≥ 3 in *CONTROL_SHELL.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | AOPT | DAF | DKF | DMF | EFS |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | DSF | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | MANGLE | MSG |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| GXC | GXT | GYC | GYT | GSL | GXCO | GXTO | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XC | XT | YC | YT | SL | XCO | XTO | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| FIO | SIGY | ETAN | BETA | PFL | PUCK | SOFT | DT |
|---|---|---|---|---|---|---|---|

***Card 8.*** This card is optional.

| EPSF23 | EPSR23 | TSMD23 | EPSF31 | EPSR31 | TSMD31 | | |
|---|---|---|---|---|---|---|---|

***Card 9.*** This card is optional.

| EF_11T | EF_11C | EF_22T | EF_22C | EF_12 | EF_23 | EF_31 | LCSS |
|---|---|---|---|---|---|---|---|

***Card 10.*** This card is optional.

| CF12 | CF13 | CF23 | SOFTC | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | GT.0.0: $E_a$, Young's modulus - longitudinal direction<br>LT.0.0: Load curve or table ID = (-EA). It is available for shells only.<br>**Load Curve.** When -EA refers to a load curve ID, it is taken as defining the uniaxial elastic stress as a function of strain behavior in the longitudinal direction. Negative data points correspond to compression and positive values to tension.<br>**Tabular Data.** When -EA refers to a table ID, it defines a load curve for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the longitudinal direction.<br>**Logarithmically Defined Tables.** If the first uniaxial elastic stress as a function of strain curve in the table |

| VARIABLE | DESCRIPTION |
|---|---|
| | corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves. |
| EB | GT.0.0: $E_b$, Young's modulus - transverse direction<br>LT.0.0: Load Curve ID or Table ID = (-EB). (shells only).<br>**Load Curve.** When -EB refers to a load curve ID, it is taken as defining the uniaxial elastic stress as a function of strain behavior in the transverse direction. Negative data points correspond to compression, and positive values to tension.<br>**Tabular Data.** When -EB corresponds to a table ID, it specifies a load curve for each strain rate value. The load curves give the uniaxial elastic stress as a function of strain behavior in the transverse direction.<br>**Logarithmically Defined Tables.** If the first uniaxial elastic stress as a function of strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* stress-strain curves. |
| EC | $E_c$, Young's modulus in *c*-direction |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | AOPT | DAF | DKF | DMF | EFS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | GT.0.0: $G_{ab}$, shear modulus in the *ab*-direction<br>LT.0.0: Load Curve ID or Table ID = (-GAB) |

| VARIABLE | DESCRIPTION |
|---|---|
| | **Load Curve.** When -GAB refers to a load curve ID, it is taken as defining the elastic shear stress as a function of shear strain behavior in the *ab*-direction.<br><br>**Tabular Data.** When -GAB corresponds to a table ID, it defines a load curve for each strain rate value. The load curves give the elastic shear stress as a function of shear strain behavior in the *ab*-direction.<br><br>**Logarithmically Defined Tables.** If the *first* elastic shear stress as a function of shear strain curve in the table corresponds to a negative strain rate, LS-DYNA assumes that the natural logarithm of the strain rate value is used for *all* shear stress-shear strain curves. |
| GBC | $G_{bc}$, shear modulus *bc* |
| GCA | $G_{ca}$, shear modulus *ca* |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES, and then, for shells only, rotated about the shell element normal by an angle MANGLE<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle (MANGLE) from a line in the plane of the element defined by the cross product of the vector **v** with the element normal<br>EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *p*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID |

| VARIABLE | DESCRIPTION |
|---|---|
| | number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| DAF | Flag to control failure of an integration point based on longitudinal (fiber) tensile failure:<br>EQ.0.0: Integration point fails if any damage variable reaches 1.0.<br>EQ.1.0: No failure of integration point due to fiber tensile failure, $da(i) = 1.0$ |
| DKF | Flag to control failure of an integration point based on longitudinal (fiber) compressive failure:<br>EQ.0.0: Integration point fails if any damage variable reaches 1.0.<br>EQ.1.0: No failure of integration point due to fiber compressive failure, $\mathrm{dkink}(i) = 1.0$. |
| DMF | Flag to control failure of an integration point based on transverse (matrix) failure:<br>EQ.0.0: Integration point fails if any damage variable reaches 1.0.<br>EQ.1.0: No failure of integration point due to matrix failure, $\mathrm{dmat}(i) = 1.0$ |
| EFS | Maximum effective strain for element layer failure. A value of unity would equal 100% strain.<br>GT.0.0: Fails when effective strain calculated assuming material is volume preserving exceeds EFS<br>LT.0.0: Fails when effective strain calculated from the full strain tensor exceeds \|EFS\| |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | DSF | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP YP ZP | Coordinates of point $p$ for AOPT = 1 and 4 |
| A1 A2 A3 | Components of vector **a** for AOPT = 2 |
| DSF | Flag to control failure of an integration point based on in-plane shear failure:<br>EQ.0.0: Integration point fails if any damage variable reaches 1.0.<br>EQ.1.0: No failure of integration point due to in-plane shear failure, $\mathrm{dsl}(i) = 1.0$ |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | MANGLE | MSG |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1 V2 V3 | Components of vector **v** for AOPT = 3 |
| D1 D2 D3 | Components of vector **d** for AOPT = 2 |
| MANGLE | Material angle in degrees for AOPT = 0 (shells only) and AOPT = 3. MANGLE may be overridden on the element card; see *ELEMENT_SHELL_BETA and *ELEMENT_SOLID_ORTHO. |
| MSG | Flag to control the output of warning messages:<br>EQ.0: Only one warning message will be written per part.<br>GT.0: All warnings are written.<br>LT.0: No warnings are written. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GXC | GXT | GYC | GYT | GSL | GXCO | GXTO | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| GXC | Fracture toughness for longitudinal (fiber) compressive failure mode.<br>GT.0.0: The given value is regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = -GXC. If referring to a load curve, the load curve gives the fracture toughness for fiber compressive failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber compressive failure mode as a function of characteristic element length. In either case, no further regularization occurs. |
| GXT | Fracture toughness for longitudinal (fiber) tensile failure mode:<br>GT.0.0: The given value is regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = -GXT. If referring to a load curve, the load curve gives the fracture toughness for fiber tensile failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber tensile failure mode as a function of characteristic element length. In either case, no further regularization occurs. |
| GYC | Fracture toughness for transverse compressive failure mode.<br>GT.0.0: The given value is regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = -GYC. If referring to a load curve, the load curve gives the fracture toughness for transverse compressive failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for transverse compressive failure mode as a function of characteristic element length. In either case, no further regularization occurs. |
| GYT | Fracture toughness for transverse tensile failure mode.<br>GT.0.0: The given value is regularized with the characteristic element length. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: Load curve or table ID = -GYT. If referring to a load curve,t eh load curve defines the fracture toughness for transverse tensile failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for transverse tensile failure mode as a function of characteristic element length. In either case, no further regularization occurs. |
| GSL | Fracture toughness for in-plane shear failure mode.<br>GT.0.0: The given value is regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = -GSL. If referring to a load curve, the load curve gives the fracture toughness for in-plane shear failure mode as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for in-plane shear failure mode as a function of characteristic element length. In either case, no further regularization occurs. |
| GXCO | Fracture toughness for longitudinal (fiber) compressive failure mode to define bilinear damage evolution.<br>GT.0.0: The given value is regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = -GXCO. If referring to a load curve, the load curve gives the fracture toughness for fiber compressive failure mode to define bilinear damage evolution as a function of characteristic element length. If referring to a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber compressive failure mode to define bilinear damage evolution as a function of characteristic element length. In either case, no further regularization occurs. |
| GXTO | Fracture toughness for longitudinal (fiber) tensile failure mode to define bilinear damage evolution.<br>GT.0.0: The given value is regularized with the characteristic element length.<br>LT.0.0: Load curve or table ID = -GXTO. If referring to a load curve, the load curve defines the fracture toughness for |

| VARIABLE | DESCRIPTION |
|---|---|
| | fiber tensile failure mode to define bilinear damage evolution as a function of characteristic element length. If a table, each strain rate value indexes a load curve that defines the fracture toughness for fiber tensile failure mode to define bilinear damage evolution as a function of characteristic element length. In either case, no further regularization occurs. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XC | XT | YC | YT | SL | XC0 | XT0 | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XC | Longitudinal compressive strength, *a*-axis (positive value):<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XC) which defines the longitudinal compressive strength as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$) |
| XT | Longitudinal tensile strength, *a*-axis:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XT) which defines the longitudinal tensile strength as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$) |
| YC | Transverse compressive strength, *b*-axis (positive value):<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-YC) which defines the transverse compressive strength as a function of transverse strain rate ($\dot{\varepsilon}_{bb}$) |
| YT | Transverse tensile strength, *b*-axis:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-YT) which defines the transverse |

| VARIABLE | DESCRIPTION |
|---|---|
| | tensile strength as a function of transverse strain rate ($\dot{\varepsilon}_{bb}$) |
| SL | Shear strength, *ab* plane:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SL) which defines the longitudinal shear strength as a function of in-plane shear strain rate ($\dot{\varepsilon}_{ab}$) |
| XCO | Longitudinal compressive strength at inflection point (positive value):<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XCO) which defines the longitudinal compressive strength at inflection point as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$). |
| XTO | Longitudinal tensile strength at inflection point:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-XTO) which defines the longitudinal tensile strength at inflection point as a function of longitudinal strain rate ($\dot{\varepsilon}_{aa}$) |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FIO | SIGY | ETAN | BETA | PFL | PUCK | SOFT | DT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| FIO | Fracture angle in pure transverse compression (in degrees, default = 53.0) |
| SIGY | In-plane shear yield stress:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-SIGY) which defines the in-plane shear yield stress as a function of in-plane shear strain rate ($\dot{\varepsilon}_{ab}$) |

| VARIABLE | DESCRIPTION |
|---|---|
| ETAN | Tangent modulus for in-plane shear plasticity:<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = (-ETAN) which defines the tangent modulus for in-plane shear plasticity as a function of in-plane shear strain rate ($\dot{\varepsilon}_{ab}$) |
| BETA | Hardening parameter for in-plane shear plasticity (0.0 ≤ BETA ≤ 1.0):<br>EQ.0.0: Pure kinematic hardening<br>EQ.1.0: Pure isotropic hardening<br>0.0 < BETA < 1.0: Mixed hardening |
| PFL | Percentage of layers which must fail before crashfront is initiated. For example, if \|PFL\| = 80.0, then 80% of the layers must fail before strengths are reduced in neighboring elements. By default, all layers must fail. A single layer fails if 1 in-plane IP fails (PFL > 0) or if 4 in-plane IPs fail (PFL < 0). |
| PUCK | Flag for evaluation and post-processing of Puck's inter-fiber-failure criterion (IFF, see Puck, Kopp and Knops [2002]).<br>EQ.0.0: No evaluation of Puck's IFF-criterion<br>EQ.1.0: Puck's IFF-criterion will be evaluated. |
| SOFT | Softening reduction factor for material strength in crashfront elements (default = 1.0). If SOFTC is defined as well, SOFTC is used to reduce the longitudinal compressive strength XC. |
| DT | Strain rate averaging option:<br>EQ.0.0: Strain rate is evaluated using a running average.<br>LT.0.0: Strain rate is evaluated using average of last 11 time steps.<br>GT.0.0: Strain rate is averaged over the last DT time units. |

**Optional Transverse Shear Failure Card.** This card is optional.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSF23 | EPSR23 | TSMD23 | EPSF31 | EPSR31 | TSMD31 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EPSF23 | Damage initiation transverse shear strain (23-plane) |
| EPSR23 | Final rupture transverse shear strain (23-plane) |
| TSMD23 | Transverse shear maximum damage; default = 0.90 (23-plane). |
| EPSF31 | Damage initiation transverse shear strain (31-plane) |
| EPSR31 | Final rupture transverse shear strain (31-plane) |
| TSMD31 | Transverse shear maximum damage; default = 0.90 (31-plane). |

**Optional Card.** This card is optional. It only applies to shell elements.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EF_11T | EF_11C | EF_22T | EF_22C | EF_12 | EF_23 | EF_31 | LCSS |
| Type | F | F | F | F | F | | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EF_11T | Tensile failure strain in longitudinal *a*-direction |
| EF_11C | Compressive failure strain in longitudinal *a*-direction |
| EF_22T | Tensile failure strain in transverse *b*-direction |
| EF_22C | Compressive failure strain in transverse *b*-direction |
| EF_12 | In-plane shear failure strain in *ab*-plane |
| EF_23 | Out-of-plane shear failure strain in *bc*-plane |
| EF_31 | Out-of-plane shear failure strain in *ca*-plane |

| VARIABLE | DESCRIPTION |
|---|---|
| LCSS | Load curve ID or table ID. If this is defined, SIGY and ETAN will be ignored.<br>**Load Curve.** When LCSS is a load curve ID, it defines the nonlinear in-plane shear stress as a function of in-plane shear strain ($\gamma_{ab}$).<br>**Tabular Data.** The table maps in-plane strain rate values ($\dot{\gamma}_{ab}$) to a load curve giving the in-plane shear stress as a function of in-plane shear strain. For strain rates below the minimum value, the curve for the lowest defined value of strain rate is used. Likewise, when the strain rate exceeds the maximum value, the curve for the highest defined value of strain rate is used.<br>**Logarithmically Defined Table.** An alternative way to invoke logarithmic interpolation between discrete strain rates is described as follows. If the *first* value in the table is negative, LS-DYNA assumes that all the table values represent the natural logarithm of a strain rate. Since the tables are internally discretized to equally space the table values, it makes good sense from an accuracy standpoint that the table values represent the natural log of strain rate when the lowest strain rate and highest strain rate differ by several orders of magnitude. There is some additional computational cost associated with invoking logarithmic interpolation. |

**Optional Card.** This card is optional. It only applies to shell elements.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CF12 | CF13 | CF23 | SOFTC | | | | |
| Type | F | F | F | F | | | | |
| Default | 1.0 | 1.0 | 1.0 | 1.0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CF12 | Coupling factor for in-plane shear (*ab*-plane) damage with the fiber damage in tension:<br>$$d_6 = 1 - [1 - d_6^*(r_{2+})](1 - d_{1+}\text{CF12})$$<br>Here, $d_6$ is the in-plane shear damage, $d_6^*$ is an intermediate damage variable needed for finding the in-plane shear damage that is a function of $r_{2+}$, $r_{2+}$ is internal value of the constitutive law representing an elastic domain threshold, and $d_{1+}$ is the fiber damage in |

| VARIABLE | DESCRIPTION |
|---|---|
| | tension. See Maimí, Camanho, Mayugo, and Dávila [2007] for more details. |
| CF13 | Scaling factor on the fiber damage that is used when determining the reduced transverse shear (*ca*-plane) resulting from the fiber damage: $c_{66} = (1 - d_1 \text{CF13}) G_{ca}$ Here, $c_{66}$ is reduced transverse shear modulus in the *ca*-plane, $G_{ca}$ is the *ca* shear modulus, and $d_1$ is the fiber damage. See Maimí, Camanho, Mayugo, and Dávila [2007] for more details. |
| CF23 | Scaling factor on the in-plane shear damage that is used when determining the reduced transverse shear (*bc*-plane) resulting from the in-plane shear damage: $c_{55} = (1 - d_6 \text{CF23}) G_{bc}$ Here, $c_{55}$ is the reduced transverse shear in the *bc*-plane, $G_{bc}$ is the *bc* shear modulus, and $d_6$ is the in-plane shear damage. See Maimí, Camanho, Mayugo, and Dávila [2007] for more details. |
| SOFTC | Softening reduction factor for XC material strength in crashfront elements. If this is not defined, XC reduces according to SOFT. |

**Remarks:**

The failure surface to limit the elastic domain is assembled by *four* sub-surfaces, representing different failure mechanisms. They are defined as follows:

1. longitudinal (fiber) tension,

$$\phi_{1+} = \frac{\sigma_{11} - v_{12}\sigma_{22}}{X_T} = 1$$

2. longitudinal (fiber) compression – (transformation to fracture plane),

$$\phi_{1-} = \frac{\langle |\sigma_{12}^m| + \mu_L \sigma_{22}^m \rangle}{S_L} = 1$$

with

$$\mu_L = -\frac{S_L \cos(2\phi_0)}{Y_C \cos^2(\phi_0)}$$
$$\sigma_{22}^m = \sigma_{11}\sin^2(\varphi^c) + \sigma_{22}\cos^2(\varphi^c) - 2|\sigma_{12}|\sin(\varphi^c)\cos(\varphi^c)$$
$$\sigma_{12}^m = (\sigma_{22} - \sigma_{11})\sin(\varphi^c)\cos(\varphi^c) + |\sigma_{12}|\left(\cos^2(\varphi^c) - \sin^2(\varphi^c)\right)$$

and

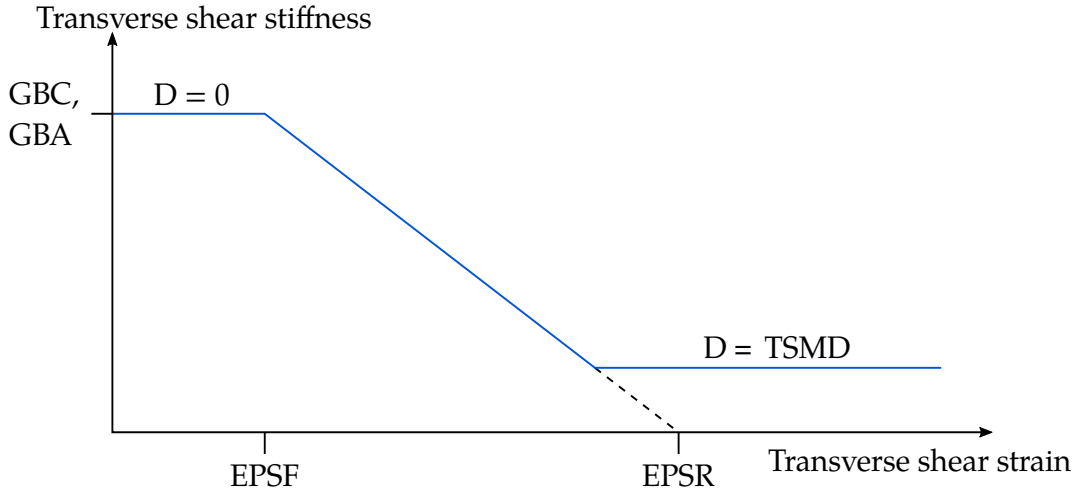



**Figure M262-1.** Linear Damage for Transverse Shear Behavior

$$\varphi^c = \arctan\left[\frac{1-\sqrt{1-4\left(\frac{S_L}{X_C}+\mu_L\right)\frac{S_L}{X_C}}}{2\left(\frac{S_L}{X_C}+\mu_L\right)}\right]$$

3. transverse (matrix) failure: perpendicular to the laminate mid-plane,

$$\phi_{2+} = \begin{cases} \sqrt{(1-g)\frac{\sigma_{22}}{Y_T}+g\left(\frac{\sigma_{22}}{Y_T}\right)^2+\left(\frac{\sigma_{12}}{S_L}\right)^2} = 1 & \sigma_{22} \geq 0 \\ \frac{\langle|\sigma_{12}|+\mu_L\sigma_{22}\rangle}{S_L} = 1 & \sigma_{22} < 0 \end{cases}$$

4. transverse (matrix) failure: transverse compression/shear,

$$\phi_{2-} = \sqrt{\left(\frac{\tau_T}{S_T}\right)^2+\left(\frac{\tau_L}{S_L}\right)^2} = 1 \quad if \quad \sigma_{22} < 0$$

with

$$\mu_T = -\frac{1}{\tan(2\phi_0)}$$

$$S_T = Y_C\cos(\phi_0)\left[\sin(\phi_0)+\frac{\cos(\phi_0)}{\tan(2\phi_0)}\right]$$

$$\theta = \arctan\left(\frac{-|\sigma_{12}|}{\sigma_{22}\sin(\phi_0)}\right)$$

$$\tau_T = \langle -\sigma_{22}\cos(\phi_0)\left[\sin(\phi_0)-\mu_T\cos(\phi_0)\cos(\theta)\right]\rangle$$

$$\tau_L = \langle \cos(\phi_0)\left[|\sigma_{12}|+\mu_L\sigma_{22}\cos(\phi_0)\sin(\theta)\right]\rangle$$

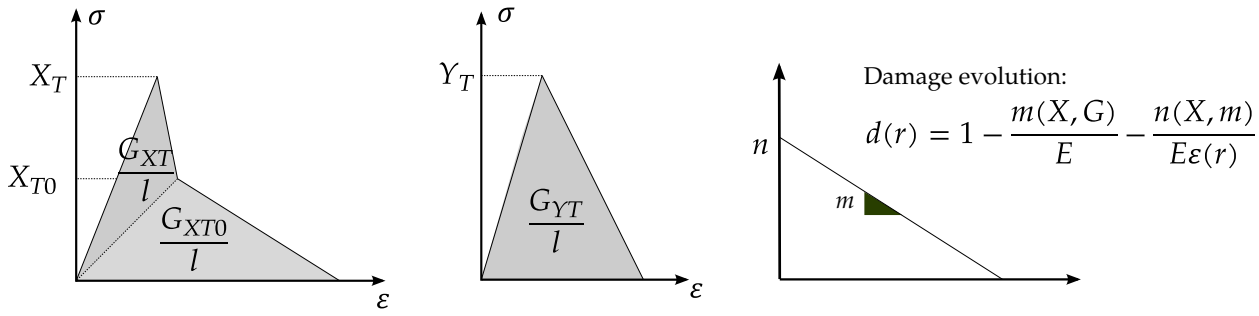



**Figure M262-2.** Damage evolution law

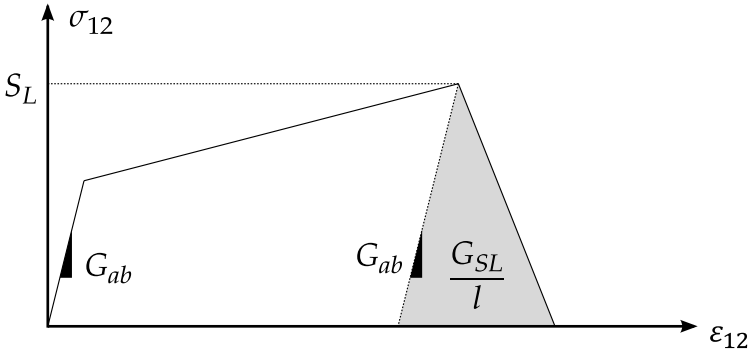



**Figure M262-3.** In-plane shear behavior

So long as the stress state is located within the failure surface the model behaves orthotropic elastic. The constitutive law is derived on basis of a proper definition for the ply complementary free energy density $G$, whose second derivative with respect to the stress tensor leads to the compliance tensor $\mathbf{H}$

$$\mathbf{H} = \frac{\partial^2 G}{\partial \sigma^2} = \begin{bmatrix} \frac{1}{(1-d_1)E_1} & -\frac{v_{21}}{E_2} & 0 \\ -\frac{v_{12}}{E_1} & \frac{1}{(1-d_2)E_2} & 0 \\ 0 & 0 & \frac{1}{(1-d_6)G_{12}} \end{bmatrix},$$

$$d_1 = d_{1+}\frac{\langle \sigma_{11} \rangle}{|\sigma_{11}|} + d_{1-}\frac{\langle -\sigma_{11} \rangle}{|\sigma_{11}|}$$

$$d_2 = d_{2+}\frac{\langle \sigma_{22} \rangle}{|\sigma_{22}|} + d_{2-}\frac{\langle -\sigma_{22} \rangle}{|\sigma_{22}|}$$

Once the stress state reaches the failure criterion, a set of scalar damage variables ($d_{1-}$, $d_{1+}$, $d_{2-}$, $d_{2+}$, $d_6$) is introduced associated with the different failure mechanisms. A bilinear (longitudinal direction) and a linear (transverse direction) damage evolution law is used to define the development of the damage variables driven by the fracture toughness and a characteristic internal element length to account for objectivity. See Figure M262-2.

To account for the characteristic non-linear in-plane shear behavior of laminated fiber-reinforced composites a 1D elasto-plastic formulation with linear hardening is coupled to a linear damage behavior once the maximum allowable stress state for shear failure is reached. See Figure M262-3.

More detailed information about this material model can be found in Maimí, Camanho, Mayugo and Dávila [2007].

When failure has occurred in all the composite layers (through-thickness integration points), the element is deleted. Elements which share nodes with the deleted element become "crashfront" elements and can have their strengths reduced by using the SOFT parameter. An earlier initiation of crashfront elements is possible by using the parameter PFL.

The number of additional integration point variables written to the LS-DYNA database is input by the *DATABASE_EXTENT_BINARY definition with the variable NEIPS (shells) and NEIPH (solids). These additional variables are tabulated below ($i$ = integration point).

When intending to initialize the stress state using *INTIAL_STRESS_*OPTION*, the stress values SIGXX, SIGYY, etc. in *INITIAL_STRESS_*OPTION* are not used, rather stresses are determined from the total strain history variables 31 to 36.

| History Variable # | Description | Value |
|---|---|---|
| 1 | Fiber tensile mode, $\phi_{1+}(i)$ | $0 \rightarrow 1$: elastic<br>1: failure criterion reached |
| 2 | Fiber compressive mode, $\phi_{1-}(i)$ | $0 \rightarrow 1$: elastic<br>1: failure criterion reached |
| 3 | Tensile matrix mode, $\phi_{2+}(i)$ | $0 \rightarrow 1$: elastic<br>1: failure criterion reached |
| 4 | Compressive matrix mode, $\phi_{2-}(i)$ | $0 \rightarrow 1$: elastic<br>1: failure criterion reached |
| 5 | Fiber damage in tension, $d_{1+}(i)$ | 0: elastic<br>1: fully damaged |
| 6 | Fiber damage in compression, $d_{1-}(i)$ | 0: elastic<br>1: fully damaged |
| 7 | Transverse damage, $d_2(i)$ | 0: elastic<br>1: fully damaged |
| 8 | In-plane shear damage, $d_6(i)$ | 0: elastic<br>1: fully damaged |

| History Variable # | Description | Value |
|---|---|---|
| 9 | Crashfront | -1: element intact<br>$10^{-8}$: element in crashfront<br>+1: element failed |
| 10 | Tensile matrix mode (Puck criteria) | $0 \rightarrow 1$: elastic<br>1: failure criterion reached |
| 11 | Compressive matrix mode (Puck criteria) | $0 \rightarrow 1$: elastic<br>1: failure criterion reached |
| 12 | Angle of fracture plane in radians (Puck criteria) | |
| 16 | Longitudinal damage, $d_1(i)$ | 0: elastic<br>1: fully damaged |
| 17 | Transverse damage in tension, $d_{2+}(i)$ | 0: elastic<br>1: fully damaged |
| 18 | Transverse damage in compression, $d_{2-}(i)$ | 0: elastic<br>1: fully damaged |
| 31 | Total strain in material 11-direction | |
| 32 | Total strain in material 22-direction | |
| 33 | Total strain in material 33-direction | |
| 34 | Total strain in material 12-direction | |
| 35 | Total strain in material 23-direction | |
| 36 | Total strain in material 31-direction | |
| 55 | Averaged strain rate in longitudinal direction | |
| 56 | Averaged strain rate in transverse direction | |
| 57 | Averaged engineering shear strain rate in in-plane direction | |

## *MAT_LOU-YOON_ANISOTROPIC_PLASTICITY

This is Material Type 263. It is based on the anisotropic yield function proposed by Lou and Yoon (Lou and Yoon, 2017). This yield function extends the original Drucker function into an anisotropic form using a fourth order linear transformation tensor. The non-associated flow rule (non-AFR) can be applied to accurately describe both the directional yield stresses and *R*-values. The anisotropic flexibility of this model can be further improved by summing up components of the anisotropic Drucker function. See the section Constitutive relations below for more details. This model is supported for shell and solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | HR | P1 | P2 | ITER |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AFR | NFUNC | AOPT | | LCID | E0 | LCF | P3 |
| Type | I | I | F | | I | F | I | |
| Default | none | 1 | none | | none | none | none | none |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | | | A1 | A2 | A3 | | |
| Type | | | | F | F | F | | |
| Default | | | | none | none | none | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | CC | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 | PCC | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

Optional card that only needs to be included if LCF < 0.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VF1 | VF2 | VF3 | VF4 | VF5 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| HR | Hardening rules (see section Hardening laws below):<br>EQ.1.0: Linear hardening (default)<br>EQ.2.0: Exponential hardening (Swift)<br>EQ.3.0: Load curve<br>EQ.4.0: Exponential hardening (Voce)<br>EQ.5.0: Exponential hardening (Gosh)<br>EQ.6.0: Exponential hardening (Hocken-Sherby) |
| P1 | Material parameter:<br>HR.EQ.1.0: Tangent modulus<br>HR.EQ.2.0: $q$, coefficient for exponential hardening law (Swift)<br>HR.EQ.4.0: $a$, coefficient for exponential hardening law (Voce)<br>HR.EQ.5.0: $q$, coefficient for exponential hardening law (Gosh)<br>HR.EQ.6.0: $a$, coefficient for exponential hardening law (Hocket-Sherby) |
| P2 | Material parameter:<br>HR.EQ.1.0: Yield stress for the linear hardening law<br>HR.EQ.2.0: $n$, coefficient for (Swift) exponential hardening<br>HR.EQ.4.0: $c$, coefficient for exponential hardening law (Voce)<br>HR.EQ.5.0: $n$, coefficient for exponential hardening law (Gosh)<br>HR.EQ.6.0: $c$, coefficient for exponential hardening law (Hocket-Sherby) |
| ITER | Iteration flag for speed:<br>EQ.0.0: Fully iterative<br>EQ.1.0: Fixed at three iterations. Generally, ITER = 0.0 is recommended. However, ITER = 1.0 is faster and may give acceptable results in most problems. |

| VARIABLE | DESCRIPTION |
|---|---|
| AFR | Flag to use associated flow rule (AFR):<br>EQ.0: Use non-AFR.<br>EQ.1: Use AFR. |
| NFUNC | Number of Drucker function components. Currently NFUNC is always set to 1. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for more complete description).<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes. The shells only the material axes are rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined **a** and **d** defined below, as with *DEFINED_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. The material directions are determined as follows: **a** is the cross product of **v** with the normal vector, **b** is the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM, or *DEFINE_COORDINATE_VECTOR). |
| LCID | Load curve ID giving the hardening law for HR = 3 |
| E0 | Material parameter:<br>HR.EQ.2.0: $\varepsilon_0$, initial yield strain for exponential hardening law (Swift) (default = 0.0)<br>HR.EQ.4.0: $b$, coefficient for exponential hardening (Voce) |

| VARIABLE | DESCRIPTION |
|---|---|
| | HR.EQ.5.0: $\varepsilon_0$, initial yield strain for exponential hardening (Gosh), Default = 0.0<br>HR.EQ.6.0: $b$, coefficient for exponential hardening law (Hocket-Sherby) |
| LCF | Fracture curve:<br>EQ.0: No fracture curves (default)<br>GT.0: Load curve or table ID of customized fracture curve/surface. If referring to a load curve ID, the fracture curve is defined as effective plastic strain as a function of triaxiality. If referring to a table ID, for each load parameter, an effective plastic strain as a function of. triaxiality curve can be defined (only applicable to solids<br>EQ.-1: Drucker ductile fracture criterion. Optional Card 7 is needed in this case. VF1, VF2 and VF3 in Card 7 will be used as $a$, $b$ and $c$ in the Drucker ductile fracture criterion. See section Fracture criteria for more details.<br>EQ.-2: DF2016 fracture criterion. Optional card 7 is needed in this case. VF1, VF2, VF3, VF4 and VF5 in Card 7 will be used as C1, C2, C3 and C in DF2016 criterion. See section Fracture criteria for more details. |
| P3 | Material parameter:<br>HR.EQ.5.0: $p$, coefficient for exponential hardening (Gosh)<br>HR.EQ.6.0: $n$, exponent for exponential hardening law (Hocket-Sherby) |
| A1, A2, A3 | Components of vector **a** for AOPT = 2.0 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3.0 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2.0 |
| C$i$ | Anisotropic parameters $c'_1$ through $c'_6$ that defines the fourth order linear transformation tensor $L'$ |
| CC | Material constant $c$ in Drucker yield function. $c$ is recommended to be 1.226 for BCC metals and 2 for FCC metals. |
| PC$i$ | Anisotropic parameters $\hat{c}_1$ through $\hat{c}_6$ defining the fourth-order linear transformation tensor $\hat{L}$ for the plastic potential in the non-AFR |

| VARIABLE | DESCRIPTION |
|---|---|
| | case (see field AFR, which is input on Card 2). |
| PCC | Material constant $\hat{c}$ in Drucker function for the plastic potential. $\hat{c}$ is recommended to be 1.226 for BCC metals and 2 for FCC metals unless calibrated otherwise. |
| VF*i* | Components of the fracture criterion included for LCF < 0. See LCF (input on Card 2) for a description. |

**Hardening laws:**

The implemented hardening laws are the following:

1. The Swift hardening law
2. The Voce hardening law
3. The Gosh hardening law
4. The Hocket-Sherby hardening law
5. A loading curve, where the yield stress is given as a function of the effective plastic strain

The Swift hardening law can be written as:

$$\sigma_y(\varepsilon_{\text{ep}}) = q(\varepsilon_0 + \varepsilon_{\text{ep}})^n \ ,$$

where $q$ and $n$ are material parameters.

The Voce equation says that the yield stress can be written in the following form:

$$\sigma_y(\varepsilon_{\text{ep}}) = a - be^{-c\varepsilon_{\text{ep}}} \ ,$$

where $a, b$and $c$ are material parameters. The Gosh equation is similar to the Swift equation. They only differ by a constant

$$\sigma_y(\varepsilon_{\text{ep}}) = q(\varepsilon_0 + \varepsilon_{\text{ep}})^n - p \ ,$$

where $q$, $\varepsilon_0$, $n$ and $p$ are material constants. The Hocket-Sherby equation resemblance the Voce equation, but with an additional parameter added

$$\sigma_y(\varepsilon_{\text{ep}}) = a - be^{-c\varepsilon_{\text{ep}}^n} \ .$$

where $a$, $b$, $c$ and $n$ are material parameters.

**Constitutive relations:**

Drucker proposed a yield function that includes the effect of the third stress invariant in the classic Von Mises yield function. Lou and Yoon (2017) extended this yield function to an anisotropic form as shown below:

$$\overline{\sigma}_y(\sigma_{ij}) = (J_2'^3 - cJ_3'^2)^{1/6} \ .$$

Here $J_2'$ and $J_3'$ are the second and third invariants of the linear transformed deviatoric stress tensor $\mathbf{s}'$:

$$\mathbf{s}' = \mathbf{L}'\boldsymbol{\sigma} \ .$$

The fourth order linear transformation tensor $\mathbf{L}'$ is given by:

$$\mathbf{L}' = \begin{bmatrix} (c_2' + c_3')/3 & -c_3'/3 & -c_2'/3 & 0 & 0 & 0 \\ -c_3'/3 & (c_1' + c_3')/3 & -c_1'/3 & 0 & 0 & 0 \\ -c_2'/3 & -c_1'/3 & (c_2' + c_1')/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_4' & 0 & 0 \\ 0 & 0 & 0 & 0 & c_5' & 0 \\ 0 & 0 & 0 & 0 & 0 & c_6' \end{bmatrix} .$$

$c_1'$, $c_2'$, $c_3'$ and $c_6'$ can be calibrated from uniaxial tensile yield stress along different directions and the balanced biaxial yield stress. $c_4'$ and $c_5'$, which are related to the through-thickness material properties, are very difficult to obtain experimentally and therefore assumed to be identical with $c_6'$.

With the non-associated flow rule (non-AFR), the plastic flow is not necessarily aligned with the yield surface normal and the *R*-values are modeled by a different plastic potential:

$$\overline{\sigma}_p(\sigma_{ij}) = (\hat{J}_2^3 - c\hat{J}_3^2)^{1/6} \ .$$

Here $\hat{J}_2$ and $\hat{J}_3$ are the second and third invariants of the linear transformed deviatoric stress tensor:

$$\hat{\mathbf{s}} = \hat{\mathbf{L}}\boldsymbol{\sigma} \ .$$

And $\hat{\boldsymbol{L}}$ is defined as:

$$\hat{\mathbf{L}} = \begin{bmatrix} (\hat{c}_2 + \hat{c}_3)/3 & -\hat{c}_3/3 & -\hat{c}_2/3 & 0 & 0 & 0 \\ -\hat{c}_3/3 & (\hat{c}_1 + \hat{c}_3)/3 & -\hat{c}_1/3 & 0 & 0 & 0 \\ -\hat{c}_2/3 & -\hat{c}_1/3 & (\hat{c}_2 + \hat{c}_1)/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{c}_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \hat{c}_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{c}_6 \end{bmatrix} .$$

The anisotropic parameters $\hat{c}_1$, $\hat{c}_2$, $\hat{c}_3$ and $\hat{c}_6$ can be calibrated with experimentally measured *R*-values along different directions.

Another approach to improve the flexibility of the yield function is to sum up $n$ components of the anisotropic Drucker functions as follows:

$$\overline{\sigma}_y(\sigma_{ij}) = \frac{1}{n}\sum_{m=1}^{n}\{[(J_2'^{(m)})^3 - c(J_3'^{(m)})^2]^{1/6}\} \ .$$

where $n$ is an integer with $n \geq 1$. The same idea can be applied to the plastic potential in the non-AFR approach, as shown in equation:

$$\overline{\sigma}_p(\sigma_{ij}) = \frac{1}{n}\sum_{m=1}^{n}\{[(\hat{J}_2^{(m)})^3 - c(\hat{J}_3^{(m)})^2]^{1/6}\} \ .$$

**Fracture criteria:**

The Drucker ductile fracture criterion is given by:

$$\overline{\sigma}_f(\sigma_{ij}) = \mathrm{a}\left(bI_1 + \left(J_2^3 - cJ_3^2\right)^{1/6}\right) = 1$$

The DF2016 fracture criterion is given by:

$$\left(\frac{\sigma_1 - \sigma_3}{\overline{\sigma}_{VM}}\right)^{C_1}\left(\left\langle\frac{f(\eta, L, C)}{f(1/3, -1, C)}\right\rangle\right)^{C_2}\bar{\varepsilon}_f^p = C_3$$

Here

$$\langle x\rangle = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

and

$$f(\eta, L, C) = \eta + C_4\frac{(3-L)}{3\sqrt{L^2+3}} + C \ .$$

## *MAT_TABULATED_JOHNSON_COOK_ORTHO_PLASTICITY

This is Material Type 264. This is an orthotropic, elastic-plastic material law with a J3-dependent yield surface and options for isotropic or orthotropic failure. This material considers tensile/compressive asymmetry in the material response, which is essential for HCP metals. It is available for solid elements and thick shell element types 3, 5, and 7.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | CP | TR | BETA | NUMINT |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCT00R | LCT00T | LCF00 | LCG00 | LCH00 | LCI00 | IFTRA | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| LCC00R | LCC00T | LCS45R | LCS45T | IFLAG | SFIEPM | NITER | AOPT |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| LCT90R | LCT45R | LCTTHR | LCC90R | LCC45R | LCCTHR | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| LCT90T | LCT45T | LCTTHT | LCC90T | LCC45T | LCCTHT | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MACF | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | MANGLE | |
|---|---|---|---|---|---|---|---|

**Card 8.** Include this card if IFTRA $\neq 0$.

| LCF90 | LCG90 | LCH90 | LCI90 | LCF45AB | LCG45AB | LCH45AB | LCI45AB |
|---|---|---|---|---|---|---|---|

**Card 9.** Include this card if IFTRA $\neq 0$.

| LCFTH | LCGTH | LCHTH | LCITH | LCF45BC | LCG45BC | LCH45BC | LCI45BC |
|---|---|---|---|---|---|---|---|

**Card 10.** Include this card if IFTRA ≠ 0.

| LCF45CA | LCG45CA | LCH45CA | LCI45CA | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | CP | TR | BETA | NUMINT |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 1.0 | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| CP | Specific heat |
| TR | Room temperature |
| BETA | Fraction of plastic work converted into heat:<br>GT.0.0: Constant value is used.<br>LT.0.0: -BETA gives a load curve ID for strain rate dependence, a table ID for temperature (TABLE) and strain rate (CURVE) dependence, a 3-dimensional table ID for temperature (TABLE_3D), strain rate (TABLE) and plastic strain (CURVE) dependence, or a 4-dimensional table ID for triaxiality (TABLE_4D), temperature (TABLE_3D), strain rate (TABLE), and plastic strain (CURVE) dependence. |
| NUMINT | Number of failed integration points before element deletion. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-200: Turns off erosion for solids. Not recommended unless used with *CONSTRAINED_TIED_NODES_FAILURE. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCT00R | LCT00T | LCF00 | LCG00 | LCH00 | LCI00 | IFTRA | |
| Type | I | I | I | I | I | I | I | |
| Default | none | 0 | 0 | 0 | 0 | 0 | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCT00R | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the 0° direction. |
| LCT00T | Table ID defining for each temperature value a load curve ID giving the (quasi-static) tensile yield stress as a function of plastic strain for that temperature in the 0° direction. |
| LCF00 | Load curve or table ID giving plastic failure strain in the 0° direction. The load curve ID defines plastic failure strain as a function of triaxiality. The table ID specifies a load curve ID for each Lode parameter, giving the plastic failure strain as a function of triaxiality for that Lode parameter. See Remark 2. |
| LCG00 | Load curve ID defining plastic failure strain (or a scale factor for determining the plastic failure strain) as a function of plastic strain rate in the 0° direction. The curve should not extrapolate to zero; otherwise, failure may occur at low strain. If the first abscissa value in the curve corresponds to a negative strain rate, the natural logarithm of the strain rate value is assumed to be used for all abscissa values. See Remark 2. |
| LCH00 | Load curve ID defining plastic failure strain (or a scale factor for determining the plastic failure strain) as a function of temperature in the 0° direction. See Remark 2. |
| LCI00 | Load curve ID, table ID, or 3D table ID giving the plastic failure |

| VARIABLE | DESCRIPTION |
|---|---|
| | strain (or a scale factor for determining the plastic failure strain) in the 0° direction. The load curve gives plastic failure strain as a function of element size. The table defines a load curve ID for each triaxiality, giving the plastic failure strain as a function of element size for that triaxiality. If referring to a three-dimensional table ID, plastic failure strain can be a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2. |
| IFTRA | Failure option:<br>EQ.0.0: Isotropic failure. In this case, LCF00, LCG00, LCH00, and LCI00 define failure in any direction, and Cards 8, 9, and 10 are not defined.<br>EQ.1.0: Orthotropic failure with original formulation. The plastic strain rate tensor is expressed in the current material system (updated for material rotation).<br>EQ.2.0: Orthotropic failure with alternative formulation. The plastic strain rate tensor is expressed in the initial material system (not updated for material rotation). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCC00R | LCC00T | LCS45R | LCS45T | IFLAG | SFIEPM | NITER | AOPT |
| Type | I | I | I | I | I | I | I | F |
| Default | 0 | 0 | 0 | 0 | 0 | 1 | 100 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCC00R | Table ID. The curves in this table define compressive yield stress as a function of plastic strain. The table specifies a load curve ID for each plastic strain rate value, giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the 0° direction. |
| LCC00T | Table ID defining for each temperature value a load curve ID giving the (quasi-static) compressive yield stress as a function of strain for that temperature. The curves in this table define compressive yield stress as a function of plastic strain in the 00-degree direction. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCS45R | Table ID. The table defines a load curve ID for each plastic strain rate value, giving the (isothermal) shear yield stress as a function of plastic strain for that rate in the 45-degree direction. |
| LCS45T | Table ID. The table defines a load curve ID for each temperature value, giving the (quasi-static) shear yield stress versus strain for that temperature. The load curves define shear yield stress as a function of plastic strain or effective plastic strain (see IFLAG) in the 45-degree direction. |
| IFLAG | Flag to specify abscissas for LCT00R, LCC00R, LCS45R, LCT90R, LCT45R, LCTTHR, LCC90R, LCC45R, LCCTHR (see Remark 1):<br>EQ.0: Compressive and shear yields are given as functions of plastic strain as defined in the remarks (default).<br>EQ.1: Compressive and shear yields are given as functions of effective plastic strain. |
| SFIEPM | Scale factor on the initial estimate of the plastic multiplier |
| NITER | Maximum number of iterations for the plasticity algorithm |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, $P$, in space and the global location of the element center; this is the **a**-direction.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector |

| VARIABLE | DESCRIPTION |
|---|---|
| | with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. Either the element's input or this keyword's input (see MANGLE) sets the angle. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying the angle, depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION). |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCT90R | LCT45R | LCTTHR | LCC90R | LCC45R | LCCTHR | | |
| Type | I | I | I | I | I | I | | |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCT90R | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the 90-degree direction |
| LCT45R | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the 45-degree direction |
| LCTTHR | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) tensile yield stress as a function of plastic strain for that rate in the through-thickness direction |
| LCC90R | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the 90-degree direction |
| LCC45R | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of |

| VARIABLE | DESCRIPTION |
|---|---|
| | plastic strain for that rate in the 45-degree direction |
| LCCTHR | Table ID defining for each plastic strain rate value a load curve ID giving the (isothermal) compressive yield stress as a function of plastic strain for that rate in the through-thickness direction |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCT90T | LCT45T | LCTTHT | LCC90T | LCC45T | LCCTHT | | |
| Type | I | I | I | I | I | I | | |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCT90T | Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress as a function of plastic strain for that rate in the 90° direction |
| LCT45T | Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress as a function of plastic strain for that rate in the 45° direction |
| LCTTHT | Table ID defining for each temperature value a load curve ID giving the (quasistatic) tensile yield stress as a function of plastic strain for that rate in the through-thickness direction |
| LCC90T | Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress as a function of plastic strain for that rate in the 90° direction |
| LCC45T | Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress as a function of plastic strain for that rate in the 45° direction |
| LCCTHT | Table ID defining for each temperature value a load curve ID giving the (quasistatic) compressive yield stress as a function of plastic strain for that rate in the through-thickness direction |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA or MANGLE rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA or MANGLE rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA or MANGLE rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA or MANGLE rotation<br>EQ.3: Switch material axes *a* and *c* after BETA or MANGLE rotation<br>EQ.4: Switch material axes *b* and *c* after BETA or MANGLE rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the procedure to obtain the final material axes. If BETA on *ELEMENT_SOLID_{OPTION} is defined, then BETA is used for the rotation for all AOPT options. Otherwise, for AOPT = 3, MANGLE input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no rotation will be performed. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | MANGLE | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| MANGLE | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO. |

**IFTRA Card 1.** Additional card for orthotropic failure if IFTRA ≠ 0.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCF90 | LCG90 | LCH90 | LCI90 | LCF45AB | LCG45AB | LCH45AB | LCI45AB |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCF90 | Load curve or table ID specifying the plastic failure strain in the 90° direction. The load curve gives the plastic failure strain as a function of triaxiality. For the table, the Lode parameter indexes plastic failure strain as a function of triaxiality curves. See Remark 2. |
| LCG90 | Load curve ID giving plastic failure strain (or a scale factor for determining the plastic failure strain) as a function of plastic strain rate in the 90° direction. The curve should not extrapolate to zero; otherwise, failure may occur at low strain. If the first abscissa value in the curve corresponds to a negative strain rate, the natural logarithm of the strain rate value is assumed to be used for all abscissa values. See Remark 2. |
| LCH90 | Load curve ID defining plastic failure strain (or a scale factor for |

| VARIABLE | DESCRIPTION |
|---|---|
| | determining the plastic failure strain) in the 90° direction as a function of temperature. See Remark 2. |
| LCI90 | Load curve ID, table ID, or 3D table ID specifying the plastic failure strain (or scale factor or a scale factor for determining the plastic failure strain) in the 90° direction. The load curve gives plastic failure strain as a function of element size. For a table, the plastic failure strain depends on triaxiality (TABLE) and element size (CURVE). For a three-dimensional table, plastic failure strain depends on the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2. |
| LCF45AB | Load curve or table ID specifying plastic failure strain in the 45° direction on the rolling (A) - transverse (B) plane. The load curve gives plastic failure strain as a function of triaxiality. For the table, the plastic failure strain in this direction depends on the Lode parameter (TABLE) and triaxiality (CURVE). See Remark 2. |
| LCG45AB | Load curve ID defining plastic failure strain (or a scale factor for determining the plastic failure strain) as a function of plastic strain rate in the 45° direction on the rolling (A) – transverse (B) plane. The curve should not extrapolate to zero; otherwise, failure may occur at low strain. If the first abscissa value in the curve corresponds to a negative strain rate, the natural logarithm of the strain rate value is assumed to be used for all abscissa values. See Remark 2. |
| LCH45AB | Load curve ID defining plastic failure strain (or scale factor for determining the plastic failure strain) as a function of temperature in the 45° direction on the rolling (A) - transverse (B) plane. See Remark 2. |
| LCI45AB | Load curve ID, table ID, or 3D table ID giving the plastic failure strain (or a scale factor for determining the plastic failure strain) in the 45° direction on the rolling (A) - transverse (B) plane. The load curve gives plastic failure strain as a function of element size. For a table, the plastic failure strain depends on triaxiality (TABLE) and element size (CURVE). For a three-dimensional table, plastic failure strain depends on the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2. |

**IFTRA Card 2.** Additional Card for the orthotropic failure if IFTRA ≠ 0.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCFTH | LCGTH | LCHTH | LCITH | LCF45BC | LCG45BC | LCH45BC | LCI45BC |
| Type | I | I | I | I | I | I | I | I |
| Default | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| LCFTH | Load curve or table ID giving the plastic failure strain in the through-thickness direction. The load curve defines plastic failure strain as a function of triaxiality. For the table, the Lode parameter indexes plastic failure strain as a function of triaxiality curves. See Remark 2. |
| LCGTH | Load curve ID defining plastic failure strain (or a scale factor for determining the plastic failure strain) as a function of plastic strain rate in the through-thickness direction. The curve should not extrapolate to zero; otherwise, failure may occur at low strain. If the first abscissa value in the curve corresponds to a negative strain rate, the natural logarithm of the strain rate value is assumed to be used for all abscissa values. See Remark 2. |
| LCHTH | Load curve ID defining plastic failure strain (or a scale factor for determining the plastic failure strain) as a function of temperature in the through-thickness direction. See Remark 2. |
| LCITH | Load curve ID, table ID, or 3D table ID defining plastic failure strain (or a scale factor for determining the plastic failure strain) in the through-thickness direction. The load curve gives plastic failure strain as a function of element size. For a table, the plastic failure strains depend upon triaxiality (TABLE) and element size (CURVE). For a three-dimensional table, plastic failure strain is a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2. |
| LCF45BC | Load curve or table ID giving plastic failure strain in the 45° direction on the transverse (B) - normal (C) plane. The load curve defines plastic failure strain as a function of triaxiality. For the table, the Lode parameter indexes each plastic failure strain as a function of triaxiality curve. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCG45BC | Load curve ID defining plastic failure strain (or a scale factor for determining plastic failure strain) as a function of plastic strain rate in the 45° direction on the transverse (B) - normal (C) plane. The curve should not extrapolate to zero; otherwise, failure may occur at low strain. If the first abscissa value in the curve corresponds to a negative strain rate, the natural logarithm of the strain rate value is assumed to be used for all abscissa values. See Remark 2. |
| LCH45BC | Load curve ID defining plastic failure strain (or a scale factor for determining plastic failure strain) as a function of temperature in the 45° direction on the transverse (B) - normal (C) plane. See Remark 2. |
| LCI45BC | Load curve ID, table ID, or 3D table ID giving plastic failure strain (or a scale factor for determining plastic failure strain) in the 45° direction on the transverse (B) - normal (C) plane. The load curve gives plastic failure strain as a function of element size. For a table, the plastic failure strains depend upon triaxiality (TABLE) and element size (CURVE). For a three-dimensional table, plastic failure strain is a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2. |

**IFTRA Card 3.** Additional Card for the orthotropic failure if IFTRA ≠ 0.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCF45CA | LCG45CA | LCH45CA | LCI45CA | | | | |
| Type | I | I | I | I | | | | |
| Default | 0 | 0 | 0 | 0 | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LF45CA | Load curve or table ID defining plastic failure strain in the 45° direction on the normal (C) - rolling (A) plane. The load curve defines plastic failure strain as a function of triaxiality. For a table, the Lode parameter indexes each plastic failure strain as a function of triaxiality curve. See Remark 2. |
| LG45CA | Load curve ID defining plastic failure strain (or a scale factor for determining plastic failure strain) as a function of plastic strain rate |

| VARIABLE | DESCRIPTION |
|---|---|
| | in the 45° direction on the normal (C) - rolling (A) plane. The curve should not extrapolate to zero; otherwise, failure may occur at low strain. If the first abscissa value in the curve corresponds to a negative strain rate, the natural logarithm of the strain rate value is assumed to be used for all abscissa values. See Remark 2. |
| LH45CA | Load curve ID defining plastic failure strain (or a scale factor for determining plastic failure strain) as a function of temperature in the 45° direction on the normal (C) - rolling (A) plane. See Remark 2. |
| LI45CA | Load curve ID, table ID, or 3D table ID giving plastic failure strain (or a scale factor for determining plastic failure strain) in the 45° direction on the normal (C) - rolling (A) plane. The load curve gives plastic failure strain as a function of element size. For a table, the plastic failure strains depend upon triaxiality (TABLE) and element size (CURVE). For a three-dimensional table, plastic failure strain is a function of the Lode parameter (TABLE_3D), triaxiality (TABLE), and element size (CURVE). See Remark 2. |

**Remarks:**

1. **IFLAG.** If IFLAG = 0, the compressive and shear curves are defined as follows:

$$\sigma_{\text{comp}}\left(\varepsilon_{p,\,\text{comp}}, \dot{\varepsilon}_{p,\,\text{comp}}\right), \quad \varepsilon_{p,\text{comp}} = \varepsilon_{\text{comp}} - \frac{\sigma_{\text{comp}}}{E}, \quad \dot{\varepsilon}_{p,\,\text{comp}} = \frac{\partial \varepsilon_{p,\text{comp}}}{\partial t}$$

where comp is one of the tension (0°,45°,90°), compression (0°,45°,90°), tension and compression thickness, or shear components.

If IFLAG = 1, the compressive and shear curves are defined as follows:

$$\sigma_{\text{comp}}\left(\dot{\lambda}, \lambda\right) \quad \dot{W}_p = \sigma_{\text{eff}}\dot{\lambda}$$

2. **Scale factors on the plastic failure strain.** Multiple curves or tables that depend on different variables (such as LCF00, LCG00, LCH00, and LCI00 for the 0° direction) can be input to specify the plastic failure strain in each direction. If more than one curve or table is included for a given direction, the additional curves or tables are assumed to be scale factors on the first one input. Thus, the plastic failure strain in a given direction is determined by multiplying the values from all the provided tables/curves for that direction.

3. **History variables.** History variables may be post-processed through additional variables. NEIPH on *DATABASE_EXTENT_BINARY sets the number of

additional variables for solids written to the d3plot and d3thdt databases. The following table lists the relevant additional variables of this material model:

| History Variable # | Description |
|---|---|
| 5 | Strain rate |
| 6 | Plastic failure strain |
| 7 | Triaxiality |
| 8 | Lode parameter |
| 9 | Plastic work |
| 10 | Damage |
| 11 | Element size |
| 12 | Temperature |
| 13 | Compressive plastic strain |
| 14 | Shear plastic strain |

### *MAT_CONSTRAINED_*OPTION*

This is Material Type 265. This special model defines material data for *CONSTRAINED_SPR2 or *CONSTRAINED_INTERPOLATION_SPOTWELD (aka SPR3) instead of in the input for the constraint. This material model is not available for standard elements. See the Sample Input below.

Available options include:

SPR2

SPR3

The input depends on the option used. SPR2 requires two cards, and SPR3 needs up to four cards.

### Card Summary:

**Card 1.** Include this card if the keyword option SPR2 is used.

| MID | RO | FN | FT | DN | DT | XIN | XIT |
|---|---|---|---|---|---|---|---|

**Card 2.** Include this card if the keyword option SPR2 is used.

| ALPHA1 | ALPHA2 | ALPHA3 | EXPN | EXPT | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if the keyword option SPR3 is used.

| MID | RO | MODEL | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if the keyword option SPR3 is used.

| STIFF | RN | RS | ALPHA1 | BETA1 | LCF | LCUPF | LCUPR |
|---|---|---|---|---|---|---|---|

**Card 5a.1.** Include this card if the keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

| STIFF2 | STIFF3 | STIFF4 | LCDEXP | GAMMA | SROPT | | |
|---|---|---|---|---|---|---|---|

***Card 5a.2.*** This card is optional. It is read if the keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

| FFN | FFB | FFS | EXFC | STIFP | MFSFC | DEFC | NPFC |
|---|---|---|---|---|---|---|---|

**Card 5b.1.** Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

| UPFN | UPFS | ALPHA2 | BETA2 | UPRN | UPRS | ALPHA3 | BETA3 |
|---|---|---|---|---|---|---|---|

**Card 5b.2.** Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

| MRN | MRS | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

**SPR2 Cards.** Include this card if the SPR2 keyword option is used.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | FN | FT | DN | DT | XIN | XIT |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| FN | Rivet strength in tension (pull-out) |
| FT | Rivet strength in pure shear |
| DN | Failure displacement in normal direction |
| DT | Failure displacement in tangential direction |
| XIN | Fraction of failure displacement at maximum normal force |
| XIT | Fraction of failure displacement at maximum tangential force |

**SPR2 Cards.** Include this card if the SPR2 keyword option is used.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | EXPN | EXPT | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ALPHA1 | Dimensionless parameter scaling the effective displacement |
| ALPHA2 | Dimensionless parameter scaling the effective displacement |
| ALPHA3 | Dimensionless parameter scaling the effective displacement. The sign of ALPHA3 can be used to choose the normal update procedure:<br>GT.0: Incremental update (default)<br>LT.0: Total update (recommended) |
| EXPN | Exponent value for load function in the normal direction |
| EXPT | Exponent value for load function in the tangential direction |

**SPR3 Cards.** Include this card if the SPR3 keyword option is used.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | MODEL | | | | | |
| Type | A | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| MODEL | Material behavior and damage model (see Remarks of *CONSTRAINED_INTERPOLATION_SPOTWELD).<br>EQ.1: SPR3 (default) |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2: SPR4 |
| | EQ.11: Same as 1 with selected material parameters as functions |
| | EQ.12: Same as 2 with selected material parameters as functions |
| | EQ.21: Same as 11 with slight modification |
| | EQ.22: Same as 12 with slight modification |
| | EQ.31: Same as 11 but with 12 more material parameters as functions |
| | EQ.41: Same as 31 with slight modification |

**SPR3 cards.** Include this card if the SPR3 keyword option is used.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | STIFF | RN | RS | ALPHA1 | BETA1 | LCF | LCUPF | LCUPR |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| STIFF | Elastic stiffness. Function ID if MODEL > 10 . |
| RN | Tensile strength factor, $R_n$. |
| | GT.0.0: Constant value unless MODEL > 10. Function ID if MODEL > 10 (see Remarks section for *CONSTRAINED_INTERPOLATION_SPOTWELD). |
| | LT.0.0: Load curve with ID \|RN\| giving $R_n$ as a function of peel ratio (see Remarks section for *CONSTRAINED_INTERPOLATION_SPOTWELD) |
| RS | Shear strength factor, $R_s$. Function ID if MODEL > 10. |
| ALPHA1 | Scaling factor $\alpha_1$. Function ID if MODEL > 10. |
| BETA1 | Exponent for plastic potential $\beta_1$. Function ID if MODEL > 10. |
| LCF | Load curve or table ID. Load curve ID describing force as a function of plastic displacement, that is, $F^0(\bar{u}^{\text{pl}})$. Table ID describing force as a function of mode mixity (table values) and plastic displacement (curves), that is, $F^0(\bar{u}^{\text{pl}}, \kappa)$. |

| VARIABLE | DESCRIPTION |
|---|---|
| LCUPF | Load curve ID describing plastic initiation displacement as a function of mode mixity, that is, $\bar{u}_0^{\mathrm{pl}}(\kappa)$. Only for MODEL = 1, 11, or 21. For MODEL = 1, LCUPF can also be a table ID giving plastic initiation displacement as a function of peel ratio (table values) and mode mixity (curves). See Remarks section for *CONSTRAINED_INTERPOLATION_SPOTWELD. |
| LCUPR | Load curve ID describing plastic rupture displacement as a function of mode mixity, that is, $\bar{u}_f^{\mathrm{pl}}(\kappa)$. Only for MODEL = 1, 11, or 21. For MODEL = 1, LCUPF can also be a table ID giving plastic initiation displacement as a function of peel ratio (table values) and mode mixity (curves). See Remarks section for *CONSTRAINED_INTERPOLATION_SPOTWELD. |

**SPR3 Cards.** Include this card if the keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

| Card 5a.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | STIFF2 | STIFF3 | STIFF4 | LCDEXP | GAMMA | SROPT | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| STIFF2 | Elastic shear stiffness. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| STIFF3 | Elastic bending stiffness. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| STIFF4 | Elastic torsional stiffness. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| LCDEXP | Load curve for damage exponent as a function of mode mixity |
| GAMMA | Scaling factor, $\gamma_1$. It is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| SROPT | Shear rotation option that defines local kinematics system:<br>EQ.0: Pure shear does not create a normal component (default). |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: Pure shear creates a normal component. |

**SPR3 Cards.** This card is optional. It is read if keyword option SPR3 is used and MODEL = 1, 11, 21, 31, or 41.

| Card 5a.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FFN | FFB | FFS | EXFC | STIFP | MFSFC | DEFC | NPFC |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| FFN | Resultant normal force at failure. FFN is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| FFB | Resultant bending force at failure. FFB is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| FFS | Resultant shear force at failure. FFS is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| EXFC | Failure function exponent. EXFC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| STIFP | Plastic stiffness. If greater than zero, this replaces LCF by a simple linear hardening law: $$F^0\left(\bar{u}^{\mathrm{pl}}\right) = 1.0 + \mathrm{STIFP} \times \bar{u}^{\mathrm{pl}} .$$ STIFP is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| MFSFC | Scaling factor for torsion term in resultant shear force. MFSC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |
| DEFC | Fading energy for damage. DEFC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |

| VARIABLE | DESCRIPTION |
|---|---|
| NPFC | Plastic displacement offset for damage initiation. NPFC is a function ID if MODEL > 30. See *CONSTRAINED_INTERPOLATION_SPOTWELD for details. |

**SPR3 Cards.** Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

| Card 5b.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | UPFN | UPFS | ALPHA2 | BETA2 | UPRN | UPRS | ALPHA3 | BETA3 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| UPFN | Plastic initiation displacement in normal direction, $\bar{u}^{\mathrm{pl},n}_{0,\mathrm{ref}}$ |
| UPFS | Plastic initiation displacement in shear direction, $\bar{u}^{\mathrm{pl},s}_{0,\mathrm{ref}}$ |
| ALPHA2 | Plastic initiation displacement scaling factor, $\alpha_2$ |
| BETA2 | Exponent for plastic initiation displacement, $\beta_2$ |
| UPRN | Plastic rupture displacement in normal direction, $\bar{u}^{\mathrm{pl},n}_{f,\mathrm{ref}}$ |
| UPRS | Plastic rupture displacement in shear direction, $\bar{u}^{\mathrm{pl},s}_{f,\mathrm{ref}}$ |
| ALPHA3 | Plastic rupture displacement scaling factor, $\alpha_3$ |
| BETA3 | Exponent for plastic rupture displacement, $\beta_3$ |

**SPR3 Cards.** Include this card if the keyword option SPR3 is used and MODEL = 2, 12, or 22.

| Card 5b.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MRN | MRS | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MRN | Proportionality factor for dependency, $m_{R_n}$ |
| MRS | Proportionality factor for dependency, $m_{R_s}$ |

### Sample Input:

With this material model it is possible to replace the following input example

```
*CONSTRAINED_SPR2
$     UPID      LPID      NSID     THICK         D        FN        FT        DN
         5         8       123       5.0       8.0      2.53       4.8       4.0
$       DT       XIN       XIT    ALPHA1    ALPHA2    ALPHA3      DENS      INTP
       7.5       0.6       0.5       0.2       0.7       1.9    7.8e-6         1
$     EXPN      EXPT     PIDVB
       8.0       8.0       999
$    XPID1     XPID2     XPID3     XPID4
         20
```

with this "split" one

```
*CONSTRAINED_SPR2
$     UPID      LPID      NSID     THICK         D        FN        FT        DN
         5         8       123       5.0       8.0      -555
$       DT       XIN       XIT    ALPHA1    ALPHA2    ALPHA3      DENS      INTP
                                                                           1
$     EXPN      EXPT     PIDVB
                           999
$    XPID1     XPID2     XPID3     XPID4
         20
*MAT_CONSTRAINED_SPR2
$      MID        RO        FN        FT        DN        DT       XIN       XIT
       555    7.8e-6      2.53       4.8       4.0       7.5       0.6       0.5
$   ALPHA1    ALPHA2    ALPHA3      EXPN      EXPT
       0.2       0.7       1.9       8.0       8.0
```

and still get the same result. Note that only the non-material data (UPID, LPID, NSID, THICK, D, INTP, PIDVB, XPID*i*) remains with the *CONSTRAINED keyword. Variables in grey are optional.

## *MAT_TISSUE_DISPERSED

This is Material Type 266. This material is an invariant formulation for dispersed orthotropy in soft tissues, e.g., heart valves, arterial walls or other tissues where one or two collagen fibers are used. The passive contribution is composed of an isotropic and two anisotropic parts. The isotropic part is a simple neo-Hookean model. The first anisotropic part is passive, with two collagen fibers to choose from: (1) a simple exponential model and (2) a more advanced crimped fiber model from Freed et al. [2005]. The second anisotropic part is active described in Guccione et al. [1993] and is used for active contraction.

> NOTE: This material model is obsolete. Please use MAT_ANISOTROPIC_HYPERELASTIC which contains most of the features of MAT_TISSUE_DISPERSED. For missing or additional features, please consult LST directly.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | F | SIGMA | MU | KAPPA | ACT | INIT |
| Type | A | F | F | F | F | F | I | I |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FID | ORTH | C1 | C2 | C3 | THETA | NHMOD | |
| Type | I | I | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ACT1 | ACT2 | ACT3 | ACT4 | ACT5 | ACT6 | ACT7 | ACT8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ACT9 | ACT10 | | | | | | |
| Type | F | F | | | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | BETA | XP | YP | ZP | A1 | A2 | A3 |
| Type | I | F | F | F | F | F | F | F |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. |
| F | Fiber dispersion parameter governs the extent to which the fiber dispersion extends to the third dimension. F = 0 and F = 1 apply to 2D splay with the normal to the membrane being in the $\beta$ and the $\gamma$-directions, respectively (see Figure M266-1). F = 0.5 applies to 3D splay with transverse isotropy. Splay will be orthotropic whenever$F \neq 0.5$. This parameter is ignored if INIT = 1. |
| SIGMA | The parameter SIGMA governs the extent of dispersion, such that as SIGMA goes to zero, the material symmetry reduces to pure transverse isotropy. Conversely, as SIGMA becomes large, the material symmetry becomes isotropic in the plane. This parameter is ignored if INIT = 1. |
| MU | MU is the isotropic shear modulus that models elastin. MU should be chosen such that the following relation is satisfied: |

| VARIABLE | DESCRIPTION |
|---|---|
| | $0.5\,(3\text{KAPPA} - 2\text{MU})/(3\text{KAPPA} + \text{MU}) < 0.5.$<br>Instability can occur for implicit simulations if this quotient is close to 0.5. A modest approach is a quotient between 0.495 and 0.497. |
| KAPPA | Bulk modulus for the hydrostatic pressure. |
| ACT | ACT = 1 indicates that an active model will be used that acts in the mean fiber-direction. The active model, like the passive model, will be dispersed by SIGMA and F, or if INIT = 1, with the *INITIAL_FIELD_SOLID keyword. |
| INIT | INIT = 1 indicates that the anisotropy eigenvalues will be given by *INITIAL_FIELD_SOLID variables in the global coordinate system (see Remark 1). |
| FID | The passive fiber model number. There are two passive models available: FID = 1 or FID = 2. They are described in Remark 2. |
| ORTH | ORTH specifies the number (1 or 2) of fibers used. When ORTH = 2 two fiber families are used and arranges symmetrically THETA degrees from the mean fiber direction and lying in the tissue plane. |
| C1-C3 | Passive fiber model parameters. |
| THETA | The angle between the mean fiber direction and the fiber families. The parameter is active only if ORTH = 2 and is particularly important in vascular tissues (e.g. arteries) |
| NHMOD | Neo-Hooke model flag<br>EQ.0.0: Original implementation (modified Neo-Hooke)<br>EQ.1.0: Standard Neo-Hooke model (as in umat45 of dyn21.f) |
| ACT1 - ACT10 | Active fiber model parameters. Note that ACT10 is an input for a time dependent load curve that overrides some of the ACTx values. See section 2 below. |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element |

| VARIABLE | DESCRIPTION |
|---|---|
| | center; this is the *a*-direction.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes determined by rotating the material axes about the element normal by an angle, BETA, from a line in the plane of the element defined by the cross product of the vector v with the element normal.<br>EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **p**, which define the centerline axis.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| BETA | Material angle in degrees for AOPT = 3, may be overridden on the element card *ELEMANT_SOLID_ORTHO. |
| XP - ZP | XP, YP and ZP define the coordinates of point P for AOPT = 1 and AOPT = 4. |
| A1 - A3 | A1, A2 and A3 define the components of vector A for AOPT = 2. |
| D1 - D3 | D1, D2 and D3 define components of vector D for AOPT = 2. |
| V1 - V3 | V1, V2 and V3 define components of vector V for AOPT = 3 and AOPT = 4. |

**Material Formulation:**

Details of the passive model can be found in Freed et al. (2005) and Einstein et al. (2005). The stress in the reference configuration consists of a deviatoric matrix term, a hydrostatic pressure term, and either one (ORTHO = 1) or two (ORTH = 2) fiber terms:

$$\mathbf{S} = \kappa J(J-1)\mathbf{C}^{-1} + \mu J^{-2/3}\mathbf{DEV}\left[\frac{1}{4}\left(\mathbf{I} - \bar{\mathbf{C}}^{-2}\right)\right] + J^{-2/3}\sum_{i=1}^{n}[\sigma_i(\lambda_i) + \varepsilon_i(\lambda_i)]\mathbf{DEV}[\mathbf{K}_i]$$

where **S** is the second Piola-Kirchhoff stress tensor, $J$ is the Jacobian of the deformation gradient, $\kappa$ is the bulk modulus, $\sigma_i$ is the passive fiber stress model used, and $\varepsilon_i$ is the corresponding active fiber model used. The operator **DEV** is the deviatoric projection:

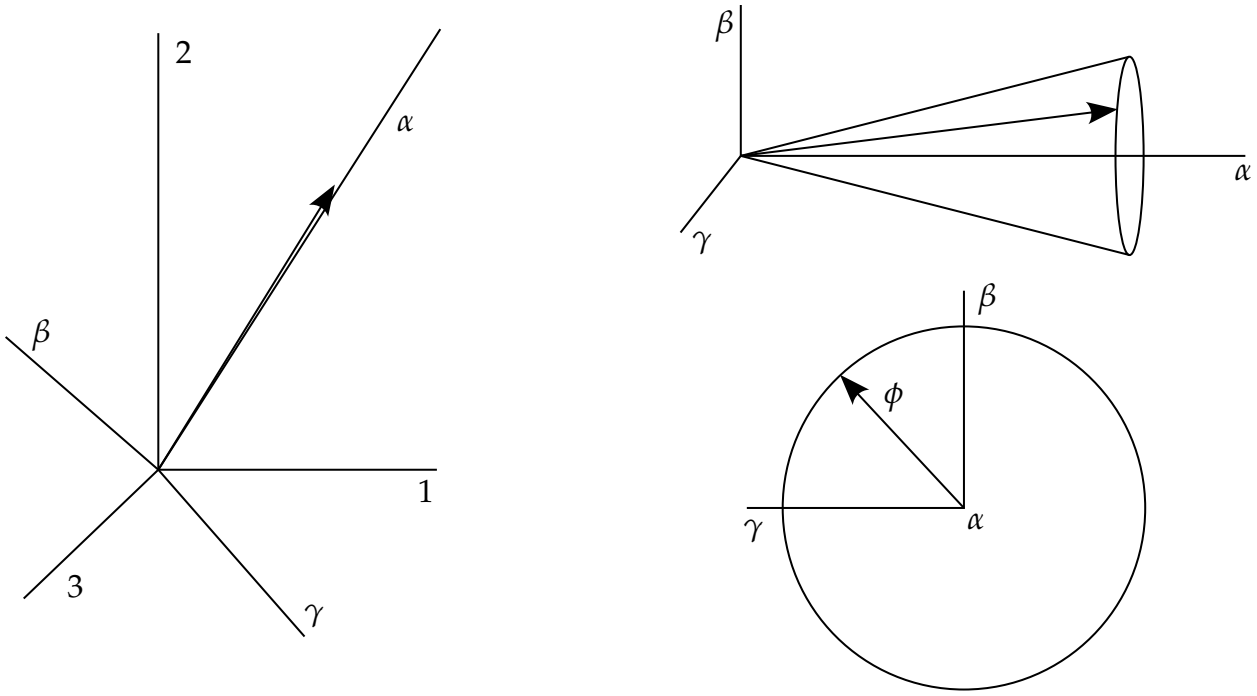



**Figure M266-1.** The plot on the left relates the global coordinates (1, 2, 3) to the local coordinates $(\alpha, \beta, \gamma)$, selected so the mean fiber direction in the reference configuration is align with the $\alpha$–axis. The plots on the right show how the unit vector for a specific fiber within the fiber distribution of a 3D tissue is oriented with respect to the mean fiber direction via angles $\theta$ and $\phi$.

$$\mathbf{DEV}[\bullet] = (\bullet) - \frac{1}{3}\mathrm{tr}[(\bullet)\mathbf{C}]\mathbf{C}^{-1}$$

where **C** is the right Cauchy-Green deformation tensor. The dispersed fourth invariant is $\lambda = \sqrt{\mathrm{tr}[\mathbf{K}\bar{\mathbf{C}}]}$, where $\bar{\mathbf{C}}$is the isochoric part of the Cauchy-Green deformation. Note that $\lambda$ is not a stretch in the classical way, since **K** embeds the concept of dispersion. **K** is called the dispersion tensor or anisotropy tensor and is given in global coordinates. The passive and active fiber models are defined in the fiber coordinate system. In effect the dispersion tensor rotates and weights these one dimensional models, such that they are both three-dimensional and in the Cartesian framework.

In the case where, the splay parameters SIGMA and F are specified, **K** is given by:

$$\mathbf{K}_i = \frac{1}{2}\mathbf{Q}_i \begin{bmatrix} 1 + e^{-2\mathrm{SIGMA}^2} & 0 & 0 \\ 0 & \mathrm{F}\left(1 - e^{-2\mathrm{SIGMA}^2}\right) & 0 \\ 0 & 0 & (1-\mathrm{F})\left(1 - e^{-2\mathrm{SIGMA}^2}\right) \end{bmatrix} \mathbf{Q}_i^T$$

where **Q** is the transformation tensor that rotates from the local to the global Cartesian system. In the case when INIT = 1, the dispersion tensor is given by

$$\mathbf{K}_i = \mathbf{Q}_i \begin{pmatrix} \chi_i^1 & 0 & 0 \\ 0 & \chi_i^2 & 0 \\ 0 & 0 & \chi_i^3 \end{pmatrix} \mathbf{Q}_i^T$$

where the $\chi$:s are given on the *INITIAL_FIELD_SOLID card. For the values to be physically meaningful $\chi_i^1 + \chi_i^2 + \chi_i^3 = 1$. It is the responsibility of the user to assure that this condition is met, no internal checking for this is done. These values typically come from diffusion tensor data taken from the myocardium.

