Viscoelasticity in LS-DYNA

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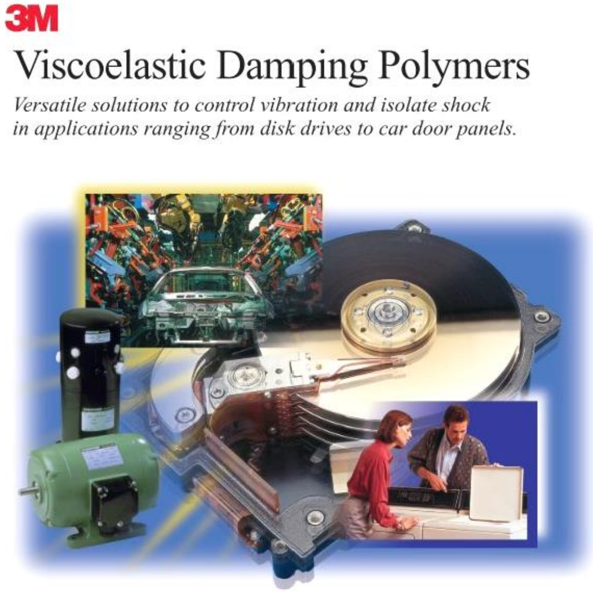
Agenda

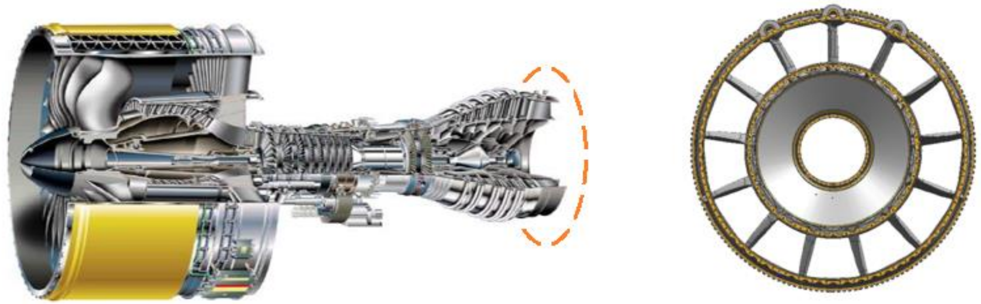
- What is Viscoelasticity ?
- How to model viscoelasticity
- ***MAT_VISCOELASTIC**
- Prony Series
- ***MAT_GENERAL_VISCOELASTIC**
- How to determine viscoelastic parameters
  - From other codes
  - Relaxation test (Input curve to LS-DYNA)
  - DMA (From DMA to Prony series)
- Shift Function
- ***MAT_HYPERELASTIC_RUBBER**
- ***MAT_ADD_INELASTICITY** - Viscoelasticity
- ***MAT_SAMP(Light)**
- Creep deformation – ***MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP**
- ***MAT_ADD_INELASTICITY** - Creep
- Summary
- Related information

What is viscoelasticity ?

- Synthetic polymers, wood, and human tissue, as well as metals at high temperature, display significant viscoelastic effects.
- Polymers
  - Straightening, untangling and breakage of polymer chains
  - Used in dampers
- Metals (Creep)
  - Bulk or grain boundary diffusion
  - Glide-controlled or climb controlled dislocation creep

- Temperature dependent

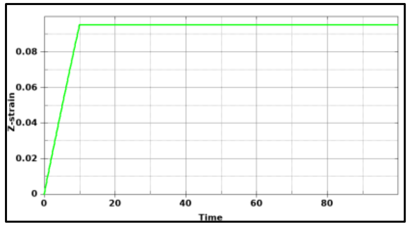




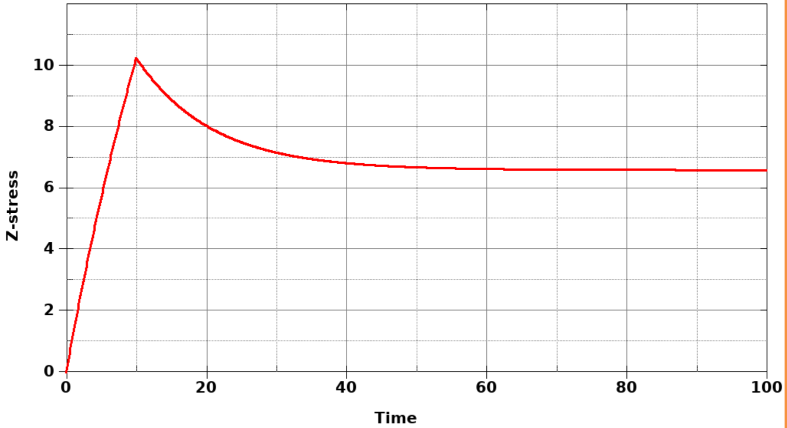
Berkeley Research Company, Berkeley California

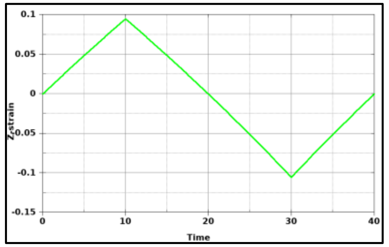
What is viscoelasticity?

- A viscoelastic material shows the following behaviour:

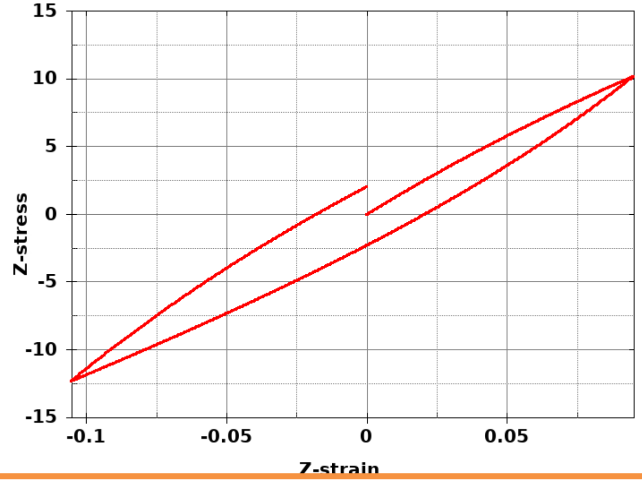


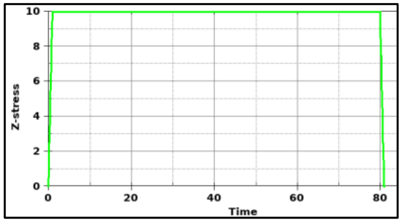
Stress relaxation under constant strain



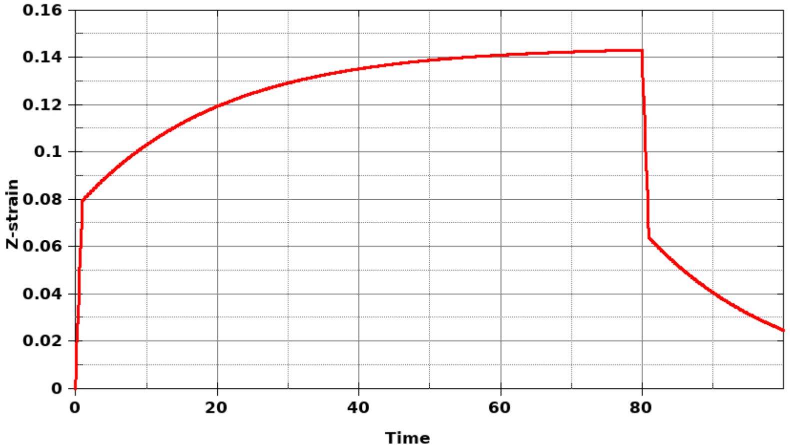


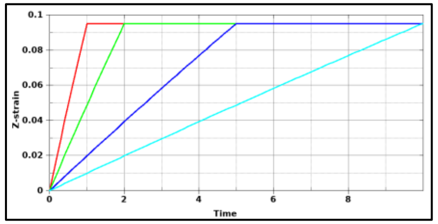
Strain-Stress hysteresis



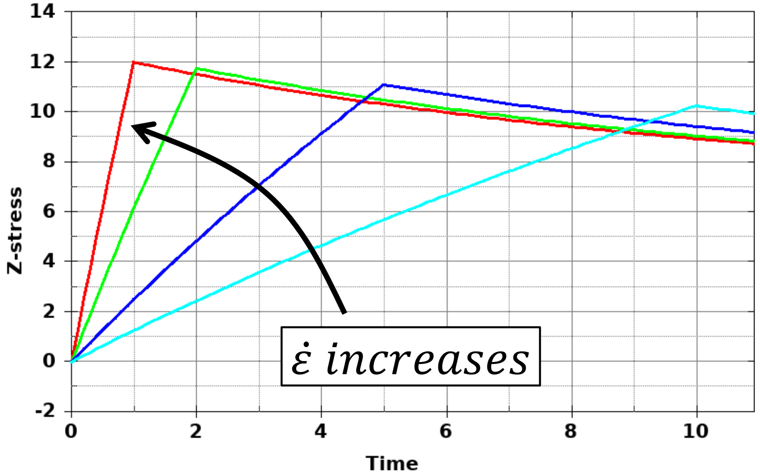


Creep under constant stress





Stiffness depends on the strain rate

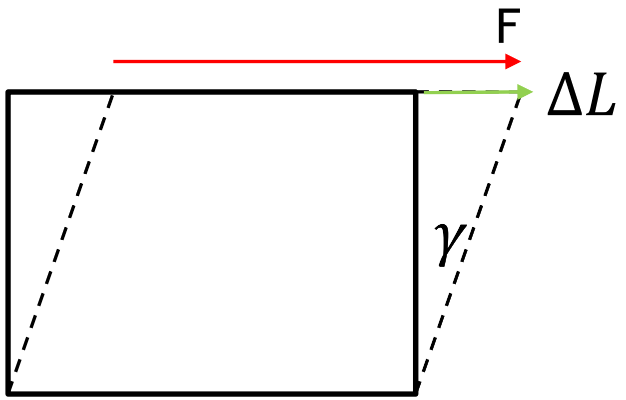


What is viscoelasticity?

