LS-OPT®

Probabilistic Analysis

Willem Roux (LSTC)

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Content

- Objectives
- Statistical Background
- Structural Mechanics
- Methodologies
- Examples
- Hands-on Tutorial

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Objectives

Modeling of Variability

- Repeatability of Response

Design Criteria

- Probability of failure
- Robustness (Variance)

Redesign

- Source of variability

BASIC STATISTICS

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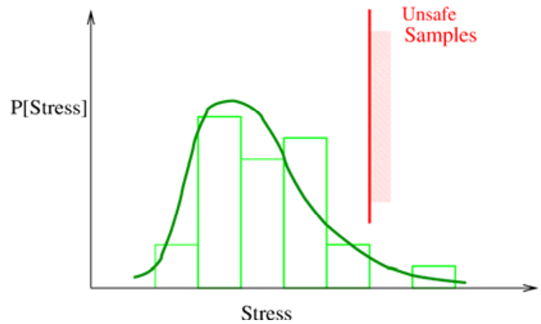
Response Variability



Response distribution

- Mean
- Standard deviation

Probability of Failure

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Basic Computations

- Mean

$$\mu = \frac{1}{n}\sum_{i=1}^{n} y_i$$

- Variance

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \mu)^2$$

- Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

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Basic Computations

Coefficient of variation

$$c.o.v = \frac{\sigma}{\mu}$$

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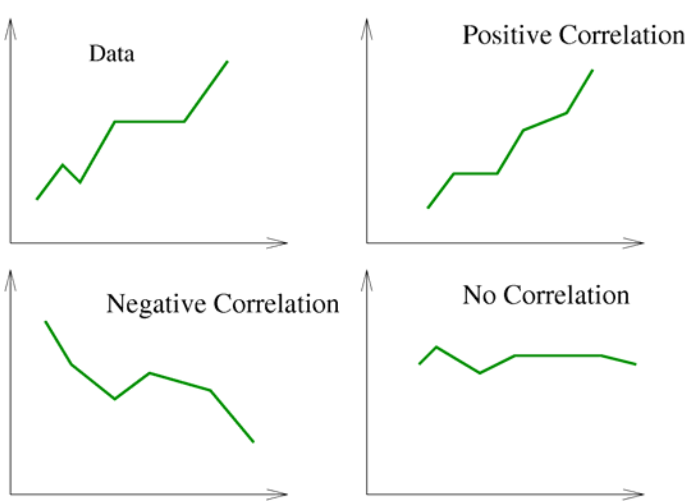
Covariance and Correlation



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Covariance and Correlation

Covariance

$$Cov(y_1, y_2) = E[(y_1 - \mu_1)(y_2 - \mu_2)]$$

Coefficient of correlation

$$\rho = \frac{Cov(y_1, y_2)}{\sigma_1 \sigma_2}$$

Always between –1 and 1

Viewer shows confidence bounds on correlation.

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Covariance and Correlation

Correlation plots are the X-ray machines of the stochastic world. It shows the interdependence of results.

Correlation is not causation though.

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Sum of Normal Variables

$$Y = X_1 + X_2$$

$$\mu_Y = \mu_1 + \mu_2$$

$$\sigma_Y^2 = \sigma_1^2 + \sigma_2^2$$

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Statistical Distributions

Beta

Binomial

Lognormal

Normal

Uniform

User defined

Weibull

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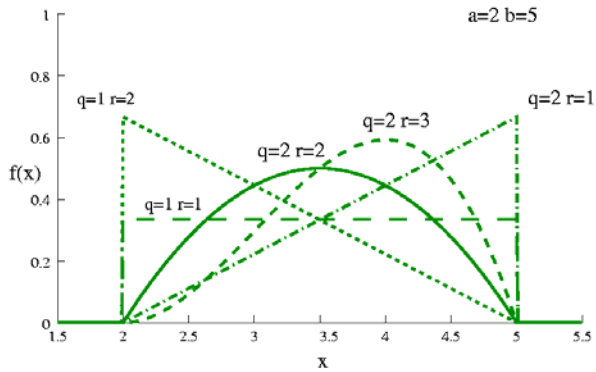
Beta Distribution