**Remarks:**

1. Passive fiber models. Currently there are two models available.

   a) If FID = 1 a crimped fiber model is used. It is solely developed for collagen fibers. Given $H_0$ and $R_0$ compute:

   $$L_0 = \sqrt{(2\pi)^2 + (H_0)^2}, \Lambda = \frac{L_0}{H_0}$$

   and

   $$E_s = \frac{E_f H_0}{H_0 + \left(1 + \frac{37}{6\pi^2} + 2\frac{L_0^2}{\pi^2}\right)(L_0 - H_0)}.$$

   Now if the fiber stretches $\lambda < \Lambda$ the fiber stress is given by:

   $$\sigma = \xi E_s(\lambda - 1)$$

   where

   $$\xi = \frac{6\pi^2\left(\Lambda^2 + \left(4\pi^2 - 1\right)\lambda^2\right)\lambda}{\Lambda\left(3H_0^2(\Lambda^2 - \lambda^2)(3\Lambda^2 + (8\pi^2 - 3)\lambda^2) + 8\pi^2(10\Lambda^2 + (3\pi^2 - 10)\lambda^2)\right)}$$

   and if $\lambda > \Lambda$ the fiber stress equals:

   $$\sigma = E_s(\lambda - 1) + E_f(\lambda - \Lambda).$$

   In Figure M266-1 the fiber stress is rendered with $H0 = 27.5, R0 = 2$ and the transition point becomes $\Lambda = 1.1$.

   b) The second fiber model available (FID = 2) is a simpler but more useful model for the general fiber reinforced rubber. The fiber stress is simply given by:

   $$\sigma = C_1\left[e^{\frac{C_2}{2}(\lambda^2 - 1)} - 1\right].$$

   The difference between the two fiber models is given in Figure M266-2.

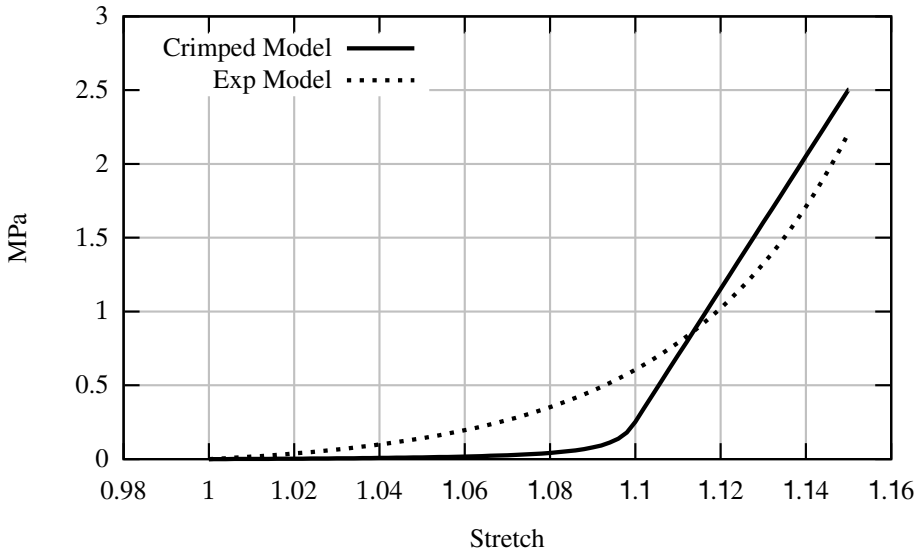



**Figure M266-2.** Visualization ofthe Crimped and the Exponential fiber models. Here $\Lambda = 1.1$ is the transition point in the crimped model.

The active model for myofibers (ACT = 1) is defined in Guccione et al. (1993) and is given by:

$$\sigma = T_{\max}\frac{Ca_0^2}{Ca_0^2 + ECa_{50}^2}C(t)$$

where

$$ECa_{50}^2 = \frac{(Ca_0)_{\max}}{\sqrt{e^{B\left(l_r\sqrt{2(\lambda-1)+1}-l_0\right)}-1}}$$

and $B$ is a constant, $(Ca_0)_{\max}$ is the maximum peak intracellular calcium concentration, $l_0$ is the sarcomere length at which no active tension develops and $l_r$ is the stress free sarcomere length. The function $C(t)$ is defined in one of two ways. First it can be given as:

$$C(t) = \frac{1}{2}(1 - \cos\omega(t))$$

where

$$\omega = \begin{cases} \pi\dfrac{t}{t_0} & 0 \le t < t_0 \\ \pi\dfrac{t - t_0 + t_r}{t_r} & t_0 \le t < t_0 + t_r \\ 0 & t_0 + t_r \le t \end{cases}$$

and $t_r = ml_R\lambda + b$. Secondly, it can also be given as a load curve. If a load curve should be used its index must be given in ACT10. Note that all variables that

correspond to ω are neglected if a load curve is used. The active parameters on Card 3 and 4 are interpreted as:

| ACT1 | ACT2 | ACT3 | ACT4 | ACT5 | ACT6 | ACT7 | ACT8 | ACT9 | ACT10 |
|---|---|---|---|---|---|---|---|---|---|
| $T_{\max}$ | $Ca_0$ | $(Ca_0)_{\max}$ | $B$ | $l_0$ | $t_0$ | $m$ | $b$ | $l_R$ | LCID |

**References:**

1. Freed AD., Einstein DR. and Vesely I., Invariant formulation for dispersed transverse isotropy in aortic heart valves – An efficient means for modeling fiber splay, Biomechan model Mechanobiol, 4, 100-117, 2005.

2. Guccione JM., Waldman LK., McCulloch AD., Mechanics of Active Contraction in Cardiac Muscle: Part II – Cylindrical Models of the Systolic Left Ventricle, J. Bio Mech, 115, 82-90, 1993.

## *MAT_EIGHT_CHAIN_RUBBER

This is Material Type 267. This is an advanced rubber-like model that is tailored for glassy polymers and similar materials. It is based on Arruda's eight chain model but enhanced with non-elastic properties. This material is available for solid and SPH elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | K | MU | N | MULL | VISPL | VISEL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| YLD0 | FP | GP | HP | LP | MP | NP | PMU |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| M1 | M2 | M3 | M4 | M5 | TIME | VCON | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| Q1 | B1 | Q2 | B2 | Q3 | B3 | Q4 | B4 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| K1 | S1 | K2 | S2 | K3 | S3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Card 8a.** This card is included if VISEL = 1. Include up to 6 of this card. This next keyword ("*") card terminates this input.

| TAU*i* | BETA*i* | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 8b.** This card is included if VISEL = 2. Include up to 6 of this card. The next keyword ("*") card terminates this input.

| TAU*i* | GAMMA*i* | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | MU | N | MULL | VISPL | VISEL |
| Type | A | F | F | F | I | I | I | I |
| Default | none | none | 0.0 | 0.0 | 0 | none | 0 | 0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Bulk modulus. To get almost incompressible behavior set K one or two orders of magnitude higher than MU. Note that the poisons ratio should be kept at a realistic value. $$v = \frac{3K - 2\text{MU}}{2(3K + \text{MU})}.$$ |
| MU | Shear modulus. MU is the product of the number of molecular chains per unit volume ($n$), Boltzmann's constant ($k$) and the absolute temperature ($T$). Thus MU $= nkT$. |
| N | Number of rigid links between crosslinks of the soft domain region. See Remark 1. |
| MULL | Parameter describing which softening algorithm that shall be used (see Remarks 1 and 2).<br>EQ.1: Strain based Mullins effect from Qi and Boyce<br>EQ.2: Energy based Mullins, a modified version of Roxburgh and Ogden model. M1, M2, and M3 must be set. |
| VISPL | Parameter describing which viscoplastic formulation that should be used; see the theory section for details (see Remark 4).<br>EQ.0: No viscoplasticity<br>EQ.1: 2 parameter standard model; K1 and S1 must be set. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2: 6 parameter G'Sells model; K1, K2, K3, S1, S2 and S3 must be set. |
| | EQ.3: 4 parameter strain hardening model; K1, K2, S1, and S2 must be set. |
| VISEL | Option for viscoelastic behavior; see the theory section for details. |
| | EQ.0: No viscoelasticity |
| | EQ.1: Free energy formulation based on Holzapfel and Ogden |
| | EQ.2: Formulation based on stiffness ratios from Simo et al. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | YLD0 | FP | GP | HP | LP | MP | NP | PMU |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| YLD0 | Initial yield stress (see Remark 4). |
| | EQ.0.0: No plasticity |
| | GT.0.0: Initial yield stress; hardening is defined separately. |
| | LT.0.0: -YLD0 is taken as the load curve ID for the yield stress as a function of effective plastic strain. |
| FP-NP | Parameters for Hill's general yield surface. For Von Mises yield criteria set FP = GP = HP = 0.5 and LP = MP = NP = 1.5. See Remark 4. |
| PMU | Kinematic hardening parameter. It usually equals MU. See Remark 5. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M1 | M2 | M3 | M4 | M5 | TIME | VCON | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | 0.0 | 9.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| M1 | Mullins constant (see Remarks 1 and 2):<br>MULL.EQ.1: Constant $A$. For the case of a dilute solution the Mullins parameter $A$ should be equal to 3.5. See Qi and Boyce [2004].<br>MULL.EQ.2: Constant M1 in the Mullins equations. M1 > 0.0 must be set. |
| M2 | Mullins constant (see Remarks 1 and 2):<br>MULL.EQ.1: Constant $B$. For a system with well dispersed particles B should somewhere around 18. See Qi and Boyce [2004].<br>MULL.EQ.2: Constant M2 in the Mullins equations. M2 > 0.0 must be set. |
| M3 | Mullins constant (see Remarks 1 and ):<br>MULL.EQ.1: Constant $Z$. Qi recommends 0.7.<br>MULL.EQ.2: Constant M3 in the Mullins equations. M3 > 0.0 must be set. |
| M4 | Mullins parameter (see Remarks 1 and 2):<br>MULL.EQ.1: Initial value of $v_s$. $v_s$ must be between 0 and 1 and must be less than $v_{ss}$ (see M5).<br>MULL.EQ.2: Not used |
| M5 | Mullins parameter (see Remarks 1 and 2):<br>MULL.EQ.1: $v_{ss}$, saturation value of $v_s$. $v_{ss}$ must be between 0 and 1 and must be greater than $v_s$ (see M4).<br>MULL.EQ.2: Not used |

| VARIABLE | DESCRIPTION |
|---|---|
| TIME | A time filter is used to smooth out the time derivative of the strain invariant over a TIME interval. Default is no smoothening but a value 100 × TIMESTEP is recommended. |
| VCON | A material constant for the volumetric part of the strain energy. The default is 9.0 but any value can be used to tailor the volumetric response. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | B1 | Q2 | B2 | Q3 | B3 | Q4 | B4 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| Q1 - B4 | Voce hardening parameters. See Remark 4. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | S1 | K2 | S2 | K3 | S3 | | |
| Type | F | F | F | F | F | F | | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K1 - S3 | Viscoplastic parameters (see Remark 4).<br>VISPL.EQ.1: K1 and S1 are used.<br>VISPL.EQ.2: K1, S1, K2, S2, K3 and S3 are used.<br>VISPL.EQ.3: K1, S1 and K2 are used. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP, YP, ZP | Coordinates for point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |

| VARIABLE | DESCRIPTION |
|---|---|
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SOLID_ORTHO. |

| Card 8a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TAU*i* | BETA*i* | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TAU*i* | Relaxation time. See Remark 3. |
| BETA*i* | Dissipating energy factors (see Holzapfel). See Remark 3. |

| Card 8b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TAU*i* | GAMMA*i* | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TAU*i* | Relaxation time. A maximum of 6 values can be used. See Remark 3. |
| GAMMA*i* | Gamma factors (see Simo). See Remark 3. |

**Remarks:**

1. **Basic Theory.** This model is based on the work done by Arruda and Boyce [1993], in particular Arruda's thesis [1992]. The eight chain rubber model is based on hyper-elasticity. It is formulated with elastic strain invariants. Strain

softening is accounted for by the parameter from $v_s$ following the work done by Qi and Boyce [2004].

The strain energy is defined in terms of the elastic deformation gradient $\mathbf{F}_e$ through the right Cauchy-Green tensor $\mathbf{C}_e = \mathbf{F}_e^T\mathbf{F}_e$ and its determinant $J_e = \det\mathbf{F}_e$ as $\Psi = \Psi_1 + \Psi_2$. $\Psi_1$ and $\Psi_2$ are the deviatoric and volumetric contributions, respectively:

$$\Psi_1 = v_s\mu\left[\sqrt{N}\Lambda_c\beta + N\ln\left(\frac{\beta}{\sinh\beta}\right)\right]$$

$$\Psi_2 = \frac{\kappa}{\nu_{\text{con}}^2}\left(\nu_{\text{con}}\ln J_e + \frac{1}{J_e^{\nu_{\text{con}}}} - 1\right)$$

Here

$$\beta = \mathcal{L}^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right)$$

with $\mathcal{L}^{-1}$ denoting the inverse of the Langevin function, $\mathcal{L}(x) = \coth x - 1/x$, and the amplified chain stretch is given by

$$\Lambda_c = \sqrt{X(v_s)\left(\bar{\lambda}^2 - 1\right) + 1},$$

where $\bar{\lambda}^2 = \text{Tr}(\bar{\mathbf{C}}_e)/3$ and $\bar{\mathbf{C}}_e = J_e^{-2/3}\mathbf{C}_e$.

Among the constant parameters, $\mu$ is the initial modulus of the soft domain, $\kappa$ is the bulk modulus, $\nu_{\text{con}}$ is a pressure influential exponent and $N$ is the number of rigid links between crosslinks of the soft domain region. $X$ is a general polynomial describing the interaction between the soft and hard phases (Qi and Boyce [2004] and Tobin and Mullins [1957]). It is given by

$$X(v_s) = 1 + A(1 - v_s) + B(1 - v_s)^2$$

where $A$ and $B$ are constants. Without the Mullins effect, $\nu_s = 1$. Otherwise, its evolution depends on the Mullins effect. See Remark 2.

The Cauchy stress is then computed as

$$\boldsymbol{\sigma} = \frac{2}{J_e}\mathbf{F}_e\frac{\partial\Psi}{\partial\mathbf{C}_e}\mathbf{F}_e{}^T = v_s\mu\frac{X(v_s)}{3J_e^{5/3}}\frac{\sqrt{N}}{\Lambda_c}\beta\left(\mathbf{B}_e - \frac{\text{tr}(\mathbf{B}_e)}{3}\mathbf{I}\right) + \frac{\kappa}{\nu_{\text{con}}J_e}\left(1 - \frac{1}{J_e^{\nu_{\text{con}}}}\right)\mathbf{I},$$

where $\mathbf{B}_e = \mathbf{F}_e\mathbf{F}_e^T$ is the left elastic Cauchy-Green tensor.

2. **Mullins Effect.** Two models for the Mullins effect are implemented.

   a) *MULL = 1*. The strain softening is developed by the evolution law taken from Boyce 2004:

$$\dot{v}_s = Z(v_{ss} - v_s)\frac{\sqrt{N}-1}{\left(\sqrt{N}-\Lambda_c^{\max}\right)^2}\dot{\Lambda}_c^{\max} \ ,$$

where $Z$ is a parameter that characterizes the evolution in $v_s$ with increasing $\dot{\Lambda}_c^{\max}$. The parameter $v_{ss}$ is the saturation value of $v_s$. Note that $\dot{\Lambda}_c^{\max}$ is the maximum of $\Lambda_c$ from the past:

$$\dot{\Lambda}_c^{\max} = \begin{cases} 0 & \Lambda_c < \Lambda_c^{\max} \\ \dot{\Lambda}_c & \Lambda_c > \Lambda_c^{\max}. \end{cases}$$

The structure now evolves with the deformation. The dissipation inequality requires that the evolution of the structure is irreversible $\dot{v}_s \geq 0$. See Qi and Boyce [2004].

b) *MULL = 2*. The energy driven model is based on Ogden and Roxburgh. When activated the strain energy is automatically transformed to a standard eight chain model, meaning the variables $Z$, $v_s$ and $X$ are automatically set to 0, 1, and 1, respectively. The stress is multiplicative split of the true stress and the softening factor $\eta$.

$$\overline{\sigma} = \eta\sigma, \quad \eta = 1 - \frac{1}{\mathrm{M1}}\mathrm{erf}\left(\frac{\Psi_1^{\max} - \Psi_1}{\mathrm{M3} - \mathrm{M2}\Psi_1^{\max}}\right) \ .$$

3. **Viscoelasticity.** Two models for viscoelasticity are implemented.

a) *VISEL = 1*. The viscoelasticity is based on work done by Holzapfel (2004)

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\beta_\alpha \frac{d}{dt}\frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \beta_\alpha \dot{\mathbf{S}}_\mathbf{1}$$

where $\alpha$ is the number of viscoelastic terms (1, …, 6).

b) *VISEL = 2.* With this option the evolution is based on work done by Simo and Hughes (2000).

$$\dot{\mathbf{Q}}_\alpha + \frac{\mathbf{Q}_\alpha}{\tau_\alpha} = 2\frac{\gamma_a}{\tau_a}\frac{d}{dt}\frac{\partial \Psi_1}{\partial \mathbf{C}_e} = \frac{\gamma_a}{\tau_a}\mathbf{S}_\mathbf{1}$$

The number of Prony terms is restricted to a maximum of 6. Also $\tau_\alpha$ and $\gamma_\alpha$ must be greater than 0. The Cauchy stress is obtained by a push forward operation on the total second Piola-Kirchhoff stress.

$$\sigma = \frac{1}{J}\mathbf{F}_e\mathbf{S}\mathbf{F}_e{}^T.$$

4. **Viscoplasticity.** Plasticity is based on the general Hills' yield surface

$$\sigma_{eff}^2 = F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{12}^2 + 2M\sigma_{23}^2 + 2N\sigma_{13}^2$$

The hardening is either based on a load curve ID (-YLD0) or an extended Voce hardening

$$\sigma_{\text{yld}} = \sigma_{\text{yld0}} + Q_1\left(1 - e^{B_1\bar{\varepsilon}}\right) + Q_2\left(1 - e^{B_2\bar{\varepsilon}}\right) + Q_3\left(1 - e^{B_3\bar{\varepsilon}}\right) + Q_4\left(1 - e^{B_4\bar{\varepsilon}}\right) \ .$$

The evolution of the elastic deformation gradient $\mathbf{F}_e$ is written as

$$\dot{\mathbf{F}}_e = (\mathbf{L} - \mathbf{L}_p)\mathbf{F}_e$$

where $\mathbf{L}$ is the spatial velocity gradient and $\mathbf{L}_\text{p}$ is the spatial (Eulerian) plastic velocity gradient which is given by the associative flow rule

$$\mathbf{L}_p = \dot{\bar{\varepsilon}}\frac{\partial f}{\partial \boldsymbol{\sigma}}$$

with $f$ being the rate independent yield surface

$$f = \sigma_{\text{eff}} - \sigma_{\text{yld}}.$$

For rate independent plasticity (VISPL = 0) the evolution of plastic strain $\bar{\varepsilon}$ follows from the consistency conditions $f \le 0$, $\dot{\bar{\varepsilon}} \ge 0$ and $f\dot{\bar{\varepsilon}} = 0$. For viscoplasticity these conditions are abandoned, but we instead invoke a constitutive equation for the effective plastic strain rate. This is the Perzyna (1966) overstress model, and three different formulations are available.

a) *VISPL = 1*. The evolution equation is

$$\dot{\bar{\varepsilon}} = \left(\frac{\max(f,0)}{K_1}\right)^{S_1} \ ,$$

where $K_1$ and $S_1$ are viscoplastic material parameters.

b) *VISPL = 2*. The evolution equation is

$$\dot{\bar{\varepsilon}} = \left[\frac{\max(f,0)}{K_1\left(1 - e^{-S_1(\bar{\varepsilon}+K_2)}\right)e^{S_2\bar{\varepsilon}^{K_3}}}\right]^{S_3} \ ,$$

where $K_1$, $K_2$, $K_3$, $S_1$, $S_2$, and $S_3$ are viscoplastic parameters.

c) *VISPL = 3*. The evolution equation is

$$\dot{\bar{\varepsilon}} = \left(\frac{\max(f,0)}{K_1}\right)^{S_1} (\bar{\varepsilon} + K_2)^{S_2}$$

where $K_1$, $K_2$, $S_1$, and $S_2$ are viscoplastic parameters.

5. **Kinematic Hardening.** The back stress is calculated similar to the Cauchy stress above but without the softening factors:

$$\beta = \frac{\mu_p}{3J}\frac{\sqrt{N}}{\Lambda_c}L^{-1}\left(\frac{\Lambda_c}{\sqrt{N}}\right)\left(\mathbf{I} - \frac{1}{3}I_p\mathbf{C}_p^{-1}\right) \ .$$

$\mu_p$ is a hardening material parameter (PMU). The total Piola-Kirchhoff stress is now given by $\mathbf{S}^* = \mathbf{S} - \boldsymbol{\beta}$ and the total stress is given by a standard push forward operation with the elastic deformation gradient.

**References:**

Qi HJ., Boyce MC., Constitutive model for stretch-induced softening of stress-stretch behavior of elastomeric materials, Journal of the Mechanics and Physics of Solids, 52, 2187-2205, 2004.

Arrude EM., Characterization of the strain hardening response of amorphous polymers, PhD Thesis, MIT, 1992.

Mullins L., Tobin NR., Theoretical model for the elastic behavior of filler reinforced vulcanized rubber, Rubber Chem. Technol., 30, 555-571, 1957.

Ogden RW. Roxburgh DG., A pseudo-elastic model for the Mullins effect in Filled rubber., Proc. R. Soc. Lond. A., 455, 2861-2877, 1999.

Simo JC., Hughes TJR., Computational Inelasticity, Springer, New York, 2000.

Holzapfel GA., Nonlinear Solid Mechanics, Wiley, New-York, 2000.

## *MAT_BERGSTROM_BOYCE_RUBBER

This is Material Type 269. This is a rubber model based on the Arruda and Boyce (1993) chain model accompanied with a viscoelastic contribution according to Bergström and Boyce (1998). The viscoelastic treatment is based on the physical response of a single entangled chain in an embedded polymer gel matrix, and the implementation is based on Dal and Kaliske (2009). This model is only available for solid elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | G | GV | N | NV | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C | M | GAM0 | TAUH | | | | |
| Type | F | F | F | F | | | | |
| Default | none | none | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Elastic bulk modulus, $K$ |
| G | Elastic shear modulus, $G$ |
| GV | Viscoelastic shear modulus, $G_v$ |
| N | Elastic segment number, $N$ |
| NV | Viscoelastic segment number, $N_v$ |

| VARIABLE | DESCRIPTION |
|---|---|
| C | Inelastic strain exponent, $c$. It should be less than zero. |
| M | Inelastic stress exponent, $m$ |
| GAM0 | Reference strain rate, $\dot{\gamma}_0$ |
| TAUH | Reference Kirchhoff stress, $\hat{\tau}$ |

**Remarks:**

The deviatoric Kirchhoff stress for this model is the sum of an elastic and viscoelastic part according to

$$\overline{\boldsymbol{\tau}} = \boldsymbol{\tau}_e + \boldsymbol{\tau}_v$$

The elastic part is governed by the Arruda-Boyce strain energy potential resulting in the following expression (after a Pade approximation of the Langevin function)

$$\boldsymbol{\tau}_e = \frac{G}{3}\frac{3-\lambda_r^2}{1-\lambda_r^2}\left(\bar{\mathbf{b}} - \frac{\mathrm{tr}(\bar{\mathbf{b}})}{3}\mathbf{I}\right)$$

Here $G$ is the elastic shear modulus, $\bar{\mathbf{b}}$ is the unimodular left Cauchy-Green tensor given by:

$$\bar{\mathbf{b}} = J^{-2/3}\mathbf{F}\mathbf{F}^T$$
$$J = \det \mathbf{F}$$

and $\lambda_r$ is the relative network stretch given by:

$$\lambda_r^2 = \frac{tr(\bar{\mathbf{b}})}{3N}$$

The viscoelastic stress is based on a multiplicative split of the unimodular deformation gradient into unimodular elastic and inelastic parts, respectively,

$$J^{-1/3}\mathbf{F} = \mathbf{F}_e\mathbf{F}_i$$

We define

$$\mathbf{b}_e = \mathbf{F}_e\mathbf{F}_e^T$$

to be the elastic left Cauchy-Green tensor. The viscoelastic stress is given as

$$\boldsymbol{\tau}_v = \frac{G_v}{3}\frac{3-\lambda_v^2}{1-\lambda_v^2}\left(\mathbf{b}_e - \frac{\mathrm{tr}(\mathbf{b}_e)}{3}\mathbf{I}\right)$$

where

$$\lambda_v^2 = \frac{\mathrm{tr}(\mathbf{b}_e)}{3N_v}$$

is the relative network stretch for the viscoelastic part. The evolution of the elastic left Cauchy-Green tensor can be written

$$\dot{\mathbf{b}}_e = \bar{\mathbf{L}}\mathbf{b}_e + \mathbf{b}_e\bar{\mathbf{L}}^T - 2\mathbf{D}_i\mathbf{b}_e$$

where the inelastic rate-of-deformation tensor is given as

$$\mathbf{D}_i = \dot{\gamma}_0(\lambda_i - 0.999)^c \left(\frac{\|\boldsymbol{\tau}_v\|}{\hat{\tau}\sqrt{2}}\right)^m \frac{\boldsymbol{\tau}_v}{\|\boldsymbol{\tau}_v\|}$$

and

$$\bar{\mathbf{L}} = \mathbf{L} - \frac{\mathrm{tr}(\mathbf{L})}{3}\mathbf{I}$$

is the deviatoric velocity gradient. The stretch of a single chain relaxing in a polymer is linked to the inelastic right Cauchy-Green tensor as

$$\lambda_i^2 = \frac{\mathrm{tr}\left(\mathbf{F}_i^T\mathbf{F}_i\right)}{3} \geq 1 \quad .$$

This stretch is available as the plastic strain variable in the post-processing of this material. The volumetric part is elastic and governed by the bulk modulus, the pressure for this model is given as

$$p = K\left(J^{-1} - 1\right) \quad .$$

## *MAT_CWM

This is Material Type 270. It is a thermo-elastic-plastic model with kinematic hardening that allows for material creation and annealing triggered by temperature. The acronym CWM stands for Computational Welding Mechanics, Lindström (2013, 2015). The model is intended to be used for simulating multistage weld processes. This model is available for solid and shell elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | LCEM | LCPR | LCSY | LCHR | LCAT | BETA |
| Type | A | F | I | I | I | I | I | F |
| Default | none | none | none | none | none | none | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TASTART | TAEND | TLSTART | TLEND | EGHOST | PGHOST | AGHOST | EPSINI |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | 0.0 |

**Optional Phase Change Card.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T2PHASE | T1PHASE | ANOPT | POSTV | DTEMP | DOSPOT | | |
| Type | F | F | F | I | F | I | | |
| Default | optional | optional | 0.0 | 0 | 0.0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Material density |
| LCEM | Load curve ID giving Young's modulus as a function of temperature |
| LCPR | Load curve ID giving Poisson's ratio as a function of temperature |
| LCSY | Load curve or table for yield stress.<br>GT.0: Load curve ID giving yield stress as a function of temperature.<br>LT.0: \|LCSY\| is a table ID giving yield curves for different temperatures. Each yield curve is a function of plastic strain. |
| LCHR | Load curve ID giving the hardening modulus as a function of temperature. LCHR is not used for LCSY < 0. The hardening modulus is then calculated from the yield curve's slope. |
| LCAT | Load curve (or table) ID giving the thermal expansion coefficient as a function of temperature (and maximum temperature up to the current time). In the case of a table, load curves are listed according to their maximum temperature. See Remark 1. |
| BETA | Fraction of isotropic hardening between 0 and 1:<br>EQ.0.0: Kinematic hardening<br>EQ.1.0: Isotropic hardening |
| TASTART | Annealing temperature start, $T_a^{\text{start}}$. See Remark 3. |
| TAEND | Annealing temperature end, $T_a^{\text{end}}$. See Remark 3. |
| TLSTART | Birth temperature start, $T_l^{\text{start}}$. See Remark 1. |
| TLEND | Birth temperature end, $T_l^{\text{end}}$. See Remark 1. |
| EGHOST | Young's modulus for ghost (quiet) material. See Remark 1. |
| PGHOST | Poisson's ratio for ghost (quiet) material. See Remark 1. |
| AGHOST | Thermal expansion coefficient for ghost (quiet) material. See Remark 1. |
| EPSINI | Initial plastic strains, uniformly distributed within the part. |

| VARIABLE | DESCRIPTION |
|---|---|
| T2PHASE | Temperature at which phase change commences. See Remark 4. |
| T1PHASE | Temperature at which phase change ends. See Remark 4. |
| ANOPT | Annealing option for thermal expansion (see Remark 3):<br>EQ.0: No modification for thermal expansion.<br>EQ.1: TAEND defines the upper limit (cut-off temperature) for evaluation of thermal expansion.<br>LT.0: \|ANOPT\| defines the upper limit (cut-off temperature) for evaluation of thermal expansion. |
| POSTV | Define additional history variables that might be useful for post-processing. See Remark 5. |
| DTEMP | Maximum temperature variation within a time step. If exceeded during the analysis at a certain integration, a local (only for the respective integration points) sub-cycling is used for the calculation of the phase transformations.<br>EQ.0.0: Not active (default)<br>GT.0.0: Active |
| DOSPOT | Activate thinning of tied shell elements when SPOTHIN > 0 on *CONTROL_CONTACT.<br>EQ.0: Spot weld thinning is inactive for shells tied to solids that use this material (default).<br>EQ.1: Spot weld thinning is active for shells tied to solids that use this material. |

**Remarks:**

1. **Material birth.** This material is initially in a quiet state, sometimes referred to as a ghost material. In this state, the material has thermo-elastic properties defined by the quiet Young's modulus, quiet Poisson's ratio, and quiet thermal expansion coefficient. These properties should represent void, meaning the Young's modulus should be small enough not to influence the surroundings but large enough to avoid numerical problems. A quiet material stress should never reach the yield point. When the temperature reaches the birth temperature, a history variable representing the indicator of the welding material is incremented. This variable follows

$$\gamma(t) = \min\left(1, \max\left[0, \frac{T_{\max} - T_l^{\text{start}}}{T_l^{\text{end}} - T_l^{\text{start}}}\right]\right)$$

where $T_{\max} = \max_{s \le t} T(s)$. This parameter is available as history variable 9 in the output database. The effective thermo-elastic material properties are interpolated as

$$E = E(T)\gamma + E_{\text{quiet}}(1-\gamma)$$
$$\nu = \nu(T)\gamma + \nu_{\text{quiet}}(1-\gamma)$$
$$\alpha = \alpha(T, T_{\max})\gamma + \alpha_{\text{quiet}}(1-\gamma)$$

where $E$, $\nu$, and $\alpha$ are the Young's modulus, Poisson's ratio and thermal expansion coefficient, respectively. Here, the thermal expansion coefficient is either a temperature-dependent curve or a collection of temperature-dependent curves ordered in a table according to maximum temperature, $T_{\max}$.

2. **Stress update.** The stress update follows a classical isotropic associative thermo-elastic-plastic approach with kinematic hardening that is summarized in the following. The explicit temperature dependence is sometimes dropped for the sake of clarity.

The stress evolution is given as

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p - \dot{\boldsymbol{\varepsilon}}_T\right)$$

where $\mathbf{C}$ is the effective elastic constitutive tensor and

$$\dot{\boldsymbol{\varepsilon}}_T = \alpha \dot{T}\mathbf{I}$$
$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\varepsilon}_p \frac{3}{2}\frac{\mathbf{s} - \boldsymbol{\kappa}}{\bar{\sigma}}$$

are the thermal and plastic strain rates, respectively. The latter expression includes the deviatoric stress

$$\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3}\text{Tr}(\boldsymbol{\sigma})\mathbf{I},$$

the back stress $\boldsymbol{\kappa}$ and the effective stress

$$\bar{\sigma} = \sqrt{\frac{3}{2}(\mathbf{s} - \boldsymbol{\kappa}) : (\mathbf{s} - \boldsymbol{\kappa})}$$

that are involved in the plastic equations. To this end, the effective yield stress is given as

$$\sigma_Y = \sigma_Y(T) + \beta H(T)\varepsilon_p$$

and plastic strains evolve when the effective stress exceeds this value. The back stress evolves as

$$\dot{\boldsymbol{\kappa}} = (1-\beta)H(T)\dot{\varepsilon}_p \frac{\mathbf{s} - \boldsymbol{\kappa}}{\bar{\sigma}}$$

where $\dot{\varepsilon}_p$ is the rate of effective plastic strain rate that follows from consistency equations.

3. **Annealing.** When the temperature reaches the start annealing temperature, the material begins assuming its virgin properties. Beyond the start annealing temperature, it behaves as an ideal elastic-plastic material but with no evolution of plastic strains. The resetting of effective plastic properties in the annealing temperature interval is done by modifying the effective plastic strain and back stress before the stress update as

$$\varepsilon_p^{n+1} = \varepsilon_p^n \max\left[0, \min\left(1, \frac{T - T_a^{\text{end}}}{T_a^{\text{start}} - T_a^{\text{end}}}\right)\right]$$

$$\boldsymbol{\kappa}^{n+1} = \boldsymbol{\kappa}^n \max\left[0, \min\left(1, \frac{T - T_a^{\text{end}}}{T_a^{\text{start}} - T_a^{\text{end}}}\right)\right]$$

Depending on the choice for parameter ANOPT, annealing may also affect the thermal expansion of the structure. A cut-off temperature can be defined for the evaluation of the thermal expansion. Above this temperature limit, further expansion is suppressed. The cut-off temperature does not necessarily coincide with the annealing temperature.

4. **Average temperature rate.** Optional Card 3 is used to set history variable 11, which is the average temperature rate by which the temperature has gone from T2PHASE to T1PHASE. To fringe this variable, the range should be set to positive values. During the simulation it is temporarily used to store the time when the material has reached temperature T2PHASE which is stored as a negative value. A strictly positive value means that the material has reached temperature T2PHASE and gone down to T1PHASE and the history variable is $(\text{T2PHASE} - \text{T1PHASE})/(\text{T1} - \text{T2})$, where T2 is the time when temperature T2PHASE is reached and T1 is the time when temperature T1PHASE is reached. Note that T2PHASE > T1PHASE and T1 > T2. A value of zero means that the element has not yet reached temperature T2PHASE. A strictly negative value means that the element has reached temperature T2PHASE but not yet T1PHASE.

5. **History variables.** This material formulation outputs additional data for post-processing to the set of history variables if requested. The parameter POSTV defines the data to be written. Its value is calculated as

$$\text{POSTV} = a_1 + 2\,a_2 + 4\,a_3 + 8\,a_4$$

Each flag $a_i$ is a binary number (can be either 1 or 0) and corresponds to one particular post-processing variable according to the following table:

| Flag | Description | Variables | # of History Variables |
|---|---|---|---|
| $a_1$ | Accumulated thermal strain | $\varepsilon_T$ | 1 |

| Flag | Description | Variables | # of History Variables |
|---|---|---|---|
| $a_2$ | Accumulated strain tensor | $\boldsymbol{\varepsilon}$ | 6 |
| $a_3$ | Plastic strain tensor | $\boldsymbol{\varepsilon}_p$ | 6 |
| $a_4$ | Equivalent strain | $\varepsilon_{\text{VM}}$ | 1 |

The above table also shows the order of output as well as the number of extra history variables associated with the particular flag. The values of these user-defined histories are reset when the temperature is in the annealing range.

In total NXH extra variables are required depending on the choice of parameter POSTV. For example, the maximum number of additional variables is NXH = 14 for POSTV = 15.

A complete list of history variables for the material is given in the following table. "Position" refers to the history variable number as listed by LS-PrePost when post-processing the d3plot database. The variable NEIPS in *DATABASE_EXTENT_BINARY must to set to output these history variables.

| Position | Description |
|---|---|
| 1 - 6 | Back stress |
| 7 | Temperature at last time step |
| 8 | Yield indicator: 1 if yielding, else 0 |
| 9 | Welding material indicator: 0 for ghost material, else 1 |
| 10 | Maximum temperature reached |
| 11 | Average temperature rate going from T2PHASE to T1PHASE |
| 12 → 11+NXH | User-defined history data as described in the preceding table |

## *MAT_POWDER

This is Material Type 271. This model is used to analyze the compaction and sintering of cemented carbides and the model is based on the works of Brandt (1998). This material is only available for solid elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | P11 | P22 | P33 | P12 | P23 | P13 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| E0 | LCK | PR | LCX | LCY | LCC | L | R |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| CA | CD | CV | P | LCH | LCFI | SINT | TZRO |
|---|---|---|---|---|---|---|---|

**Card 3.1.** Include this card if SINT = 1.

| LCFK | LCFS2 | DV1 | DV2 | DS1 | DS2 | OMEGA | RGAS |
|---|---|---|---|---|---|---|---|

**Card 3.2.** Include this card if SINT = 1.

| LCPR | LCFS3 | LCTAU | ALPHA | LCFS1 | GAMMA | L0 | LCFKS |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | P11 | P22 | P33 | P12 | P23 | P13 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| RO | Mass density |
| PIJ | Initial compactness tensor $P_{ij}$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0 | LCK | PR | LCX | LCY | LCC | L | R |
| Type | F | I | F | I | I | I | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| E0 | Initial anisotropy variable $e$ (value between 1 and 2) |
| LCK | Load curve ID for bulk modulus $K$ as function of relative density $d$ |
| PR | Poisson's ratio, $\nu$ |
| LCX | Load curve ID for hydrostatic compressive yield $X$ as function of relative density $d$ |
| LCY | Load curve for uniaxial compressive yield $Y$ as function of relative density $d$ |
| LCC | Load curve ID for shear yield $C_0$ as function of relative density $d$ |
| L | Yield surface parameter $L$ relating hydrostatic compressive yield to point on hydrostatic axis with maximum strength |
| R | Yield surface parameter $R$ governing the shape of the yield surface |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CA | CD | CV | P | LCH | LCFI | SINT | TZRO |
| Type | F | F | F | F | I | I | F | F |
| Default | none | none | none | none | none | none | 0.0 | none |

| VARIABLE | DESCRIPTION |
|---|---|
| CA | Hardening parameter $c_a$ |
| CD | Hardening parameter $c_d$ |
| CV | Hardening parameter $c_v$ |
| P | Hardening exponent $p$ |
| LCH | Load curve ID giving back stress parameter, $H$, as function of hardening parameter $e$ |
| LCFI | Load curve ID giving plastic strain evolution angle, $\phi$, as function of relative volumetric stress |
| SINT | Activate sintering:<br>EQ.0.0: Sintering off<br>EQ.1.0: Sintering on |
| TZRO | Absolute zero temperature, $T_0$ |

**Sintering Card 1.** Additional card for SINT = 1.

| Card 3.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCFK | LCFS2 | DV1 | DV2 | DS1 | DS2 | OMEGA | RGAS |
| Type | I | I | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| LCFK | Load curve ID for viscous compliance, $f_K$, as function of relative density, $d$ |
| LCFS2 | Load curve ID for viscous compliance, $f_{S2}$, as function of temperature, $T$ |
| DV1 | Volume diffusion coefficient $d_{V1}$ |
| DV2 | Volume diffusion coefficient $d_{V2}$ |
| DS1 | Surface diffusion coefficient $d_{S1}$ |
| DS2 | Surface diffusion coefficient $d_{S2}$ |
| OMEGA | Blending parameter $\omega$ |
| RGAS | Universal gas constant, $R_{gas}$ |

**Sintering Card 2.** Additional card for SINT = 1.

| Card 3.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCPR | LCFS3 | LCTAU | ALPHA | LCFS1 | GAMMA | L0 | LCFKS |
| Type | I | I | I | F | I | F | F | I |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| LCPR | Load curve ID for viscous Poisson's ratio, $\nu^v$, as a function of relative density, $d$ |
| LCFS3 | Load curve ID for evolution of mobility factor, $f_{S3}$, as function of temperature, $T$ |
| LCTAU | Load curve for relaxation time, $\tau$, as function of temperature, $T$ |
| ALPHA | Thermal expansion coefficient, $\alpha$ |
| LCFS1 | Load curve ID for sintering stress scaling, $f_{S1}$, as function of relative density, $d$ |

| VARIABLE | DESCRIPTION |
|---|---|
| GAMMA | Surface energy density, $\gamma$, which affects sintering stress |
| L0 | Grain size, $l_0$, which affects sintering stress |
| LCFKS | Load curve ID scaling bulk modulus, $f_{\mathrm{KS}}$, as function of temperature $T$ |

**Remarks:**

This model is intended to be used in two stages. During the first step, the compaction of a powder specimen is simulated, after which the results are dumped to file, and in a subsequent step, the model is restarted to simulate the sintering of the compacted specimen. In the following, an overview of the two different models is given; for a detailed description, we refer to Brandt (1998). The progressive stiffening in the material during compaction makes it more or less necessary to run double precision, and with constraint contacts to avoid instabilities; unfortunately, this currently limits the use of this material to the SMP version.

The powder compaction model makes use of a multiplicative split of the deformation gradient into a plastic and elastic part according to

$$\mathbf{F} = \mathbf{F}_e\mathbf{F}_p \ ,$$

where the plastic deformation gradient maps the initial reference configuration to an intermediate relaxed configuration

$$\delta\tilde{\mathbf{x}} = \mathbf{F}_p\delta\mathbf{X}$$

and subsequently, the elastic part maps this onto the current loaded configuration

$$\delta\mathbf{x} = \mathbf{F}_e\delta\tilde{\mathbf{x}} \ .$$

The compactness tensor, $\mathbf{P}$, then maps the intermediate configuration onto a virtual fully compacted configuration

$$\delta\bar{\mathbf{x}} = \mathbf{P}\delta\tilde{\mathbf{x}}$$

and we define the relative density as

$$d = \det\mathbf{P} = \frac{\rho}{\bar{\rho}} \ ,$$

where $\rho$ and $\bar{\rho}$ denote the current and fully compacted density, respectively. The elastic properties depend highly on the relative density through the bulk modulus $K(d)$, but the Poisson's ratio is assumed constant.

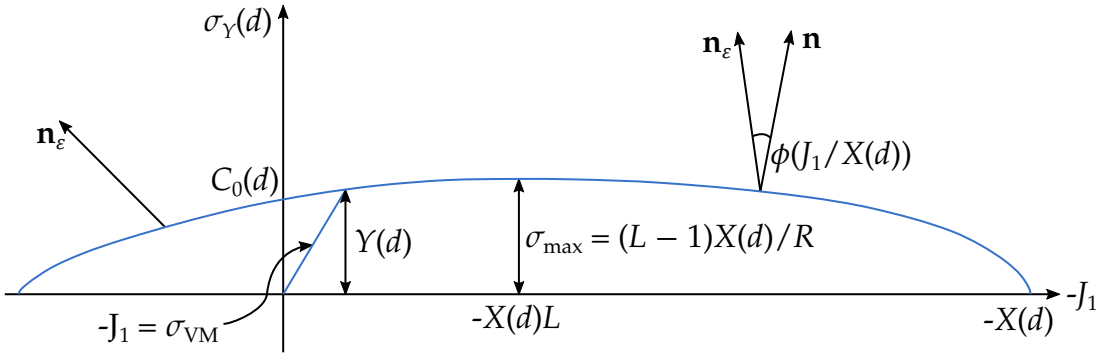



**Figure M271-1.** Yield surface

The yield surface is represented by two functions in the Rendulic plane according to

$$\sigma_Y(d) = \begin{cases} \frac{1}{2}C_0(d) - C_1(d)J_1 - C_2(d)J_1^2 & J_1 \geq LX(d) \\ \dfrac{\sqrt{[(L-1)X(d)]^2 - [J_1 - LX(d)]^2}}{R} & J_1 < LX(d) \end{cases}$$

and is in this way capped in both compression and tension. Here

$$J_1 = 3\sigma^m = \mathrm{Tr}(\sigma) \ .$$

The polynomial coefficients in the expression above are chosen to give continuity at $J_1 = LX(d)$ and to give the uniaxial compressive strength $Y(d)$. Yielding is assumed to occur when the equivalent stress (note the definition) equals the yield stress

$$\sigma_{\mathrm{eq}} = \frac{\sigma_{\mathrm{VM}}}{\sqrt{3}} = \sqrt{\frac{1}{2}\mathbf{s}:\mathbf{s}} \leq \sigma_Y(d) \ ,$$

where

$$\mathbf{s} = \underbrace{\boldsymbol{\sigma} - \sigma^m \mathbf{I}}_{\boldsymbol{\sigma}^d} - \boldsymbol{\kappa}$$

in which the last term is the back stress to be described below. The yield surface does not depend on the third stress invariant. The plastic flow is non-associated, and its direction is given by

$$\mathbf{n}_\varepsilon = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \mathbf{n} \ ,$$

where

$$\mathbf{n} = \frac{\sigma_Y(d)}{\sigma_{\max}} \begin{pmatrix} \dfrac{\partial \sigma_Y}{\partial J_1} \\ 1 \end{pmatrix}$$

is the normal to the yield surface as depicted in the Rendulic plane above (note the sign of $J_1$). The angle $\phi$ is a function of and defined only for positive values of the relative

volumetric stress $J_1/X(d) > 0$; for negative values $\phi$ is determined internally to achieve smoothness in the plastic flow direction and to avoid numerical problems at the tensile cap point. The above equations are for illustrative purposes; from now on, the plastic flow direction is generalized to a second-order tensor. The plastic flow rule is then

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\lambda}\mathbf{n}_\varepsilon, \qquad \dot{\varepsilon}_p^m = \frac{1}{3}\mathrm{Tr}\left(\dot{\boldsymbol{\varepsilon}}_p\right), \qquad \dot{\boldsymbol{\varepsilon}}_p^d = \dot{\boldsymbol{\varepsilon}}_p - \dot{\varepsilon}_p^m\mathbf{I} \ .$$

The evolution of the compactness tensor is directly related to the evolution of plastic strain as

$$\dot{\mathbf{P}} = -\frac{1}{2}\left(\dot{\boldsymbol{\varepsilon}}_p\mathbf{P} + \mathbf{P}\dot{\boldsymbol{\varepsilon}}_p\right) \ ,$$

and thus, the relative density is given by

$$\dot{d} = -3\dot{\varepsilon}_p^m d \ .$$

The back stress is assumed coaxial with the deviatoric part of the compactness tensor and given by

$$\boldsymbol{\kappa} = J_1 H(e)\left(\mathbf{P} - \frac{\mathrm{Tr}(\mathbf{P})}{3}\mathbf{I}\right),$$

where $e$ is a measure of intensity of anisotropy. This takes a value between 1 and 2 and evolves with plastic strain and plastic work according to

$$\dot{e} = c_a\sqrt{\frac{1}{2}\dot{\boldsymbol{\varepsilon}}_p^d{:}\,\dot{\boldsymbol{\varepsilon}}_p^d} - c_v J_1\dot{\varepsilon}_p^m W(d, J_1) + c_d\dot{\boldsymbol{\varepsilon}}_p^d{:}\,\boldsymbol{\sigma}W(d, J_1) \ ,$$

where

$$W(d, J_1) = -\left[\frac{J_1}{X(d)}\right]^p \int_{d_0}^{d} \frac{X(\xi)}{3\xi}d\xi$$

and $d_0$ is the density in the initial uncompressed configuration. The stress update is completed by the rate equation of stress

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}(d){:}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_p\right) \ ,$$

where $\mathbf{C}(d)$ is the elastic constitutive matrix.

The sintering model is a thermo and viscoelastic model where the evolution of the mean and deviatoric stress can be written as

$$\dot{\sigma}^m = 3K^s\left(\dot{\varepsilon}^m - \dot{\varepsilon}_T - \dot{\varepsilon}_p^m\right)$$
$$\dot{\boldsymbol{\sigma}}^d = 2G^s\left(\dot{\boldsymbol{\varepsilon}}^d - \dot{\boldsymbol{\varepsilon}}_p^d\right)$$

The thermal strain rate is given by the thermal expansion coefficient as

$$\dot{\varepsilon}_T = \alpha\dot{T} \ ,$$

and the bulk and shear modulus are the same as for the compaction model with the exception that they are scaled by a temperature curve

$$K^s = f_{\mathrm{KS}}(T)K(d)$$
$$G^s = \frac{3(1-2\nu)}{2(1+\nu)}K^s$$

The inelastic strain rates are different from the compaction model and are here given by

$$\dot{\boldsymbol{\varepsilon}}_p = \frac{\boldsymbol{\sigma}^d}{2G^v} + \frac{\sigma^m - \sigma^s}{3K^v}\mathbf{I}$$

which results in viscoelastic behavior that depends on the viscous compliance and sintering stress. The viscous bulk compliance can be written

$$\frac{1}{K^v} = 3f_K(d)\left\{d_{\mathrm{V1}}\exp\left[-\frac{d_{\mathrm{V2}}}{R_{\mathrm{gas}}(T-T_0)}\right] + \omega d_{\mathrm{S1}}\exp\left[-\frac{d_{\mathrm{S2}}}{R_{\mathrm{gas}}(T-T_0)}\right]\right\}\left[1+f_{\mathrm{S2}}(T)\xi\right]$$

from which the viscous shear compliance is modified with the aid of the viscous Poisson's ratio

$$\frac{1}{G^v} = \frac{2[1+\nu^v(d)]}{3[1-2\nu^v(d)]}\frac{1}{K^v} \ .$$

The mobility factor $\xi$ evolves with temperature according to

$$\dot{\xi} = \frac{f_{\mathrm{S3}}(T)\dot{T} - \xi}{\tau(T)} \ ,$$

and the sintering stress is given as

$$\sigma^s = f_{\mathrm{S1}}(d)\frac{\gamma}{l_0} .$$

All this is accompanied by, again, the evolution of relative density given as

$$\dot{d} = -3\dot{\varepsilon}_p^m d \ .$$

### History Variables:

The following table lists the additional history variables available for this material. See NEIPH on *DATABASE_EXTENT_BINARY to output additional history variables.

| History Variable # | Description |
|---|---|
| 1 | Relative density |

## *MAT_RHT

This is Material Type 272. This model is used to analyze concrete structures subjected to impulsive loadings; see Riedel et.al. (1999) and Riedel (2004).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | SHEAR | ONEMPA | EPSF | B0 | B1 | T1 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| A | N | FC | FS* | FT* | Q0 | B | T2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| E0C | E0T | EC | ET | BETAC | BETAT | PTF | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| GC* | GT* | XI | D1 | D2 | EPM | AF | NF |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| GAMMA | A1 | A2 | A3 | PEL | PCO | NP | ALPHA0 |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | SHEAR | ONEMPA | EPSF | B0 | B1 | T1 |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| SHEAR | Elastic shear modulus |

| VARIABLE | DESCRIPTION |
|---|---|
| ONEMPA | Unit conversion factor defining 1 MPa in the pressure units used. It can also be used for automatic generation of material parameters for a given compressive strength (see remarks).<br>EQ.0: defaults to 1.0<br>EQ.-1: parameters generated in m, s and kg (Pa)<br>EQ.-2: parameters generated in mm, s and tonne (MPa)<br>EQ.-3: parameters generated in mm, ms and kg (GPa)<br>EQ.-4: parameters generated in in, s and dozens of slugs (psi)<br>EQ.-5: parameters generated in mm, ms and g (MPa)<br>EQ.-6: parameters generated in cm, μs and g (Mbar)<br>EQ.-7: parameters generated in mm, ms and mg (kPa) |
| EPSF | Eroding plastic strain (default is 2.0) |
| B0 | Parameter for polynomial EOS |
| B1 | Parameter for polynomial EOS |
| T1 | Parameter for polynomial EOS |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | N | FC | FS* | FT* | Q0 | B | T2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A | Failure surface parameter *A* |
| N | Failure surface parameter *N* |
| FC | Compressive strength |
| FS* | Relative shear strength |
| FT* | Relative tensile strength |
| Q0 | Lode angle dependence factor |

| VARIABLE | DESCRIPTION |
|---|---|
| B | Lode angle dependence factor |
| T2 | Parameter for polynomial EOS |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E0C | E0T | EC | ET | BETAC | BETAT | PTF | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| E0C | Reference compressive strain rate |
| E0T | Reference tensile strain rate |
| EC | Break compressive strain rate |
| ET | Break tensile strain rate |
| BETAC | For compressive strain rate dependence $F_r^c$ (optional):<br>GT.0.0: Exponent $\beta_c$ in the equation in the remarks below.<br>LT.0.0: Load curve ID = \|BETAC\| for the curve that defines $F_r^c$ as a function of strain rate $\dot{\varepsilon}_p$. This option supports logarithmic interpolation. When the first abscissa value is negative, LS-DYNA assumes that all the values represent the natural logarithm of a strain rate. |
| BETAT | For tensile strain rate dependence $F_r^t$ (optional):<br>GT.0.0: Exponent $\beta_t$ in the equation in the remarks below.<br>LT.0.0: Load curve ID = \|BETAT\| for the curve that defines $F_r^t$ as a function of strain rate $\dot{\varepsilon}_p$. This option supports logarithmic interpolation. When the first abscissa value is negative, LS-DYNA assumes that all the values represent the natural logarithm of a strain rate. |
| PTF | Pressure influence on plastic flow in tension (default is 0.001) |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GC* | GT* | XI | D1 | D2 | EPM | AF | NF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GC* | Compressive yield surface parameter |
| GT* | Tensile yield surface parameter |
| XI | Shear modulus reduction factor |
| D1 | Damage parameter |
| D2 | Damage parameter |
| EPM | Minimum damaged residual strain |
| AF | Residual surface parameter |
| NF | Residual surface parameter |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAMMA | A1 | A2 | A3 | PEL | PCO | NP | ALPHA0 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| GAMMA | Gruneisen gamma |
| A1 | Hugoniot polynomial coefficient |
| A2 | Hugoniot polynomial coefficient |
| A3 | Hugoniot polynomial coefficient |
| PEL | Crush pressure |
| PCO | Compaction pressure |

| VARIABLE | DESCRIPTION |
|---|---|
| NP | Porosity exponent |
| ALPHA | Initial porosity |

**Remarks:**

In the RHT model, the shear and pressure part is coupled in which the pressure is described by the Mie-Gruneisen form with a polynomial Hugoniot curve and a $p$-$\alpha$ compaction relation. For the compaction model, we define a history variable representing the porosity $\alpha$ that is initialized to $\alpha_0 > 1$. This variable represents the current fraction of density between the matrix material and the porous concrete and will decrease with increasing pressure, that is, the reference density is expressed as $\alpha\rho$. The evolution of this variable is given as

$$\alpha(t) = \max\left(1, \min\left\{\alpha_0, \min_{s\le t}\left[1 + (\alpha_0 - 1)\left(\frac{p_{\text{comp}} - p(s)}{p_{\text{comp}} - p_{\text{el}}}\right)^N\right]\right\}\right) ,$$

where $p(t)$ indicates the pressure at time $t$. This expression also involves the initial pore crush pressure $p_{\text{el}}$, compaction pressure $p_{\text{comp}}$ and porosity exponent $N$. For later use, we define the cap pressure, or current pore crush pressure, as

$$p_c = p_{\text{comp}} - \left(p_{\text{comp}} - p_{\text{el}}\right)\left(\frac{\alpha - 1}{\alpha_0 - 1}\right)^{1/N} .$$

The remainder of the pressure (EOS) model is given in terms of the porous density $\rho$ and specific internal energy $e$ (with respect to the porous density). Depending on user inputs, it is either governed by ($B_0 > 0$)

$$p(\rho, e) = \frac{1}{\alpha}\begin{cases}(B_0 + B_1\eta)\alpha\rho e + A_1\eta + A_2\eta^2 + A_3\eta^3 & \eta > 0\\ B_0\alpha\rho e + T_1\eta + T_2\eta^2 & \eta < 0\end{cases}$$

or ($B_0 = 0$)

$$p(\rho, e) = \Gamma\rho e + \frac{1}{\alpha}p_H(\eta)\left[1 - \frac{1}{2}\Gamma\eta\right]$$
$$p_H(\eta) = A_1\eta + A_2\eta^2 + A_3\eta^3$$

together with

$$\eta(\rho) = \frac{\alpha\rho}{\alpha_0\rho_0} - 1 .$$

For the shear strength description, we use

$$p^* = \frac{p}{f_c}$$

as the pressure normalized with the compressive strength parameter. We also use **s** to denote the deviatoric stress tensor and $\dot{\varepsilon}_p$ the plastic strain rate. The effective plastic strain is thus denoted $\varepsilon_p$ and can be viewed as such in the post processor of choice.

For a given stress state and rate of loading, the elastic-plastic yield surface for the RHT model is given by

$$\sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, \varepsilon_p^*) = f_c \sigma_y^*(p^*, F_r(\dot{\varepsilon}_p, p^*), \varepsilon_p^*) R_3(\theta, p^*)$$

and is the composition of two functions and the compressive strength parameter $f_c$. The first describes the pressure dependence for principal stress conditions $\sigma_1 < \sigma_2 = \sigma_3$ and is expressed in terms of a failure surface and normalized plastic strain as

$$\sigma_y^*(p^*, F_r, \varepsilon_p^*) = \sigma_f^* \left( \frac{p^*}{\gamma}, F_r \right) \gamma$$

with

$$\gamma = \varepsilon_p^* + (1 - \varepsilon_p^*) F_e F_c \, .$$

The failure surface is given as

$$\sigma_f^*(p^*, F_r) = \begin{cases} A \left[ p^* - \dfrac{F_r}{3} + \left( \dfrac{A}{F_r} \right)^{-1/n} \right]^n & 3p^* \geq F_r \\ \dfrac{F_r f_s^*}{Q_1} + 3p^* \left( 1 - \dfrac{f_s^*}{Q_1} \right) & F_r > 3p^* \geq 0 \\ \dfrac{F_r f_s^*}{Q_1} - 3p^* \left( \dfrac{1}{Q_2} - \dfrac{f_s^*}{Q_1 f_t^*} \right) & 0 > 3p^* > 3p_t^* \\ 0 & 3p_t^* > 3p^* \end{cases}$$

in which $p_t^*$ is the failure cut-off pressure

$$p_t^* = \frac{F_r Q_2 f_s^* f_t^*}{3(Q_1 f_t^* - Q_2 f_s^*)} \, ,$$

$F_r$ is a dynamic increment factor, and

$$Q_1 = R_3 \left( \frac{\pi}{6}, 0 \right)$$
$$Q_2 = Q(p^*)$$

In these expressions, $f_t^*$ and $f_s^*$ are the tensile and shear strength of the concrete relative to the compressive strength $f_c$ and the $Q$ values are introduced to account for the tensile and shear meridian dependence. Further details are given in the following.

To describe reduced strength on shear and tensile meridian the factor

$$R_3(\theta,p^*)=\frac{2(1-Q^2)\cos\theta+(2Q-1)\sqrt{4(1-Q^2)\cos^2\theta+5Q^2-4Q}}{4(1-Q^2)\cos^2\theta+(1-2Q)^2}$$

is introduced, where $\theta$ is the Lode angle given by the deviatoric stress tensor $\mathbf{s}$ as

$$\cos 3\theta=\frac{27\det(\mathbf{s})}{2\overline{\sigma}(\mathbf{s})^3}$$

$$\overline{\sigma}(\mathbf{s})=\sqrt{\frac{3}{2}\mathbf{s}:\mathbf{s}}$$

The maximum reduction in strength is given as a function of relative pressure

$$Q=Q(p^*)=Q_0+Bp^*\ .$$

Finally, the strain rate dependence is given by

$$F_r(\dot{\varepsilon}_p,p^*)=\begin{cases}F_r^c & 3p^*\geq F_r^c\\ F_r^c-\dfrac{3p^*-F_r^c}{F_r^c+F_r^tf_t^*}\left(F_r^t-F_r^c\right) & F_r^c>3p^*\geq -F_r^tf_t^*\\ F_r^t & -F_r^tf_t^*>3p^*\end{cases}$$

in which

$$F_r^c\left(\dot{\varepsilon}_p\right)=\begin{cases}\left(\dfrac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0^c}\right)^{\beta_c} & \dot{\varepsilon}_p^c\geq\dot{\varepsilon}_p\\ \gamma_c\sqrt[3]{\dot{\varepsilon}_p} & \dot{\varepsilon}_p>\dot{\varepsilon}_p^c\end{cases}$$

$$F_r^t\left(\dot{\varepsilon}_p\right)=\begin{cases}\left(\dfrac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0^t}\right)^{\beta_t} & \dot{\varepsilon}_p^t\geq\dot{\varepsilon}_p\\ \gamma_t\sqrt[3]{\dot{\varepsilon}_p} & \dot{\varepsilon}_p>\dot{\varepsilon}_p^t\end{cases}$$

The parameters involved in these expressions are given as ($f_c$ is in MPa below)

$$\beta_c=\frac{4}{20+3f_c}$$

$$\beta_t=\frac{2}{20+f_c}$$

$\gamma_c$ and $\gamma_t$ are determined from continuity requirements. However, $\beta_c$ and $\beta_t$ can be directly input with BETAC and BETAT. $F_r^c\left(\dot{\varepsilon}_p\right)$ and $F_r^t\left(\dot{\varepsilon}_p\right)$ can also be defined directly as curves; see the descriptions for the less than zero case of BETAC and BETAT.

The elastic strength parameter used above is given by

$$F_e(p^*)=\begin{cases}g_c^* & 3p^*\geq F_r^cg_c^*\\ g_c^*-\dfrac{3p^*-F_r^cg_c^*}{F_r^cg_c^*+F_r^tg_t^*f_t^*}\left(g_t^*-g_c^*\right) & F_r^cg_c^*>3p^*\geq -F_r^tg_t^*f_t^*\\ g_t^* & -F_r^tg_t^*f_t^*>3p^*\end{cases}$$

while the cap of the yield surface is represented by

$$F_c(p^*) = \begin{cases} 0 & p^* \geq p_c^* \\ \sqrt{1 - \left(\dfrac{p^* - p_u^*}{p_c^* - p_u^*}\right)^2} & p_c^* > p^* \geq p_u^* \\ 1 & p_u^* > p^* \end{cases}$$

where

$$p_c^* = \frac{p_c}{f_c}$$
$$p_u^* = \frac{F_r^c g_c^*}{3} + \frac{G^* \varepsilon_p}{f_c}$$

The hardening behavior is described linearly with respect to the plastic strain, where

$$\varepsilon_p^* = \min\left(\frac{\varepsilon_p}{\varepsilon_p^h}, 1\right)$$
$$\varepsilon_p^y = \frac{\sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, \varepsilon_p^*)(1 - F_e F_c)}{\gamma 3G^*}$$

here

$$G^* = \xi G$$

where $G$ is the shear modulus of the virgin material and $\xi$ is a reduction factor representing the hardening in the model.

When hardening states reach the ultimate strength of the concrete on the failure surface, damage is accumulated during further inelastic loading controlled by plastic strain. To this end, the plastic strain at failure is given as

$$\varepsilon_p^f = \begin{cases} D_1[p^* - (1-D)p_t^*]^{D_2} & p^* \geq (1-D)p_t^* + \left(\dfrac{\varepsilon_p^m}{D_1}\right)^{1/D_2} \\ \varepsilon_p^m & (1-D)p_t^* + \left(\dfrac{\varepsilon_p^m}{D_1}\right)^{1/D_2} > p^* \end{cases}$$

The damage parameter is accumulated with plastic strain according to

$$D = \int_{\varepsilon_p^h}^{\varepsilon_p} \frac{d\varepsilon_p}{\varepsilon_p^f}$$

and the resulting damage surface is given as

$$\sigma_d(p^*, \mathbf{s}, \dot{\varepsilon}_p) = \begin{cases} \sigma_y(p^*, \mathbf{s}, \dot{\varepsilon}_p, 1)(1-D) + D f_c \sigma_r^*(p^*) & 0 \leq p^* \\ \sigma_y\left(p^*, \mathbf{s}, \dot{\varepsilon}_p, 1\right)\left(1 - D - \dfrac{p^*}{p_t^*}\right) & (1-D)p_t^* \leq p^* < 0 \end{cases}$$

where

$$\sigma_r^*(p^*) = A_f\{p^*\}^{n_f} .$$

Plastic flow occurs in the direction of deviatoric stress, meaning

$$\dot{\varepsilon}_p \sim \mathbf{s} ,$$

but for tension there is an option to set the parameter PFC to a number corresponding to the influence of plastic volumetric strain. If $\lambda \leq 1$ is used to denote this parameter, then for the special case of $\lambda = 1$

$$\dot{\varepsilon}_p \sim \mathbf{s} - p\mathbf{I} .$$

This was introduced to reduce noise in tension that was observed on some test problems. A failure strain can be used to erode elements with severe deformation which by default is set to 200%.

For simplicity, automatic generation of material parameters is available using ONEMPA < 0; no other parameters are needed. If FC = 0 then the 35 MPa strength concrete in Riedel (2004) is generated in the units specified by the value of ONEMPA. For FC > 0 FC then specifies the actual strength of the concrete in the units specified by the value of ONEMPA. The other parameters are generated by interpolating between the 35 MPa and 140 MPa strength concretes as presented in Riedel (2004). Any automatically generated parameter may be overridden by the user; one of these parameters may be the initial porosity ALPHA0 of the concrete.

For post-processing, the following history variables may be of interest:

| History Variable | Description |
|---|---|
| 2 | Internal energy per volume ($\rho e$) |
| 3 | Porosity value ($\alpha$) |
| 4 | Damage value ($D$) |

or as an alternative use a material history list

*DEFINE_MATERIAL_HISTORIES Properties

| Label | Attributes | Description |
|---|---|---|
| Damage | - - - - | Damage value $D$ |

## *MAT_CONCRETE_DAMAGE_PLASTIC_MODEL

An alternate name for this keyword is:

### *MAT_CDPM

This is Material Type 273. CDPM is a damage plastic concrete model based on Grassl et al. (2011, 2013) and Grassl and Jirásek (2006). This model aims to simulate the failure of concrete structures subjected to dynamic loadings. It describes the characterization of the failure process subjected to multi-axial and rate-dependent loading. The model is based on effective stress plasticity and includes a damage model based on plastic and elastic strain measures. This material model is available only for solid elements.

This material model includes many parameters for the advanced user, but most have default values based on experimental tests. They might not be useful for all concrete and load paths, but the values provide a good starting point. If the default values are not good enough, see the remarks at the end for a discussion of these parameters.

More details on this material can be found at: https://petergrassl.com/research/damage-plasticity/

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | ECC | QH0 | FT | FC |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | 0.2 | AUTO | 0.3 | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HP | AH | BH | CH | DH | AS | DF | FC0 |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.5 | 0.08 | 0.003 | 2.0 | 1.0E-6 | 15.0 | 0.85 | AUTO |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TYPE | BS | WF | WF1 | FT1 | STRFLG | FAILFLG | EFC |
| Type | F | F | F | F | F | F | F | F |
| Default | 0.0 | 1.0 | none | 0.15 × WF | 0.3 × FT | 0.0 | 0.0 | 1.0E-4 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E | Young's modulus. The sign determines if an anisotropic (E positive) or an isotropic (E negative) damage formulation is used. See Remark 1. The Young's modulus is taken as the absolute value of this parameter. See Remark 3. |
| PR | Poisson's ratio |
| ECC | Eccentricity parameter. See Remark 2. EQ.0.0: ECC is calculated from Jirásek and Bazant (2002) as $$\text{ECC} = \frac{1+\epsilon}{2-\epsilon}, \qquad \epsilon = \frac{f_t(f_{bc}^2 - f_c^2)}{f_{bc}(f_c^2 - f_t^2)}, \qquad f_{bc} = 1.16 f_c$$ |
| QH0 | Initial hardening defined as $FC_0/FC$ where $FC_0$ is the compressive stress at which the initial yield surface is reached. |
| FT | Uniaxial tensile strength (stress), $f_t$. See Remarks 2 and 3. |
| FC | Uniaxial compression strength (stress), $f_c$. See Remarks 2 and 3. |
| HP | Hardening parameter, $H_p$. The default, HP = 0.5, is the value used in Grassl et al. (2011) for a strain-rate-dependent material response (STRFLG = 1). For applications without a strain rate effect (STRFLG = 0), a value of HP = 0.01 is recommended, which has been used in Grassl et al. (2013). See Remark 2. |
| AH | Hardening ductility parameter 1, $A_h$. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| BH | Hardening ductility parameter 2, $B_h$. See Remark 2. |
| CH | Hardening ductility parameter 3, $C_h$. See Remark 2. |
| DH | Hardening ductility parameter 4, $D_h$. See Remark 2. |
| AS | Ductility parameter during damage, $A_s$. See Remark 3. |
| DF | Flow rule parameter, $D_f$. See Remark 2. |
| FC0 | Rate-dependent parameter, $f_{c0}$. It is only needed if STRFLG = 1. The recommended value is 10 MPa, which has to be entered consistently with the system of units used. See Remark 4. |
| TYPE | Flag for damage type (see Remark 3):<br>EQ.0.0: Linear damage formulation<br>EQ.1.0: Bilinear damage formulation<br>EQ.2.0: Exponential damage formulation<br>EQ.3.0: No damage<br>The best results are obtained with the bilinear formulation. |
| BS | Damage ductility exponent during damage, $B_s$. See Remark 3. |
| WF | Tensile threshold value for linear damage formulation, $w_f$. It controls the tensile softening branch of the exponential tensile damage formulation. See Remark 3. |
| WF1 | Tensile threshold value for the second part of the bilinear damage formulation, $w_{f1}$. The default is 0.15 × WF. See Remark 3. |
| FT1 | Tensile strength threshold value for the bilinear damage formulation, $f_{t1}$. The default is 0.3 × FT. See Remark 3. |
| STRFLG | Strain rate flag:<br>EQ.1.0: Strain rate dependent (see Remark 4)<br>EQ.0.0: Not strain rate dependent |
| FAILFLG | Failure flag.<br>EQ.0.0: Not active, meaning no erosion<br>GT.0.0: Active. Ane element erodes if $\omega_t$ and $\omega_c$ equal 1 in FAILFLG percent of the integration points. For |

| VARIABLE | DESCRIPTION |
|---|---|
| | example, if FAILFLG = 0.60, 60% of all integration points must fail before erosion. |
| EFC | Parameter controlling the compressive damage softening branch of the exponential compressive damage formulation, $\varepsilon_{fc}$. See Remark 3. |

**Remarks:**

1. **Stress depending on the damage model.** The stress for the anisotropic damage plasticity model (E > 0 in the input) is defined as

$$\boldsymbol{\sigma} = (1-\omega_t)\boldsymbol{\sigma}_t + (1-\omega_c)\boldsymbol{\sigma}_c$$

where $\boldsymbol{\sigma}_t$ and $\boldsymbol{\sigma}_c$ are the positive and negative part of the effective stress, $\boldsymbol{\sigma}_{eff}$, determined in the principal stress space. The scalar functions $\omega_t$ and $\omega_c$ are damage parameters.

The stress for the isotropic damage plasticity model (E < 0 in the input) is defined as

$$\boldsymbol{\sigma} = (1-\omega_t)\boldsymbol{\sigma}_{eff}$$

The effective stress, $\boldsymbol{\sigma}_{eff}$, is defined according to the damage mechanics convention as

$$\boldsymbol{\sigma}_{eff} = \mathbf{D}_e : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p)$$

2. **Plasticity.** The yield surface is described by the Haigh-Westergaard coordinates: the volumetric effective stress, $\sigma_v$, the norm of the deviatoric effective stress, $\rho$, and the Lode angle, $\theta$. The following equation gives the yield surface:

$$f_p(\sigma_v,\rho,\theta,\kappa) = \left[[1-q_1(\kappa)]\left(\frac{\rho}{\sqrt{6}f_c}+\frac{\sigma_v}{f_c}\right)^2 + \sqrt{\frac{3}{2}}\frac{\rho}{f_c}\right]^2 + m_0 q_1(\kappa)^2 q_2(\kappa)\left[\frac{\rho}{\sqrt{6}f_c}r(\cos\theta)+\frac{\sigma_v}{f_c}\right] - q_1^2(\kappa)q_2^2(\kappa)\,.$$

The variables $q_1$ and $q_2$ depend on the hardening variable $\kappa$. The parameter $f_c$ is the uniaxial compressive strength.

The following function controls the shape of the deviatoric part:

$$r(\cos\theta) = \frac{4\left(1-e^2\right)\cos^2\theta + (2e-1)^2}{2(1-e^2)\cos\theta + (2e-1)\sqrt{4(1-e^2)\cos^2\theta + 5e^2 - 4e}}$$

where $e$ is the eccentricity parameter (ECC). The parameter $m_0$ is the friction parameter. It is defined as:

$$m_0 = \frac{3(f_c^2 - f_t^2)}{f_c f_t}\frac{e}{e+1},$$

where $f_t$ is the tensile strength.

The flow rule is non-associative, meaning that the plastic flow's direction is not normal to the yield surface. This aspect is essential for modeling concrete because an associative flow rule gives an overestimated maximum stress. The plastic potential is given by:

$$g(\sigma_v, \rho, \kappa) = \left\{[1 - q_1(\kappa)]\left(\frac{\rho}{\sqrt{6}f_c} + \frac{\sigma_v}{f_c}\right)^2 + \sqrt{\frac{3}{2}}\frac{\rho}{f_c}\right\}^2 + q_1(\kappa)\left(\frac{m_0\rho}{\sqrt{6}f_c} + \frac{m_g(\sigma_v,\kappa)}{f_c}\right),$$

where

$$m_g(\sigma_v, \kappa) = A_g(\kappa)B_g(\kappa)f_c e^{\frac{\sigma_v - q_2 f_t/3}{B_g f_c}}$$

and

$$A_g = \frac{3f_t q_2(\kappa)}{f_c} + \frac{m_0}{2}, \qquad B_g = \frac{q_2(\kappa)}{3}\frac{1 + f_t/f_c}{\ln\dfrac{A_g}{3q_2 + \dfrac{m_0}{2}} + \ln\left(\dfrac{D_f + 1}{2D_f - 1}\right)}$$

The hardening laws $q_1$ and $q_2$ control the shape of the yield surface and the plastic potential. They are defined as:

$$q_1(\kappa) = \begin{cases} q_{h0} + (1 - q_{h0})(\kappa^3 - 3\kappa^2 + 3\kappa) - H_p(\kappa^3 - 3\kappa^2 + 2\kappa) & \text{if } \kappa < 1 \\ 1 & \text{if } \kappa \geq 1 \end{cases}$$

$$q_2(\kappa) = \begin{cases} 1 & \text{if } \kappa < 1 \\ 1 + H_p(\kappa - 1) & \text{if } \kappa \geq 1 \end{cases}$$

The evolution for the hardening variable is given by

$$\dot{\kappa} = \frac{4\dot{\lambda}\cos^2\theta}{x_h(\sigma_v)}\left\|\frac{dg}{d\sigma}\right\|$$

It sets the rate of the hardening variable to the norm of the plastic strain rate scaled by a ductility measure, which is defined as:

$$x_h(\sigma_v) = \begin{cases} A_h - (A_h - B_h)e^{-\frac{R_h(\sigma_v)}{C_h}} & \text{if } R_h(\sigma_v) \geq 0 \\ E_h e^{\frac{R_h(\sigma_v)}{F_h}} + D_h & \text{if } R_h(\sigma_v) < 0 \end{cases}$$

Here,

$$E_h = B_h - D_h, \qquad F_h = \frac{(B_h - D_h)C_h}{A_h - B_h}, \qquad R_h(\sigma_v) = -\frac{\sigma_v}{f_c} - \frac{1}{3}$$

3. **Damage.** Damage initializes when the equivalent strain, $\tilde{\varepsilon}$, reaches the threshold value $\varepsilon_0 = f_t/E$, where the equivalent strain is defined as

$$\tilde{\varepsilon} = \frac{\varepsilon_0 m_0}{2}\left[\frac{\rho}{\sqrt{6}f_c}r(\cos\theta) + \frac{\sigma_V}{f_c}\right] + \sqrt{\frac{\varepsilon_0^2 m_0^2}{4}\left(\frac{\rho}{\sqrt{6}f_c}r(\cos\theta) + \frac{\sigma_V}{f_c}\right)^2 + \frac{3\varepsilon_0^2\rho^2}{2f_c^2}}$$

A stress-inelastic displacement law describes tensile damage. For linear, bilinear, and exponential damage types, the stress value $f_t$ and the displacement value $w_f$ must be defined. Additional parameters $f_{t1}$ and $w_{f1}$ must be defined for the bilinear type. Figure M273-1 illustrates how the input parameters control the stress softening for the different damage models.

