Constrained Multidisciplinary Topology Optimization

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Abstract

For multidisciplinary topology optimization it can be difficult to select the weights for each load case. This becomes even harder if there are multiple design considerations per case. But if constraint values can be defined, then the problem is solvable, because the problem is transformed into one of satisfying the constraints. The most difficult constraint to control is that of the crash pulse, because the existing linear methodologies cannot be used – solutions such as multipoint strategies and spatial kernels must be introduced instead. The NVH constraints are however linear and solving the NVH constraints in combination with the crash pulse becomes a two-level problem. In this paper we show multidisciplinary design optimization considering constraints from impact, linear statics, and frequency load cases.

Introduction

For vehicle design one has a multitude of design considerations, of which some are likely in conflict. For example, the standard topology objective of minimum compliance may increase the lethality of the impact forces. There is therefore a need for multidisciplinary topology optimization methodologies.

Topology optimization [1,2,3] finds the layout of a structure supporting the required load by starting with a ground structure within which the required structural topology or load path must be found. Much of the current research in topology optimization is driven by additive manufacturing [2,3].

Multi-objective topology optimization is important in many cases, with modern direction of investigation varying from vehicle design as in this study to the design of pelvic prostheses [4]. A current approach of doing multidisciplinary topology optimization is by choosing weights for each load case and doing a trade-off study. This is workable if only there is only a single design consideration per load case. Specifically, it won't work when designing for a NVH (Noise, Vibration and Harshness) load case considering both the fundamental frequency and the first bending/torsion mode frequency, together with an impact load case considering both an energy requirement and a limit on the peak acceleration, because two load case weights cannot be used to control four constraints.

Designing for vehicle occupant protection typically consider a force-displacement curve in some form. For example, the energy E and the load resistance R of a part are related as $E = \int R(u)dx$ with u the displacement. The main technological problem is that of design for the crash pulse and the energy absorbed as described for example in [5]. Design sensitivity information (the derivatives of the constraints and objective with respect to the design variables) is normally used to solve design optimization problems; for example Weider and Schumacher [6] computed topological derivatives considering both material and geometrical nonlinearities. A design sensitivity implementation is however not feasible for impact problems and an alternative method such as surrogate modeling which does not require analytical gradients is required.

Multipoint methods such as surrogate modeling or numerical derivatives can be used as substitutes for the design sensitivity information. This is solved by using a spatial kernel approach [7] allowing the use of a multipoint scheme. The multipoint and the related metamodeling schemes were first developed by the Livermore Software Technology team in work that started before 2015 [8,9,10,11]. Through our clients such as Honda Research Institute it became known to others. The method has been shown to be able to control the crash pulse and maximize energy absorption [5].

Constrained multi-disciplinary topology optimization

Solving general problems using a dual problem or a saddle point

Considering the load resistance of a part -- the energy E is $E = \int R(u)dx$ with R the resistance and u the displacement. Designing for maximum energy absorption in occupant protection is often stated as $max_{\boldsymbol{x}}E(\boldsymbol{x})$ with $\boldsymbol{x}$ the topology variables. This will however not yield the desired result, because a structure must be at its minimum energy state to be stable -- simply maximizing the energy can result in infinite displacements and other instabilities. To obtain a stable problem you must solve for $min_{\boldsymbol{x}}E(\boldsymbol{x})$, which is the exact opposite.

The solution is to introduce some additional variables $\boldsymbol{\xi}$ (known as the spatial kernel), and to solve the max-min or saddle point optimization problem

$$max_{\boldsymbol{\xi}}min_{\boldsymbol{x}}E(\boldsymbol{\xi},\boldsymbol{x})$$

One therefore has two saddle directions: a load-bearing structure is found by computing the minimum energy state using $\boldsymbol{x}$, while energy absorption of the structure is maximized by solving for the spatial kernel variables $\boldsymbol{\xi}$.