Lower and Upper Value

Two Shape parameters

Versatile



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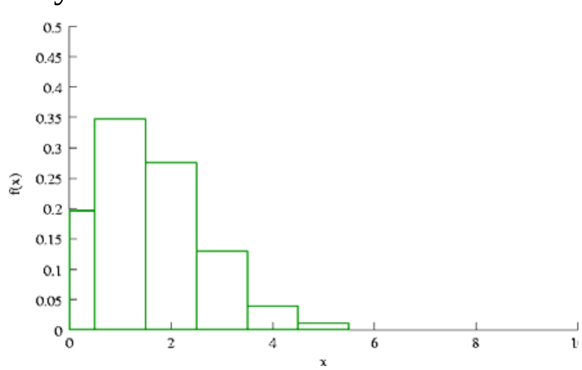
Binomial Distribution

Discrete Distribution

Probability of Event and Number of Trials



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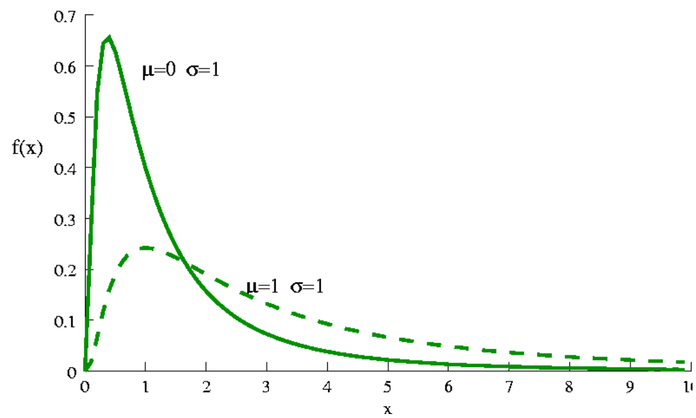
Lognormal Distribution

Mean and Standard Deviation



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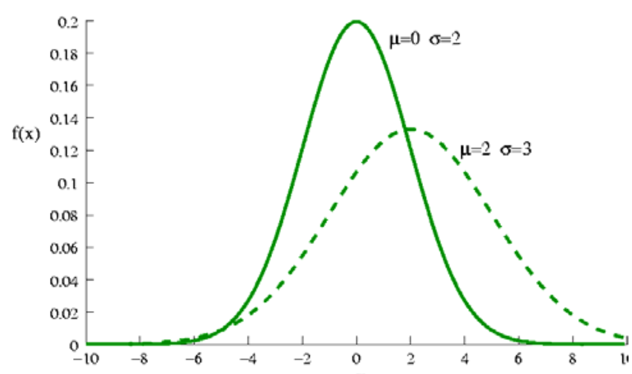
Normal Distribution

Mean and Standard Deviation



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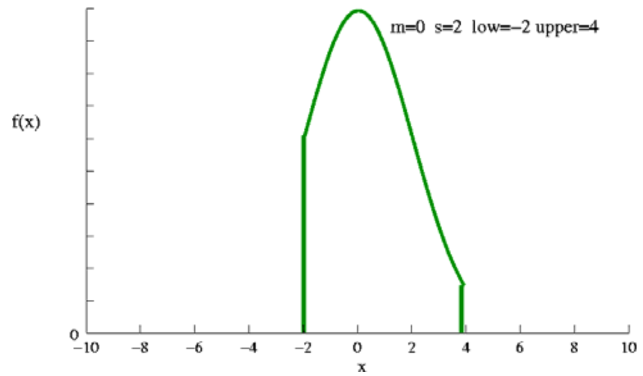
Truncated Normal

Mean, standard, deviation, lower and upper bound



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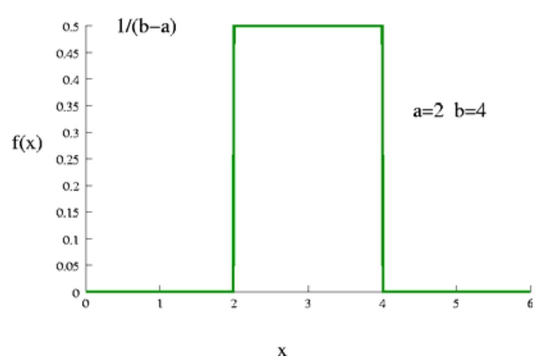
Uniform Distribution