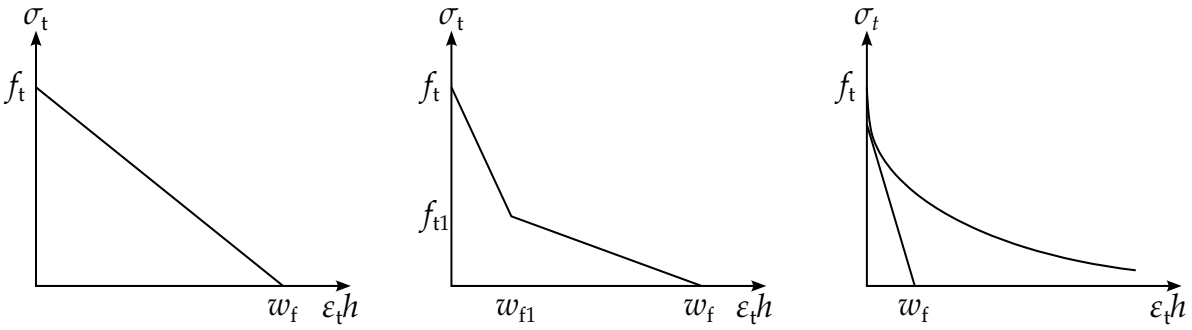



**Figure M273-1.** Stress softening due to damage in tension. The figures from left to right show this behavior for linear, bilinear, and exponential damage, respectively.

The variable $h$ in Figure M273-1 is a mesh-dependent measure used to convert strains to displacements. The variable $\varepsilon_t$ is called the inelastic tensile strain and is defined as the sum of the irreversible plastic strain $\varepsilon_p$ and the reversible strain $w_t(\varepsilon - \varepsilon_p)$ (in compression $w_c(\varepsilon - \varepsilon_p)$).

A damage ductility measure, $x_s$, models the influence of multi-axial stress states on the softening:

$$x_s = 1 + (A_s - 1)R_s^{B_s}$$

Here, $A_s$ and $B_s$ are input parameters, and

$$R_s = \begin{cases} -\dfrac{\sqrt{6}\sigma_V}{\rho} & \text{if } \sigma_V \le 0 \\ 0 & \text{if } \sigma_V > 0 \end{cases}$$

The inelastic strain is then modified according to:

$$\varepsilon_i = \frac{\varepsilon_i}{x_s}$$

An exponential stress-inelastic strain law controls compressive damage. Stress value $f_c$ and inelastic strain $\varepsilon_{fc}$ need to be specified. Figure M273-2 illustrates

how the input parameters affect stress softening. A small value of $\varepsilon_{fc}$, such as 1.0E-4 (the default), causes a brittle form of damage.

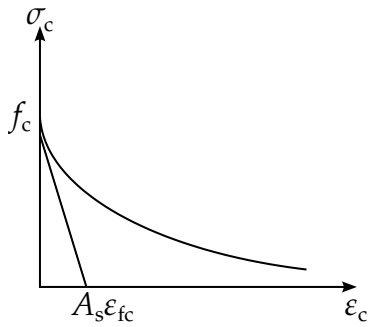



**Figure M273-2.** Stress softening due to damage in compression

4. **Strain rate.** Concrete is strongly rate dependent. If the loading rate increases, the tensile and compressive strengths increase and are more prominent in tension than in compression. $\alpha_r \geq 1$ models this dependency. The rate dependency is included by scaling both the equivalent strain rate and the inelastic strain. The rate parameter is defined by

$$\alpha_r = (1 - X_{\text{compression}})\alpha_{rt} + X_{\text{compression}}\alpha_{rc} \ ,$$

where $X_{\text{compression}}$ is a continuous compression measure (= 1 means only compression, = 0 means only tension). For tension:

$$\alpha_{rt} = \begin{cases} 1 & \text{if} \quad \dot{\varepsilon}_{max} < 30 \times 10^{-6}\ s^{-1} \\ \left(\dfrac{\dot{\varepsilon}_{max}}{\dot{\varepsilon}_{t0}}\right)^{\delta_t} & \text{if } 30 \times 10^{-6} < \dot{\varepsilon}_{max} < 1\ s^{-1} \\ \beta_t \left(\dfrac{\dot{\varepsilon}_{max}}{\dot{\varepsilon}_{t0}}\right)^{\frac{1}{3}} & \text{if} \quad \dot{\varepsilon}_{max} > 1\ s^{-1} \end{cases}$$

where $\delta_t = \frac{1}{1+8f_c/f_{c0}}$, $\beta_t = e^{6\delta_t - 2}$, and $\dot{\varepsilon}_{t0} = 1 \times 10^{-6}\ s^{-1}$. For compression, the corresponding rate factor is given by:

$$\alpha_{rc} = \begin{cases} 1 & \text{if} \quad |\dot{\varepsilon}_{min}| < 30 \times 10^{-6}\ s^{-1} \\ \left[S\dfrac{|\dot{\varepsilon}_{min}|}{\dot{\varepsilon}_{c0}}\right]^{1.026\delta_c} & \text{if } 30 \times 10^{-6} < |\dot{\varepsilon}_{min}| < 1\ s^{-1} \\ \beta_c \left[\dfrac{|\dot{\varepsilon}_{min}|}{\dot{\varepsilon}_{c0}}\right]^{\frac{1}{3}} & \text{if} \quad |\dot{\varepsilon}_{min}| > 30\ s^{-1} \end{cases}$$

where $\delta_c = \frac{1}{5+9f_c/f_{c0}}$, $\beta_c = e^{6.156\delta_c - 2}$, and $\dot{\varepsilon}_{c0} = 30 \times 10^{-6}\ s^{-1}$. $f_{c0}$ is an input parameter. A recommended value is 10 MPa.

5. **History variables.** Extra history variables of interest are listed in the following table. Set NEIPH in *DATABASE_EXTENT_BINARY to request these variables.

| History Variable # | Description |
|---|---|
| 1 | Hardening variable, $\kappa$. See Remark 2. |
| 15 | Damage in tension, $\omega_t$. See Remark 1. |
| 16 | Damage in compression, $\omega_c$. See Remark 1. |

## *MAT_PAPER

This is Material Type 274. This is an orthotropic elastoplastic model for paper materials, based on Xia (2002) and Nygards (2009). It is available for solid and shell elements. Solid elements use a hyperelastic-plastic formulation, while shell elements use a hypoelastic-plastic formulation. The material is available for explicit and implicit simulations; see Remark 5.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E1 | E2 | E3 | PR21 | PR32 | PR31 |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| G12 | G23 | G13 | E3C | CC | TWOK | | ROT |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| S01 | A01 | B01 | C01 | S02 | A02 | B02 | C02 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| S03 | A03 | B03 | C03 | S04 | A04 | B04 | C04 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| S05 | A05 | B05 | C05 | PRP1 | PRP2 | PRP4 | PRP5 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| ASIG | BSIG | CSIG | TAU0 | ATAU | BTAU | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | E2 | E3 | PR21 | PR32 | PR31 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Material density |
| E*i* | Young's modulus in direction *i*, $E_i$ |
| PR*ij* | Elastic Poisson's ratio $\nu_{ij}$ |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G12 | G23 | G13 | E3C | CC | TWOK | | ROT |
| Type | F | F | F | F | F | F | | F |
| Default | none | none | none | none | none | none | | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| G*ij* | Elastic shear modulus in direction, $G_{ij}$ |
| E3C | Elastic compression parameter |
| CC | Elastic compression exponent |
| TWOK | Exponent in in-plane yield surface |

| VARIABLE | DESCRIPTION |
|---|---|
| ROT | Option for two-dimensional solids (shell element forms 13, 14, or 15):<br>EQ.0.0: No rotation of material axes (default). Direction of material axes are solely defined by AOPT. It is only possible to rotate in shell-plane.<br>EQ.1.0: Rotate coordinate system around material 1-axis such that 2-axis coincides with shell normal. This rotation is done in addition to AOPT.<br>EQ.2.0: Rotate coordinate system around material 2-axis such that 1-axis coincides with shell normal. This rotation is done in addition to AOPT. |

**In plane Yield Surface Card 1.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S01 | A01 | B01 | C01 | S02 | A02 | B02 | C02 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

**In plane Yield Surface Card 2.**

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S03 | A03 | B03 | C03 | S04 | A04 | B04 | C04 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

**In plane Yield Surface Card 3.**

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S05 | A05 | B05 | C05 | PRP1 | PRP2 | PRP4 | PRP5 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | 1/2 | 2/15 | 1/2 | 2/15 |

| VARIABLE | DESCRIPTION |
|---|---|
| S0*i* | $i^{th}$ in-plane plasticity yield parameter<br>LT.0.0: \|S0*i*\| is a load curve ID; see Remark 1. |
| A0*i* | $i^{th}$ in-plane plasticity hardening parameter |
| B0*i* | $i^{th}$ in-plane plasticity hardening parameter |
| C0*i* | $i^{th}$ in-plane plasticity hardening parameter |
| PRP1 | Tensile plastic Poisson's ratio in direction 1 |
| PRP2 | Tensile plastic Poisson's ratio in direction 2 |
| PRP4 | Compressive plastic Poisson's ratio in direction 1 |
| PRP5 | Compressive plastic Poisson's ratio in direction 2 |

**Out of Plane and Transverse Shear Yield Surface Card.**

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ASIG | BSIG | CSIG | TAU0 | ATAU | BTAU | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| ASIG | Out-of-plane plasticity yield parameter |

| VARIABLE | DESCRIPTION |
|---|---|
| BSIG | Out-of-plane plasticity hardening parameter |
| CSIG | Out-of-plane plasticity hardening parameter |
| TAU0 | Transverse shear plasticity yield parameter |
| ATAU | Transverse shear plasticity hardening parameter |
| BTAU | Transverse shear plasticity hardening parameter |

**Orthotropic Parameter Card 1.**

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | MACF | XP | YP | ZP | A1 | A2 | A3 |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a-**direction. This option is for solid elements only.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR<br>EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of |

| VARIABLE | DESCRIPTION |
|---|---|
| | the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_*OPTION*). |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, if AOPT = 3, the BETA input on Card 8 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

**Orthotropic Parameter Card 2.**

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | |
| Type | F | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 3. It may be overridden on the element card; see *ELEMENT_SHELL_BETA or *ELEMENT_SOLID_ORTHO. |

**Remarks:**

1. **Hardening function.** Each hardening function, $q_i$ (note that $q_6 = q_3$), is given by a load curve if $S_i^0 < 0$, otherwise

$$q_i\left(\varepsilon_p^f\right) = S_i^0 + A_i^0 \tanh\left(B_i^0 \varepsilon_p^f\right) + C_i^0 \varepsilon_p^f.$$

2. **Material model for solid elements.** The stress-strain relationship for solid elements is based on a multiplicative split of the deformation gradient into an elastic and a plastic part

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_p \ .$$

The elastic Green strain is formed as

$$\mathbf{E}_e = \frac{1}{2}\left(\mathbf{F}_e^{\mathrm{T}} \mathbf{F}_e - \mathbf{I}\right) \ ,$$

and the 2nd Piola-Kirchhoff stress as

$$\mathbf{S} = \mathbf{C}\mathbf{E}_e \ ,$$

where the constitutive matrix is taken as orthotropic and can be represented in Voigt notation by its inverse as

$$\mathbf{C}^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\frac{v_{21}}{E_2} & -\frac{v_{31}}{E_3} & & & \\ -\frac{v_{12}}{E_1} & \frac{1}{E_2} & -\frac{v_{32}}{E_3} & & & \\ -\frac{v_{13}}{E_1} & -\frac{v_{23}}{E_2} & \frac{1}{E_3} & & & \\ & & & \frac{1}{G_{12}} & & \\ & & & & \frac{1}{G_{23}} & \\ & & & & & \frac{1}{G_{13}} \end{bmatrix} .$$

In out-of-plane compression the stress is modified according to

$$S_{33} = C_{31}E^e_{11} + C_{32}E^e_{22} + \begin{cases} E_3 E^e_{33}, & E^e_{33} \geq 0 \\ E^c_3[1 - \exp(-C_c E^e_{33})], & E^e_{33} < 0 \end{cases}$$

Three yield surfaces are present: in-plane, out-of-plane, and transverse shear. The in-plane yield surface is given as (see Remark 1)

$$f = \sum_{i=1}^{6} \left[ \frac{\max(0, S{:}N_i)}{q_i\left(\varepsilon^f_p\right)} \right]^{2k} - 1 \leq 0 \ ,$$

with the six yield plane normals (in strain Voigt notation)

$$N_1 = \left[ \frac{1}{\sqrt{1+v^2_{1p}}} \quad -\frac{v_{1p}}{\sqrt{1+v^2_{1p}}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^{\mathrm{T}} ,$$

$$N_2 = \left[ -\frac{v_{2p}}{\sqrt{1+v^2_{2p}}} \quad \frac{1}{\sqrt{1+v^2_{2p}}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^{\mathrm{T}} ,$$

$$N_3 = \left[ 0 \quad 0 \quad 0 \quad \sqrt{2} \quad 0 \quad 0 \right]^{\mathrm{T}} ,$$

$$N_4 = -\left[ \frac{1}{\sqrt{1+v^2_{4p}}} \quad -\frac{v_{4p}}{\sqrt{1+v^2_{4p}}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^{\mathrm{T}} ,$$

$$N_5 = -\left[ -\frac{v_{5p}}{\sqrt{1+v^2_{5p}}} \quad \frac{1}{\sqrt{1+v^2_{5p}}} \quad 0 \quad 0 \quad 0 \quad 0 \right]^{\mathrm{T}} ,$$

$$N_6 = -N_3 \ .$$

The yield planes describe the following states

| Plane | Stress State |
|---|---|
| 1 | Tension in material direction 1 |
| 2 | Tension in material direction 2 |
| 3 | Positive shear in the 1, 2-direction |
| 4 | Compression in material direction 1 |
| 5 | Compression in material direction 2 |
| 6 | Negative shear in the 1, 2-direction |

The out-of-plane surface is given as

$$g = \frac{-S_{33}}{A_\sigma + B_\sigma \exp\left(-C_\sigma \varepsilon_p^g\right)} - 1 \leq 0 \ ,$$

and the transverse shear surface is

$$h = \frac{\sqrt{S_{13}^2 + S_{23}^2}}{\tau_0 + [A_\tau - \min(0, S_{33})\, B_\tau]\varepsilon_p^h} - 1 \leq 0 \ .$$

The flow rule is given by the evolution of the plastic deformation gradient

$$\dot{\mathbf{F}}_p = \mathbf{L}_p \mathbf{F}_p \ ,$$

where the plastic velocity gradient is given as

$$\mathbf{L}_p = \begin{bmatrix} \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{11}} & \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{12}} & \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{13}} \\ \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{12}} & \dot{\varepsilon}_p^f \dfrac{\partial f}{\partial S_{22}} & \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{23}} \\ \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{13}} & \dot{\varepsilon}_p^h \dfrac{\partial h}{\partial S_{23}} & \dot{\varepsilon}_p^g \dfrac{\partial g}{\partial S_{33}} \end{bmatrix} \ ,$$

and where it is implicitly assumed that the involved derivatives in the expression of the velocity gradient is appropriately normalized.

3. **Material model for shell elements.** The stress-strain relationship for shell elements is based on an additive split of the rate of deformation into an elastic and a plastic part

$$\mathbf{D} = \mathbf{D}_e + \mathbf{D}_p \ ,$$

and the rate of Cauchy stress is given by

$$\dot{\sigma} = \mathbf{C}\mathbf{D}_e \ .$$

In out-of-plane compression the stress rate is modified according to

$$\dot{\sigma}_{33} = C_{31} D_{11}^e + C_{32} D_{22}^e + D_{33}^e \begin{cases} E_3, & \varepsilon_{33}^e \geq 0 \\ E_3^c C_c \exp(-C_c \varepsilon_{33}^e), & \varepsilon_{33}^e < 0 \end{cases}$$

For shell elements, $D_{33}^p = 0$, and only two yield surfaces are present: the in-plane yield surface (see Remark 1),

$$f = \sum_{i=1}^{6} \left[ \frac{\max(0, \sigma : N_i)}{q_i\left(\varepsilon_{\mathrm{P}}^f\right)} \right]^{2k} - 1 \le 0 \ ,$$

and the transverse-shear yield surface,

$$h = \frac{\sqrt{\sigma_{13}^2 + \sigma_{23}^2}}{\tau_0 + [A_\tau - \min(0, \sigma_{33})\, B_\tau]\varepsilon_{\mathrm{P}}^h} - 1 \le 0 \ .$$

For this case, the plastic flow rule is given by

$$\dot{\boldsymbol{\varepsilon}}_p = \mathbf{D}_p = \mathbf{L}_p \ ,$$

where the plastic velocity gradient is given as

$$\mathbf{L}_p = \begin{bmatrix} \dot{\varepsilon}_{\mathrm{P}}^f \dfrac{\partial f}{\partial \sigma_{11}} & \dot{\varepsilon}_{\mathrm{P}}^f \dfrac{\partial f}{\partial \sigma_{12}} & \dot{\varepsilon}_{\mathrm{P}}^h \dfrac{\partial h}{\partial \sigma_{13}} \\ \dot{\varepsilon}_{\mathrm{P}}^f \dfrac{\partial f}{\partial \sigma_{12}} & \dot{\varepsilon}_{\mathrm{P}}^f \dfrac{\partial f}{\partial \sigma_{22}} & \dot{\varepsilon}_{\mathrm{P}}^h \dfrac{\partial h}{\partial \sigma_{23}} \\ \dot{\varepsilon}_{\mathrm{P}}^h \dfrac{\partial h}{\partial \sigma_{13}} & \dot{\varepsilon}_{\mathrm{P}}^h \dfrac{\partial h}{\partial \sigma_{23}} & 0 \end{bmatrix} .$$

4. **History variables.** The *Effective Plastic Strain* is

$$\varepsilon_p = \sqrt{\left(\varepsilon_p^f\right)^2 + \left(\varepsilon_p^g\right)^2 + \left(\varepsilon_p^h\right)^2} \ .$$

The other history variables are listed below.

| History Variable # | Solid Elements | Shell Elements |
|---|---|---|
| 1 | $\varepsilon_p^f$ | $Q_{11}$ in element to material rotation tensor |
| 2 | $\varepsilon_p^g$ | $Q_{12}$ in element to material rotation tensor |
| 3 | $\varepsilon_p^h$ | $\varepsilon_p^f$ |
| 4 | | $\varepsilon_p^h$ |

5. **Tangent stiffness.** The shell hypoelastic-plastic formulation produces a symmetric tangent stiffness. For the solid hyperelastic-plastic formulation, the tangent stiffness is nonsymmetric. However, unless LCPACK on *CONTROL_IMPLICIT_SOLVER is set to 3, a simplified symmetric tangent will be used for solid elements. This simplified tangent is based on the assumption of small elastic strains. For some problems, using the nonsymmetric tangent significantly improves the convergence rate.

## *MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC

This is Material Type 275, a smooth viscoelastic viscoplastic model based on the works of Hollenstein et.al. [2013, 2014] and Jabareen [2015]. The stress response is rheologically represented by HJR (Hollenstein-Jabareen-Rubin) elements in parallel (see Figure M275-1), where each element exhibits combinations of viscoelastic and viscoplastic characteristics. The model is based on large displacement hyper-elastoplasticity and the numerical implementation is strongly objective; this together with the smooth characteristics makes it especially suitable for implicit analysis.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K | | | | | |
| Type | A | F | F | | | | | |

**HJR Element Cards.** At least 1 and optionally up to 6 cards should be input. The next keyword ("*") card terminates this input.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A0 | B0 | A1 | B1 | M | KAPAS | KAPA0 | SHEAR |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K | Elastic bulk modulus |
| A0 | Rate-dependent understress viscoplastic parameter |
| B0 | Rate-independent understress plasticity parameter |
| A1 | Rate-dependent overstress viscoplastic parameter |
| B1 | Rate-independent overstress plasticity parameter |
| M | Exponential hardening parameter |

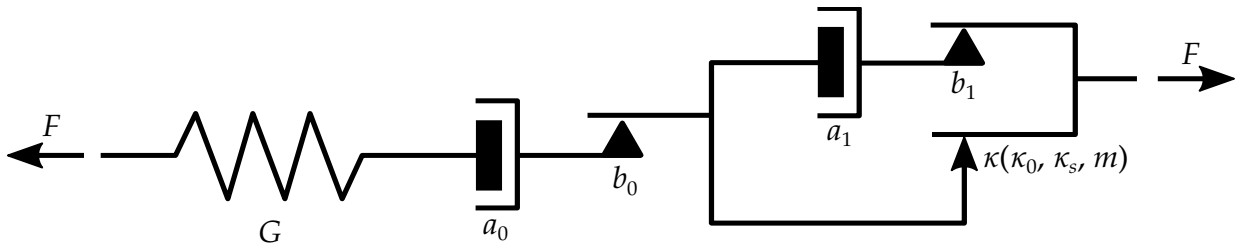



**Figure M275-1.** Rheological representation of an HJR element, including the associated parameters

| VARIABLE | DESCRIPTION |
|---|---|
| KAPAS | Saturated yield strain |
| KAPA0 | Initial yield strain |
| SHEAR | Elastic shear modulus |

**Remarks:**

The Cauchy stress for this smooth viscoelastic viscoplastic material is given by

$$\sigma = K(J-1)\mathbf{I} + \sum_{i=1}^{6} \mathbf{s}_i \quad ,$$

where $K$ is the elastic bulk modulus provided on the first card and $J = \det(\mathbf{F})$ is the relative volume with $\boldsymbol{F}$ being the total deformation gradient. The deviatoric stresses, $\boldsymbol{s}_i$, are coming from the *HJR (Hollenstein-Jabareen-Rubin)* elements in parallel. Up to 6 such elements can be defined for the deviatoric response and a rheological representation of one is shown in Figure M275-1. Each element is associated with 8 material parameters that are provided on the optional cards and characterize its inelastic response. All this allows for a wide range of stress strain relationships. The critical part involves estimating parameters for a given test suite. Some elaboration on the physical interpretation of the individual parameters in the context of uniaxial stress is given following a general description of the model.

We analyze one HJR element by letting $\overline{\mathbf{B}}$ denote the associated isochoric elastic left Cauchy-Green tensor. Define

$$\widetilde{\mathbf{B}} = \overline{\mathbf{B}} - \tfrac{1}{3}\alpha\mathbf{I}, \text{ where } \alpha = \mathrm{tr}(\overline{\mathbf{B}}) \quad .$$

The evolution of $\overline{\mathbf{B}}$ is given by

$$\dot{\overline{\mathbf{B}}} = \mathbf{L}\overline{\mathbf{B}} + \overline{\mathbf{B}}\mathbf{L}^{\mathrm{T}} - \frac{2}{3}\mathrm{tr}(\mathbf{D})\overline{\mathbf{B}} - \dot{\Gamma}\mathbf{A}, \text{ where } \mathbf{A} = \overline{\mathbf{B}} - \left[\frac{3}{\mathrm{tr}(\overline{\mathbf{B}}^{-1})}\right]\mathbf{I} \quad ,$$

where $\mathbf{D}$ is the rate-of-deformation and $\dot{\Gamma}$ governs the inelastic deformation. The functional form of $\dot{\Gamma}$ is summarized in the following set of equations

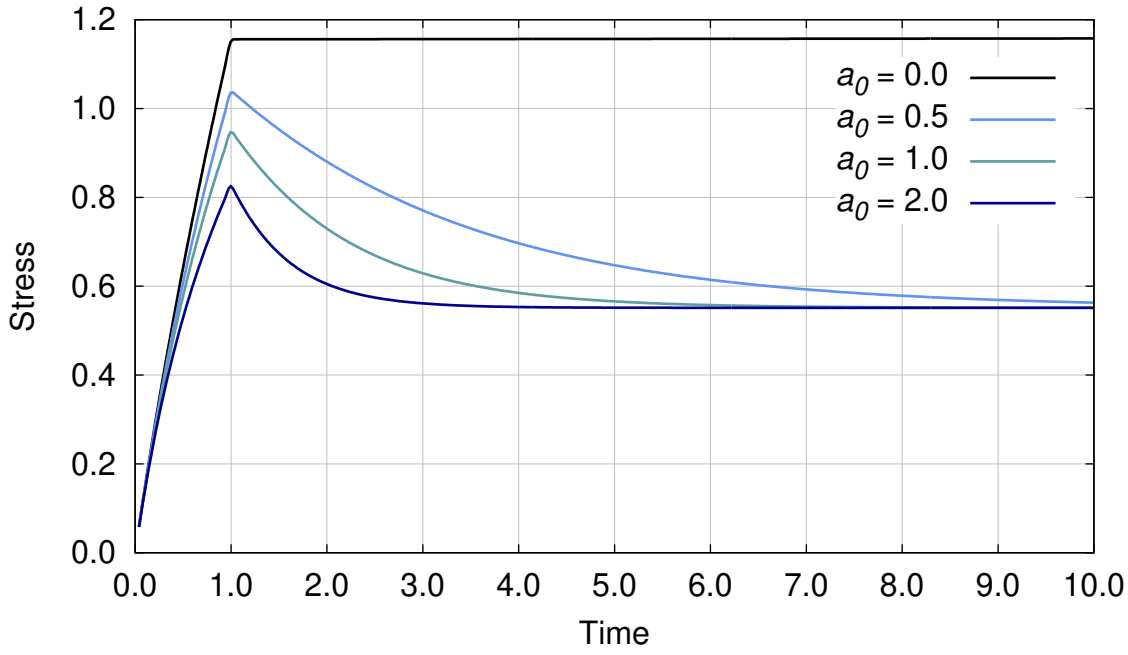



**Figure M275-2.** Influence of parameter $a_0$ on stress relaxation

$$\dot{\Gamma} = \dot{\Gamma}_0 + \langle g \rangle \dot{\Gamma}_1$$
$$\dot{\Gamma}_i = a_i + b_i \dot{\varepsilon}, \qquad i = 0,1$$
$$g = 1 - \frac{\kappa}{\tilde{\gamma}}$$

where

$$\langle g \rangle = \max(0, g)$$
$$\dot{\varepsilon} = \sqrt{\frac{2}{3} \tilde{\mathbf{D}} : \tilde{\mathbf{D}}}$$
$$\tilde{D} = D - \frac{1}{3} \mathrm{tr}(\mathbf{D}) \mathbf{I}$$
$$\tilde{\gamma} = \sqrt{\frac{3}{8} \tilde{\mathbf{B}} : \tilde{\mathbf{B}}}$$
$$\dot{\kappa} = m \dot{\Gamma}_1 \langle g \rangle (\kappa_s - \kappa)$$

A hyperelastic law with a strain energy potential for the distortional deformation given by

$$\psi(\alpha) = \frac{G}{2}(\alpha - 3)$$

yields a contribution to the deviatoric Cauchy stress of

$$\mathbf{s} = G J^{-1} \tilde{\mathbf{B}} \ .$$

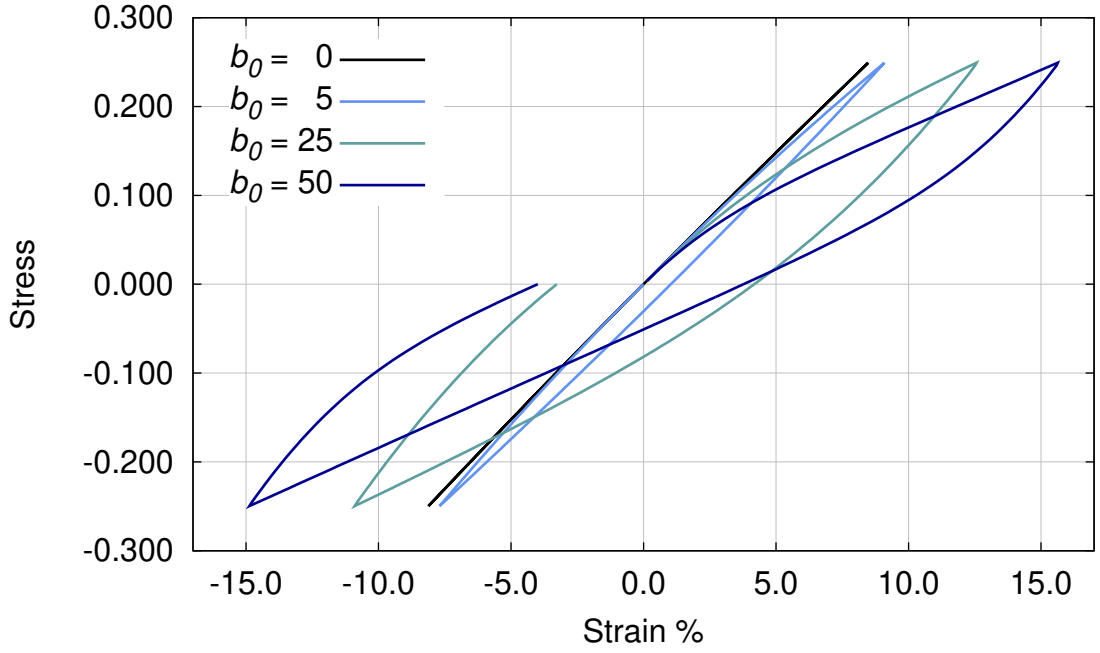



**Figure M275-3.** Influence of $b_0$ in cyclic loading

In uniaxial stress at constant total distortional rate of deformation $\pm\dot{\varepsilon}$ (tension or compression), these equations can be reduced to scalar correspondents

$$\frac{\dot{\bar{b}}}{\bar{b}} = 2\left(\pm\dot{\varepsilon} - \dot{\Gamma}\frac{\bar{b}\sqrt{\bar{b}}-1}{2\bar{b}\sqrt{\bar{b}}+1}\right)$$

$$\tau = G\left(\bar{b} - \frac{1}{\sqrt{\bar{b}}}\right) \tag{M275.1}$$

where $\bar{b}$ is the component of $\overline{\mathbf{B}}$ in the direction of deformation and $\tau$ is the uniaxial Kirchhoff stress. The evolution of $\Gamma$ follows the equations above with

$$\tilde{\gamma} = \tfrac{1}{2}\left|\bar{b} - 1/\sqrt{\bar{b}}\right|.$$

Even though analytical solutions may be out of reach, this would be the basis for estimating as well as interpreting the material parameters. Obviously the shear modulus $G$ (SHEAR) provides the elastic deviatoric stiffness, for a purely elastic material just define one such parameter and leave out all the other parameters on the same card. If several cards are used, the effective elastic shear stiffness is the sum of the contributions from each of the corresponding HJR elements. An interesting observation is that the stress in a HJR element saturates to a value given by the solution of $\bar{b}$ to

$$\begin{aligned}\bar{b}\sqrt{\bar{b}}\left(\pm 2 - \left\{b_0 + b_1 + \frac{a_0+a_1}{\dot{\varepsilon}}\right\}\right) \pm 2\sqrt{\bar{b}}\kappa_s\left(b_1 + \frac{a_1}{\dot{\varepsilon}}\right) \\ + \left(\pm 1 + \left\{b_0 + b_1 + \frac{a_0+a_1}{\dot{\varepsilon}}\right\}\right) = 0\end{aligned} \tag{M275.2}$$

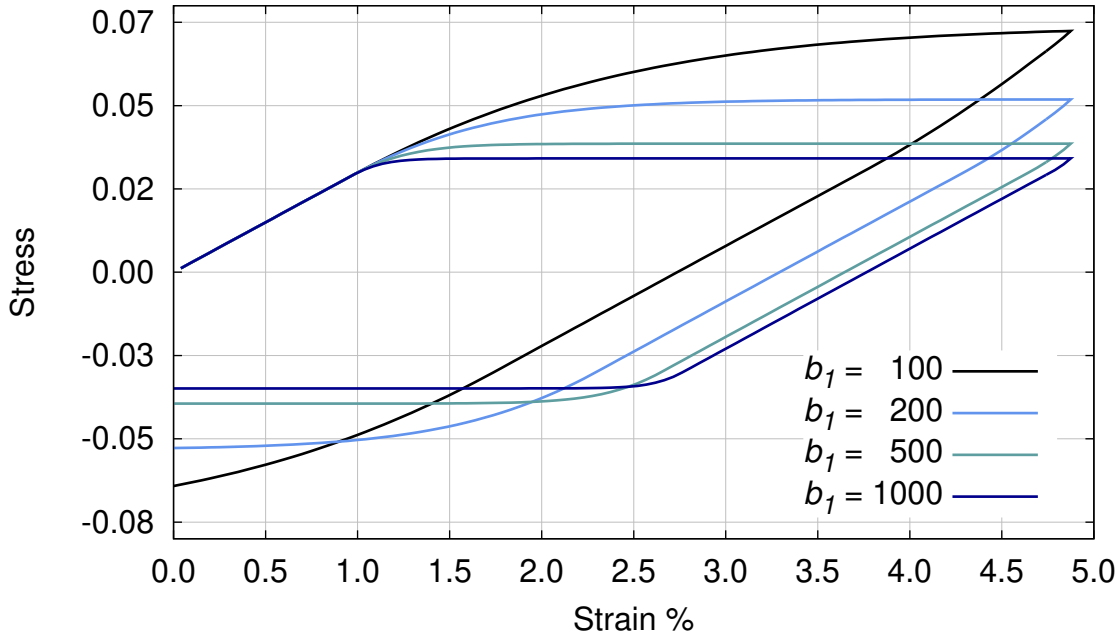



**Figure M275-4.** Effect of $b_1$ in cyclic loading

in tension (+) and compression (−), followed by application of (M275.1) above, assuming that

$$b_0 + b_1 + \frac{a_0 + a_1}{\dot{\varepsilon}} > 2$$

in tension and

$$b_0 + b_1 + \frac{a_0 + a_1}{\dot{\varepsilon}} > 1$$

in compression. This expression will be used in special cases below when examining each inelastic material parameter individually; the material parameters above are input on the HJR element cards as A0, B0, A1, B1 and KAPAS.

A Maxwell material is obtained by providing an element with a nonzero $a_0$ (A0) and with other parameters set to zero. This parameter should be interpreted as the viscoelastic relaxation coefficient determining the rate at which the stress relaxes to zero (see parameter BETA in *MAT_VISCOELASTIC). In Figure M275-2 a stress relaxation is shown for a strain controlled problem using two HJR elements and normalized material parameters using a bulk modulus of $K = 1$. For the first element $G = 0.5$ and for the other $G = 1$ while $a_0$ varies; all other parameters are zero. The engineering strain is ramped to 50% from $t = 0$ to $t = 1$ and then kept constant. The response is very similar to other viscoelastic models in LS-DYNA. Not surprisingly, a HJR element with $a_0 > 0$ (and $a_1 = b_1 = 0$) will always relax to zero stress, which follows from (M275.1) and (M275.2); thus the relaxed stress in this case comes from the purely elastic element. A general viscoelastic material can be obtained by putting several such HJR elements in parallel, in analogy to *MAT_GENERAL_VISCOELASTIC.

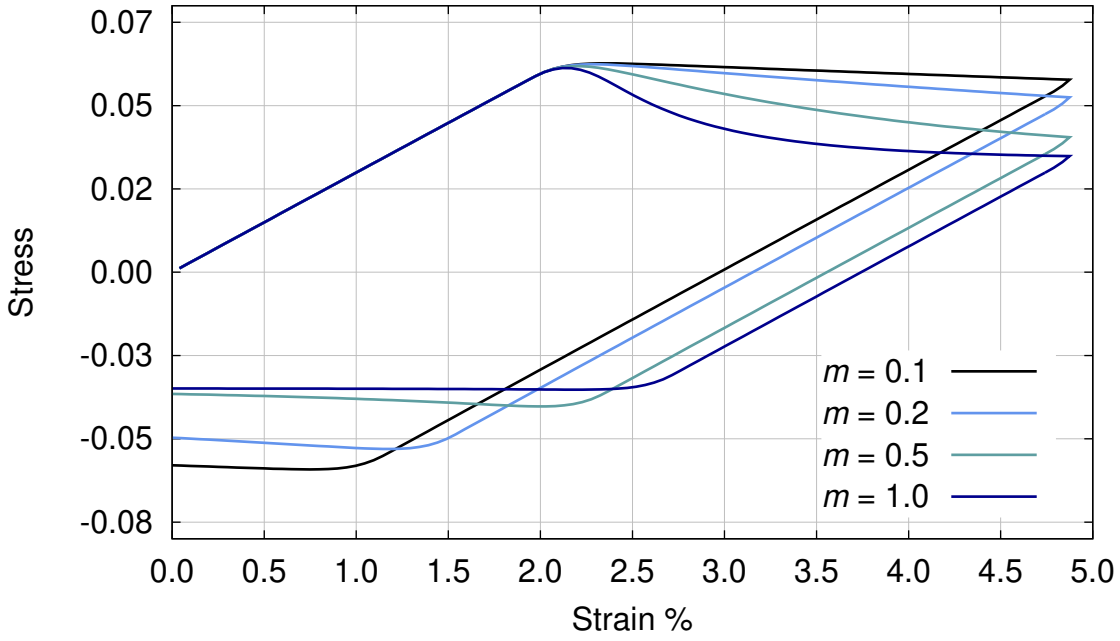



**Figure M275-5.** Softening response in cyclic loading for various values of $m$

For a nonzero $b_0$ (B0) with other parameters set to zero, a rate independent plastic response is obtained exhibiting zero yield stress, that is, inelastic strains develop immediately upon loading. From (M275.2) the value of $b_0$ determines the saturated stress value for the associated HJR element by (M275.1) and

$$\bar{b} = \left(\frac{b_0 \pm 1}{b_0 \mp 2}\right)^{2/3}$$

in tension (+) and compression (−), respectively. A smooth response is obtained that is characterized by hysteresis as shown in Figure M275-3. The same material parameters as in the previous example are used with the exception of varying $b_0$ instead of $a_0$. The deformation is controlled by a cyclic Cauchy stress between $-0.25$ and 0.25; for larger $b_0$ a hysteresis is observed. It should however be mentioned that the hysteresis vanishes as $b_0 \to \infty$ as the stress for the second element saturates quickly to a small value, so it is not trivial to quantitatively estimate the amount of hysteresis for a given parameter setting and deformation.

Rate independent plasticity with a nonzero yield stress can be obtained by a nonzero $b_1$ (B1) in combination with parameters $\kappa_0$ (KAPA0), $\kappa_s$ (KAPAS) and $m$ (M). The yield stress in the sense of von Mises is given by

$$\sigma_Y = 2GJ^{-1}\kappa$$

from which $\kappa$ is interpreted as the current yield strain. Here $b_1$ determines the amount of overstress through (M275.1) and (M275.2), requiring the solution of a non-trivial

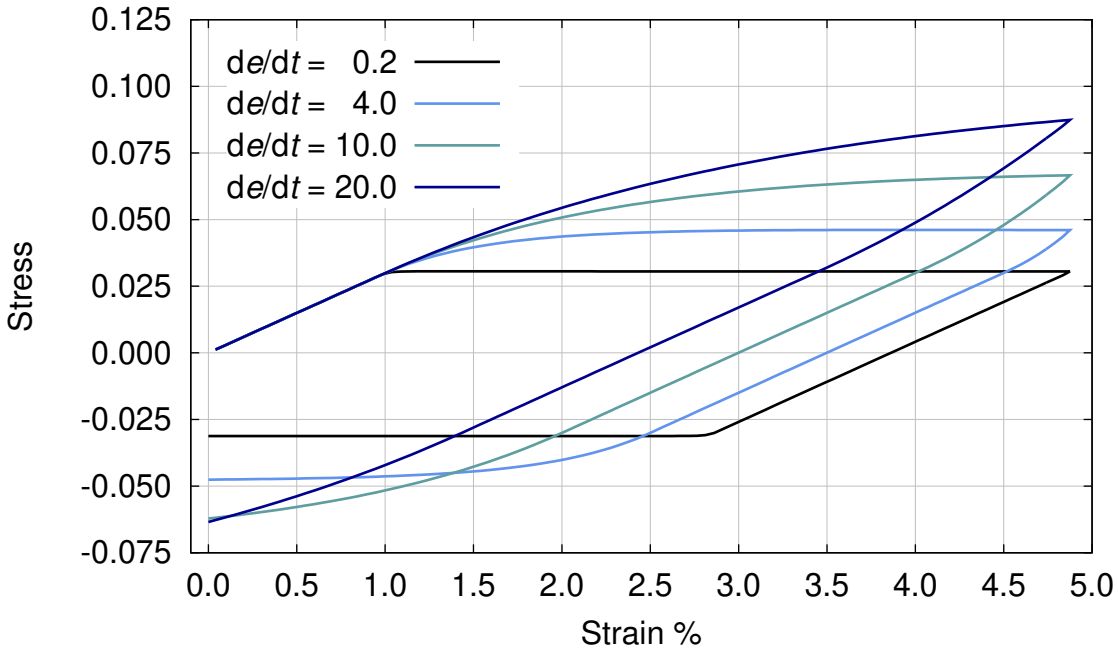



**Figure M275-6.** Strain rate dependence for $a_1 = 1000$ and $b_1 = 10$

polynomial equation. This is exemplified in Figure M275-4 using one HJR element with $K = 1, G = 1.5, \kappa_0 = \kappa_s = 0.01$ and $m = 0$. The engineering strain is ramped up to 5% and down to 0 and $b_1$ is varied with all other parameters zero; the response tends to an elastic-perfectly plastic as $b_1$ increases. The saturated stress value for $b_1 \to \infty$ can be calculated as

$$\bar{b} = \left[ \left( \frac{1}{2} + \sqrt{\frac{1}{4} \mp \frac{8\kappa_s^3}{27}} \right)^{1/3} + \left( \frac{1}{2} - \sqrt{\frac{1}{4} \mp \frac{8\kappa_s^3}{27}} \right)^{1/3} \right]^2 \tag{M275.3}$$

employing (M275.1).

Isotropic strain hardening ($\kappa_s > \kappa_0$) or softening ($\kappa_s < \kappa_0$) is obtained with $m > 0$; $\kappa$ tends exponentially towards $\kappa_s$ at a rate determined by $m$. Using $b_1 = 1000$ (meaning very little overstress), $\kappa_0 = 0.02$, and $\kappa_s = 0.01$ while varying $m$, the softening response in Figure M275-5 is obtained. The rate at which the element hardens is difficult to quantitatively estimate, but presumably it depends not only on $m$ but also on $b_1$. It is important to note however that for small to moderate $b_1$ the model appears to harden with $m = 0$, which is due to larger overstress. The hardening determined by $m$ can be determined from a loading, unloading and reloading cycle to detect how the the yield strain $\kappa$ changes; see Hollenstein et.al. [2013].

Finally, $a_1$ (A1) is the viscoplastic parameter determining how stress responds to change in strain rate. Its interpretation is very similar to that of $a_0$; stress increases with increasing loading rate and relaxes to the saturated stress value given by (M275.1) and (M275.2). In Figure M275-6 a rate dependency is illustrated for $K = 1, G = 1.5, \kappa_0 = \kappa_s = 0.01$ and

$m = 0$, where we have set $a_1 = 1000$ and $b_1 = 10$. The engineering strain rate varies from 0.2 to 20 and for small strain rates (M275.3) can be used for estimating the saturated stress, but in general (M275.2) must be used.

Putting several HJR elements in parallel can thus provide a fairly general combination of viscoelastic/viscoplastic response with isotropic hardening/softening, but this of course requires a rich test suite and a good way of estimating the material parameters. Presumably it is often sufficient to neglect some effects and work with only a subset of the material parameters.

For post-processing, the effective plastic strain in this model is defined as

$$\varepsilon_p = \sqrt{\frac{2}{3}\boldsymbol{\varepsilon}_p : \boldsymbol{\varepsilon}_p} \ ,$$

where

$$\boldsymbol{\varepsilon}_p = \boldsymbol{\varepsilon}_t - \boldsymbol{\varepsilon}_e$$

is a crude estimation of the difference between total and elastic strain. We set

$$\boldsymbol{\varepsilon}_t = \frac{1}{2J}\left[\mathbf{B} - \frac{1}{3}\mathrm{tr}(\mathbf{B})\mathbf{I}\right]$$
$$\boldsymbol{\varepsilon}_e = \frac{1}{2G}\left[\boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})\mathbf{I}\right]$$

where

$$\mathbf{B} = J^{-2/3}\mathbf{F}\mathbf{F}^{\mathrm{T}}$$

and $G$ here is the sum of all shear moduli defined on the HJR element cards. Note that this does not correspond to the traditional measure of effective plastic strain which should be accounted for when validating results.

## *MAT_CHRONOLOGICAL_VISCOELASTIC

This is Material Type 276. This material model provides a general viscoelastic Maxwell model having up to 6 terms in the Prony series expansion and is useful for modeling dense continuum rubbers and solid explosives. It is similar to Material Type 76 but allows the incorporation of aging effects on the material properties. Either the coefficients of the Prony series expansion or a relaxation curve may be specified to define the viscoelastic deviatoric and bulk behavior.

The material model can also be used for laminated shells with either an elastic or viscoelastic layer. To activate the laminated shell, set the formulation flag on *CONTROL_SHELL. With the laminated option, a user-defined integration rule is needed.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BULK | PCF | EF | TREF | A | B |
| Type | A | F | F | F | F | F | F | F |

**Relaxation Curve.** If fitting is done from a relaxation curve, specify fitting parameters on this card. *Otherwise*, if constants are set on Viscoelastic Constant Cards, *LEAVE THIS CARD BLANK*.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCID | NT | BSTART | TRAMP | LCIDK | NTK | BSTARTK | TRAMPK |
| Type | F | I | F | F | F | I | F | F |

**Viscoelastic Constant Cards.** Up to 12 cards may be input. The next keyword ("*") card terminates this input. These cards are not needed if relaxation data is defined. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included. If an elastic layer is defined, only G*i* and K*i* need to be defined (note in an elastic layer only one card is needed).

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| BULK | Elastic bulk modulus |
| PCF | Tensile pressure elimination flag for solid elements only. If set to unity, tensile pressures are set to zero. |
| EF | Elastic flag:<br>EQ.0: Layer is viscoelastic.<br>EQ.1: Layer is elastic. |
| TREF | Reference temperature for shift function (must be greater than zero) |
| A | Chronological coefficient $\alpha(t_a)$. See Remarks below. |
| B | Chronological coefficient $\beta(t_a)$. See Remarks below. |
| LCID | Load curve ID for deviatoric behavior if constants, $G_i$ and $\beta_i$, are determined using a least squares fit. See Figure M76-1 for an example relaxation curve. |
| NT | Number of terms in shear fit. The default is 6. Fewer than NT terms will be used if the fit produces one or more negative shear moduli. Currently, the maximum number is 6. |
| BSTART | In the fit, $\beta_1$ is set to zero, $\beta_2$ is set to BSTART, $\beta_3$ is 10 times $\beta_2$, $\beta_4$ is 10 times $\beta_3$ , and so on. If zero, BSTART is determined by an iterative trial and error scheme. |
| TRAMP | Optional ramp time for loading |
| LCIDK | Load curve ID for bulk behavior if constants, $K_i$, and $\beta_{K_i}$ are determined via a least squares fit. See Figure M76-1 for an example relaxation curve. |
| NTK | Number of terms desired in bulk fit. The default is 6. Currently, the maximum number is 6. |
| BSTARTK | In the fit, $\beta_{K_1}$ is set to zero, $\beta_{K_2}$ is set to BSTARTK, $\beta_{K_3}$ is 10 times $\beta_{K_2}$, $\beta_{K_4}$ is 10 times $\beta_{K_3}$, and so on. If zero, BSTARTK is determined |

| VARIABLE | DESCRIPTION |
|---|---|
| | by an iterative trial and error scheme. |
| TRAMPK | Optional ramp time for bulk loading |
| G*i* | Optional shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Optional shear decay constant for the $i^{th}$ term |
| K*i* | Optional bulk relaxation modulus for the $i^{th}$ term |
| BETAK*i* | Optional bulk decay constant for the $i^{th}$ term |

**Remarks:**

The Cauchy stress, $\sigma_{ij}$, is related to the strain rate by

$$\sigma_{ij}(t) = -p\delta_{ij} + \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau}d\tau$$

For this model, it is postulated that the mathematical form is preserved in the constitutive equation for aging; however two new material functions, $g_0'(t_a)$ and $g_1'(t_a,t)$ are introduced to replace $g_0$ and $g_1(t)$, which is expressed in terms of a Prony series as in material model 76, *MAT_GENERAL_VISCOELASTIC. The aging time is denoted by $t_a$.

$$\sigma_{ij}(t_a,t) = -p\delta_{ij} + \int_0^t g'_{ijkl}(t_a,t-\tau)\frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau}d\tau$$

where

$$g'_{ijkl}(t_a,t) = \alpha(t_a)g_{ijkl}[\beta(t_a)t] \ .$$

Here $\alpha(t_a)$ and $\beta(t_a)$ are two new material properties that are functions of the aging time $t_a$. The material properties functions $\alpha(t_a)$ and $\beta(t_a)$ will be determined using experimental results. For determination of $\alpha(t_a)$ and $\beta(t_a)$, the above equations can be written in the following form

$$\log\left(\sigma_{ij}-p\delta_{ij}\right)_{t_a,t} = \log\alpha(t_a) + \log\left(\sigma_{ij}-p\delta_{ij}\right)_{t_a=0,t\to\xi}$$
$$\log\xi = \log\beta(t_a) + \log t$$

Therefore, if one plots the stress as a function of time on log-log scales, with the vertical axis being the stress and the horizontal axis being the time, then the stress-relaxation curve for any aged time history can be obtained directly from the stress-relaxation curve at $t_a = 0$ by imposing a vertical shift and a horizontal shift on the stress-relaxation curves. The vertical shift and the horizontal shift are $\log\alpha(t_a)$ and $\log\beta(t_a)$ respectively.

## *MAT_ADHESIVE_CURING_VISCOELASTIC

This is Material Type 277. It is useful for modeling adhesive materials during chemical curing. This material model provides a general viscoelastic Maxwell model having up to 16 terms in the Prony series expansion. It is similar to Material Type 76, but the viscoelastic properties not only depend on the temperature but also on an internal variable representing the state of cure for the adhesive. The kinematics of the curing process depend on temperature as well as on temperature rate and follow the Kamal model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | K1 | K2 | C1 | C2 | M | N |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CHEXP1 | CHEXP2 | CHEXP3 | LCCHEXP | LCTHEXP | R | TREFEXP | DOCREXP |
| Type | F | F | F | I | I | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | WLFTREF | WLFA | WLFB | LCG0 | LCK0 | IDOC | INCR | QCURE |
| Type | F | F | F | I | I | F | I | F |

**Viscoelastic Constant Cards.** Up to 16 cards may be input. A keyword ("*") card terminates this input if fewer than 16 cards are used. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETAG*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| K1 | Parameter $k_1$ for Kamal model |
| K2 | Parameter $k_2$ for Kamal model |
| C1 | Parameter $c_1$ for Kamal model |
| C2 | Parameter $c_2$ for Kamal model |
| M | Exponent $m$ for Kamal model |
| N | Exponent $n$ for Kamal model |
| CHEXP1 | Quadratic parameter $\gamma_2$ for chemical shrinkage |
| CHEXP2 | Linear parameter $\gamma_1$ for chemical shrinkage |
| CHEXP3 | Constant parameter $\gamma_0$ for chemical shrinkage |
| LCCHEXP | \|LCCHEXP\| is a load curve ID defining the coefficient for chemical shrinkage $\gamma(\alpha)$ as a function of the state of cure, $\alpha$. If set, parameters CHEXP1, CHEXP2 and CHEXP3 are ignored. See Remark 1 below. |
| LCTHEXP | \|LCTHEXP\| is a load curve ID or table ID defining the coefficient of thermal expansion $\beta(\alpha, T)$ as a function of cure, $\alpha$, and temperature, $T$. If referring to a load curve, parameter $\beta(T)$ is a function of temperature, $T$. |
| R | Gas constant, $R$, for Kamal model |
| TREFEXP | Reference temperature, $T_0$, for secant form of thermal expansion. See Remark 1 below. |
| DOCREXP | Reference degree of cure, $\alpha_0$, for sequential form of chemical expansion. See Remark 1 below. |
| WLFTREF | Reference temperature for either the Arrhenius or Williams-Landel-Ferry shift function (must be greater than zero for the shift function). Set to zero (along with WLFA and WLFB) to not apply scaling. See Remark 2. |

| VARIABLE | DESCRIPTION |
|---|---|
| WLFA | Coefficient for the Arrhenius and the Williams-Landel-Ferry shift functions. Set to zero (along with WLFTREF and WLFB) to not apply scaling. See Remark 2. |
| WLFB | Coefficient for the Williams-Landel-Ferry shift function. Set to zero for the Arrhenius shift function or to not apply scaling (to not apply scaling also set WLFTREF and WLFA to zero). See Remark 2. |
| LCG0 | Load curve ID defining the instantaneous shear modulus, $G_0$, as a function of state of cure |
| LCK0 | Load curve ID defining the instantaneous bulk modulus, $K_0$, as a function of state of cure |
| IDOC | Initial degree of cure, $\alpha_I$ |
| INCR | Switch between incremental and total stress formulation:<br>EQ.0: Total form (default)<br>EQ.1: Incremental form (recommended) |
| QCURE | Heat generation factor, relating the heat generated in one time step with the increment of the degree of cure in that step |
| G*i* | Shear relaxation modulus for the $i^{th}$ term for fully cured material |
| BETAG*i* | Shear decay constant for the $i^{th}$ term for fully cured material |
| K*i* | Bulk relaxation modulus for the $i^{th}$ term for fully cured material. |
| BETAK*i* | Bulk decay constant for the $i^{th}$ term for fully cured material |

**Remarks:**

1. **Material Formulation.** Within this material formulation an internal variable $\alpha$ has been included to represent the degree of cure for the adhesive. The evolution equation for this variable is given by the Kamal model and reads

$$\frac{\mathrm{d}\alpha}{\mathrm{dt}} = \left(k_1 \exp\left(\frac{-c_1}{RT}\right) + k_2 \exp\left(\frac{-c_2}{RT}\right)\alpha^m\right)(1-\alpha)^n \ .$$

The chemical reaction of the curing process results in a shrinkage of the material. The coefficient of the chemical shrinkage $\gamma(\alpha)$ can either be given by a load curve or by using the quadratic expression

$$\gamma(\alpha) = \gamma_2\alpha^2 + \gamma_1\alpha + \gamma_0 \ .$$

For positive values of the parameter LCCHEXP, a differential form is used to compute the chemical strains:

$$d\varepsilon^{\text{ch}} = \gamma(\alpha)d\alpha \ .$$

Otherwise a secant form defines the strains:

$$\varepsilon^{\text{ch}} = \gamma(\alpha)(\alpha - \alpha_0) - \gamma(\alpha_I)(\alpha_I - \alpha_0) \ .$$

Consequently, the definition of $\gamma(\alpha)$ as quadratic expression goes along the secant formulation.

Analogously, the thermal strains are either defined in a secant or differential form, depending on the load curve parameter LCTHEXP. For positive values of that parameter, the differential form is applied, otherwise the secant form is used. For the latter the reference temperature $T_0$ is identified with the input parameter TREFEXP. In both cases the coefficient of thermal expansion can be given as table depending on degree of cure and temperature.

Finally, the Cauchy stress, $\sigma_{ij}$, is related to the strain rate by

$$\sigma_{ij}(t) = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau}d\tau \ .$$

The relaxation functions $g_{ijkl}(t-\tau)$ are represented in this material formulation by up to 16 terms (not including the instantaneous modulus $G_0$) of the Prony series:

$$\mathrm{g}(t,\alpha) = G_0(\alpha) - \sum_i G_i(\alpha) + \sum_i G_i(\alpha)\, e^{-\beta_i t} \ .$$

For the sake of simplicity, a constant ratio $G_i(\alpha)/G_0(\alpha)$ for all degrees of cure is assumed. Consequently, it suffices to define one term $G_0(\alpha)$ as a function of the degree of cure and further coefficients for the fully cured state of the adhesive:

$$\mathrm{g}(t,\alpha) = G_0(\alpha)\left(1 - \sum_i \frac{G_{i,\alpha=1.0}}{G_{0,\alpha=1.0}}\left(1 - e^{-\beta_i t}\right)\right) \ .$$

2. **Temperature Effect on the Stress Relaxation.** A possible temperature effect on the stress relaxation (see Remark 1) is accounted for by the Williams-Landel-Ferry (WLF) shift function or the Arrhenius shift function. For details on this function, please see material formulation 76, *MAT_GENERAL_VISCOELASTIC. If all three values (WLFTREF, WLFA, and WLFB) are nonzero, the WLF function is used; the Arrhenius function is used if WLFB is zero; and no scaling is applied if all three values are zero.

## *MAT_CF_MICROMECHANICS

This is Material Type 278 developed for draping and curing analysis of pre-impregnated (prepreg) carbon fiber sheets. This material model is a mixture of *MAT_234 [Tabiei et. al.] and *MAT_277. It was developed with the collaboration of Professor Tabiei from UC. *MAT_234 provides the reorientation and locking phenomenon of fibers while *MAT_-277 provides the viscoelastic behavior of epoxy resin. Both the epoxy resin and the fiber orientation and deformation contribute to the overall stress.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E1 | E2 | G12 | G23 | EU | C |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| EKA | EUA | VMB | EKB | THL | TA | THI1 | THI2 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| W | SPAN | THICK | H | AREA | E3 | PR13 | PR23 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| AOPT | A1 | A2 | A3 | XP | YP | ZP | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| VYARN | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| K1 | K2 | C1 | C2 | M | N | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| CHEXP1 | CHEXP2 | CHEXP3 | LCCHEXP | LCTHEXP | R | TREF | DOCREF |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| WLFTREF | WLFA | WLFB | LCG0 | LCK0 | IDOC | INCR | QCURE |
|---|---|---|---|---|---|---|---|

**Card 10.** Up to 14 cards may be input. The next keyword ("*") card terminates this input.

| G*i* | BETAG*i* | K*i* | BETAK*i* | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | E2 | G12 | G23 | EU | C |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| E1 | Young's modulus in the yarn's axial direction, $E_1$ |
| E2 | Young's modulus in the yarn's transverse direction, $E_2$ |
| G12 | Shear modulus of the yarns, $G_{12}$ |
| G23 | Transverse shear modulus |
| EU | Ultimate strain at failure |
| C | Coefficient of friction between the fibers |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EKA | EUA | VMB | EKB | THL | TA | THI1 | THI2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EKA | Elastic constant of element "a" |

| VARIABLE | DESCRIPTION |
|---|---|
| EUA | Ultimate strain of element "a" |
| VMB | Damping coefficient of element "b" |
| EKB | Elastic constant of element "b" |
| THL | Yarn locking angle |
| TA | Transition angle of locking |
| THI1 | Initial braid angle 1 |
| THI2 | Initial braid angle 2 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | W | SPAN | THICK | H | AREA | E3 | PR13 | PR23 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| W | Fiber width |
| SPAN | Span between the fibers |
| THICK | Real fiber thickness |
| H | Effective fiber thickness |
| AREA | Fiber cross-sectional area |
| E3 | Young's modulus, $E_3$, in the "thickness" direction as defined by the 3rd axis of the material coordinate system (solids only) |
| PR13 | Transverse Poisson's ratio $\nu_{13}$ (solids only) |
| PR23 | Transverse Poisson's ratio $\nu_{23}$ (solids only) |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | A1 | A2 | A3 | XP | YP | ZP | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes option (see MAT_*OPTION* TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.<br>EQ.3.0: Locally orthotropic material axes for each integration point determined by rotating the material axes about the element normal by an angle, B*i* (see *PART_COMPOSITE), from a line in the plane of the element defined by the cross product of the vector **v** with the element normal.<br>EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, **p**, which define the centerline axis. This option is for solid elements only.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| A1, A2, A3 | Components of vector **a** for AOPT = 2.0 |
| XP, YP, ZP | Coordinates of point **p** for AOPT = 1.0 and 4.0 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3.0 and 4.0 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2.0 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VYARN | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| VYARN | Volume fraction of yarn |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | K2 | C1 | C2 | M | N | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K1 | Parameter $k_1$ for Kamal model |
| K2 | Parameter $k_2$ for Kamal model |
| C1 | Parameter $c_1$ for Kamal model |
| C2 | Parameter $c_2$ for Kamal model |
| M | Exponent $m$ for Kamal model |
| N | Exponent $n$ for Kamal model |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CHEXP1 | CHEXP2 | CHEXP3 | LCCHEXP | LCTHEXP | R | TREF | DOCREF |
| Type | F | F | F | I | I | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CHEXP1 | Quadratic parameter $\gamma_2$ for chemical shrinkage |
| CHEXP2 | Quadratic parameter $\gamma_1$ for chemical shrinkage |
| CHEXP3 | Quadratic parameter $\gamma_0$ for chemical shrinkage |
| LCCHEXP | Load curve ID to define the coefficient for chemical shrinkage $\gamma(\alpha)$ as a function of the state of cure $\alpha$. If set, parameters CHEXP1, CHEXP2, and CHEXP3 are ignored. |
| LCTHEXP | Load curve ID or table ID defining the instantaneous coefficient of thermal expansion $\beta(\alpha, T)$ as a function of cure $\alpha$ and temperature $T$. If referring to a load curve, parameter $\beta(T)$ is a function of temperature $T$. |
| R | Gas constant $R$ for Kamal model |
| TREF | Reference temperature $T_0$ for secant form of thermal expansion |
| DOCREF | Reference degree of cure $\alpha_0$ for sequential form of chemical expansion |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | WLFTREF | WLFA | WLFB | LCG0 | LCK0 | IDOC | INCR | QCURE |
| Type | F | F | F | I | I | F | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| WLFTREF | Reference temperature for WLF shift function |
| WLFA | Parameter $A$ for WLF shift function |

| VARIABLE | DESCRIPTION |
|---|---|
| WLFB | Parameter $B$ for WLF shift function |
| LCG0 | Load curve ID defining the instantaneous shear modulus $G_0$ as a function of state of cure |
| LCK0 | Load curve ID defining the instantaneous bulk modulus $K_0$ as a function of state of cure |
| IDOC | Initial degree of cure |
| INCR | Flag for stress formulation:<br>EQ.0: Total formulation (default)<br>EQ.1: Incremental formulation (recommended) |
| QCURE | Heat generation factor, relating the heat generated in one time step with the increment of the degree of cure in that step |

**Viscoelastic Constant Cards.** Up to 14 cards may be input. A keyword ("*") card terminates this input if fewer than 14 cards are used. The number of terms for the shear behavior may differ from that for the bulk behavior: simply insert zero if a term is not included.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETAG*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Shear relaxation modulus for the $i^{th}$ term for fully cured material |
| BETAG*i* | Shear decay constant for the $i^{th}$ term for fully cured material |
| K*i* | Bulk relaxation modulus for the $i^{th}$ term for fully cured material |
| BETAK*i* | Bulk decay constant for the $i^{th}$ term for fully cured material |

## *MAT_COHESIVE_PAPER

This is Material Type 279. This is a cohesive model for paper materials and can be used only with cohesive element formulations; see the variable ELFORM in *SECTION_SOLID and *SECTION_SHELL.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | EN0 | ET0 | EN1 | ET1 |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T0N | DN | T1N | T0T | DT | T1T | E3C | CC |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ASIG | BSIG | CSIG | FAILN | FAILT | | | |
| Type | F | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| ROFLG | Flag for whether density is specified per unit area or volume:<br>EQ.0: Specifies density per unit volume (default)<br>EQ.1: Specifies the density is per unit area for controlling the mass of cohesive elements with an initial volume of zero. |
| INTFAIL | The number of integration points required for the cohesive element to be deleted. The value of INTFAIL may range from 1 to 4 with 1 the recommended value.<br>LT.0.0: Employs a Newton-Cotes integration scheme. The element will be deleted when \|INTFAIL\| integration points have failed.<br>EQ.0.0: Employs a Newton-Cotes integration scheme. The element will *not* be deleted even if it satisfies the failure criterion.<br>GT.0.0: Employs a Gauss integration scheme. The element will be deleted when INTFAIL integration points have failed. |
| EN0 | The initial tensile stiffness (units of stress / length) normal to the plane of the cohesive element. |
| EN1 | The final tensile stiffness (units of stress / length) normal to the plane of the cohesive element. |
| ET0 | The initial stiffness (units of stress / length) tangential to the plane of the cohesive element. |
| ET1 | The final stiffness (units of stress / length) tangential to the plane of the cohesive element. |
| T0N | Peak tensile traction in normal direction. |
| DN | Scale factor (unit of length). |
| T1N | Final tensile traction in normal direction. |
| T0T | Peak tensile traction in tangential direction. If negative, the absolute value indicates a curve with respect to the normal traction. |
| DT | Scale factor (unit of length). If negative, the absolute value indicates a curve with respect to the normal stress. |

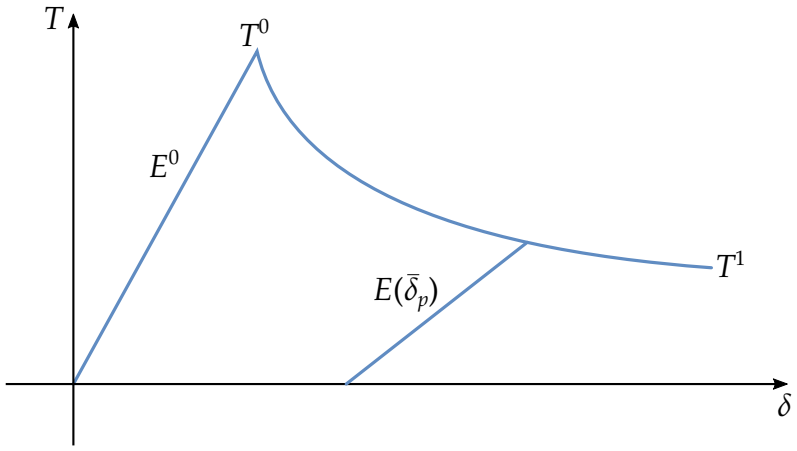



**Figure M279-1.** Traction-separation law

| VARIABLE | DESCRIPTION |
|---|---|
| T1T | Final traction in tangential direction. If negative, the absolute value indicates a curve with respect to the normal traction. |
| E3C | Elastic parameter in normal compression. |
| CC | Elastic parameter in normal compression. |
| ASIG | Plasticity hardening parameter in normal compression. |
| BSIG | Plasticity hardening parameter in normal compression. |
| CSIG | Plasticity hardening parameter in normal compression. |
| FAILN | Maximum effective separation distance in normal direction. Beyond this distance failure occurs. |
| FAILT | Maximum effective separation distance in tangential direction. Beyond this distance failure occurs. |

**Remarks:**

In this elastoplastic cohesive material, the normal and tangential directions are treated separately, but can be connected by expressing the in-plane traction parameters as functions of the normal traction. In the normal direction the material uses different models in tension and compression.

**Normal tension:**

Assume the total separation is an additive split of the elastic and plastic separation

$$\delta = \delta_e + \delta_p \, .$$

In normal tension ($\delta_e > 0$) the elastic traction is given by

$$T = E\delta_e = E(\delta - \delta_p) \geq 0,$$

where the tensile normal stiffness

$$E = (E_N^0 - E_N^1)\exp\left(\frac{-\bar{\delta}_p}{\delta_N}\right) + E_N^1 ,$$

depends on the effective plastic separation in the normal direction

$$\bar{\delta}_p = \int |\mathrm{d}\delta_p| .$$

Yield traction for tensile loads in normal direction is given by

$$T_{\text{yield}} = (T_N^0 - T_N^1)\exp\left(\frac{-\bar{\delta}_p}{\delta_N}\right) + T_N^1 \geq 0,$$

and yielding occurs when $T > T_{\text{yield}} \geq 0$. The above elastoplastic model gives the traction-separation law depicted in Figure M279-1.

**Normal compression:**

In normal compression the elastic traction is

$$T = E_3^c[1 - \exp(-C_c\delta_e)] \leq 0,$$

and the yield traction is

$$T_{\text{yield}} = -\left[A_\sigma + B_\sigma \exp\left(-C_\sigma\bar{\delta}_p\right)\right] \leq 0,$$

with yielding if $T < T_{\text{yield}} \leq 0$.

**Tangential traction:**

Assume the total separation is an additive split of the elastic and plastic separation in each in-plane direction

$$\delta_i = \delta_e^i + \delta_p^i, \qquad i = 1,2.$$

The elastic traction is given by

$$T_i = E\delta_e^i = E(\delta_i - \delta_p^i),$$

where the tensile normal stiffness

$$E = (E_T^0 - E_T^1)\exp\left(\frac{-\bar{\delta}_p}{\delta_T}\right) + E_T^1,$$

depends on the effective plastic separation

$$\bar{\delta}_p = \int \mathrm{d}\delta_p , \qquad \mathrm{d}\delta_p = \sqrt{\left(\mathrm{d}\delta_p^1\right)^2 + \left(\mathrm{d}\delta_p^2\right)^2}.$$

Yield traction is given by

$$T_{\text{yield}} = \left(T_T^0 - T_T^1\right) \exp\left(\frac{-\bar{\delta}_p}{\delta_T}\right) + T_T^1,$$

and yielding occurs when

$$T_1^2 + T_2^2 - T_{yield}^2 \geq 0.$$

The plastic flow increment follows the flow rule

$$\mathrm{d}\delta_p^i = \frac{T_i}{\sqrt{T_1^2 + T_2^2}} \mathrm{d}\delta_p.$$

The above elastoplastic model gives the traction-separation law depicted in Figure M279-1.

**History variables**

This material uses five history variables. Effective separation in the tangential direction is saved as Effective Plastic Strain. History variable 1 and 2 indicates the plastic separation in each tangential direction. Effective plastic separation and plastic separation in the normal direction are saved as history variable 3 and 4, respectively.

***MAT_GLASS**_*{OPTION}*

Available options include:

<BLANK>

STOCHASTIC

SPM

This is Material Type 280. It is a smeared fixed crack model with a selection of different brittle, stress-state dependent failure criteria such as Rankine, Mohr-Coulomb, or Drucker-Prager. The model incorporates up to 2 (orthogonal) cracks per integration point, simultaneous failure over element thickness, and crack closure effects. It is available for shell elements and thick shell types 1, 2 and 6. It is only available for explicit analysis.

The STOCHASTIC keyword option allows spatially varying tensile strength behavior. See *DEFINE_STOCHASTIC_VARIATION for additional information.

The SPM keyword option invokes the additional use of the Glass Strength Prediction Model (GSPM) developed by Rudshaug et al. [2023]. See Remark 11.