- A viscoelastic material behaves both as an elastic solid and a fluid

- Elastic solid material (Hookean)

$$F \sim \Delta L$$

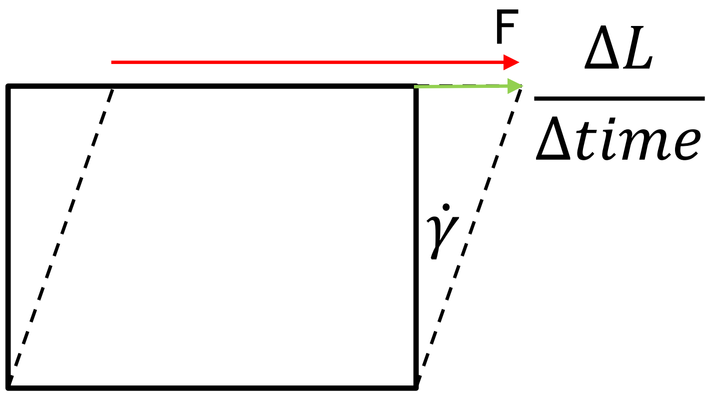


Elastic constant

$$\tau = G\gamma$$

- Fluid material (Newtonian)

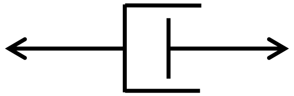
$$F \sim \frac{\Delta L}{\Delta time}$$

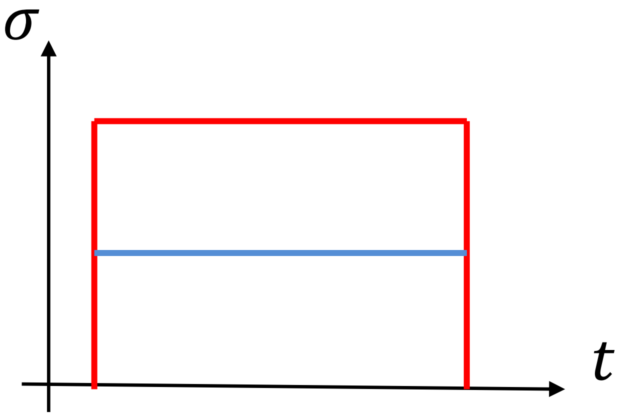
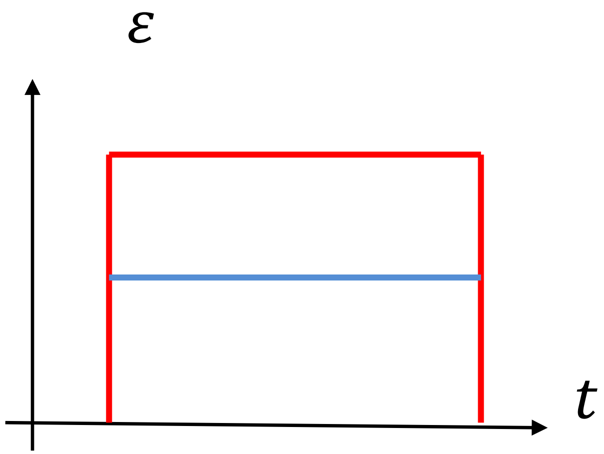


Viscosity

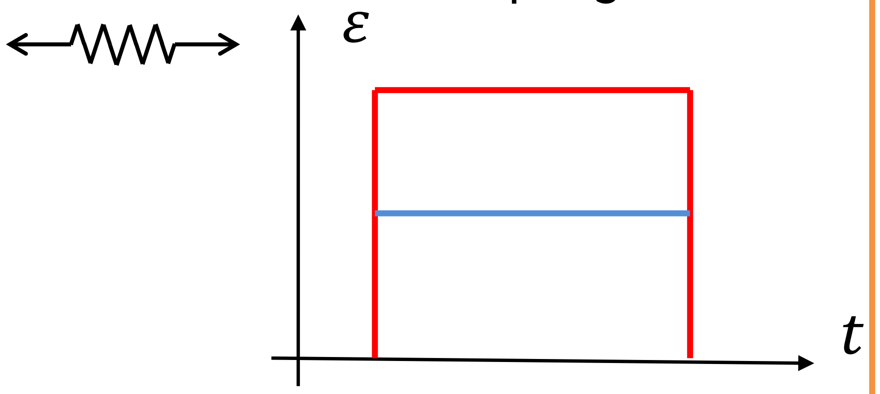
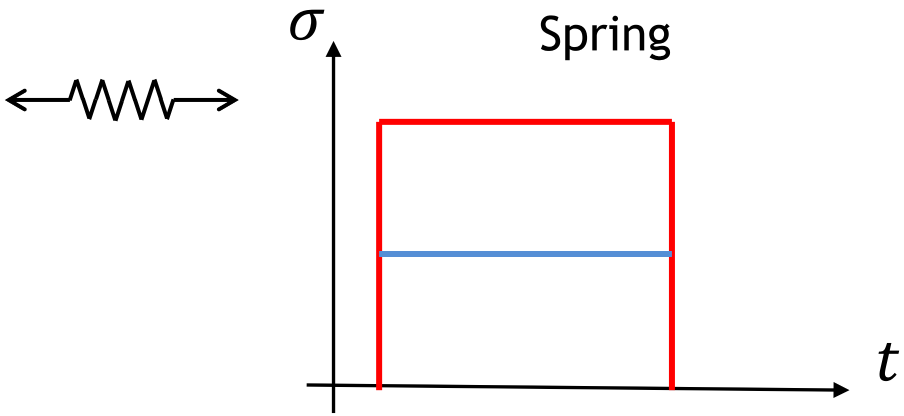
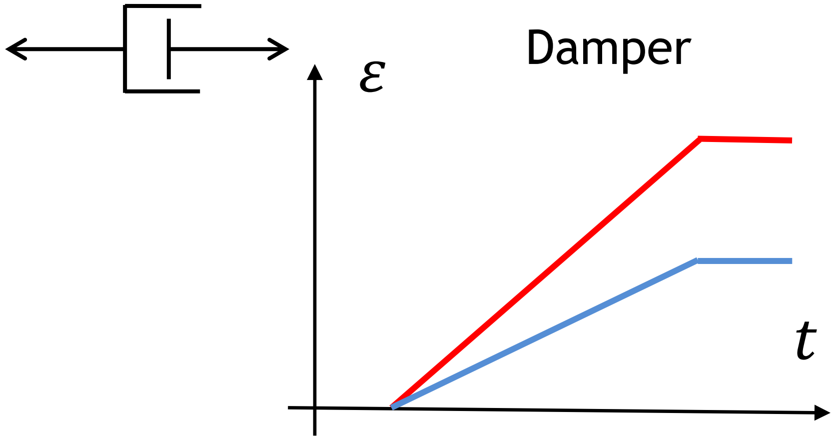
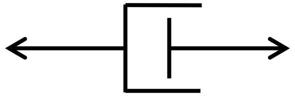
$$\tau = \eta\dot{\gamma}$$

How to model viscoelasticity?

- Elastic part  $\sigma = E\varepsilon$
- Viscous part  $\sigma = \eta\dot{\varepsilon}$

Creep (constant stress)	Stress relaxation (Constant strain)
	

How to model viscoelasticity?

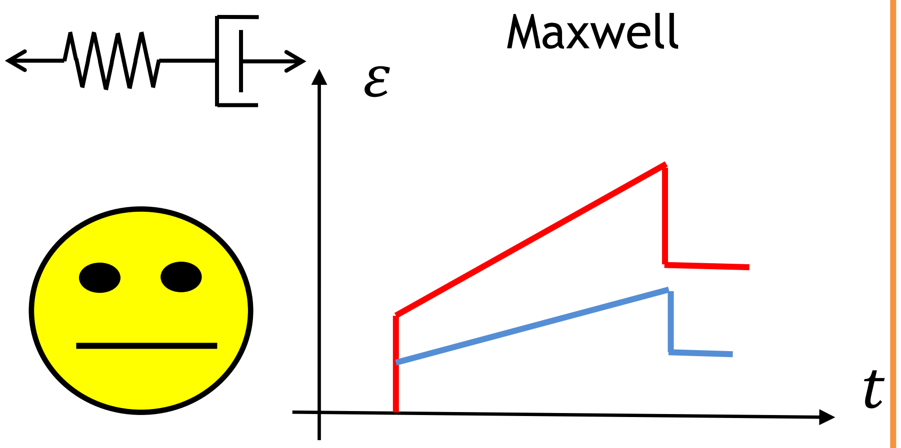
Creep (constant stress)	Stress relaxation (Constant strain)
Spring	Spring
	
Damper	Damper
	

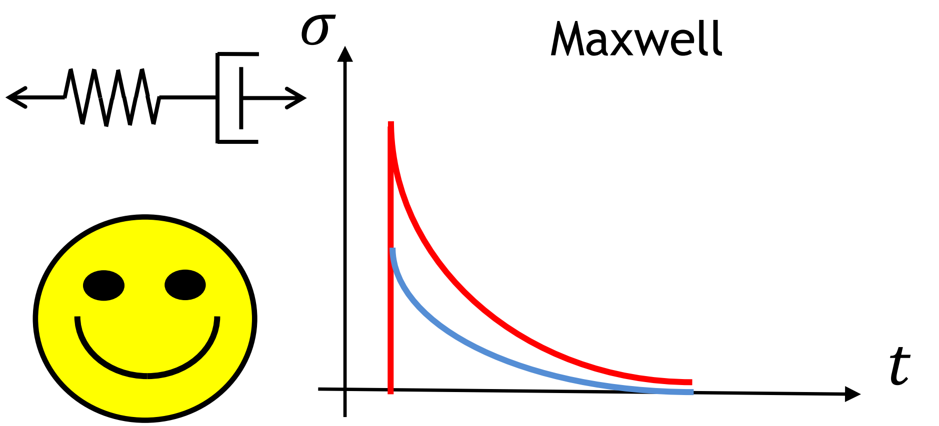
Instantaneous strain -> infinite stress in the viscous part

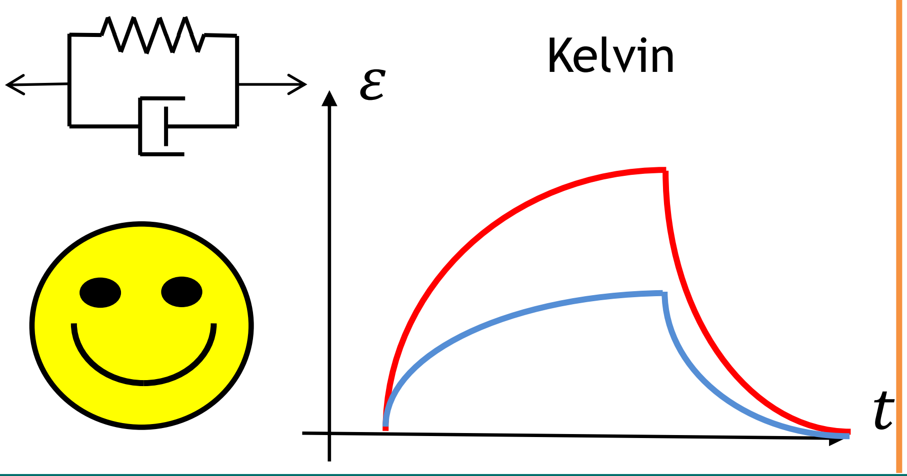
How to model viscoelasticity?