In general, and specifically for Multidisciplinary Design Optimization (MDO) problems, one does not design merely for energy absorption. A generalized problem can be solved considering the dual problem:

$$min_{\boldsymbol{\xi}}F(\boldsymbol{\xi}(\boldsymbol{x}))$$

where $\boldsymbol{x}$ is computed using

$$min_{\boldsymbol{x}}f(\boldsymbol{x}(\boldsymbol{\xi}))$$

with *f* usually taken as the compliance, or the negative value of fundamental frequency, which means the analyst only must specify *F*. In such a case one can maximize energy absorption using F while maximize stiffness using *f*. Similarly, you can minimize mass using *F* while maximizing stiffness using *f*.

Introducing constraints into the dual problem

We have the standard objective is

$$f(\boldsymbol{x})$$

and the constraints as

$$g_i(\boldsymbol{x}) \leq 0$$

The constraints can be split into two sets -- for the one set design sensitivity information can be analytically computed

$$g_i^{\text{ana}}(\boldsymbol{x}) \leq 0 \text{ with } i = 1, \dots, n$$

while the other set requires the computation of numerical derivatives using the spatial kernel in the upper problem

$$g_j^{\text{num}}(\boldsymbol{x}) \leq 0 \text{ with } j = 1, \dots, m$$

Adding the Lagrange multipliers to the objective gives us the Lagrange function

$$L(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\xi}) = f(\boldsymbol{x}) + \sum_i \lambda_i g_i^{\text{ana}}(\boldsymbol{x}) + \sum_j \xi_j g_j^{\text{num}}(\boldsymbol{x})$$

The constraints needing numerical derivatives are given special treatment. A spatial kernel

$$s(\boldsymbol{\xi}) = \sum_j \xi_j S_j(\boldsymbol{\zeta})$$

is introduced to satisfy these constraints. The kernel is composed of basis functions referring to $\boldsymbol{\zeta}$ the spatial coordinates associated with variable $\boldsymbol{x}$ and is applied to $f(\boldsymbol{x})$which is the function that generates the load bearing structure, which yields

$$L(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\xi}) = [1 + s(\boldsymbol{\xi})] f(\boldsymbol{x}) + \sum_i \lambda_i g_i^{\text{ana}}(\boldsymbol{x})$$

The current implementation of the spatial kernel is slightly different, because we roll up all the spatial kernel functions into a surface written as a summation over both the basis functions and the elements as:

$$h(\boldsymbol{x}) = \frac{1}{n} \sum_{e=1}^{N} \frac{x_e}{\exp(\xi_0 + \xi_1 S_1(\zeta_e) + \xi_2 S_2(\zeta_e) + \cdots)} = 1$$

with x_e a spatial value at element e ($e = 1, \dots, N$), and $\boldsymbol{\xi}$ solved to satisfy the constraints. See reference [7] for the details of an implementation.

Expansion to MDO

For multidisciplinary optimization we have the objective as

$$f(\boldsymbol{x}) = \sum_{lc} w_{lc} f_{lc}(\boldsymbol{x})$$

in which it should be noted that the load case weights can be used to solve for a subset of the constraints. The constraints are split into two sets as described before -- for the one set design sensitivity information can be computed, while the other set requires the computation of numerical derivatives using the spatial kernel in the upper problem.

As before, we can add the Lagrange multipliers to the objective giving the Lagrange function

$$L(\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{\xi}) = \sum_{lc} w_{lc} [1 + s(\boldsymbol{\xi})] f_{lc}(\boldsymbol{x}) + \sum_i \lambda_i g_i^{\text{ana}}(\boldsymbol{x})$$

which contains the high-level variables $[\boldsymbol{w}, \boldsymbol{\lambda}, \boldsymbol{\xi}]$ used to solve for the constraints.

Solving the dual problem

The dual problem is solved as an upper level problem in the Lagrange multipliers (including weight and spatial kernel variables) and a lower level problem in the topology variables. The lower level problem is solved using the projected subgradient method [12] considering the Lagrange multipliers, while the upper level problem can be solved using finite differences or surrogate models.