**Card Summary:**

**Card 1.** This card is required.

| MID | RO | E | PR | | | IMOD | ILAW |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| FMOD | FT | FC | AT | BT | AC | BC | FTSCL |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| SFSTI | SFSTR | CRIN | ECRCL | NCYCR | NIPF | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card when using the SPM keyword option.

| NUMIT | FDMIN | FDMAX | KCRIT | FDENS | ACMN | ACSTD | ACMIN |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card when using the SPM keyword option.

| ACMAX | JUMAR | KTH | V0 | N | TSCL | EXPA | NINC |
|---|---|---|---|---|---|---|---|

**Card 6.** Include this card when using the SPM keyword option.

| FPERC | | | | | | | |
|---|---|---|---|---|---|---|---|

***Card 7.*** This card is optional.

| EPSCR | ENGCRT | RADCRT | RATENL | RFILTF | FRACEN | CTRACK | GRPFT |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | | | IMOD | ILAW |
| Type | A | F | F | F | | | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density, $\rho$ |
| E | Young's modulus, $E$ |
| PR | Poisson's ratio, $\nu$ |
| IMOD | Flag to choose degradation procedure when critical stress is reached:<br>EQ.0.0: Softening in NCYCR load steps. Define SFSTI, SFSTR, and NCYCR (default).<br>EQ.1.0: Damage model for softening. Define ILAW, AT, BT, AC, and BC. |
| ILAW | Flag to choose damage evolution law if IMOD = 1.0 (see Remark 5):<br>EQ.0.0: Same damage evolution for tensile and compressive failure (default)<br>EQ.1.0: Different damage evolution for tensile failure and compressive failure. |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FMOD | FT | FC | AT | BT | AC | BC | FTSCL |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| FMOD | Flag to choose between failure criteria (see Remark 1):<br>EQ.0.0: Rankine maximum stress (default)<br>EQ.1.0: Mohr-Coulomb<br>EQ.2.0: Drucker-Prager<br>EQ.10.0: Rankine with modified compressive failure<br>EQ.11.0: Mohr-Coulomb with modified compressive failure<br>EQ.12.0: Drucker-Prager with modified compressive failure |
| FT | Tensile strength, $f_t$.<br>GT.0.0: Constant value<br>LT.0.0: Load curve ID = \|FT\|, which gives tensile strength as a function of effective strain rate (RFILTF is recommended). If used with FTSCL > 0, \|FT\| specifies a curve for tensile strength vs. strain rate, and FTSCL scales the strength values from that curve as long as the material is intact. If cracked, neighbors get non-scaled values from that curve. RATENL is set to zero in that case. Logarithmic interpolation between strain rates is assumed if the first abscissa value in the curve is negative; in this case, all the abscissa values are assumed to represent the natural logarithm of a strain rate.<br>Note that a spatially-varying scale factor can be applied to FT through *INITIAL_STRESS_SHELL by setting history variable #13 or through the STOCHASTIC keyword option by using *DEFINE_STOCHASTIC_VARIATION. This scale factor scales FT regardless of RATENL. As a result, depending on the value of RATENL, both this scale factor and FTSCL may scale FT. |
| FC | Compressive strength, $f_c$. |
| AT | Tensile damage evolution parameter, $\alpha_t$. AT controls the loss of load-carrying capacity during damage evolution. Larger values of |

| VARIABLE | DESCRIPTION |
|---|---|
| | AT lead to a stronger reduction of the residual stress level. With increasing tensile strain, the stress approaches $\sigma_\infty = (1 - \alpha_t)f_t$. |
| BT | Tensile damage evolution parameter, $\beta_t$. It controls the softening velocity for tensile failure. |
| AC | Compressive damage evolution parameter, $\alpha_c$. AC controls the loss of load-carrying capacity during damage evolution. Larger values of AC lead to a stronger reduction of the residual stress level. With increasing compressive strain, the stress approaches $\sigma_\infty = (1 - \alpha_c)f_c$. |
| BC | Compressive damage evolution parameter $\beta_c$. It controls the softening velocity for compressive failure. |
| FTSCL | Scale factor for the tensile strength (default = 1.0):<br>$\mathrm{FT}_{\mathrm{mod}} = \mathrm{FTSCL} \times \mathrm{FT}$<br>If RATENL = 0.0 (see Card 4), then the tensile strength drops to its original value, FT, as soon as the first crack happens in the associated part. In this case, FTSCL > 1.0 can be helpful in modeling high-force peaks in impact events.<br><br>If RATENL ≠ 0.0, the tensile strength of an element is evaluated depending on the smoothed effective strain rate when a crack forms in a neighboring element (see Remark 7). |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SFSTI | SFSTR | CRIN | ECRCL | NCYCR | NIPF | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| SFSTI | Scale factor for stiffness after failure. For example, SFSTI = 0.001 means that stiffness is reduced to 0.1% of the elastic stiffness at failure. |
| SFSTR | Scale factor for stress in case of failure. For example, SFSTR = 0.01 means that stress is reduced to 1% of the failure stress at failure. |
| ICRIN | Flag for crack strain initialization:<br>EQ.0.0: Initial crack strain is the strain at failure (default). |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1.0: Initial crack strain is zero. |
| ECRCL | Crack strain necessary to reactivate certain stress components after crack closure |
| NCYCR | Number of cycles in which the stress is reduced to SFSTR × failure stress |
| NIPF | Number of failed through-thickness integration points needed to fail *all* through-thickness integration points for IMOD = 0 |

This card is included if the SPM keyword option is used.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | NUMIT | FDMIN | FDMAX | KCRIT | FDENS | ACMN | ACSTD | ACMIN |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| NUMIT | Number of virtual flaw maps |
| FDMIN | Minimum flaw depth |
| FDMAX | Maximum flaw depth |
| KCRIT | Critical stress intensity |
| FDENS | Flaw density |
| ACMN | Flaw depth-to-half-length ratio, mean value |
| ACSTD | Flaw depth-to half-length ratio, standard deviation |
| ACMIN | Flaw depth-to-half-length ratio, minimum value |

This card is included if the SPM keyword option is used.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ACMAX | JUMAR | KTH | V0 | N | TSCL | EXPA | NINC |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| ACMAX | Flaw depth-to-half-length ratio, maximum value |
| JUMAR | Area of a jumbo glass plate |
| KTH | Stress intensity threshold for subcritical crack growth |
| V0 | Terminal velocity of subcritical crack growth |
| N | Subcritical crack growth exponent |
| TSCL | Time scaling factor, TSCL > 0. Scales the time step increment used to calculate the subcritical crack growth velocity: $v = \frac{\mathrm{d}a}{\mathrm{d}t} = \mathrm{V0}\left(\frac{K_\mathrm{I}}{K_\mathrm{IC}}\right)^\mathrm{N}$ $\mathrm{d}t = (t_{i+1} - t_i)\times\mathrm{TSCL}$ |
| EXPA | Exponent for moving exponential averaging, 0<= EXPA <= 1 $\overline{K}_{\mathrm{I},i} = \mathrm{EXPA}\times K_{\mathrm{I},i} + (1 - \mathrm{EXPA})\overline{K}_{\mathrm{I},i-1}$ |
| NINC | Defines the number of increments to run per GSPM update, NINC ≥ 1. NINC = 1 means that GSPM runs at each increment, while NINC = 10 results in one GSPM running every tenth increment. |

This card is included if the SPM keyword option is used.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FPERC | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FPERC | Failure percentile, $0 < \text{FPERC} \leq 1$. When the number of virtual glass plates with fracture initiation exceeds FPERC × NUMIT, fracture initiation is triggered in the simulation. |

Optional card.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EPSCR | ENGCRT | RADCRT | RATENL | RFILTF | FRACEN | CTRACK | GRPFT |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| EPSCR | Critical value to trigger element deletion. This can be useful to get rid of highly distorted elements.<br>GT.0.0: EPSCR is effective critical strain.<br>LT.0.0: \|EPSCR\| is critical crack opening displacement. |
| ENGCRT | Critical energy for nonlocal failure criterion (see Remark 6):<br>GT.0.0: Constant value.<br>LT.0.0: \|ENGCRT\| refers to a *DEFINE_FUNCTION giving the critical energy as a function of the minimum distance of the center of impact to the edge of the windshield. Thus, the critical energy value can now depend on this distance, for example, larger in the middle area and smaller toward the edge. |
| RADCRT | Critical radius for nonlocal failure criterion; see Remark 6. |
| RATENL | Quasi-static strain rate threshold variable, which activates a nonlocal, strain rate dependent tensile strength adaption; see Remark 7. |
| RFILTF | Smoothing factor on the effective strain rate for the evaluation of the current tensile strength if RATENL > 0.0; see Remark 7.<br>$\dot{\varepsilon}_n^{\text{avg}} = \text{RFILTF} \times \dot{\varepsilon}_{n-1}^{\text{avg}} + (1 - \text{RFILTF}) \times \dot{\varepsilon}_n$ |
| FRACEN | Fracture energy (units of stress × length). An alternative orthotropic damage model with linear softening is invoked with this |

| VARIABLE | DESCRIPTION |
|---|---|
| | option. Values smaller than $0.5 \times f_t \times \frac{f_t}{E} \times l_e$ (element size) lead to immediate failure. This is the area under the elastic stress-displacement line until $f_t$ is reached. Only larger values result in actual residual energy after crack initiation. Variables SFSTI, SFSTR, and NCYCR are ignored with this option. You can specify a spatially varying scale factor for FRACEN by setting history variable #14 with *INITIAL_STRESS_SHELL. |
| CTRACK | Flag for optional crack tracking algorithm (see Remark 10):<br>EQ.0.0: Inactive<br>EQ.1.0: Active |
| GRPFT | Optional group number for strength reduction. If several parts use *MAT_280 with potentially different material parameters, giving them the same value of GRPT causes them to experience tensile strength reduction by FTSCL at the same time (RATENL = 0). |

**Remarks:**

1. **Plane stress failure criteria.** The underlying material behavior before failure is isotropic, small strain linear elasticity with Young's modulus, $E$, and Poisson's ratio, $\nu$. Asymmetric (tension-compression dependent) failure happens as soon as one of the following plane stress failure criteria is violated.

   a) *Rankine Maximum Stress.* For FMOD = 0, a maximum stress criterion (Rankine) is used where principal stresses, $\sigma_1$ and $\sigma_2$, are bound by tensile strength, $f_t$, and compressive strength, $f_c$, as follows:

   $$-f_c < \{\sigma_1, \sigma_2\} < f_t$$

   b) *Mohr-Coulomb.* With FMOD = 1, the Mohr-Coulomb criterion with expressions in four different categories is used:

   $$\sigma_1 > 0 \text{ and } \sigma_2 > 0: \qquad \max\left(\frac{\sigma_1}{f_t}, \frac{\sigma_2}{f_t}\right) < 1$$

   $$\sigma_1 < 0 \text{ and } \sigma_2 < 0: \qquad \max\left(-\frac{\sigma_1}{f_c}, -\frac{\sigma_2}{f_c}\right) < 1$$

   $$\sigma_1 > 0 \text{ and } \sigma_2 < 0: \qquad \frac{\sigma_1}{f_t} - \frac{\sigma_2}{f_c} < 1$$

   $$\sigma_1 < 0 \text{ and } \sigma_2 > 0: \qquad -\frac{\sigma_1}{f_c} + \frac{\sigma_2}{f_t} < 1$$

c) *Drucker-Prager.* For FMOD = 2, the plane stress Drucker-Prager criterion is given by:

$$\frac{1}{2f_c}\left[\left(\frac{f_c}{f_t}-1\right)(\sigma_1+\sigma_2)+\left(\frac{f_c}{f_t}+1\right)\sqrt{\sigma_1^2+\sigma_2^2-\sigma_1\sigma_2}\right]<1$$

The modified versions, FMOD = 10, 11, 12, change what is considered tensile and compressive failure. The form of the failure stays the same for each type. In each case, a line with slope $-f_c/f_t$ distinguishes the difference between the two types of failure. See Figure M280-1 for an example of how the tensile and compressive failures are modified for the Rankine failure criterion.

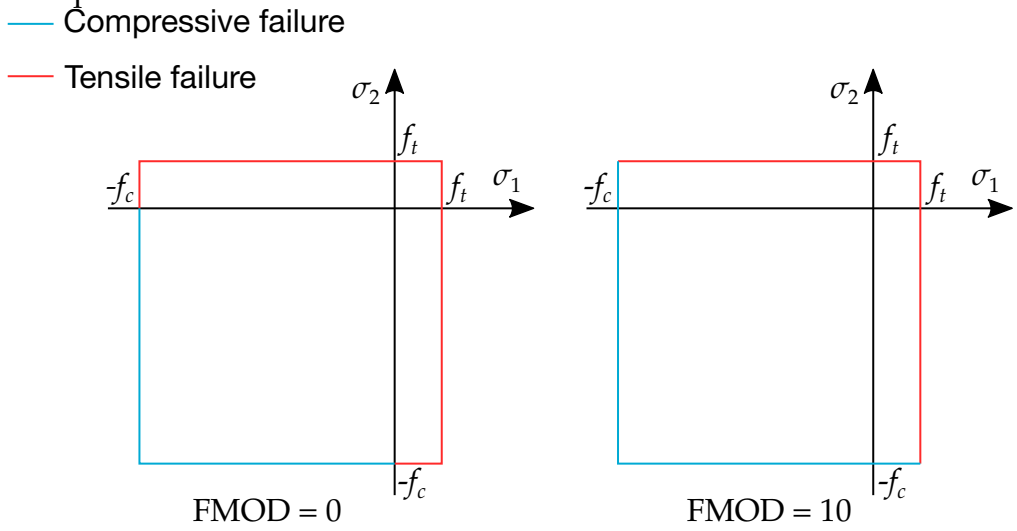



**Figure M280-1.** Rankine failure criterion. With FMOD = 10, the form of the failure criterion does not change but what is considered compressive failure is modified.

2. **Crack formation.** As soon as failure happens in the tensile regime, a crack occurs perpendicular to the maximum principal stress direction. That means a crack coordinate system is set up and stored, defined by a relative angle with respect to the element coordinate system. Appropriate stress and stiffness tensor components (e.g. normal to the crack) are reduced according to SFSTR and SFSTI if IMOD = 0. The stress reduction takes place in a period of NCYCR time step cycles. For IMOD = 1.0 the stress and stiffness tensor are reduced by a damage model (see Remark 5). A second crack orthogonal to the first crack is possible which can open and close independently from the first one, further reducing the element stiffness.

3. **Crack closure.** To deal with crack closure, the current strain in the principal stress direction is stored as the initial crack strain (ICRIN = 0, default), or the initial crack strain is set to zero (ICRIN = 1). After failure, the crack strain is tracked, so that later crack closure will be detected. If that is the case, appropriate stress and stiffness tensor components (e.g., compressive) are reactivated so

that, e.g., under pressure, a load could be carried and cause nonzero stress perpendicular to the crack.

4. **Number of failed integration points.** If the critical number of failed integration points (NIPF) in one element is reached, all integration points over the element thickness fail as well. The default value of NIPF=1 resembles the fact, that a crack in a glass plate immediately runs through the thickness.

5. **Damage model.** IMOD = 1 invokes a damage model for stress and stiffness softening. The corresponding evolution law for ILAW = 0 is given by:

$$D = \begin{cases} 0 & \text{for } \kappa \le \kappa^0 \\ 1 - \dfrac{\kappa^0}{\kappa}\left(1 - \alpha_{t,c} + \alpha_{t,c} e^{-\beta_{t,c}(\kappa - \kappa^0)}\right) & \text{otherwise} \end{cases}$$

meaning tensile and compressive failure are treated in the same fashion.

However, with ILAW = 1 the damage evolution for tensile failure is given by:

$$D = \begin{cases} 0 & \text{for } \kappa \le \kappa^0 \\ 1 - \dfrac{\kappa^0}{\kappa}\left(1 - \alpha_{t} + \alpha_{t} e^{-\beta_{t}(\kappa - \kappa^0)}\right) & \text{otherwise} \end{cases}$$

while damage for compressive failure evolves as (more delayed stress reduction):

$$D = \begin{cases} 0 & \text{for } \kappa \le \kappa^0 \\ 1 - \dfrac{\kappa^0}{\kappa}(1 - \alpha_c) - \alpha_c e^{-\beta_c(\kappa - \kappa^0)} & \text{otherwise} \end{cases}$$

6. **Nonlocal failure.** Use ENGCRT and RADCRT to specify a nonlocal failure criterion. This criterion is mainly intended for windshield impact. The same procedure is used as in *MAT_ADD_EROSION; see Remark 1i there.

7. **Strain-rate-dependent, nonlocal tensile strength.** If RATENL > 0.0, all elements in the appropriate part are initialized with a tensile strength of FTSCL × FT. If one integration point in an element fails, the tensile strength in the neighboring elements is set to $\widetilde{FT}$ where

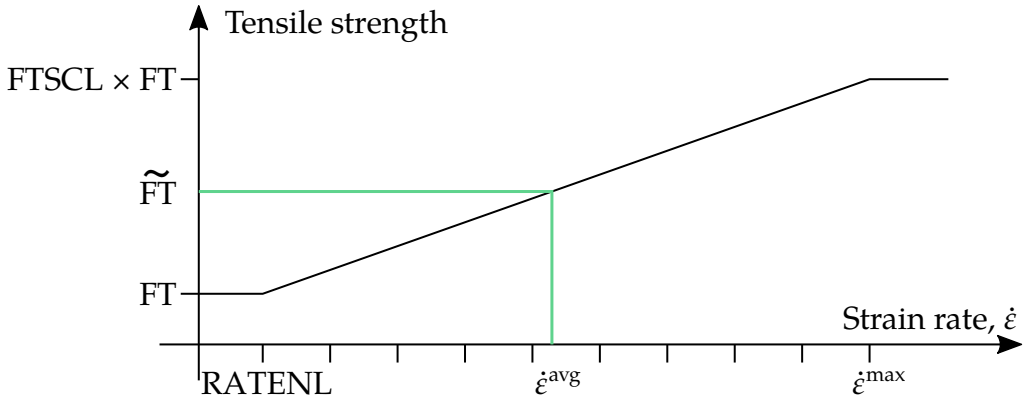



**Figure M280-2.** Modified tensile strength of elements neighboring an element that has at least one failed integration point as a function of strain rate when RATENL > 0.0. $\dot{\varepsilon}^{max} = 10^9 \times$ RATENL.

$$\widetilde{FT} = \begin{cases} FT & \text{if } \dot{\varepsilon}^{avg} \le RATENL \\ FT + (FTSCL \times FT - FT)\dfrac{\ln\left(\dfrac{\dot{\varepsilon}^{avg}}{RATENL}\right)}{\ln\left(\dfrac{\dot{\varepsilon}^{max}}{RATENL}\right)} & \text{if } RATENL < \dot{\varepsilon}^{avg} < \dot{\varepsilon}^{max} \\ FTSCL \times FT & \text{if } \dot{\varepsilon}^{avg} > \dot{\varepsilon}^{max} \end{cases}$$

Here $\dot{\varepsilon}^{max} = 10^9 \times$ RATENL. See Figure M280-2 for a plot of this tensile strength as a function of average strain rate. The average strain rate in this case is calculated as:

$$\dot{\varepsilon}_n^{avg} = RFILTF \times \dot{\varepsilon}_{n-1}^{avg} + (1 - RFILTF) \times \dot{\varepsilon}_n$$

where $n$ is the time step.

8. **Element based (or stochastically varied) tensile strength.** You can define a spatially varying tensile strength with history variable #13 which is a scale factor on the strength. The history variable #13 value can be filled with *INITIAL_STRESS_SHELL or can come from *DEFINE_STOCHASTIC_VARIATION.

9. **Vector plot of crack direction.** Extra history variables #15, #16, and #17 store the global coordinates of the first crack direction. These coordinates can be used to represent the crack as a vector, such as in LS-PrePost with Post→Vector→Hist. var. cosine.

10. **Crack tracking algorithm.** Cracks often follow the shell element meshing directions, e.g., it is often observed that cracks are "trapped" in element rows and

cannot travel freely through the structure. This phenomenon is also called directional mesh-bias dependency. To alleviate this issue, a nonlocal crack tracking algorithm can be invoked with CTRACK = 1. Among other things, it weakens neighboring elements in the first crack direction more strongly. Currently this option must be used in conjunction with the nonlocal model discussed in Remark 7 (RATENL > 0.0) or the other nonlocal option invoked with FT < 0.0. To make this option work with MPP, the whole glass part must be put on one processor, such as by using *CONTROL_MPP_DECOMPOSITION_ARRANGE_PARTS with TYPE = 10.

11. **GSPM.** The glass strength prediction model is a Monte-Carlo-based fracture initiation predictor. It combines the theories of linear elastic fracture mechanics (LEFM) and sub-critical crack growth (SCG) to generate a representative sample of virtual glass plates that are monitored during the simulation. Each virtual glass plate has the same geometry as the respective glass part and includes a unique set of surface flaws. The flaw distribution parameters (NUMIT, FDMIN, FDMAX, FDENS, ACMN, ACSTD, ACMIN, ACMAX, and JUMAR) combined with the LEFM and SCG parameters (KCRIT, KTH, V0, and N) determine the strength distribution of the representative sample of virtual glass plates. The virtual glass plates are monitored at every NINC increment when the flaw status is updated. If the stress intensity factor is critical for a flaw, fracture initiates for the corresponding virtual glass plate. The number of virtual glass plates with fracture initiation exceeding FPERC × NUMIT triggers fracture initiation in the simulation. Selecting the failure percentile value, FPERC, enables determining the location within the generated representative sample of the virtual glass plate used to trigger fracture initiation. If time scaling is used in the simulation, the parameter TSCL can be used to scale the time increment calculated in the GSPM. EXPA is a moving average exponent that can be used to exponentially average the stress intensity factor to account for noise.

12. **History variables.** This material has the following additional history variables that can be output to the d3plot file.

| History Variable # | Description |
|---|---|
| 1 | Crack flag:<br>EQ.0: No crack<br>EQ.1: One crack<br>EQ.2: Two cracks<br>EQ.-1: Failed under compression |
| 2 | Direction of 1st principle stress as angle in radians with respect to the element direction. The shell normal defines the positive angle direction. The 1st crack direction is perpendicular to the direction of 1st principle stress. |

| History Variable # | Description |
|---|---|
| 3 | Angle in radians that defines the orthogonal to the 2nd crack direction (with respect to the element direction). |
| 4 | Failure criterion value, see Remark 1 |
| 7 | Current tensile strength value |
| 8 | Effective strain rate (if FT < 0 or RATENL > 0 is used) |
| 9 | Crack opening displacement (1st crack) |
| 10 | Time to failure / GSPM initial crack flag |
| 11 | Damage in 1st crack direction (only with FRACEN > 0) |
| 12 | Damage in 2nd crack direction (only with FRACEN > 0) |
| 13 | Scale factor for tensile strength |
| 14 | Scale factor for fracture energy |
| 15 | Global $x$-coordinate of 1st crack direction |
| 16 | Global $y$-coordinate of 1st crack direction |
| 17 | Global $z$-coordinate of 1st crack direction |

## *MAT_SHAPE_MEMORY_ALLOY

This is Material Type 291, a micromechanics-inspired constitutive model for shape-memory alloys that accounts for the initiation and saturation of phase. This model is based on Kelly, Stebner, and Bhattacharya (2016) and is available for solid elements only.

### Card Summary:

**Card 1.** This card is required.

| MID | RHO | EM | EA | PRM | PRA | AOPT | STYPE |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| CPM | CPA | LH | TC | TMF | TMS | TAS | TAF |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| A1I | A2I | BI | CI | KI | MI | KL | ML |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| A1S | A2S | BS | CS | KS | MS | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| D0L | D0M | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MACF | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
|---|---|---|---|---|---|---|---|

**Card 8.1.** Include this card if STYPE = 1.

| N11 | N22 | N33 | N44 | N55 | N66 | N12 | N23 |
|---|---|---|---|---|---|---|---|

**Card 8.2.** Include this card if STYPE = 1.

| N34 | N45 | N56 | N13 | N24 | N35 | N46 | N14 |
|---|---|---|---|---|---|---|---|

**Card 8.3.** Include this card if STYPE = 1.

| N25 | N36 | N15 | N26 | N16 | | | |
|---|---|---|---|---|---|---|---|

***Card 9.*** This card is optional.

| KP | MP | KC | MC | | | | |
|---|---|---|---|---|---|---|---|

***Card 10.*** This card is optional.

| DOP | QP | NP | QL | NL | QM | NM | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHO | EM | EA | PRM | PRA | AOPT | STYPE |
| Type | A | F | F | F | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RHO | Mass density |
| EM | Martensite Young's modulus |
| EA | Austenite Young's modulus |
| PRM | Martensite Poisson's ratio |
| PRA | Austenite Poisson's ratio |
| AOPT | Material axes option (see MAT_*OPTION*TROPIC_ELASTIC, particularly the Material Directions section, for details):<br>EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and the global location of the element center; this is the **a**-direction.<br>EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on the value of MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. |
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_OPTION). |
| STYPE | Initiation/saturation surface type (see Remark 1): |
| | EQ.0: Uses strain invariants (default) |
| | EQ.1: Uses principal strains |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CPM | CPA | LH | TC | TMF | TMS | TAS | TAF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CPM | Martensite volumetric heat capacity (density times specific heat capacity) |
| CPA | Austenite volumetric heat capacity (density times specific heat capacity) |

| VARIABLE | DESCRIPTION |
|---|---|
| LH | Volumetric latent heat of transformation (density times specific latent heat) |
| TC | Thermodynamic temperature |
| TMF | Martensite finish temperature, optional; see Remark 2. |
| TMS | Martensite start temperature, optional; see Remark 2. |
| TAS | Austenite start temperature, optional; see Remark 2. |
| TAF | Austenite finish temperature, optional; see Remark 2. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A1I | A2I | BI | CI | KI | MI | KL | ML |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| A1I | Tension/compression asymmetry for initiation surface |
| A2I | Tension/compression asymmetry for initiation surface |
| BI | Radius for initiation surface |
| CI | Eccentricity of initiation surface with respect to material direction |
| KI | Coefficient in initiation energy |
| MI | Exponent in initiation energy |
| KL | Coefficient in volume fraction energy |
| ML | Exponent in volume fraction energy |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A1S | A2S | BS | CS | KS | MS | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| A1S | Tension/compression asymmetry for saturation surface |
| A2S | Tension/compression asymmetry for saturation surface |
| BS | Radius for saturation surface |
| CS | Eccentricity of saturation surface with respect to material direction |
| KS | Coefficient in saturation energy |
| MS | Exponent in saturation energy |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D0L | D0M | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D0L | Initial driving force for volume fraction transformation |
| D0M | Initial driving force for martensite strain transformation |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements:<br>EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation<br>Figure M2-2 indicates when LS-DYNA applies MACF during the process to obtain the final material axes. If BETA on *ELEMENT_SOLID_*{OPTION}* is defined, then that BETA is used for the rotation for all AOPT options. Otherwise, for AOPT = 3, the BETA input on Card 7 rotates the axes. For all other values of AOPT, the material axes will be switched as specified by MACF, but no BETA rotation will be performed. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 3. This angle may be overridden on the element card; see *ELEMENT_SOLID_ORTHO. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Anisotropy Parameter Cards.** This card and the following two cards are included if STYPE = 1.

| Card 8.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | N11 | N22 | N33 | N44 | N55 | N66 | N12 | N23 |
| Type | F | F | F | F | F | F | F | F |

| Card 8.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | N34 | N45 | N56 | N13 | N24 | N35 | N46 | N14 |
| Type | F | F | F | F | F | F | F | F |

| Card 8.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | N25 | N36 | N15 | N26 | N16 | | | |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| N*ij* | Additional anisotropy parameters for initiation/saturation surface, relative to material axis given by AOPT. Used for STYPE = 1. |

**Plasticity Parameter Card 1.** The following two cards are optional.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | KP | MP | KC | MC | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| KP | Coefficient in plastic energy |

| VARIABLE | DESCRIPTION |
|---|---|
| MP | Exponent in plastic energy |
| KC | Coefficient in coupling energy |
| MC | Exponent in coupling energy |

**Plasticity Parameter Card 2.** This card is optional.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D0P | QP | NP | QL | NL | QM | NM | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| D0P | Initial driving force for plastic transformation |
| QP | Isotropic hardening coefficient in plastic kinetic relation |
| NP | Isotropic hardening exponent in plastic kinetic relation |
| QL | Isotropic hardening coefficient in volume fraction kinetic relation |
| NL | Isotropic hardening exponent in volume fraction kinetic relation |
| QM | Isotropic hardening coefficient in martensite kinetic relation |
| NM | Isotropic hardening exponent in martensite kinetic relation |

**Remarks:**

1. **Material model.** The total strain $\boldsymbol{\varepsilon}$ is composed of elastic strain $\boldsymbol{\varepsilon}_e$, martensite strain $\boldsymbol{\varepsilon}_m$, and plastic strain $\boldsymbol{\varepsilon}_p$ according to the additive split

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_e + \lambda \boldsymbol{\varepsilon}_m + \boldsymbol{\varepsilon}_p \ ,$$

where $0 \leq \lambda \leq 1$ is the volume fraction of martensite. Initially, the material is only composed of austenite, that is, $\lambda = 0$. The material is assumed to be isotropic elastic

$$\boldsymbol{\sigma} = \mathbf{C}(\lambda)\boldsymbol{\varepsilon}_e \ ,$$

and the martensite strain is assumed to be trace-free

$$\mathrm{tr}(\boldsymbol{\varepsilon}_m) = 0 \ .$$

Given the total strain $\boldsymbol{\varepsilon}$ and temperature $T$, this model finds $\lambda$, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_p$ that minimize the mechanical energy

$$U = W + D \ ,$$

where $W$ is the Helmholtz free energy and $D$ is the dissipated energy. The Helmholtz free energy here is given by

$$W\left(\boldsymbol{\varepsilon},\lambda,\boldsymbol{\varepsilon}_m,\boldsymbol{\varepsilon}_p,T\right) = \frac{1}{2}\boldsymbol{\sigma}\colon\boldsymbol{\varepsilon}_e + \lambda\omega(T) - c(\lambda)T\ln\left(\frac{T}{\mathrm{TC}}\right) + \lambda G_I(\boldsymbol{\varepsilon}_m) + G_S(\lambda\boldsymbol{\varepsilon}_m) + G_\lambda(\lambda) + G_P\left(\boldsymbol{\varepsilon}_p\right) + \lambda G_C\left(\boldsymbol{\varepsilon}_m,\boldsymbol{\varepsilon}_p\right),$$

where

$$\omega(T) = \mathrm{LH}\frac{T-\mathrm{TC}}{\mathrm{TC}} \ ,$$
$$c(\lambda) = \lambda\times\mathrm{CPM} + (1-\lambda)\times\mathrm{CPA} \ .$$

The functions $G_I$ and $G_S$ denote initiation energy and saturation energy, respectively, and are defined as

$$G_i(\mathbf{X}) = \begin{cases} \mathrm{K}i\times\max(g_i(\mathbf{X}),0)^{\mathrm{M}i+1} & \mathrm{K}i \geq 0 \\ -\mathrm{K}i\times\log(\max(g_i(\mathbf{X}),1))^{\mathrm{M}i+1} & \mathrm{K}i < 0 \end{cases}, \qquad i = I,S,$$

where the functions of $g_I$ and $g_S$ depends on the parameter STYPE and $\mathbf{X}$ is a tensor. Negative values of $g_i$lead to no contribution to the free energy. Thus, $g_i \leq 0$ defines the set of admissible values for $\lambda$ and $\varepsilon_m$.

For STYPE = 0, we have

$$g_i(\mathbf{X}) = -1 + \frac{1}{\mathrm{B}i}\left[\left(\frac{1}{2}\mathbf{X}\colon\mathbf{X}\right)^{\frac{3}{2}} - \mathrm{A1}i\times\det(\mathbf{X}) - \mathrm{C}i\times\left|\mathbf{n}^T\bullet\mathbf{X}\,\mathbf{n}\right|^3\right], \qquad i = I,S,$$

Where $\mathrm{B}i > 0$. The direction vector, $\mathbf{n}$, is given by the main material direction defined with AOPT.

For STYPE = 1, we have

$$g_i(\mathbf{X}) = -1 + \frac{1}{\mathrm{B}_i^{\mathrm{A2}i+1}}\sum_{n=1,2,3}\left[|\mu_n(\mathbf{N}\colon\mathbf{X})| - \mathrm{A1}i\times\mu_n(\mathbf{N}\colon\mathbf{X})\right]^{\mathrm{A2}i+1}, \qquad i = I,S,$$

for $-1 < \mathrm{A1}i < 1$, $\mathrm{A2}i > 0$, $\mathrm{B}i > 0$, and the principal values $\mu_n$. The anisotropy tensor, $\mathbf{N}$, is relative to the main material direction, $\mathbf{n}$, defined with AOPT.

The function $G_\lambda$ denotes martensite volume fraction energy and is defined as

$$G_\lambda(\lambda) = \frac{\mathrm{KL}\times\lambda^{\mathrm{ML}+1}}{\mathrm{ML}+1}, \qquad \mathrm{KL},\mathrm{ML} \geq 0.$$

Thus, the amount of stored energy in the system can increase with increasing volume fraction.

The function $G_P$ denotes plastic strain energy and is defined as

$$G_P(\boldsymbol{\varepsilon}_p) = \mathrm{KP} \times \|\boldsymbol{\varepsilon}_p\|^{\mathrm{MP}+1}, \qquad \mathrm{KP}, \mathrm{MP} \geq 0,$$

and $G_C$ denotes coupling energy and is defined as

$$G_C(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p) = -\mathrm{KC} \times \|\boldsymbol{\varepsilon}_p\|^{\mathrm{MC}+1} \boldsymbol{\varepsilon}_m : \boldsymbol{\varepsilon}_p, \qquad \mathrm{KC}, \mathrm{MC} \geq 0.$$

The driving forces for $\lambda$, $\boldsymbol{\varepsilon}_m$ and $\boldsymbol{\varepsilon}_p$ are defined as

$$\begin{aligned} d_\lambda &= -\frac{\partial W}{\partial \lambda} = -\frac{1}{2}\mathbf{C}'(\lambda)\boldsymbol{\varepsilon}_e : \boldsymbol{\varepsilon}_e + \boldsymbol{\sigma} : \boldsymbol{\varepsilon}_m - \omega(T) + c'(\lambda) T \ln\left(\frac{T}{\mathrm{TC}}\right) \\ &\qquad - G_I(\boldsymbol{\varepsilon}_m) - \boldsymbol{\varepsilon}_m : G_s'(\lambda \boldsymbol{\varepsilon}_m) - G_\lambda'(\lambda) - G_C(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p), \\ \mathbf{d}_{\boldsymbol{\varepsilon}_m} &= -\frac{\partial W}{\partial \boldsymbol{\varepsilon}_m} = \lambda\left(\boldsymbol{\sigma} - G_I'(\boldsymbol{\varepsilon}_m) - G_s'(\lambda \boldsymbol{\varepsilon}_m) - \frac{\partial G_C}{\partial \boldsymbol{\varepsilon}_m}(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p)\right), \\ \mathbf{d}_{\boldsymbol{\varepsilon}_p} &= -\frac{\partial W}{\partial \boldsymbol{\varepsilon}_p} = \boldsymbol{\sigma} - G_P'(\boldsymbol{\varepsilon}_p) - \lambda \frac{\partial G_C}{\partial \boldsymbol{\varepsilon}_p}(\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_p), \end{aligned}$$

and typically govern the evolution of $\lambda$, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_p$ through evolution equations

$$\begin{aligned} \dot{\lambda} &= f_\lambda(d_\lambda), \\ \dot{\boldsymbol{\varepsilon}}_m &= \mathbf{f}_{\boldsymbol{\varepsilon}_m}(\mathbf{d}_{\boldsymbol{\varepsilon}_m}), \\ \dot{\boldsymbol{\varepsilon}}_p &= \mathbf{f}_{\boldsymbol{\varepsilon}_p}(\mathbf{d}_{\boldsymbol{\varepsilon}_p}). \end{aligned}$$

In this model, $\lambda$, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_m$ can, however, evolve freely, and the evolution is instead postulated to satisfy the kinetic relations:

$$\begin{aligned} \dot{\lambda} &= 0 && \text{if } |d_\lambda| < \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NL}}, \\ d_\lambda \dot{\lambda} &\geq 0, \\ |d_\lambda| &\leq \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NL}}, \end{aligned}$$

for volume fraction,

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}}_m &= 0 && \text{if } \|\mathbf{d}_{\boldsymbol{\varepsilon}_m}\| < \lambda\left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NM}}\right), \\ \mathbf{d}_{\boldsymbol{\varepsilon}_m} : \dot{\boldsymbol{\varepsilon}}_m &\geq 0, \\ \|\mathbf{d}_{\boldsymbol{\varepsilon}_m}\| &\leq \lambda\left(\frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NM}}\right) && \text{if } g_i(\boldsymbol{\varepsilon}_m) \leq 0. \end{aligned}$$

for martensite strain, and

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}}_p &= 0 && \text{if } \|\mathbf{d}_{\boldsymbol{\varepsilon}_p}\| < \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NP}}, \\ \mathbf{d}_{\boldsymbol{\varepsilon}_p} : \dot{\boldsymbol{\varepsilon}}_p &\geq 0, \\ \|\mathbf{d}_{\boldsymbol{\varepsilon}_p}\| &\leq \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p\|^{\mathrm{NP}}. \end{aligned}$$

for plastic strain. The norm is here defined as

$$\|\mathbf{d}\| = \sqrt{\mathbf{d}:\mathbf{d}},$$

and the scaling factor $\sqrt{1.5}$ implies that D0M and D0P corresponds to the von Mises stress at which the martensite and plastic strains start to develop.

The above kinetic relations correspond to the rate of dissipation

$$\dot{D} = d_\lambda \dot{\lambda} + \mathbf{d}_{\boldsymbol{\varepsilon}_m}:\dot{\boldsymbol{\varepsilon}}_m + \mathbf{d}_{\boldsymbol{\varepsilon}_p}:\dot{\boldsymbol{\varepsilon}}_p \geq 0,$$

and are incorporated in the model by minimizing the mechanical energy over one time-step

$$\begin{aligned}
\Delta U &= \int_t^{t+\Delta t} \dot{U} ds = \int_t^{t+\Delta t} (\dot{W} + \dot{D}) ds = \Delta W + \int_t^{t+\Delta t} \dot{D} ds \\
&\leq \Delta W + \int_t^{t+\Delta t} \left( \left( \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NL}} \right) |\dot{\lambda}| + \lambda \left( \frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NM}} \right) \|\dot{\boldsymbol{\varepsilon}}_m\| \right. \\
&\qquad \left. + \left( \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NP}} \right) \|\dot{\boldsymbol{\varepsilon}}_p\| \right) ds \\
&\approx \Delta W + \left( \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NL}} \right) |\lambda_{t+\Delta t} - \lambda_t| \\
&\qquad + \lambda_t \left( \frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NM}} \right) \|\boldsymbol{\varepsilon}_m^{t+\Delta t} - \boldsymbol{\varepsilon}_m^t\| \\
&\qquad + \left( \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NP}} \right) \|\boldsymbol{\varepsilon}_p^{t+\Delta t} - \boldsymbol{\varepsilon}_p^t\|,
\end{aligned}$$

with respect to $\lambda_{t+\Delta t}$, $\boldsymbol{\varepsilon}_m^{t+\Delta t}$, and $\boldsymbol{\varepsilon}_p^{t+\Delta t}$. The evolution of $\lambda$, $\boldsymbol{\varepsilon}_m$, and $\boldsymbol{\varepsilon}_p$ is thus constrained by the time step.

Minimizing the mechanical energy over one time step with respect to $\lambda_{t+\Delta t}$, $\boldsymbol{\varepsilon}_m^{t+\Delta t}$, and $\boldsymbol{\varepsilon}_p^{t+\Delta t}$ gives the optimality constraints

$$\begin{aligned}
\frac{\partial U_{t+\Delta t}}{\partial \lambda_{t+\Delta t}} &= -d_\lambda^{t+\Delta t} + \left( \mathrm{D0L} + \mathrm{QL}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NL}} \right) \mathrm{sign}(\lambda_{t+\Delta t} - \lambda_t) = 0, \\
\frac{\partial U_{t+\Delta t}}{\partial \boldsymbol{\varepsilon}_m^{t+\Delta t}} &= -\mathbf{d}_{\boldsymbol{\varepsilon}_m}^{t+\Delta t} + \lambda_t \left( \frac{\mathrm{D0M}}{\sqrt{1.5}} + \mathrm{QM}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NM}} \right) \frac{(\boldsymbol{\varepsilon}_m^{t+\Delta t} - \boldsymbol{\varepsilon}_m^t)}{\|\boldsymbol{\varepsilon}_m^{t+\Delta t} - \boldsymbol{\varepsilon}_m^t\|} = \mathbf{0}, \\
\frac{\partial U_{t+\Delta t}}{\partial \boldsymbol{\varepsilon}_p^{t+\Delta t}} &= -\mathbf{d}_{\boldsymbol{\varepsilon}_p}^{t+\Delta t} + \left( \frac{\mathrm{D0P}}{\sqrt{1.5}} + \mathrm{QP}\|\boldsymbol{\varepsilon}_p^t\|^{\mathrm{NP}} \right) \frac{(\boldsymbol{\varepsilon}_p^{t+\Delta t} - \boldsymbol{\varepsilon}_p^t)}{\|\boldsymbol{\varepsilon}_p^{t+\Delta t} - \boldsymbol{\varepsilon}_p^t\|} = \mathbf{0},
\end{aligned}$$

and incorporating the trace-free condition on $\boldsymbol{\varepsilon}_m$ and $\boldsymbol{\varepsilon}_p$ gives

$$\begin{aligned}
\frac{\partial U_{t+\Delta t}}{\partial \lambda_{t+\Delta t}} &= 0, \\
\frac{\partial U_{t+\Delta t}}{\partial (\boldsymbol{\varepsilon}_q^{t+\Delta t})_i} - \frac{\partial U_{t+\Delta t}}{\partial (\boldsymbol{\varepsilon}_q^{t+\Delta t})_3} &= 0, \qquad i = 1,2,
\end{aligned}$$

$$\frac{\partial U_{t+\Delta t}}{\partial\left(\varepsilon_q^{t+\Delta t}\right)_i}=0, \qquad i=4,5,6,$$

for $q=m,p$.

2. **Transition temperatures.** The initial (zero stress) austenite and martensite start and finish temperatures, TMF ≤ TMS ≤ TAS ≤ TAF, can be given instead of the parameters *"TC"* , KL, and D0L. The temperatures are defined as

$$\begin{aligned}
\mathrm{TMS} &= \mathrm{TC}\left(1-\frac{\mathrm{D0L}}{\mathrm{LH}}\right)\\
\mathrm{TMF} &= \mathrm{TC}\left(1-\frac{\mathrm{D0L}+\mathrm{KL}}{\mathrm{LH}}\right)\\
\mathrm{TAS} &= \mathrm{TC}\left(1+\frac{\mathrm{D0L}-\mathrm{KL}}{\mathrm{LH}}\right)\\
\mathrm{TAF} &= \mathrm{TC}\left(1+\frac{\mathrm{D0L}}{\mathrm{LH}}\right)
\end{aligned}$$

which gives

$$\begin{aligned}
\mathrm{TC} &= \frac{1}{2}(\mathrm{TMS}+\mathrm{TAF})\\
\frac{\mathrm{KL}}{\mathrm{LH}} &= 1-\frac{\mathrm{TMF}+\mathrm{TAS}}{\mathrm{TMS}+\mathrm{TAF}}\\
\frac{\mathrm{D0L}}{\mathrm{LH}} &= \frac{\mathrm{TAF}-\mathrm{TMS}}{\mathrm{TAF}+\mathrm{TMS}}
\end{aligned}$$

Thus, if 0 < TMF ≤ TMS ≤ TAS ≤ TAF are given as keyword input, then *"TC"* , KL, and D0L are calculated as above, and their keyword values are ignored.

3. **Heat generation.** The internal energy is

$$\epsilon = W+\eta T \ ,$$

and the entropy is defined as

$$\eta = -\frac{\partial W}{\partial T} = -\lambda\frac{\mathrm{LH}}{\mathrm{TC}}+c(\lambda)\left(1+\ln\left(\frac{T}{\mathrm{TC}}\right)\right) \ .$$

From the differential of $W$, we have

$$\begin{aligned}
\dot{W} &= \frac{\partial W}{\partial \boldsymbol{\varepsilon}}\dot{\boldsymbol{\varepsilon}}+\frac{\partial W}{\partial \lambda}\dot{\lambda}+\frac{\partial W}{\partial \boldsymbol{\varepsilon}_m}\dot{\boldsymbol{\varepsilon}}_m+\frac{\partial W}{\partial \boldsymbol{\varepsilon}_p}\dot{\boldsymbol{\varepsilon}}_p+\frac{\partial W}{\partial T}\dot{T}\\
&= \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}-d_\lambda\dot{\lambda}-\mathbf{d}_{\boldsymbol{\varepsilon}_m}:\dot{\boldsymbol{\varepsilon}}_m-\mathbf{d}_{\boldsymbol{\varepsilon}_p}:\dot{\boldsymbol{\varepsilon}}_p-\eta\dot{T}\\
&= \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}-\dot{D}-\eta\dot{T}
\end{aligned}$$

and thus

$$\dot{\epsilon} = \dot{W}+\dot{\eta}T+\eta\dot{T} \ = \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}-\dot{D}-\eta\dot{T}+(\dot{\eta}T+\eta\dot{T}) = \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}-\dot{D}-\dot{\eta}T \ .$$

Combining this with the energy balance

$$\dot{\epsilon} = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} + \nabla \cdot (k \nabla T) \ ,$$

we get

$$c(\lambda)\dot{T} = \nabla \cdot (k \nabla T) + Q \ ,$$

with the volumetric heat generation rate

$$Q = T \frac{\mathrm{LH}}{\mathrm{TC}} \dot{\lambda} - T c'(\lambda) \dot{\lambda} \left( 1 + \ln \left( \frac{T}{\mathrm{TC}} \right) \right) + \dot{D} \ .$$

4. **History variables.** The following history variables are available:

| History Variable # | Description |
|---|---|
| 1-6 | Strain in local coordinate system ($\boldsymbol{\varepsilon}$) |
| 7-12 | Martensite strain in local system ($\boldsymbol{\varepsilon}_m$) |
| 13 | Martensite volume fraction ($\lambda$) |
| 14 | Volumetric heat generation (Q) |
| 15-20 | Stress in local system ($\boldsymbol{\sigma}$) |
| 21-26 | Plastic strain in local system ($\boldsymbol{\varepsilon}_p$) |

In a thermal analysis, *MAT_291 can be coupled with *MAT_THERMAL_ISOTROPIC_TD_LC with load curves HCLC/TCLC depending on history variable 13, and load curve TGRLC depending on history variable 14. Note that *MAT_291 uses volumetric quantities for CPA, CPM, and LH, while the thermal materials use specific quantities, meaning the volumetric quantity divided by density.

## *MAT_ELASTIC_PERI

This is Material Type 292. This material is valid for modeling brittle elastic materials with peridynamics solids. Material failure is captured through a bond-based peridynamics model. See Ren et al 2017 for details about this model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | GT | GS | PSX | PSY | PSZ |
| Type | A | I | F | F | F | F | F | F |
| Default | none | none | none | $10^{20}$ | $10^{20}$ | 0.0 | 0.0 | 0.0 |

**Thermal Residual Stress Analysis Card.** This card is optional. It is only needed for performing a thermally-induced residual stress analysis.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GF | ALPHAT | | | | | | |
| Type | I | F | | | | | | |
| Default | 0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Material density |
| E | Young's modulus |
| GT | Fracture energy release rate |
| GS | Fracture energy release rate for compression |
| PSX | Initial strain along the *x*-axis |
| PSY | Initial strain along the *y*-axis |

| VARIABLE | DESCRIPTION |
|---|---|
| PSZ | Initial strain along the *z*-axis |
| GF | Flag to perform a thermal residual stress analysis during dynamic relaxation (see Remark1):<br>EQ.0: No thermal residual stress analysis.<br>GT.0: Perform thermal residual stress analysis. GF is a function ID for the function defining the thermal field: f(x,y,z,t). x, y, and z give the position, and t is time.<br>LT.0: Perform thermal residual stress analysis. Read peritprofile.txt which gives the initial temperature field for the thermal residual stress analysis. |
| ALPHAT | Isotropic thermal expansion coefficient for residual stress analysis |

**Remarks:**

1. **Thermal residual stress analysis.** Peridynamics supports performing a thermally-induced residual stress analysis during the dynamic relaxation phase. To perform this analysis, set GF to a nonzero value. The sign of GF determines how the temperature is provided for the analysis. If GF > 0, it refers to the ID of a function giving the thermal field as a function of position and time. GF < 0 causes reading file peritprofile.txt, which must provide the initial temperature over the region. This file must have the following format:

```
x, y, z, temp
```

where x, y, and z are position coordinates, and temp is the temperature at that position.

**References:**

B Ren, CT Wu, E Askari (2017) A 3D discontinuous Galerkin finite element method with the bond-based peridynamics model for dynamic brittle failure analysis, International Journal of Impact Engineering 99, 14-25.

## *MAT_ELASTIC_PERI_LAMINATE

This is Material Type 292A. This material is for modeling unidirectional fiber reinforced polymer laminates with peridynamics. Each lamina is modeled as a transversely isotropic material while the matrix is assumed to be isotropic. See Ren et al 2018 for details about this model.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E1 | E2 | PR12 | G12 | | |
| Type | I/A | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FOPT | FC1 | FC2 | FCC1 | FCC2 | FCD | FCDC | |
| Type | I | F | F | F | F | F | F | |
| Default | none | none | none | none | none | none | none | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | | | | | |
| Type | F | F | F | | | | | |
| Default | none | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Material density |

| VARIABLE | DESCRIPTION |
|---|---|
| E1 | Young's modulus-longitudinal direction for one lamina (1-direction) |
| E2 | Young's modulus-transverse direction for one lamina (2-direction) |
| PR12 | Poisson's ratio in the lamina plane |
| G12 | Shear modulus in the 12-direction |
| FOPT | Failure criterion type for FC1, FC2, FCC1, FCC2, FCD, and FCDC:<br>EQ.1: Energy release rate<br>EQ.2: Failure stretch ratio for tension (recommended) |
| FC1 | Tension failure criterion for longitudinal direction, 1-direction |
| FC2 | Tension failure criterion for transverse direction, 2-direction |
| FCC1 | Compression failure criterion for longitudinal direction, 1-direction |
| FCC2 | Compression failure criterion for transverse direction, 2-direction |
| FCD | Tension delamination failure criterion |
| FCDC | Compression delamination failure criterion |
| V1, V2, V3 | Components of the reference fiber direction in the global coordinate system |

**References:**

B Ren, CT Wu, P Seleson, D Zeng, D Lyu (2018) A peridynamic failure analysis of fiber-reinforced composite laminates using finite element discontinuous Galerkin approximations, International Journal of Fracture 214 (1), 49-68.

## *MAT_COMPRF

This is Material Type 293. This material models the behavior of pre-impregnated (pre-preg) composite fibers during the high-temperature preforming process. In addition to providing stress and strain, it also provides warp and weft yarn directions and stretch ratios after the forming process. The major applications of the model are for materials used in lightweight automobile parts.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ET | EC | PR | G121 | G122 | G123 |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G124 | G125 | G126 | GAMMAL | VF | EF3 | VF23 | EM |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VM | EPSILON | THETA | BULK | G | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Continuum equivalent mass density. |
| ET | Tensile modulus along the fiber yarns, corresponding to the slope of the curve in Figure M293-2 in the Stable Modulus region from a uniaxial tension test. See Remark 6. |
| EC | Compression modulus along the fiber yarns, reversely calculated using bending tests when all the other material properties are determined. See Remark 6. |

| VARIABLE | DESCRIPTION |
|---|---|
| PR | Poisson's ratio. See Remark 6. |
| G12*i* | Coefficients for the bias-extension angle change-engineering stress curve in Figure M293-3. G121 to G126 corresponds to the 6th-order to 1st-order factors of the loading curve. See Remark 6. |
| GAMMAL | Shear locking angle, in degrees. See Remark 6. |
| VF | Fiber volume fraction in the prepreg composite. |
| EF3 | Transverse compression modulus of the dry fiber. |
| VF23 | Transverse Poisson's ratio of the dry fiber |
| EM | Young's modulus of the cured resin. |
| VM | Poisson's ratio of the cured resin |
| EPSILON | Stretch ratio at the end of the undulation stage during the uniaxial tension test. Example shown in Figure M293-2. See Remark 6. |
| THETA | Initial angle offset between the fiber direction and the element direction. To reduce simulation error, when building the model, the elements should be aligned to the same direction as much as possible. |
| BULK | Bulk modulus of the prepreg material |
| G | Shear modulus of the prepreg material |

**Remarks:**

1. **Fiber and resin properties.** The dry fiber properties, EF3 and VF23, and the cure resin properties, EM and VM, are used to calculate the through-thickness elastic modulus of the prepreg using the rule of mixture. These properties will not affect the in-plane deformation of the prepreg during the preforming simulation.

2. **Shear locking.** In most of the preforming cases, the angle between the fiber yarns will not reach the shear-locking state. This model is not designed for, and, therefore, not recommended for simulating shear locking.

3. **History variables.** History variable 1 represents the angle between warp/weft yarns. History variables 2 and 3 are the stretch ratio of fibers in the 1 and 2 directions, respectively.

4. **BULK and G.** BULK and G are used by the contact algorithm. Changing these parameters will not affect the final simulation result significantly (but it may affect the time step).

5. **Model description.** Woven composite prepregs are characterized using a non-orthogonal coordinate system having two principal directions: one aligned with the longitudinal warp yarns and the other with the transverse weft yarns. Prior to deformation the warp and weft yarns are orthogonal. The directions and the fiber stretch ratios are determined from the deformation gradient. In Figure M293-1, the angles $\alpha$ and $\beta$ refer to the relative of the rotation of the warp yarn coordinate to the local corotational $x$ coordinate and the angle between the warp and weft yarns, respectively [2,3,4].

The stress from material deformation is divided into two parts: (1) stress caused by the fiber stretch, $\boldsymbol{\sigma}^f$, as shown in Figure M293-1 (a); (2) stress caused by the fiber rotation, $\boldsymbol{\sigma}^m$, as shown in Figure M293-1 (b). The total stress tensor, $\boldsymbol{\sigma}$, in the local corotational $x-y$ coordinate system is the sum where the components are given below [3]:

$$\sigma_{xx}^f = \sigma_1^f \cos^2\alpha + \sigma_2^f \cos^2(\alpha+\beta) \tag{1}$$

$$\sigma_{xy}^f = \sigma_{yx}^f = \frac{1}{2}\sigma_1^f \sin 2\alpha + \frac{1}{2}\sigma_2^f \sin 2(\alpha+\beta) \tag{2}$$

$$\sigma_{yy}^f = \sigma_1^f \sin^2\alpha + \sigma_2^f \sin^2(\alpha+\beta) \tag{3}$$

$$\sigma_{xx}^m = \frac{\sigma_1^m + \sigma_2^m}{2} + \frac{\sigma_1^m - \sigma_2^m}{2}\cos(2\alpha+\beta) \tag{4}$$

$$\sigma_{xy}^m = \sigma_{yx}^m = \frac{\sigma_1^m - \sigma_2^m}{2}\sin(2\alpha+\beta) \tag{5}$$

$$\sigma_{yy}^m = \frac{\sigma_1^m + \sigma_2^m}{2} - \frac{\sigma_1^m - \sigma_2^m}{2}\cos(2\alpha+\beta) \tag{6}$$

$$\sigma_{xx} = \sigma_{xx}^f + \sigma_{xx}^m \tag{7}$$

$$\sigma_{xy} = \sigma_{yx} = \sigma_{xy}^f + \sigma_{xy}^m \tag{8}$$

$$\sigma_{yy} = \sigma_{yy}^f + \sigma_{yy}^m \tag{9}$$

6. **Material property characterization.** The non-orthogonal stress components caused by yarn stretch and rotation at various deformation states will be characterized via a set of experiments, which are uniaxial tension, bias-extension and cantilever beam bending tests. All the tests need to be performed at the preforming temperature. See references [1] and [3] for more details.

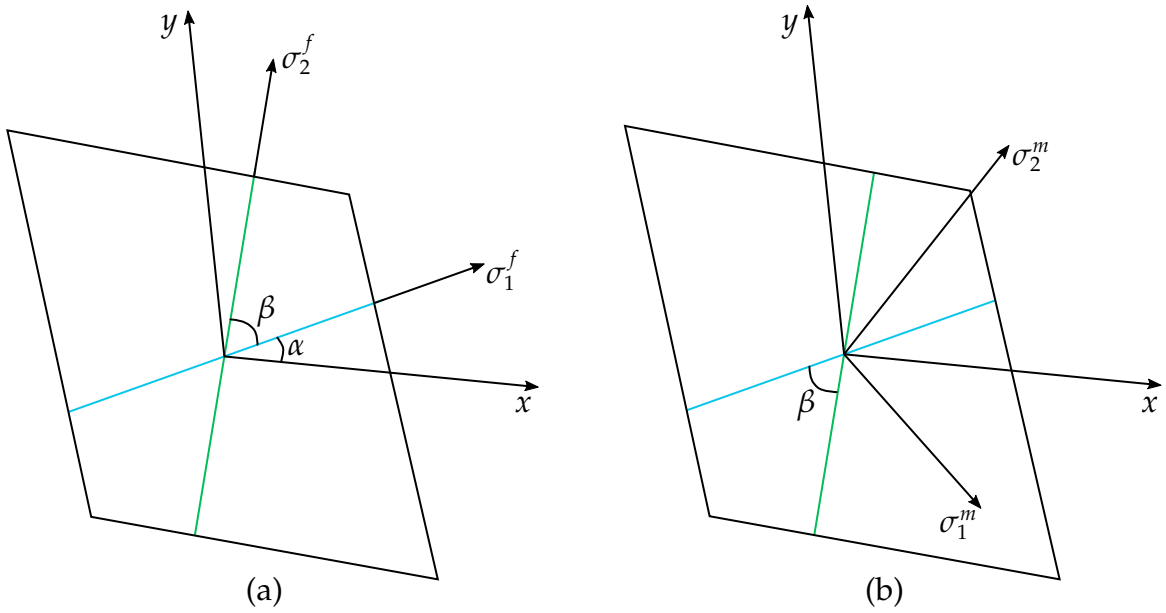



**Figure M293-1.** Stress components caused by (a) stretch in fiber directions and (b) rotation of the fibers [3].

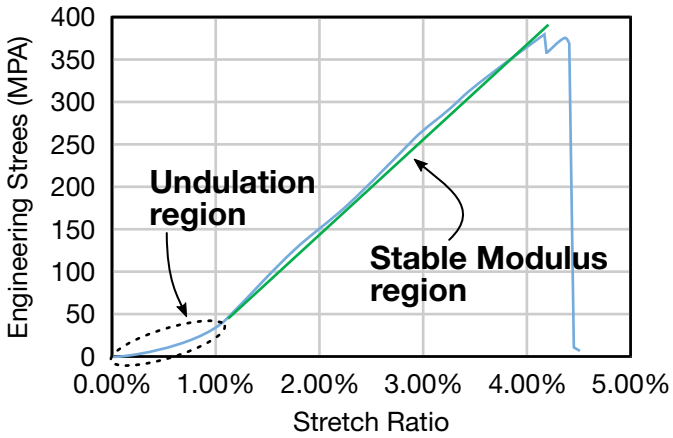



**Figure M293-2.** An example of the engineering stress as a function of stretch ratio from the uniaxial tension test [3].

The uniaxial tension test is used to obtain the fiber direction undulation strains and the stable tensile moduli, together with the in-plane Poisson's ratio (PR). A typical test result is shown in Figure M293-2. From the stretch ratio-engineering stress curve, the tensile modulus, ET, and the stretch ratio at the end of undulation, EPSILON, can be captured.

The bias-extension test is used to characterize the shear behavior of the composite needed for fields G12*i*. The test procedure comes from the benchmark test literature [1]. An example of the bias-extension test angle change-engineering stress curve is shown in Figure M293-3.

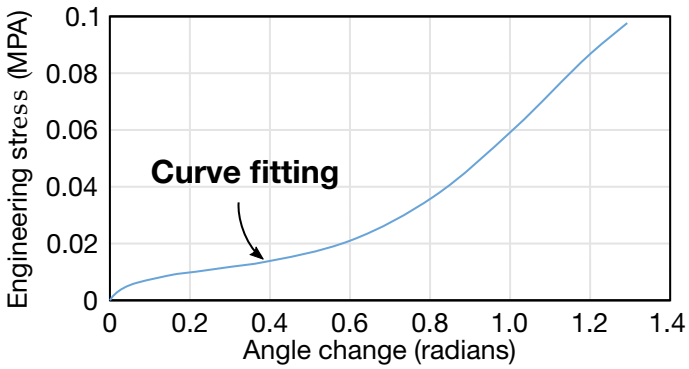



**Figure M293-3.** An example of the angle change-engineering stress curve from the bias-extension test. The curve fit for this example is $y = -0.29x^6 + 1.09x^5 - 1.68x^4 + 1.37x^3 - 0.56x^2 + 0.12x$ . For this example curve the inputs into LS-DYNA are G121 = −0.29, G122 = 1.09, G123 = −1.68, G124 = 1.37, G125 = −0.56, and G126 = −0.12 [3].

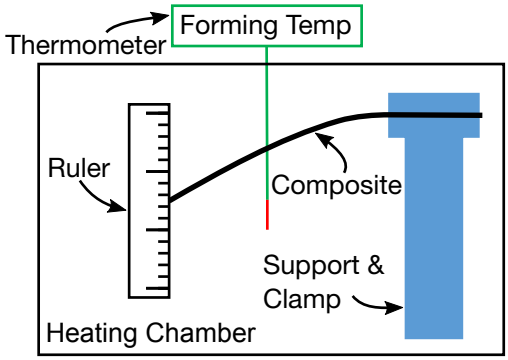



**Figure M293-4.** Bending test setup [3]

The angle change is calculated by using the equation [1]:

$$\gamma = \frac{\pi}{2} - 2\cos^{-1}\frac{D+d}{\sqrt{2}D}$$

where $d$ is the cross-head displacement and $D$ is the difference between the original height and the original width of the sample. This equation holds only before the shear locking angle, specified in field GAMMAL, which is measured directly at the end of the test, so the curve should end when the fiber yarn angle reaches the shear locking state.

The bending test should be performed to characterize the compression modulus along the yarn directions, as specified in the EC field. The test setup is shown in Figure M293-4. The composite specimen is held in a clamp and deforms under its own gravity. During the test, the composite is heated to the preforming temperature and the tip displacement is recorded. Due to the nonlinearity of the tensile modulus, the compression modulus is reversely calculated using a simulation: it is adjusted until the simulation leads to similar tip displacement to the

real experiment case. The starting point for the compression modulus iteration can be set as about 100X of the shear modulus when the warp and weft yarns are perpendicular to each other.

7. **Element type.** The material model is available for shell elements with OSU = 1 and INN = 2 in the CONTROL_ACCURACY card. It is recommended to use a double-precision version of LS-DYNA.

**References:**

[1] J. Cao, R. Akkerman, P. Boisse, J. Chen, H.S. Cheng, E.F. de Graaf, J.L. Gorczyca, P. Harrison, G. Hivet, J. Launay, W. Lee, L. Liu, S.V. Lomov, A. Long, E. de Luycker, F. Morestin, J. Padvoiskis, X.Q. Peng, J. Sherwood, Tz. Stoilova, X.M. Tao, I. Verpoest, A. Willems, J. Wiggers, T.X. Yu, B. Zhu, Characterization of mechanical behavior of woven fabrics: Experimental methods and benchmark results, Composites Part A: Applied Science and Manufacturing, Volume 39, Issue 6, 2008, Pages 1037-1053, ISSN 1359-835X.

[2] Pu Xue, Xiongqi Peng, Jian Cao, A non-orthogonal constitutive model for characterizing woven composites, Composites Part A: Applied Science and Manufacturing, Volume 34, Issue 2, 2003, Pages 183-193, ISSN 1359-835X.

[3] Weizhao Zhang, Huaqing Ren, Biao Liang, Danielle Zeng, Xuming Su, Jeffrey Dahl, Mansour Mirdamadi, Qiangsheng Zhao, Jian Cao, A non-orthogonal material model of woven composites in the preforming process, CIRP Annals - Manufacturing Technology, Volume 66, Issue 1, 2017, Pages 257-260, ISSN 0007-8506.

[4] X.Q. Peng, J. Cao, A continuum mechanics-based non-orthogonal constitutive model for woven composite fabrics, Composites Part A: Applied Science and Manufacturing, Volume 36, Issue 6, 2005, Pages 859-874, ISSN 1359-835X.

## *MAT_ANISOTROPIC_HYPERELASTIC

This is Material Type 295 which includes a collection of *(nearly-in)compressible, (an)isotropic, hyperelastic* material models primarily aimed at describing the mechanical behavior of biological soft tissues. Some of the material models may also be used to analyze a wider class of materials including fiber-reinforced elastomers and stretchable fabrics.

The constitutive laws are implemented in a modular fashion. Each module may be invoked at most once, however, the order of modules is interchangeable. Each module may comprise of different models. Consequently, one may easily change models in a module and include additional modules to account for more complex material behavior within the same keyword. Extending an existing module with a new model or even including a new module is straightforward and does not require a new material keyword.

### Card Summary:

**Card 1.** This card is required.

| MID | RHO | AOPT | | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** ISOtropic module. This card and all related cards below are required.

| TITLE | ITYPE | BETA | NU | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.1a.** Include this card if ITYPE = ±1.

| MU1 | MU2 | MU3 | MU4 | MU5 | MU6 | MU7 | MU8 |
|---|---|---|---|---|---|---|---|

**Card 2.2a.** Include this card if ITYPE = ±1.

| ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
|---|---|---|---|---|---|---|---|

**Card 2.1b.** Include this card if ITYPE = -2.

| C1 | C2 | C3 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.1c.** Include this card if ITYPE = ±3.

| K1 | K2 | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** ANISOtropic module. This card and all related cards below are optional.

| TITLE | ATYPE | INTYPE | NF | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.1.** Include this card if ATYPE = ±1. Include a pair of this card and one of the following two cards for each fiber family $i = 1,\ldots,NF$, that is, $2 \times NF$ cards in total.

| THETA | A | B | | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.2a.** Include this card if FTYPE = 1.

| FTYPE | FCID | K1 | K2 | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.2b.** Include this card if FTYPE = 2.

| FTYPE | FCID | E | RONORM | HONORM | | | |
|---|---|---|---|---|---|---|---|

**Card 3.3.** Include this card if INTYPE = 1.

| K1 | K2 | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** ACTIVE module. This and all related cards below are optional and may only be used in combination with the ANISOtropic module.

| TITLE | ACTYPE | ACDIR | ACID | ACTHR | SF | SS | SN |
|---|---|---|---|---|---|---|---|

**Card 4.1a.** Include this card if ACTYPE = 1.

| T0 | CA2ION | CA2IONM | N | TAUMAX | ST | B | L0 |
|---|---|---|---|---|---|---|---|

**Card 4.2a.** Include this card if ACTYPE = 1.

| L | DTMAX | MR | TR | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.1b.** Include this card if ACTYPE = 2.

| T0 | CA2ION | CA2IONM | N | TAUMAX | ST | B | L0 |
|---|---|---|---|---|---|---|---|

**Card 4.2b.** Include this card if ACTYPE = 2.

| L | ETA | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.1c.** Include this card if ACTYPE = 3.

| T0 | CA2ION | CA2ION50 | N | TAUMAX | ST | L | ETA |
|---|---|---|---|---|---|---|---|

**Card 4.1d.** Include this card if ACTYPE = 4.

| T0 | CA2ION50 | CA2IONM | N | TAUMAX | ST | CA2ION0 | TCA |
|---|---|---|---|---|---|---|---|

**Card 4.2d.** Include this card if ACTYPE = 4.

| L | ETA | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.1e.** Include this card if ACTYPE = 5.

| FSEID | FLID | FVID | ALPHAID | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is optional and must be used in combination with the ANISOtropic module only.

| XP | YP | ZP | A1 | A2 | A3 | MACF | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is optional and must be used in combination with the ANISOtropic module only.

| V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RHO | AOPT | | | | | |
| Type | A | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RHO | Mass density |
| AOPT | Material axes option (see *MAT_002 for a more complete description): |

EQ.0.0: Locally orthotropic with material axes determined by element nodes. The **a**-direction is from node 1 to node 2 of the element. The **b**-direction is orthogonal to the **a**-direction and is in the plane formed by nodes 1, 2, and 4. For shells only, the material axes are then rotated about the normal vector to the surface of the shell by the angle BETA.

EQ.1.0: Locally orthotropic with material axes determined by a

| VARIABLE | DESCRIPTION |
|---|---|
| | point, *P*, in space and the global location of the element center; this is the **a**-direction. This option is for solid elements only. |
| | EQ.2.0: Globally orthotropic with material axes determined by vectors **a** and **d** input below, as with *DEFINE_COORDINATE_VECTOR |
| | EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedrons. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle BETA. BETA may be set in the keyword input for the element or in the input for this keyword. Note that for solids, the material axes may be switched depending on the choice of MACF. The switch may occur before or after applying BETA depending on MACF. |
| | EQ.4.0: Locally orthotropic in a cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *P*, which define the centerline axis. This option is for solid elements only. |
| | LT.0.0: \|AOPT\| is a coordinate system ID (see *DEFINE_COORDINATE_*OPTION*). |

**Isotropic Module Card.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TITLE | ITYPE | BETA | NU | | | | |
| Type | A10 | I | F | F | | | | |
| Default | none | none | 0.0 | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TITLE | Module title which must be set to ISO |
| ITYPE | Type of isotropic model (see Remarks 1 and 2):<br>EQ.±1: Compressible/nearly-incompressible Ogden [12] (see Remark 4)<br>EQ.-2: Yeoh [13]<br>EQ.±3: Compressible/nearly-incompressible Holzapfel-Ogden [1], [7] |
| BETA | Volumetric response function coefficient |
| NU | Poisson's ratio (see Remark 3) |

**Ogden Model Card 1.** This card is only defined if ITYPE = ±1.

| Card 2.1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MU1 | MU2 | MU3 | MU4 | MU5 | MU6 | MU7 | MU8 |
| Type | F | F | F | F | F | F | F | F |

**Ogden Model Card 2.** This card is only defined if ITYPE = ±1.

| Card 2.2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA1 | ALPHA2 | ALPHA3 | ALPHA4 | ALPHA5 | ALPHA6 | ALPHA7 | ALPHA8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MU*i* | Ogden moduli, with $i = 1,\ldots,8$ |
| ALPHA*i* | Ogden constants, with $i = 1,\ldots,8$ |

**Yeoh Model Card.** This card is only defined if ITYPE = -2.

| Card 2.1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| C*i* | Yeoh moduli, with *i* = 1,2,3 |

**Holzapfel-Ogden Model Card.** This card is only defined if ITYPE = ±3.

| Card 2.1c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | K2 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K1 | Holzapfel-Ogden modulus |
| K2 | Holzapfel-Ogden constant |

**Anisotropic Module Card.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TITLE | ATYPE | INTYPE | NF | | | | |
| Type | A10 | I | I | I | | | | |
| Default | none | none | none | none | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TITLE | Module title which must be set to ANISO |

| VARIABLE | DESCRIPTION |
|---|---|
| ATYPE | Type of anisotropic model:<br>EQ.±1: General structure tensor-based; see Holzapfel et al. [8] (see Remark 5) |
| INTYPE | Type of interaction/coupling (see Remarks 6 and 7):<br>EQ.0: None<br>EQ.1: Holzapfel-Ogden [1], [5] |
| NF | Number of fiber families (see Remarks 5 and 6) |

**General Structure Tensor-Based Model Card A.** This card is only defined if ATYPE = ±1. Include a pair of this card and one of the 2 cards following this card for each fiber family $i = 1,\ldots,$NF, that is, 2 × NF cards in total.

| Card 3.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | THETA | A | B | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| THETA | Mean fiber family orientation angle with respect to the **a** material axis in the **ab** material plane in degrees |
| A | First structure tensor parameter |
| B | Second structure tensor parameter |

**Holzapfel-Gasser-Ogden Model Card.** This card is only defined if FTYPE = 1.

| Card 3.2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | FCID | K1 | K2 | | | | |
| Type | I | I | F | F | | | | |

**Freed-Doehring Model Card.** This card is only defined if FTYPE = 2.

| Card 3.2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FTYPE | FCID | E | R0NORM | H0NORM | | | |
| Type | I | I | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FTYPE | Type of fiber model:<br>EQ.1: Holzapfel-Gasser-Ogden [6]<br>EQ.2: Freed-Doehring [2] |
| FCID | Curve ID defining the fiber stress as a function of fiber stretch, default if nonzero. |
| K1 | Holzapfel-Gasser-Ogden modulus |
| K2 | Holzapfel-Gasser-Ogden constant |
| E | Fiber modulus |
| R0NORM | Initial crimp/coil amplitude normalized with respect to the initial fiber radius ($R_0/r_0$) |
| H0NORM | Initial crimp/coil wavelength normalized with respect to the initial fiber radius ($H_0/r_0$) |

**Holzapfel-Ogden Coupling Model Card(s).** These cards are only defined if INTYPE = 1.

| Card 3.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | K2 | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| K1 | Coupling modulus between the fiber and sheet directions |
| K2 | Coupling constant between the fiber and sheet directions |

**Active Module Card.**

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TITLE | ACTYPE | ACDIR | ACID | ACTHR | SF | SS | SN |
| Type | A10 | I | I | I | F | F | F | F |
| Default | none | none | 0 | 0 | 0.0 | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| TITLE | Module title which must be set to ACTIVE |
| ACTYPE | Type of active model:<br>EQ.1: Guccione-Waldman-McCulloch [4]<br>EQ.2: Guccione-Waldman-McCulloch [4] and Hunter-Nash-Sands [9]<br>EQ.3: Hunter-Nash-Sands [9]<br>EQ.4: Hunter-Nash-Sands [9] and Hunter-McCulloch-ter Keurs [10]<br>EQ.5: Martins-Pato-Pires [14] |
| ACDIR | Direction of active tension:<br>EQ.0: Active tension develops along the mean fiber orientation of all fiber families.<br>GT.0: Active tension develops along the mean orientation of the ACDIR$^{th}$ fiber family. |
| ACID | Activation curve ID (takes priority over T0 for ACTYPE = 1, 2, 3, or 4 when defined, see Remark 8) |
| ACTHR | (De/re)activation threshold (see Remark 8) |
| SF | Active stress scaling factor in the fiber direction (see Remark 9) |
| SS | Active stress scaling factor in the transverse sheet direction (see Remark 9) |

| VARIABLE | DESCRIPTION |
|---|---|
| SN | Active stress scaling factor in the transverse normal direction (see Remark 9) |

**Guccione-Waldman-McCulloch Model Card 1.** This card is only defined if AC-TYPE = 1.

| Card 4.1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T0 | CAION | CAIONM | N | TAUMAX | ST | B | L0 |
| Type | F | F | F | F | F | F | F | F |

**Guccione-Waldman-McCulloch Model Card 2.** This card is only defined if AC-TYPE = 1.

| Card 4.2a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L | DTMAX | MR | TR | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| T0 | Starting time of active stress development |
| CA2ION | Intercellular calcium ion concentration |
| CA2IONM | Maximum intercellular calcium ion concentration |
| N | Hill coefficient |
| TAUMAX | Peak isometric tension under maximum activation |
| ST | Active fiber stress scaling factor in the transverse directions (see Remark 9) |
| B | Shape coefficient |
| L0 | Sarcomere length with no active tension |
| L | Reference (stress-free) sarcomere length |

| VARIABLE | DESCRIPTION |
|---|---|
| DTMAX | Time to peak tension |
| MR | Slope of linear relaxation versus sarcomere length relation |
| TR | Time intercept of linear relaxation as a function of sarcomere length relation |

**Guccione-Waldman-McCulloch and Hunter-Nash-Sands Model Card 1.** This card is only defined if ACTYPE = 2.

| Card 4.1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T0 | CA2ION | CA2IONM | N | TAUMAX | ST | B | L0 |
| Type | F | F | F | F | F | F | F | F |

**Guccione-Waldman-McCulloch and Hunter-Nash-Sands Model Card 2.** This card is only defined if ACTYPE = 2.

| Card 4.2b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L | ETA | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| T0 | Starting time of active stress development |
| CA2ION | Intercellular calcium ion concentration |
| CA2IONM | Maximum intercellular calcium ion concentration |
| N | Hill coefficient |
| TAUMAX | Peak isometric tension under maximum activation |
| ST | Active fiber stress scaling factor in the transverse directions (see Remark 9) |
| B | Shape coefficient |

| VARIABLE | DESCRIPTION |
|---|---|
| L0 | Sarcomere length with no active tension |
| L | Reference (stress-free) sarcomere length |
| ETA | Scaling parameter |

**Hunter-Nash-Sands Model Card 1.** This card is only defined if ACTYPE = 3.

| Card 4.1c | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T0 | CA2ION | CA2ION50 | N | TAUMAX | ST | L | ETA |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| T0 | Starting time of active stress development |
| CA2ION | Intercellular calcium ion concentration |
| CA2ION50 | Intercellular calcium ion concentration at half of peak isometric tension |
| N | Hill coefficient |
| TAUMAX | Peak isometric tension under maximum activation |
| ST | Active fiber stress scaling factor in the transverse directions (see Remark 9) |
| L | Reference (stress-free) sarcomere length |
| ETA | Scaling parameter |

**Hunter-Nash-Sands and Hunter-McCullogh-ter Keurs Model Card A.** This card is only defined if ACTYPE = 4.

| Card 4.1d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T0 | CA2ION50 | CA2IONM | N | TAUMAX | ST | CA2ION0 | TCA |
| Type | F | F | F | F | F | F | F | F |

**Hunter-Nash-Sands and Hunter-McCullogh-ter Keurs Model Card B.** This card is only defined if ACTYPE = 4.

| Card 4.2d | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | L | ETA | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| T0 | Starting time of active stress development |
| CA2ION50 | Intercellular calcium ion concentration at half of peak isometric tension |
| CA2IONM | Maximum intercellular calcium ion concentration |
| N | Hill coefficient |
| TAUMAX | Peak isometric tension under maximum activation |
| ST | Active fiber stress scaling factor in the transverse directions (see Remark 9) |
| CA2ION0 | Intercellular calcium ion concentration at rest |
| TCA | Shape coefficient |
| L | Reference (stress-free) sarcomere length |
| ETA | Scaling parameter |

**Martins-Pato-Pires Model Card A.** This card is only defined if ACTYPE = 5.

| Card 4.1e | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FSEID | FLID | FVID | ALPHAID | | | | |
| Type | I | I | I | I | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FSEID | Serial stress function ID (see Remark 10) |
| FLID | Normalized force-contractile stretch curve ID |
| FVID | Normalized force-contractile stretch rate curve ID |
| ALPHAID | Activation curve ID |

**Local Coordinate System Card A.** These cards are only defined in combination with the ANISOtropic module.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MACF | |
| Type | F | F | F | F | F | F | I | |

**Local Coordinate System Card B.**

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | BETA | REF |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MACF | Material axes change flag for solid elements (see *MAT_002 for more details): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.-4: Switch material axes *b* and *c* before BETA rotation<br>EQ.-3: Switch material axes *a* and *c* before BETA rotation<br>EQ.-2: Switch material axes *a* and *b* before BETA rotation<br>EQ.1: No change, default<br>EQ.2: Switch material axes *a* and *b* after BETA rotation<br>EQ.3: Switch material axes *a* and *c* after BETA rotation<br>EQ.4: Switch material axes *b* and *c* after BETA rotation |
| V1, V2, V3 | Components of vector **v** for AOPT = 3 and 4 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |
| BETA | Material angle in degrees for AOPT = 0 (shells and thick shells only) and AOPT = 3 (all element types). This angle may be overridden on the element card; see *ELEMENT_SHELL_BETA, *ELEMENT_TSHELL_BETA, and *ELEMENT_SOLID_ORTHO. |
| REF | Use reference geometry to initialize the stress tensor. The reference geometry is defined by the keyword: *INITIAL_FOAM_REFERENCE_GEOMETRY.<br>EQ.0.0: Off<br>EQ.1.0: On |

**Remarks:**

1. **Volumetric strain energy function.** The pure volumetric part of the strain energy function is defined as part of the ISOtropic module.

2. **compressible and nearly incompressible models.** Depending on the sign of ITYPE and ATYPE, several formulations are available. Negative model numbers indicate that the corresponding part of the strain energy function is considered isochoric. Furthermore, the sign of FTYPE and INTYPE is directly linked to ATYPE. For example, if ATYPE is negative, both fiber and fiber interaction models are in their isochoric form. Consequently, compressible and nearly incompressible anisotropy is obtained by using both INTYPE and ATYPE with or without a sign, respectively.

3. **Incompressibility limit.** While there is no strict lower bound on the Poisson's ratio ($v_L$), for nearly incompressible materials, LS-DYNA will issue a warning message if $v < v_L = 0.49$.

4. **Special cases of the compressible/nearly incompressible Ogden model.** The following described special cases of the OGDEN Model (ITYPE = ±1).

   a) The (nearly-in)compressible neo-Hookean model is obtained as special case of ITYPE = (-)1 with $\mu_1 > 0$, $\alpha_1 = 1$.

   b) The Mooney-Rivlin model is obtained as a special case of ITYPE = -1 with $\mu_i\alpha_i > 0$ for $i = 1,2$, $\alpha_1 = 2$, and $\alpha_2 = -2$.

   c) By setting $\beta = -1$ and ITYPE = -1, one obtains an equivalent formulation with *MAT_077_O.