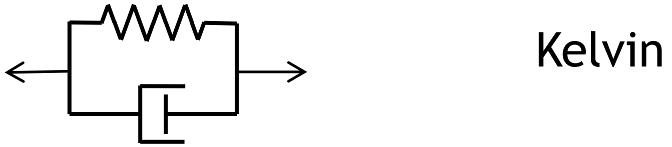
Creep (constant stress)

Stress relaxation (Constant strain)





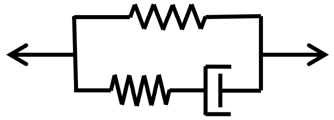


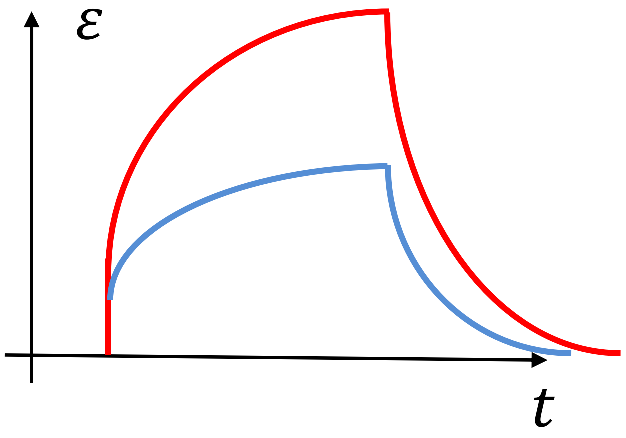


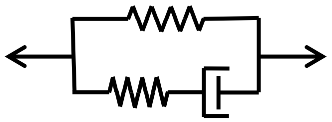
Instantaneous strain -> infinite stress in the viscous part

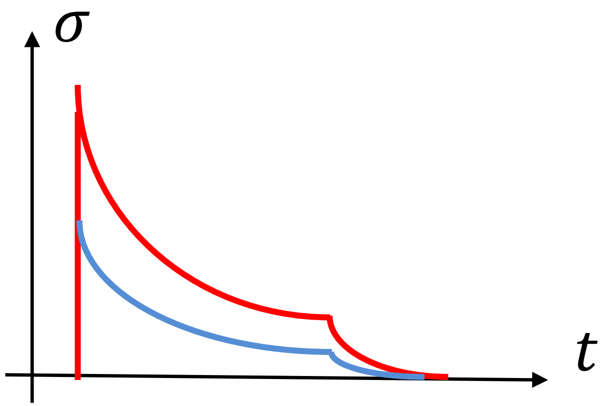
How to model viscoelasticity?

Creep (constant stress)	Stress relaxation (Constant strain)
Standard viscoelastic	Standard viscoelastic



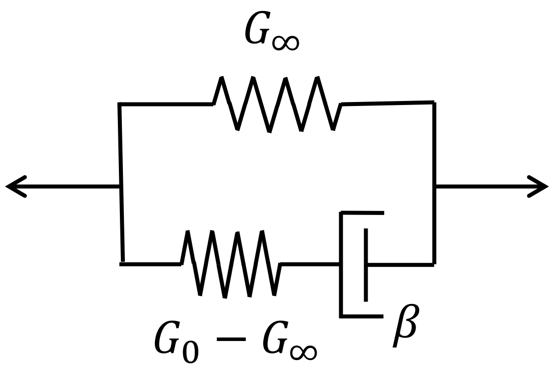






*MAT_VISCOELASTIC – *MAT_006

- Standard viscoelastic model – Spring and Maxwell element in parallel



$$G(t) = G_\infty + (G_0 - G_\infty)\exp(-\beta t)$$

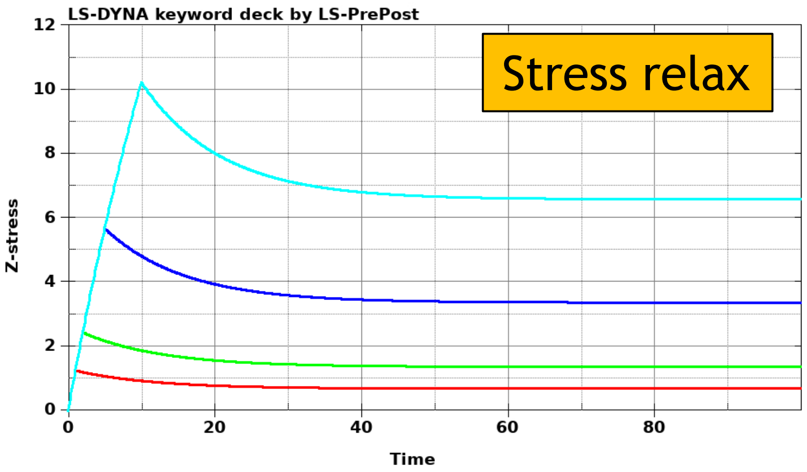
***MAT_006** — *MAT_VISCOELASTIC

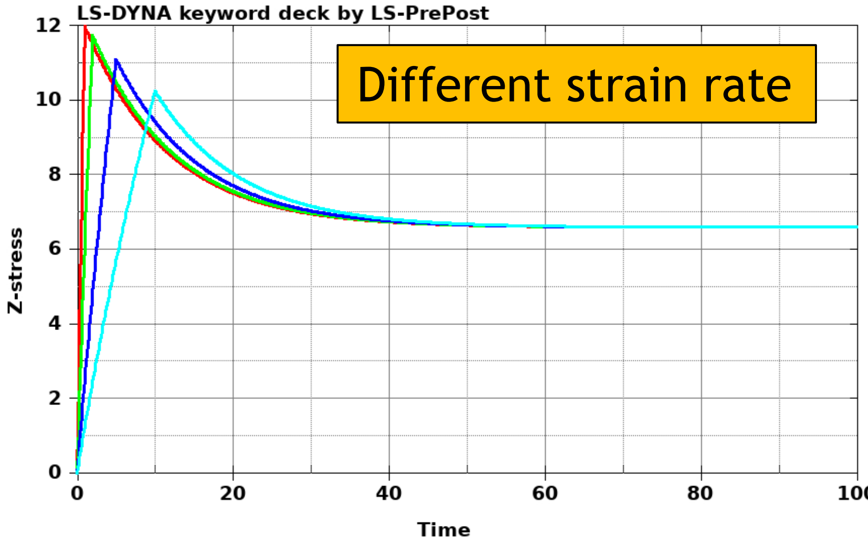
*MAT_VISCOELASTIC

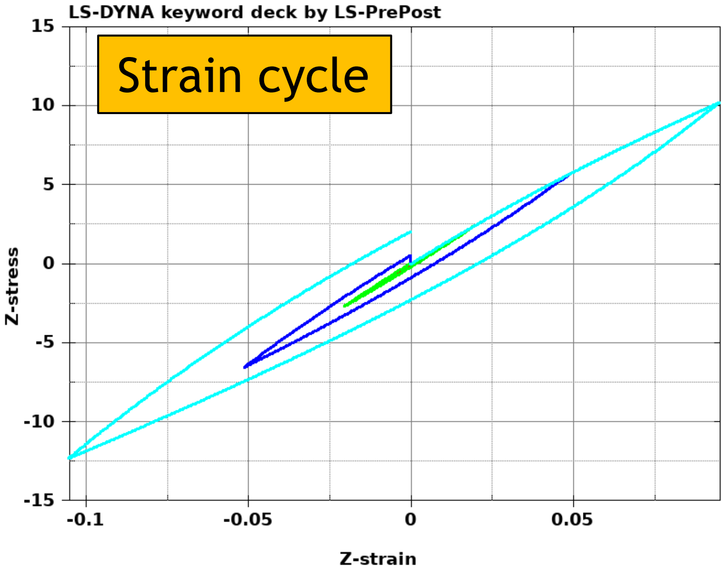
Card 1	1	2	3	4	5	6	7	8
Variable	MID	RO	BULK	G0	GI	BETA		
Type	A	F	F	F	F	F		

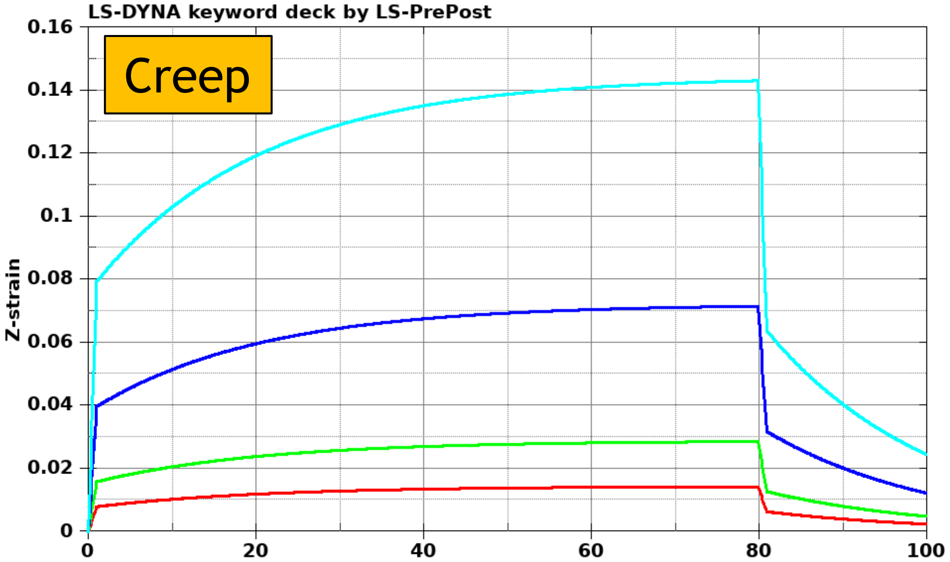
- All input can be temperature dependent

*MAT_VISCOELASTIC – *MAT_006









Prony Series

- Prony series are parallel Maxwell elements
- The stress response from one element is

$$\tau(t) = \gamma_0 G e^{-\beta t}$$

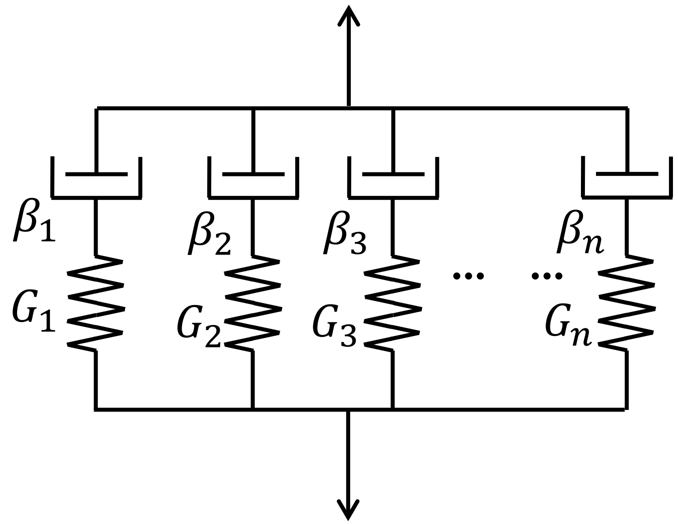
- The stress response from *n* parallel maxwell elements

$$\tau(t) = \gamma_0 \sum_{i=1}^{n} G_i \, e^{-\beta_i t}$$

- If the strain is constant the stress relaxation modulus becomes

$$\frac{\tau(t)}{\gamma_0} = G(t) = \sum_{i=1}^{n} G_i \, e^{-\beta_i t}$$

- Depending on the type of strain, the Prony series can be formulated as e.g. tensile/compression *E*, volumetric *K*, shear *G*



*MAT_GENERAL_VISCOELASTIC – *MAT_076

Stress relaxation input Parameter fit in LS-DYNA	Prony series input
▪ Relaxation modulus as a function of time from a stress relaxation test ▪ Linear elastic Bulk modulus or viscoelastic bulk modulus from a hydrostatic stress relaxation test	▪ Up to 18 prony series ▪ Shear viscoelasticity G_i, β_i ▪ Bulk viscoelasticity K_i, β_i

- The material model can also be used with laminated shells. Either an elastic or viscoelastic layer can be defined with the laminated formulation.
- The Arrhenius or Williams-Landel-Ferry (WLF) shift functions account for the effects of the temperature on the stress relaxation