Lower and Upper Bound



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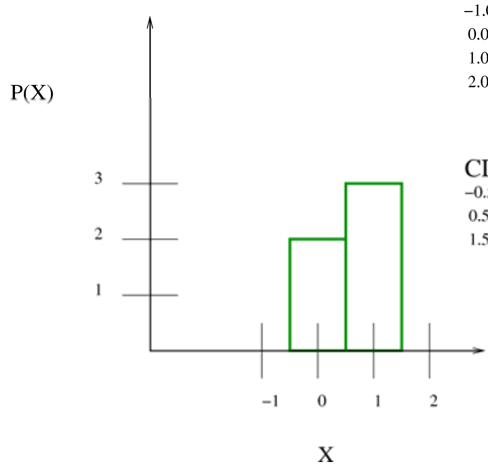
User Defined



PDF File
−1.0 0.0
0.0 2.0
1.0 3.0
2.0 0.0

CDF File
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0.5 0.4
1.5 1.0

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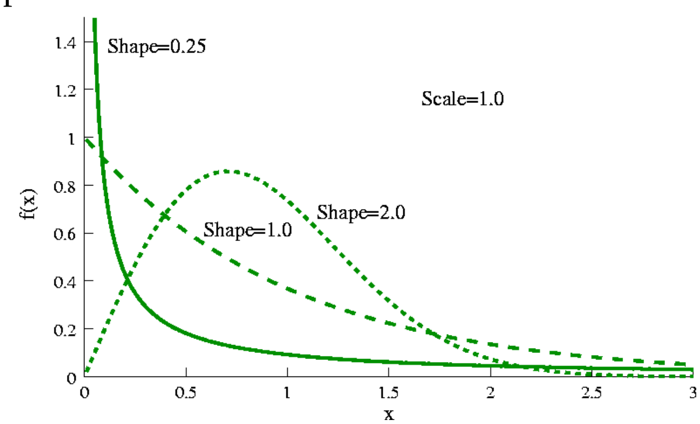
Weibull

Shape and Scale



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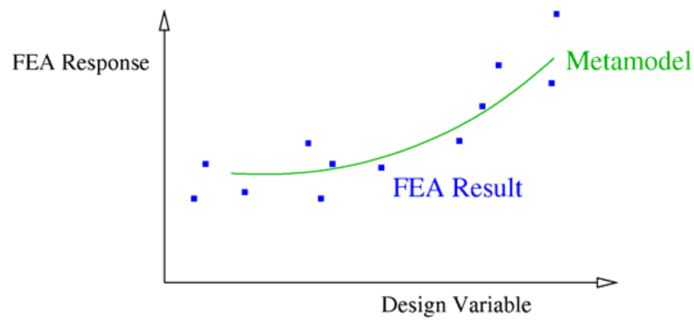
Response Surfaces

- Approximations to structural behavior
- Build from responses from selected set of designs
  - Experimental design
- Multi-dimensional Least Squares Fit



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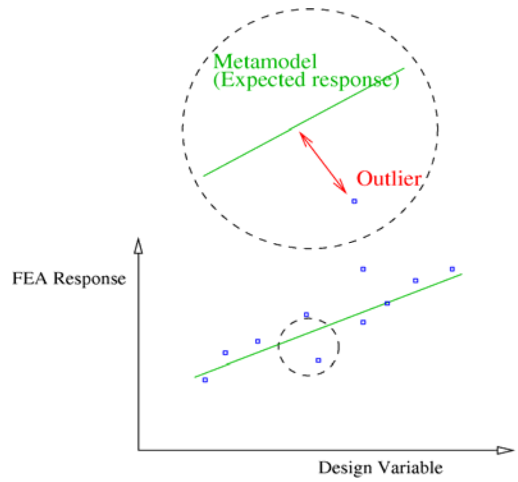
Outliers/Residuals



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METHODOLOGIES I

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Probabilistic Analysis

- Monte Carlo
- Monte Carlo using Metamodels
- Reliability Based Design Optimization
- Robust Parameter Design
- Outlier/bifurcation investigation
- Metal Forming