5. **General structure tensor.** Considering the anisotropic part of the strain energy function, one may distinguish angular integration (AI) and general structure tensor (GST) based models. Owing to their numerical efficacy, currently, all models in LS-DYNA rely on the general structure tensor [7], [8]. Parameters defining the structure tensor and fiber models need to be provided for each fiber family. Consequently, one may use different structure tensors and/or fiber models to describe the behavior of the individual fiber families.

   The model proposed by Freed *et al.* [3] is a special case of the general structure tensor-based models assuming rotational symmetric fiber dispersion, determining the parameters A and B using a normal distribution, and invoking the fiber model introduced by Freed and Doehring [2].

6. **Fiber families.** Characteristic material directions within the plane are defined by fiber families. The number of fiber families for INTYPE = 0 is currently limited to 3. If INTYPE = 1, the number of fiber families is limited to 2, representing the fiber and sheet directions, respectively. The fiber and sheet directions form an orthonormal basis.

7. **Coupling (quasi-)invariants.** To further enhance the material model, coupling (quasi-) invariants associated with pairs of directions may be included in the strain-energy function. Following the formulation in Holzapfel and Ogden [7] and Eriksson *et al.* [1], a single coupling invariant defined between the orthonormal fiber and sheet directions is included with INTYPE = 1.

8. **Onset of active stress.** The input for this model gives several different methods for triggering the activation and deactivation of active stress development.

   a) For ACTYPE = 1, 2, 3, and 4, T0 specifies the time at which active stress development is activated. This method does not include deactivation. The other methods take priority over setting T0.

   b) For all ACTYPE options, you can specify ACID and ACTHR. ACID represents the evolution of either the calcium ion concentration (ACTYPE = 1, 2,

3, or 5) or the transmembrane potential (ACTYPE = 4) over time. ACTHR is a threshold value for one of these quantities depending on ACTYPE. When the calcium ion concentration or transmembrane potential from the curve exceeds the threshold, the active stress development is activated. When it is less than the threshold, the active stress development is deactivated. With this method, the active stress development can be reactivated again when the value in the curve exceeds the threshold.

c) For all ACTYPE methods, if you set up a coupled problem with the electrophysiology solver, ACTHR again gives the threshold value for the calcium ion concentration (ACTYPE = 1, 2, 3, or 5) or transmembrane potential (ACTYPE = 4). The electrophysiology solver provides the value to compare to the threshold to activate and deactivate the active stress development. As with ACID, the active stress development is activated when the value exceeds ACTHR and deactivates when the value is less than ACTHR. With this method, the active stress development can be reactivated again when the value exceeds the threshold.

9. **Active stress development.** Active stress is developed along direction(s) defined by ACDIR and may be scaled using the scaling factors SF, SS, and SN. For ACTYPE < 5, if SS and SN are zero, they are reset internally to ST.

Depending on ACDIR active stress may develop along one or multiple fiber families. Consider a single fiber family with unit fiber orientation vector $\mathbf{e}_f$. Let $\tau_A$ be the active stress. Then, the active stress tensor in the local fiber frame is:

$$\boldsymbol{\tau}_A = \tau_A\left(\mathrm{SF}\ \mathbf{e}_f \otimes \mathbf{e}_f + \mathrm{SS}\ \mathbf{e}_s \otimes \mathbf{e}_s + \mathrm{SN}\ \mathbf{e}_n \otimes \mathbf{e}_n\right) \ .$$

Here $\mathbf{e}_s$ and $\mathbf{e}_n$ are the unit vectors in the sheet and normal directions that form a basis with $\mathbf{e}_f$.

If active tension develops along multiple fiber families, then the active stress tensor is:

$$\boldsymbol{\tau}_A = \sum_{i=1}^{\mathrm{NF}} \boldsymbol{\tau}_{A_i} \ .$$

In the above NF is the number of fiber families and $\boldsymbol{\tau}_{A_i}$ is the active stress tensor for the $i^{\text{th}}$ fiber family.

10. **Serial stress function.** The serial stress function needs to be expressed in terms of the fiber stretch $\lambda$ and contractile stretch $\lambda^{\mathrm{CE}}$. Thus, the elastic stretch in the serial element $\lambda^{\mathrm{SE}}$ needs to be eliminated using the multiplicative decomposition of the fiber stretch, that is, $\lambda = \lambda^{\mathrm{CE}}\lambda^{\mathrm{SE}}$.

11. **History variables.** The history variables are listed in the table below. The default number of history variables depends on the used modules. If only the

mandatory ISOtropic module is used, the number of history variables is 9. Including the ANISOtropic and ACTIVE modules in a hierarchical fashion yields an additional 12 and 9 history variables, that is, making the total number of history variables 21 and 30, respectively.

| History Variable # | Definition |
|---|---|
| 1-9 | Deformation gradient (column-wise storage) |
| 10-15 | First two rows of the rotation matrix defining the material coordinate system **a-b-c** |
| 16-18 | Fiber stretch in each fiber family |
| 19-21 | Fiber stress in each fiber family |
| 22-24 | Active fiber stress in each fiber family |
| 25 | Calcium ion concentration at $t^n$ if ACTYPE = 1,2,3,5)<br>Transmembrane potential at $t^n$ if ACTYPE = 4 |
| 26 | Calcium ion concentration at $t^{n-1}$ if ACTYPE = 1,2,3,5)<br>Transmembrane potential at $t^{n-1}$ if ACTYPE = 4 |
| 27 | Time since onset of activation |
| 28-30 | Contractile stretch in the dashpot at $t^n$ if ACTYPE = 5 |

**References:**


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## *MAT_ANAND_VISCOPLASTICITY

This is Material Type 296. This viscoplastic model by Professor Anand uses a set of evolution equations instead of a loading-unloading criterion to describe dislocation motion and the hardening or softening behavior of materials. This model can be applied to simulate solders used in electronic packaging.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | YM | PR | ALPHA | A1 | RATIOQR | XI |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M | S0 | H0 | A2 | SBAR | N | | TREF |
| Type | F | F | F | F | F | F | | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| YM | Young's modulus:<br>GT.0.0: Constant value<br>LT.0.0: \|YM\| refers to a load curve ID for a curve giving the Young's modulus as a function of temperature. |
| PR | Poisson's ratio:<br>GT.0.0: Constant value<br>LT.0.0: \|PR\| refers to a load curve ID for a curve giving the Poisson's ratio as a function of temperature. |
| ALPHA | Coefficient of thermal expansion, $\alpha$ |
| A1 | Pre-exponential factor, $A$ |

| VARIABLE | DESCRIPTION |
|---|---|
| RATIOQR | Ratio of the activation energy, $Q$ (J/mol), to the universal gas constant, $R$ (J/mol/K) |
| XI | Multiplier of stress, $\xi$ |
| M | Strain rate sensitivity, $m$ |
| S0 | Initial value of deformation resistance, $s_0$ |
| H0 | Hardening/softening constant, $H_0$ |
| A2 | Strain rate sensitivity of hardening or softening, $a$ |
| SBAR | Coefficient of deformation resistance saturation value, $\bar{s}$ |
| N | Strain rate sensitivity of deformation resistance saturation value, $n$ |
| TREF | Reference temperature, $T_{\text{ref}}$ |

**Remarks:**

In the Anand model, the equivalent stress $\sigma$ is proportional to the deformation resistance $s$ and depends upon the temperature $T$ and the equivalent elastic strain $\dot{\varepsilon}^p$ as

$$\sigma = c(T, \dot{\varepsilon}^p)s \ ,$$

where $c$ is a material parameter defined by

$$c = \frac{1}{\xi}\sinh^{-1}\left[\left(\frac{\dot{\varepsilon}^p}{A}\exp(\frac{Q}{RT})\right)^m\right] \ .$$

The material parameter $c$ depends on material constants defined in the variable list above and the universal gas constant $R$.

The above equations can be rearranged to express the equivalent inelastic strain rate $\dot{\varepsilon}^p$ in terms $\sigma$, $T$, and $s$ as

$$\dot{\varepsilon}^p = A\exp\left(-\frac{Q}{RT}\right)\left[\sinh(\xi\frac{\sigma}{s})\right]^{1/m} \ .$$

This is called the flow equation.

The rate of deformation resistance $\dot{s}$ is defined as

$$\dot{s} = H\dot{\varepsilon}^p \ ,$$

where

$$H = H_0 \left| 1 - \frac{s}{s_s} \right|^a sign\left(1 - \frac{s}{s_s}\right) .$$

In the above equation, $s_s$ is the deformation resistance saturation value which is defined as

$$s_s = \bar{s}\left[\frac{\dot{\varepsilon}^p}{A}\exp(\frac{Q}{RT})\right]^n .$$

With the equivalent inelastic strain rate $\dot{\varepsilon}^p$, the inelastic strain components can be computed based on a normality hypothesis of the Prandtl-Reuss flow law:

$$\dot{\boldsymbol{\varepsilon}}^p = \sqrt{\frac{3}{2}}\dot{\varepsilon}^p \mathbf{N} .$$

In the above equation, the direction of plastic flow **N** is defined as

$$\mathbf{N} = \sqrt{\frac{3}{2}}\frac{\mathbf{S}}{\sigma} ,$$

where **S** is the deviatoric part of the stress $\sigma$.

The Cauchy Stress **T** for this model is

$$\mathbf{T} = J^{e-1}\mathbf{R}^e\mathbf{M}^e{\mathbf{R}^e}^T ,$$

where

$$\mathbf{M}^e = \mathbf{C}[\mathbf{E}^e - \alpha(T - T_0)] .$$

$T_0$ is the initial temperature. **C** is defined as

$$\mathbf{C} \stackrel{\text{def}}{=} 2G\left(\mathbb{I}^s - \frac{1}{3}\mathbf{I}\otimes\mathbf{I}\right) + K\mathbf{I}\otimes\mathbf{I} .$$

### References:

[1] Brown, Stuart B., Kwon H. Kim, and Lallit Anand. "An internal variable constitutive model for hot working of metals." International journal of plasticity 5.2 (1989): 95-130.

[2] Lallit Anand, Constitutive equations for hot-working of metals, International Journal of Plasticity, (1985), 213-231.

## *MAT_DMN_COMPOSITE_FRC

This is Material Type 303. It is a machine-learning-based multiscale material model for analysis of injection-molded fiber-reinforced composites (FRC). The multiscale material model can predict the macroscopic material responses (stress, equivalent plastic strain, etc.) based on the microscopic material information. Using this material model requires providing the geometric descriptors for material microstructures (i.e., fiber orientation tensor, fiber volume fraction, etc.) and the material properties of each base material (i.e., fiber and matrix), respectively. Only certain constitutive laws described in Remarks 2 and 3 are supported for the base materials. The FIBAND (FIBer-distribution-based Anisotropic Damage) model is available for predicting the failure of injection-molded composites with arbitrary fiber distributions.

To obtain the geometrical information for the microstructures, use an injection molding simulation software, such as Moldex3D. LS-PrePost can import the injection molding results into models (see Remark 6 and Workflow to import fiber data from Moldex3D).

This model is supported for MPP/SMP double precision versions. Currently, this 3D multiscale material model supports explicit dynamic finite element analysis using eight-node hexahedral solid elements, four-node tetrahedral solid elements, SPG elements, and types 25 and 26 shell elements.

### Card Summary:

**Card 1.** This card is required.

| MID | | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| FVF | RO | RF | RM | FL | FD | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| F_E | F_PR | ISO | DAM | | | S_F | S_M |
|---|---|---|---|---|---|---|---|

**Card 3.1.** Include this card if ISO = 1 in Card 3.

| F_EL | F_ET | F_PRTL | F_PRTT | F_GLT | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| M_E | M_PR | M_SY | M_H1 | M_H2 | M_H3 | ITC | |
|---|---|---|---|---|---|---|---|

**Card 4.1.** Include this card if ITC = 1 or 3 in Card 4.

| M_EC | M_PRC | M_SYC | M_H1C | M_H2C | M_H3C | PT | PC |
|---|---|---|---|---|---|---|---|

**Card 5.** Include this card if DAM = 1 or 2 on Card 3.

| D_C | | | | D_ERO | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| LCIDT | LCIDC | LCFS | LCFA | LCSRS | LCSRA | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | | | | | | | |
| Type | A | | | | | | | |
| Default | none | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FVF | RO | RF | RM | FL | FD | | |
| Type | F | F | F | F | F | F | | |
| Default | none | none | none | none | none | none | | |

| VARIABLE | DESCRIPTION |
|---|---|
| FVF | $\varphi^f$, fiber volume fraction. If RF and RM are given, a nonzero FVF must be specified to calculate the mass density of the overall fiber-reinforced composite. This value can be overwritten by *INI- |

| VARIABLE | DESCRIPTION |
|---|---|
| | TIAL_STRESS_SHELL or *INITIAL_STRESS_SOLID. See Remark 4. |
| RO | $\rho^c$, mass density of the overall fiber-reinforced composite. This value will be neglected if RF and RM are given, respectively. |
| RF | $\rho^f$, mass density of the fiber phase |
| RM | $\rho^m$, mass density of the matrix phase |
| FL | Fiber length. Alternatively, if you want to specify the fiber aspect ratio, set FL to the fiber aspect ratio and FD to 1.0. |
| FD | Fiber diameter. Alternatively, if you want to directly specify the fiber aspect ratio, set FD to 1.0 and FL to the fiber aspect ratio. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | F_E | F_PR | ISO | DAM | | | S_F | S_M |
| Type | F | F | I | I | | | F | F |
| Default | none | none | 0 | 0 | | | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| F_E | $E^f$, Young's modulus of the fiber phase if the fiber property is isotropic. See Remark 2. |
| F_PR | $v_{tl}^f$, Poisson's ratio of the fiber phase if the fiber property is isotropic. |
| ISO | Flag for anisotropy of the fiber phase:<br>EQ.0: Isotropic fiber material property<br>EQ.1: Transversely isotropic fiber material property |
| DAM | Flag for the composite failure model:<br>EQ.0: Do not consider material damage.<br>EQ.1: Use the FIBAND model. See Remark 4. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.2: Material damage occurs when the failure strength of the fiber or failure strain of the matrix is reached. See Remark 5. |
| S_F | Failure strength of the fiber phase (ignored unless DAM = 2) |
| S_M | Failure strain of the matrix phase (ignored unless DAM = 2) |

**Transversely Isotropic Fiber Material Card.** Include this card if ISO = 1 on Card 3.

| Card 3.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | F_EL | F_ET | F_PRTL | F_PRTT | F_GLT | | | |
| Type | F | F | F | F | F | | | |
| Default | none | none | none | none | none | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| F_EL | $E_l^f$, Young's modulus of the fiber phase along the fiber's longitudinal direction, $l$. See Remark 2. |
| F_ET | $E_t^f$, Young's modulus of the fiber phase along the fiber's transverse direction $t$. Note that the transversely isotropic model becomes isotropic by setting $E_t^f = E_l^f$, $v_{tt}^f = v_{tl}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1 + v_{tl}^f\right)\right]$. |
| F_PRTL | $v_{tl}^f$, Poisson's ratio of the fiber phase |
| F_PRTT | $v_{tt}^f$, Poisson's ratio of the fiber phase. Note that the transversely isotropic model becomes isotropic by setting $E_t^f = E_l^f$, $v_{tt}^f = v_{tl}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1 + v_{tl}^f\right)\right]$. |
| F_GLT | $G_{lt}^f$, shear modulus of the fiber phase in the $lt$ direction. Note that the transversely isotropic model becomes isotropic by setting $E_t^f = E_l^f$, $v_{tt}^f = v_{tl}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1 + v_{tl}^f\right)\right]$. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M_E | M_PR | M_S1 | M_S2 | M_S3 | M_S4 | ITC | |
| Type | F | F | F | F | F | F | I | |
| Default | none | none | none | none | none | none | 0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| M_E | $E^m$, Young's modulus of the matrix phase. See Remark 3. |
| M_PR | $v^m$, Poisson's ratio of the matrix phase |
| M_S1 | $s_1^m$, plastic yielding parameter of the matrix phase |
| M_S2 | $s_2^m$, plastic yielding parameter of the matrix phase |
| M_S3 | $s_3^m$, plastic yielding parameter of the matrix phase |
| M_S4 | $h_0^m$, plastic yielding parameter of the matrix phase |
| ITC | Option for the elastoplastic material law for the matrix phase.<br>EQ.0: No tension-compression asymmetry in material properties<br>EQ.1: Use tension-compression asymmetric material properties<br>EQ.2: Use a viscoplastic formulation to account for strain rate effects, where a table can define the yield strength as a function of the equivalent plastic strain for various strain rates<br>EQ.3: Use tension-compression asymmetric material properties in a viscoplastic formulation to account for strain rate effects |

**Tension-Compression Asymmetry Card.** Include this card if ITC = 1 or 3.

| Card 4.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M_EC | M_PRC | M_S1C | M_S2C | M_S3C | M_S4C | PT | PC |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| M_EC | $E^m$, Young's modulus of the matrix phase in compression |
| M_PRC | $v^m$, Poisson's ratio of the matrix phase in compression |
| M_S1C | $s_1^m$, plastic yielding parameter of the matrix phase in compression |
| M_S2C | $s_2^m$, plastic yielding parameter of the matrix phase in compression |
| M_S3C | $s_3^m$, plastic yielding parameter of the matrix phase in compression |
| M_S4C | $h_0^m$, plastic yielding parameter of the matrix phase in compression |
| PT | Absolute value of the tensile mean stress threshold beyond which the tensile material properties are adopted. If the mean stress $(\sigma_{XX} + \sigma_{YY} + \sigma_{ZZ})/3$ falls within the range [−PC, PT], a weighted average of the tensile and compressive material properties is used for the matrix phase. See Remark 3. |
| PC | Absolute value of the compressive mean stress threshold beyond which compressive material properties are adopted. |

**Material Failure Card.** Include this card if DAM = 1 or 2 on Card 3.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D_C | | | | D_ERO | | | |
| Type | F | | | | F | | | |
| Default | none | | | | optional | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D_C | Critical damage threshold value |
| D_ERO | Maximum damage value beyond which the element is eroded. Element erosion does not occur if this value is left empty. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCIDT | LCIDC | LCFS | LCFA | LCSRS | LCSRA | | |
| Type | I | I | I | I | I | I | | |
| Default | none | none | none | none | 0 | 0 | | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCIDT | Load curve or table ID. The load curve is available for ITC = 0 and 1 while the table is available for ITC = 2 and 3.<br><br>**Load Curve.** When LCIDT is a load curve ID, data points representing the accumulated equivalent plastic strain and the corresponding yield strength for the matrix phase are respectively given in the first column and the second column of the corresponding load curve in *DEFINE_CURVE. If ITC = 1 is specified in Card 4, this load curve describes the matrix material in tension only. See Remark 3.<br><br>**Tabular Data.** If ITC = 2 or 3 is specified in Card 4, LCIDT is treated as a table ID. Data points representing different strain rates are given in one column of the corresponding table in *DEFINE_TABLE, followed by the definitions of load curves for the yield strength for the matrix phase as a function of effective plastic strain at each given strain rate value. See *DEFINE_TABLE. Linear interpolation of the yield strengths at different given strain rates is used by default. If the strain rate values fall out of range, extrapolation is not used; instead, either the first or last curve determines the yield strength as a function of effective plastic strain, which depends on whether the strain rate falls below the minimum given value or exceeds the maximum given value, respectively. If ITC = 3 is specified in Card 4, this table describes the matrix material in tension only.<br><br>**Logarithmically-Defined Tables.** If ITC = 2 or 3 is specified in Card 4, LCIDT refers to a table ID. In addition, if the first value in |

| VARIABLE | DESCRIPTION |
|---|---|
| | the table is negative, all data points in the table represent the natural logarithm of strain rates, and logarithmic interpolation of the yield strengths at discrete given strain rates is used. Note that this option works only when the lowest strain rate has a value less than 1.0. For values greater than or equal to 1.0, use the LOG_INTERPOLATION option. |
| LCIDC | ID for a load curve or table. Similar to LCIDT, LCIDC is the ID of a load curve or table that contains the accumulated equivalent plastic strain and the yield strength for the matrix phase in compression. This parameter is available for ITC = 1 (load curve) and 3 (table). The description for the load curve and table is the same as for LCIDT but for compression. See Remark 3. |
| LCFS | ID of a load curve giving the equivalent failure strain, $\bar{\varepsilon}^F$, (ordinate) as a function of the stress triaxiality state (abscissa). This curve should only be defined when DAM = 1. |
| LCFA | ID of a load curve giving the failure anisotropy factor, $\bar{\beta}^F$, (ordinate) as a function of the stress triaxiality state (abscissa). $\bar{\beta}^F$ falls within the range [0,1]. This curve should only be defined when DAM = 1. |
| LCSRS | ID of a load curve giving scaling factor SRS (ordinate) as a function of strain rate (abscissa). The factor SRS scales the failure strain, $\bar{\varepsilon}^F$, defined with LCFS. It is optional. |
| LCSRA | ID of a load curve giving scaling factor SRA (ordinate) as a function of strain rate (abscissa). The factor SRA scales the failure anisotropy factor, $\bar{\beta}^F$, defined with LCFA. It is optional. |

**Remarks:**

1. **Mechanistic machine learning-based multiscale simulation.** The core algorithm of this multiscale simulation is the DMN (Deep Material Network) based upon a mechanistic machine learning technique [1, 2, 3, 4, 5, 6]. As described in [4, 5, 6], the machine learning model creation requires an "offline" training process, which involves the learning of composite material physics hidden in the high-fidelity RVE simulation-based data. To cover a wide variety of material microstructures of fiber-reinforced composites, we adopted a transfer learning method [3, 4, 6] to generate different networks based on the actual material microstructure information at each integration point of the finite element mesh. In LS-DYNA, we have implemented a trained DMN database. It can effectively

predict the highly nonlinear macroscopic material behaviors. The computational cost of DMN is orders-of-magnitude lower than finite element simulation of high-fidelity 3D RVEs containing complex material microstructures. The overall concurrent multiscale simulation framework enabled by DMN is depicted in Figure M303-1.

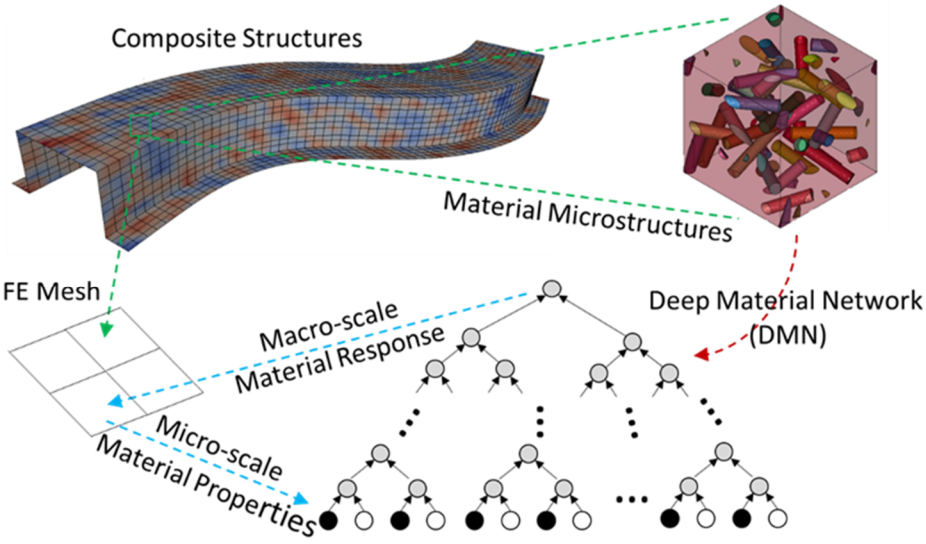



**Figure M303-1.** Schematic of the simulation framework for concurrent multiscale nonlinear analysis of injection-molded fiber-reinforced composite structures.

Different from conventional material models, this machine learning-based multiscale material model is data-driven, so its simulation capability can be continuously enhanced as more high-quality training data for fiber-reinforced composites are supplied in the future. Enhanced DMN databases with new functions will be available in future releases.

2. **Constitutive laws for fiber materials.** By default, the constitutive behaviors of the fiber material are modeled with isotropic elasticity. If you set ISO to 1 on Card 3, then the fiber material is modeled with a transversely isotropic elasticity. Its symmetric compliance matrix takes the following form:

$$\begin{bmatrix} 1/E_l^f & -v_{tl}^f/E_t^f & -v_{tl}^f/E_t^f & 0 & 0 & 0 \\ -v_{tl}^f/E_t^f & 1/E_t^f & -v_{tt}^f/E_t^f & 0 & 0 & 0 \\ -v_{tl}^f/E_t^f & -v_{tt}^f/E_t^f & 1/E_t^f & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{lt}^f & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\left(1+v_{tl}^f\right)/E_l^f & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{lt}^f \end{bmatrix}$$

Here $E_l^f$ and $E_t^f$ are the Young's moduli along the fiber's longitudinal ($l$) and transverse ($t$) directions, respectively; $v_{tl}^f$and $v_{tt}^f$ are the Poisson's ratios; and $G_{lt}^f$ is the shear modulus. If the five material parameters given in Card 3.1 satisfy $E_l^f = E_t^f$, $v_{tl}^f = v_{tt}^f$, and $G_{lt}^f = E_l^f / \left[2\left(1 + v_{tl}^f\right)\right]$, then the elastic fiber's material model becomes isotropic which is the same as simply providing the two material constants in Card 3.

3. **Constitutive laws for matrix materials.** The matrix materials are modeled with an associated elastoplastic constitutive model. If ITC = 0 in Card 4, then the material properties given in Card 4 and/or LCIDT in Card 6 will be used for the matrix material, and Card 4.1 should not appear in the input file. However, if ITC = 1 or 3 is specified in Card 4, the model for matrix material considers tension-compression asymmetry. In this case, Card 4 and/or LCIDT specify tensile material properties while Card 4.1 and/or LCIDC specify compressive material properties. By default, the sign of the mean stress $(\sigma_{XX} + \sigma_{YY} + \sigma_{ZZ})/3$ determines tension with a positive sign indicating that the material is in tension. Numerically, an abrupt transition from the tensile and compressive yield surfaces may cause convergence difficulty. To avoid this numerical issue, we can assign small positive numbers (e.g., a small percentage of the yield strength) to PT and PC in Card 4.1 which define a mean stress range $[-\text{PC}, \text{PT}]$ for which weighted averaged values of the tensile and compressive properties are used in the simulation.

   Three von Mises yield functions with different hardening laws are available, including:

   a) A yield function based on the following isotropic hardening law:

   $$s_Y^m = s_1^m + s_2^m \bar{\varepsilon}_P^m - s_3^m \exp(-h_0^m \bar{\varepsilon}_P^m)$$

   where $s_Y^m$ denotes the current yield strength for the matrix phase, $\bar{\varepsilon}_P^m$ denotes the accumulated equivalent plastic strain of the matrix material, and plastic yielding parameters $h_0^m$, $s_1^m$, $s_2^m$, and $s_3^m$ are defined in Card 4 or Card 4.1. If $s_3^m = 0$, the yield function becomes equivalent to a linear hardening law with a hardening coefficient $s_2^m$ an initial yield strength $s_1^m$. Otherwise, $s_1^m - s_3^m$ represents the initial yield strength.

   b) A yield function based on an input hardening curve. The curve ID is given as LCIDT or LCIDC in Card 6, where the curve data's first and second columns represent, respectively, the accumulated equivalent plastic strain, $\bar{\varepsilon}_P^m$, and the corresponding yield strength, $s_Y^m$.

   c) A yield function based on an input hardening table. The table ID is given as LCIDT or LCIDC in Card 6, where the associated curves provide the accumulated equivalent plastic strain $\bar{\varepsilon}_P^m$ and the corresponding yield strength $s_Y^m$ at different strain rates.

If a curve or table is provided in Card 6, the corresponding yield strength and hardening parameters defined in Card 4 or 4.1 are ignored.

Based on feedback and requests, we will develop other constitutive laws for the base materials to capture more complex behaviors of composites, such as material failures.

4. **FIBAND model for short-fiber-reinforced composites.** The FIBer-distribution-based ANisotropic Damage (FIBAND) model predicts the stress triaxiality and strain-rate-dependent anisotropic failures of injection-molded fiber-reinforced composites. DAM = 1 on Card 3 enables this model.

   For unidirectionally aligned short-fiber-reinforced composites (UD-SFRC), larger values of the equivalent failure strain, $\bar{\varepsilon}^F$, correspond to a higher failure strength, and larger values of the factor, $\bar{\beta}^F$, correspond to higher degrees of anisotropy in the failure strength. These two parameters are defined as functions of the stress triaxiality state through load curves LCFS and LCFA in Card 6. Note that $\bar{\beta}^F$ falls within the range [0,1]. The factors defined by LCSRS and LCSRA can scale $\bar{\varepsilon}^F$ and $\bar{\beta}^F$, respectively. In addition, the critical damage threshold value, D_C on Card 5, falls within the range [0,1]. Higher values correspond to faster damage growth rates that are suitable for capturing more brittle failures.

   For injection-molded composites with more complex fiber orientation states, FIBAND models the anisotropic failure of the composite by representing its microstructure as an aggregate of UD-SFRC unit cells with various orientations. The stress triaxiality and strain-rate-dependent damage initiation and growth are predicted based on the parameters in Cards 5 and 6 in conjunction with the local fiber distribution state. The local fiber distribution state is parametrized by the second-order fiber orientation tensor and the fiber volume fraction, which are mapped from injection molding data (see Remark 6 and Workflow to import fiber data from Moldex3D).

5. **Failure and damage based on fiber strength and matrix strain.** Setting DAM = 2 invokes a failure model in which damage begins to occur when the fiber failure strength (S_F) or matrix failure strain (S_M) is reached. Like for DAM = 1, D_C must be defined on Card 5 to provide the critical damage.

6. **Heterogeneous distributions of fiber orientations and fiber volume fractions.** Due to the manufacturing process, fiber-reinforced composites contain heterogeneous distributions of material microstructures, such as different fiber orientations, fiber volume fractions, and thermally/chemically-induced residual stresses at different locations of the composite structure. This information can be obtained from either experimental measurements or injection molding simulation software packages. If these microstructure data are available, they can be used as initial conditions in LS-DYNA by defining the keyword *INITIAL_-

STRESS_SOLID or *INITIAL_STRESS_SHELL, depending on the finite element formulations adopted in the macroscale numerical model.

a) If solid finite elements (e.g., eight-node hexahedron or four-node tetrahedron elements) are used, the *INITIAL_STRESS_SOLID keyword can be used to define the fiber information. As shown in the following example, 6 history variables initialize the components of the symmetric fiber orientation tensor, $(A_{XX})_e^p$, $(A_{YY})_e^p$, $(A_{XY})_e^p$, $(A_{YZ})_e^p$, $(A_{XZ})_e^p$, and the fiber volume fraction, $(fvf)_e^p$, at each integration point of the finite element mesh. The subscript $e = 1, 2, 3, \ldots$ denotes the finite element index, and the superscript $p = 1, 2, 3, \ldots$ denotes the integration point index. Note that, the component $(A_{ZZ})_e^p$ of the fiber orientation tensor is not provided in this keyword because it can be easily calculated based on its relationship with the $(A_{XX})_e^p$ and $(A_{YY})_e^p$ components, i.e., $(A_{ZZ})_e^p = 1.0 - (A_{YY})_e^p - (A_{XX})_e^p$. *MAT_303 offers an effective method of capturing the effects of the manufacturing-process-induced residual stress field on the mechanical performance. If the residual stress effects need to be considered, *INITIAL_STRESS_SOLID can provide the six residual stress components at each integration point. See *INITIAL_STRESS_SOLID for details.

```
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$  Define Fiber Information in *INITIAL_STRESS_SOLID
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*KEYWORD
$
$...>....1....>....2....>....3....>....4....>....5....>....6....>....7....>....8
*INITIAL_STRESS_SOLID
$       EID      NINT     NHISV     LARGE
          1         1         6         1
$......>.......1.......>.......2.......>.......3.......>.......4.......>.......5
$           SIGXX           SIGYY           SIGZZ           SIGXY           SIGYZ
              0.0             0.0             0.0             0.0             0.0
$           SIGZX             EPS           HISV1           HISV2           HISV3
              0.0             0.0      (A_XX)_1^1      (A_YY)_1^1      (A_XY)_1^1
$           HISV4           HISV5           HISV6
            (A_YZ)_1^1      (A_XZ)_1^1      (fvf)_1^1

…

$       EID      NINT     NHISV     LARGE
          e         1         6         1
$           SIGXX           SIGYY           SIGZZ           SIGXY           SIGYZ
              0.0             0.0             0.0             0.0             0.0
$           SIGZX             EPS           HISV1           HISV2           HISV3
              0.0             0.0      (A_XX)_e^1      (A_YY)_e^1      (A_XY)_e^1
$           HISV4           HISV5           HISV6
            (A_YZ)_e^1      (A_XZ)_e^1      (fvf)_e^1

…
```

```
$
*END
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

b) If shell finite elements (formulations 25 or 26 that support the use of 3D constitutive laws) are used, the *INITIAL_STRESS_SHELL keyword can be used to define the fiber information. If we assume that the one-point quadrature scheme is adopted in the in-plane direction while 3 integration points exist along the shell thickness direction (note: you can define the number of through-thickness integration points), then we need to define 6 history variables at each of the 3 integration points of every shell finite element. These history variables include the components of the symmetric fiber orientation tensor, $\left(A_{XX}\right)_e^p$, $\left(A_{YY}\right)_e^p$, $\left(A_{XY}\right)_e^p$, $\left(A_{YZ}\right)_e^p$, $\left(A_{XZ}\right)_e^p$, and the fiber volume fraction, $(fvf)_e^p$. The subscript $e = 1, 2, 3, \ldots$ denotes the finite element index, and the superscript $p = 1, 2, 3, \ldots$ denotes the integration point index. Please refer to the following example.

This material also supports a random short fiber orientation state. This state refers to the condition where the reinforcing fibers are distributed uniformly in all in-plane directions, resulting in isotropic material behavior in all in-plane directions. To obtain this state, set $(A_{xx})_e^p = \left(A_{yy}\right)_e^p = 0.5$. All other components of the fiber orientation tensor are zero.

*MAT_303 offers an effective method of capturing the effects of the manufacturing-process-induced residual stress field on the mechanical performance. If the residual stress effects need to be considered, *INITIAL_STRESS_SHELL can provide the six residual stress components at each integration point. See *INITIAL_STRESS_SHELL for details.

```
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
$$$$  Define Fiber Information in *INITIAL_STRESS_SHELL
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
*KEYWORD
$
$...>....1....>....2....>....3....>....4....>....5....>....6....>....7....>....8
*INITIAL_STRESS_SHELL
$      EID    NPLANE    NTHICK     NHISV    NTENSR     LARGE
         e         1         3         6         0         1
$......>.......1.......>.......2.......>.......3.......>.......4.......>.......5
$             T           SIGXX           SIGYY           SIGZZ           SIGXY
        -0.77459             0.0             0.0             0.0             0.0
$          SIGYZ           SIGZX             EPS
             0.0             0.0             0.0
$          HISV1           HISV2           HISV3           HISV4           HISV5
           (A_XX)_e^1      (A_YY)_e^1      (A_XY)_e^1      (A_YZ)_e^1      (A_XZ)_e^1
$          HISV6
           (fvf)_e^1
$             T           SIGXX           SIGYY           SIGZZ           SIGXY
         0.00000             0.0             0.0             0.0             0.0
```

```
$           SIGYZ           SIGZX             EPS
              0.0             0.0             0.0
$           HISV1           HISV2           HISV3           HISV4           HISV5
            (A_XX)_e^2      (A_YY)_e^2      (A_XY)_e^2      (A_YZ)_e^2      (A_XZ)_e^2
$           HISV6
            (fvf)_e^2
$             T             SIGXX           SIGYY           SIGZZ           SIGXY
          0.77459             0.0             0.0             0.0             0.0
$           SIGYZ           SIGZX             EPS
              0.0             0.0             0.0
$           HISV1           HISV2           HISV3           HISV4           HISV5
            (A_XX)_e^3      (A_YY)_e^3      (A_XY)_e^3      (A_YZ)_e^3      (A_XZ)_e^3
$           HISV6
            (fvf)_e^3
…

$
*END
$
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

### Workflow to import fiber data from Moldex3D:

LS-PrePost 4.11 or newer versions support the mapping of Moldex3D injection molding simulation results (fiber orientation tensor $A_{ij}$ and the fiber concentration $c$) onto LS-DYNA mechanical models. The mapped data will be exported as material history state variables in the initial stress keyword cards *INITIAL_STRESS_SHELL or *INITIAL_STRESS_SOLID (see Remark 6). In addition, if warpage data are incorporated in the mapping, a new mesh considering the initial warpage deformation will be created automatically. These mapped data will be used by LS-DYNA when the multiscale material model for fiber-reinforced composites, *MAT_DMN_COMPOSITE_FRC, is used in the finite element analysis. For this discussion, Moldex3D provides the data for the "source" parts which are mapped by LS-PrePost onto the "target" LS-DYNA parts.

LS-PrePost provides an advanced option and a basic option for data mapping. We recommend the advanced option because it is more efficient and offers more functionalities (such as considering molding-induced warpage and residual stress effects) than the basic option. Depending on which LS-PrePost mapping option is chosen, the injection molding simulation result data can be exported in different file formats from Moldex3D:

1. The basic option requires exporting a .k format file for the mesh data (element connectivity and nodal coordinates), a .o2d file for the fiber orientation tensor at each element, and a .fcd file for the fiber concentration at each node from Moldex3D as follows:

   a) Click *Results* in the top toolbar of the Moldex3D software, and select *FEA interface*.

   b) In *FEA Interface Function Option*, choose *LS-Dyna* as the stress solver, and then select *fiber concentration output* and *fiber orientation output*. Next, click

*Export* to create the .k, .o2d, and .fcd files, which are located in the project folder.

Note that the above procedure creates two .k files (LS-DYNA format mesh files) simultaneously, but the one that treats each element as a part should be discarded. The other .k file contains the finite element mesh information and will be used in LS-PrePost.

2. If the LS-PrePost advanced option is chosen, a similar procedure can be followed, but only a single .mdx format file needs to be exported. The .k, .o2d, and .fcd files are no longer required from Moldex3D.

In both cases, to map the data from these Moldex3D files to an LS-DYNA finite element model, click *Application* in the top toolbar of LS-PrePost, select *Plastics Analysis*, and click *Import Data from Moldex3D*. A Moldex3D dialog box will appear, as shown in Figure M303-2.

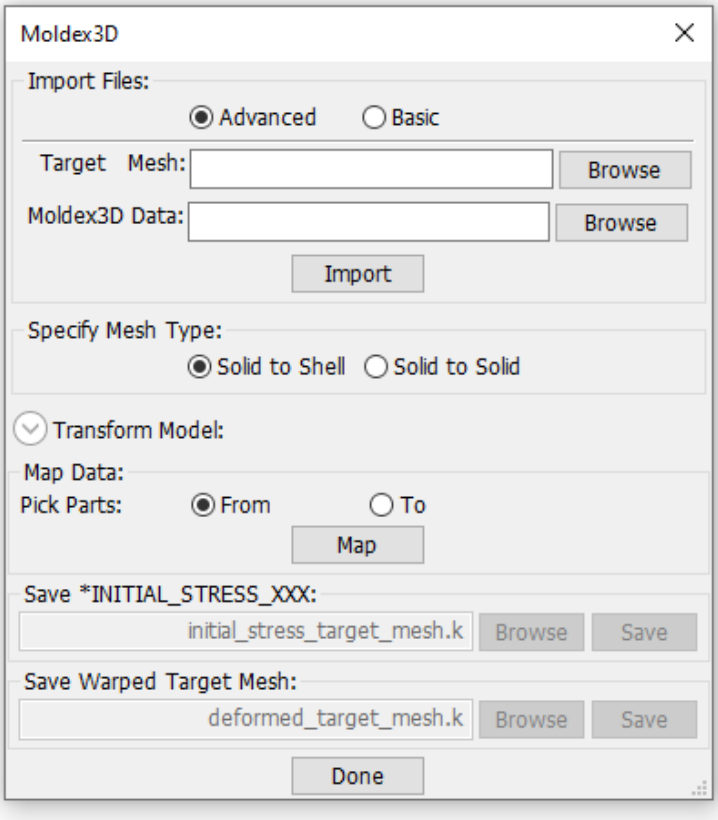



**Figure M303-2.** Moldex3D mapping GUI in LS-PrePost. Steps 1 through 5 below use this GUI.

To map the information from these Moldex3D files (source) to an LS-DYNA finite element model (target), perform the following procedure in LS-PrePost:

1. **Import files.**

   a) Click the *Browse* button under *Target Mesh* and select the .k file for the LS-DYNA mechanical model. The .k file must contain the element connectivity and nodal coordinates for the target parts to be used in the LS-DYNA structural analysis.

   b) If using the advanced option, click the *Browse* button for *Moldex3D Data* and select the .mdx file exported from Moldex3D. The .mdx file contains the mesh, fiber orientation tensor distribution, fiber concentration (i.e., fiber volume fraction × 100), manufacturing process-induced residual stress, and warpage data for the source parts that are used in the Moldex3D injection molding analysis.

      If using the basic option, the following steps import the Moldex3D files:

      i) Click the *Browse* button under *Moldex3D Mesh* and select the .k file exported from Moldex3D. The .k file must contain the element connectivity and nodal coordinates for the source parts that are used in the injection molding analysis in Moldex3D.

      ii) Click the *Browse* button under *Moldex3D O2D* and select the .o2d file exported from Moldex3D. The .o2d file contains the fiber orientation tensor distribution in the source part.

      iii) Click the *Browse* button under *Moldex3D FCD* and select the .fcd file exported from Moldex3D. The .fcd file contains the fiber concentration (i.e., fiber volume fraction × 100) distribution in the source part. If the .fcd file is not available, this step can be skipped, and the fiber volume fraction distribution is considered homogeneous in the target part based on the parameter FVF given in *MAT_DMN_COMPOSITE_FRC.

   c) Click the button *Import* so the source and target models will be imported and visualized in the same global coordinate system.

2. **Specify the mesh type.** Choose *Solid to Shell* if the target part is discretized by shell finite elements or *Solid to Solid* if the target part is discretized by solid finite elements.

   Note that, in the current LS-PrePost mapping function, only solid elements can be used in the source parts from Moldex3D.

   For *Solid to Shell*, LS-PrePost automatically identifies the number of through-thickness integration points defined in the keyword card *SECTION_SHELL for shell finite elements in the target part. If the number of through-thickness

integration points is not defined for the target part, LS-PrePost will use two through-thickness integration points in the solid-to-shell mapping.

3. **Transform the model.** If the source part and the target part are oriented in the same direction in the global coordinate system, this step should be skipped. Otherwise, a rigid body rotation needs to be performed to rotate the source part into the same direction as the target part.

   To perform this transformation, click *Transform Model*, and pick three nodes (S1, S2, S3) in the source model that define a plane (plane A), and then pick three nodes (T1, T2, T3) in the target model that define a plane (plane B). LS-PrePost automatically applies a 3D rigid body rotation that transforms plane A to plane B when it maps the Moldex3D data from the source part to the target part in subsequent steps.

   After the transformation, nodes S1 and T1 will have identical global coordinates. While the coordinate of node S2 can be different from node T2 after the transformation, the line S1-S2 (i.e., the straight-line connecting node S1 and node S2) must be parallel to the straight line T1-T2 after the transformation. Similarly, the transformed coordinates of nodes S3 and T3 can be different, but the line S1-S3 must be parallel to the line T1-T3 after the transformation.

4. **Map the data.** To map the data:

   a) Click *From* in *Pick Parts* and select the source part from the Moldex3D source model.

   b) Click *To* in *Pick Parts* and select the target part from the LS-DYNA model.

   c) Click the *Map* button to map the data from the source part to the target part.

5. **Save the mapped results.**

   a) Under *Save *INITIAL_STRESS_XXX*, click *Browse* to specify the path and filename for a new keyword file, and then click *Save*. These steps create a new keyword file containing the cards *INITIAL_STRESS_SHELL or *INITIAL_STRESS_SOLID, which should be included in the LS-DYNA finite element analysis.

   b) Under *Save Warped Target Mesh*, click *Browse* to specify the path and filename for a new keyword file, and then click *Save*. These steps produce a new keyword file containing the warped target mesh. This file should be included in the LS-DYNA finite element analysis. Note that this step is only necessary when the Moldex3D .mdx file contains warpage data.

   c) Click *Done* to exit the Moldex3D mapping function.

6. **Visualization.** To visualize the data, click *Post*, choose *Fringe Component*, and select either the source or target part to view contour plots of the original or mapped data, respectively. To view residual stress and shell thickness data, use *Dynain* in the *Fringe Component* GUI. Fiber orientation and volume fraction data can be selected in *History* in the *Fringe Componen*t GUI. To visualize warpage displacement data, use *User* in the *Fringe Component* GUI.

**References:**

[1] Liu, Z., C.T. Wu, and M. Koishi, "A deep material network for multiscale topology learning and accelerated nonlinear modeling of heterogeneous materials," *Computer Methods in Applied Mechanics and Engineering*, Vol. 345, pp. 1138-1168, (2019).

[2] Liu, Z. and C.T. Wu, "Exploring the 3D architectures of deep material network in data-driven multiscale mechanics," *Journal of the Mechanics and Physics of Solids*, Vol. 127, pp. 20-46, (2019).

[3] Liu, Z., C.T. Wu, and M. Koishi, "Transfer learning of deep material network for seamless structure-property predictions," *Computer Mechanics*, Vol. 64, pp. 451-465, (2019).

[4] Liu, Z., H. Wei, T. Huang, and C.T. Wu, "Intelligent multiscale simulation based on process-guided composite database," (2020). https://arxiv.org/abs/2003.09491

[5] Wei, H., C.T. Wu, D. Lyu, W. Hu, F.H. Rouet, K. Zhang, P. Ho, H. Oura, M. Nishi, T. Naito, and L. Shen, "Multiscale simulation of short-fiber-reinforced composites from computational homogenization to mechanistic machine learning in LS-DYNA," 13th European LS-DYNA Conference, Ulm, Germany (2021). Vol. 64, pp. 451-465, (2019). https://www.dynalook.com/conferences/13th-european-ls-dyna-conference-2021

[6] Wei, H., Wu, C. T., Hu, W., Su, T. H., Oura, H., Nishi, M., Naito, T., Chung, S., Shen, L., "LS-DYNA Machine Learning–Based Multiscale Method for Nonlinear Modeling of Short Fiber–Reinforced Composites." Journal of Engineering Mechanics, 149(3), 04023003, (2023).

[7] Wei, H., Hu, W., Wu, C. T., Pavia, F., "AI-empowered LS-DYNA ICME simulation technique for multiscale predictive modeling of composites." American Society for Composites (ASC_ 39th Annual Technical Conference & US-Japan Joint Symposium, October 21-23, 2024, San Diego, California, USA.

## *MAT_HOT_PLATE_ROLLING

This is Material Type 305. This model is for hot rolling of steel. It can only be used with solid elements for explicit simulation. The model contains the following features: work hardening, dynamic softening, static recovery, and static recrystallization. Input parameters are calibrated from Gleeble tests at various deformation rates and temperatures; see Schill et. al. [2021] and references therein.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | E | PR | ALPHAT | BETA | VP | TOL |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| YB | QDEF | R | A | B | MINRT | POST | ODESOL |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| ASIG0 | BSIG0 | ASIGS | BSIGS | ASIGSS | BSIGSS | AEPS | BEPS |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| THRES | M | ALPHA | NUD | U0 | K | NU | BNU |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| T50 | N | A50 | D | GSF | P | Q | QREX |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | E | PR | ALPHAT | BETA | VP | TOL |
| Type | A | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 1.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see*PART). |
| RO | Mass density |
| E | Young's modulus |
| PR | Poisson's ratio |
| ALPHAT | Thermal expansion coefficient, $\alpha_T$. |
| BETA | Mixed hardening parameter, $0 \leq \beta \leq 1$. $\beta = 0$ for isotropic and $\beta = 1$ for kinematic hardening. |
| VP | Formulation for rate effects in plasticity update:<br>EQ.0.0: No plastic strain rate dependence in yield stress (default)<br>EQ.1.0: Plastic strain rate dependence in yield stress. Slower but more stable (recommended). |
| TOL | Multiplication factor (must be > 0.0) on tolerance criteria for plasticity and annealing iterations.<br>LT.1.0: Increases accuracy at greater computational cost<br>EQ.1.0: Default value<br>GT.1.0: Decreases accuracy at less computational cost |

**Work Hardening and Dynamic Softening Parameters Card.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | YB | QDEF | R | A | B | MINRT | POST | ODESOL |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| BY | Work hardening parameter, $B_y$. See Work Hardening and Dynamic Softening. |

| VARIABLE | DESCRIPTION |
|---|---|
| QDEF | Work hardening activation energy, $Q_{def}$. See Work Hardening and Dynamic Softening. |
| R | Work hardening gas constant, $R$. See Work Hardening and Dynamic Softening. |
| A | Dynamic softening parameter, $a$. See Work Hardening and Dynamic Softening. |
| B | Dynamic softening parameter, $b$. See Work Hardening and Dynamic Softening. |
| MINRT | Work hardening minimum (plastic) strain rate, $\dot{\varepsilon}_{min}$, in Zener-Hollomon parameter. See Work Hardening and Dynamic Softening. |
| POST | Save additional history variables for post-processing with POST = 1 |
| ODESOL | Solver for static recovery stress:<br>EQ.0.0: Trapezoidal rule (default)<br>EQ.1.0: Heun's method: Faster but less stable. |

**Second Work Hardening and Dynamic Softening Parameters Card.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ASIG0 | BSIG0 | ASIGS | BSIGS | ASIGSS | BSIGSS | AEPS | BEPS |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| ASIG0, ASIGS | Parameters $a_i$, $i = 0, s, ss$, to calculate $\sigma_0$, $\sigma_s$, and $\sigma_{ss}$, respectively, from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening. |
| BSIG0, BSIGS | Parameters $b_i$, $i = 0, s, ss$, to calculate $\sigma_0$, $\sigma_s$, and $\sigma_{ss}$, respectively, from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening. |

| VARIABLE | DESCRIPTION |
|---|---|
| AEPS | Parameter $a_{\varepsilon_s}$ used to calculate the saturation strain, $\varepsilon_s$, for dynamic relaxation from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening. |
| BEPS | Parameter $a_{\varepsilon_s}$ used to calculate the saturation strain, $\varepsilon_s$, for dynamic relaxation from the Zener-Hollomon parameter. See Work Hardening and Dynamic Softening. |

**Static Recovery Parameters Card.** See Static Recovery and Static Recrystallization.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | THRES | M | ALPHA | NUD | U0 | K | NU | BNU |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| THRES | Static recovery strain rate threshold. THRES > 0 turns off dynamic softening, meaning sets A = 0. |
| M | Taylor factor, $M$, for static recovery stress |
| ALPHA | $\alpha$ parameter for static recovery stress |
| NUD | Debye frequency, $\nu_D$, for static recovery stress |
| U0 | Activation energy, $U_0$, for static recovery stress |
| K | Boltzmann constant, $k$ |
| NU | Interaction volume, $\nu$, for static recovery stress |
| BNU | Burger's vector, $b_\nu$, for static recovery stress |

**Static Recrystallization Parameters Card.** See Static Recovery and Static Recrystallization.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T50 | N | A50 | D | GSF | P | Q | QREX |
| Type | F | F | F | F | F | F | F | F |
| Default | none | none | none | none | none | none | none | none |

| VARIABLE | DESCRIPTION |
|---|---|
| T50 | Time required to reach 50% static recrystallization |
| N | Static recrystallization exponent |
| A50 | Scale parameter for strain dependent recrystallization time:<br>EQ.0.0: T50 parameter is used for recrystallization time.<br>GT.0.0: T50 parameter is ignored. Recrystallization time is calculated with A50, D, GSF, P, Q, and QREX.<br>LT.0.0: T50 parameter is used. Recrystallization factor and combined recovery stress calculated using A50, D, GSF, P, Q, QREX are additional history variables if POST = 1. |
| D | Length parameter for strain dependent recrystallization time |
| GSF | Exponent for strain dependent recrystallization time |
| P | Exponent for strain dependent recrystallization time |
| Q | Exponent for strain dependent recrystallization time |
| QREX | Activation energy for strain dependent recrystallization time |

**Material Model:**

This material model uses a hypo-elastoplastic formulation

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}\dot{\boldsymbol{\varepsilon}}_e = \mathbf{C}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_T - \dot{\boldsymbol{\varepsilon}}_p\right) \ ,$$

with thermal strain rate

$$\dot{\boldsymbol{\varepsilon}}_T = \alpha_T \dot{T}\mathbf{I} \ ,$$

and plastic strain rate

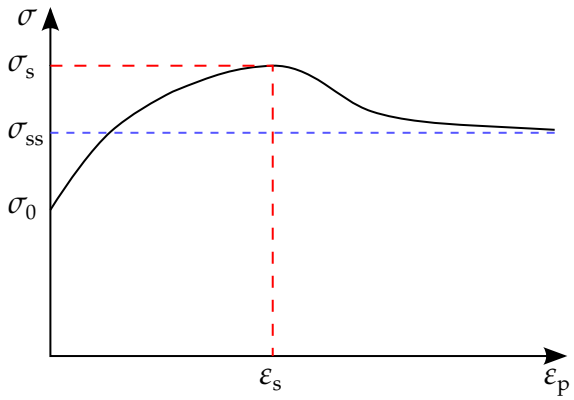



**Figure M305-1.** Stress-strain with work hardening and dynamic softening

$$\dot{\boldsymbol{\varepsilon}}_p = \dot{\varepsilon}_p \frac{3}{2} \frac{\mathbf{s} - \boldsymbol{\alpha}}{\sigma_{\text{VM}}} \ ,$$

where $\mathbf{s}$ is the deviatoric stress, $\boldsymbol{\alpha}$ is the back stress, $\varepsilon_p$ is the effective plastic strain, and

$$\sigma_{\text{VM}} = \sqrt{\frac{3}{2}(\mathbf{s} - \boldsymbol{\alpha}) : (\mathbf{s} - \boldsymbol{\alpha})} \ ,$$

is the Von Mises stress.

Using a mixed kinematic isotropic-kinematic hardening with mixing factor $\beta \in [0,1]$ and a nonlinear hardening function $h(T, \varepsilon_p, \dot{\varepsilon}_p)$ , the back stress evolves according to

$$\dot{\boldsymbol{\alpha}} = \beta H \dot{\varepsilon}_p \frac{\mathbf{s} - \boldsymbol{\alpha}}{\sigma_{\text{VM}}} \ ,$$

where $H = \partial h / \partial \varepsilon_p$ is the material hardening. The yield stress becomes

$$\sigma_y = h + \beta(\sigma_0 - h) \ ,$$

where $\sigma_0$ is the initial yield stress. We will discuss the nonlinear hardening function in Work Hardening and Dynamic Softening below.

We will next discuss the models for work hardening, dynamic recrystallization, static recovery, and static recrystallization.

**Work Hardening and Dynamic Softening**

To begin this discussion, we need to introduce the Zener-Hollomon parameter. The initial yield stress ($\sigma_0$), the saturation or peak stress ($\sigma_s$), the steady state stress ($\sigma_{ss}$), and the saturation strain ($\varepsilon_s$) which describe the stress-strain curve in the work hardening and dynamic softening regime (see Figure M305-1) can depend on the deformation temperature and strain rate. The Zener-Hollomon provides this dependence. The parameter is given by

$$Z = \max(\dot{\varepsilon}, \dot{\varepsilon}_{\text{min}}) \exp\left(\frac{Q_{\text{def}}}{RT}\right) \ .$$

The stresses then have the general form of:

$$\sigma_i = a_i \ln Z + b_i, \qquad i = 0, s, ss \ ,$$

and the saturation strain, similarly, is given by

$$\varepsilon_s = a_{\varepsilon_s} \ln Z + b_{\varepsilon_s} \ .$$

In the above, $\dot{\varepsilon}$ is the effective strain rate, and $\dot{\varepsilon}_{\min}$ is the minimum strain rate for which the parameter fit is done to prevent unphysical values of $\sigma_i$ and $\varepsilon_s$. If VP = 1 the effective *plastic* strain rates are used here. $Q_{\text{def}}$ is the activation energy, $R$ is the gas constant, and $T$ is the temperature.

The work hardening model is based on the interplay between storage and annihilation of dislocations described by the Estrin and Mecking model

$$\sigma_{\text{EM}} = \sqrt{\sigma_s^2 - (\sigma_s^2 - \sigma_0^2)\exp(-2B_y\varepsilon_p)} \ ,$$

where $B_y$ is a material parameter and $\varepsilon_p$ is the effective plastic strain.

We include the effect of softening due to dynamic recrystallization in the prediction of the flow stress beyond a saturation strain, $\varepsilon_s$. To model this effect, we introduce the dynamic recrystallization fraction

$$X_{drx} = \begin{cases} 0, & \varepsilon_p < \varepsilon_s \\ 1 - \exp\left(-a\left(\varepsilon_p - \varepsilon_s\right)^b\right), & \varepsilon_p \geq \varepsilon_s \end{cases}$$

The transient flow stress due to work hardening and dynamic softening is, then, predicted using a mixture law between the steady state stress, $\sigma_{ss}$, and the Estrin Mecking stress, $\sigma_{\text{EM}}$:

$$\sigma = \sigma_{\text{EM}} - X_{\text{drx}}(\sigma_s - \sigma_{ss}) \ .$$

The resulting hardening function then becomes

$$h = (1 - X_{\text{drx}})\sigma_{\text{EM}} + X_{\text{drx}}\sigma_{ss}.$$

**Static Recovery and Static Recrystallization**

After deformation, the material softens due to static recovery and static recrystallization. The static recovery stress, $\sigma_{\text{srx}}$, is modeled by

$$\sigma_{\text{srx}} = \sigma_0 + \Delta\sigma$$

where $\sigma_0$ is the initial yield stress and $\Delta\sigma$ is the change in stress due to dislocation climb. $\Delta\sigma$ changes with time by

$$\frac{d\Delta\sigma}{dt} = -\frac{64\Delta\sigma^2}{9M^3\alpha^2E(T)}\nu_D \exp\left(-\frac{U_0}{RT}\right)\sinh\left(\frac{\Delta\sigma\nu b_\nu^3}{kT}\right) \ .$$

At the start of recovery $\Delta\sigma = \sigma_{\text{VM}} - \sigma_0$. $M$, $\alpha$, $\nu_D$, and $b_v$ are physical constants related to the properties of the FCC iron lattice, $U_0$ is the activation energy for climb, $\nu$ is the

interaction volume, and $R$ and $k$ are the universal gas constant and Boltzmann constant, respectively.

Static recovery starts when the material is under plastic load and the plastic strain rate is lower than THRES. It stops when the plastic strain rate is higher than THRES.

Softening is assumed to be caused by recrystallized grain growth. Deformed structure with high dislocation density is replaced with new grains with a low dislocation density and constant stress $\sigma_0$. The recrystallized fraction is described by the static recrystallization fraction $X_{\text{srx}}$ which is described via a standard JMAK expression

$$X_{\text{srx}} = 1 - \exp\left(-0.693\left(\frac{t - t_{\text{start}}}{t_{50}}\right)^n\right) ,$$

where $t$ is the total time, $t_{\text{start}}$ is the start time of the recovery and $t_{50}$ is the time required to reach 50% recrystallization.

Finally, the combined recovery stress state is expressed by a law of mixtures

$$\sigma_r = X_{\text{srx}}\sigma_0 + (1 - X_{\text{srx}})\sigma_{\text{srx}}.$$

If A50 $\neq$ 0, the recrystallization time $t_{50}$ may be calculated from

$$t_{50} = |A_{50}|(\varepsilon_p^{\text{tstart}})^p \left(\dot{\varepsilon}_p^{\text{tstart}} \exp\left(\frac{Q_{\text{def}}}{RT}\right)\right)^q d^{G_{\text{sf}}} \exp\left(\frac{Q_{\text{rex}}}{RT}\right) ,$$

where $\varepsilon_p^{\text{tstart}}$ is the plastic strain at start of recovery. For A50 > 0, the parameter T50 is ignored, and the material routine calculates $t_{50}$ with the above expression. For A50 < 0, the material routine uses the parameter T50, but, if POST = 1, LS-DYNA additionally calculates history variables $X_{\text{srx}}^{\text{post}}$ and $\sigma_r^{\text{post}}$ with $t_{50}$ from the expression above.

During static recovery, stress and effective plastic strain is annealed, meaning stress and back stress is scaled with

$$\gamma = \frac{\sigma_r - \sigma_0}{\sigma_{\text{VM}} - \sigma_0}, \qquad 0 \leq \gamma \leq 1,$$

and the annealed plastic strain solves

$$\sigma_y(\varepsilon_p^{\text{annealed}}) = \gamma\sigma_y(\varepsilon_p).$$

**History variables:**

The following history variables are available by default:

| History Variable # | Description |
|---|---|
| 1-6 | Back stress |

| History Variable # | Description |
|---|---|
| 7 | Temperature |
| 8 | Plastic strain rate |
| 9 | Plastic strain at start of recovery |
| 10 | Time since start of recovery ($t - t_{\text{start}}$). It is set to -1 when inactive. |
| 11 | Static recovery stress, $\sigma_{\text{srx}}$ |
| 12 | Combined recovery stress, $\sigma_r$ |

The following additional variables are available if POST = 1:

| History Variable # | Description |
|---|---|
| 13 | Initial yield stress, $\sigma_0$ |
| 14 | Saturation stress, $\sigma_s$ |
| 15 | Steady state stress, $\sigma_{ss}$ |
| 16 | Saturation strain, $\varepsilon_s$ |
| 17 | Dynamic recrystallization fraction, $X_{\text{drx}}$ |
| 18 | Yield stress, $\sigma_y$ |
| 19 | Static recrystallization fraction, $X_{\text{srx}}$ |

The following additional variables are available if POST = 1 and A50 < 0:

| History Variable # | Description |
|---|---|
| 20 | Derived static recrystallization fraction, $X_{\text{srx}}^{\text{post}}$ |
| 21 | Derived combined recovery stress, $\sigma_r^{\text{post}}$ |

**References:**

Schill, M., Karlsson, J., Magnusson, H., Huyan, F., Nosar, N.S., Lagergren, J., Narström, T., and Johansson, F. "Simulation of Hot Plate Rolling using LS-DYNA," *13th European LS-DYNA Conference* (2021).

## *MAT_GENERALIZED_ADHESIVE_CURING

This is Material Type 307. It incorporates a modular approach for modeling adhesive materials during chemical curing. This material model provides a general viscoelastic Maxwell model defined by its Prony series expansion of up to 18 terms that considers the effects of temperature and degree of cure. It is supported for solid and cohesive solid elements.

### Card Summary:

**Card 1.** This card is required.

| MID | RO | GASC | IDOC | INCR | QCURE | TZERO | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| CKOPT | CK1 | CK2 | CK3 | CK4 | CK5 | CK6 | CK7 |
|---|---|---|---|---|---|---|---|

**Card 2.1.** Include this card if |CKOPT| > 3 and |CKOPT| < 11.

| CK8 | CK9 | CK10 | CK11 | CK12 | CK13 | CK14 | CK15 |
|---|---|---|---|---|---|---|---|

**Card 2.2.** Include this card if CKOPT = 5, 6, 9, or 10.

| CK16 | CK17 | CK18 | CK19 | CK20 | CK21 | CK22 | CK23 |
|---|---|---|---|---|---|---|---|

**Card 3.** Include this card if CKOPT < 0.

| CDOPT | CD1 | CD2 | CD3 | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include this card if CKOPT < 0.

| CTGOPT | CTG1 | CTG2 | CTG3 | | | | |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| CEOPT | CE1 | CE2 | CE3 | CE4 | | | |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| TEOPT | TE1 | TE2 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| THOPT | TH1 | TH2 | TH3 | TH4 | TH5 | TH6 | TH7 |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| TVOPT | TV1 | TV2 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| PHOPT | PH1 | PH2 | PH3 | PH4 | PH5 | PH6 | |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is required.

| PVOPT | PV1 | PV2 | PV3 | PV4 | | | |
|---|---|---|---|---|---|---|---|

**Card 11.** This card is required.

| PL1OPT | PL11 | PL12 | PL13 | PL14 | PL15 | | |
|---|---|---|---|---|---|---|---|

**Card 12.** This card is required.

| PL2OPT | PL21 | PL22 | PL23 | PL24 | PL25 | PL26 | |
|---|---|---|---|---|---|---|---|

**Card 13.** This card is required.

| DAOPT | DAEVO | DATRIA | DA1 | DA2 | DA3 | DA4 | DA5 |
|---|---|---|---|---|---|---|---|

**Card 14.** This card is required.

| DA6 | DA7 | DA8 | DA9 | DA10 | DA11 | PDA1 | PGEL |
|---|---|---|---|---|---|---|---|

**Card 15a.** The keyword reader assumes the input deck includes this version of Card 15 if, in the first instantiation of this card, the value in the first entry is ≥ 0.0. Input up to 18 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 18 cards. The number of terms for the shear behavior may differ from that for the bulk behavior: insert zero to exclude a term.

| G*i* | BETAG*i* | K*i* | BETAK*i* | | | | |
|---|---|---|---|---|---|---|---|

**Card 15b.** The keyword reader assumes the input deck includes this version of Card 15 if the value in the first entry is < 0.0

| VISOPT | NUE | | | | | | |
|---|---|---|---|---|---|---|---|

**Card 16a.** Include this card if VISOPT = -1 on Card 15b. Input up to 13 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 13 cards.

| E*i* | BETA*i* | E*j* | BETA*j* | | | | |
|---|---|---|---|---|---|---|---|

**Card 16b.** Include this card if VISOPT = -2. Include up to 13 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 13 cards.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| G*i* | BETA*i* | G*j* | BETA*j* | | | | |

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | GASC | IDOC | INCR | QCURE | TZERO | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| GASC | Gas constant, $R$ |
| IDOC | Initial degree of cure, $p_I$:<br>EQ.0.0: Uncured initial state, $p_I = 0$<br>GT.0.0: Uniformly distributed initial state of cure with $p_I$ = IDOC<br>LT.0.0: Use *INITIAL_STRESS_OPTION to define a potentially nonuniform distribution for the initial state of cure (history variable #2). |
| INCR | Switch between incremental and total stress formulation:<br>EQ.1: Incremental form (default, recommended)<br>EQ.2: Total form |
| QCURE | Heat generation factor, relating the heat generated in one time step with the increment of the degree of cure in that step |
| TZERO | Temperature value with respect to the temperature scale used in the input deck for a temperature of 0 K. See Remark 1. |

**Curing Kinetics Card 1**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CKOPT | CK1 | CK2 | CK3 | CK4 | CK5 | CK6 | CK7 |
| Type | I | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CKOPT | Curing kinetics option (see Remark 2 for details):<br>EQ.0: No curing kinetics<br>EQ.1: Generalized model, with load curves for pre-exponential factors<br>EQ.2: Extended Kamal model<br>EQ.-2: Extended Kamal model with diffusion control<br>EQ.3: Kamal model<br>EQ.-3: Kamal model with diffusion control<br>EQ.4: Three-species reaction kinetics model<br>EQ.-4: Three-species reaction kinetics model with diffusion control<br>EQ.5: Five-species reaction kinetics model<br>EQ.6: Five-species reaction kinetics model<br>EQ.7: Three-species reaction kinetics model<br>EQ.8: Three-species reaction kinetics model<br>EQ.9: Four-species reaction kinetics model<br>EQ.10: Five-species reaction kinetics model<br>EQ.11: Model-free kinetics |
| CK*i* | $i^{th}$ curing kinetics model parameter. The meaning of the parameter depends on the choice of CKOPT. For details, see Remark 2. |

**Curing Kinetics Card 2.** Additional card for |CKOPT| > 3 and |CKOPT| < 11.

| Card 2.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CK8 | CK9 | CK10 | CK11 | CK12 | CK13 | CK14 | CK15 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CK*i* | $i^{th}$ curing kinetics model parameter. The meaning of the parameter depends on the choice of CKOPT. For details, see Remark 2. |

**Curing Kinetics Card 3.** Additional card for CKOPT = 5, 6, 9, or 10.

| Card 2.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CK16 | CK17 | CK18 | CK19 | CK20 | CK21 | CK22 | CK23 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| CK*i* | $i^{th}$ curing kinetics model parameter. The meaning of the parameter depends on the choice of CKOPT. For details, see Remark 2. |

**Curing Kinetics Diffusion Control Card 1.** Additional card for CKOPT < 0.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CDOPT | CD1 | CD2 | CD3 | | | | |
| Type | I | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CDOPT | Diffusion control mechanism option (see Remark 16):<br>EQ.1: Williams-Landel-Ferry (WLF) type<br>EQ.2: Arrhenius shift-like mechanism<br>EQ.3: Combined Williams-Landel-Ferry (WLF) and Arrhenius |

| VARIABLE | DESCRIPTION |
|---|---|
| | shift-like mechanism |
| CD*i* | $i^{th}$ diffusion control model parameter. The meaning of the parameter depends on the choice of CDOPT. For details, see Remark 16. |

**Curing Kinetics Diffusion Control Card 2.** Additional card for CKOPT < 0.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CTGOPT | CTG1 | CTG2 | CTG3 | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CTGOPT | Glass transition temperature for diffusion control (see Remark 16):<br>EQ.1: DiBenedetto equation<br>EQ.2: Hesekamp equation |
| CTGD*i* | $i^{th}$ parameter for calculating glass transition temperature. The meaning of the parameter depends on the choice of CTOPT. For details, see Remark 16. |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CEOPT | CE1 | CE2 | CE3 | CE4 | | | |
| Type | I | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| CEOPT | Chemical expansion option (see Remark 3 for details):<br>EQ.0: No chemical expansion<br>EQ.1: Differential form with load curve input<br>EQ.2: Secant form with load curve input<br>EQ.3: Secant form with a polynomial expression |
| CE*i* | $i^{th}$ chemical expansion model parameter. The meaning of the |

| VARIABLE | DESCRIPTION |
|---|---|
| | parameter depends on the choice of CEOPT. For details, see Remark 3. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TEOPT | TE1 | TE2 | | | | | |
| Type | I | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TEOPT | Thermal expansion option (see Remark 4):<br>EQ.0: No thermal expansion<br>EQ.1: Differential form with load curve input<br>EQ.2: Secant form with load curve input |
| TE*i* | $i^{th}$ thermal expansion parameter. The meaning of the parameter depends on the choice of TEOPT. For details, see Remark 4. |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | THOPT | TH1 | TH2 | TH3 | TH4 | TH5 | TH6 | TH7 |
| Type | I | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| THOPT | Option for the horizontal temperature shift of the viscoelastic master curve as given by the Prony series expansion (see Remarks 5 and 6):<br>EQ.0: No temperature shift<br>EQ.1: Williams-Landel-Ferry (WLF) shift function<br>EQ.2: Arrhenius shift function<br>EQ.3: Combined WLF (above glass transition temperature $T_g$) and Arrhenius (below $T_g$) shift function<br>EQ.4: As 3, but with a glass transition temperature $T_g(p)$ as a |

| VARIABLE | DESCRIPTION |
|---|---|
| | function of the degree of cure $p$<br>EQ.5: Direct input of shift factors as a function of temperature<br>EQ.6: Combined extended WLF (above $T_g(p)$) and exponential shift function (below $T_g(p)$) |
| TH$i$ | $i^{th}$ shifting parameter. The meaning of the parameter depends on the choice of THOPT. For details, see Remark 6. |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TVOPT | TV1 | TV2 | | | | | |
| Type | I | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TVOPT | Option for the vertical temperature shift of the master viscoelastic curve as given by the Prony series expansion. See Remarks 5 and 7 for details.<br>EQ.0: No temperature shift<br>EQ.1: Shifting of the complete master curves $G(t)$<br>EQ.2: Shifting of all terms $G_i$ and $K_i$, but not $G_\infty$ and $K_\infty$ |
| TV$i$ | The meaning of the shifting parameters depends on the choice of TVOPT. For details, see Remark 7. |

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PHOPT | PH1 | PH2 | PH3 | PH4 | PH5 | PH6 | |
| Type | I | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| PHOPT | Option for the horizontal shift due to curing effects of the master viscoelastic curve as given by the Prony series expansion (see Remarks 5 and 8): |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0: No shift<br>EQ.1: Eom model<br>EQ.2: Direct input of shift factors as function of the degree of cure |
| PH*i* | $i^{th}$ shifting parameter. The meaning of the parameter depends on the choice of PHOPT. For details, see Remark 8. |

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PV0PT | PV1 | PV2 | PV3 | PV4 | | | |
| Type | I | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PVOPT | Option for the vertical shift of the master viscoelastic curve due to curing effects as given by the Prony series expansion (see Remarks 5 and 9 for details):<br>EQ.0: No shift<br>EQ.1: Input of instantaneous moduli $G_0$ and $K_0$ as a function of degree of cure $p$. Assumption of constant ratios $G_i(p)/G_0(p)$ and $K_i(p)/K_0(p)$.<br>EQ.2: Input of long-term moduli $G_\infty(p)$ and $K_\infty(p)$ as functions of degree of cure $p$ and of scaling functions for other moduli $G_i$ and $K_i$. |
| PV*i* | $i^{th}$ shifting parameter. The meaning of the parameter depends on the choice of PVOPT. For details, see Remark 9. |

| Card 11 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PL10PT | PL11 | PL12 | PL13 | PL14 | PL15 | | |
| Type | I | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| PL1OPT | Option for yield function description (see Remarks 10 and 11):<br>EQ.0: No plasticity<br>EQ.1: Version of Toughened Adhesive Polymer model (TAPO) with cap in tension and Drucker & Prager in compression with distortional hardening under plastic flow<br>EQ.2: Version of Toughened Adhesive Polymer model (TAPO) with cap in tension and von Mises in compression<br>EQ.3: Version of Toughened Adhesive Polymer model (TAPO) with cap in tension and Drucker & Prager in compression with distortional hardening under temperature change. |
| PL1*i* | $i^{th}$ yield surface parameter. The meaning of the parameter depends on the choice of PL1OPT. For details, see Remark 11. |

| Card 12 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | PL20PT | PL21 | PL22 | PL23 | PL24 | PL25 | PL26 | PL27 |
| Type | I | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| PL2OPT | Option for yield stress description (see Remarks 10 and 12 for details):<br>EQ.0: No plasticity<br>EQ.1: Tabular input for yield stress as a function of curing, temperature, and plastic strains.<br>EQ.2: Tabular input for initial yield stress as a function of curing and temperature and hardening as a function of curing, temperature, and plastic strains.<br>EQ.3: Load curve inputs for effects of curing on initial yield stress and on hardening. Load curve input for temperature dependence of initial yield stress. Tabular input for hardening as a function of temperature and strain<br>EQ.4: Load curve inputs for effects of curing and temperature on the parameters for the yield stress definitions in the Toughened Adhesive Polymer model (TAPO) |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.5: Yield stress definitions in the Toughened Adhesive Polymer model (TAPO). No influence of temperature or curing. |
| PL2*i* | $i^{th}$ yield stress parameter. The meaning of the parameter depends on the choice of PL2OPT. For details, see Remark 12. |

| Card 13 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DAOPT | DAEVO | DATRIA | DA1 | DA2 | DA3 | DA4 | DA5 |
| Type | I | I | I | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| DAOPT | Material damaging option (damage parameter $D_1$), defines the strain thresholds $\gamma_c$ and $\gamma_f$ for damage initiation and rupture (see Remark 13):<br>EQ.0: No material damage<br>EQ.1: Version of Toughened Adhesive Polymer model (TAPO): Strain threshold exponential function of triaxiality. Load curve inputs for temperature and cure dependence.<br>EQ.2: Version of Toughened Adhesive Polymer model (TAPO). Same as 1, but without temperature and cure dependence.<br>EQ.3: Version of Toughened Adhesive Polymer model (TAPO). Same as 1, but with additional strain rate dependency. |
| DAEVO | Effective strain measure used for material damage evolution (see Remark 13):<br>EQ.0: Arc length of damage plastic strain<br>EQ.1: Arc length of plastic strain<br>EQ.2: Arc length of viscoelastic-plastic strain rate |
| DATRIAX | Triaxiality flag for calculation of strain thresholds $\gamma_c$ and $\gamma_f$ for damage initiation and rupture of material damage option (see Remark 13):<br>EQ.0: Use triaxiality factor only in tension |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: Use triaxiality factor in tension and compression |
| DA*i* | $i^{th}$ material damage parameter. The meaning of the parameter depends on the choice of DAOPT for the evolution of damage parameter $D_1$. For details, see Remark 13. |

| Card 14 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | DA6 | DA7 | DA8 | DA9 | DA10 | DA11 | PDA1 | PDA2 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| DA*i* | $i^{th}$ material damage parameter. The meaning of the parameter depends on the choice of DAOPT for the evolution of damage parameter $D_1$. For details, see Remark 13. |
| PDA1 | Parameter for the (pre-) damage formulation due to for example viscous fingering. It defines the damage parameter $D_2$ as function of the thickness strain $\varepsilon_{33}$ and the degree of cure $p$. For details, see Remark 14.<br>EQ.0: No damage<br>GT.0: Use exponential approach<br>LT.0: Load curve input with ID \|PDA1\| input for $D_2(\varepsilon_{33})$ |
| PGEL | Gelation point $p_{gel}$ as needed to switch between evolution of damage parameters $D_1$ and $D_2$. For details, see Remark 13 and Remark 14. |

**Viscoelastic Constant Card.** The keyword reader assumes the input deck includes this version of Card 15 if, in the first instantiation of this card, the value in the first entry is ≥ 0.0. Input up to 18 instantiations of this card. The next keyword ("*") card terminates this input if using fewer than 18 cards. The number of terms for the shear behavior may differ from that for the bulk behavior: insert zero to exclude a term.

| Card 15a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETAG*i* | K*i* | BETAK*i* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Shear relaxation modulus for the $i^{th}$ term |
| BETAG*i* | Shear decay constant for the $i^{th}$ term |
| K*i* | Bulk relaxation modulus for the $i^{th}$ term |
| BETAK*i* | Bulk decay constant for the $i^{th}$ term |

**Viscoelastic Option Card.** The keyword reader assumes the input deck includes this version of Card 15 if the value in the first entry is < 0.0.

| Card 15b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | VISOPT | PR | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| VISOPT | Viscous option determining the input of the Prony series:<br>EQ.-1: Prony series input for Young's modulus $E(t)$. Prony series for shear modulus $G(t)$ and bulk modulus $K(t)$ are derived from it, assuming a constant Poisson's ratio.<br>EQ.-2: Prony series input for shear modulus $G(t)$. The Prony series for the bulk modulus $K(t)$ is derived from it, assuming a constant Poisson's ratio. |
| PR | Constant Poisson's ratio $\nu$ |

**Viscoelastic Constant Cards for VISOPT = -1.** Include up to 13 instantiations of this card if VISOPT = -1. See Remark 5.

| Card 16a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E*i* | BETA*i* | E*j* | BETA*j* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| E*i* | Relaxation modulus for the $i^{th}$ term |
| BETA*i* | Decay constant for the $i^{th}$ term |
| E*j* | Relaxation modulus for the $j^{th}$ term |
| BETA*j* | Decay constant for the $j^{th}$ term |

**Viscoelastic Constant Cards for VISOPT = -2.** Include up to 13 instantiations of this card if VISOPT = -2. See Remark 5.

| Card 16b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | G*i* | BETA*i* | G*j* | BETA*j* | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| G*i* | Shear relaxation modulus for the $i^{th}$ term |
| BETA*i* | Decay constant for the $i^{th}$ term |
| G*j* | Shear relaxation modulus for the $j^{th}$ term |
| BETA*j* | Decay constant for the $j^{th}$ term |

**Remarks:**

1. **Temperature scale.** This material formulation requires providing the material data with respect to a consistent temperature unit. For all the curing kinetics models described in Remark 2, except CKOPT = 1, it is necessary to define the

temperature $T$ in Kelvin. Consequently, if considering curing, all temperature-dependent input should be given for temperature data in Kelvin.