The important algorithm settings are the step size (desired mass flow) for the lower level problem and the trust region bounds (move limits) for the upper level problem. The two settings are linked. If the convergence is too noisy then they can both be reset to a smaller value such as “0.25*Default” – in which it must be noted that “Default” is an allowed and recommended part of the expression.

Examples

Benchmark example with a displacement and fundamental frequency constraint.
This is an academic example chosen for benchmark and verification purposes. It can be found as part example problems of LS-TaSC™ version 4.2. The structure is designed for two load cases – supporting the load as shown, as well as for the fundamental frequency. Three constraints are placed on the design: the displacement must be less than 0.008, the 2nd harmonic of the structure must be larger than 80, and the 3rd harmonic of the structure must be larger than 120. Note that the frequency load case has two constraints, which means the problem cannot be solved using the load cases weights. The results are shown in Figure 1.

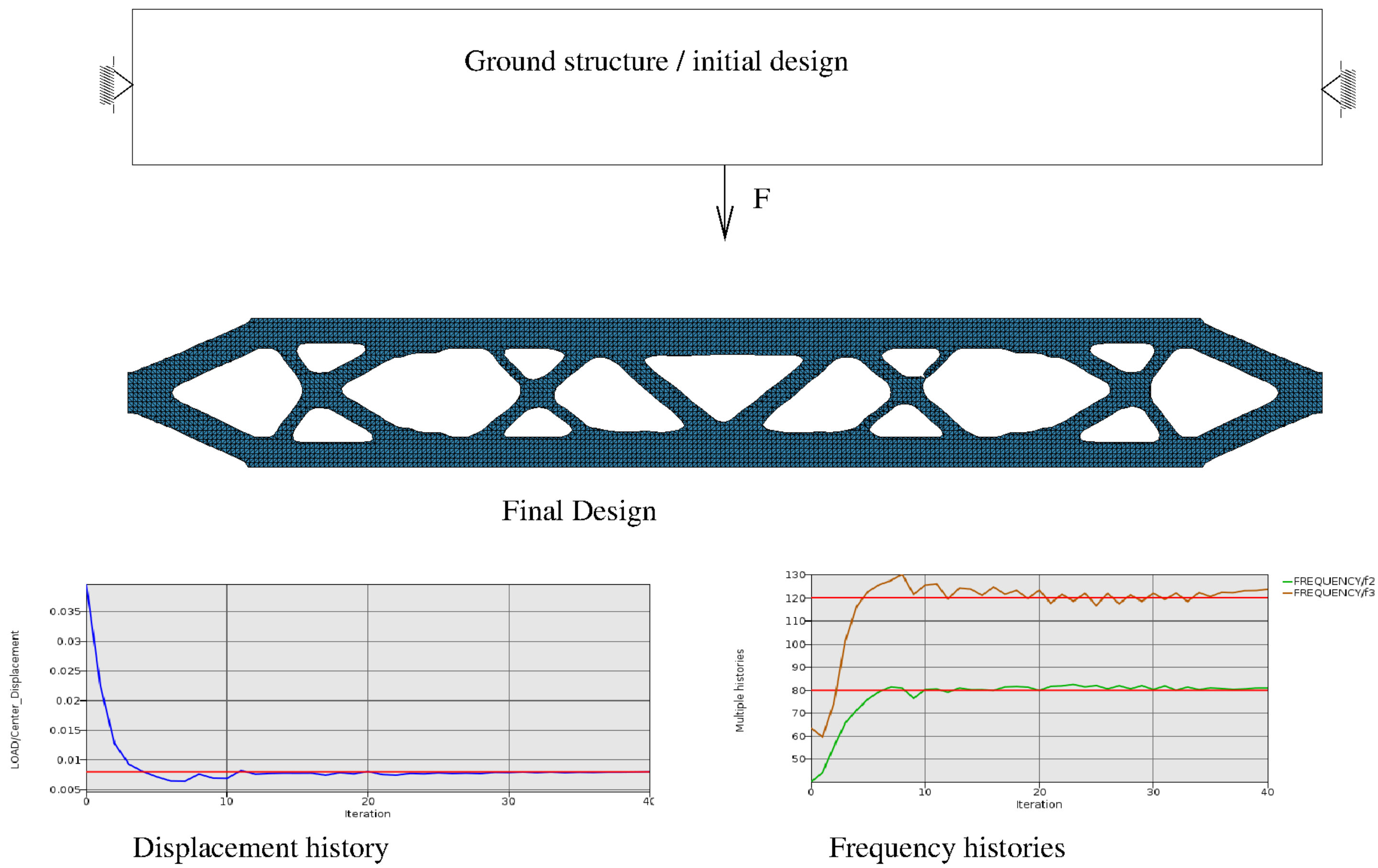


Figure 1 Benchmark example. The ground structure is shown on top with the final design below. The histories of the displacement and frequency constraints show that design process satisfied the constraints. Note that the frequency load case has two constraints, which means the problem cannot be solved using the load cases weights.

Nonlinear Beam Example
For this problem we study the trade-off between the natural frequency and a nonlinear displacement. The multidisciplinary problem is stated as minimizing displacement while subject to a lower bound of 750 on the natural frequency. This problem required a reduction in the optimization step size.

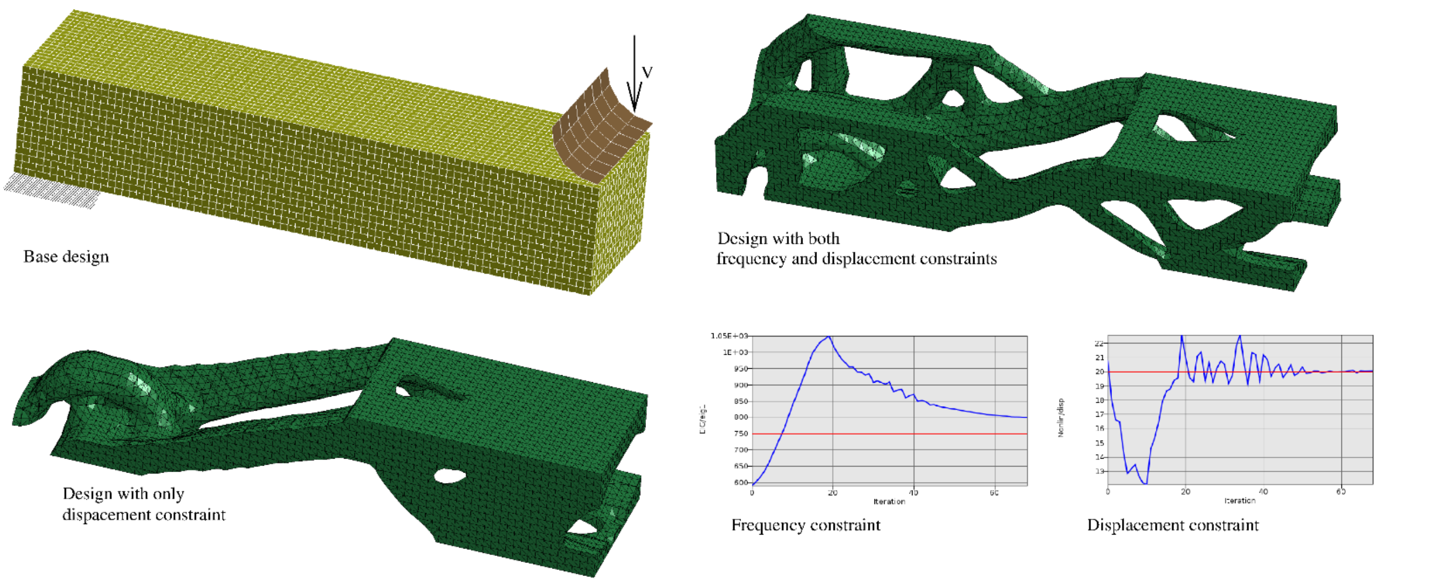
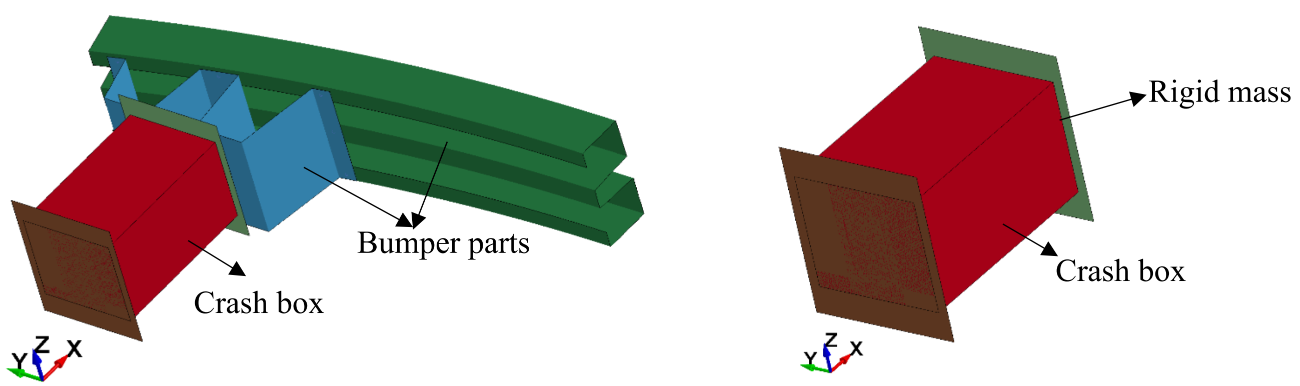