*MAT_GENERAL_VISCOELASTIC – *MAT_076 - Example

- Trying to match the input from MAT_006

$$G_0 = 50\ MPa\ and\ G_\infty = 25\ MPa, \beta = 0.1$$

- Using 2 series is enough

Prony series *MAT_006

$$G_1 e^{-\beta_1 t} + G_2 e^{-\beta_2 t} = G_\infty + (G_0 - G_\infty) e^{-\beta t}$$

$$G_1 + G_2 e^{-\beta_2 t} = G_\infty + (G_0 - G_\infty) e^{-\beta t}$$

$$G_1 = 25\ MPa\ and\ G_2 = 25\ MPa, \beta_1 = 0\ and\ \beta_2 = 0.1\ 1/s$$

How to determine viscoelastic parameters

- Data or input parameters from e.g. other codes
  - The decay coefficient, β_i, in LS-DYNA is ${}^{1}/_{\tau_i}$ where τ_i is the relaxation time for a maxwell unit
  - The relaxation time, τ_i, is the time it takes for the stress to reduce to ${}^{1}/_{e}$ ~ 0.369 from its original value $\tau_i = {}^{G_i}/_{\eta_i}$ where η_i is the viscosity

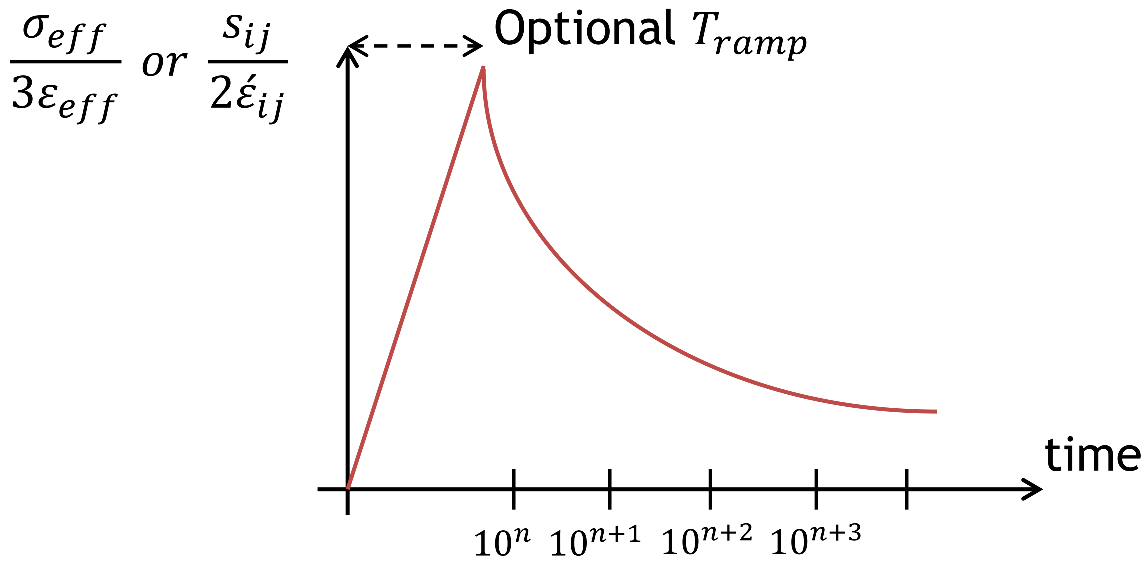
Stress relaxation parameters	LS-DYNA
"Other" code $G(t) = G_0(1 - \sum_{i=1}^{n} g_i(1 - e^{-t/\tau_i}))$	$G(t) = \sum_{i=1}^{n} G_i\, e^{-\beta_i t}$ $G_i = g_i G_0$ $\beta_i = {}^{1}/_{\tau_i}$

If $\sum_{i=1}^{n} g_i < 1$ the remaining part is represented by a spring in LS-DYNA

$G_{spring} = G_0(1 - \sum_{i=1}^{n} g_i)$ and $\beta_{spring} = 0$

How to determine viscoelastic parameters

- In e.g. ***MAT_GENERAL_VISCOELASTIC**, LS-DYNA can determine Prony Series parameters from a stress relaxation test .



β_i values can be input or determined through iterative scheme.
$\beta_1 = 0.$

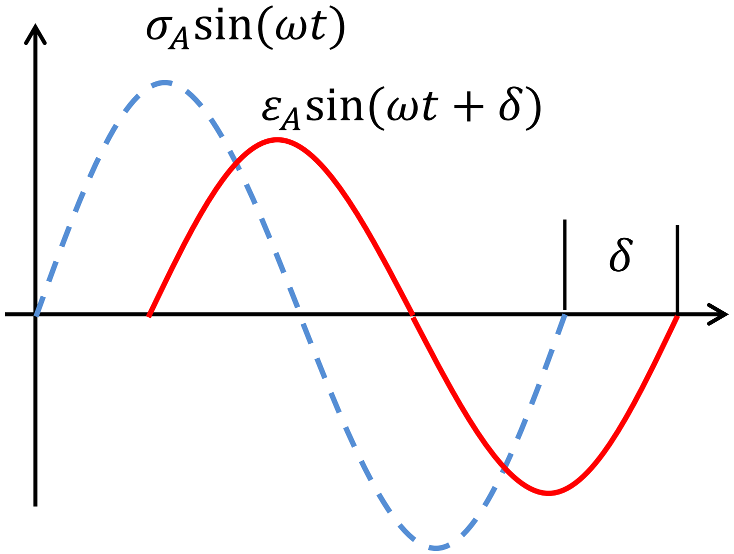
```
 number of terms used in fit=       5
1     shear constant=   1.2926E+02    beta =    0.0000E+00
2     shear constant=   4.3869E-01    beta =    2.2959E-04
3     shear constant=   1.0856E+02    beta =    2.2959E-03
4     shear constant=   2.0418E+00    beta =    2.2959E-02
5     shear constant=   0.0000E+00    beta =    2.2959E-01

*** final fit ***
           time         test value       fit  value
      1.0000E+01        2.3741E+02       2.3741E+02
      1.0000E+02        2.1618E+02       2.1618E+02
      1.0000E+03        1.4053E+02       1.4053E+02
      1.0000E+04        1.2930E+02       1.2930E+02

*****************
```

In d3hsp file

- Fit works best if the input is smooth and time is equally spaced on the logarithmic scale
- User defined number of Prony Series
- Optional Prony Series fit for *K* as well

How to determine viscoelastic parameters

- Viscoelastic parameters can be determined through a DMA (Dynamic Mechanical Analysis) test.
- A test specimen (e.g compression, bending, shear) is subjected to various frequencies and the phase shift between the applied stress/strain and resulting strain/stress is observed.

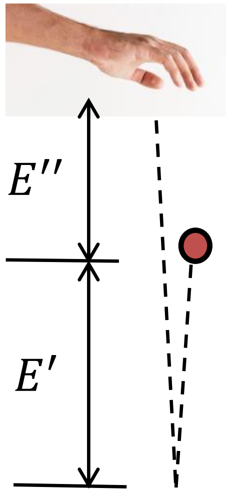


$$E^* = \frac{\sigma(\omega, t)}{\varepsilon(\omega, t)} = \cdots \equiv E' + iE''$$

E' is the storage modulus

E'' is the loss modulus

$$\tan(\delta) = \frac{E''}{E'}$$



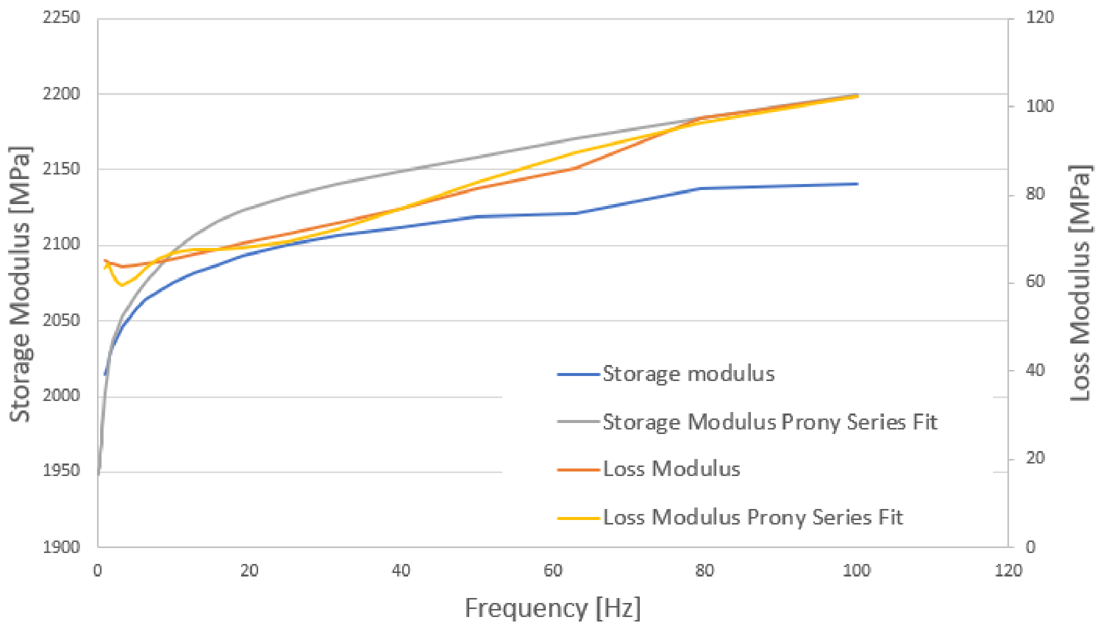
- The DMA teats can be done at varying temperatures and/or frequencies to determine the transition temperatures and loss and storage moduli dependencies.

From DMA to Prony Series

- Prony series terms can be calculated by curve approximation according to e.g. Ferry*

$$G' = G_0 + \sum_{i=1}^{N} \frac{G_i \left(\frac{\omega}{\beta_i}\right)^2}{1 + \left(\frac{\omega}{\beta_i}\right)^2}$$

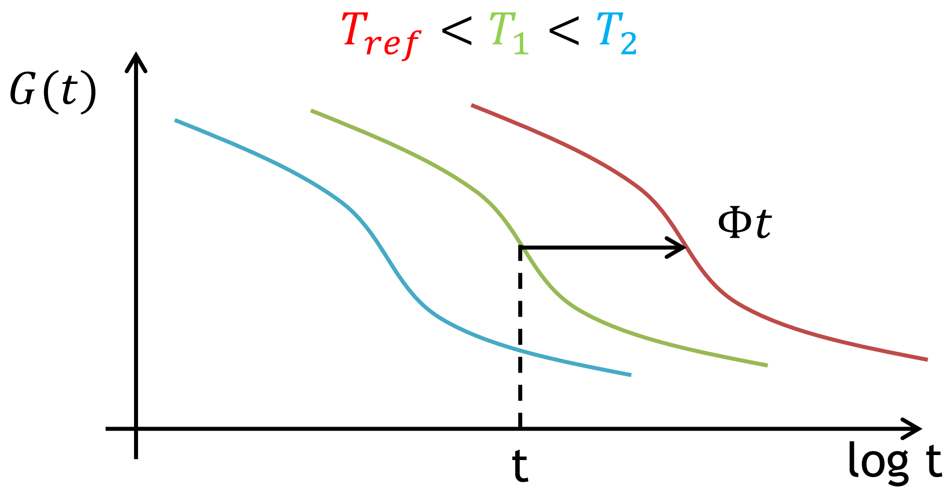
$$G'' = \sum_{i=1}^{N} \frac{G_i \frac{\omega}{\beta_i}}{1 + \left(\frac{\omega}{\beta_i}\right)^2}$$



* Ferry, J.D. (1980). *Viscoelastic properties of polymers*. 3rd edition: John Wiley & Sons.