TZERO enables including this material model in a simulation set up in the Celsius temperature scale. It defines the temperature value $T_{0K}$ in the user system for 0 K. Thus, to run a simulation with the Celsius scale, set $T_{0K}$ to approximately -273. For a model using the Kelvin scale, $T_{0K}$ is 0.

For all temperature-dependent values in this material, LS-DYNA uses the modified temperature

$$T = T_{\text{user}} - T_{0K} \ ,$$

where $T_{\text{user}}$ refers to the temperature value in the simulation.

2. **Curing kinetics.** This material formulation includes an internal variable $p$ to represent the degree of cure for the adhesive. In all cases, it is the result of a set of chemical reactions. The number of species in the reaction, the number of reaction steps, and the reaction kinetics applied depend on the choice of the curing kinetics option CKOPT.

All options CKOPT except 1 use the Arrhenius formula:

$$K_i^{\text{chem}}(T) = k_i \exp\left(-\frac{Q_i}{RT}\right) \ .$$

In the above, $R$ is the universal gas constant. GASC in Card 1 sets $R$.

For the positive values of CKOPT, $K_i^{\text{chem}}(T)$ is used as an effective parameter in the given kinetics models below, that is, $K_i(T) = K_i^{\text{chem}}(T)$. The negative options CKOPT = -2,-3,-4 feature a diffusion control mechanism, which introduces an additional term $K^{\text{diff}}(T)$ to calculate the values $K_i(T)$ as

$$K_i(T) = \left(\frac{1}{K_i^{\text{chem}}(T)} + \frac{1}{K^{\text{diff}}(T)}\right)^{-1}$$

The definition of the diffusion control effect $K_i^{\text{diff}}$ depends on the input of Cards 3 and 4 and is discussed in Remark 16.

A table at the end of this remark gives the input structure for the parameters used by the different CKOPT options.

a) *Two-species reaction kinetics model (CKOPT = 1, 2,-2, 3 and -3)*

We can directly give the evolution equation in terms of the degree of cure $p$ by identifying it with the product $c_2$ of a chemical reaction with two chemical species. In the most general form, it reads

$$\dot{p} = K_1(T)(1-p)^{n_1} + K_2(T)p^{m_2}(1-p)^{n_2} \ .$$

The functions $K_1(T)$ and $K_2(T)$ are the load curves for CKOPT = 1. The other options follow the equations stated above. The standard Kamal model (CKOPT = 3 and -3) introduces a simplification to the above equations with $n_1 = n_2 = n$.

b) *Three-species reaction kinetics model (CKOPT = 4 and -4)*

This option represents a system of chemical reactions involving three chemical species, A, B, and C, with two reaction steps ($n^{\text{th}}$ order with autocatalysis). We denote the concentrations of the species with $c_1$, $c_2$, and $c_3$. The following gives the evolution equations for the concentrations of the reactant, $c_1$, and intermediate, $c_2$:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_2)c_1^{n_1}\\ \dot{c}_2 &= \quad K_1(T)(1.+k_{c1}c_2)c_1^{n_1} - K_2(T)(1.+k_{c2}c_3)c_2^{n_2}\end{aligned}$$

with input parameters $k_{ci}$ and $n_i$. The identity $c_3 = 1. - c_1 - c_2$ eliminates the concentration $c_3$ of product species C from the equations. Therefore, the algorithm internally only uses the concentrations $c_1$ and $c_2$. Thus, this model requires initial values $c_{1,0}$ and $c_{2,0}$.

Finally, we determine the degree of cure by a linear combination:

$$\begin{aligned}p &= F_1(1.-c_1) + (1.-F_1)(1.-c_1-c_2)\\ &= 1.-c_1-c_2+F_1c_2\end{aligned}$$

with an additional factor $F_1$.

c) *Five-species reaction kinetics models (CKOPT = 5 and 6)*

These options represent systems of chemical reactions with five chemical species A, B, C, D, and E with concentrations $c_1$, $c_2$, $c_3$, $c_4$, and $c_5$. The four reaction steps ($n^{\text{th}}$ order with autocatalysis) of the system result in evolution equations for the reactant $c_1$ and intermediates $c_2$, $c_3$, and $c_4$ as follows

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_2)c_1^{n_1}\\ \dot{c}_2 &= \quad K_1(T)(1.+k_{c1}c_2)c_1^{n_1} - K_2(T)\left(1.+k_{c2}\tilde{c}_{\text{Opt}}\right)c_2^{n_2}\\ \dot{c}_3 &= \quad K_2(T)\left(1.+k_{c2}\tilde{c}_{\text{Opt}}\right)c_2^{n_2} - K_3(T)(1.+k_{c3}c_4)c_3^{n_3}\\ \dot{c}_4 &= \quad K_3(T)(1.+k_{c3}c_4)c_3^{n_3} - K_4(T)(1.+k_{c4}c_5)c_4^{n_4}\end{aligned}$$

with input parameters $k_{ci}$ and $n_i$. The identity $c_5 = 1. - c_1 - c_2 - c_3 - c_4$ eliminates the concentration $c_5$ of the product E from the system. Consequently, the algorithm internally only uses the concentrations $c_1$, $c_2$, $c_3$, and $c_4$ internally and requires input of their initial values $c_{1,0}, c_{2,0}, c_{3,0}$ an, d $c_{4,0}$.

The options CKOPT = 5 and 6 only differ in the species used in the autocatalysis in the second reaction step. For option CKOPT = 5, we implemented an autocatalysis by D. Thus, the value of $\tilde{c}_{\text{Opt}}$ in the above equations is the

concentration $c_4$ ($\tilde{c}_{\text{CKOPT=5}} = c_4$). We use an autocatalysis by C as the second reaction step for CKOPT = 6. Consequently, $\tilde{c}_{\text{Opt}}$ is $c_3$ ($\tilde{c}_{\text{CKOPT=6}} = c_3$).

Finally, we determine the degree of cure $p$ by a linear combination of the concentrations with scaling factors $F_1$, $F_2$, and $F_3$:

$$p = (1. - c_1 - c_2 - c_3 - c_4) + F_1(c_2 + c_3 + c_4) + F_2(c_3 + c_4) + F_3(c_4)$$

d) *Three-species reaction kinetics model (CKOPT = 7)*

This option implements another system of chemical reactions involving three chemical species A, B, and C (concentrations denoted by $c_1$, $c_2$, and $c_3$). It involves three reaction steps. The first reaction step (A → B) is an $n^{\text{th}}$ order reaction with autocatalysis by B. The reaction from the intermediate B to the product C follows the extended Kamal model. The evolution equations thus read

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1. + k_{c1}c_2)c_1^{n_1} \\ \dot{c}_2 &= \quad K_1(T)(1. + k_{c1}c_2)c_1^{n_1} - K_2(T)c_2^{n_2}c_3^{m_2} - K_3(T)c_2^{n_3}\end{aligned}$$

with input parameters $k_{c1}$, $m_2$ and $n_i$. The identity $c_3 = 1. - c_1 - c_2$ removes concentration $c_3$ of the product C from the equations. Therefore, the algorithm internally only uses the concentrations $c_1$ and $c_2$ and requires initial values $c_{1,0}$ and $c_{2,0}$.

Finally, a linear combination with the factor $F_1$ determines the degree of cure:

$$p = 1. - c_1 - c_2 + F_1 c_2$$

e) *Three-species reaction kinetics model (CKOPT = 8)*

This option represents the third system of chemical reactions with three chemical species A, B, and C (concentrations denoted by $c_1, c_2, c_3$). It involves two reaction steps. The first reaction step (A → B) follows the Prout-Tompkins equation, the second (B→ C) is described by an $n^{\text{th}}$ order reaction. The evolution equations read as:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)c_1^{n_1}c_2^{\text{m}_1} \\ \dot{c}_2 &= \quad K_1(T)c_1^{n_1}c_2^{\text{m}_1} - K_2(T)c_2^{n_2}\end{aligned}$$

with input parameters $m_1$ and $n_i$. The identity $c_3 = 1. - c_1 - c_2$ replaces the concentration $c_3$ of the product. Thus, the internal calculation does not use $c_3$. The calculation requires initial values $c_{1,0}$ and $c_{2,0}$.

Finally, a linear combination determines the degree of cure:

$$p = 1. - c_1 - c_2 + F_1 c_2$$

with an additional factor $F_1$.

f) *Four-species reaction kinetics model (CKOPT = 9)*

This option implements a reduced version of the system of chemical reactions defined by CKOPT = 6. This option only considers four chemical species A, B, C, and D and three reaction steps. The following gives the evolution equations for the reactant $c_1$ and intermediates $c_2$ and $c_3$ as:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_2)c_1^{n_1}\\ \dot{c}_2 &= \quad K_1(T)(1.+k_{c1}c_2)c_1^{n_1} - K_2(T)(1.+k_{c2}c_3)c_2^{n_2}\\ \dot{c}_3 &= \quad K_2(T)(1.+k_{c2}c_3)c_2^{n_2} - K_3(T)(1.+k_{c3}c_4)c_3^{n_3}\end{aligned}$$

with input parameters $k_{ci}$ and $n_i$. The identity $c_4 = 1. - c_1 - c_2 - c_3$ eliminates the concentration $c_4$ of the product D, which allows expressing the system in terms of the concentrations $c_1, c_2$ and $c_3$. Initial values $c_{1,0}, c_{2,0}$ and $c_{3,0}$ are needed to solve the system.

A linear combination determines the degree of cure $p$:

$$p = (1. - c_1 - c_2 - c_3) + F_1(c_2 + c_3) + F_2(c_3) \quad .$$

g) *Five-species reaction kinetics model (CKOPT = 10)*

Option CKOPT = 10 models a system of chemical reactions with five chemical species: A, B, C, D, and E with concentrations $c_1$, $c_2$, $c_3$, $c_4$, and $c_5$. The first reaction involves the reactant A and the product D ($n^{\text{th}}$ order with autocatalysis). The second reaction changes species B into species E through an intermediate species C. Modeling this reaction involves two reaction steps ($n^{\text{th}}$ order with autocatalysis by C and an $n^{\text{th}}$ order reaction). The evolution equations are:

$$\begin{aligned}\dot{c}_1 &= -K_1(T)(1.+k_{c1}c_4)c_1^{n_1}\\ \dot{c}_2 &= -K_2(T)(1.+k_{c2}c_3)c_2^{n_2}\\ \dot{c}_3 &= \quad K_2(T)(1.+k_{c2}c_3)c_2^{n_2} - K_3(T)c_3^{n_3}\end{aligned}$$

with input parameters $k_{ci}$ and $n_i$. The equations $c_4 = 1. - c_1$ and $c_5 = 1. - c_2 - c_3$ give the concentrations $c_4$ and $c_5$ of the products D and E. Consequently, these equations reduce the system to three unknown concentrations, $c_1, c_2$, and $c_3$. Therefore, solving the system requires the input of initial values $c_{1,0}, c_{2,0}$ and $c_{3,0}$.

The following equation determines the degree of cure $p$ from the concentrations and factors $F_1$ and $F_2$:

$$p = F_1(1. - c_1) + (1. - F_1)\big(F_2(1. - c_2) + (1. - F_2)(1. - c_2 - c_3)\big)$$

h) *Model-free kinetics approach (CKOPT = 11)*

This option allows for a direct, tabulated input of the evolution equation governing the curing process. This choice for CKOPT requires inputting

load curves for a logarithmic scaling function $\ln(A'(p))$ and the activation energy $Q(p)$ as functions of the degree of cure $p$. The differential equation then reads:

$$\dot{p} = \exp\left(\ln\left(A'(p)\right)\right) \times \exp\left(-\frac{Q(p)}{RT}\right)$$

| CKOPT | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CK1 | $K_1(T)$ | $k_1$ | $k_1$ | $k_1$ | $k_1$ | $k_1$ | $k_1$ | $k_1$ | $k_1$ | $k_1$ | $\ln A'(p)$ |
| CK2 | $K_2(T)$ | $k_2$ | $k_2$ | $k_2$ | $k_2$ | $k_2$ | $k_2$ | $k_2$ | $k_2$ | $k_2$ | $Q(p)$ |
| CK3 | $m_2$ | $Q_1$ | $Q_1$ | $Q_1$ | $k_3$ | $k_3$ | $k_3$ | $Q_1$ | $k_3$ | $k_3$ | |
| CK4 | $n_1$ | $Q_2$ | $Q_2$ | $Q_2$ | $k_4$ | $k_4$ | $Q_1$ | $Q_2$ | $Q_1$ | $Q_1$ | |
| CK5 | $n_2$ | $m_2$ | $m_2$ | $n_1$ | $Q_1$ | $Q_1$ | $Q_2$ | $n_1$ | $Q_2$ | $Q_2$ | |
| CK6 | | $n_1$ | $n$ | $n_2$ | $Q_2$ | $Q_2$ | $Q_3$ | $m_1$ | $Q_3$ | $Q_3$ | |
| CK7 | | $n_2$ | | $k_{c1}$ | $Q_3$ | $Q_3$ | $n_1$ | $n_2$ | $n_1$ | $n_1$ | |
| CK8 | | | | $k_{c2}$ | $Q_4$ | $Q_4$ | $n_2$ | $F_1$ | $n_2$ | $n_2$ | |
| CK9 | | | | $F_1$ | $n_1$ | $n_1$ | $m_2$ | $c_{1,0}$ | $n_3$ | $n_3$ | |
| CK10 | | | | $c_{1,0}$ | $n_2$ | $n_2$ | $n_3$ | $c_{2,0}$ | $k_{c1}$ | $k_{c1}$ | |
| CK11 | | | | $c_{2,0}$ | $n_3$ | $n_3$ | $k_{c1}$ | | $k_{c2}$ | $k_{c2}$ | |
| CK12 | | | | | $n_4$ | $n_4$ | $F_1$ | | $k_{c3}$ | $F_1$ | |
| CK13 | | | | | $k_{c1}$ | $k_{c1}$ | $c_{1,0}$ | | $F_1$ | $F_2$ | |
| CK14 | | | | | $k_{c2}$ | $k_{c2}$ | $c_{2,0}$ | | $F_2$ | $c_{1,0}$ | |
| CK15 | | | | | $k_{c3}$ | $k_{c3}$ | | | $c_{1,0}$ | $c_{2,0}$ | |
| CK16 | | | | | $k_{c4}$ | $k_{c4}$ | | | $c_{2,0}$ | $c_{3,0}$ | |
| CK17 | | | | | $F_1$ | $F_1$ | | | $c_{3,0}$ | | |
| CK18 | | | | | $F_2$ | $F_2$ | | | | | |
| CK19 | | | | | $F_3$ | $F_3$ | | | | | |
| CK20 | | | | | $c_{1,0}$ | $c_{1,0}$ | | | | | |

| CKOPT | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| CK21 | | | | | $c_{2,0}$ | $c_{2,0}$ | | | | | |
| CK22 | | | | | $c_{3,0}$ | $c_{3,0}$ | | | | | |
| CK23 | | | | | $c_{4,0}$ | $c_{4,0}$ | | | | | |

3. **Chemical shrinkage.** The chemical reaction of the curing process results in shrinkage of the material. Three options are available to model this behavior.

   For CEOPT = 1 and 2, the coefficient of chemical shrinkage, $\gamma(p)$, is specified with a load curve. For CEOPT = 3, the coefficient is given by the following quadratic expression:

   $$\gamma(p) = \gamma_2 p^2 + \gamma_1 p + \gamma_0 \, .$$

   For CEOPT = 1, this load curve is used to compute the chemical strains by the following differential form:

   $$d\varepsilon^{\text{ch}} = \gamma(p) dp.$$

   CEOPT = 2 and 3 invoke a secant form, such that the strains are computed as:

   $$\varepsilon^{\text{ch}} = \gamma(p) \times (p - p_R) - \gamma(p_I) \times (p_I - p_R) \, ,$$

   with a reference degree of cure $p_R$ and initial degree of cure $p_I$.

   The following table summarizes the input structure.

| CEOPT | CE1 | CE2 | CE3 | CE4 |
|---|---|---|---|---|
| 1 | $\gamma(p)$ | | | |
| 2 | $\gamma(p)$ | $p_R$ | | |
| 3 | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $p_R$ |

4. **Thermal expansion.** Like the strains resulting from chemical shrinkage discussed in Remark 3, the thermal strains are either defined in a secant or differential form. In both cases the coefficient of thermal expansion $\eta(p, T)$ can be given as function of degree of cure $p$ and temperature $T$ and requires the input by of two-dimensional tabular data.

   Option TEOPT = 1 refers to the differential form

   $$d\varepsilon^{\text{th}} = \eta(p, T) \, dT \quad .$$

   TEOPT = 2 invokes the secant formulation which requires the specification of an additional reference temperature $T_R$

$$\varepsilon^{\text{th}} = \eta(p,T) \times (T - T_R) - \eta(p,T_I) \times (T_I - T_R) \ .$$

Coefficient $\eta(p,T)$ is specified with a 2D table (*DEFINE_TABLE_2D) whose ID is provided by parameter TE1. The values given in the table input correspond to the degree of cure and the abscissa of the referenced curve to temperature. If a load curve is reference by parameter TE1, the coefficient $\eta$ is assumed to be a function of temperature.

The following table summarizes the input structure.

| TEOPT | TE1 | TE2 |
|---|---|---|
| 1 | $\eta(p,T)$ | |
| 2 | $\eta(p,T)$ | $T_R$ |

5. **Stress relaxation.** The Cauchy stress, $\sigma_{ij}$, is related to the strain rate by:

$$\sigma_{ij}(t) = \int_0^t g_{ijkl}(t-\tau)\frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau} d\tau \ .$$

The relaxation functions $g_{ijkl}(t-\tau)$ are represented in this material formulation by terms of the Prony series for the shear modulus $G$ and the bulk modulus $K$ as functions of time $t$. For the shear modulus $G$, the series expansion is given by:

$$G(t) = G_\infty + \sum_{i=1}^{n_G} G_i e^{-\beta_i^G t} = G_0 - \sum_{i=1}^{n_G} G_i + \sum_{i=1}^{n_G} G_i e^{-\beta_i^G t},$$

with shear relaxation moduli $G_i$ and decay constants $\beta_i^G$. The relation between the shear equilibrium modulus $G_\infty$ and the instantaneous shear modulus $G_0$ is given by

$$G_\infty = G_0 - \sum_i G_i.$$

A similar Prony series definition is expected for the bulk modulus $K(t)$:

$$K(t) = K_\infty + \sum_{i=1}^{n_K} K_i e^{-\beta_i^K t} = K_0 - \sum_{i=1}^{n_K} K_i + \sum_{i=1}^{n_K} K_i e^{-\beta_i^K t},$$

with bulk relaxation moduli $K_i$ and decay constants $\beta_i^K$.

This material model provides several options for inputting the Prony series data. If the first entry in Card 15 is non-negative, the input strategy defaults to VISOPT = 0. For this option, provide the terms for individual Prony series for $G(t)$ and $K(t)$ with up to 18 Prony series terms for each ($n_G \le 18$, $n_K \le 18$).

If the first entry in Card 15 is negative, it represents the option VISOPT. A negative value VISOPT < 0 implies the following coupling between the Prony series terms $G_i$ and $K_i$:

$$K_i = \frac{2+2\,\nu}{3(1-2\,\nu\,)} G_i \quad \text{and} \quad \beta_i^K = \beta_i^G\,.$$

Note that Poisson's ratio $\nu$ in the above equation is constant. This approach allows accounting for up to 25 Prony series terms ($n_G = n_K \leq 25$). For VIOPT = -1, LS-DYNA interprets the input relaxation constants as terms $E_i$ for Young's modulus $E(t)$ and internally translates them into the necessary constants $G_i$. For VIOPT = -2, LS-DYNA assumes a direct input of the shear relaxation moduli $G_i$.

In most applications the viscoelastic properties depend on temperature and degree of cure. In this material, shifting functions acting on the moduli $G_i$, $G_0$ and $G_\infty$ (vertical shifting) and on the decay constants $\beta_i$ (horizontal shifting) apply these dependencies. Note that, if not stated otherwise, LS-DYNA applies the same shifting operations to the shear and bulk moduli. Cards 7 to 10 set the shifting functions. We discuss these functions in Remarks 6, 7, 8 and 9.

6. **Horizontal temperature shift.** You can account for a possible temperature effect on the stress relaxation (see Remark 5) by a horizontal shift operation on the relaxation curve, implemented by the scaling of the decay constants $\beta_i$ with a factor $a_T(\,T)$.

For THOPT = 1, the Williams-Landel-Ferry (WLF) shift function is used:

$$\ln\big(a_T(T)\big) = \frac{-A(T-T_R)}{B+T-T_R}$$

with constant parameters $A$ and $B$ and the reference temperature $T_R$.

THOPT = 2 invokes the Arrhenius shift function which requires the input of a reference temperature $T_R$ and one parameter $C$:

$$\ln\big(a_T(T)\big) = C\left(\frac{1}{T}-\frac{1}{T_R}\right)$$

For many adhesive materials, the qualitative behavior of the temperature dependence changes with the glass transition temperature $T_G$ from an Arrhenius- to a WLF-type description of the shifting. THOPT = 3 provides this behavior:

$$\ln\big(a_T(T)\big) = \begin{cases} C\left(\dfrac{1}{T}-\dfrac{1}{T_G}\right) & T \leq T_G \\ \dfrac{-A(T-T_G)}{B+T-T_G} & T > T_G \end{cases}$$

It has been proposed in literature to extend this option by a curing that depends on the glass transition temperature $T_G = T_G(p)$, such that the shifting factor reads

$$\ln\left(a_T(T,p)\right) = \begin{cases} C\left(\frac{1}{T} - \frac{1}{T_G(p)}\right) & T \le T_G(p) \\ \frac{-A(T - T_G(p))}{B + T - T_G(p)} & T > T_G(p) \end{cases}$$

This feature corresponds to THOPT = 4. The glass transition temperature must be input as a load curve.

THOPT = 6 replaces the Arrhenius shift function with an exponential approach. Above the glass transition temperature $T_G$, an extension of the WLF-type shift function is used:

$$a_T(T,p) = \begin{cases} \left(1 - D\,(T - T_G(p))\right)^C & T \le T_G(p) \\ \exp\left(\min\left(\frac{-A_1(T - T_G(p))}{B_1 + T - T_G(p)}, \frac{-A_2(T - T_G(p))}{B_2 + T - T_G(p)}\right)\right) & T > T_G(p) \end{cases}$$

For the options discussed so far, no difference in temperature dependence is made between the shear and bulk moduli. The same scaling is applied to both Prony series expansions.

Finally, THOPT = 5 allows defining direct input for scaling factors $a_T^G$ and $a_T^K$ for the shear and bulk moduli, respectively. Load curve or table IDs are expected as input. The load curves (either referenced by the table or by the input) define the logarithm of the factors, that is, $\ln\left(a_T^G\right)$ and $\ln\left(a_T^G\right)$, as functions of temperature. In case of a table ID input, an additional dependence on the degree of cure can be accounted for.

The parameters input for the different options are shown in the following table.

| THOPT | TH1 | TH2 | TH3 | TH4 | TH5 | TH6 | TH7 |
|---|---|---|---|---|---|---|---|
| 1 | $A$ | $B$ | $T_R$ | | | | |
| 2 | $C$ | $T_R$ | | | | | |
| 3 | $A$ | $B$ | $C$ | $T_G$ | | | |
| 4 | $A$ | $B$ | $C$ | $T_G(p)$ | | | |
| 5 | $\ln(a_T^G(T))$ / $\ln(a_T^G(p,T))$ | $\ln(a_T^K(T))$ / $\ln(a_T^K(p,T))$ | | | | | |
| 6 | $A_1$ | $B_1$ | $C$ | $D$ | $A_2$ | $B_2$ | $T_G(p)$ |

7. **Vertical temperature shift.** To model the effect of temperature on the viscoelastic response, you can apply a vertical shift to the master relaxation curve (see Remark 5). The shear relaxation moduli ($G_i$ and $G_\infty$) and bulk relaxation moduli ($K_i$ and $K_\infty$) are scaled by temperature dependent scaling factors $b_T^G(T)$ and $b_T^K(T)$, respectively, to achieve this shift. The input parameters for the factors need to be load curves. Here, parameter TV1 refers to $b_T^G(T)$ and TV2 to $b_T^K(T)$.

   For TVOPT = 1 the entire relaxation curve is scaled. In contrast, TVOPT = 2 causes shifting of only the time dependent terms of the Prony series and, consequently, only the moduli $G_i$ and $K_i$ are scaled.

8. **Horizontal $p$-shift.** The effect of curing on the viscoelastic property of an adhesive material can be modelled by a horizontal shift of the relaxation curve (see Remark 5), meaning by scaling the decay moduli $\beta_i$. The scaling factors are denoted in this case by $a_c$.

   For PHOPT = 1, an analytical expression based on Eom et al is implemented

$$\log(a_c(p)) = \begin{cases} c(p - p_{\text{gel}}) + a_{\text{gel}} & p < p_{\text{gel}} \\ a_{\text{gel}} H^{(p-p_{\text{gel}})} \left( \dfrac{p_f - p}{p_f - p_{\text{gel}}} \right)^m & p \geq p_{\text{gel}} \end{cases}$$

   with $p_{\text{gel}}$ and $a_{\text{gel}}$ being properties at the gelation point of the material. This shift is applied to both the shear and bulk moduli.

   PHOPT = 2 offers the possibility of a direct input of the scaling factors as functions of degree of cure. Here, load curves defining $\log\left(a_c^G\right)$ for shifting the shear curve and $\log\left(a_c^K\right)$ for shifting the bulk curve are expected.

   The set of input parameters is summarized in the following table.

| PHOPT | PH1 | PH2 | PH3 | PH4 | PH5 | PH6 |
|---|---|---|---|---|---|---|
| 1 | $p_{\text{gel}}$ | $a_{\text{gel}}$ | $c$ | $H$ | $p_f$ | $m$ |
| 2 | $\log\left(a_c^G(p)\right)$ | $\log\left(a_c^K(p)\right)$ | | | | |

9. **Vertical $p$-shift.** We have implemented two different approaches to represent the effect of curing on the viscoelasticity through vertical shifting operations. The vertical shifting operations apply to the master curves $G(t)$ and $K(t)$ as defined in Remark 5.

   The first approach (PVOPT = 1) is taken from *MAT_277 and assumes a constant ratio $G_i(p)/G_0(p)$ for all degrees of cure. Consequently, it suffices to define one

term $G_0(p)$ as a function of the degree of cure and further coefficients for the fully cured state of the adhesive:

$$G(t,p) = G_0(p)\left(1 - \sum_i \frac{G_{i,p=1.0}}{G_{0,\mathrm{p}=1.0}}\left(1 - e^{-\beta_i t}\right)\right) .$$

PVOPT = 2 distinguishes the effect of curing on the equilibrium moduli from its effect on the time-depending terms of the Prony series. Consequently, load curve IDs are expected to define $G_\infty(p)$ and $K_\infty(p)$ as well as scaling factors $b_c^G(p)$ and $b_c^K(p)$. The latter are applied to all $G_i$ and $K_i$, respectively. This is also reflected by the input structure shown in the following table.

| PVOPT | PV1 | PV2 | PV3 | PV4 |
|---|---|---|---|---|
| 1 | $G_0(p)$ | $K_0(p)$ | | |
| 2 | $G_\infty(p)$ | $K_\infty(p)$ | $b_c^G(p)$ | $b_c^K(p)$ |

10. **Plasticity.** This material features an isotropic plasticity formulation with a non-associated flow rule closely related to the TAPO model implemented in *MAT_252. Both, the yield criterion $F$ as well as the flow potential $F^*$, are defined in terms of invariants $\tilde{I}_1$ and $\tilde{J}_2$of the effective stress tensor:

$$\tilde{\sigma} = \sigma/(1 - D_1)(1 - D_2),$$

where the evolution of the damage parameters $D_1$ and $D_2$ is defined separately.

The general form of $F$ and $F^*$ in this model is given by

$$F = f\left(\tilde{I}_1, \tilde{J}_2, r, T\right) - \tau_Y^2(p,T,r) = 0$$
$$F^* = f^*\left(\tilde{I}_1, \tilde{J}_2\right) - \tau_Y^2(p,T,r)$$

The yield surface $f$ and yield strength $\tau_Y$ are functions of the arc length of the damage plastic strain rate $\dot{r}$, which is defined by means of the arc length of the plastic strain rate $\dot{\gamma}_\mathrm{v}$ as in Lemaitre [1992]:

$$\dot{r} = (1 - D_1)\dot{\gamma}_\mathrm{v} = (1 - D_1)\sqrt{2\,\mathrm{tr}(\dot{\varepsilon}^\mathrm{p})^2}.$$

The plastic strain rate $\dot{\varepsilon}^\mathrm{p}$ is given by the non-associated flow rule

$$\dot{\varepsilon}^\mathrm{p} = \lambda \frac{\partial F^*}{\partial \sigma}.$$

The expressions for $f$ and $f^*$ or in other words the form of yield surface and flow potential are determined by the choice of parameters in Card 11. The yield strength computation is defined by Card 12. For details, see Remarks 11, 12, 13, and 14.

11. **Yield surface.** The yield surface definition is controlled by choice of parameter PL1OPT in Card 11. For the currently available options PL1OPT = 1, 2 or 3, the same flow potential is assumed:

$$f^*(\tilde{I}_1, \tilde{J}_2) = \tilde{J}_2 + \frac{a_2^*}{3}\langle \tilde{I}_1 \rangle^2 \ ,$$

where $a_2^*$ is a user-defined material parameter.

Choosing PL1OPT = 1 results in a cap model in tension and nonlinear Drucker & Prager in compression with a distortional hardening under plastic flow. There is no temperature dependence for function $f$ in this case, which reads:

$$f(\tilde{I}_1, \tilde{J}_2, r) = \tilde{J}_2 + \frac{1}{\sqrt{3}} a_1(r)\ \tau_0\ \tilde{I}_1 + \frac{a_2(r)}{3}\langle \tilde{I}_1 \rangle^2 \ .$$

Distortional hardening is introduced by phenomenological descriptions for parameters $a_1(r)$ and $a_2(r)$:

$$\begin{aligned} a_1(r) &= a_{10} + a_1^{\mathrm{H}} r \\ a_2(r) &= \max\left(a_{20} + a_2^{\mathrm{H}} r, 0.0\right) \end{aligned}$$

PL1OPT = 2 does not consider distortional hardening and refers to a cap model in tension and a von Mises yield function in compression:

$$f(\tilde{I}_1, \tilde{J}_2) = \tilde{J}_2 + \frac{a_{20}}{3}\left\langle \tilde{I}_1 + \frac{\sqrt{3} a_{10} \tau_0}{2\, a_{20}} \right\rangle^2 - \frac{a_{10}^2\ \tau_0^2}{4\, a_{20}}.$$

Finally, PL1OPT = 3 refers to a temperature dependent yield surface. Equivalently to PL1OPT = 1 a cap model in tension and nonlinear Drucker & Prager in compression is used, but the distortional hardening is defined with respect to the current temperature:

$$f(\tilde{I}_1, \tilde{J}_2, T) = \tilde{J}_2 + \frac{1}{\sqrt{3}} a_{10} \tau_0 \tilde{I}_1 + \frac{a_2(T)}{3}\langle \tilde{I}_1 \rangle^2,$$

with the simple linear temperature dependence

$$a_2(T) = a_{20}\left(1 - m_{a2}\ (T - T_0)\right).$$

Input parameters for the different options can be found in the following table:

| PL1OPT | PL11 | PL12 | PL13 | PL14 | PL15 |
|---|---|---|---|---|---|
| 1 | $a_{10}$ | $a_{20}$ | $a_2^*$ | $a_1^H$ | $a_2^H$ |
| 2 | $a_{10}$ | $a_{20}$ | $a_2^*$ | | |
| 3 | $a_{10}$ | $a_{20}$ | $a_2^*$ | $m_{a2}$ | $T_0$ |

12. **Yield strength.** The yield strength $\tau_Y$ is defined by the parameters in Card 12. Different options are available to define temperature and degree of cure dependent hardening behavior. The most general option (PL2OPT = 1) is a three-dimensional tabular input for $\tau_Y(p,T,r)$, employing *DEFINE_TABLE_3D. Here $p$ is the degree of cure, $T$ is the temperature, and $r$ is the damage plastic strain.

For PL2OPT = 2, initial strength $\tau_0$ and hardening $R$ are defined independently with tabular data. Their sum represents the current yield strength:

$$\tau_Y(p,T,r) = \tau_0(p,T) + R(p,T,r).$$

A two-dimensional table (*DEFINE_TABLE_2D) is required to define $\tau_0(p,T)$ as a function of degree of cure $p$ and temperature $T$. The hardening part $R(p,T,r)$ naturally requires a three-dimensional table (*DEFINE_TABLE_3D).

PL2OPT = 3 employs the same split between initial and hardening part as the second option, but it is further assumed, that the effect of curing can be modelled by different scaling operations $\chi_c(p)$ and $\phi_c(p)$:

$$\tau_Y(p,T,r) = \tau_{0\theta}(T)\chi_c(p) + R_\theta(T,r)\phi_c(p).$$

The input only requires a two-dimensional tabular input for the hardening $R_\theta(T,r)$ and three load curve definitions (see *DEFINE_CURVE) for $\tau_{0\theta}(T)$, $\chi_c(p)$ and $\phi_c(p)$.

PL2OPT = 4 only differs from PL2OPT = 3 in the input of the temperature dependent hardening part $R_\theta(T,r)$. Instead of tabular data, an exponential hardening behavior is assumed. When PL2OPT = 4 is invoked, the following analytical expression for $R_\theta(T,r)$ is used:

$$R_\theta(T,r) = H_\theta(T)r + q_\theta(T)\left(1 - e^{-b_\theta(T)r}\right) \ .$$

Temperature dependencies for the parameters $H_\theta$, $q_\theta$, and $b_\theta$ requires load curve input.

The simplest version is invoked by PL2OPT = 5, where the yield strength $\tau_Y(r)$ is a function solely of the plastic strain data. Again, an exponential hardening is assumed:

$$\tau_Y(r) = \tau_0 + Hr + q\left(1 - e^{-br}\right).$$

The input of the parameters is shown in the following table.

| PL2OPT | PL21 | PL22 | PL23 | PL24 | PL25 | PL26 |
|---|---|---|---|---|---|---|
| 1 | $\tau_Y(p,T,r)$ | | | | | |
| 2 | $\tau_0(p,T)$ | $R(p,T,r)$ | | | | |
| 3 | $\tau_{0\theta}(T)$ | $\chi_c(p)$ | $R_\theta(T,r)$ | $\phi_c(p)$ | | |

| PL2OPT | PL21 | PL22 | PL23 | PL24 | PL25 | PL26 |
|---|---|---|---|---|---|---|
| 4 | $\tau_{0\theta}(T)$ | $\chi_c(p)$ | $q_\theta(T)$ | $b_\theta(T)$ | $H_\theta(T)$ | $\phi_c(p)$ |
| 5 | $\tau_0$ | $q$ | $B$ | $H$ | | |

13. **Material damage.** Material damage can occur for this material when in a solid-like state. The material becomes solid-like when the current degree of cure $p$ reaches the gelation point $p_{\text{gel}}$, given by parameter PGEL. A different damage mechanism occurs in the liquid phase, as discussed in Remark 14.

The material damage is described in terms of the damage parameter $D_1$. Its evolution is based on the approach in Lemaitre [1985]. For $p \geq p_{\text{gel}}$, the general formulation can be defined in terms of a chosen strain measure $\zeta$ as follows:

$$\dot{D}_1 = \dot{D}_1(\zeta, \dot{\zeta}) = n \left\langle \frac{\zeta - \gamma^{\text{c}}}{\gamma^{\text{f}} - \gamma^{\text{c}}} \right\rangle^{n-1} \frac{\dot{\zeta}}{\gamma^{\text{f}} - \gamma^{\text{c}}} \ .$$

The parameter DAEVO defines the strain measure $\zeta$. For DAEVO = 0, the arc length of the damage plastic strain rate is used: $\dot{\zeta} = \dot{r}$. The arc length of plastic strain rate $\dot{\gamma}_{\text{v}}$ governs the damage evolution for DAEVO = 1, that is, $\dot{\zeta} = \dot{\gamma}_{\text{v}}$. DAEVO = 2 employs the viscoelastic-plastic strain rate $\dot{\gamma}$ as strain rate measure:

$$\dot{\zeta} = \dot{\gamma} = \sqrt{2\text{tr}((\dot{\boldsymbol{\varepsilon}}^{\text{vp}})^2)}, \qquad \dot{\boldsymbol{\varepsilon}}^{\text{vp}} = \dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{\text{th}} - \dot{\boldsymbol{\varepsilon}}^{\text{ch}} \ .$$

The strains at the thresholds $\gamma_{\text{c}}$ and $\gamma_{\text{f}}$ for damage initiation and rupture depend on a function $\xi(\eta)$ of the triaxiality $\eta$. This function $\xi(\eta)$ determines if triaxiality is considered only under tensile loading or under tensile and compressive loading. Consequently, there are two choices available: For DATRIAX = 0, the Macauley bracket is used ($\xi(\eta) = \langle\eta\rangle$), whereas DATRIX = 1 reduces $\xi$ to the identity ($\xi(\eta) = \eta$).

The particular equations for the strain thresholds $\gamma_{\text{c}}$ and $\gamma_{\text{f}}$ are determined by the damage option parameter DAOPT. Choosing DAOPT = 1 allows for temperature and degree of cure dependence:

$$\gamma^{\text{c}} = \left(d_1^{\text{c}} + d_2^{\text{c}}\left(e^{-d_3\xi(\eta)}\right)\right) d_\theta(T)\beta(p)$$
$$\gamma^{\text{f}} = \left(d_1 + d_2\left(e^{-d_3\xi(\eta)}\right)\right) d_\theta(T)\delta(p)$$

Functions $d_\theta(T)$, $\beta(p)$, and $\delta(p)$ each require a load curve input. These functions are omitted in the simplified option DAOPT = 2, for which the strain thresholds reduce to

$$\gamma^{\text{c}} = \left(d_1^{\text{c}} + d_2^{\text{c}}\left(e^{-d_3\xi(\eta)}\right)\right)$$
$$\gamma^{\text{f}} = \left(d_1 + d_2\left(e^{-d_3\xi(\eta)}\right)\right)$$

Strain rate effects for the definition of the thresholds are incorporated into option DAOPT = 3 following Johnson and Cook [1985], which leads to

$$\gamma^{\mathrm{c}} = \left(d_1^{\mathrm{c}} + d_2^{\mathrm{c}}\left(e^{-d_3\xi(\eta)}\right)\right) d_\theta(T)\beta(p)\left(1 + d_4\left\langle \ln \dot{\gamma}/\dot{\gamma}_0 \right\rangle\right)$$
$$\gamma^{\mathrm{f}} = \left(d_1 + d_2\left(e^{-d_3\xi(\eta)}\right)\right) d_\theta(T)\delta(p)\left(1 + d_4\left\langle \ln \dot{\gamma}/\dot{\gamma}_0 \right\rangle\right)$$

The parameter input is summarized in the following table:

| DAOPT | DA1 | DA2 | DA3 | DA4 | DA5 | DA6 |
|---|---|---|---|---|---|---|
| 1 | $n$ | $d_1^c$ | $d_2^c$ | $d_1$ | $d_2$ | $d_3$ |
| 2 | $n$ | $d_1^c$ | $d_2^c$ | $d_1$ | $d_2$ | $d_3$ |
| 3 | $n$ | $d_1^c$ | $d_2^c$ | $d_1$ | $d_2$ | $d_3$ |

| DAOPT | DA7 | DA8 | DA9 | DA10 | DA11 |
|---|---|---|---|---|---|
| 1 | $d_\theta(T)$ | $\beta(p)$ | $\delta(p)$ | | |
| 2 | | | | | |
| 3 | $d_\theta(T)$ | $\beta(p)$ | $\delta(p)$ | $d_4$ | $\dot{\gamma}_0$ |

14. **Viscous fingering.** Viscous fingering can occur if the connected partners (partially) separate while the connecting adhesive is still in the liquid phase, resulting in an incomplete bonding of the partners. For this material model we implemented a rather simple phenomenological approach. An additional damage parameter $D_2$ models the effect of viscous fingering. $D_2$ accounts for the reduction of the effective adhesive area. Furthermore, we assume that $D_2$ can be expressed as function $\delta_A(\varepsilon_{33})$ of the thickness strain $\varepsilon_{33}$ of the element.

    In the liquid phase (meaning $p < p_{\mathrm{gel}}$), this damage mechanism is active if parameter PDA1 is nonzero. For positive values of PDA1, the parameter is interpreted as scalar input $\beta_A$ for an exponential approach:

    $$D_2 = \delta_A(\varepsilon_{33}) = 1 - \exp(-\beta_A\varepsilon_{33})$$

    Here, only positive values of $\varepsilon_{33}$ are considered. Within the compressive regime, $\varepsilon_{33} < 0$, the damage mechanism is not active, and $D_2 = 0$.

    Alternatively, a negative input for PDA1 implies a direct input of $D_2 = \delta_A(\varepsilon_{33})$ as a load curve with ID = |PDA1|. Not that this load curve can also be extended to the compressive regime, $\varepsilon_{33} < 0$, to account for the effect of liquid material possibly being pressed out of the adhesive zone. Any damage resulting from

compressive strains is neutralized as soon as the material cures and reaches the gelation point.

As soon as the degree of cure exceeds the gelation point $p_{gel}$, the mechanism is stopped and the damage parameter $D_2$ remains constant. The value for $p_{gel}$ is to be given as input parameter PGEL.

15. **Histories variables.** The most important history variables are listed in the following table:

| History Variable # | Description |
|---|---|
| 1 | Temperature, $T$ |
| 2 | Degree of cure, $p$ |
| 3 | Chemical expansion |
| 4 | Thermal expansion |
| 5 | Initial temperature, $T_0$ |
| 6 | Material damage parameter, $D_1$ |
| 7 | Viscous fingering damage parameter, $D_2$ |
| 8 | Effective strain measure, $\zeta$, for material damage |
| 9 | Thickness strain $\varepsilon_{33}$ |
| 10 | Current effective shear modulus, $G(t)$ |
| 11 | Current effective bulk modulus, $K(t)$ |
| 12 | Concentration $c_1$ of species A |
| 13 | Concentration $c_2$ of species B |
| 14 | Concentration $c_3$ of species C |
| 15 | Concentration $c_4$ of species D |

16. **Diffusion control mechanism definition.** Diffusion processes in the material may affect the curing process of the material. For example, they may slow down the chemical reaction for temperatures around the glass transition temperature, $T_G$, which is itself a function of the degree of cure, $p$. Parameter CDOPT sets the model giving the diffusion effect, $K^{\text{diff}}(T)$, discussed in Remark 2. CTGOPT defines the relationship between $T_G(p)$ and $p$ needed by the CDOPT models.

We begin with a discussion of the models specified with CTGOPT because $T_G(p)$ is needed by the diffusion effect models. Currently, the material distinguishes between two different models for $T_G(p)$: DiBenedetto and Hesekamp. The DiBenedetto model (CTGOPT = 1) requires the input of the glass transition

temperatures for uncured ($T_{G0}$) and for fully cured ($T_{G\infty}$) material. With a transition parameter $\lambda$, the DiBenedetto equations then reads:

$$T_G(p) = T_{G0} + \lambda\, p\, \frac{T_{G\infty} - T_{G0}}{1 - (1-\lambda)p}.$$

The Hesekamp equation (CTGOPT = 2) is given in terms of the glass transition temperature for uncured material ($T_{G0}$) and two algorithmic parameters ($g_1$ and $g_2$):

$$T_G(p) = T_{G0} \exp\left(\frac{g_1\, p}{g_2 - p}\right).$$

The following table summarizes how the necessary parameters are defined in the input of Card 4.

| CTGOPT | CTG1 | CTG2 | CTG3 |
|---|---|---|---|
| 1 | $T_{G0}$ | $T_{G\infty}$ | $\lambda$ |
| 2 | $T_{G0}$ | $g_1$ | $g_2$ |

With $T_G(p)$ defined, we can describe the models for $K^{\text{diff}}(T)$ set with CDOPT. CDOPT = 1 invokes a function similar to the Williams-Landel-Ferry (WLF) shift function:

$$K^{\text{diff}}(T) = k^{\text{diff}} \exp\left(\frac{c_1(T - T_G(p))}{c_2 + T - T_G(p)}\right).$$

CDOPT = 2 models the diffusion effect with an equation closely related to the Arrhenius shift function:

$$K^{\text{diff}}(T) = k^{\text{diff}} \exp\left(\frac{-c_1 T_g(p)^2}{c_2}\left(\frac{1}{T} - \frac{1}{T_G(p)}\right)\right).$$

Finally, CDOPT = 3 combines the above approaches and switches formulations at the glass transition temperature:

$$K^{\text{diff}}(T) = \begin{cases} k^{\text{diff}} \exp\left(\dfrac{c_1(T - T_G(p))}{c_2 + T - T_G(p)}\right) & T \le T_G(p) \\ k^{\text{diff}} \exp\left(\dfrac{-c_1 T_g(p)^2}{c_2}\left(\dfrac{1}{T} - \dfrac{1}{T_G(p)}\right)\right) & T > T_G(p) \end{cases}$$

In all three of the above equations, the input parameter $k^{\text{diff}}$ defines the diffusion constant at the glass transition temperature ($k^{\text{diff}} = K^{\text{diff}}(T_g(p))$). In addition, these models require two material parameters, $c_1$ and $c_2$. The following table shows the parameters input in Card 3:

| CDOPT | CD1 | CD2 | CD3 |
|---|---|---|---|
| 1/2/3 | $k^{\text{diff}}$ | $c_1$ | $c_2$ |

## *MAT_RRR_POLYMER

This is Material Type 317. It is for analysis of isotropic polymers, such as thermoplastics. This rheological network model was developed to incorporate rate, relaxation, and recovery effects in plastics up to yield plateau. Damping and creep effects spanning from milliseconds to years can be represented. It works for both the explicit and implicit solver and uses a numerically efficient implementation. Only solid elements are supported.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ESTR1 | EEND1 | EELIM1 | PR1 | | | | |
| Type | F | F | F | F | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ESTR2 | EEND2 | EELIM2 | PR2 | | | | |
| Type | F | F | F | F | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ESTR3 | EEND3 | EELIM3 | PR3 | | | | |
| Type | F | F | F | F | | | | |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MSTR1 | MEND1 | ECLIM1 | SGLIM1 | A1 | PRV1 | | |
| Type | F | F | F | F | F | F | | |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MSTR2 | MEND2 | ECLIM2 | SGLIM2 | A2 | PRV2 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| ESTR*i* | Starting Young's modulus, $E_{s_i}$, in link $i$, $i = 1,2,3$. |
| EEND*i* | Ending Young's modulus, $E_{e_i}$, in link $i$, $i = 1,2,3$. |
| EELIM*i* | Elastic limit, $\bar{\varepsilon}_{e_i}$, in link $i$, $i = 1,2,3$. |
| PR*i* | Poisson ratio, $\nu_i$, in link $i$, $i = 1,2,3$. |
| MSTR*i* | Starting exponent, $m_{s_i}$, in link $i$, $i = 1,2$. |
| MEND*i* | Ending exponent, $m_{e_i}$, in link $i$, $i = 1,2$. |
| ECLIM*i* | Creep strain limit, $\bar{\varepsilon}_{c_i}$, in link $i$, $i = 1,2$. |
| SGLIM*i* | Effective stress limit, $\bar{\sigma}_i$, in link $i$, $i = 1,2$. |
| A*i* | Reference creep strain rate, $\alpha_i$, in link $i, i = 1,2$. |
| PRV*i* | Viscous Poisson ratio, $\mu_i$, in link $i, i = 1,2$. Default: 0.5. |

**Remarks:**

1. **Material model.** The material model is due to M. Lindvall and is composed of two viscoelastic links and one purely elastic link. Each link is characterized by its own set of parameters resulting in a Cauchy stress, $\boldsymbol{\sigma}_i$ with $i = 1,2,3$, so the complete stress is given by:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3.$$

Each link is a hypo-elasto-viscoelastic model with a stress rate on the form[5]

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}(\varepsilon_e)(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_c).$$

Here $\mathbf{C}(\varepsilon_e)$ is the isotropic Hooke tensor which depends on the effective elastic strain:

$$\varepsilon_e = \sqrt{\frac{1}{1+2\mu^2}\boldsymbol{\varepsilon}_e^{\text{dev}}:\boldsymbol{\varepsilon}_e^{\text{dev}}}.$$

The elastic strain quantities are given by $\boldsymbol{\varepsilon}_e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_c$, $\varepsilon_e^{\text{mean}} = \frac{1}{3}\boldsymbol{\varepsilon}_e:\mathbf{I}$, and $\boldsymbol{\varepsilon}_e^{\text{dev}} = \boldsymbol{\varepsilon}_e - \varepsilon_e^{\text{mean}}\mathbf{I}$. $\mu$ is the viscous Poisson ratio. The exact expression for $\mathbf{C}(\varepsilon_e)$ is indirectly defined by a strain-dependent Young's modulus $E(\varepsilon_e)$ and constant Poisson ratio, $\nu$:

$$E(\varepsilon_e) = E_s + (E_e - E_s)\tanh\left(\frac{\varepsilon_e}{\bar{\varepsilon}_e}\right).$$

$\bar{\varepsilon}_e$ is the elastic limit, and $E_s$ and $E_e$ are the starting and ending Young's moduli. These are all input on Cards 2 through 4, except for $\mu$ which is input on Cards 5 and 6.

The creep strain tensor evolves as[6]

$$\dot{\boldsymbol{\varepsilon}}_c = a\left(\frac{\sigma^{\text{eff}}}{\overline{\sigma}}\right)^{m(\varepsilon_c)}\frac{(1+\mu)\boldsymbol{\sigma} + 3\mu p\mathbf{I}}{\sigma^{\text{eff}}},$$

where

$$p = -\frac{1}{3}\boldsymbol{\sigma}:\mathbf{I}$$
$$\mathbf{s} = \boldsymbol{\sigma} + p\mathbf{I}$$
$$\sigma^{\text{eff}} = \sqrt{(1+\mu)\boldsymbol{\sigma}:\boldsymbol{\sigma} - 9\mu p^2}$$

$\sigma^{\text{eff}}$ is the effective stress. $\overline{\sigma}$ is the effective stress limit, $\mu$ is the viscous Poisson ratio, and $a$ is the reference creep strain rate. These are all input on Cards 5 and 6. The exponent $m(\varepsilon_c)$ depends on the effective creep strain as

[5] For the sake of convenience, we drop the link subscripts and superscripts, and also emphasize that the rates are to be interpreted as objective.

[6] For elastic link #3, $a = 0$, meaning there is no creep strain.

$$m(\varepsilon_c) = m_s + (m_e - m_s)\tanh\left(\frac{\varepsilon_c}{\bar{\varepsilon}_c}\right),$$

where

$$\varepsilon_c = \sqrt{\frac{1}{1+2\mu^2}\,\boldsymbol{\varepsilon}_c : \boldsymbol{\varepsilon}_c}.$$

In the above, $m_s$, $m_e$, and $\bar{\varepsilon}_c$ are input parameters (see Cards 5 and 6). $m_s$ and $m_e$ are the starting and ending exponents. $\bar{\varepsilon}_c$ is the creep strain limit.

2. **History variables.** This material model outputs 10 history variables. To output the history variables, set the variable NEIPH in *DATABASE_EXTENT_BINARY.

| History Variable # | Definition |
|---|---|
| 1 | Effective elastic strain for link 1, $\varepsilon_e^1$ |
| 2 | Effective elastic strain for link 2, $\varepsilon_e^2$ |
| 3 | Effective elastic strain for link 3, $\varepsilon_e^3$ |
| 4 | Effective creep strain for link 1, $\varepsilon_c^1$ |
| 5 | Effective creep strain for link 2, $\varepsilon_c^2$ |
| 6 | Effective exponent for link 1, $m(\varepsilon_c^1)$ |
| 7 | Effective exponent for link 2, $m(\varepsilon_c^2)$ |
| 8 | Effective von Mises stress for link 1, $\sigma_1^{\text{eff}}$ |
| 9 | Effective von Mises stress for link 2, $\sigma_2^{\text{eff}}$ |
| 10 | Effective von Mises stress for link 3, $\sigma_3^{\text{eff}}$ |

**References:**

Borrvall, T., and Lindvall, M. “A Pragmatic Approach to the Modelling of Nonlinear Rheological Networks for Polymers,” *North American LS-DYNA User Forum 2023*.

## *MAT_TNM_POLYMER

This is Material Type 318. It is for the analysis of isotropic polymers, such as thermoplastics. It works for both the explicit and implicit solvers. This keyword is supported for solid elements and some thick shell elements (ELFORM = 3, 5, and 7). However, 2D continuum elements (shell formulations 13, 14, and 15) are *not* supported for implicit.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MUA | THETAH | LAMBL | KAPPA | TAUHA | A | MA | N |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MUBI | MUBF | BETA | TAUHB | MB | MUC | Q | ALPHA |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | THETA0 | IBULK | IG | TSSTIF | GAMMA0 | | | |
| Type | F | F | F | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| MUA | Shear modulus for network A, $\mu_A$ |
| THETAH | Temperature factor, $\hat{\theta}$ |
| LAMBL | Locking stretch, $\lambda_L$ |
| KAPPA | Bulk modulus, $\kappa$ |
| TAUHA | Flow resistance of network A, $\hat{\tau}_A$ |
| A | Pressure dependence of flow, $a$ |
| MA | Stress exponential of network A, $m_A$ |
| N | Temperature exponential, $n$ |
| MUBI | Initial shear modulus for network B, $\mu_{Bi}$ |
| MUBF | Final shear modulus for network B, $\mu_{Bf}$ |
| BETA | Evolution rate of shear modulus for network B, $\beta$ |
| TAUHB | Flow resistance of network B, $\hat{\tau}_B$ |
| MB | Stress exponential of network B, $m_B$ |
| MUC | Shear modulus for network C, $\mu_C$ |
| Q | Relative contribution of $I_1$ and $I_2$ on network C, $q$ |
| ALPHA | Thermal expansion coefficient |
| THETA0 | Reference temperature, $\theta_0$ |
| IBULK | Internal bulk modulus |
| IG | Internal shear modulus |
| TSSTIF | Transversal stiffness for shells |
| GAMMA0 | Reference strain rate |

**Remarks:**

1. **Material model.** The material model is due to J. Bergström and consists of a rheologic network of three hyperelastic springs: A, B, and C. The springs act in

parallel so that the total deformation gradient ($\mathbf{F}^{\text{tot}}$), and thus the total strain, is the same for each of them. The deformation gradient is made up of both a thermal part, $\mathbf{F}^{\text{th}}$, and a mechanical part, $\mathbf{F}$, in a multiplicative manner: $\mathbf{F}^{\text{tot}} = \mathbf{F}\mathbf{F}^{\text{th}}$. The thermal part takes the form $\mathbf{F}^{\text{th}} = (1 + \alpha(\theta - \theta_0))\mathbf{I}$, with $\alpha$ as the thermal expansion coefficient, $\theta$ as the temperature, $\theta_0$ as a reference temperature, and $\mathbf{I}$ as the unit tensor. The mechanical part, $\mathbf{F}$, depends on the network.

In network A, $\mathbf{F}$ is multiplicatively decomposed into elastic and viscoplastic parts

$$\mathbf{F} = \mathbf{F}_A^e \mathbf{F}_A^v \ .$$

The Cauchy stress is defined by a temperature-dependent variant of the Arruda-Boyce eight-chain model

$$\boldsymbol{\sigma}_A = \frac{\mu_A}{J_A^e \overline{\lambda_A^e}} \left(1 + \frac{\theta - \theta_0}{\hat{\theta}}\right) \frac{\mathcal{L}^{-1}\left(\frac{\overline{\lambda_A^e}}{\lambda_L}\right)}{\mathcal{L}^{-1}\left(\frac{1}{\lambda_L}\right)} \text{dev}(\mathbf{b}_A^e) + \kappa(J_A^e - 1)\mathbf{I} \ ,$$

where $\mu_A$ is the (constant) shear modulus, and $\kappa$ is the bulk modulus. $\mathcal{L}^{-1}$ is the inverse of the Langevin function $\mathcal{L}(x) = \coth(x) - 1/x$. In the above,

$$\begin{aligned} J_A^e &= \det(\mathbf{F}_A^e) \\ \mathbf{b}_A^e &= (J_A^e)^{-\frac{2}{3}} \mathbf{F}_A^e (\mathbf{F}_A^e)^T \\ \overline{\lambda_A^e} &= \sqrt{\frac{\text{tr}(\mathbf{b}_A^e)}{3}} \end{aligned}$$

$\mathbf{b}_A^e$ is the Cauchy-Green strain tensor. $\overline{\lambda_A^e}$ is the so-called chain stretch, while $\lambda_L$ is the chain-locking stretch.

Similar to network A, the Cauchy stress for network B is given by the eight-chain model

$$\boldsymbol{\sigma}_B = \frac{\mu_B}{J_B^e \overline{\lambda_B^e}} \left(1 + \frac{\theta - \theta_0}{\hat{\theta}}\right) \frac{\mathcal{L}^{-1}\left(\frac{\overline{\lambda_B^e}}{\lambda_L}\right)}{\mathcal{L}^{-1}\left(\frac{1}{\lambda_L}\right)} \text{dev}(\mathbf{b}_B^e) + \kappa(J_B^e - 1)\mathbf{I} \ ,$$

where now

$$\begin{aligned} \mathbf{F} &= \mathbf{F}_B^e \mathbf{F}_B^v \\ \mathbf{b}_B^e &= (J_B^e)^{-\frac{2}{3}} \mathbf{F}_B^e (\mathbf{F}_B^e)^T \\ J_B^e &= \det(\mathbf{F}_B^e) \\ \overline{\lambda_B^e} &= \sqrt{\frac{\text{tr}(\mathbf{b}_B^e)}{3}} \ . \end{aligned}$$

However, unlike in network A, the shear modulus, $\mu_B$, in network B evolves with plastic strain from a starting value $\mu_{Bi}$ to a final value $\mu_{Bf}$ according to

$$\dot{\mu}_B = -\beta(\mu_B - \mu_{Bf})\dot{\gamma}_A \ ,$$

Here, $\dot{\gamma}_A$ is the viscoplastic flow rate in network A defined by

$$\dot{\gamma}_A = \dot{\gamma}_0 \left(\frac{\tau_A}{\hat{\tau}_A + aR(p_A)}\right)^{m_A} \left(\frac{\theta}{\theta_0}\right)^n \ ,$$

with pressure $p_A = -\operatorname{tr}(\boldsymbol{\sigma}_A)/3$ and von Mises-like stress $\tau_A = \sqrt{\operatorname{dev}(\boldsymbol{\sigma}_A):\operatorname{dev}(\boldsymbol{\sigma}_A)}$. $\hat{\tau}_A$ is the flow resistance, and $a$, $\beta$, $m_A$, $n$, and $\dot{\gamma}_0$ are other given material parameters. $R(x) = (x + |x|)/2$ is a ramp function.

The viscoplastic deformation gradient in network A is

$$\dot{\mathbf{F}}_A^v = \dot{\gamma}_A \mathbf{F}_A^{e\,-1} \operatorname{dev}(\boldsymbol{\sigma}_A)\mathbf{F}/\tau_A \ ,$$

and a similar relation holds for $\mathbf{F}_B^v$.

In network C, the Cauchy stress is, again, defined by a variant of the eight-chain model

$$(1+q)\boldsymbol{\sigma}_C = \frac{\mu_C}{J\bar{\lambda}}\left(1 + \frac{\theta - \theta_0}{\hat{\theta}}\right)\frac{\mathcal{L}^{-1}\left(\frac{\bar{\lambda}}{\lambda_L}\right)}{\mathcal{L}^{-1}\left(\frac{1}{\lambda_L}\right)}\operatorname{dev}(\mathbf{b}) + \kappa(J_B^e - 1)\mathbf{I} + q\frac{\mu_C}{J}\left(I_1\mathbf{b} - \frac{2I_2}{3}\mathbf{I} - \mathbf{b}^2\right) \ ,$$

where

$$\mathbf{b} = (J)^{-\frac{2}{3}}\mathbf{F}(\mathbf{F})^T$$
$$J = \det(\mathbf{F})$$
$$\bar{\lambda} = \sqrt{\frac{\operatorname{tr}(\mathbf{b})}{3}} \ .$$

$I_1$ and $I_2$ are the 1st and 2nd invariants of $\mathbf{b}$. The parameter $q$ controls the influence of these invariants. $\mu_C$ is the (constant) shear modulus.

The total stress is the sum of the stress in the network:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_A + \boldsymbol{\sigma}_B + \boldsymbol{\sigma}_C \ .$$

2. **History variables.** This material model outputs 21 history variables. To output the history variables, set the variable NEIPH in *DATABASE_EXTENT_BINARY. History variables #1-9 are the components of the viscoplastic deformation gradient, $\mathbf{F}_A^v$, in network A. Similarly, history variables #10-18 are the components of $\mathbf{F}_B^v$. History variable #19 is the shear modulus, $\mu_B$. In addition, for implicit simulations, history variables #20 and #21 are the accumulated plastic strains $\gamma_A$ and $\gamma_B$.

## *MAT_IFPD

This is Material Type 319. It is for modeling fluid particles for incompressible free surface flow with incompressible SPG. It was developed to predict the shape evolution of solder joints during the electronic reflow process. See Pan et al 2020 for details.

**WARNING:** The *MAT_319 keyword name cannot be used in the input deck in R13. For R13, you must use *MAT_IFPD as the keyword name. For releases after September 2021, *MAT_319 can be used in the input deck.

**NOTE:** This material only works for ISPG element formulations set on *SECTION_FPD.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | DYNVIS | SFTEN | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| DYNVIS | Dynamic viscosity of the fluid |
| SFTEN | Surface tension coefficient of the fluid |

**References:**

Pan, X., Wu, C.T., and Hu, W. “Incompressible Smoothed Particle Galerkin (ISPG) Method for an Efficient Simulation of Surface Tension and Wall Adhesion Effects in the 3D Reflow Soldering Process,” *16th International LS-DYNA Users Conference* (2020).

## *MAT_COHESIVE_GASKET

This is Material Type 326 developed for analysis of gaskets. This material model can only be used with cohesive elements. Also, a gasket thickness must be set; see the variable ELFORM and GASKETT in *SECTION_SOLID.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | ROFLG | INTFAIL | | | | |
| Type | A | F | F | I | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LC | UC | ETEN | | | | | |
| Type | F | F | F | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ETSR | | | | | | | |
| Type | F | | | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EMEM | PR | PS | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label be specified (see *PART). |
| RO | Mass density |

| VARIABLE | DESCRIPTION |
|---|---|
| ROFLG | Flag for whether density is specified per unit area or volume:<br>EQ.0: Density is per unit volume (default).<br>EQ.1: Density is per unit area for controlling the mass of cohesive elements with an initial volume of zero |
| INTFAIL | Quadrature rule. Note that this material has no failure<br>LE.0: 2 × 2 Newton-Cotes quadrature<br>GT.0: 2 × 2 Gaussian quadrature |
| LC | Main load curve ID defining the pressure as function of closure, $p = p(c)$ |
| UC | Table ID defining the unloading curves |
| ETEN | Tensile stiffness |
| ETSR | Transverse shear stiffness |
| EMEM | Membrane stiffness |
| PR | Membrane Poisson ratio |
| PS | Membrane plane stress or plain strain assumption:<br>EQ.0: Plane stress (default)<br>EQ.1: Plane strain |

**Remarks:**

1. **Cohesive Elements for Modeling Gaskets.** A gasket is a mechanical seal placed between two mating surfaces to prevent leakage. A gasket is typically thin in comparison to the length and width of its surface. This makes it cumbersome to model with solid elements, since these require good aspect ratios. Cohesive elements, however, are less sensitive to this kind of geometric quality and are, therefore, better suited for modeling gaskets. To use cohesive elements for modeling gaskets, the normal (the local 3-direction) of the cohesive element must be aligned with the gasket thickness direction, and the mid-surface of the cohesive element (the local 1-, and 2-direction) must coincide with the gasket surface.

2. **Material Model.** The strains pertaining to the normal are defined by: $\varepsilon_{13} = \delta_1$, $\varepsilon_{23} = \delta_2$, and $\varepsilon_{33} = \delta_3$. $\delta_i$ is the separation in local direction $i = 1,2,3$, meaning

the relative displacement between the top and bottom face of the cohesive element measured along local direction $i$. In particular, $-\varepsilon_{33}$ is the so-called gasket closure, $c$. The so-called membrane strains, $\varepsilon_{11}$, $\varepsilon_{12}$, and $\varepsilon_{22}$, in the plane orthogonal to the normal follow the usual definition $\varepsilon_{ij} = (\partial_j u_i + \partial_i u_j)/2$, where $u_i$, $i = 1,2$, is the local displacement of the mid-surface.

The cohesive gasket material model is comprised of the following three uncoupled material models:

a) Isotropic linear elastic membrane stress:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = D \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix}$$

with $D$ as the constitutive matrix for either plane strain or stress.

b) Isotropic linear elastic transverse stress:

$$\sigma_{i3} = E_{TSR}\delta_i, \qquad i = 1,2$$

c) A user defined pressure-closure relation of the form (see Remark 3):

$$\begin{aligned} -\sigma_{33} \equiv p = f(c), &\qquad c > 0 \\ \sigma_{33} = 0, &\qquad \text{otherwise} \end{aligned}$$

3. **Pressure-Closure Relation.** The pressure-closure relation, $f$, is an important feature of the material model. It can be used to include mechanical effects that are typical for gaskets, such as hysteresis. It consists of a main loading curve and one or more unloading curves.

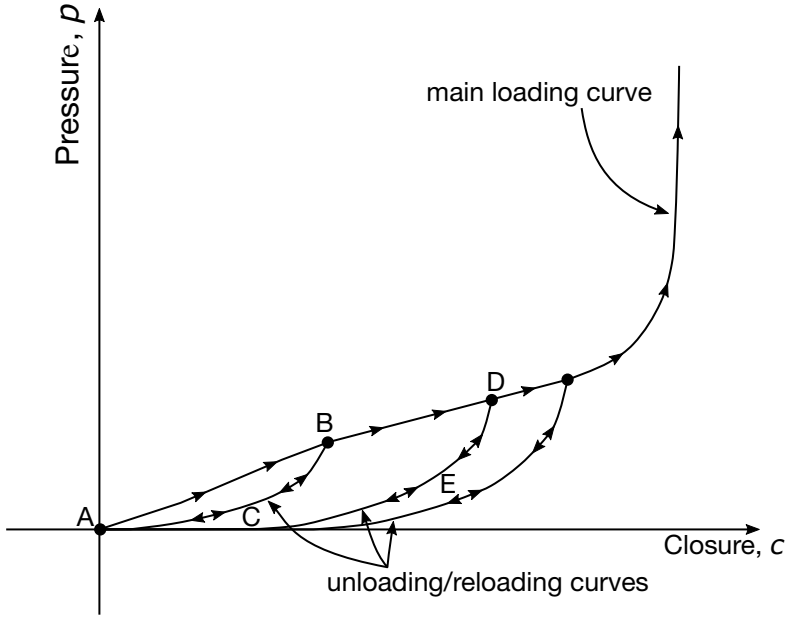



**Figure M326-1.** Schematic pressure-closure response

Figure M326-1 gives a schematic of a pressure-closure response. As the gasket is compressed the closure, $c$, increases and the pressure, $p$, follows the main loading curve from A to, say, B. Now, if the gasket for some reason is unloaded at B, $c$ will then decrease and $p$ follows the unloading curve BCA back to the initial configuration A. If the gasket is then reloaded it will follow the path ACB back to B. Then, when $c$ exceeds the closure value at point B, it will continue on the main loading curve until some new point D, where unloading takes place. The new unloading then follows the path DEA back to A. If unloading occurs between two unloading curves, interpolation is used to determine the pressure.

Which unloading path to follow and where to switch to the main loading curve is determined by keeping track of the maximum value of $c$, called $c_{\max}$, at the last unloading point. If the closure becomes negative, that is, the gasket is subject to tension rather than compression, its stiffness is given by $E_{TEN}$. This is mostly for numerical stability. The unloading curves must be input using *DEFINE_TABLE_2D, with the first dependency being maximum closure and the second closure, meaning $p = p(c_{max}, c)$. Also, the unloading curves should be in a normalized form giving zero pressure for zero closure and unit pressure for unit closure.

4. **History Variables.** This material model outputs the maximum closure as history variable #1 to the post-processing database. Therefore, NEIPH and NEIPS must be set in *DATABASE_EXTENT_BINARY.

## *MAT_IMPACT_ORTHOTROPIC

This is Material Type 331. This material is intended for capturing the behavior of composite materials subjected to hypervelocity impact. This implementation is a transition of some material models from the AMMHIS (Advanced Material Models for Hypervelocity Impact Simulations) project, originally developed in Ansys® Autodyn [1-3], to LS-DYNA.

We patterned the input for this material after *MAT_054. The input here contains many, but not all, of the same input fields. The orthotropic stiffness properties, the material axes' definition, and the failure inputs are specified in a very similar way to *MAT_054. This material has additional input parameters for failure and strength modeling.

*MAT_331 is used together with *EOS_LINEAR_POLYNOMIAL for a coupling between deviatoric and volumetric response. For failure modeling, a brittle damage model is available, meaning a maximum material stress/strain criterion (see Remark 2). Regarding strength modeling, isotropic plasticity (von Mises plasticity) is available.

*MAT_331 is supported for solid reduced integration element type (ELFORM = 1 on *SECTION_SOLID).

### Card Summary:

**Card 1.** This card is required.

| MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| GAB | GBC | GCA | | AOPT | 2WAY | | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | MANGLE | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| V1 | V2 | V3 | D1 | D2 | D3 | DFAILM | DFAILS |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| TFAIL | | | | | DFAILT | EFSTYPE | EFS |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| | XT | | YT | SC | | | |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| S_CA | S_CB | DFAILSCA | DFAILSCB | FTYPE | FMAA | FMBB | FMCC |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| FMBA | FMCA | FMCB | FSHR | FSMAX | STYPE | YLD | EFPS |
|---|---|---|---|---|---|---|---|

***Card 9.*** This card is optional.

| EROSION | | | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | EA | EB | EC | PRBA | PRCA | PRCB |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| EA | $E_a$, Young's modulus along the longitudinal direction |
| EB | $E_b$, Young's modulus along the transverse direction |
| EC | $E_c$, Young's modulus along the normal direction |
| PRBA | $\nu_{ba}$, Poisson's ratio *ba* |
| PRCA | $\nu_{ca}$, Poisson's ratio *ca* |
| PRCB | $\nu_{cb}$, Poisson's ratio *cb* |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | GBC | GCA | | AOPT | 2WAY | | |
| Type | F | F | F | | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| GAB | $G_{ab}$, shear modulus *ab* |
| GBC | $G_{bc}$, shear modulus *bc* |
| GCA | $G_{ca}$, shear modulus *ca* |
| AOPT | Material axes option (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description): |

EQ.0.0: Locally orthotropic with material axes determined by element nodes 1, 2, and 4, as with *DEFINE_COORDINATE_NODES.

EQ.1.0: Locally orthotropic with material axes determined by a point in space and the global location of the element center; this is the *a*-direction.

EQ.2.0: Globally orthotropic with material axes determined by vectors defined below, as with *DEFINE_COORDINATE_VECTOR.

EQ.3.0: Locally orthotropic material axes determined by a vector **v** and the normal vector to the plane of the element. The plane of a solid element is the midsurface between the inner surface and outer surface, defined by the first four nodes and the last four nodes of the connectivity of the element, respectively. Thus, for solid elements, AOPT = 3 is only available for hexahedra. **a** is determined by taking the cross product of **v** with the normal vector, **b** is determined by taking the cross product of the normal vector with **a**, and **c** is the normal vector. Then **a** and **b** are rotated about **c** by an angle. The angle may be set in the keyword input for the element as BETA or in the input for this keyword as MANGLE.