Figure 2 Nonlinear beam example. Shown are the base structure and two designs. The one design used only a displacement constraint and the other both a displacement and natural frequency constraint. The constraint histories for the latter case is also shown.

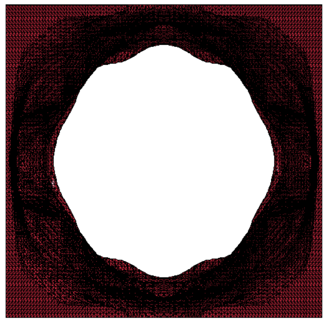
Crash Box Example

This problem is to perform lightweight optimization of an automotive crash box by considering crashworthiness characteristics along with NVH characteristics. These two characteristics yield a competition between deformability necessary for improving energy absorption and rigidity for improving NVH characteristics of the crash box. A solid design part connected to simplified bumper parts is used as the baseline design of the crash box subject to high-speed collision. However, this FE model cannot be used directly for conducting modal frequency analysis due to the involvement of bumper parts in the model. Therefore, the solid design part of the crash box with rigid mass attached at its front end is used for modal frequency analysis. A multidisciplinary optimization is conducted to minimize mass of the crash box with a constraint on energy absorption ability and two constraints on the frequencies of the first two bending modes with mode tracking. The energy absorption ability of the crash box should meet at least same maximum energy absorption as a reference shell-structured crash box, which is E^{*} = 52 KJ. The frequencies of the first two bending modes in y-direction and z-direction should be larger than 0.73 and 0.71 respectively. Results are shown in Figure 3, Figure 4, and Figure 5.