SHIFT FUNCTION/Time-Temperature superposition

- If the stress relaxation modulus shape as a function of time is not changed but only shifted with a temperature change, it implies that a time- temperature superposition is possible.
- The stress relaxation modulus is then determined at a reference temperature and then "time-shifted" as the temperature is modified.
- $\Phi(T)$ is the scalefactor that the time need to be shifted in order to get the same response at temperature T as with T_{ref}.



- Increasing the deformation rate will shift the curve to higher temperatures

SHIFT FUNCTION/Time-Temperature superposition

- Two types of shift functions are typically used in LS-DYNA viscoelastic models, the Arrhenius and the Williams-Landel-Ferry (WLF).

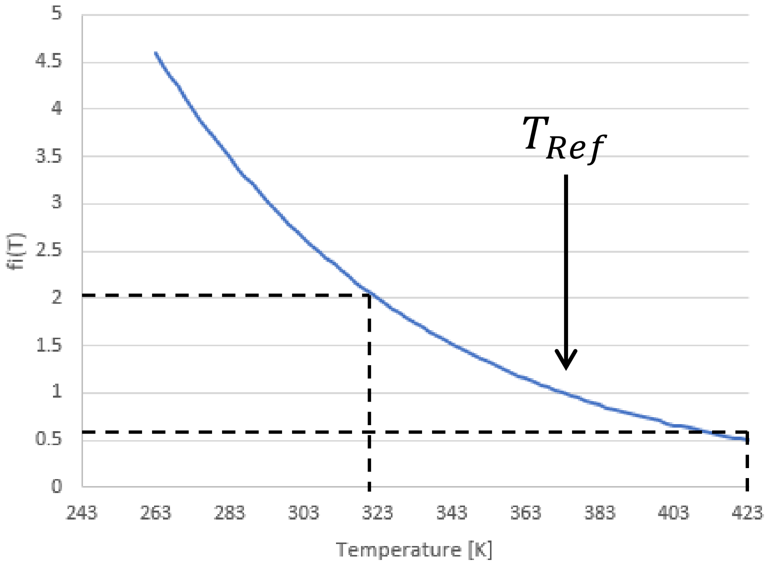
$$\Phi_{ARR}(T) = exp\left[-A\left(\frac{1}{T} - \frac{1}{T_{Ref}}\right)\right]$$

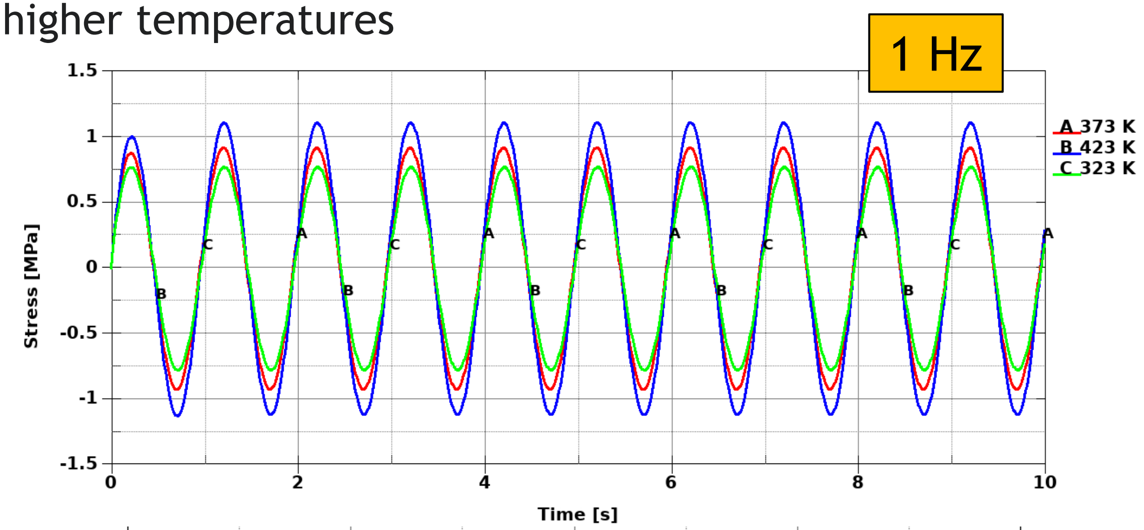
$$\Phi_{WLF}(T) = exp\left[-A\,\frac{T - T_{Ref}}{B + T - T_{Ref}}\right]$$

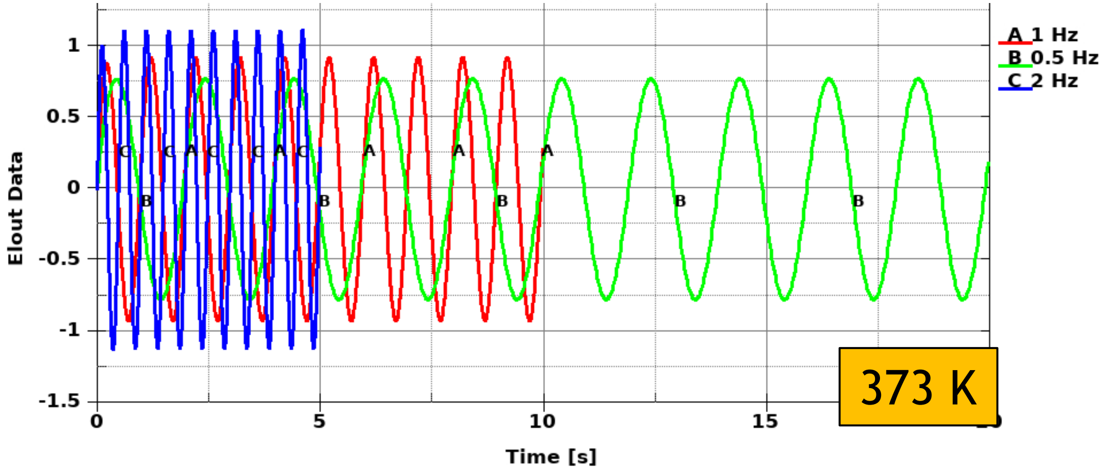
- Increasing the temperatures will lower $\Phi(T)$, thus decreasing the timescale
- Increasing the rate of deformation will shift the curve to higher temperatures, thus decreasing the timescale

Shift function example

- DMA compression with prescribed strain with frequency 1 Hz
- Increasing the rate of deformation will shift the curve to higher temperatures
- Increasing the temperature with 50 deg will yield the same response as 2 Hz
- Decreasing the temperature with 50 deg will yield the same response as 0.5 Hz



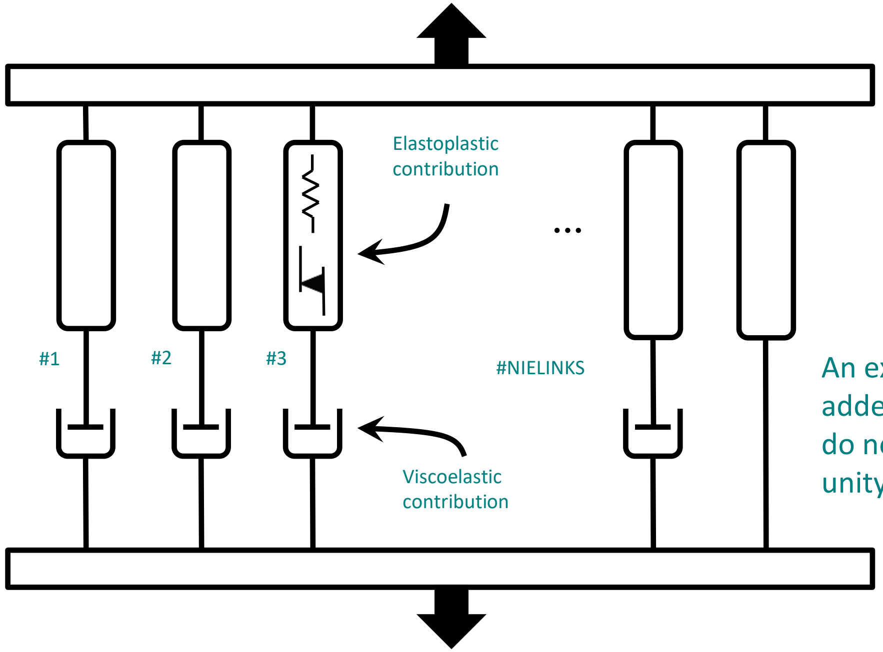




Materials | Add-on Inelasticity

- Inelastic contribution from userdefined number of link groups with a weight factor.
- Intention is not to replace any material model but to complement with potentially missing features and accept an incurred cost when doing so.
- To amend ”any” material model with an inelasticity feature
  - Plasticity
    - von Mises isotropic hardening viscoplasticity
  - Creep
    - Norton creep variants
  - Viscoelasticity
    - Prony series
    - Bulk and Shear decay
    - Arrhenius or WLF Shift function for both Shear and Bulk relaxation
    - Shift function can be separate for all links
  - Explicit and implicit, shells and solids

Borrvall, Bengzon



An extra link is added if weights do not sum to unity

$$\sigma = \sum_{i=1}^{NLINKS(+1)} w_i \sigma_i$$

*MAT_ADD_INELASTICITY compared to *MAT_GENERAL_VISCOELASTIC

```
*MAT_ELASTIC
$      mid        ro         e         pr
         1    7.8e-9  210000.0        0.2
*MAT_ADD_INELASTICITY
$      mid  nielinks
        12
```

Link 1

```
$  nielaws    weight
        10.3
$      law     model
         6         1
$    betag     betak
      0.05
```