EQ.4.0: Locally orthotropic in cylindrical coordinate system with the material axes determined by a vector **v**, and an originating point, *p*, which define the centerline axis.

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| 2WAY | Flag to turn on two-way fiber action (see Remark 2):<br>EQ.0.0: Unidirectional behavior, meaning fibers run only in *a*-direction.<br>EQ.1.0: Two-way fiber behavior, meaning fibers run in both the *a*- and *b*-directions. The meaning of the fields DFAILM, DFAILT, XT, and YT are altered if this flag is set. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | MANGLE | |
| Type | F | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| MANGLE | Material angle in degrees for AOPT = 3. MANGLE may be overridden on the element card; see BETA on *ELEMENT_SOLID_ORTHO. |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | V1 | V2 | V3 | D1 | D2 | D3 | DFAILM | DFAILS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| V1, V2, V3 | Components of vector **v** for AOPT = 3 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2 |

| VARIABLE | DESCRIPTION |
|---|---|
| DFAILM | Maximum strain for the matrix straining in tension. For unidirectional fibers, this is in the *b*- and *c*-directions, while for two-way fibers, this is in the *c*-direciton. |
| DFAILS | Maximum shear strain in the *ab*-plane |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TFAIL | | | | | DFAILT | EFSTYPE | EFS |
| Type | F | | | | | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TFAIL | Time step size criterion for element deletion that occurs as long as EROSION = 1.0:<br>LE.0.0: No element deletion due to time step size.<br>GT.0.0.and.LE.0.1: An element is deleted when its time step is smaller than the given value.<br>GT.0.1: An element is deleted when the quotient of the actual time step and the original time step drops below the given value.<br>Note that this field is ignored if EROSION ≠ 1.0. |
| DFAILT | Maximum fiber strain under tension in the fiber longitudinal direction. A value of 1 is 100% tensile strain. If the two-way fiber flag is set, then DFAILT is the fiber tensile failure strain in the *a*- and *b*-directions; otherwise, it is only in the *a*-direction. |
| EFSTYPE | Type of effective strain:<br>EQ.1.0: Instantaneous effective strain, $\varepsilon_{\text{eq}} = \sqrt{\frac{2}{3}\lvert\boldsymbol{\varepsilon}' : \boldsymbol{\varepsilon}'\rvert}$ , where $\boldsymbol{\varepsilon}'$ is the deviatoric strain tensor.<br>EQ.2.0: Incremental effective strain, $\varepsilon_{\text{eq}}^{n+1} = \varepsilon_{\text{eq}}^{n} + \sqrt{\frac{2}{3}\mathrm{d}\boldsymbol{\varepsilon}:\mathrm{d}\boldsymbol{\varepsilon}}$ , |

| VARIABLE | DESCRIPTION |
|---|---|
| | where ε is the strain tensor. |
| EFS | Effective failure strain used when erosion is enabled with EROSION = 1.0 on Card 9. If the effective strain (see EFSTYPE) is greater than the effective failure strain, the element erodes if EROSION = 1. Note that this field is ignored if EROSION ≠ 1.0. |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | XT | | YT | SC | | | |
| Type | | F | | F | F | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XT | Tensile strength of the fibers in the fiber longitudinal direction. For unidirectional fibers (TWOWAY = 0.0), this is along the *a*-direction, while for two-way fiber behavior (TWOWAY = 1.0), this is along the *a*- and *b*-directions. |
| YT | Tensile strength of the matrix transverse to the fibers. For unidirectional fibers (TWOWAY = 0.0), this is along the *b*- and *c*-directions, while for two-way fiber behavior (TWOWAY = 1.0), this is along the *c*-direction. |
| SC | Shear strength in the *ab*-plane |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | S_CA | S_CB | DFAILSCA | DFAILSCB | FTYPE | FMAA | FMBB | FMCC |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| S_CA | Shear strength in the *ca*-plane |
| S_CB | Shear strength in the *cb*-plane |
| DFAILSCA | Maximum shear strain in the *ca*-plane |

| VARIABLE | DESCRIPTION |
|---|---|
| DFAILSCB | Maximum shear strain in the *cb*-plane |
| FTYPE | Post-failure option (see Remark 2):<br>EQ.1.0: Isotropic post-failure response. All the directions behave the same after failure; the failed elements can only carry bulk compressive stresses.<br>EQ.2.0: Orthotropic post-failure response. The three spatial directions behave differently after failure, requiring the definition of additional post-failure parameters. These parameters include the post-failure mode in each material direction and the residual shear strength of the failed material. |
| FMAA | Failure mode in the failed *a*-direction (FTYPE = 2 only):<br>EQ.1.0: Bulk<br>EQ.2.0: *aa* only |
| FMBB | Failure mode in the failed *b*-direction (FTYPE = 2 only):<br>EQ.1.0: Bulk<br>EQ.2.0: *bb* only |
| FMCC | Failure mode in the failed *c*-direction (FTYPE = 2 only):<br>EQ.1.0: Bulk<br>EQ.2.0: *cc* only |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FMBA | FMCA | FMCB | FSHR | FSMAX | STYPE | YLD | EFPS |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| FMBA | Failure mode in the failed *ba* plane (FTYPE = 2 only):<br>EQ.1.0: Bulk<br>EQ.2.0: *ba* and *aa* only |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.3.0: *ba* and *bb* only<br>EQ.4.0: *ba* and *cc* only |
| FMCA | Failure mode in the failed *ca* plane (FTYPE = 2 only):<br>EQ.1.0: Bulk<br>EQ.2.0: *ca* and *aa* only<br>EQ.3.0: *ca* and *bb* only<br>EQ.4.0: *ca* and *cc* only |
| FMCB | Failure mode in the failed *cb* plane (FTYPE = 2 only):<br>EQ.1.0: Bulk<br>EQ.2.0: *cb* and *aa* only<br>EQ.3.0: *cb* and *bb* only<br>EQ.4.0: *cb* and *cc* only |
| FSHR | Residual shear stiffness fraction; 0.0 ≤ FSHR ≤ 1.0. For the orthotropic post-failure response, the residual shear modulus is set to the specified value of FHSR times the intact shear modulus. |
| FSMAX | Maximum residual shear stress, 0.0 ≤ FSMAX ≤ 1.0e20. For the orthotropic post-failure response, the maximum shear stress allowed in a failed element is recommended to be a value less than or equal to the failure shear stress. |
| STYPE | Strength model option (see Remark 3):<br>EQ.1.0: Isotropic plasticity (von Mises yield criterion) |
| YLD | Yield stress |
| EFPS | Effective plastic strain at failure used when erosion is enabled with EROSION = 1.0 on Card 9. If the effective plastic strain is greater than the effective plastic strain at failure, the element erodes if EROSION = 1.0. Note that this field is ignored if EROSION ≠ 1.0. |

This card is optional.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | EROSION | | | | | | | |
| Type | F | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| EROSION | Flag to enable element erosion under certain conditions:<br>EQ.0.0: Elements are not eroded even if they meet at least one erosion criterion.<br>EQ.1.0: Elements are eroded when an erosion criterion is met.<br>An element erodes when at least one of the following conditions is met:<br>• The effective strain is greater than the effective strain at failure, EFS.<br>• The effective plastic strain is greater than the effective plastic strain at failure, EFPS.<br>• The time step is less than the minimum time step based on the criterion specified by TFAIL.<br>• Any of the failure model criteria mentioned in Remark 2 is reached. |

**Remarks:**

1. **Material model.** This discussion has been adapted from the Autodyn Composite Modeling Guide. For orthotropic materials, the incremental constitutive relation in component form, after splitting the strain increments into their deviatoric and volumetric parts and given that the constitutive matrix is symmetric, can be written as follows:

$$\begin{bmatrix} \Delta\sigma_{aa} \\ \Delta\sigma_{bb} \\ \Delta\sigma_{cc} \\ \Delta\sigma_{ab} \\ \Delta\sigma_{bc} \\ \Delta\sigma_{ca} \end{bmatrix} = \begin{bmatrix} C_{aa} & C_{ab} & C_{ca} & 0 & 0 & 0 \\ C_{ab} & C_{bb} & C_{bc} & 0 & 0 & 0 \\ C_{ca} & C_{bc} & C_{cc} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{ab} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{bc} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{ca} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon^{\mathrm{d}}_{aa} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{bb} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{cc} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{ab} \\ \Delta\varepsilon^{\mathrm{d}}_{bc} \\ \Delta\varepsilon^{\mathrm{d}}_{ca} \end{bmatrix}$$

where $\Delta\sigma_{aa}$, $\Delta\sigma_{bb}$, $\Delta\sigma_{cc}$, $\Delta\sigma_{ab}$, $\Delta\sigma_{bc}$, and $\Delta\sigma_{ca}$ are the components of the incremental stress vector, $\Delta\varepsilon^{\mathrm{d}}_{aa}$, $\Delta\varepsilon^{\mathrm{d}}_{bb}$, $\Delta\varepsilon^{\mathrm{d}}_{cc}$, $\Delta\varepsilon^{\mathrm{d}}_{ab}$, $\Delta\varepsilon^{\mathrm{d}}_{bc}$, and $\Delta\varepsilon^{\mathrm{d}}_{ca}$ are the components of the incremental deviatoric strain vector, and $\Delta\varepsilon_{\mathrm{vol}}$ is the volumetric strain increment. $C_{aa}$, $C_{bb}$, $C_{cc}$, $C_{ab}$, $C_{bc}$, and $C_{ca}$ are the terms of the constitutive matrix computed from the Young's moduli and Poisson's ratios. $G_{ab}$, $G_{bc}$, and $G_{ca}$ are the shear moduli, which are given directly in the input.

Restrictions apply to the elastic constants that can be used for an orthotropic material, resulting from the requirement that the sum of work done by all stress components must be positive to avoid the creation of energy. The first condition states that the elastic constants are positive:

$$E_{aa}, E_{bb}, E_{cc}, G_{ab}, G_{bc}, G_{ca} > 0$$

Second, the determinant of the stiffness matrix should be positive:

$$\Delta = 1 - \nu_{ca}\nu_{ac} - \nu_{bc}\nu_{cb} - \nu_{ab}\nu_{ba} - 2\nu_{ac}\nu_{ba}\nu_{cb} > 0$$

Finally, the requirement for positive stiffness leads to:

$$\nu_{ca} < \sqrt{\frac{E_{cc}}{E_{aa}}}, \qquad \nu_{ab} < \sqrt{\frac{E_{aa}}{E_{bb}}}, \qquad \nu_{bc} < \sqrt{\frac{E_{bb}}{E_{cc}}}$$

These conditions are tested during each cycle, with an error message thrown if any are violated.

The first three incremental stress components can be written as:

$$\Delta\sigma_{aa} = \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}}\,(C_{aa} + C_{ab} + C_{ca}) + C_{aa}\Delta\varepsilon^{\mathrm{d}}_{aa} + C_{ab}\Delta\varepsilon^{\mathrm{d}}_{bb} + C_{ca}\Delta\varepsilon^{\mathrm{d}}_{cc}$$

$$\Delta\sigma_{bb} = \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}}\,(C_{ab} + C_{bb} + C_{bc}) + C_{ab}\Delta\varepsilon^{\mathrm{d}}_{aa} + C_{bb}\Delta\varepsilon^{\mathrm{d}}_{bb} + C_{bc}\Delta\varepsilon^{\mathrm{d}}_{cc}$$

$$\Delta\sigma_{cc} = \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}}\,(C_{ca} + C_{bc} + C_{cc}) + C_{ca}\Delta\varepsilon^{\mathrm{d}}_{aa} + C_{bc}\Delta\varepsilon^{\mathrm{d}}_{bb} + C_{cc}\Delta\varepsilon^{\mathrm{d}}_{cc}$$

The pressure increment, $\Delta p$, is given as:

$$\Delta p = -\frac{1}{3}(\Delta\sigma_{aa} + \Delta\sigma_{bb} + \Delta\sigma_{cc})$$

Substituting the first three incremental stress components into the pressure increment equation, the pressure increment can be written analytically as:

$$\Delta p = -\frac{1}{9}[C_{aa} + C_{bb} + C_{cc} + 2(C_{ab} + C_{bc} + C_{ca})]\Delta\varepsilon_{\text{vol}} - \frac{1}{3}(C_{aa} + C_{ab} + C_{ca})\Delta\varepsilon^{\text{d}}_{aa} - \frac{1}{3}(C_{ab} + C_{bb} + C_{bc})\Delta\varepsilon^{\text{d}}_{bb} - \frac{1}{3}(C_{ca} + C_{bc} + C_{cc})\Delta\varepsilon^{\text{d}}_{cc}$$

The pressure increment is given in relation to the volumetric strain increment, $\Delta\varepsilon_{\text{vol}}$, and the deviatoric strain increments, $\Delta\varepsilon^{\text{d}}_{aa}$, $\Delta\varepsilon^{\text{d}}_{bb}$, and $\Delta\varepsilon^{\text{d}}_{cc}$. The last three terms constitute the *orthotropic pressure correction term*, $\Delta p_{\text{corr}}$.

In this model, the following steps are implemented:

a) *Compute pressure.* As long as bulk or directional failure has not occurred (see Remark 2), both a linear pressure increment and a total pressure update due to nonlinear shock effects are computed. For the linear pressure increment,

$$\Delta p_{\text{l}} = -k_{\text{eff}}\Delta\varepsilon_{\text{vol}} + \Delta p_{\text{corr}}$$

where the effective bulk modulus, $k_{\text{eff}}$, is:

$$k_{\text{eff}} = \frac{1}{9}[C_{aa} + C_{bb} + C_{cc} + 2(C_{ab} + C_{bc} + C_{ca})]$$

To include nonlinear effects, a total pressure update is computed using *EOS_LINEAR_POLYNOMIAL:

$$p_{\text{total}} = p_{\text{EOS}} + \sum_{n=1}^{\#\text{ of cycles}} \Delta p_{\text{corr},n}$$

Here, $\Delta p_{\text{corr},n}$ is the $n^{\text{th}}$ cycle's pressure correction.

b) *Deviatoric stress update.* Assuming that $\{\sigma_n\}_{6\times1}$ and $\{\sigma_{n+1}\}_{6\times1}$ are the (6 × 1) stress vectors from the previous and current cycles, respectively, the stress update is given as:

$$\{\sigma_{n+1}\}_{6\times1} = \{\sigma_n\}_{6\times1} + \{\Delta\sigma\}_{6\times1}$$

where $\{\Delta\sigma\}_{6\times1}$ is the (6 × 1) vector of stress increments. The subscripts that give the size of the vectors will be omitted in the following, for readability reasons.

The constitutive relation is used in its incremental form to compute the vector of total stress increments $\{\Delta\sigma\}$ as follows:

$$\{\Delta\boldsymbol{\sigma}\} = [\mathbf{C}]\{\Delta\boldsymbol{\varepsilon}\}$$

where $[\mathbf{C}]$ is the (6 × 6) constitutive matrix that is computed based on the orthotropic material properties and $\{\Delta\boldsymbol{\varepsilon}\}$ is the vector of strain increments given as:

$$\{\Delta\boldsymbol{\varepsilon}\} = \begin{Bmatrix} \Delta\boldsymbol{\varepsilon}_{\text{normal}} \\ \Delta\boldsymbol{\varepsilon}_{\text{shear}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{d}_{\text{normal}} \\ \mathbf{d}_{\text{shear}} \end{Bmatrix} \mathrm{d}t$$

where $\begin{Bmatrix} \mathbf{d}_{\text{normal}} \\ \mathbf{d}_{\text{shear}} \end{Bmatrix}$ are the (3 × 1) vector components of the (6 × 1) vector of rate of deformation. A factor of 0.5 is used for the shear terms $\{\mathbf{d}_{\text{shear}}\}$.

Using the linear increment for pressure, the normal components of the deviatoric stress vector increments, $\Delta s_{ii}$, with $i = a, b, c$, are given as:

$$\Delta s_{ii} = \Delta\sigma_{ii} + \Delta p_1 \qquad \text{with } i = a, b, c$$

while the deviatoric shear stress components are equal to the corresponding total shear stress terms

$$\Delta s_{ij} = \Delta\sigma_{ij} \qquad \text{with } i, j = a, b, c \text{ and } i \neq j$$

Thus, the update of the deviatoric stress vector is:

$$\{\mathbf{s}_{n+1}\} = \{\mathbf{s}_n\} + \{\Delta\mathbf{s}\}$$

c) *Total stress update.* As long as bulk or directional failure has not occurred (see Remark 2), using the total pressure update and the deviatoric stress vector update, the total stress vector is given as:

$$\sigma_{ii,n+1} = s_{ii,n+1} - p_{\text{total},n+1}$$
$$\sigma_{ij,n+1} = s_{ij,n+1}$$

with $i, j = a, b, c$ and $i \neq j$.

d) *Energy update.* When no bulk or directional failure has occurred (see Remark 2), the nonlinear part of the work done by pressure results in the shock energy, $E_{\text{pnl},n+1}$, that appears in the computation of internal energy:

$$E_{\text{pnl,n+1}} = E_{\text{p},n+1} - E_{\text{plin,n+1}}$$

$E_{\text{p},n+1}$ is the internal energy at the current cycle modified by the pressure work:

$$E_{\text{p},n+1} = E_n - 0.5(V_{n+1} - V_n)\,[(p_{\text{EOS},n} + q_n) + (p_{\text{EOS},n+1} + q_{n+1})]$$

Here, $E_n$ is the internal energy at cycle $n$, $V_n$ is the volume at cycle $n$, and $V_{n+1}$ is the volume at cycle $n + 1$, respectively. $p_{\text{EOS},n}$, $q_n$, $p_{\text{EOS},n+1}$, and $q_{n+1}$ are the EOS pressures and bulk viscosities at cycles $n$ and $n + 1$.

$E_{\text{plin,n+1}}$ is the linear counterpart of $E_{\text{p},n+1}$, given by:

$$E_{\text{plin},n+1} = E_n + \Delta E_{\text{plin}}$$

where $\Delta E_{\text{plin}}$ is the internal energy change due to the linear part of pressure work, given by:

$$\Delta E_{\text{plin}} = -0.5\left[p_{\text{total},n} + k_{\text{eff}}\left(\frac{V_0}{V_{n+1}} - 1\right)\right](V_{n+1} - V_n)$$

The internal energy, $E_{n+1}$, depends on the work done by both pressure and stress in a cycle, as follows

$$E_{n+1} = E_n + 0.5\,(V_{n+1} + V_n)\,\Delta u + E_{\text{pnl},n+1}$$

$\Delta u$ is the specific strain energy increment, meaning the strain energy increment per unit volume, given as:

$$\Delta u = 0.5[\mathbf{d} \cdot (\boldsymbol{\sigma}_{n+1} + \boldsymbol{\sigma}_n)]\,\mathrm{d}t$$

If failure occurs, $E_{\text{pnl},n+1} = 0$, and the internal energy is given by:

$$E_{n+1} = E_{\text{plin},n+1} + 0.5\,(V_{n+1} + V_n)\,\Delta u - 0.5\,(q_n + q_{n+1})(V_{n+1} - V_n)$$

2. **Brittle damage model.** For brittle materials, the orthotropic material stress/strain failure initiation model is available [1]. This model is useful for materials that are likely to fail along predefined material planes: for instance, where failure occurs, parallel to the interface, between two layers in a laminated plate.

For these failure criteria, the following assumptions are made:

- The main failure modelling assumption is that even failed material will continue to resist compressive loads. Thus, compressive failure is not taken into consideration.
- If the tensile stress or strain exceeds the maximum stress or strain limits at one integration point, a failure flag is defined for this integration point. Because this material is currently only available for reduced-order integration solid elements, where only one integration point is used, a failure flag at one integration point means a failure flag for the element.
- This model allows different tensile and shear failure stresses and strains for each of the principal (material) directions.

In the case of strain failure:

- For one-way fibers, DFAILT is the tensile strain limit used for the fiber *a*-direction, while DFAILM is the tensile strain limit used for the matrix in the *b*- and *c*-directions.
- For two-way fibers, DFAILT is the tensile strain limit used for the fiber *a*- and *b*-directions, while DFAILM is the tensile strain limit used for the remaining matrix *c*-direction.
- Three shear strain limits exist: DFAILS, DFAILSCB, and DFAILSCA.

In the case of stress failure:

- For one-way fibers, XT is the tensile stress limit used for the fiber *a*-direction, while YT is the tensile stress limit used for the matrix in the *b*- and *c*-directions.
- For two-way fibers, XT is the tensile stress limit used for the fiber *a*- and *b*-directions, while YT is the tensile stress limit used for the remaining matrix *c*-direction.
- Three shear stress limits exist: SC, S_CB, and S_CA.

Failure is initiated if any of the principal material stresses or strains exceed their respective failure levels. For shear failures, the shear stresses and strains on planes parallel to the principal directions are checked against the inputted maximum shear stresses and strains.

The failure data input includes an option to use isotropic (FTYPE = 1) or orthotropic (FTYPE =2) post-failure response.

a) *Isotropic post-failure response.* If the isotropic post-failure option is selected, failed elements can only carry bulk compressive stresses. In this case, after failure is initiated in an element, the following occurs:

   i) All shear moduli are set to zero.

   ii) The orthotropic elastic incremental stress-strain relations are applied.

   iii) The average stress, i.e., pressure, is recomputed, using the normal calculation:

$$p_{\text{total}} = -\frac{1}{3}(\sigma_{aa} + \sigma_{bb} + \sigma_{cc})$$

   iv) If the element is in compression, the principal stresses are set equal to the average stress, i.e. pressure:

$$\sigma_{ii} = -p_{\text{total}}$$

   If the element is in tension, all principal stresses, and therefore, the average stress, i.e., pressure, are set to zero:

$$\sigma_{ii} = -p_{\text{total}} = 0$$

   v) Shear stress components are set to zero.

   vi) Any tensile stresses are set to zero for completely failed directions.

b) *Orthotropic post-failure response.* The post-failure behavior of laminated composite materials is better represented by the orthotropic post-failure option. This option was developed specifically to simulate performance and failure, including delamination in fiber-reinforced composite materials.

If the orthotropic option is selected, you define several post-failure parameters, including the post-failure response mode for failure in each material direction and the failed material's residual shear strength.

For all failure modes, on failure initiation, the stress in the failed material directions is set to zero. In addition, unless bulk failure is selected, the stresses in material directions orthogonal to the failed direction are reduced to account for the loss in the Poisson effect from strain in the failed direction. In subsequent cycles, element tensile stresses are only allowed in non-failed directions and if an element fails in two or more material directions, bulk failure is assumed. If bulk failure is selected, the tensile stresses in all materials directions are set to zero if the material fails in the specified material direction or plane.

Following failure initiation, the stiffness and strength properties of the failed element are modified depending on the failure initiation modes described below.

i) *Delamination.* In an axisymmetric simulation, the *cc*-direction is assumed to be through the thickness of the laminate, and the *bb*-direction is the hoop direction. Delamination can result from excessive through-thickness tensile stresses and/or strains or from excessive shear stress and/or strain in the *ca*-plane. If failure is initiated in either of these two modes, the stress in the *cc*-direction is instantaneously set to zero, and the strain in the *cc*-direction at failure is stored. Subsequently, if the tensile material strain in the *cc*-direction exceeds the failure strain, the material stiffness matrix is modified as

$$\begin{bmatrix} \Delta\sigma_{aa} \\ \Delta\sigma_{bb} \\ 0 \\ \Delta\sigma_{ab} \\ \Delta\sigma_{bc} \\ \Delta\sigma_{ca} \end{bmatrix} = \begin{bmatrix} C_{aa} & C_{ab} & 0 & 0 & 0 & 0 \\ C_{ab} & C_{bb} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha G_{ab} & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha G_{bc} & 0 \\ 0 & 0 & 0 & 0 & 0 & aG_{ca} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon^{\mathrm{d}}_{aa} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{bb} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{cc} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{ab} \\ \Delta\varepsilon^{\mathrm{d}}_{bc} \\ \Delta\varepsilon^{\mathrm{d}}_{ca} \end{bmatrix}$$

This stiffness modification does not allow tensile through-thickness stresses, while tensile in-plane stresses are maintained. Also note that the delamination will, in practice, be associated with a reduction in shear stiffness. Often, in the absence of appropriate material

data, a nominal value of 20% is typically used for the residual shear stiffness.

ii) *In-plane failure*. In an axisymmetric simulation, the *aa*- and *bb*-directions are assumed to be in the plane of the composite (in other words, in fiber directions). If failure is initiated in these two modes, the stress in the failed direction is instantaneously set to zero, and the strain in the failed direction at failure is stored. Subsequently, if the tensile material strain in the failed direction exceeds the failure strain, the material stiffness matrix is modified as

$$\begin{bmatrix} 0 \\ \Delta\sigma_{bb} \\ \Delta\sigma_{cc} \\ \Delta\sigma_{ab} \\ \Delta\sigma_{bc} \\ \Delta\sigma_{ca} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{bb} & C_{bc} & 0 & 0 & 0 \\ 0 & C_{bc} & C_{cc} & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha G_{ab} & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha G_{bc} & 0 \\ 0 & 0 & 0 & 0 & 0 & aG_{ca} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon^{\mathrm{d}}_{aa} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{bb} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{cc} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{ab} \\ \Delta\varepsilon^{\mathrm{d}}_{bc} \\ \Delta\varepsilon^{\mathrm{d}}_{ca} \end{bmatrix}$$

for *aa*-failure, and

$$\begin{bmatrix} \Delta\sigma_{aa} \\ 0 \\ \Delta\sigma_{cc} \\ \Delta\sigma_{ab} \\ \Delta\sigma_{bc} \\ \Delta\sigma_{ca} \end{bmatrix} = \begin{bmatrix} C_{aa} & 0 & C_{ca} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ C_{ca} & 0 & C_{cc} & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha G_{ab} & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha G_{bc} & 0 \\ 0 & 0 & 0 & 0 & 0 & aG_{ca} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon^{\mathrm{d}}_{aa} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{bb} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{cc} + \frac{1}{3}\Delta\varepsilon_{\mathrm{vol}} \\ \Delta\varepsilon^{\mathrm{d}}_{ab} \\ \Delta\varepsilon^{\mathrm{d}}_{bc} \\ \Delta\varepsilon^{\mathrm{d}}_{ca} \end{bmatrix}$$

for *bb*-failure.

This stiffness modification does not allow tensile stresses in the failed directions. Also note that these failure modes will, in practice, be associated with a reduction in shear stiffness. Often, in the absence of appropriate material data, a nominal value of 20% is typically used for the residual shear stiffness.

iii) *Combined failure.* The combined effect of failure in all three material directions is represented by a change in the material stiffness and

strength to isotropic with no stress deviators and no tensile material stresses.

3. **Plasticity modeling assumptions.** The von Mises strength model is used. The orthotropic material will initially respond elastically to loading until a stress state exceeds the constant von Mises yield value. Then the material starts to deform plastically until failure is reached. For the computation of effective plastic strain, the shear modulus used is the average of the three shear moduli along the three material axes.

4. **History variables.** This material has the following history variables. Note that NEIPH on *DATABASE__EXTENT_BINARY specifies the number of additional history variables output for solid elements.

| History Variable # | Description |
|---|---|
| 1 | Internal energy |
| 2 | Bulk viscosity |
| 3 | Volume |
| 4 | Orthotropic correction term in the *a*-direction |
| 5 | Orthotropic correction term in the *b*-direction |
| 6 | Orthotropic correction term in the *c*-direction |
| 7 | Material status in the *a*-direction:<br>EQ.0: failed<br>EQ.1: elastic<br>EQ.2: plastic |
| 8 | Material status in the *b*-direction:<br>EQ.0: failed<br>EQ.1: elastic<br>EQ.2: plastic |
| 9 | Material status in the *c*-direction:<br>EQ.0: failed<br>EQ.1: elastic<br>EQ.2: plastic |

| History Variable # | Description |
|---|---|
| 10 | Material status in the *ba*-plane:<br>EQ.0: failed<br>EQ.1: elastic<br>EQ.2: plastic |
| 11 | Material status in the *ca*-plane:<br>EQ.0: failed<br>EQ.1: elastic<br>EQ.2: plastic |
| 12 | Material status in the *cb*-plane:<br>EQ.0: failed<br>EQ.1: elastic<br>EQ.2: plastic |
| 13 | Bulk failure flag:<br>EQ.-1: total bulk resistance<br>EQ.0: no bulk resistance<br>EQ.1: only compressive bulk resistance |

**References:**

[1] Autodyn Composite Modeling Guide, Autodyn 2026R1.

[2] R.A. Clegg, D.M. White, W. Riedel, W. Harwick (2006). "Hypervelocity impact damage prediction in composites: Part I – material model and characterization". *International Journal of Impact Engineering* 33 (1-12), pp. 190-200. URL: https://doi.org/10.1016/j.ijimpeng.2006.09.055.

[3] W. Riedel, H. Nahme, D.M. White, R.A. Clegg (2006). "Hypervelocity impact damage prediction in composites: Part II – experimental investigations and simulations". *International Journal of Impact Engineering* 33 (1-12), pp. 670-680. URL: https://doi.org/10.1016/j.ijimpeng.2006.09.052.

## *MAT_ALE_VACUUM

See *MAT_VACUUM or *MAT_140.

***MAT_ALE_GAS_MIXTURE**_{OPTION}

Available settings of *OPTION* are:

<BLANK>

ADV

With *OPTION1* set to <BLANK>, this keyword is exactly the same as *MAT_GAS_MIXTURE or *MAT_148. See that manual page for the input format,

The keyword format described below is only for *OPTION1* set to ADV. It is exclusively used as a component of *AIRBAG_SALE. It was created to support more than 8 gas species.

**Data Cards for the ADV Keyword Option Only:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | IADIAB | RUNIV | NSPS | | | | |
| Type | A | I | F | I | | | | |
| Default | none | 0 | 0.0 | none | | | | |

Include NSPS instantiations of this card, one for each gas species.

| Cards 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MOLWT | CPMOL | B | C | | | | |
| Type | F | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified. |
| IADIAB | Flag to turn on/off adiabatic compression logic for an ideal gas. See Remark 5 of *MAT_148.<br>EQ.0: Off (default) |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.1: On |
| RUNIV | Universal gas constant in per-mole unit (8.31447 J/(mole × K)). |
| NSPS | Number of gas species |
| MOLWT | Molecular weight of each ideal gas in the mixture (mass-unit/mole). |
| CPMOL | Heat capacity at constant pressure in per-mole unit. These are nominal heat capacity values typically at STP. These are denoted by the variable *A* in the equation in Remark 2 of *MAT_148. |
| B | First-order coefficient for a temperature-dependent heat capacity at constant pressure. These are denoted by the variable *B* in the equation in Remark 2 of *MAT_148. |
| C | Second-order coefficient for a temperature-dependent heat capacity at constant pressure. These are denoted by the variable *C* in the equation in Remark 2 of *MAT_148. |

**Remarks:**

1. **Element energy update.** For the ADV keyword option, pressure work always gives the element energy update. Without the keyword option, the ideal gas gamma law, by default, gives the element energy update. However, this method can be changed to pressure work by setting PDV to 1. Please refer to Remark 6 of *MAT_GAS_MIXTURE for explanations.

## *MAT_ALE_VISCOUS

This may also be referred to as MAT_ALE_03. This "fluid-like" material model is very similar to Material Type 9 (*MAT_NULL). It allows the modeling of non-Newtonian fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. If inviscid material is modeled, the deviatoric or viscous stresses are zero, and the equation of state supplies the pressures (or diagonal components of the stress tensor). All *MAT_ALE_ cards apply only to ALE elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | | RN |
| Type | A | F | F | F | F | F | | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0). See Remark 4. |
| MULO | Dynamic viscosity (see Remark 1):<br>EQ.0.0: Inviscid fluid is assumed.<br>GT.0.0: If MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient, $\mu$. Otherwise if MUHI > 0.0, then MULO and MUHI are the lower and upper dynamic viscosity limit values for a power-law-like variable viscosity model.<br>LT.0.0: -MULO is a load curve ID defining dynamic viscosity as a function of equivalent strain rate. |
| MUHI | Dynamic viscosity:<br>EQ.0.0: Only MULO is used to define the dynamic viscosity, default<br>LT.0.0: The viscosity can be defined by the user in the file dyn21.F with a routine called f3dm9ale_userdef1. The |

| VARIABLE | DESCRIPTION |
|---|---|
| | file is part of the general usermat package. Note that in this case MULO is a parameter for the subroutine. |
| | GT.0.0: This is the upper dynamic viscosity limit. This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | Variable dynamic viscosity multiplier. See Remark 6. |
| RN | Variable dynamic viscosity exponent. See Remark 6. |

**Remarks:**

1. **Deviatoric viscous stress.** The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity. For example, in SI unit system, $\mu$ has a unit of [Pa × s].

2. **Hourglass control issues.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient QM should be small (in the range $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Null material properties.** Null material has no yield strength and behaves in a fluid-like manner.

4. **Numerical cavitation.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with small values of viscosity exponent.** If the viscosity exponent is less than 1.0 (RN $< 1.0$), then RN $- 1.0 < 0.0$. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MUHI is then used as the viscosity value.

6. **Empirical dynamic viscosity.** The empirical variable dynamic viscosity is typically modeled as a function of *equivalent shear rate* based on experimental data.

$$\mu(\dot{\bar{\gamma}}') = \mathrm{RK} \times \dot{\bar{\gamma}}'^{(\mathrm{RN}-1)}$$

For an incompressible fluid, this may be written equivalently as

$$\mu(\dot{\bar{\varepsilon}}') = \mathrm{RK} \times \dot{\bar{\varepsilon}}'^{(\mathrm{RN}-1)}$$

The "overbar" denotes a scalar equivalence. The "dot" denotes a time derivative or rate effect. And the "prime" symbol denotes deviatoric or volume preserving components. The *equivalent shear rate* components may be related to the basic definition of (small-strain) strain rate components as follows:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \Rightarrow \dot{\varepsilon}'_{ij} = \dot{\varepsilon}_{ij} - \delta_{ij}\left(\frac{\dot{\varepsilon}_{kk}}{3}\right)$$
$$\dot{\gamma}_{ij} = 2\dot{\varepsilon}_{ij}$$

Typically, the 2nd invariant of the deviatoric strain rate tensor is defined as:

$$I_{2\dot{\varepsilon}'} = \frac{1}{2}\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right]$$

The equivalent (small-strain) deviatoric strain rate is defined as:

$$\dot{\bar{\varepsilon}}' \equiv 2\sqrt{I_{2\dot{\varepsilon}'}} = \sqrt{2\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right]} = \sqrt{4\left[\dot{\varepsilon}'_{12}{}^2 + \dot{\varepsilon}'_{23}{}^2 + \dot{\varepsilon}'_{31}{}^2\right] + 2\left[\dot{\varepsilon}'_{11}{}^2 + \dot{\varepsilon}'_{22}{}^2 + \dot{\varepsilon}'_{33}{}^2\right]}$$

In non-Newtonian literatures, the *equivalent shear rate* is sometimes defined as

$$\dot{\bar{\gamma}} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{4[\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{31}^2] + 2[\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2]}$$

It turns out that, (a) for incompressible materials ($\dot{\varepsilon}_{kk} = 0$), and (b) the shear terms are equivalent when $i \neq j \rightarrow \dot{\varepsilon}_{ij} = \dot{\varepsilon}'_{ij}$, the *equivalent shear rate* is algebraically equivalent to the *equivalent (small-strain) deviatoric strain rate*.

$$\dot{\bar{\varepsilon}}' = \dot{\bar{\gamma}}'$$

## *MAT_ALE_MIXING_LENGTH

This may also be referred to as *MAT_ALE_04. This viscous "fluid-like" material model is an advanced form of *MAT_ALE_VISCOUS. It allows the modeling of fluid with constant or variable viscosity and a *one-parameter mixing-length turbulence model*. The variable viscosity is a function of an equivalent deviatoric strain rate. The equation of state supplies the pressures for the stress tensor. All *MAT_ALE_cards apply only to ALE elements.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | | RN |
| Type | A8 | F | F | F | F | F | | F |
| Defaults | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | 0.0 |

**Internal Flow Card.**

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCI | C0 | C1 | C2 | C3 | C4 | C5 | C6 |
| Type | F | F | F | F | F | F | F | F |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

**External Flow Card.**

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCX | D0 | D1 | D2 | E0 | E1 | E2 | |
| Type | F | F | F | F | F | F | F | |
| Defaults | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0) |
| MULO | Dynamic viscosity:<br>GE.0.0: if MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient, $\mu$. Otherwise if MUHI > 0.0, then MULO and MUHI are the lower and upper dynamic viscosity limit values.<br>LT.0.0: -MULO is a load curve ID defining dynamic viscosity as a function of equivalent strain rate. |
| MUHI | Upper dynamic viscosity limit (default = 0.0) if MULO > 0. This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | Variable dynamic viscosity multiplier (see Remark 6 of MAT_ALE_VISCOUS). The viscosity is computed as $\mu(\overline{\dot{\varepsilon}'}) = \text{RK} \times \overline{\dot{\varepsilon}'}^{(\text{RN}-1)}$, where the equivalent deviatoric strain rate is $\overline{\dot{\varepsilon}'} = \sqrt{\frac{2}{3}\left[\dot{\varepsilon}'_{11}{}^2 + \dot{\varepsilon}'_{22}{}^2 + \dot{\varepsilon}'_{33}{}^2 + 2\left(\dot{\varepsilon}'_{12}{}^2 + \dot{\varepsilon}'_{23}{}^2 + \dot{\varepsilon}'_{31}{}^2\right)\right]}$ . |
| RN | Variable dynamic viscosity exponent (see RK) |
| LCI | Characteristic length, $l_{ci}$, of the internal turbulent domain |
| C0 - C6 | Internal flow mixing length polynomial coefficients. The one-parameter turbulent mixing length is computed as $l_m = l_{ci}\left[C_0 + C_1\left(1 - \frac{y}{l_{ci}}\right) + \cdots + C_6\left(1 - \frac{y}{l_{ci}}\right)^6\right]$ |
| LCX | Characteristic length, $l_{cx}$, of the external turbulent domain |
| D0 - D2 | External flow mixing length polynomial coefficients. If $y \le l_{cx}$, then the mixing length is computed as $l_m = \left[D_0 + D_1 y + D_2 y^2\right]$. |
| E0 - E2 | External flow mixing length polynomial coefficients. If $y > l_{cx}$, then the mixing length is computed as $l_m = \left[E_0 + E_1 y + E_2 y^2\right]$. |

**Remarks:**

1. **Deviatoric Viscous Stress.** The null material must be used with an equation of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = \mu\dot{\varepsilon}'_{ij}$$
$$\left[\frac{N}{m^2}\right] \approx \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity with unit of [Pa × s].

2. **Hourglass Control Issues.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general for fluid(s), the hourglass coefficient QM should be small (in the range $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Null Material Properties.** The null material has no yield strength and behaves in a fluid-like manner.

4. **Numerical Cavitation.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with Small Value of Viscosity Exponent.** If the viscosity exponent is less than 1.0 (RN $< 1.0$) then RN $- 1.0 < 0.0$. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Turbulent Viscosity.** Turbulence is treated simply by considering its effects on viscosity. Total effective viscosity is the sum of the laminar and turbulent viscosities, $\mu_{\text{eff}} = \mu_l + \mu_t$ where $\mu_{\text{eff}}$ is the effective viscosity, and $\mu_t$ is the turbulent viscosity. The turbulent viscosity is computed based on the Prandtl's Mixing Length Model,

$$\mu_t = \rho l_m^2 |\nabla \mathbf{v}| \ .$$

## *MAT_ALE_INCOMPRESSIBLE

See *MAT_160.

## *MAT_ALE_HERSCHEL

This may also be referred to as MAT_ALE_06. This is the Herschel-Buckley model. It is an enhancement to the power law viscosity model in *MAT_ALE_VISCOUS (*MAT_-ALE_03). Two additional input parameters, the yield stress threshold and critical shear strain rate, can be specified to model "rigid-like" material for low strain rates.

It allows the modeling of non-viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. All *MAT_ALE_ cards apply only to ALE element formulation.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | | RN |
| Type | A | F | F | F | F | F | | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | | 0.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GDOTC | TAO0 | | | | | | |
| Type | F | F | | | | | | |
| Default | none | none | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0); see Remark 4. |
| MULO | There are 4 possible cases (see Remark 1):<br>1. If MULO = 0.0, then an inviscid fluid is assumed.<br>2. If MULO > 0.0, and MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient $\mu$. |

| VARIABLE | DESCRIPTION |
|---|---|
| | 3. If MULO > 0.0, and MUHI > 0.0, then MULO and MUHI are lower and upper viscosity limit values for a power-law-like variable viscosity model.<br>4. If MULO is negative (for example, MULO = -1), then a user-input data load curve (with LCID = 1) defining dynamic viscosity as a function of equivalent strain rate is used. |
| MUHI | Upper dynamic viscosity limit (default = 0.0). This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | $k$, consistency factor (see Remark 6) |
| RN | $n$, power law index (see Remark 6) |
| GDOTC | $\dot{\gamma}_c$, critical shear strain rate (see Remark 6) |
| TAO0 | $\tau_0$, yield stress (see Remark 6) |

**Remarks:**

1. **Viscous stress.** The null material must be used with an equation-of-state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity. For example, in SI unit system, $\mu$ has a unit of [Pa-s].

2. **Hourglass control.** The null material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general, for fluid(s), the hourglass coefficient QM should be small (in the range of $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Yield strength.** Null material has no yield strength and behaves in a fluid-like manner.

4. **Pressure cut-off.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above a certain magnitude, it should no longer be able to resist this

dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Viscosity.** If the viscosity exponent is less than 1.0, $RN < 1.0$, then $RN - 1.0 < 0.0$. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Herschel-buckley model.** The Herschel-Buckley model employs a large viscosity to model the "rigid-like" behavior for low shear strain rates ($\dot{\gamma} < \dot{\gamma}_c$).

$$\mu(\dot{\gamma}) = \tau_0 \frac{(2-\dot{\gamma}/\dot{\gamma}_c)}{\dot{\gamma}_c} + k[(2-n) + (n-1)\frac{\dot{\gamma}}{\dot{\gamma}_c}]$$

A power law is used once the yield stress is passed.

$$\mu(\dot{\gamma}) = \frac{\tau_0}{\dot{\gamma}} + k(\frac{\dot{\gamma}}{\dot{\gamma}_c})^{n-1}$$

The shear strain rate is:

$$\dot{\gamma} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{4[\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{31}^2] + 2[\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2]}$$

### *MAT_ISPG_CARREAU

This is Material Type 1 for ISPG. The Carreau model attempts to describe a wide range of fluids by establishing a curve-fit to piece together functions for both Newtonian and shear-thinning ($n < 1$) non-Newtonian laws.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | VISC0 | SFTEN | VISC_LIM | LAMBDA | N | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| VISC0 | Zero-shear viscosity of the fluid, $\eta_0$ |
| SFTEN | Surface tension coefficient of the fluid |
| VISC_LIM | Infinite-shear viscosity, $\eta_\infty$ |
| LAMBDA | Time constant, $\lambda$ |
| N | Power-law index, $n$ |
| ALPHA | Ratio of the activation energy to thermodynamic constant |

| VARIABLE | DESCRIPTION |
|---|---|
| TREF | Reference temperature in Kelvin, $T_\alpha$. The defaults is 273.15 K. |

**Remarks:**

1. **Viscosity.** In the Carreau model, the viscosity is described as:

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)(1 + \dot{\gamma}^2\lambda^2)^{(n-1)/2}$$

where $\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}}$ is the second invariant of the rate-of-deformation tensor $\mathbf{D} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial j}\right)\mathbf{e}_i \otimes \mathbf{e}_j$, $\lambda$ is a time constant, $n$ is the power-law index, and $\eta_0$ and $\eta_\infty$ are the zero- and infinite-shear viscosities, respectively. The total viscosity is calculated as

$$\mu = H(T)\eta(\dot{\gamma}) \ ,$$

where $H(T)$ is the temperature dependence. It is described by an Arrhenius law as:

$$H(T) = \exp\left[\alpha\left(\frac{1}{T - T_0} - \frac{1}{T_\alpha - T_0}\right)\right] \ .$$

where $\alpha$ is the ratio of the activation energy to the thermodynamic constant. $T_\alpha$ is a reference temperature in Kelvin with a default value of 273.15 K. $T_0$ is the temperature shift in Kelvin. It is hard-coded as 0 K. If the parameter $\alpha$ is set to 0, the temperature dependence will be ignored because $H(T) = 1$.

2. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

### *MAT_ISPG_CROSSMODEL

This is Material Type 2 for ISPG. The Cross model attempts to describe the shear-rate dependence across the Newtonian region and the shear-thinning region.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | VISC0 | SFTEN | | LAMBDA | n | |
| Type | A | F | F | F | | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| VISC0 | Zero-shear viscosity of the fluid, $\eta_0$ |
| SFTEN | Surface tension coefficient of the fluid |
| LAMBDA | Natural time. $\lambda$ |
| N | Power-law index, $n$ |
| ALPHA | Ratio of the activation energy to thermodynamic constant, $\alpha$ |
| TREF | Reference temperature in Kelvin, $T_\alpha$. The default value is 273.15 K. |

**Remarks:**

1. **Viscosity.** The Cross model describes the viscosity as:

$$\eta = \frac{\eta_0}{1 + (\lambda\dot{\gamma})^{1-n}}$$

where $\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}}$ is the second invariant of the rate-of-deformation tensor $\mathbf{D} = \frac{1}{2}\left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial j}\right)\mathbf{e}_i \otimes \mathbf{e}_j$ , $\lambda$ is a time constant, $n$ is the power-law index, and $\eta_0$ is the zero-shear-rate viscosity. The total viscosity is calculated as

$$\mu = H(T)\eta(\dot{\gamma})$$

where $H(T)$ is the temperature dependence. It is described by an Arrhenius law as:

$$H(T) = \exp\left[\alpha\left(\frac{1}{T - T_0} - \frac{1}{T_\alpha - T_0}\right)\right]$$

where $\alpha$ is the ratio of the activation energy to the thermodynamic constant. $T_\alpha$ is a reference temperature in Kelvin with a default value of 273.15 K. $T_0$ is the temperature shift in Kelvin. It is hardcoded as 0 K. If the parameter $\alpha$ is set to 0, the temperature dependence will be ignored because $H(T) = 1$.

2. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

## *MAT_ISPG_ISO_NEWTONIAN

This is Material Type 3 for ISPG. This material type models the Newtonian flow behavior of an incompressible free surface flow. We developed it to predict the shape evolution of solder joints during the electronic reflow process. See Pan et al. 2020 for details.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | DYNVIS | SFTEN | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHA | TREF | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| DYNVIS | Dynamic viscosity of the fluid, $\eta$ |
| SFTEN | Surface tension coefficient of the fluid |
| ALPHA | Ratio of the activation energy to thermodynamic constant, $\alpha$ |
| TREF | Reference temperature in Kelvin, $T_\alpha$. The default value is 273.15 K. |

**Remarks:**

1. **Viscosity.** The total viscosity is calculated as

$$\mu = H(T)\eta$$

where $H(T)$ is the temperature dependence. It is described by an Arrhenius law as:

$$H(T) = \exp\left[\alpha\left(\frac{1}{T - T_0} - \frac{1}{T_\alpha - T_0}\right)\right]$$

where $\alpha$ is the ratio of the activation energy to the thermodynamic constant. $T_\alpha$ is a reference temperature in Kelvin with a default value of 273.15 K. $T_0$ is the temperature shift in Kelvin. It is hardcoded as 0 K. If the parameter $\alpha$ is set to 0, the temperature dependence will be ignored because $H(T) = 1$.

2. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

**References:**

Pan, X., Wu, C.T., and Hu, W. "Incompressible Smoothed Particle Galerkin (ISPG) Method for an Efficient Simulation of Surface Tension and Wall Adhesion Effects in the 3D Reflow Soldering Process," 16th International LS-DYNA Users Conference (2020).

## *MAT_ISPG_CROSS_CASTRO_MACOSKO

This is Material Type 4 for ISPG. The Cross Castro Macosko model attempts to describe the effects of shear rate, temperature, and degree of cure on the viscosity of a reactive fluid. The degree of cure is described by the Kama-Sourour model, which is the most used model to describe the curing kinetics of thermoset compounds.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | B | SFTEN | TAU | N | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TREF | | | | | | | |
| Type | F | | | | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | ALPHAG | C1 | C2 | | | | | |
| Type | F | F | F | | | | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | M_KS | N_KS | A1 | A2 | T1 | T2 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| B | Coefficient for calculating the zero-shear viscosity of the fluid, $B$ |
| SFTEN | Surface tension coefficient of the fluid |
| TAU | Shear stress at the transition from Newtonian to non-Newtonian flow, $\tau^*$ |
| N | Power-law index, $n$ |
| TREF | Reference temperature for calculating the zero-shear viscosity, $T_{\text{ref}}$ |
| ALPHAG | Gel point at which flow is no longer possible, $\alpha_g$ |
| C1 | Experimental constant, $c_1$ |
| C2 | Experimental constant, $c_2$ |
| M_KS | Reaction order in the Kamal-Sourour model, $m_{\text{ks}}$ |
| N_KS | Reaction order in the Kamal-Sourour model, $n_{\text{ks}}$ |
| A1 | Fitted rate coefficient, $A_1$ |
| A2 | Fitted rate coefficient, $A_2$ |
| T1 | Activation temperature, $T_1$ |
| T2 | Activation temperature, $T_2$ |

**Remarks:**

1. **Viscosity.** In the Cross Castro Macosko model, the viscosity is described as:

$$\eta(T,\dot{\gamma},\alpha) = \frac{\eta_0(T)}{1+\left(\frac{\eta_0(T)\dot{\gamma}}{\tau^*}\right)^{1-n}}\left(\frac{\alpha_g}{\alpha_g-\alpha}\right)^{(c_1+c_2\alpha)}$$

Here, $\eta_0(T)$ is the temperature-dependent zero-shear viscosity of the fluid:

$$\eta_0(T) = B\exp\left(\frac{T_{\text{ref}}}{T}\right) ,$$

$\dot{\gamma}$ is the second invariant of the rate of deformation tensor, **D**:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}} \ ,$$

and $\alpha$ is the degree of cure. $B$, $T_{\text{ref}}$, $\alpha_g$, $c_1$, and $c_2$ are input parameters.

2. **Degree of cure.** The Kamal-Sourour model describes the conversion rate of $\alpha$:

$$\frac{d\alpha}{dt} = (k_1 + k_2\alpha^{m_{\text{ks}}})(1-\alpha)^{n_{\text{ks}}}$$

with

$$k_1 = A_1 \exp\left(-\frac{E_1}{RT}\right)$$
$$k_2 = A_2 \exp\left(-\frac{E_2}{RT}\right)$$

Here, $m_{\text{ks}}$ and $n_{\text{ks}}$ are the reaction orders, $k_1$ and $k_2$ are the Arrhenius rate constants, $A_1$ and $A_2$ are fitted rate coefficients, $E_1$ and $E_2$ are the activation energies, $R$ is the universal gas law constant of 8.314 $\text{J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$, and $T$ is the cure temperature. The activation temperature is obtained by dividing the activation energy by the universal gas law constant, such as $T_1 = E_1/R$.

3. **Enabling temperature dependence.** To consider temperature dependence, use either *ISPG_CONTROL_SOLUTION or *LOAD_THERMAL_LOAD_CURVE. To obtain the temperature by solving the energy governing equations, enable a combined flow and thermal analysis with *ISPG_CONTROL_SOLUTION. Temperatures defined with *LOAD_THERMAL_LOAD_CURVE are applied only during a flow-only analysis *LOAD_THERMAL_LOAD_CURVE is ignored during a combined flow and thermal analysis.

## *MAT_ISPG_CROSS_WLF_MODEL

This is Material Type 5 for ISPG. The Cross WLF model attempts to describe the temperature and shear rate dependency of the viscosity for thermoplastic materials This model combines the Williams-Landel-Ferris (WLF) equation with a power-law dependence on shear rate. It captures the non-Arrhenius behavior of visocity near the glass transition temperature.

> **NOTE:** This material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the main input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | | SFTEN | | TAU | N | |
| Type | A | F | | F | | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | | D1 | TGLASS | | A1 | A2 | | |
| Type | | F | F | | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Fluid density |
| SFTEN | Surface tension coefficient of the fluid |
| TAU | Critical stress level at the transition to shear thinning, $\tau^*$ |
| N | Power-law index, $n$ |
| D1 | Constant $D_1$ in the zero-shear-rate viscosity equation |
| TGLASS | Glass transition temperature, $T^*$ |

| VARIABLE | DESCRIPTION |
|---|---|
| A1 | Constant $A_1$ in the zero-shear-rate viscosity equation |
| A2 | Constant $A_2$ in the zero-shear-rate viscosity equation |

**Remarks:**

1. **Viscosity.** In this model, the melt viscosity, $\eta$, is given by:

$$\eta = \frac{\eta_0}{1 + \left(\frac{\eta_0 \dot{\gamma}}{\tau^*}\right)^{1-n}}$$

In the above, $\eta_0$ is the zero-shear-rate viscosity and $\dot{\gamma}$ is the second invariant of the rate of deformation tensor, **D**:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\mathbf{D}:\mathbf{D}}$$

$\tau^*$ is the critical stress level at the transition to shear thinning, and $n$ is the power-law index in the high-shear-rate regime.

The zero-shear-rate viscosity is given by the following:

$$\eta_0 = D_1 \exp\left[-\frac{A_1(T - T^*)}{A_2 + (T - T^*)}\right]$$

Here, $T$ is the temperature in Kelvin, $T^*$ is the glass transition temperature, and $D_1$, $A_1$, and $A_2$ are constants.

## *MAT_SPH_VISCOUS

This may also be referred to as *MAT_SPH_01. This "fluid-like" material model is very similar to Material Type 9 (*MAT_NULL). It models viscous fluids with constant or variable viscosity. The variable viscosity is a function of an equivalent deviatoric strain rate. If an inviscid material is modeled, the deviatoric or viscous stresses are zero, and the equation of state supplies the pressures (or diagonal components of the stress tensor).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | PC | MULO | MUHI | RK | RC | RN |
| Type | A | F | F | F | F | F | F | F |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| PC | Pressure cutoff (≤ 0.0). See Remark 4. |
| MULO | Dynamic viscosity (see Remark 1):<br>EQ.0.0: Inviscid fluid is assumed.<br>GT.0.0: If MUHI = 0.0 or is not defined, then this is the traditional constant dynamic viscosity coefficient, $\mu$. Otherwise, if MUHI > 0.0, then MULO and MUHI are the lower and upper dynamic viscosity limit values for a power-law-like variable viscosity model.<br>LT.0.0: -MULO is a load curve ID defining dynamic viscosity as a function of equivalent strain rate. |
| MUHI | Dynamic viscosity:<br>EQ.0.0: Only MULO is used to define the dynamic viscosity, default<br>LT.0.0: The viscosity can be defined by the user in the file dyn21.F with a routine called f3dm9ale_userdef1. The file is part of the general usermat package. Note that in |

| VARIABLE | DESCRIPTION |
|---|---|
| | this case MULO is a parameter for the subroutine.<br>GT.0.0: This is the upper dynamic viscosity limit. This is defined only if RK and RN are defined for the variable viscosity case. |
| RK | Variable dynamic viscosity multiplier. See Remark 6. |
| RC | Cross viscosity model:<br>RC.GT.0.0: Use the Cross viscosity model which overwrites all other options. The values of MULO, MUHI, RK, and RN are used in the Cross viscosity model. See Remark 7.<br>RC.LE.0.0: Use a viscosity model based on the above fields. See Remark 6. |
| RN | Variable dynamic viscosity exponent. See Remark 6. |

**Remarks:**

1. **Deviatoric viscous stress.** This material must be used with an equation of state. Pressure cutoff is negative in tension. A (deviatoric) viscous stress of the form

$$\sigma'_{ij} = 2\mu\dot{\varepsilon}'_{ij}$$

$$\left[\frac{N}{m^2}\right] \sim \left[\frac{N}{m^2}s\right]\left[\frac{1}{s}\right]$$

is computed for nonzero $\mu$ where $\dot{\varepsilon}'_{ij}$ is the deviatoric strain rate. $\mu$ is the dynamic viscosity. For example, in the SI unit system, $\mu$ has units of [Pa × s].

2. **Hourglass control issues.** This material has no shear stiffness and hourglass control must be used with care. In some applications, the default hourglass coefficient might lead to significant energy losses. In general, for fluid(s), the hourglass coefficient QM should be small (in the range of $10^{-4}$ to $10^{-6}$ for the standard default IHQ choice).

3. **Null material properties.** This material has no yield strength and behaves in a fluid-like manner because it is based on the null material.

4. **Numerical cavitation.** The pressure cut-off, PC, must be defined to allow for a material to "numerically" cavitate. In other words, when a material undergoes dilatation above certain magnitude, it should no longer be able to resist this dilatation. Since dilatation stress or pressure is negative, setting PC limit to a very

small negative number would allow for the material to cavitate once the pressure in the material goes below this negative value.

5. **Issues with small values of viscosity exponent.** If the viscosity exponent is less than 1.0 (RN < 1.0), then RN − 1.0 < 0.0. In this case, at very low equivalent strain rate, the viscosity can be artificially very high. MULO is then used as the viscosity value.

6. **Empirical dynamic viscosity.** The empirical variable dynamic viscosity is typically modeled as a function of *equivalent shear rate* based on experimental data:

$$\mu(\dot{\bar{\gamma}}') = \mathrm{RK} \times \dot{\bar{\gamma}}'^{(\mathrm{RN}-1)} \ .$$

For an incompressible fluid, this may be written equivalently as

$$\mu(\dot{\bar{\varepsilon}}') = \mathrm{RK} \times \dot{\bar{\varepsilon}}'^{(\mathrm{RN}-1)} \ .$$

The "overbar" denotes a scalar equivalence, the "dot" denotes a time derivative or rate effect, and the "prime" symbol denotes deviatoric or volume preserving components. The *equivalent shear rate* components may be related to the basic definition of (small-strain) strain rate components as follows:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \Rightarrow \dot{\varepsilon}'_{ij} = \dot{\varepsilon}_{ij} - \delta_{ij}\left(\frac{\dot{\varepsilon}_{kk}}{3}\right)$$
$$\dot{\gamma}_{ij} = 2\dot{\varepsilon}_{ij}$$

Typically, the 2nd invariant of the deviatoric strain rate tensor is defined as:

$$I_{2\dot{\varepsilon}'} = \frac{1}{2}\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right] \ .$$

The equivalent (small-strain) deviatoric strain rate is defined as:

$$\dot{\bar{\varepsilon}}' \equiv 2\sqrt{I_{2\dot{\varepsilon}'}} = \sqrt{2\left[\dot{\varepsilon}'_{ij}\dot{\varepsilon}'_{ij}\right]} = \sqrt{4\left[\dot{\varepsilon}'_{12}{}^2 + \dot{\varepsilon}'_{23}{}^2 + \dot{\varepsilon}'_{31}{}^2\right] + 2\left[\dot{\varepsilon}'_{11}{}^2 + \dot{\varepsilon}'_{22}{}^2 + \dot{\varepsilon}'_{33}{}^2\right]} \ .$$

In non-Newtonian literatures, the *equivalent shear rate* is sometimes defined as

$$\dot{\bar{\gamma}} \equiv \sqrt{\frac{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}{2}} = \sqrt{2\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}} = \sqrt{4[\dot{\varepsilon}_{12}^2 + \dot{\varepsilon}_{23}^2 + \dot{\varepsilon}_{31}^2] + 2[\dot{\varepsilon}_{11}^2 + \dot{\varepsilon}_{22}^2 + \dot{\varepsilon}_{33}^2]} \ .$$

It turns out that when (a) the material is incompressible material ($\dot{\varepsilon}_{kk} = 0$) and (b) the shear terms are equivalent (when $i \neq j \rightarrow \dot{\varepsilon}_{ij} = \dot{\varepsilon}'_{ij}$), the *equivalent shear rate* is algebraically equivalent to the *equivalent (small-strain) deviatoric strain rate*:

$$\dot{\bar{\varepsilon}}' = \dot{\bar{\gamma}}' \ .$$

7. **Cross viscous model.** The Cross viscous model is one of simplest and most used model for shear-thinning behavior. With shear-thinning behavior, the fluid's viscosity decreases with increasing local shear rate, $\dot{\bar{\gamma}}$. Thus, using the Cross viscous model, the dynamic viscosity $\mu$ is defined as a function of $\dot{\bar{\gamma}}$:

$$\mu(\dot{\bar{\gamma}}') = \mathrm{MUHI} + (\mathrm{MULO} - \mathrm{MUHI})/(1.0 + \mathrm{RK} \times \dot{\bar{\gamma}}')^{\mathrm{RN}-1} \ .$$

Here RK and RN are two positive fitting parameters, and MULO and MUHI are the limiting values of the viscosity at low and high shear rates, respectively. RK, RN, MULO and MUHI are fields from the keyword input.

## *MAT_SPH_INCOMPRESSIBLE_FLUID

This may also be referred to as *MAT_SPH_02. This material is only used for the implicit incompressible SPH formulation (FORM = 13 in *CONTROL_SPH).

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | MU | GAMMA1 | GAMMA2 | STENS | | |
| Type | A | F | F | F | F | F | | |
| Defaults | none | none | none | 0.0 | 0.0 | 0.0 | | |

This card is optional.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | CP | LAMBDA | | | | | | |
| Type | F | F | | | | | | |
| Default | 0.0 | 0.0 | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density |
| MU | Dynamic viscosity<br>LT.0.0: \|MU\| is a load curve of dynamic viscosity as a function of temperature. See *DEFINE_CURVE.<br>Note that MU must be set to a nonzero value because inviscid flow is not permitted for the incompressible SPH formulation. |
| GAMMA1 | Numerical surface tension coefficient. For water, we recommend a coefficient of $\gamma_1 = 1000\ \mathrm{m/s^2}$. GAMMA1 is only used if IMAT = 0 in *CONTROL_SPH_INCOMPRESSIBLE. |
| GAMMA2 | Numerical surface tension coefficient. For water, we recommend a |

| VARIABLE | DESCRIPTION |
|---|---|
| | coefficient of $\gamma_2 = 1$ m/s$^2$. GAMMA2 is only used if IMAT = 0 in *CONTROL_SPH_INCOMPRESSIBLE. |
| STENS | Physical surface tension coefficient. It is only used if IMAT = 1 in *CONTROL_SPH_INCOMPRESSIBLE. |
| CP | Fluid specific heat. It is used to calculate heat transfer coefficients if IHTC = 1 in *CONTROL_SPH_INCOMPRESSIBLE. |
| LAMBDA | Fluid thermal conductivity. It is used to calculate heat transfer coefficients if IHTC = 1 in *CONTROL_SPH_INCOMPRESSIBLE. |

**Remarks:**

The surface tension coefficients, GAMMA1 and GAMMA2, are purely numerical and are based on a normalized version of the algorithm presented in [1]. If IMAT = 1 in *CONTROL_SPH_INCOMPRESSIBLE, surface tension is calculated based on the physical surface tension properties of the fluid.

**References:**

[1] Akinci, N., Akinci, G. & Teschner, M., "Versatile surface tension and adhesion for SPH fluids," *ACM Transactions on Graphics (TOG)*, 32.6 182 (2013).

## *MAT_SPH_INCOMPRESSIBLE_STRUCTURE

This may also be referred to as *MAT_SPH_03. This material is only used for the implicit incompressible SPH formulation (FORM = 13 in *CONTROL_SPH) and should be assigned to structures sampled with the *DEFINE_SPH_MESH_SURFACE keyword.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | RO | BETA | ROUGH | ADH | | | |
| Type | A | F | F | F | F | | | |
| Defaults | none | none | 0.0 | 0.0 | 0.0 | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| RO | Mass density. This should be set to the rest density of the fluid. The actual mass of the structure will be calculated from the parent surfaces sampled with the *DEFINE_SPH_MESH_SURFACE keyword. |
| BETA | Numerical surface adhesion coefficient. For water, a value of $\beta = 1000\ \mathrm{m/s^2}$ is recommended. Only used if IMAT = 0 in *CONTROL_SPH. |
| ROUGH | Surface roughness coefficient. A friction force between the structure and the fluid is generated based on the viscosity of the fluid scaled by this coefficient. A value between 0.0 and 10.0 is usually recommended. |
| ADH | Surface adhesion scaling coefficient. It is only used if IMAT = 1 in *CONTROL_SPH. An attractive force between fluid and structure is calculated based on surface tension forces in the fluid and then scaled by ADH. |

### Remarks:

The surface adhesion coefficient is purely numerical and is based on a normalized version of the algorithm presented in [1].

## References:

[1] Akinci, N., Akinci, G. & Teschner, M. (2013). Versatile surface tension and adhesion for SPH fluids. ACM Transactions on Graphics (TOG) 32.6 182.

## *MAT_SPRING_ELASTIC

This is Material Type 1 for discrete elements (*ELEMENT_DISCRETE). This model provides a translational or rotational elastic spring located between two nodes. Only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | K | | | | | | |
| Type | A | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| K | Elastic stiffness (force/displacement) or (moment/rotation) |

**Remarks:**

Rotational displacement is measured in radians.

### *MAT_DAMPER_VISCOUS

This is Material Type 2 for discrete elements (*ELEMENT_DISCRETE). This material provides a linear translational or rotational damper located between two nodes. Only one degree of freedom is then connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | DC | | | | | | |
| Type | A | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| DC | Damping constant (force/displacement rate) or (moment/rotation rate) |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_ELASTOPLASTIC

This is Material Type 3 for discrete elements (*ELEMENT_DISCRETE). This material provides an elastoplastic translational or rotational spring with isotropic hardening located between two nodes. Only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | K | KT | FY | | | | |
| Type | A | F | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| K | Elastic stiffness (force/displacement) or (moment/rotation) |
| KT | Tangent stiffness (force/displacement) or (moment/rotation) |
| FY | Yield (force) or (moment) |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_NONLINEAR_ELASTIC

This is Material Type 4 for discrete elements (*ELEMENT_DISCRETE). This material provides a nonlinear elastic translational and rotational spring with arbitrary force as a function of displacement and moment as a function of rotation, respectively. Optionally, strain rate effects can be considered through a velocity dependent scale factor or defining a table of curves. With the spring located between two nodes, only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCD | LCR | | | | | |
| Type | A | I | I | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCD | Load curve ID (see *DEFINE_CURVE) describing force as a function of displacement or moment as a function of rotation relationship. The load curve *must* define the response in the negative and positive quadrants and pass through point (0,0). Negative data point(s) must come first in the curve definition, where negative values represent compression in the case of a translational spring.<br>LCD may also be a table ID (see *DEFINE_TABLE). The table gives for each loading rate a load curve ID defining the force-displacement (or moment-rotation) curve. Values between the data points are computed by linear interpolation. If a table ID is specified, LCR will be ignored. |
| LCR | Optional load curve describing scale factor on force or moment as a function of relative velocity or rotational velocity, respectively. |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_DAMPER_NONLINEAR_VISCOUS

This is Material Type 5 for discrete elements (*ELEMENT_DISCRETE). This material provides a viscous translational damper with an arbitrary force as a function of velocity dependency or a rotational damper with an arbitrary moment as a function of rotational velocity dependency. With the damper located between two nodes, only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCDR | | | | | | |
| Type | A | I | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCDR | Load curve ID defining force as a function of rate-of-displacement relationship or a moment as a function of rate-of-rotation relationship. The load curve *must* define the response in the negative and positive quadrants and pass through point (0,0). |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_GENERAL_NONLINEAR

This is Material Type 6 for discrete elements (*ELEMENT_DISCRETE). This material provides a general nonlinear translational or rotational spring with arbitrary loading and unloading definitions. Optionally, hardening or softening can be defined. With the spring located between two nodes, only one degree of freedom is connected.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCDL | LCDU | BETA | TYI | CYI | | |
| Type | A | I | I | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCDL | Load curve or table ID giving force/torque as a function of displacement/rotation (curve) or as a function of velocity and displacement/rotation (table) for loading; see Figure MS6-1. |
| LCDU | Load curve or table ID giving force/torque as a function of displacement/rotation (curve) or as a function of velocity and displacement/rotation (table) for unloading; see Figure MS6-1. |
| BETA | Hardening parameter, $\beta$:<br>EQ.0.0: Tensile and compressive yield with strain softening (negative or zero slope allowed in the force as a function of displacement load curves). TYI and CYI are not implemented for this option.<br>NE.0.0: Kinematic hardening without strain softening<br>EQ.1.0: Isotropic hardening without strain softening |
| TYI | Initial yield force in tension ( > 0) |
| CYI | Initial yield force in compression ( < 0) |

**Remarks:**

1. **Load curves.** Load curve points are in the format (displacement, force) or (rotation, moment). The points must be in order starting with the most negative

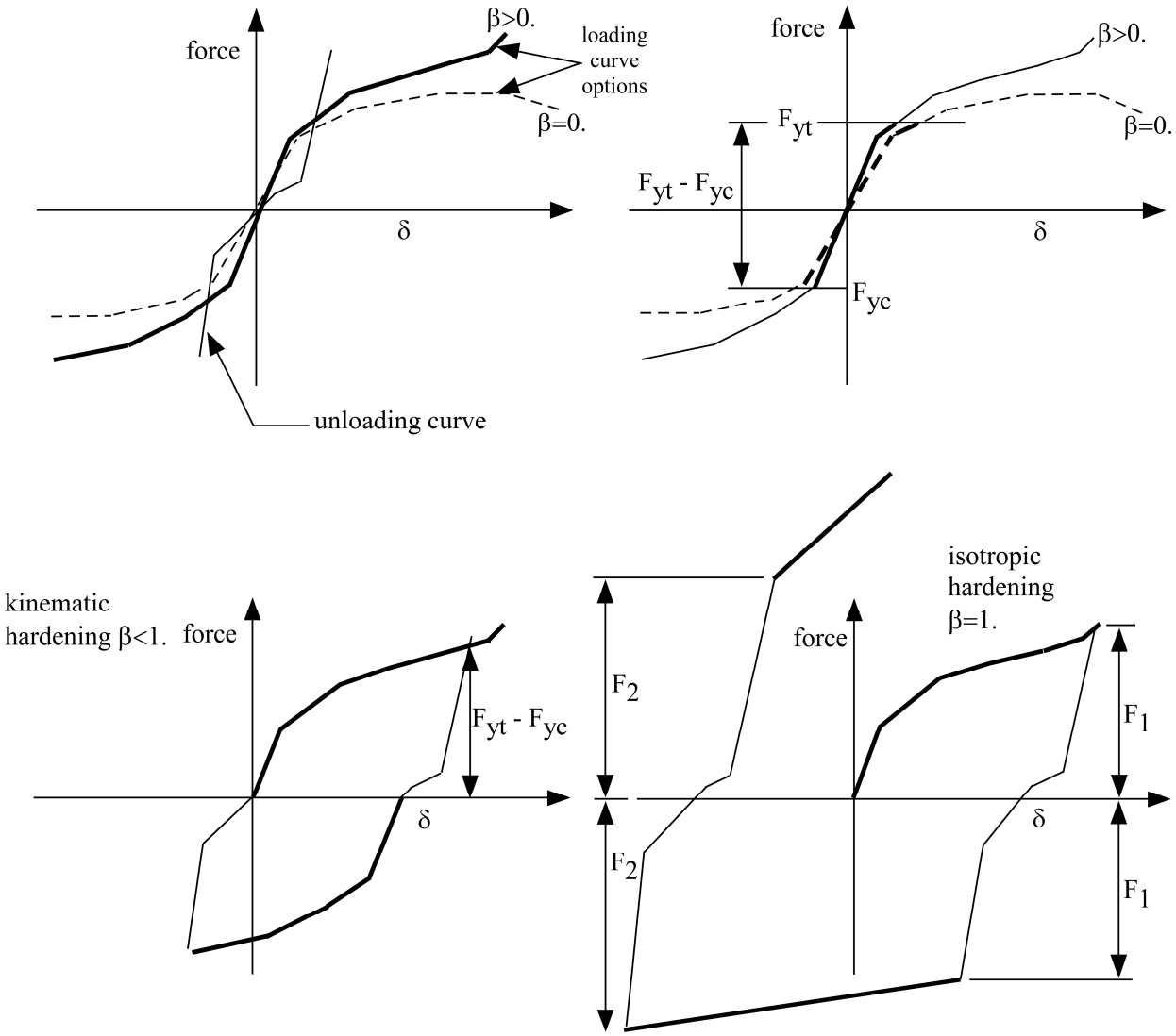



**Figure MS6-1.** General nonlinear material for discrete elements

(compressive) displacement or rotation and ending with the most positive (tensile) value. The curves need not be symmetrical.

The displacement origin of the "unloading" curve is arbitrary since it will be shifted as necessary as the element extends and contracts. On reverse yielding the "loading" curve will also be shifted along the displacement re or. rotation axis.

2. **Initial tensile and compressive yield forces.** The initial tensile and compressive yield forces (TYI and CYI) define a range within which the element remains elastic (meaning the "loading" curve is used for both loading and unloading). If at any time the force in the element exceeds this range, the element is deemed to have yielded, and at all subsequent times the "unloading" curve is used for unloading.

3. **Rotational displacement.** Rotational displacement is measured in radians.

## *MAT_SPRING_MAXWELL

This is Material Type 7 for discrete elements (*ELEMENT_DISCRETE). This material provides a three-parameter Maxwell viscoelastic translational or rotational spring. Optionally, a cutoff time with a remaining constant force/moment can be defined.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | K0 | KI | BETA | TC | FC | COPT | |
| Type | A | F | F | F | F | F | F | |
| Default | none | none | none | none | $10^{20}$ | 0.0 | 0.0 | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| K0 | $K_0$, short-time stiffness |
| KI | $K_\infty$, long-time stiffness |
| BETA | Decay parameter, $\beta$ |
| TC | Cut off time. After this time, a constant force/moment is transmitted. |
| FC | Force/moment after cutoff time |
| COPT | Time implementation option:<br>EQ.0.0: Incremental time change<br>NE.0.0: Continuous time change |

**Remarks:**

1. **Stiffness.** The time-varying stiffness, $K(t)$, may be described in terms of the input parameters as

$$K(T) = K_\infty + (K_0 - K_\infty)\exp(-\beta t) \ .$$

This equation was implemented by Schwer [1991] as either a continuous function of time or incrementally following the approach of Herrmann and Peterson

[1968]. The continuous function of time implementation has the disadvantage of the energy absorber's resistance decaying with increasing time even without deformation. The advantage of the incremental implementation is that an energy absorber must undergo some deformation before its resistance decays, meaning there is no decay until impact, even in delayed impacts. The disadvantage of the incremental implementation is that very rapid decreases in resistance cannot be easily matched.

2. **Rotational displacement.** Rotational displacement is measured in radians.

## *MAT_SPRING_INELASTIC

This is Material Type 8 for discrete elements (*ELEMENT_DISCRETE). This material provides an inelastic tension or compression only, translational or rotational spring. Optionally, a user-specified unloading stiffness can be taken instead of the maximum loading stiffness.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | LCFD | KU | CTF | | | | |
| Type | A | I | F | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| LCFD | Load curve ID describing arbitrary force/torque as a function of displacement/rotation relationship. This curve must be defined in the positive force-displacement quadrant regardless of whether the spring acts in tension or compression. |
| KU | Unloading stiffness (optional). The maximum of KU and the maximum loading stiffness in the force/displacement or the moment/rotation curve is used for unloading. |
| CTF | Flag for compression/tension:<br>EQ.-1.0: Tension only<br>EQ.1.0: Compression only (default) |

**Remarks:**

Rotational displacement is measured in radians.