Probabilistic Analysis

Variables

Control Variables (Design Parameters)

Nominal value controlled by designer

- Gauge
- Shape

Noise Variables (Environment)

Values not controlled by designer but can vary

- Load
- Yield stress

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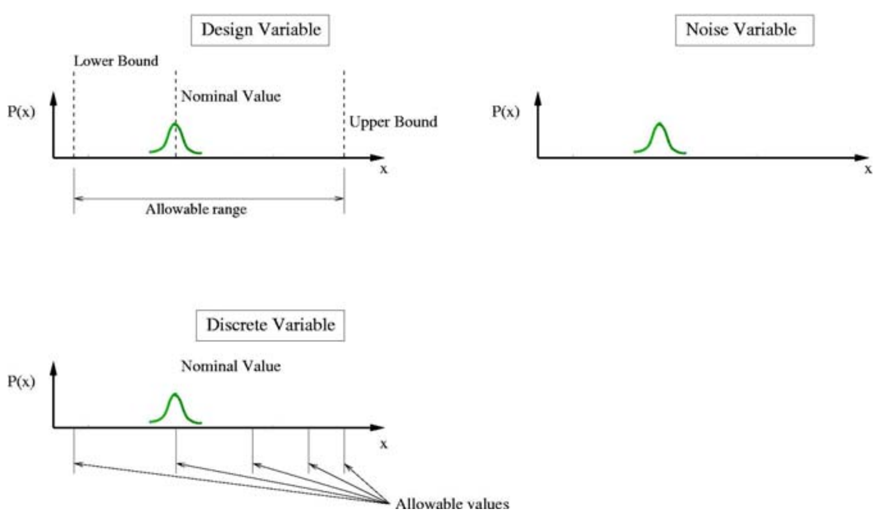
Probabilistic Analysis

Probabilistic Variables

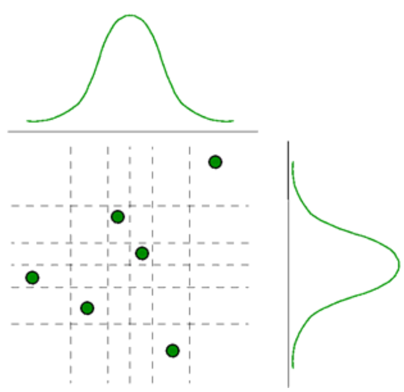


Monte Carlo Analysis

- Random process
- Large number of FE runs (100+)
- Sampling
  - Random
  - Latin Hypercube (Structured Monte Carlo)

Latin Hypercube Sampling

- Each partition has equal probability.



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Monte Carlo Using Metamodels

- Response Surface / Neural Network
- Medium number of FE runs (10 – 30+)
- Monte Carlo analysis conducted using metamodel and a large number of points (10^6)
- Identify design variable contributions

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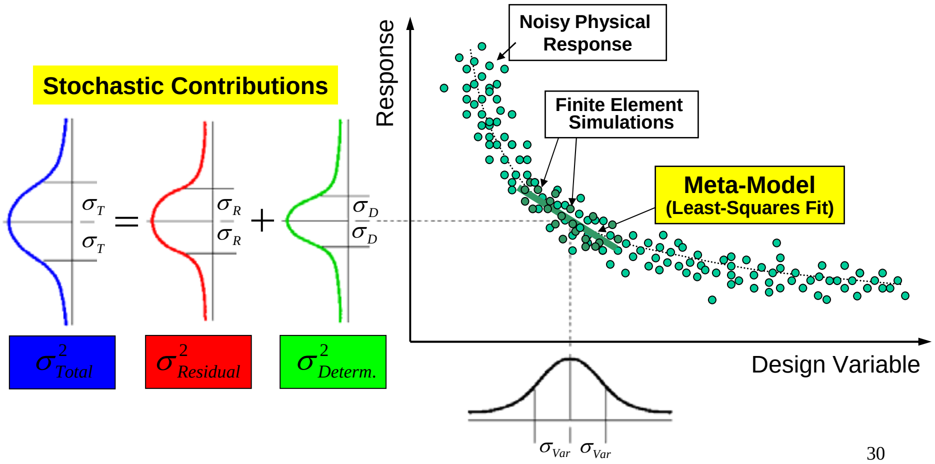
Metamodel-based Monte Carlo