Link 2

```
$  nielaws    weight
        10.5
$      law     model
         6         1
$    betag     betak
     0.005
```

```
*MAT_GENERAL_VISCOELASTIC
$      mid        ro      bulk
         2    7.8e-9 116666.67
$ blank card

$        g     betag
   43750.0     0.005
   26250.0      0.05
   17500.0
```

$$G = \frac{E}{2(1+\nu)} = 87500\ MPa$$

$$K = \frac{E}{3(1-2\nu)} = 116666.67\ MPa$$

$$w_1 = \frac{G_1}{G} = \frac{26250}{87500} = 0.3$$

$$w_2 = \frac{G_2}{G} = \frac{43750}{87500} = 0.5$$

An extra link is added if weights do not sum to unity

$$w_3 = 1 - w_1 - w_2 = 0.2$$

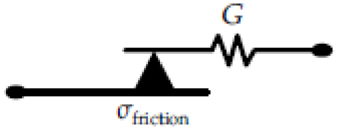
$$G_3 = 0.2 * 87500 = 17500$$

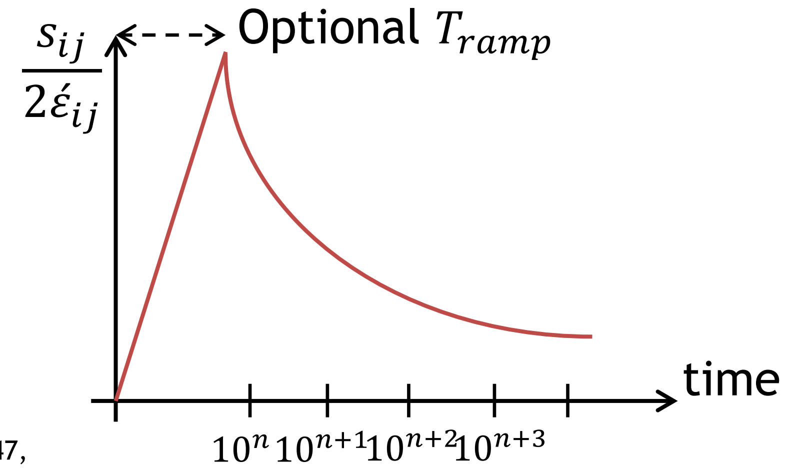
*MAT_HYPERELASTIC_RUBBER

- Many material models in LS-DYNA has “Viscoelasticity as outlined by Christensen*”
- Viscoelastic effects are taken into account by adding a series of Prony terms which are added to the strain energy stress

$$\sigma_v = \sum_{i=1}^{N} \sigma_v^i \qquad\qquad \sigma = \sigma_w + \sigma_v$$

- To avoid a constant shear modulus from this viscoelastic formulation, a term in the series is included only when $\beta > 0$.
- Input can be either as Prony terms G_i and β_i or determined from a deviatoric stress relaxation curve similar to **`*MAT_76`** using a least square fit.
- Each prony link can have a spring and slider in series for frequency independent damping

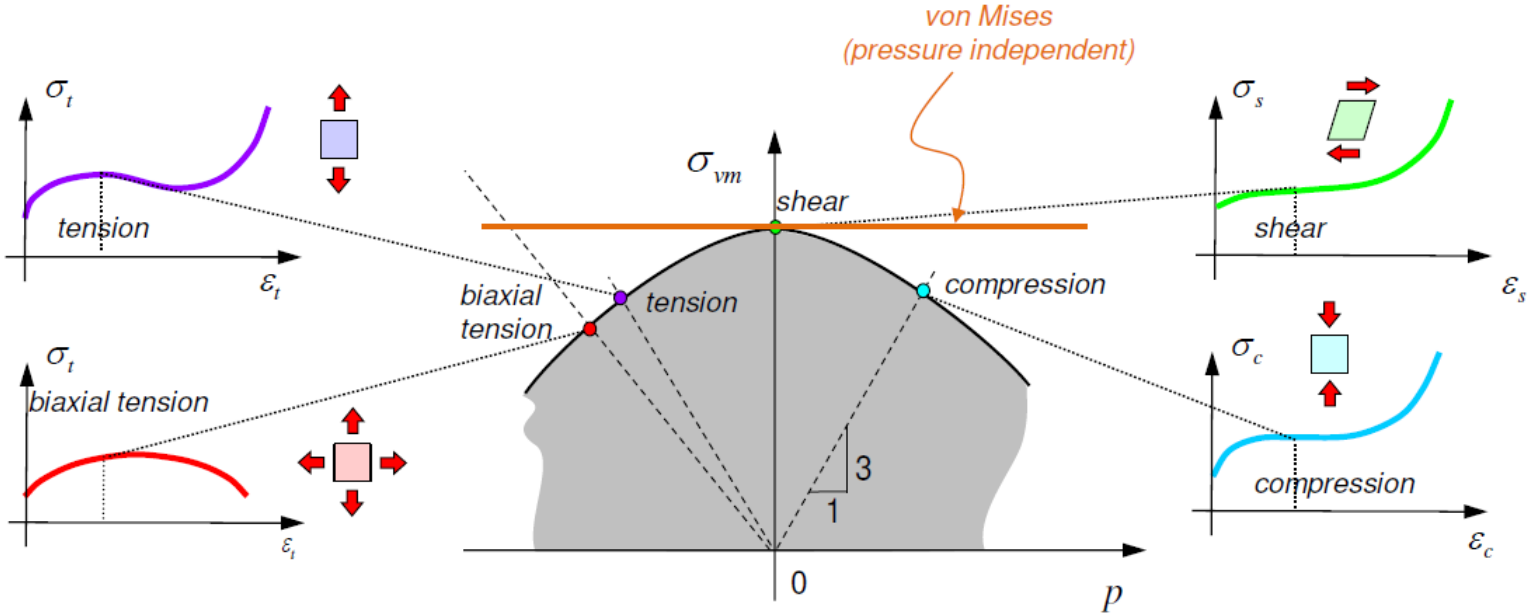




Christensen, R.M. “ Nonlinear Theory of Viscoelasticity for Application to Elastomers,”Journal of Applied Mechanics, Volume 47, American Society of Mechanical Engineers, pages 762-768, December 1980.

*MAT_SAMP and *MAT_SAMP_LIGHT

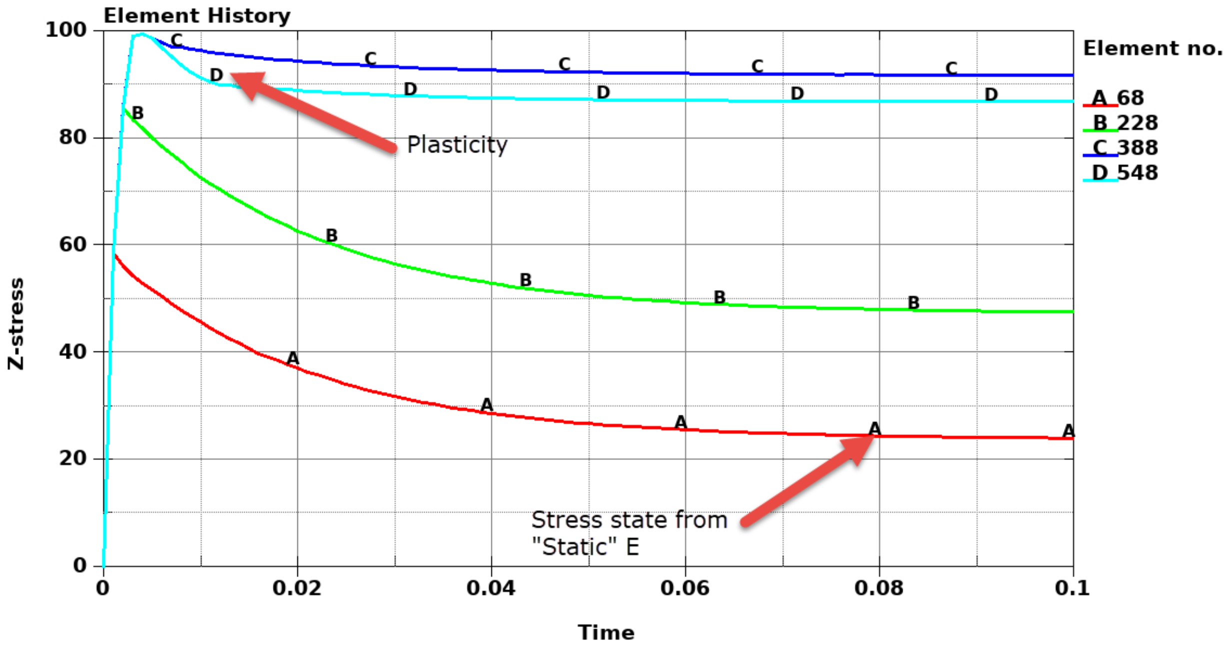
- SAMP – "Semi Analytical Model for Polymers"
  - Pressure dependency
  - Tension/Compression asymmetry
  - Viscoplasticity
  - Viscoelasticity
  - Non- associated plasticity through plastic poissons ratio



*MAT_SAMP and *MAT_SAMP_LIGHT

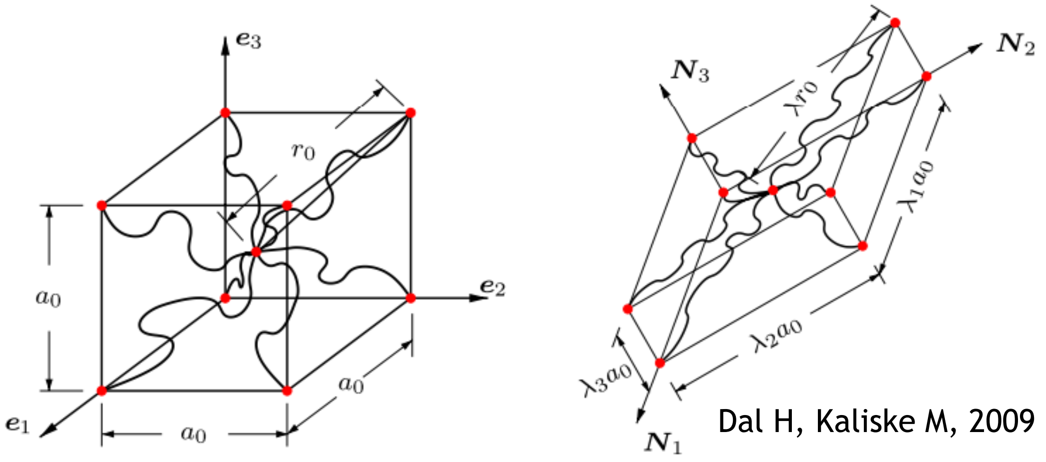
- The viscoelasticity is governed by a decay coefficient β and a strainrate dependent Young's Modulus $E\big(\dot{\varepsilon}(t)\big)$.

$$\dot{\sigma}(t) = -\beta\sigma(t) + E(\dot{\varepsilon}(t))\dot{\varepsilon}(t)$$

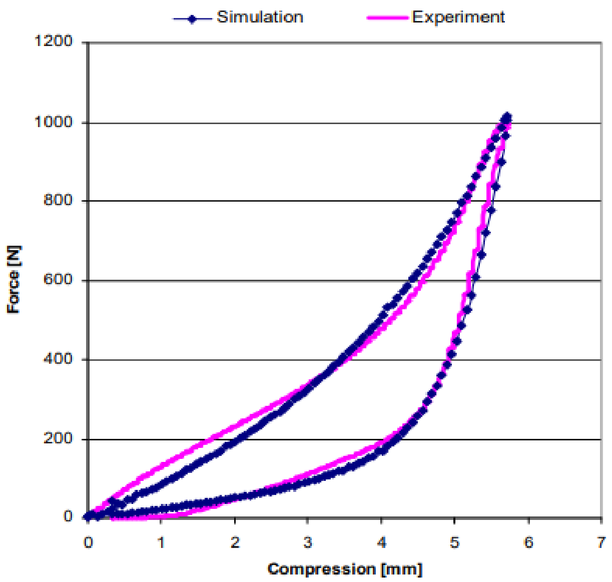


*MAT_EIGHT_CHAIN_RUBBER

- Tailored for Rubber and Glassy Polymers
- The basic theory is taken from Arruda's thesis from 1992 enhanced with
  - Mullins effect
  - Viscoelasticity
  - Plasticity
  - Viscoplasticity
- Viscoelasticity with up to six Maxwell elements



Dal H, Kaliske M, 2009

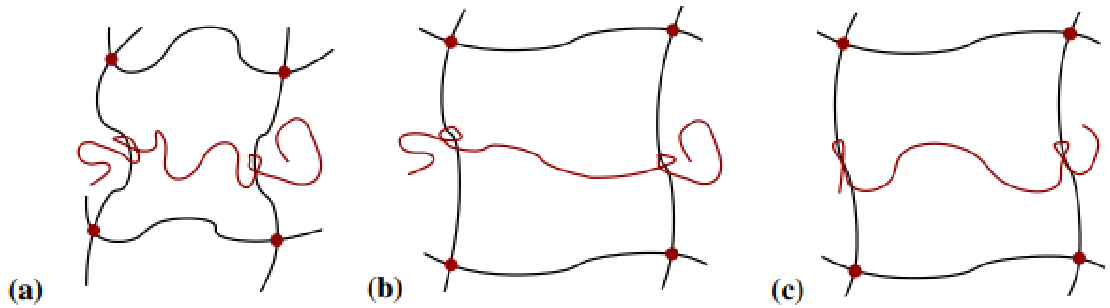


Arruda EM., Characterization of the strain hardening response of amorphous polymers, PhD. Thesis, MIT, 1992

Olsson T. and Nilsson L., A new advance visco-elastoplastic eight chain rubber model for LS-DYNA, 8 th European LS-DYNA Users Conference, Strasbourg – May 2011

*MAT_BERGSTROM_BOYCE

- Rubber model based on the Arruda and Boyce (1992) chain model
- Viscoelastic contribution according to Bergström and Boyce (1998)
  - physical response of a single entangled chain in an embedded polymer gel matrix
- Implementation based on Dal and Kaliske (2009).