## *MAT_SPRING_TRILINEAR_DEGRADING

This is Material Type 13 for discrete elements (*ELEMENT_DISCRETE). This material allows concrete shearwalls to be modeled as discrete elements under applied seismic loading. It represents cracking of the concrete, yield of the reinforcement, and overall failure. Under cyclic loading, the stiffness of the spring degrades, but the strength does not.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | DEFL1 | F1 | DEFL2 | F2 | DEFL3 | F3 | FFLAG |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| DEFL1 | Deflection at the point where concrete cracking occurs |
| F1 | Force corresponding to DEFL1 |
| DEFL2 | Deflection at the point where reinforcement yields |
| F2 | Force corresponding to DEFL2 |
| DEFL3 | Deflection at complete failure |
| F3 | Force corresponding to DEFL3 |
| FFLAG | Failure flag |

## *MAT_SPRING_SQUAT_SHEARWALL

This is Material Type 14 for discrete elements (*ELEMENT_DISCRETE). This material allows squat shear walls to be modeled using discrete elements. The behavior model captures concrete cracking, reinforcement yield, and ultimate strength followed by degradation of strength finally leading to collapse.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | A14 | B14 | C14 | D14 | E14 | LCID | FSD |
| Type | A | F | F | F | F | F | I | F |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| A14 | Material coefficient *A* |
| B14 | Material coefficient *B* |
| C14 | Material coefficient *C* |
| D14 | Material coefficient *D* |
| E14 | Material coefficient *E* |
| LCID | Load curve ID referencing the maximum strength envelope curve |
| FSD | Sustained strength reduction factor |

**Remarks:**

Material coefficients *A*, *B*, *C*, and *D* are empirically defined constants for setting the shape of the polynomial curves that govern the cyclic behavior of the discrete element. The loading and unloading paths use different polynomial relationships, allowing energy absorption through hysteresis. Coefficient *E* determines the "jump" from the loading path to the unloading path (or vice versa) when a full hysteresis loop is not completed. The load curve referenced is used to define the force-displacement characteristics of the shear wall under monotonic loading. The polynomials defining the cyclic behavior refer to this curve. On the second and subsequent loading/unloading cycles, the shear wall has reduced strength. The variable FSD is the sustained strength reduction factor.

## *MAT_SPRING_MUSCLE

This is Material Type S15 for discrete elements (*ELEMENT_DISCRETE). This material is a Hill-type muscle model with activation. The LS-DYNA implementation is due to Dr. J. A. Weiss.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | L0 | VMAX | SV | A | FMAX | TL | TV |
| Type | A | F | F | F | F | F | F | F |
| Default | none | 1.0 | none | 1.0 | none | none | 1.0 | 1.0 |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | FPE | LMAX | KSH | | | | | |
| Type | F | F | F | | | | | |
| Default | 0.0 | none | none | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Material identification. A unique number or label must be specified (see *PART). |
| L0 | Initial muscle length, $L_0$ |
| VMAX | Maximum CE shortening velocity, $V_{\max}$ |
| SV | Scale factor, $S_v$, for $V_{\max}$ as a function of active state:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| A | Activation level as a function of time function $a(t)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of A is used. |

| VARIABLE | DESCRIPTION |
|---|---|
| FMAX | Peak isometric force, $F_{max}$ |
| TL | Active tension as a function of length function, $f_{TL}(L)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| TV | Active tension as a function of velocity function, $f_{TV}(V)$:<br>LT.0.0: Absolute value gives load curve ID.<br>GE.0.0: Constant value of 1.0 is used. |
| FPE | Normalized force, $f_{PE}$, as a function of length for parallel elastic element:<br>LT.0.0: Absolute value gives load curve ID.<br>EQ.0.0: Exponential function is used (see Remarks).<br>GT.0.0: Constant value of 0.0 is used. |
| LMAX | Relative length when $F^{PE}$ reaches $F_{max}$. Required if FPE = 0.0 above. See Remarks. |
| KSH | Constant, $K_{sh}$, governing the exponential rise of $F^{PE}$. Required if FPE = 0.0 above. See Remarks. |

**Remarks:**

The material behavior of the muscle model is adapted from the original model proposed by Hill [1938]. Reviews of this model and extensions can be found in Winters [1990] and Zajac [1989]. The most basic Hill-type muscle model consists of a contractile element (CE) and a parallel elastic element (PE) (Figure MS15-1). An additional series elastic element (SEE) can be added to represent tendon compliance.

The main assumptions of the Hill model are that the contractile element is entirely stress free and freely distensible in the resting state and is described exactly by Hill's equation (or some variation). When the muscle is activated, the series and parallel elements are elastic, and the whole muscle is a simple combination of identical sarcomeres in series and parallel. The main criticism of Hill's model is that the division of forces between the parallel elements and the division of extensions between the series elements is arbitrary and cannot be made without introducing auxiliary hypotheses. However, these criticisms apply to *any* discrete element model. Despite these limitations, the Hill model has

become extremely useful for modeling musculoskeletal dynamics, as illustrated by its widespread use today.

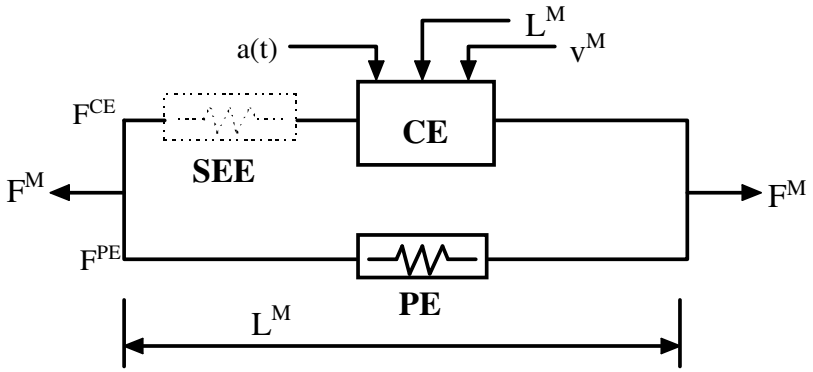



**Figure MS15-1.** Discrete model for muscle contraction dynamics, based on a Hill-type representation. The total force is the sum of passive force $F^{PE}$ and active force $F^{CE}$. The passive element (PE) represents energy storage from muscle elasticity, while the contractile element (CE) represents force generation by the muscle. The series elastic element (SEE), shown in dashed lines, is often neglected when a series tendon compliance is included. Here, $a(t)$ is the activation level, $L^M$ is the length of the muscle, and $V^M$ is the shortening velocity of the muscle.

When the contractile element (CE) of the Hill model is inactive, the entire resistance to elongation is provided by the PE element and the tendon load-elongation behavior. As activation is increased, force then passes through the CE side of the parallel Hill model, providing the contractile dynamics. The original Hill model accommodated only full activation - this limitation is circumvented in the present implementation by using the modification suggested by Winters (1990). The main features of his approach were to realize that the CE force-velocity input force equals the CE tension-length output force. This yields a three-dimensional curve to describe the force-velocity-length relationship of the CE. If the force-velocity $y$-intercept scales with activation, then given the activation, length and velocity, the CE force can be determined.

Without the SEE, the total force in the muscle FM is the sum of the force in the CE and the PE because they are in parallel:

$$F^M = F^{PE} + F^{CE} \quad .$$

The relationships defining the force generated by the CE and PE as a function of $L^M$ (length of the muscle), $V^M$ (shortening velocity of the muscle) and $a(t)$ are often scaled by $F_{max}$, the peak isometric force (p. 80, Winters 1990), $L_0$, the initial length of the muscle (p. 81, Winters 1990), and $V_{max}$, the maximum unloaded CE shortening velocity (p. 80, Winters 1990). From these, dimensionless length and velocity can be defined as:

$$L = L^M / L_0$$
$$V = \frac{V^M}{V_{\max} \times S_v[a(t)]}$$

Here, $S_v$ scales the maximum CE shortening velocity $V_{\max}$ and changes with activation level $a(t)$. This has been suggested by several researchers, that is, Winters and Stark [1985]. The activation level specifies the level of muscle stimulation as a function of time. Both have values between 0 and 1. The functions $S_v[a(t)]$ and $a(t)$ are specified using load curves in LS-DYNA, but the default values of $S_v = 1$ and $a(t) = 0$ can also be used. Note that $L$ is always positive and that $V$ is positive for lengthening and negative for shortening.

The relationship between $F^{CE}$, $V$ and $L$ was proposed by Bahler et al. [1967]. A three-dimensional relationship between these quantities is now considered standard for computer implementations of Hill-type muscle models [Winters 1990]. It can be written in dimensionless form as:

$$F^{CE} = a(t) \times F_{\max} \times f_{TL}(L) \times f_{TV}(V) \, .$$

Here, $f_{TL}(L)$ and $f_{TV}(V)$ are the tension-length and tension-velocity functions for active skeletal muscle. Thus, if current values of $L^M$, $V^M$, and $a(t)$ are known, then $F^{CE}$ can be determined (Figure MS15-1).

If FPE = 0.0, the force in the parallel elastic element, $F^{PE}$, is determined directly from the current length of the muscle using an exponential relationship [Winters 1990]:

$$f_{PE} = \frac{F^{PE}}{F_{MAX}} = \begin{cases} 0 & L \le 1 \\ \dfrac{1}{\exp(K_{sh}) - 1} \left\{ \exp\left[ \dfrac{K_{sh}}{L_{max}} (L - 1) \right] - 1 \right\} & L > 1 \end{cases}$$

Here, $L_{max}$ is the dimensionless length at which the force $F_{max}$ occurs, and $K_{sh}$ is a dimensionless shape parameter controlling the rate of rise of the exponential. Alternatively, the user can define a custom $f_{PE}$ curve giving tabular values of normalized force as a function of dimensionless length as a load curve.

For computation of the total force developed in the muscle $F^M$, the functions for the tension-length $f_{TL}(L)$ and force-velocity $f_{TV}$ relationships used in the Hill element must be defined. These relationships have been available for over 50 years but have been refined to allow for behavior such as active lengthening. The active tension-length curve $f_{TL}(L)$ describes the fact that isometric muscle force development is a function of length, with the maximum force occurring at an optimal length. According to Winters, this optimal length is typically around $L = 1.05$, and the force drops off for shorter or longer lengths, approaching zero force for $L = 0.4$ and $L = 1.5$. Thus the curve has a bell-shape. Because of the variability in this curve between muscles, the user must specify the function $f_{TL}(L)$ using a load curve, specifying pairs of points representing the normalized force (with values between 0 and 1) and normalized length $L$. See Figure MS15-2.

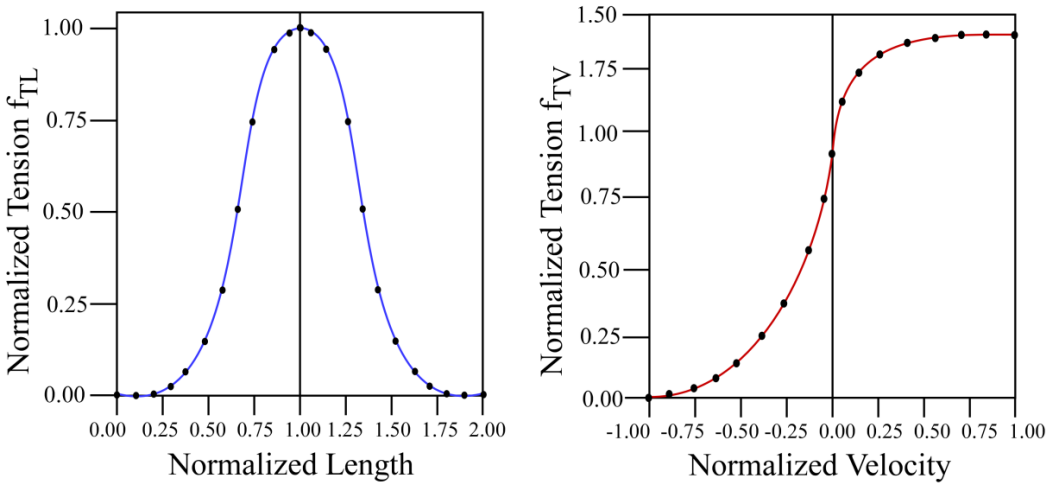



**Figure MS15-2.** Typical normalized tension-length (TL) and tension-velocity (TV) curves for skeletal muscle.

The active tension-velocity relationship $f_{\mathrm{TV}}(V)$ used in the muscle model is mainly due to the original work of Hill. Note that the dimensionless velocity $V$ is used. When $V = 0.0$, the normalized tension is typically chosen to have a value of 1.0. When $V$ is greater than or equal to 0.0, muscle lengthening occurs. As $V$ increases, the function is typically designed so that the force increases from a value of 1.0 and asymptotes towards a value near 1.4 ass shown in Figure MS15-2. When $V$ is less than zero, muscle shortening occurs and the classic Hill equation hyperbola is used to drop the normalized tension to 0.0 as shown in Figure MS15-2. The user must specify the function $f_{\mathrm{TV}}(V)$ using a load curve, specifying pairs of points representing the normalized tension (with values between 0.0 and 1.0) and normalized velocity $V$.

## *MAT_SEATBELT_{OPTION}

This is Material Type B01. It defines a seat belt material.

Available options include:

2D

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | MID | MPUL | LLCID | ULCID | LMIN | CSE | DAMP | E |
| Type | A | F | I | I | F | F | F | F |
| Default | 0 | 0.0 | 0 | 0 | 0.0 | 0.0 | 0.1 | 0.0 |

**Bending/Compression Parameter Card.** Additional card for E > 0.0.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | A | I | J | AS | F | M | R | |
| Type | F | F | F | F | F | F | F | |
| Default | 0.0 | 0.0 | 2*I | A | $10^{20}$ | $10^{20}$ | 0.05 | |

**2D Card.** Additional 1st card for the 2D keyword option.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1DOFF | FORM | ECOAT | TCOAT | SCOAT | EB | PRBA | PRAB |
| Type | I | I | F | F | F | F | F | F |
| Default | 0 | 0 | 0.0 | 0.0 | 0.0 | -0.1 | 0.3 | PRBA |

**2D Card.** Optional 2nd card for the 2D keyword option.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | GAB | | | | | | | |
| Type | F | | | | | | | |
| Default | ↓ | | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| MID | Belt material number. A unique number or label must be specified (see *PART). |
| MPUL | Mass per unit length |
| LLCID | Curve or table ID for loading. LLCID can be either a single curve (force as a function of engineering strain) or a table defining a set of strain-rate dependent load curves. |
| ULCID | Load curve identification for unloading (force as a function of engineering strain) |
| LMIN | Minimum length (for elements connected to slip rings and retractors); see Remark 4. |
| CSE | Compressive stress elimination option for nonzero FORM that applies to shell elements only (see Remark 6):<br>EQ.0.0: Don't eliminate compressive stresses in the shell fabric.<br>EQ.1.0: Eliminate compressive stresses in the shell fabric.<br>Note that for FORM = 0, the solver automatically determines whether or not to eliminate the compressive stresses. |
| DAMP | Optional Rayleigh damping coefficient, which applies to shell elements only. A coefficient value of 0.10 is the default corresponding to 10% of critical damping. Sometimes, smaller or larger values work better. |
| E | Young's modulus for bending/compression stiffness, when positive, the optional card is invoked. See Remark 5. |
| A | Cross-sectional area for bending/compression stiffness; see remarks. |

| VARIABLE | DESCRIPTION |
|---|---|
| I | Area moment of inertia for bending/compression stiffness; see Remark 5. |
| J | Torsional constant for bending/compression stiffness; see Remark 5. |
| AS | Shear area for bending/compression stiffness; see Remark 5. |
| F | Maximum force in compression/tension; see Remark 5. |
| M | Maximum torque; see Remark 5. |
| R | Rotational mass scaling factor; see Remark 5. |
| P1DOFF | Part ID offset for internally created 1D, bar-type, belt parts for 2D seatbelt of this material, that is, the IDs of newly created 1D belt parts will be P1DOFF + 1, P1DOFF + 2, … If zero, the maximum ID of user-defined parts is used as the part ID offset. |
| FORM | Formulation of the translated fabric material; see FORM of *MAT_FABRIC for details. FORM = 0 was used since R8 and non-zero FORM is available since r137465/dev. |
| ECOAT | Young's modulus of coat material for FORM = -14; see *MAT_FABRIC for details.<br>EQ.0.0: ECOAT is the Young's modulus determined by LS-DYNA.<br>GT.0.0: ECOAT is the Young's modulus to be used for coat material.<br>LT.0.0: \|ECOAT\| is the ratio of coat material's Young's modulus to that of the fabric shell which is determined by LS-DYNA. |
| TCOAT | Thickness of coat material for FORM = -14; see *MAT_FABRIC for details. |
| SCOAT | Yield stress of coat material for FORM = -14; see *MAT_FABRIC for details. If not defined, the coat material is assumed to be elastic. |
| EB | Young's modulus along transverse direction; see *MAT_FABRIC for details. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.0.0: The Young's modulus along transverse direction is 10% of the Young's determined by LS-DYNA based on the loading curve, LLCID. |
| | LT.0.0: \|EB\| is the ratio of Young's modulus along the transverse direction to the Young's modulus determined by LS-DYNA based on the loading curve, LLCID. |
| | GT.0.0: EB is the Young's modulus along the transverse direction. |
| PRBA (PRAB) | Minor (Major) Poisson's ratio *ba* (*ab*) direction |
| GAB | Shear modulus in the *ab* direction. Set to a very small value for an isotropic elastic material; see *MAT_FABRIC. If defined to be zero, a default value of $EA/(2 \times (1 + PRBA))$ will be used where EA is the Young's modulus along the longitudinal direction and is set to 1% of the Young's modulus determined by LS-DYNA according to the loading curve, LLCID. |

**Remarks:**

1. **Loading and unloading.** Each belt material defines stretch characteristics and mass properties for a set of belt elements. The user enters a load curve for loading, the points of which are (Strain, Force). Strain is defined as engineering strain, that is,

$$\text{Strain} = \frac{\text{current length}}{\text{initial length}} - 1.0$$

Another similar curve is entered to describe the unloading behavior. Both load curves should start at the origin (0,0) and contain positive force and strain values only. The belt material is tension only with zero forces being generated whenever the strain becomes negative. The first non-zero point on the loading curve defines the initial yield point of the material. On unloading, the unloading curve is shifted along the strain axis until it crosses the loading curve at the "yield" point from which unloading commences. If the initial yield has not yet been exceeded or if the origin of the (shifted) unloading curve is at negative strain, the original loading curves will be used for both loading and unloading. If the strain is less than the strain at the origin of the unloading curve, the belt is slack and no force is generated. Otherwise, forces will then be determined by the unloading curve for unloading and reloading until the strain again exceeds yield after which the loading curves will again be used.

2. **Damping.** A small amount of damping is automatically included. This reduces high frequency oscillation, but, with realistic force-strain input characteristics and loading rates, does not significantly alter the overall forces-strain performance. The damping forced opposes the relative motion of the nodes and is limited by stability:

$$D = \frac{0.1 \times \text{ mass} \times \text{relative velocity}}{\text{time step size}}$$

In addition, the magnitude of the damping force is limited to one-tenth of the force calculated from the force-strain relationship and is zero when the belt is slack. Damping forces are not applied to elements attached to sliprings and retractors.

3. **Nodal masses.** MPUL, the mass per unit length, is used to calculate the nodal masses during initialization.

4. **Minimum length.** LMIN, the "minimum" length, controls the shortest length allowed in any element. It also determines when an element passes through sliprings or is absorbed into the retractors. A large LMIN causes elements to easily pass through the sliprings. A small LMIN leads to a smaller time step and possible instability for 2D belts. One tenth of a typical initial element length is a good choice for a 1D belt. For a 2D belt, a larger value of 0.3 can be used for better robustness and a larger time step.

5. **Bending and compression stiffness for 1D elements.** Since one-dimensional elements do not possess any bending or compression stiffness, dynamic analysis is mandatory during an implicit analysis that includes belts. However, one-dimensional belt elements *can* be used in implicit statics by associating them with bending/compression properties with the first optional card. Two-dimensional belt elements are not supported with this feature.

To achieve bending and compression stiffness in one-dimensional belts, the belt element is overlayed with a Belytschko-Schwer beam element (see *SECTION_BEAM, ELFORM = 2, for a more comprehensive description of fields A, I, J, and AS) with circular cross section. These elements have 6 degrees of freedom, including rotational degrees of freedom. The material used in this context is an elastic-ideal-plastic material where the elastic part is governed by the Young's modulus, $E$. Two yield values, F (the maximum compression/tension force) and M (the maximum torque), are used as upper bounds for the resultants. The bending/compression forces and moments from this contribution are added to the force from the seatbelt itself. Since the main purpose is to eliminate the singularities in bending and compression, it is recommended to choose the bending and compression properties in the optional card carefully so as not to significantly influence the overall response.

For the sake of completeness, this feature is also supported by the explicit integrator; therefore, a rotational nodal mass is needed. Each of the two nodes of an element gets a contribution from the belt that is calculated as RMASS = R × (MASS/2) × I/A, where MASS indicates the total translational mass of the belt element, and R is a scaling factor input by the user. The translational mass is not modified. The bending and compression properties do not affect the stable time step. If the belts are used *without* sliprings, then incorporating this feature is virtually equivalent to adding Belytschko-Schwer beams on top of conventional belt elements as part of the modelling strategy. If sliprings *are* used, this feature is necessary to properly support the flow of material through the sliprings and the swapping of belt elements across the sliprings. Retractors cannot be used with this feature.

6. **Compressive stress elimination.** From versions R8 through R11, eliminating the compressive stresses was always determined by the solver. As of R12, for nonzero FORM, CSE, which had existed prior to R8, was reused to specify the behavior for stress elimination.

# *MAT_THERMAL

The *MAT_THERMAL cards allow thermal properties to be defined in coupled structural/thermal and thermal only analyses; see *CONTROL_SOLUTION. Thermal properties must be defined for all elements in such analyses.

Thermal material properties are specified by a thermal material ID number (TMID). This number is independent of the material ID number (MID) defined on all other *MAT_… property cards. In the same analysis identical TMID and MID numbers may exist. The TMID and MID numbers are related through the *PART card.

Available thermal materials are:

*MAT_THERMAL_ISOTROPIC

*MAT_THERMAL_ORTHOTROPIC

*MAT_THERMAL_ISOTROPIC_TD

*MAT_THERMAL_ORTHOTROPIC_TD

*MAT_THERMAL_DISCRETE_BEAM

*MAT_THERMAL_CHEMICAL_REACTION

*MAT_THERMAL_CWM

*MAT_THERMAL_ORTHOTROPIC_TD_LC

*MAT_THERMAL_ISOTROPIC_PHASE_CHANGE

*MAT_THERMAL_ISOTROPIC_TD_LC

*MAT_THERMAL_USER_DEFINED

*MAT_THERMAL_CHEMICAL_REACTION_ORTHOTROPIC

## *MAT_THERMAL_ISOTROPIC

This is Thermal Material Type 1. With this material, isotropic thermal properties can be defined.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | TLAT | HLAT | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HC | TC | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density:<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 2.<br>GT.0: Load curve ID giving thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier:<br>EQ.0.0: No heat generation |
| TLAT | Phase change temperature |
| HLAT | Latent heat |

| VARIABLE | DESCRIPTION |
|---|---|
| HC | Specific heat |
| TC | Thermal conductivity |

**Remarks:**

1. **Supported Load Curves.** *DEFINE_CURVE_FUNCTION is fully supported for *MAT_THERMAL_ISOTROPIC (added in revision 113488).

2. **Thermal Generation Rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION. It has units $W/m^3$ in the SI units system.

**Example:**

```
*MAT_THERMAL_ISOTROPIC
      1     2700.       210        1.0
      904.      222.
*define_curve_function
210
if(lc211,lc10,lc12,lc11)
*define_curve
211
0,-200
2.0,-200
*define_curve
10
0,1.43e+07
100,1.43e+07
*define_curve
11
0,2.43e+07
100,2.43e+07
*define_curve
12
0,3.43e+07
100,3.43e+07
```

### *MAT_THERMAL_ORTHOTROPIC

This is Thermal Material Type 2. It allows orthotropic thermal properties to be defined.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
| Type | A | F | F | F | F | F | F | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HC | K1 | K2 | K3 | | | | |
| Type | F | F | F | F | | | | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density:<br>EQ.0.0: Default to structural density |

| VARIABLE | DESCRIPTION |
|---|---|
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve ID defining thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID giving thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier:<br>EQ.0.0: No heat generation |
| AOPT | Material axes definition:<br>EQ.0.0: Locally orthotropic with material axes by element nodes N1, N2 and N4<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and global location of element center<br>EQ.2.0: Globally orthotropic with material axes determined by vectors<br>EQ.3.0: Locally orthotropic with first material axis orthogonal to element normal (defined by element nodes N1, N2 and N4) and to a vector **d**. The third material direction corresponds to element normal.<br>EQ.4.0: Local orthogonal in cylindrical coordinates with the material axes determined by a vector **d**, and an originating point, $P$, which define the centerline axis. |
| TLAT | Phase change temperature |
| HLAT | Latent heat |
| HC | Specific heat |
| K1 | Thermal conductivity, $K_1$, in local $x$-direction |
| K2 | Thermal conductivity, $K_2$, in local $y$-direction |
| K3 | Thermal conductivity, $K_3$, in local $z$-direction |
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 and 4 |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2, 3 and 4 |

**Remarks:**

1. **Thermal Generation Rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION. It has units $W/m^3$ in the SI units system.

## *MAT_THERMAL_ISOTROPIC_TD

This is Thermal Material Type 3. The isotropic properties can be temperature dependent. The temperature dependency is defined by specifying a minimum of two and a maximum of eight data points. You should define the properties for the temperature range that the material will see in the analysis.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | TLAT | HLAT | | |
| Type | A | F | F | F | F | F | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |

| VARIABLE | DESCRIPTION |
|---|---|
| TRO | Thermal density:<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve ID giving thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier:<br>EQ.0.0: No heat generation |
| TLAT | Phase change temperature |
| HLAT | Latent heat |
| T1, …, T8 | Temperatures: T1, ..., T8 |
| C1, …, C8 | Specific heat at: T1, …, T8 |
| K1, …, K8 | Thermal conductivity at: T1, …, T8 |

**Remarks:**

1. **Thermal Generation Rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION. It has units $W/m^3$ in the SI units system.

## *MAT_THERMAL_ORTHOTROPIC_TD

This is Thermal Material Type 4. It allows temperature dependent orthotropic properties to be defined. The temperature dependency is defined by specifying a minimum of two and a maximum of eight data points. The properties must be defined for the temperature range that the material will see in the analysis.

### Card Summary:

**Card 1.** This card is required.

| TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| (K1)1 | (K1)2 | (K1)3 | (K1)4 | (K1)5 | (K1)6 | (K1)7 | (K1)8 |
|---|---|---|---|---|---|---|---|

**Card 5.** This card is required.

| (K2)1 | (K2)2 | (K2)3 | (K2)4 | (K2)5 | (K2)6 | (K2)7 | (K2)8 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| (K3)1 | (K3)2 | (K3)3 | (K3)4 | (K3)5 | (K3)6 | (K3)7 | (K3)8 |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| D1 | D2 | D3 | | | | | |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
| Type | A | F | F | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density:<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve ID defining thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier:<br>EQ.0.0: No heat generation |
| AOPT | Material axes definition (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes by element nodes N1, N2 and N4<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and global location of element center<br>EQ.2.0: Globally orthotropic with material axes determined by vectors<br>EQ.3.0: Locally orthotropic with first material axis orthogonal to element normal (defined by element nodes N1, N2 and N4) and to a vector **d**- Third material direction corresponds to element normal. |

| VARIABLE | DESCRIPTION |
|---|---|
| | EQ.4.0: Local orthogonal in cylindrical coordinates with the material axes determined by a vector **d**, and an originating point, $P$, which define the centerline axis. |
| TLAT | Phase change temperature |
| HLAT | Latent heat |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| T1, ..., T8 | Temperatures: T1, ..., T8 |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| C1, ..., C8 | Specific heat at T1, ..., T8 |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | (K1)1 | (K1)2 | (K1)3 | (K1)4 | (K1)5 | (K1)6 | (K1)7 | (K1)8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| (K1)1, ..., (K1)8 | Thermal conductivity $K_1$ in the local $x$-direction at T1, ..., T8 |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | (K2)1 | (K2)2 | (K2)3 | (K2)4 | (K2)5 | (K2)6 | (K2)7 | (K2)8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| (K2)1, ..., (K2)8 | Thermal conductivity $K_2$ in the local $y$-direction at T1, ..., T8 |

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | (K3)1 | (K3)2 | (K3)3 | (K3)4 | (K3)5 | (K3)6 | (K3)7 | (K3)8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| (K3)1, ..., (K3)8 | Thermal conductivity $K_3$ in the local $z$-direction at T1, ..., T8 |

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2, 3 and 4 |

**Remarks:**

1. **Thermal Generation Rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION and has units $W/m^3$ in the SI units system.

## *MAT_THERMAL_DISCRETE_BEAM

This is Thermal Material Type 5. It defines properties for discrete beams. It is only applicable when used with ELFORM = 6 on *SECTION_BEAM.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | | | | | | |
| Type | A | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HC | TC | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density:<br>EQ.0.0: default to structural density. |
| HC | Specific heat<br>HC = (heat transfer coefficient) × (beam cross-sectional area)<br>[W/K] = [W / $m^2$ K] × [$m^2$] |
| TC | Thermal conductance (SI units are W/K) |

### Remarks:

A beam cross-sectional area is not defined on *SECTION_BEAM for an ELFORM = 6 discrete beam. Heat transfer calculations require a beam cross-sectional area. Therefore, the cross-sectional area is lumped into the value entered for HC.

## *MAT_THERMAL_CHEMICAL_REACTION

This is thermal material type 6. The chemical species making up this material undergo specified chemical reactions. A maximum of 8 species and 8 chemical reactions can be defined. The thermal material properties of a finite element undergoing chemical reactions are calculated based on a mixture law consisting of those chemical species currently present in the element. The dependence of the chemical reaction rate on temperature is described by the Arrhenius equation. Time step splitting is used to couple the system of ordinary differential equations describing the chemical reaction kinetics to the system of partial differential equations describing the diffusion of heat.

### Card Summary:

**Card 1.** This card is required.

| TMID | NCHSP | NCHRX | ICEND | CEND | GASC | FID | MF |
|---|---|---|---|---|---|---|---|

**Card 2.** This card must be included but all parameters can be set to 0 if no filler material is present.

| RHOf | LCCf | LCKf | VFf | | | | |
|---|---|---|---|---|---|---|---|

**Card 3.** Include one card for each of the NCHSP species.

| RHO*i* | LCC*i* | LCK*i* | VF*i* | MW*i* | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** Include one card for each of the NCHSP species.

| RC*i*1 | RC*i*2 | RC*i*3 | RC*i*4 | RC*i*5 | RC*i*6 | RC*i*7 | RC*i*8 |
|---|---|---|---|---|---|---|---|

**Card 5.** Include one card for each of the NCHSP species.

| RX*i*1 | RX*i*2 | RX*i*3 | RX*i*4 | RX*i*5 | RX*i*6 | RX*i*7 | RX*i*8 |
|---|---|---|---|---|---|---|---|

**Card 6.** This card is required.

| LCZ1 | LCZ2 | LCZ3 | LCZ4 | LCZ5 | LCZ6 | LCZ7 | LCZ8 |
|---|---|---|---|---|---|---|---|

**Card 7.** This card is required.

| E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | NCHSP | NCHRX | ICEND | CEND | GASC | FID | MF |
| Type | A | I | I | I | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material ID. A unique number or label must be specified (see *PART). |
| NCHSP | Number of chemical species (maximum 8) |
| NCHRX | Number of chemical reactions (maximum 8) |
| ICEND | Species number controlling reaction termination |
| CEND | Concentration for reaction termination. Reactions are terminated when concentration of species ICEND exceeds CEND. |
| GASC | Gas constant: 1.987 cal/(mol K), 8.314 J/(mol K) |
| FID | Function ID for user specified chemical reaction rate equation for a single reaction model with two species |
| MF | ODE solver method:<br>EQ.0: Default<br>EQ.1: An alternative ODE solver for stiff differential equations |

**Filler Material Properties.** This card is used to the material properties for the filler material, such as carbon fiber mat. This card must be included but all parameters can be set to 0 if no filler material is present.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RHOf | LCCf | LCKf | VFf | | | | |
| Type | F | I | I | F | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| RHOf | Density of the filler material |
| LCCf | Load curve ID specifying the specific heat as a function of temperature for the filler material |
| LCKf | Load curve ID specifying the thermal conductivity as a function of temperature for the filler material |
| VFf | Volume fraction of the filler material. The remaining volume is occupied by the reacting chemicals. |

**Chemical Species Cards.** Include one card for each of the NCHSP species. These cards set species properties. The dummy index *i* is the species number and is equal to 1 for the first species card, 2 for the second, and so on.

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RHO*i* | LCC*i* | LCK*i* | VF*i* | MW*i* | | | |
| Type | F | I | I | F | F | | | |

**Reaction Cards.** Include one card for each of the NCHSP species. Each field contains the species's coefficient for one of the NCHRX chemical reactions. See Card 3 for explanation of the species index *i*.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RC*i*1 | RC*i*2 | RC*i*3 | RC*i*4 | RC*i*5 | RC*i*6 | RC*i*7 | RC*i*8 |
| Type | F | F | F | F | F | F | F | F |

**Reaction Rate Exponent Cards.** Include one card for each of the NCHSP species. Each field contains the specie's rate exponent for one of the NCHRX chemical reactions. See Card 3 for explanation of the species index *i*.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RX*i*1 | RX*i*2 | RX*i*3 | RX*i*4 | RX*i*5 | RX*i*6 | RX*i*7 | RX*i*8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| RHO*i* | Density of the $i^{th}$ species |
| LCC*i* | Load curve ID specifying the specific heat as a function of temperature for the $i^{th}$ species |
| LCK*i* | Load curve ID specifying the thermal conductivity as a function of temperature for the $i^{th}$ species |
| VF*i* | Initial fraction of the $i^{th}$ species relative to the other reacting chemicals. Note that $\sum_i \mathrm{VF}_i = 1$. |
| MW*i* | Molecular weight of the $i^{th}$ species |
| RC*ij* | Reaction coefficient $n_{ij}$ for species $i$ in reaction $j$. Leave blank for undefined reactions |
| RX*ij* | Rate exponent $p_{ij}$ for species $i$ in reaction $j$. Leave blank for undefined reactions. |

**Pre-exponential Factor Card.** Each field contains the natural logarithm of its corresponding reaction's pre-exponential factor.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCZ1 | LCZ2 | LCZ3 | LCZ4 | LCZ5 | LCZ6 | LCZ7 | LCZ8 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| LCZ*j* | Load curve defining data pairs of $(\text{temperature}, \ln Z_j)$ where $Z_j$ is the pre-exponential factor for reaction $j$. Leave blank for undefined reactions. |

**Activation Energy Card.** Each field contains the activation energy value for its corresponding reaction.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| E*j* | Activation energy for reaction *j*. Leave blank for undefined reactions. |

**Heat of Reaction Card.** Each field contains the heat of reaction value for its corresponding reaction.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| Q*j* | Heat of reaction for reaction *j*. Leave blank for undefined reactions. |

### Rate Model for a Single Reaction:

Chemical reactions are usually expressed in *chemical equation notation*; for example, a chemical reaction involving two reactants and two products is

$$aA + bB \rightarrow gG + hH, \tag{MT6.1}$$

where A, B, G, and H are chemical species such as NaOH or HCl, and $a$, $b$, $g$, and $h$ are integers called *stoichiometric numbers*, indicating the number of molecules involved in a single reaction.

*The rate of reaction is the number of individual reactions per unit time*. Using a stoichiometric identity, which is just an accounting relation, the rate of reaction is proportional to the rate of change in the concentrations of the species involved in the reaction. For the chemical reaction in Equation (MT6.1), the relation between concentration and rate, $r$, is,

$$r = -\frac{1}{a}\frac{\mathrm{d}[\mathrm{A}]}{\mathrm{d}t} = -\frac{1}{b}\frac{\mathrm{d}[\mathrm{B}]}{\mathrm{d}t} = +\frac{1}{g}\frac{\mathrm{d}[\mathrm{G}]}{\mathrm{d}t} = +\frac{1}{h}\frac{\mathrm{d}[\mathrm{H}]}{\mathrm{d}t}, \quad \text{(MT6.2)}$$

where [X] denotes the concentration of species X, and the sign depends on whether or not the species is an input, in which case the sign is negative, or a product, in which case the sign is positive.

### The Model

This thermal material model (T06) is built on the assumption that the reaction rate $r_j$ of reaction $j$ depends on the concentration of the input species according to

$$r_j = k_j(T) \prod_i [\mathrm{X}_i]^{p_{ij}} ,$$

where $i$ ranges over all species, and, for each species, the exponent, $p_{ij}$, is determined by empirical measurement but may be approximated by the stoichiometric number associated with $\mathrm{X}_i$. The proportionality constant, $k_j$, is related to the cross-section for the reaction, and it depends on temperature through the Arrhenius equation:

$$k_j = Z_j(T) \exp\left(-\frac{E_j}{RT}\right) ,$$

where $Z_j(T)$ is experimentally determined (see Card 6), $E_j$ is the activation energy (see Card 7), $R$ is the gas constant, and $T$ is temperature.

As an example, for the chemical reaction of Equation (MT6.1)

$$r = Z(T) \exp\left(-\frac{E}{RT}\right) [\mathrm{A}]^{\alpha}[\mathrm{B}]^{\beta} ,$$

where the stoichiometric numbers have been used instead of experimentally determined exponents.

The rate of heat generation (exothermic) and absorption (endothermic) associated with a reaction is calculated by multiplying the heat of reaction, $Q_i$, by its rate.

### User-Defined Single Reaction Model:

In addition to the standard model described in the previous section this material model also allows the user to specify a chemical reaction rate equation. In case of a single reaction model with two species, the equation can be defined using the *DEFINE_FUNCTION keyword. Here, the equation can be given as function of the current temperature and concentration of the second species. Consequently, the input may read as follows:

```
*DEFINE_FUNCTION
1, reaction rate
chemrx(temp,conc)= …
```

Note that in this case the actual argument names are used for the interpretation of the function: the material model expects the temperature argument to be names "temp" and the concentration of the second species to be denoted by "conc".

### Rate Model for a System of Reactions:

For a system of coupled chemical reactions, the change in concentration of a species is the sum of all the contributions from each individual chemical reaction:

$$\frac{\mathrm{d}[\mathrm{X}_i]}{\mathrm{d}t} = \sum_j n_{ij} r_j \quad .$$

The index $j$ runs over all reactions; $n_{ij}$ is the stoichiometric number for species $\mathrm{X}_i$ in reaction $j$; and $r_j$ is the rate of reaction $j$. The sign of $n_{ij}$ is positive for reactions that have $\mathrm{X}_i$ as a product and negative for reactions that involve $\mathrm{X}_i$ as an input.

### Example:

Consider the following system of reactions (three reactions and three species):

$$\mathrm{A} \rightarrow \mathrm{B} \quad \Rightarrow \quad r_1 = -\frac{\mathrm{d}[\mathrm{A}]}{\mathrm{d}t} = \frac{\mathrm{d}[\mathrm{B}]}{\mathrm{d}t} = k_1[\mathrm{A}] \quad \Rightarrow \quad \boxed{\begin{aligned} \dot{[\mathrm{A}]} &= -k_1[\mathrm{A}] \\ \dot{[\mathrm{B}]} &= +k_1[\mathrm{A}] \end{aligned}}$$

$$\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C} \quad \Rightarrow \quad r_2 = -\frac{\mathrm{d}[\mathrm{A}]}{\mathrm{d}t} = -\frac{\mathrm{d}[\mathrm{B}]}{\mathrm{d}t} = \frac{\mathrm{d}[\mathrm{C}]}{\mathrm{d}t} = k_2[\mathrm{A}][\mathrm{B}] \quad \Rightarrow \quad \boxed{\begin{aligned} \dot{[\mathrm{A}]} &= -k_2[\mathrm{A}][\mathrm{B}] \\ \dot{[\mathrm{B}]} &= -k_2[\mathrm{A}][\mathrm{B}] \\ \dot{[\mathrm{C}]} &= +k_2[\mathrm{A}][\mathrm{B}] \end{aligned}}$$

$$2\mathrm{B} \rightarrow \mathrm{C} \quad \Rightarrow \quad r_3 = -\frac{1}{2}\frac{\mathrm{d}[\mathrm{B}]}{\mathrm{d}t} = \frac{\mathrm{d}[\mathrm{C}]}{\mathrm{d}t} = k_3[\mathrm{B}]^2 \quad \Rightarrow \quad \boxed{\begin{aligned} \dot{[\mathrm{B}]} &= -2k_3[\mathrm{B}]^2 \\ \dot{[\mathrm{C}]} &= +k_3[\mathrm{B}]^2 \end{aligned}}$$

The identities $\mathrm{X}_1 = A$, $\mathrm{X}_2 = B$, and $\mathrm{X}_3 = C$ allow deducing the rate exponents $p_{ij}$, where both indices $i$ (species) and $j$ (reaction) range from 1 to 3. The nonzero values are

$$\begin{aligned} p_{11} &= 1.0 \\ p_{12} &= 1.0 \\ p_{22} &= 1.0 \\ p_{23} &= 2.0 \end{aligned} \tag{MT6.3}$$

These values are needed as input parameters $\mathrm{RX}_{ij}$ (see card 5 in example input below).

The time evolution equations are,

(MT6.4)

$$\frac{\mathrm{d}[\mathrm{B}]}{\mathrm{d}t} = \sum n_{2j} r_j = +k_1[\mathrm{A}] - k_2[\mathrm{A}][\mathrm{B}] - 2k_3[\mathrm{B}]^2$$
$$\frac{\mathrm{d}[\mathrm{C}]}{\mathrm{d}t} = \sum n_{3j} r_j = \qquad\quad + k_2[\mathrm{A}][\mathrm{B}] + \; k_3[\mathrm{B}]^2.$$

**Equivalent Units (Normalized Units):**

The concentrations are often scaled so that each unit of reactant yields one unit of product. Systems for which each species is assigned *its own* unit of concentration based on stoichiometric considerations are *equivalent unit systems*.

Being unit-agnostic, LS-DYNA is capable of working in equivalent units. However, care must be taken so that units are treated consistently, as applying a unit scaling to the time evolution equations can be nontrivial.

1. For each reaction, the experimentally measured pre-exponential coefficients carry units that depend on the reaction itself. For instance, the pre-exponential factors $Z_1$, $Z_2$, and $Z_3$ for the reactions $\mathrm{A} \rightarrow \mathrm{B}$, $\mathrm{A} + \mathrm{B} \rightarrow \mathrm{C}$, and $2\mathrm{B} \rightarrow \mathrm{C}$ respectively will have units of

$$[Z_1] = \frac{1}{[\text{time}]} \times \frac{1}{[\text{Concentration of A}]}$$
$$[Z_2] = \frac{1}{[\text{time}]} \times \frac{1}{[\text{Concentration of A}]} \times \frac{1}{[\text{Concentration of B}]}$$
$$[Z_3] = \frac{1}{[\text{time}]} \times \left\{ \frac{1}{[\text{Concentration of B}]} \right\}^2.$$

   Note that each pre-factor has a different dimensionality.

2. The equations in (MT6.2), which relate rate to concentration change, are logically inconsistent unless all species are measured using the *same* units for concentration. A species-dependent system of equivalent units would require the insertion of additional conversion factors into (MT6.2) thereby changing the form of the time-evolution equations.

> To avoid unit consistency issues, we recommend that reactions be defined in the same unit system that was used to measure their empirical values.

**Example of Equivalent Units:**

The following system of reactions:

$$A \rightarrow B$$
$$A + B \rightarrow C$$
$$2B \rightarrow C$$

changes species A into species C through an intermediate which is species B. For each unit of species C that is produced, the reaction consumes two units of species A (1 unit from the 1st and 1 unit from the 2nd equation). Since this set of chemical formulae corresponds to the curing of epoxy, which is a nearly volume-preserving process, it is customary to work in a system of equivalent units that correspond to species volume fractions.

The following set of equivalent units, then, is used in the published literature:

1. Whatever the starting concentration of species A is, all units are uniformly rescaled so that $[A] = 1$ at time zero. Per the boxed remark above, since the constants were measured with respect to these units, this consideration does not introduce new complexity.

2. Since the process preserves volume, and since one particle of species C replaces two particles of species A (and one particle of B replace one of A), the units of concentration for species C are doubled.

$$\tilde{C} = 2[C]$$

Under this transformation the rate relation for C is

$$r_2 = r_3 = \frac{d[C]}{dt} = \frac{1}{2}\frac{d\tilde{C}}{dt} .$$

The time evolution Equations (MT6.4) become, (note [C] has been replaced by $\tilde{C}$):

$$\begin{aligned}
\frac{d[A]}{dt} &= \sum n_{1j} r_j &&= -k_1[A] - k_2[A][B] \\
\frac{d[B]}{dt} &= \sum n_{2j} r_j &&= +k_1[A] - k_2[A][B] - 2k_3[B]^2 \\
\frac{d\tilde{C}}{dt} = 2\frac{d[C]}{dt} &= 2\sum n_{3j} r_j = \sum \tilde{n}_{3j} r_j &&= +2k_2[A][B] + 2k_3[B]^2
\end{aligned}$$

The coefficients $n_{ij}$ or $\tilde{n}_{ij}$, respectively, should be identically copied from the above system as reaction coefficients $RC_{ij}$ (for Card 4):

| Variable | RC11 | RC12 | RC13 | RC14 | RC15 | RC16 | RC17 | RC18 |
|---|---|---|---|---|---|---|---|---|
| Value | -1 | -1 | 0 | | | | | |

| Variable | RC21 | RC22 | RC23 | RC24 | RC25 | RC26 | RC27 | RC28 |
|---|---|---|---|---|---|---|---|---|
| Value | +1 | -1 | -2 | | | | | |
| Variable | RC31 | RC32 | RC33 | RC34 | RC35 | RC36 | RC37 | RC38 |
| Value | 0 | +2 | +2 | | | | | |

The exponents $RX_{ij}$ are likewise picked off (Equations (MT6.3)) for next set of cards in format 5:

| Variable | RX11 | RX12 | RX13 | RX14 | RX15 | RX16 | RX17 | RX18 |
|---|---|---|---|---|---|---|---|---|
| Value | +1 | +1 | 0 | | | | | |
| Variable | RX21 | RX22 | RX23 | RX24 | RX25 | RX26 | RX27 | RX28 |
| Value | 0 | +1 | +2 | | | | | |
| Variable | RX31 | RX32 | RX33 | RX34 | RX35 | RX36 | RX37 | RX38 |
| Value | 0 | 0 | 0 | | | | | |

## *MAT_THERMAL_CWM

This is Thermal Material Type 7. It is a thermal material with temperature dependent properties that allows for material creation triggered by temperature. The acronym CWM stands for Computational Welding Mechanics and the model is intended to be used for simulating multistage weld processes in combination with the mechanical counterpart, *MAT_CWM.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | HDEAD | TDEAD | TLAT | HLAT |
| Type | A | F | F | F | F | F | F | F |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCHC | LCTC | TLSTART | TLEND | TISTART | TIEND | HGHOST | TGHOST |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density:<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE):<br>GT.0: Load curve ID defining thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature.<br>This feature is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION and has units $W/m^3$ in the SI units system. |

| VARIABLE | DESCRIPTION |
|---|---|
| TGMULT | Thermal generation rate multiplier:<br>EQ.0.0: No heat generation |
| HDEAD | Specific heat for inactive material before birth time |
| TDEAD | Thermal conductivity for inactive material before birth time |
| TLAT | Phase change temperature |
| HLAT | Latent heat |
| LCHC | Load curve (or table) for specific heat as function of temperature (and maximum temperature up to current time) |
| LCTC | Load curve for thermal conductivity as function of temperature |
| TLSTART | Birth temperature of material start |
| TLEND | Birth temperature of material end |
| TISTART | Birth time start |
| TIEND | Birth time end |
| HGHOST | Specific heat for ghost (quiet) material |
| TGHOST | Thermal conductivity for ghost (quiet) material |

**Remarks:**

This material is initially in a quiet state, sometimes referred to as a *ghost material*. In this state the material has the thermal properties defined by the quiet specific heat (HGHOST) and quiet thermal conductivity (TGHOST). These should represent the void, for example, by picking a relatively small thermal conductivity.

However, the ghost specific heat must be chosen with care since the temperature must be allowed to increase at a reasonable rate due to the heat from the weld source. When the temperature reaches the birth temperature, a history variable representing the indicator of the welding material is incremented. This variable follows

$$\gamma(t) = \min\left[1, \max\left(0, \frac{T_{\max} - T_l^{\text{start}}}{T_l^{\text{end}} - T_l^{\text{start}}}\right)\right],$$

where $T_{\max} = \max\{T(s) | s < t\}$.

The effective thermal material properties are interpolated as

$$\tilde{c}_p = c_p(T, T_{\max})\gamma + c_p^{\text{quiet}}(1-\gamma)$$
$$\tilde{\mu} = \mu(T)\gamma + \mu^{\text{quiet}}(1-\gamma)$$

where $c_p$ and $\mu$ are the specific heat and thermal conductivity, respectively. Here, the specific heat, $c_p$, is either a temperature dependent curve, or a collection of temperature dependent curves, ordered in a table according to maximum temperature $T_{\max}$.

The time parameters for creating the material provide additional formulae for the final values of the thermal properties. Before the birth time $t_i^{\text{start}}$ of the material has been reached, the specific heat $c_p^{\text{dead}}$ and thermal conductivity $\mu^{\text{dead}}$ are used. The default values, that is, the values used if no user input is given, are

$$c_p^{\text{dead}} = 10^{10} c_p(T, T_{\max})$$
$$\mu^{\text{dead}} = 0$$

Thus, the final values of the thermal properties read

$$c_p = \begin{cases} c_p^{\text{dead}} & t \le t_i^{\text{start}} \\ \tilde{c}_p \dfrac{t - t_i^{\text{start}}}{t_i^{\text{end}} - t_i^{\text{start}}} + c_p^{\text{dead}} \dfrac{t - t_i^{\text{end}}}{t_i^{\text{start}} - t_i^{\text{end}}} & t_i^{\text{start}} < t \le t_i^{\text{end}} \\ \tilde{c}_p & t_i^{\text{end}} < t \end{cases}$$

$$\mu = \begin{cases} \mu^{\text{dead}} & t \le t_i^{\text{start}} \\ \tilde{\mu} \dfrac{t - t_i^{\text{start}}}{t_i^{\text{end}} - t_i^{\text{start}}} + \mu^{\text{dead}} \dfrac{t - t_i^{\text{end}}}{t_i^{\text{start}} - t_i^{\text{end}}} & t_i^{\text{start}} < t \le t_i^{\text{end}} \\ \tilde{\mu} & t_i^{\text{end}} < t \end{cases}$$

These parameters allow you to control when the welding layer becomes active and thereby define a multistage welding process. Prior to the birth time, the temperature is kept more or less constant due to the large specific heat, and, thus, the material is prevented from being created

## *MAT_THERMAL_ORTHOTROPIC_TD_LC

This is Thermal Material Type 8. With this model, orthotropic thermal properties that are dependent on temperature (and/or mechanical history variables and/or external variables) can be specified with load curves. The properties must be defined for the temperature (and/or history variable and/or external variable) range that the material sees in the analysis. See *LOAD_EXTERNAL_VARIABLE for defining external variables and Remark 2.

### Card Summary:

**Card 1a.** Include this card if ITGHSV = 0 (see Card 2).

| TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
|---|---|---|---|---|---|---|---|

**Card 1b.** Include this card if |ITGHSV| > 0 (see Card 2).

| TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| LCC | LCK1 | LCK2 | LCK3 | ILCCHSV | ILCKHSV | ITGHSV | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card is required.

| D1 | D2 | D3 | | | | | |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

Include this card if ITGHSV = 0 (see Card 2).

| Card 1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
| Type | A | F | I | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density.<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve ID defining thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier.<br>EQ.0.0: No heat generation |
| AOPT | Material axes definition (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes by element nodes N1, N2 and N4<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and global location of element center<br>EQ.2.0: Globally orthotropic with material axes determined by vectors<br>EQ.3.0: Locally orthotropic with first material axis orthogonal to element normal (defined by element nodes N1, N2 and N4) and to a vector **d**- Third material direction corresponds to element normal.<br>EQ.4.0: Local orthogonal in cylindrical coordinates with the material axes determined by a vector **d**, and an originating point, *P*, which define the centerline axis. |
| TLAT | Phase change temperature |
| HLAT | Latent heat |

Include this card if |ITGHSV| > 0 (see Card 2).

| Card 1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | AOPT | TLAT | HLAT | |
| Type | A | F | I | F | F | F | F | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density.<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate curve/table ID (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve giving thermal generation rate as a function of the mechanical history variable specified by ITGHSV.<br>EQ.0: Use mechanical history variable specified by ITGHSV times constant multiplier value TGMULT.<br>LT.0: Table of load curves for different temperatures. Each curve gives the thermal generation rate as a function of the mechanical history variable specified by ITGHSV. |
| TGMULT | Thermal generation rate multiplier. Defines a volumetric heat rate ($W/m^3$ in SI units system).<br>EQ.0.0: No heat generation |
| AOPT | Material axes definition (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes by element nodes N1, N2 and N4<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and global location of element center<br>EQ.2.0: Globally orthotropic with material axes determined by vectors<br>EQ.3.0: Locally orthotropic with first material axis orthogonal to element normal (defined by element nodes N1, N2 |

| VARIABLE | DESCRIPTION |
|---|---|
| | and N4) and to a vector **d**- Third material direction corresponds to element normal.<br>EQ.4.0: Local orthogonal in cylindrical coordinates with the material axes determined by a vector **d**, and an originating point, $P$, which define the centerline axis. |
| TLAT | Phase change temperature |
| HLAT | Latent heat |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCC | LCK1 | LCK2 | LCK3 | ILCCHSV | ILCKHSV | ITGHSV | |
| Type | I | I | I | I | I | I | I | |

| VARIABLE | DESCRIPTION |
|---|---|
| LCC | Load curve ID defining specific heat as a function of temperature or external variable (see Remark 2), or if \|ILCCHSV\| > 0:<br>GT.0: Load curve as a function of mechanical history variable specified by ILCCHSV.<br>LT.0: Table of load curves for different temperatures. Each curve is a function of the mechanical history variable specified by ILCCHSV. |
| LCK*i* | Load curve ID defining thermal conductivity, $K_i$ (i = 1,2,3), in the local $(x,y,z)$-direction as a function of temperature or external variable (see Remark 2), or if \|ILCKHSV\| > 0:<br>GT.0: Load curve giving thermal conductivity in the local direction as a function of the mechanical history variable specified by ILCKHSV.<br>LT.0: Table of load curves for different temperatures. Each curve gives thermals conductivity in the local direction as a function of the mechanical history variable specified by ILCKHSV. |
| ILCCHSV | Optional:<br>GT.0: Mechanical history variable # used by LCC. |

| VARIABLE | DESCRIPTION |
|---|---|
| | LT.0: As above but \|ILCCHSV\| = 1 through 6 means the history variable is one of the six stress components, \|ILCCHSV\| = 7 means the history variable is the plastic strain, and \|ILCCHSV\| = 7 + $k$ means the history variable is history variable $k$. |
| ILCKHSV | Optional:<br>GT.0: Mechanical history variable # used by LCK1, LCK2, LCK3.<br>LT.0: As above but \|ILCKHSV\| = 1 through 6 means the history variable is one of the six stress components, \|ILCKHSV\| = 7 means the history variable is the plastic strain, and \|ILCKHSV\| = 7 + $k$ means the history variable is history variable $k$. |
| ITGHSV | Optional:<br>GT.0: Mechanical history variable # used by TGRLC.<br>LT.0: As above but \|ITGHSV\| = 1 through 6 means the history variable is one of the six stress components, \|ITGHSV\| = 7 means the history variable is the plastic strain, and \|ITGHSV\| = 7 + $k$ means the history variable is history variable $k$. |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point $p$ for AOPT = 1 and 4 |

| VARIABLE | DESCRIPTION |
|---|---|
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2, 3 and 4 |

**Remarks:**

1. **Thermal generation rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION and has units $W/m^3$ in the SI units system

2. **Effect of external variables.** By default, material properties can be defined as a function of the temperature field, but this material also supports material definitions based on a given distribution of an external variable (see *LOAD_EXTERNAL_VARIABLE). In that case, one or more of the load curves, LCC, LCK1, LCK2, and LCK3, are evaluated based on the external variable data instead of temperature. To do this, set ITMP on *LOAD_EXTERNAL_VARIABLE to the material property index for the desired thermal material property. The following table lists the material property indices:

| Property index | Property name | Load curve |
|---|---|---|
| 1 | Specific heat | LCC |
| 2 | Thermal conductivity in the $x$-direction | LCK1 |
| 3 | Thermal conductivity in the $y$-direction | LCK2 |
| 4 | Thermal conductivity in the $z$-direction | LCK3 |

## *MAT_THERMAL_ISOTROPIC_PHASE_CHANGE

This is Thermal Material Type 9. With this material, temperature dependent isotropic properties with phase change can be defined. The latent heat of the material is defined together with the solid and liquid temperatures. The temperature dependency is defined by specifying a minimum of two and a maximum of eight data points. The properties must be defined for the temperature range that the material will see in the analysis.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | | | | |
| Type | A | F | F | F | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | T1 | T2 | T3 | T4 | T5 | T6 | T7 | T8 |
| Type | F | F | F | F | F | F | F | F |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | C1 | C2 | C3 | C4 | C5 | C6 | C7 | C8 |
| Type | F | F | F | F | F | F | F | F |

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | K1 | K2 | K3 | K4 | K5 | K6 | K7 | K8 |
| Type | F | F | F | F | F | F | F | F |

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | SOLT | LIQT | LH | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density:<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 2.<br>GT.0: Load curve ID defining thermal generation rate as a function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier:<br>EQ.0.0: No heat generation |
| T1, …, T8 | Temperatures: T1, …, T8 |
| C1, …, C8 | Specific heat at T1, …, T8 |
| K1, …, K8 | Thermal conductivity at T1, …, T8 |
| SOLT | Solid temperature, $T_S$ (must be < $T_L$) |
| LIQT | Liquid temperature, $T_L$ (must be > $T_S$) |
| LH | Latent heat |

**Remarks:**

1. **Phase Change.** During phase change, meaning between the solid and liquid temperatures, the specific heat of the material will be enhanced to account for the latent heat as follows:

$$c(t) = m\left[1 - \cos 2\pi\left(\frac{T - T_S}{T_L - T_S}\right)\right], \qquad T_S < T < T_L$$

Here $m$ is a multiplier such that the latent heat $\lambda$ is given by:

$$\lambda = \int_{T_S}^{T_L} c(T)dT \ .$$

Here $c(T)$ is the specific heat.

2. **Thermal Generation Rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION and has units $W/m^3$ in the SI units system.

## *MAT_THERMAL_ISOTROPIC_TD_LC

This is Thermal Material Type 10. With this model, isotropic thermal properties that are dependent on temperature (and/or mechanical history variables and/or external variables) can be specified with load curves. The properties must be defined for the temperature (and/or history variable and/or external variable) range that the material sees in the analysis. See *LOAD_EXTERNAL_VARIABLE for defining external variables and Remark 2.

### Card Summary:

**Card 1a.** Include this card if TGHSV = 0.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TMID | TRO | TGRLC | TGMULT | TLAT | HLAT | | |

**Card 1b.** Include this card if TGHSV ≠ 0.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| TMID | TRO | TGRLC | TGMULT | TLAT | HLAT | | |

**Card 2.** This card is required.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| HCLC | TCLC | HCHSV | TCHSV | TGHSV | | | |

### Data Card Definitions:

Include this card if TGHSV = 0 (see Card 2).

| Card 1a | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | TLAT | HLAT | | |
| Type | A | F | I | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density.<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve ID defining thermal generation rate as a |

| VARIABLE | DESCRIPTION |
|---|---|
| | function of time<br>EQ.0: Thermal generation rate is the constant multiplier, TGMULT.<br>LT.0: \|TGRLC\| is a load curve ID defining thermal generation rate as a function of temperature. |
| TGMULT | Thermal generation rate multiplier.<br>EQ.0.0: No heat generation |
| TLAT | Phase change temperature |
| HLAT | Latent heat |

This card is included if |TGHSV| > 0 (see Card 2).

| Card 1b | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | TGRLC | TGMULT | TLAT | HLAT | | |
| Type | A | F | I | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| TRO | Thermal density.<br>EQ.0.0: Default to structural density |
| TGRLC | Thermal generation rate curve/table ID (see *DEFINE_CURVE). See Remark 1.<br>GT.0: Load curve specifying thermal generation rate as a function of the mechanical history variable specified by TGHSV.<br>EQ.0: Use mechanical history variable specified by ITGHSV times constant multiplier value TGMULT.<br>LT.0: Table of load curves for different temperatures. Each curve specifies the thermal generation rate as a function of the mechanical history variable specified by TGHSV. |

| VARIABLE | DESCRIPTION |
|---|---|
| TGMULT | Thermal generation rate multiplier. Defines a volumetric heat rate (W/m^3 in SI units system).<br>EQ.0.0: No heat generation |
| TLAT | Phase change temperature |
| HLAT | Latent heat |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HCLC | TCLC | HCHSV | TCHSV | TGHSV | | | |
| Type | I | I | I | I | I | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| HCLC | Load curve ID specifying specific heat as a function of temperature or external variable (see Remark 2), or, if \|HCHSV\| > 0:<br>GT.0: Load curve specifying the specific heat as a function of the mechanical history variable specified by HCHSV.<br>LT.0: Table of load curves for different temperatures. Each curve specifies the specific heat as a function of the mechanical history variable specified by HCHSV. |
| TCLC | Load curve ID specifying thermal conductivity as a function of temperature or external variable (see Remark 2), or if \|TCHSV\| > 0:<br>GT.0: Load curve giving thermal conductivity as a function of the mechanical history variable specified by TCHSV.<br>LT.0: Table of load curves for different temperatures. Each curve is a function of the mechanical history variable specified by TCHSV. |
| HCHSV | Optional:<br>GT.0: Mechanical history variable # used by HCLC.<br>LT.0: As above but \|HCHSV\| = 1 through 6 means that the variable is one of the six stress components, \|HCHSV\| = 7 means that the variable is the plastic strain, and |

| VARIABLE | DESCRIPTION |
|---|---|
| | \|HCHSV\| = 7 + $k$ means that the variable is history variable $k$. |
| TCHSV | Optional:<br>GT.0: Mechanical history variable # used by TCLC.<br>LT.0: As above but \|TCHSV\| = 1 through 6 means the variable is one of the six stress components, \|TCHSV\| = 7 means the variable is the plastic strain, and \|TCHSV\| = 7 + $k$ means that the variable is history variable $k$. |
| TGHSV | Optional:<br>GT.0: Mechanical history variable # used by TGRLC.<br>LT.0: As above but \|TGHSV\| = 1 through 6 means the variable is one of the six stress components, \|TGHSV\| = 7 means the variable is the plastic strain, and \|TGHSV\| = 7 + $k$ means that the variable is history variable $k$. |

**Remarks:**

1. **Thermal generation rate.** TGRLC is similar to the volumetric heat generation rate in *LOAD_HEAT_GENERATION. It has units $W/m^3$ in the SI units system.

2. **Effect of external variables.** By default, material properties can be defined as a function of the temperature field, but this material also supports material definitions based on a given distribution of an external variable (see *LOAD_EXTERNAL_VARIABLE). In that case, HCLC and/or TCLC are evaluated based on the external variable data. To do this, set ITMP on *LOAD_EXTERNAL_VARIABLE to the material property index for the desired thermal material property. The following table lists the material property indices:

| Property index | Property name | Load curve |
|---|---|---|
| 1 | Specific heat | HCLC |
| 2 | Thermal conductivity | TCLC |

## *MAT_THERMAL_USER_DEFINED

These are Thermal Material Types 11 - 15. You can supply your own subroutines. Please consult Appendix H for more information.

### Card Summary:

**Card 1.** This card is required.

| TMID | RO | MT | LMC | NVH | AOPT | IORTHO | IHVE |
|---|---|---|---|---|---|---|---|

**Card 1.1.** This card is included if IORTHO = 1.

| XP | YP | ZP | A1 | A2 | A3 | | |
|---|---|---|---|---|---|---|---|

**Card 1.2.** This card is included if IORTHO = 1.

| D1 | D2 | D3 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 2.** Up to 4 of this card can be included to set LMC parameters. This input ends at the next keyword ("*") card.

| P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
|---|---|---|---|---|---|---|---|

### Data Card Definitions:

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | RO | MT | LMC | NVH | AOPT | IORTHO | IHVE |
| Type | A | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| RO | Thermal mass density |
| MT | User material type (11-15 inclusive) |
| LMC | Length of material constants array. LMC must not be greater than 32. |

| VARIABLE | DESCRIPTION |
|---|---|
| NVH | Number of history variables |
| AOPT | Material axes option of orthotropic materials (see MAT_*OPTION*-TROPIC_ELASTIC for more details). Set if IORTHO = 1.0.<br>EQ.0.0: Locally orthotropic with material axes by element nodes N1, N2 and N4<br>EQ.1.0: Locally orthotropic with material axes determined by a point, *P*, in space and global location of element center<br>EQ.2.0: Globally orthotropic with material axes determined by vectors<br>EQ.3.0: Locally orthotropic with first material axis orthogonal to element normal (defined by element nodes N1, N2 and N4) and to a vector **d**- Third material direction corresponds to element normal.<br>EQ.4.0: Local orthogonal in cylindrical coordinates with the material axes determined by a vector **d**, and an originating point, *P*, which define the centerline axis.<br>LT.0.0: The absolute value of AOPT is a coordinate system ID number (CID on *DEFINE_COORDINATE_NODES, *DEFINE_COORDINATE_SYSTEM or *DEFINE_COORDINATE_VECTOR). |
| IORTHO | Set to 1.0 if the material is orthotropic. |
| IHVE | Set to 1.0 to activate exchange of history variables between mechanical and thermal user material models. |

**Orthotropic Card 1.** Additional card read in when IORTHO = 1.

| Card 1.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | XP | YP | ZP | A1 | A2 | A3 | | |
| Type | F | F | F | F | F | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| XP, YP, ZP | Coordinates of point *P* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |

**Orthotropic Card 2.** Additional card read in when IORTHO = 1.

| Card 1.2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| D1, D2, D3 | Components of vector **d** for AOPT = 2, 3 and 4 |

**Material Parameter Cards.** Set up to 8 parameters per card. Include up to 4 cards. This input ends at the next keyword ("*") card.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| P1 | First material parameter |
| ⋮ | ⋮ |
| PLMC | LMC$^{th}$ material parameter |

**Remarks:**

1. **IHVE.** The IHVE = 1 option makes it possible for a thermal user material subroutine to read the history variables of a mechanical user material subroutine defined for the same part and vice versa. If the integration points for the thermal and mechanical elements are not coincident, then extrapolation/interpolation is used to calculate the value when reading history variables.

2. **TITLE.** Option TITLE is supported

3. **Units Transformation.** Transformation of units using *INCLUDE_TRANSFORM is only supported for the RO field and the vectors on Cards 1.1 and 1.2.

## *MAT_THERMAL_CHEMICAL_REACTION_ORTHOTROPIC

This is Thermal Material Type 17. The chemical species making up this material undergo specified chemical reactions. The chemical reaction kinetics is the same as for thermal material *MAT_T06, but the thermal conductivity is assumed to be orthotropic. A maximum of 8 species and 8 chemical reactions can be defined. The orthotropic thermal material properties of a finite element undergoing chemical reactions are calculated based on a mixture law consisting of those chemical species currently present in the element. The dependence of the chemical reaction rate on temperature is described by the Arrhenius equation. Time step splitting is used to couple the system of ordinary differential equations describing the chemical reaction kinetics to the system of partial differential equations describing the diffusion of heat.

### Card Summary:

**Card 1.** This card is required.

| TMID | NCHSP | NCHRX | ICEND | CEND | GASC | FID | MF |
|---|---|---|---|---|---|---|---|

**Card 2.** This card is required.

| AOPT | XP | YP | ZP | A1 | A2 | A3 | |
|---|---|---|---|---|---|---|---|

**Card 3.** This card is required.

| D1 | D2 | D3 | | | | | |
|---|---|---|---|---|---|---|---|

**Card 4.** This card must be included, but all parameters can be set to 0 if no filler material is present.

| RHOf | LCCf | LCK1f | LCK2f | LCK3f | VFf | | |
|---|---|---|---|---|---|---|---|

**Card 5.** Include one card for each of the NCHSP species.

| RHO*i* | LCC*i* | LCK1*i* | LCK2*i* | LCK3*i* | VF*i* | MW*i* | |
|---|---|---|---|---|---|---|---|

**Card 6.** Include one card for each of the NCHSP species.

| RC*i*1 | RC*i*2 | RC*i*3 | RC*i*4 | RC*i*5 | RC*i*6 | RC*i*7 | RC*i*8 |
|---|---|---|---|---|---|---|---|

**Card 7.** Include one card for each of the NCHSP species.

| RX*i*1 | RX*i*2 | RX*i*3 | RX*i*4 | RX*i*5 | RX*i*6 | RX*i*7 | RX*i*8 |
|---|---|---|---|---|---|---|---|

**Card 8.** This card is required.

| LCZ1 | LCZ2 | LCZ3 | LCZ4 | LCZ5 | LCZ6 | LCZ7 | LCZ8 |
|---|---|---|---|---|---|---|---|

**Card 9.** This card is required.

| E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
|---|---|---|---|---|---|---|---|

**Card 10.** This card is required.

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
|---|---|---|---|---|---|---|---|

**Data Card Definitions:**

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | NCHSP | NCHRX | ICEND | CEND | GASC | FID | MF |
| Type | A | I | I | I | F | F | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified (see *PART). |
| NCHSP | Number of chemical species (maximum 8) |
| NCHRX | Number of chemical reactions (maximum 8) |
| ICEND | Species number controlling reaction termination |
| CEND | Concentration for reaction termination |
| GASC | Gas constant: 1.987 cal/(mol K), 8314. J/(mol K) |
| FID | Function ID for user specified chemical reaction rate equation for a single reaction model with two species |
| MF | ODE solver method:<br>EQ.0: Default<br>EQ.1: An alternative ODE solver |

**Material axis definition.** This card sets the material axes for the orthotropic heat conduction properties.

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | AOPT | XP | YP | ZP | A1 | A2 | A3 | |
| Type | I | F | F | F | F | F | F | |

| Card 3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | D1 | D2 | D3 | | | | | |
| Type | F | F | F | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| AOPT | Material axes definition (see *MAT_*OPTION*TROPIC_ELASTIC for a more complete description):<br>EQ.0.0: Locally orthotropic with material axes by element nodes N1, N2 and N4<br>EQ.1.0: Locally orthotropic with material axes determined by a point in space and global location of element center<br>EQ.2.0: Globally orthotropic with material axes determined by vectors<br>EQ.3.0: Locally orthotropic with first material axis orthogonal to element normal (defined by element nodes N1, N2 and N4) and to a vector **d**- Third material direction corresponds to element normal.<br>EQ.4.0: Local orthogonal in cylindrical coordinates with the material axes determined by a vector **d**, and an originating point, *P*, which define the centerline axis. |
| XP, YP, ZP | Coordinates of point *p* for AOPT = 1 and 4 |
| A1, A2, A3 | Components of vector **a** for AOPT = 2 |
| D1, D2, D3 | Components of vector **d** for AOPT = 2, 3 and 4 |

**Filler Material Properties.** This card sets the material properties for the filler material, such as carbon fiber. This card must be included, but all parameters can be set to 0 if no filler material is present.

| Card 4 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RHOf | LCCf | LCK1f | LCK2f | LCK3f | VFf | | |
| Type | F | I | I | I | I | F | | |

| VARIABLE | DESCRIPTION |
|---|---|
| RHOf | Density of the filler material |
| LCCf | Load curve ID specifying the specific heat as a function of temperature for the filler material |
| LCK1f | Load curve ID specifying thermal conductivity $K_1$, in the local $x$-direction, as a function of temperature for the filler material |
| LCK2f | Load curve ID specifying thermal conductivity $K_2$, in the local $y$-direction, as a function of temperature for the filler material |
| LCK3f | Load curve ID specifying thermal conductivity $K_3$, in the local $z$-direction, as a function of temperature for the filler material |
| VFf | Volume fraction of the filler material. The remaining volume is occupied by the reacting chemicals. |

**Chemical Species Cards.** Include one card for each of the NCHSP species. These cards set properties for each species. The dummy index *i* is the species number and is equal to 1 for the first species card, 2 for the second, and so on.

| Card 5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RHO*i* | LCC*i* | LCK1*i* | LCK2*i* | LCK3*i* | VF*i* | MW*i* | |
| Type | F | I | I | I | I | F | F | |

**Reaction Cards.** Include one card for each of the NCHSP species. Each field contains the species' coefficient for one of the NCHRX chemical reactions. See Card 5 for explanation of the species index *i*.

| Card 6 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RC*i*1 | RC*i*2 | RC*i*3 | RC*i*4 | RC*i*5 | RC*i*6 | RC*i*7 | RC*i*8 |
| Type | F | F | F | F | F | F | F | F |

**Reaction Rate Exponent Cards.** Include one card for each of the NCHSP species. Each field contains the species' rate exponent for one of the NCHRX chemical reactions. See Card 5 for explanation of the species index *i*.

| Card 7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | RX*i*1 | RX*i*2 | RX*i*3 | RX*i*4 | RX*i*5 | RX*i*6 | RX*i*7 | RX*i*8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| RHO*i* | Density of the $i^{th}$ species |
| LCC*i* | Load curve ID specifying the specific heat as a function of temperature for the $i^{th}$ species |
| LCK1*i* | Load curve ID specifying thermal conductivity $K_1$, in the local $x$-direction, as a function of temperature for the $i^{th}$ species |
| LCK2*i* | Load curve ID specifying thermal conductivity $K_2$, in the local $y$-direction, as a function of temperature for the $i^{th}$ species |
| LCK3*i* | Load curve ID specifying thermal conductivity $K_3$, in the local $z$-direction, as a function of temperature for the $i^{th}$ species |
| VF*i* | Initial fraction of the $i^{th}$ species relative to the other reacting chemicals. Note that $\sum_i \mathrm{VF}_i = 1$. |
| MW*i* | Molecular weight of the $i^{th}$ species |
| RC*ij* | Reaction coefficient for species *i* in reaction *j*. Leave blank for undefined reactions. |

| VARIABLE | DESCRIPTION |
|---|---|
| RX*ij* | Rate exponent for species *i* in reaction *j*. Leave blank for undefined reactions. |

**Pre-exponential Factor Card.** Each field contains the natural logarithm of its corresponding reaction's pre-exponential factor.

| Card 8 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | LCZ1 | LCZ2 | LCZ3 | LCZ4 | LCZ5 | LCZ6 | LCZ7 | LCZ8 |
| Type | I | I | I | I | I | I | I | I |

| VARIABLE | DESCRIPTION |
|---|---|
| LCZ*j* | Load curve defining data pairs of $(\text{temperature}, \ln Z_j)$ where $Z_j$ is the pre-exponential factor for reaction *j*. Leave blank for undefined reactions. |

**Activation Energy Card.** Each field contains the activation energy value for its corresponding reaction.

| Card 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | E1 | E2 | E3 | E4 | E5 | E6 | E7 | E8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| E*j* | Activation energy for reaction *j*. Leave blank for undefined reactions. |

**Heat of Reaction Card.** Each field contains the heat of reaction value for its corresponding reaction.

| Card 10 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 |
| Type | F | F | F | F | F | F | F | F |

| VARIABLE | DESCRIPTION |
|---|---|
| Q*j* | Heat of reaction for reaction *j*. Leave blank for undefined reactions. |

**Remarks:**

See the remarks for *MAT_T06.

## *MAT_THERMAL_ISPG

This is Thermal Material Type 18. It is only available for ISPG elements. With this material, isotropic thermal properties can be defined.

> **NOTE:** This thermal material only works for ISPG element formulations set on *SECTION_ISPG. It may only be in the ISPG input deck included in the LS-DYNA input deck with *INCLUDE_ISPG.

| Card 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | TMID | TRO | | | | | | |
| Type | I | F | | | | | | |

| Card 2 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Variable | HC | TC | | | | | | |
| Type | F | F | | | | | | |

| VARIABLE | DESCRIPTION |
|---|---|
| TMID | Thermal material identification. A unique number or label must be specified. (see *PART) |
| TRO | Thermal density:<br>EQ.0.0: Default to fluid density |
| TH | Specific heat |
| TC | Thermal conductivity |