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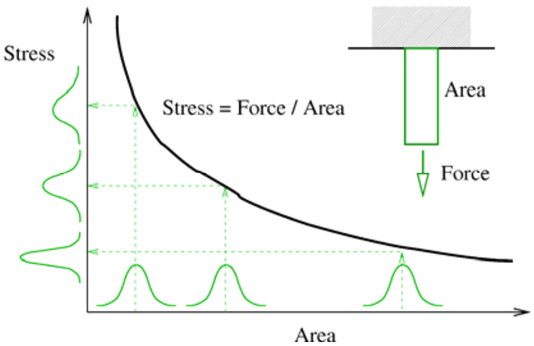
Contribution Analysis

Contribution of variation of variable to variation of response

$$\sigma_{g,i} = \frac{\partial G}{\partial x_i}\sigma_{x,i}$$

Variance of response

$$\sigma_g^2 = \sum_{i=1}^{n}\sigma_{x,i}^2$$



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EXAMPLES I

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Probabilistic Analysis

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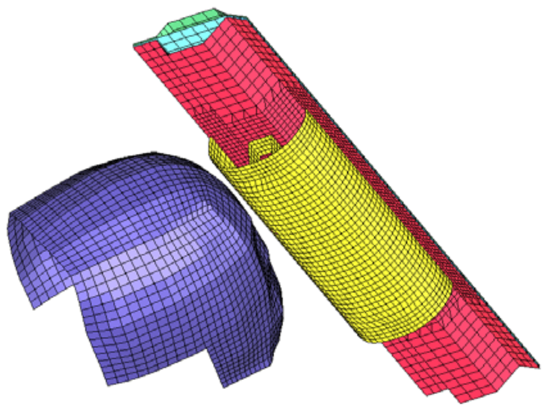
Head Impact Problem



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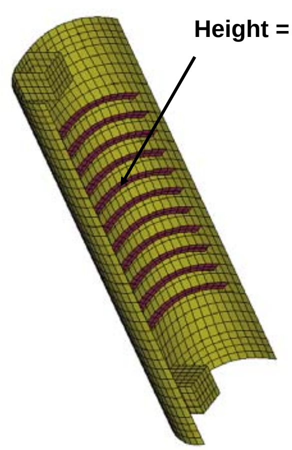
Head Impact Problem

Monitor: HIC-d

(Head Injury Criterion)

Variables:

- Horizontal Angle of impact
  - 15 degrees
  - 10% standard deviation
- Rib height
  - 12.5mm
  - 5% standard deviation



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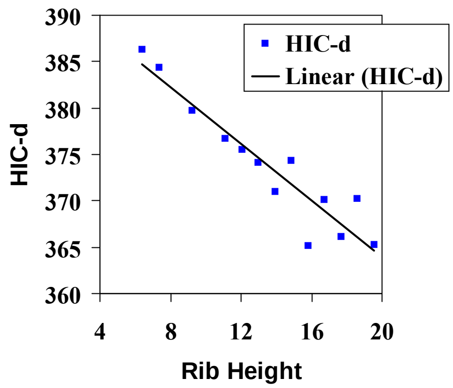
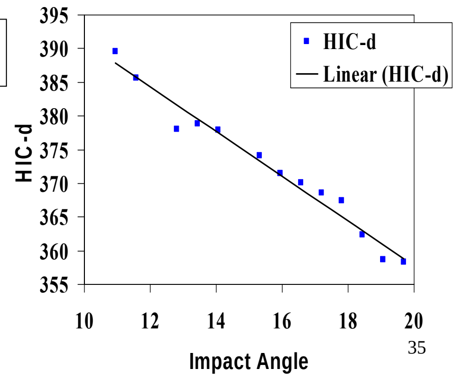
Variation

Vary one variable at a time to investigate curvature.

Linear response with some scatter (noise).