Undeformed Deformed Relaxed

Arruda EM., Characterization of the strain hardening response of amorphous polymers, PhD. Thesis, MIT, 1992

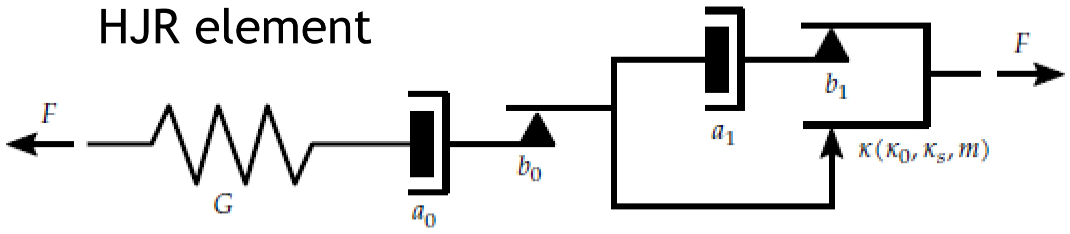
Bergström JS, Boyce MC, Constitutive modeling of the large strain time-dependent behavior of elastomers. J Mech Phys Solids 46:931–954, 1998

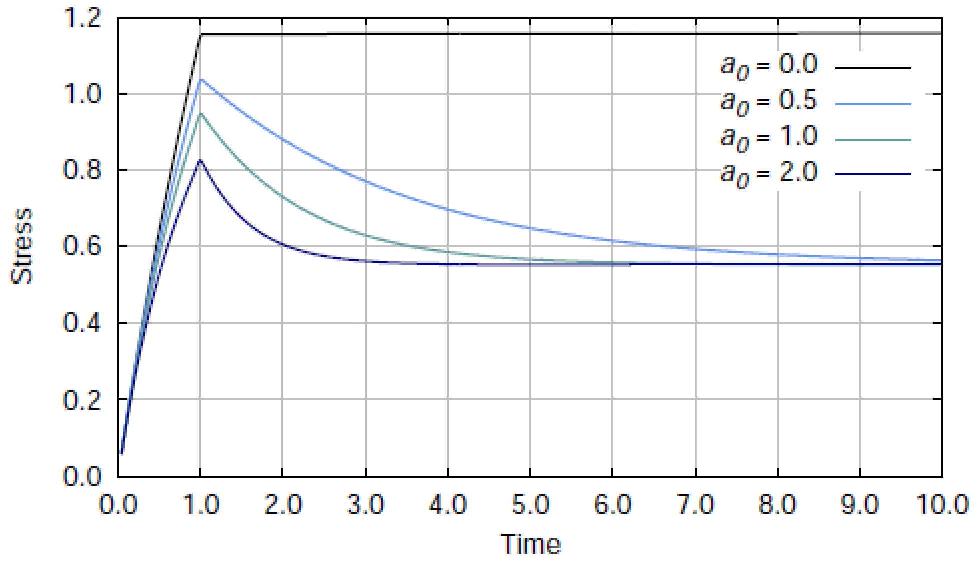
Dal H, Kaliske M, Bergström–Boyce model for nonlinear finite rubber viscoelasticity: theoretical aspects and algorithmic treatment for the FE method, Comput Mech, 44:809-823, 2009

*MAT_SMOOTH_VISCOELASTIC_VISCOPLASTIC

- Deviatoric response from up to 6 parallel HJR (Hollenstein-Jabareen-Rubin)
- 6 parameters for each element yields challenging parameter identification
- Suitable for implicit analysis due to strong objective implementation and smooth formulation

HJR element





Hollenstein M., M. Jabareen, and M.B. Rubin, Modelling a smooth elastic-inelastic transition with a strongly objective numerical integrator needing no iteration, Comput. Mech., Vol. 52, pp. 649-667, (2013).

Hollenstein M., M. Jabareen, and M.B. Rubin, Erratum to: Modelling a smooth elasticinelastic transition with a strongly objective numerical integrator needing no iteration, Comput. Mech., 2014).

Jabareen, M., “Strongly objective numerical implementation and generalization of a unified large inelastic deformation model with a smooth elastic-inelastic Transition”, submitted to Int. J. Solids and Struct. (2015).

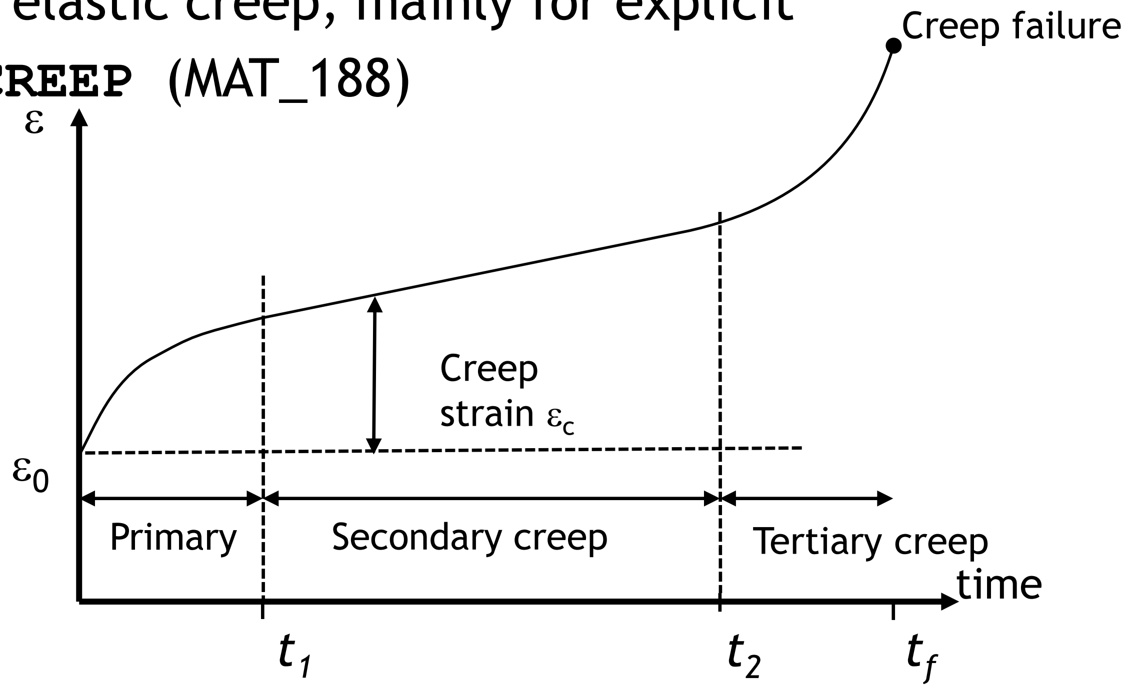
Creep

- Inelastic time dependent strain under constant (moderate) stress

$$\varepsilon = \varepsilon_e + \varepsilon_c + \varepsilon_t + \varepsilon_p$$

- Phenomena is more pronounced at high temperatures
- Material models for creep
  - ***MAT_UNIFIED_CREEP** - simple model for elastic creep, mainly for explicit
  - ***MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP** (MAT_188)
  - ***MAT_ADD_INELASTICITY**
- Norton creep model is popular
  Varying *m* can model Primary and Secondary creep

$$\varepsilon_c = A\sigma^n t^m$$



*MAT_THERMO_ELASTO_VISCOPLASTIC_CREEP - *MAT_188

- Temperature dependent parameters for
  - Young’s modulus, Poisson’s ratio, thermal expansion coefficient
  - Plasticity
  - Strain-rate dependent plasticity
  - Creep
- Isotropic, kinematic or mixed hardening possible
- Creep modelled by Norton’s power law ($CRPLAW = 1$) in rate form,

$$\dot{\varepsilon}^c = A(\tau^e)^B t^m$$

or Garafalo’s hyperbolic sine equation ($CRPLAW = 0$)

$$\dot{\varepsilon}^c = A[\sinh(B\tau^e)]^m \exp\left(-\frac{Q}{T}\right)$$

Creep - ***MAT_ADD_INELASTICITY**

- No temperature dependent input
- MODEL = 1: Norton incremental formulation

$$\dot{\varepsilon}_c = A\bar{\sigma}^m t^n$$

  - Similar to MAT_188
- MODEL = 2: Norton total formulation

$$\varepsilon_c = A\bar{\sigma}^m t^n$$

  - Similar to MAT_115 (***MAT_UNIFIED_CREEP**) but also suited for implicit
- MODEL = 3: Norton – Bailey incremental formulation

$$\dot{\varepsilon}_c = \left(A \left(\frac{\bar{\sigma}}{\sigma_0} \right)^n \left(\frac{T}{T_0} \right)^p \left((m+1)(\varepsilon_0 + \varepsilon_c) \right)^m \right)^{\frac{1}{m+1}}$$

Postprocessing of material history variables

- In general, additional results are output as history variables (*NEIPH*, *NEIPS* on ***DATABASE_EXTENT_BINARY**)
- Different materials may output results in different order – history variable #1 can have different meaning for different material models
- In order to simplify post-processing of results, the keyword ***DEFINE_MATERIAL_HISTORIES** was introduced.
  - The user can specify the order of appearance of the additional results