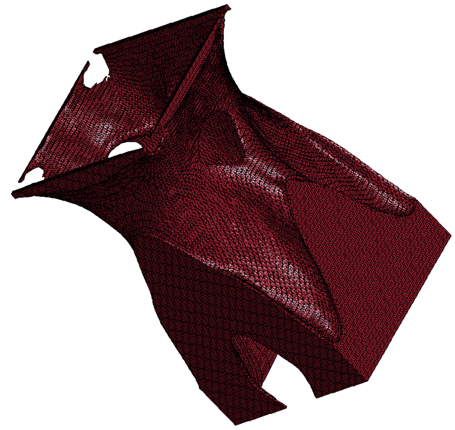


FE model for collision analysis FE model for modal frequency analysis

Final design of crash box

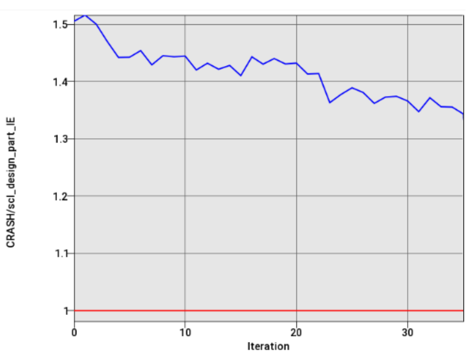


Front view of crash box

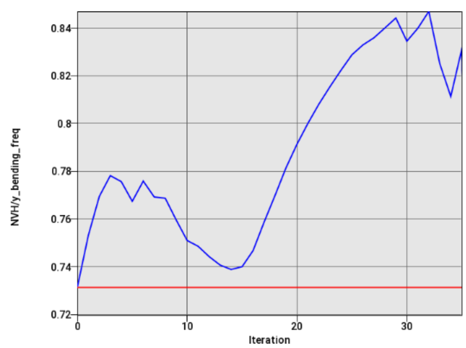


View of crash box

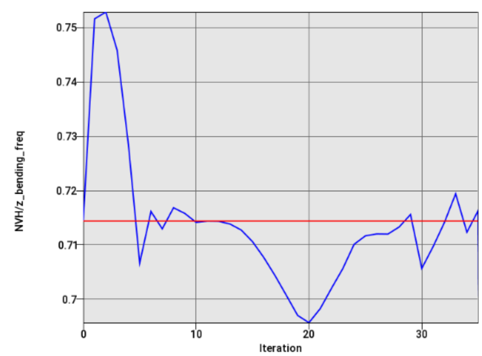
Figure 3 Crash box example. The FE models for two load cases are shown on top with the final design below displayed in different views.



Internal energy history

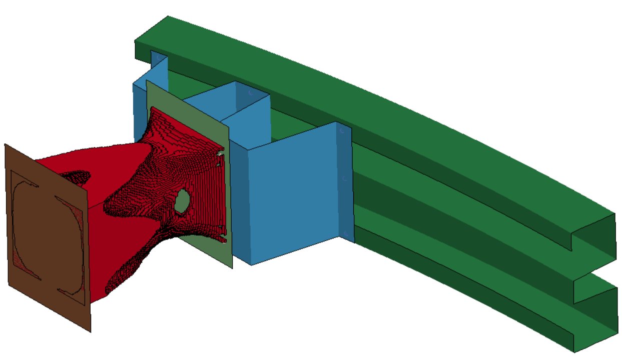


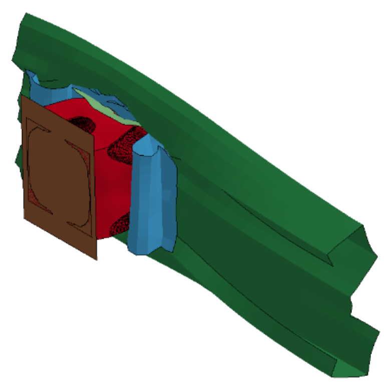
Frequency history of the 1st bending mode in y-direction



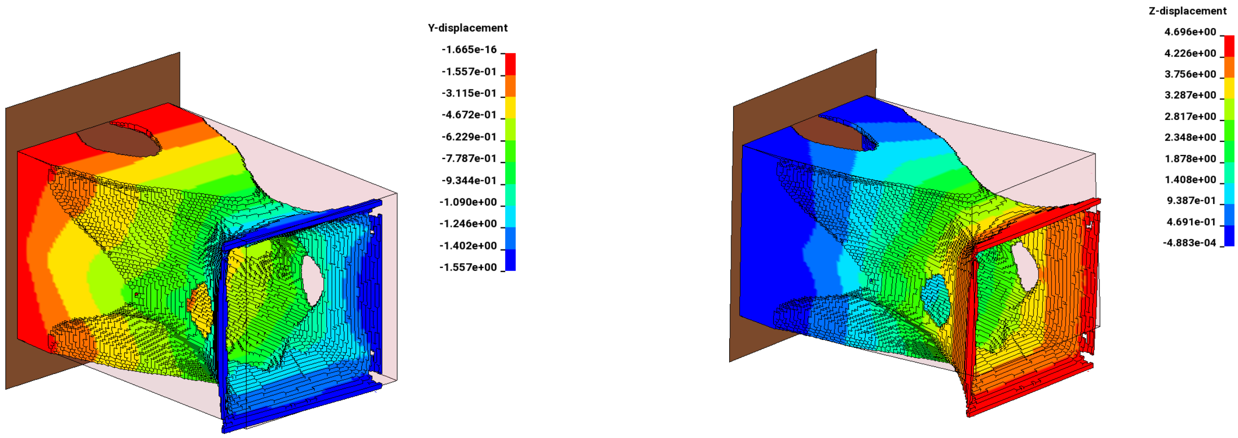
Frequency history of the 1st bending mode in z-direction

Figure 4 Histories of the internal energy and two frequency constraints.





Deformation of crash box and bumper at t = 0 s (left) and t = 30 s (right)



First two bending modes in y-direction (left) and z-direction (right)

Figure 5 The deformed structures for two load cases.

Summary and Conclusions

The paper showed how multidisciplinary topology optimization problems are formulated and solved by formulating a Lagrange function from the multitude of objectives and the constraints. This allows us to solve huge multidisciplinary topology optimization problems incorporating both occupant safety and NVH constraints.

References

1. J. Deaton, R. Grandhi, A survey of structural and multidisciplinary continuum topology optimization: post 2000, Struct. Multidiscip. Optim. 49(1): 1–38, 2014.
2. J. Liu, A. Gaynor, S. Z. K. Chen, K. Suresh, A. Takezawa, L. Li, J. Kato, J. Tang, C. C. L. Wang, L. Cheng, X. Liang, A. C. To, Current and future trends in topology optimization for additive manufacturing, Struct. Multidiscip. Optim. 57(6): 2457–2483, 2018.
3. J. Liu, Y. Ma, A survey of manufacturing oriented topology optimization methods, Adv. Eng. Softw. 100: 161–175, 2016.
4. T. Iqbal, L. Wang, D. Li, E. Dong, H. Fan, J. Fu, C. Hu, A general multi-objective topology optimization methodology developed for customized design of pelvic prostheses, Medical Engineering and Physics, 69: 8– 16, 2019. https://doi.org/10.1016/j.medengphy.2019.06.008.
5. I. Gandikota, G. Yi, W. Roux, Crashworthiness and lightweight optimization of an automotive crash box using LS-TaSC, FEA Information Engineering Solutions, October 2019.
6. K. Weider, A. Schumacher, Adjoint method for topological derivatives for optimization tasks with material and geometrical nonlinearities, in: R. H. et al (Ed.), Proceedings of the 6th International Conference on Engineering Optimization, Springer, Cham, 2018.
7. W. Roux, G. Yi, I. Gandikota, A spatial kernel approach for topology optimization, Computer Methods in Applied Mechanics and Engineering, 361: 112794, 2020. https://doi.org/10.1016/j.cma.2019.112794
8. W. Roux, Structural topology optimization using numerical derivatives, Japan patent 6548532. Filed September, 2015. Issued July 1, 2019.
9. W. Roux, Constrained topology optimization using multi-point perturbations of global variables, USNCCM 13, 2015. doi:10.13140/RG.2.2.35972.88969.
10. W. Roux, A feasible minimum energy state method for constrained, multi-disciplinary topology optimization problems, USNCCM 14, 2017. doi:10.13140/RG.2.2.24228.83845.
11. W. Roux, The LS-TaSC multipoint method for constrained topology optimization, in: 14th International LS-DYNA® Users Conference, Dearborn MI, 12-14 June, 2016.
12. W. Roux, G. Yi, I. Gandikota, Implementation of the projected subgradient method in LS-TaSC in: 15th International LS-DYNA Users Conference, Dearborn MI, 10-12 June, 2018.