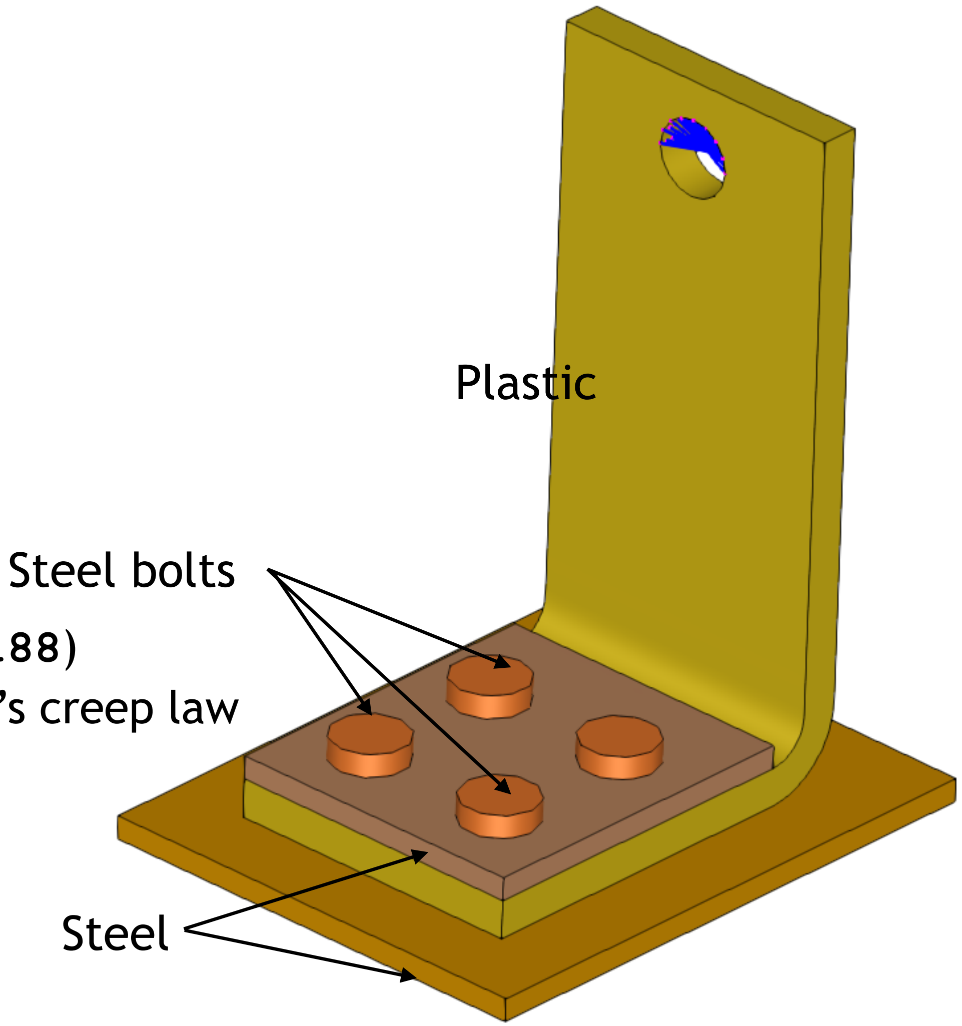
- Additional history variables are output for the creep models
  - ***MAT_188** can output effective creep strain
  - Also available from ***MAT_ADD_INELASTICITY**
- Use

```
*DEFINE_MATERIAL_HISTORIES
Effective Creep Strain
```

to request the effective creep strain as history variable #1

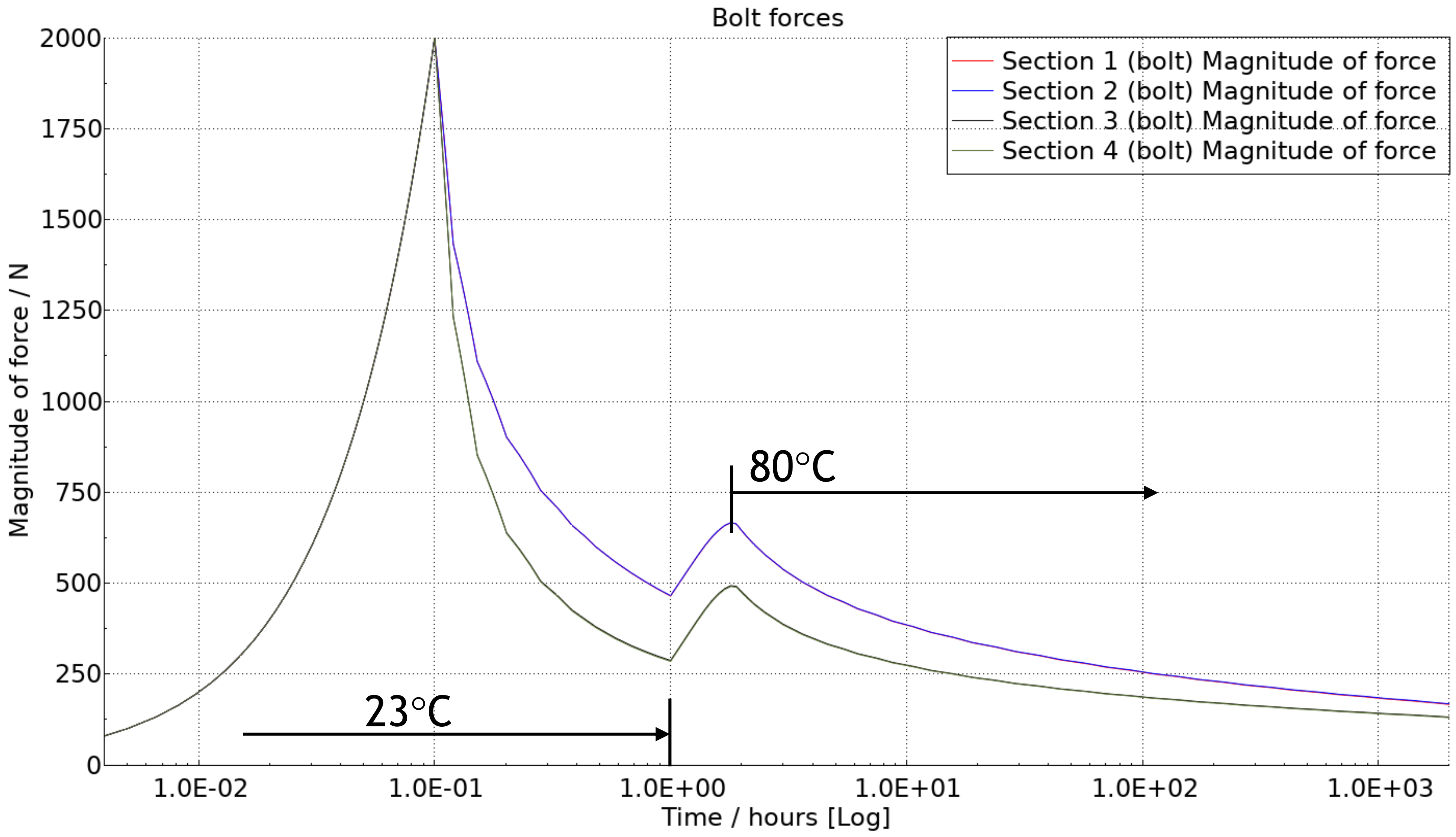
Example Creep

- Creep loading of plastic / steel assembly
- Steel support
  - Fully constrained underside
  - Estimated temperature dependent material properties (**`*MAT_106`**)
- Steel bolts
  - Pre-tension to 2 kN
  - Estimated temperature dependent (elastic) material properties (**`*MAT_106`**)
- Plastic bracket
  - Estimated temperature dependent material properties (**`*MAT_188`**)
  - Estimated (temperature independent) creep data, using Norton’s creep law
  - Time unit: hours
- Two load steps:
  - Pre-tension @ 23°C
  - Creep loading, ramp up to 80°C, hold for 20 hours



Creep loading of plastic / steel assembly

- Bolt force history



Summary

- Viscoelastic response is a combination av elastic and viscous behavior under loading
- Viscoelastic behavior is common in plastics, rubber, human tissue and metals at high temperature.
- A viscoelastic material shows:
  - Creep deformation under constant load
  - Stress relaxation under constant strain
  - Energy dissipation under cyclic strain-stress loading
  - Strain rate dependent stiffness
- Viscoelasticity is often modeled by adding one or several Maxwell elements in parallel called Prony series
- The Prony Series formulation is found in many material models, e.g. **`*MAT_76`** and **`*MAT_77`**, but can also be added to any material model through **`*MAT_ADD_INELASTICITY`**
- Prony series parameter identification can be estimated from stress- relaxation tests input, tabulated material data or e.g. DMA tests
- Temperature dependency can be modeled by doing a time- temperature shift using Arrhenius or WLF shift functions
- Material models tailored for creep behaviour are e.g. **`*MAT_188`**, but it can also be added to any material model through **`*MAT_ADD_INELASTICITY`**

Related Webinars and papers

- Material modeling course (LS-DYNA and Userdefined materials)
- Material modeling and failure models
  - DYNAmore Express: **Introduction to Material Characterization**, DYNAmore GmbH Webinar 2020-05-22, https://youtu.be/23ascSSRnzQ
  - **Ductile failure modeling in LS-DYNA**, DYNAmore Nordic Webinar 2020-08-20. (slides available for customers on files.dynamore.se)
  - DYNAmore Express: **Recent developments in GISSMO**, DYNAmore GmbH Webinar 2020-04-30, https://youtu.be/r90zTSQJk9U
  - **DIEM and the No-Copy option in LS-DYNA**, DYNAmore Nordic video, 2020-08-11
- Plastics
  - DYNAmore Express: **Modeling Plastics in LS DYNA (Part 1)** - Isotropic Modelling of Thermoplastics, DYNAmore GmbH Webinar 2020-06-19, https://youtu.be/LN_FlM-c9As
  - DYNAmore Express: **Modeling plastics in LS DYNA (Part 2)** - Anisotropic Modelling of Thermoplastics, DYNAmore GmbH Webinar 2020-06-26, https://youtu.be/WiL4K-5pvRU
- Calibration
  - **Parameter identification using regular curve matching**, DYNAmore Nordic video, https://www.youtube.com/watch?v=mOWoqcKtTt4&t=12s
  - **Full-Field Calibration**, DYNAmore Nordic video, https://www.youtube.com/watch?v=hQJ9rM3blXU&t=178s
- Dynalook.com
  - Papers from the 2020 International LS-DYNA conference are now available
- This and previous webinars by Dynamore Nordic on our customer client area

Thank you!

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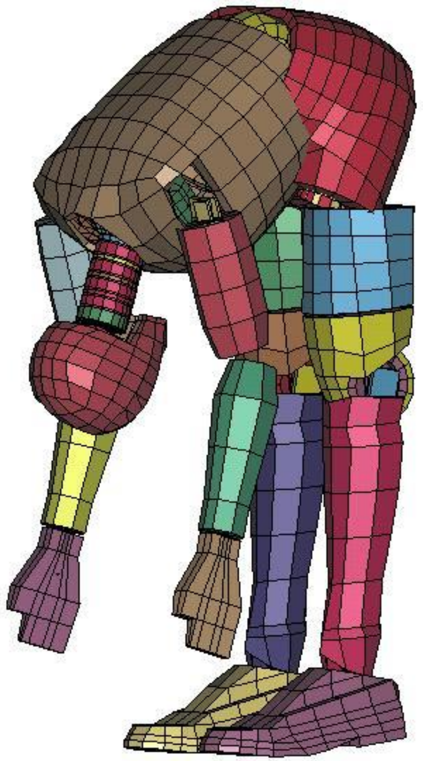

Thank you!

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