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Variation

Reponses nearly linear

Quadratic Surface should fit accurately

Range of response surface is 2**σ**

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Response Variation

Baseline Design: **HIC-d = 374.4**

Monte Carlo Analysis: **150 FE analyses**

Quadratic Response Surface: **60 FE analyses.**

Outliers have standard deviation of 2.35

	Monte Carlo (150 simulations)	Metamodel (60 simulations)
Mean	373.9	373.9
Standard deviation		
• Structural	4.85	4.21 (87%)
• Structural + Outlier		4.82

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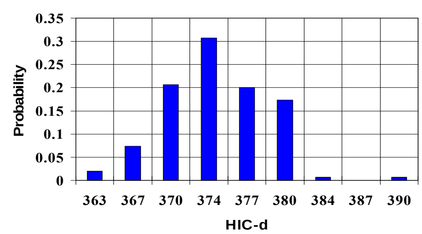
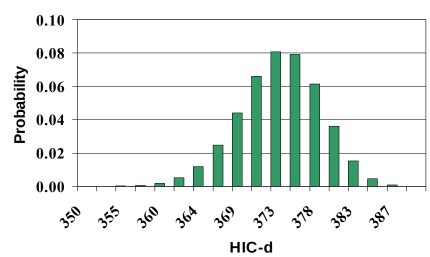
Probability of Value

Direct Monte Carlo Analysis



Metamodel



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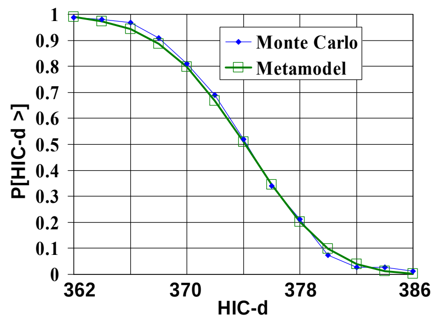
Probability of Exceeding Bound



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Probabilistic Analysis

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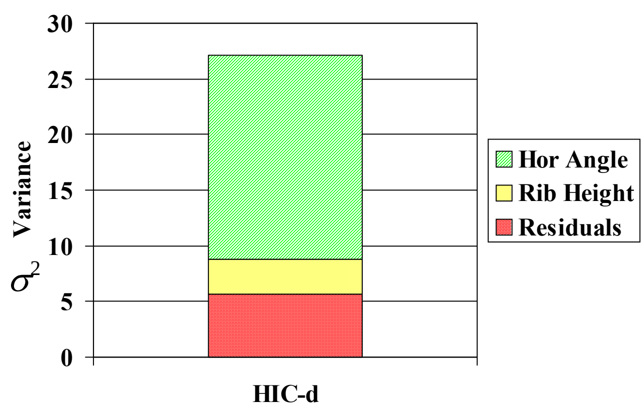
Contribution Analysis



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Tutorial I

Reliability Analysis Tutorial

- RELIABILITY/MONTE_CARLO
- RELIABILITY/METAMODEL

METHODOLOGIES II

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Probabilistic Analysis

- Monte Carlo
- Monte Carlo using Metamodels
- Reliability Based Design Optimization
- Robust Parameter Design
- Outlier/bifurcation investigation
- Metal Forming

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RBDO

Reliability Based Design Optimization includes uncertainty of variables and responses into optimization

Requires:

- Statistical distribution of variable
- Probabilistic bound on constraint.

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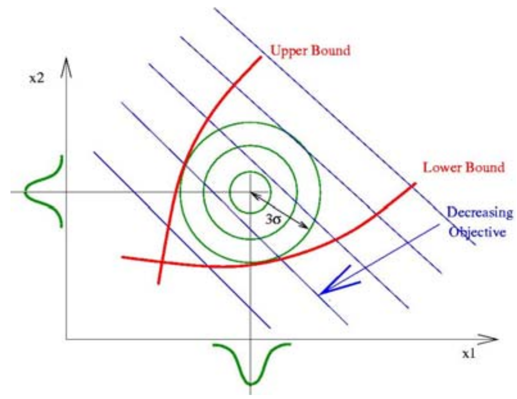
RBDO: Probabilistic bound

Underlying implementation shifts constraint bound with number of standard deviations corresponding to probability.



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Robust Parameter Design

Robust parameter design creates designs insensitive to the variation of specific inputs.

Two sets of variables:

- Noise variables
- Control (normal) variables

Probabilistic Analysis

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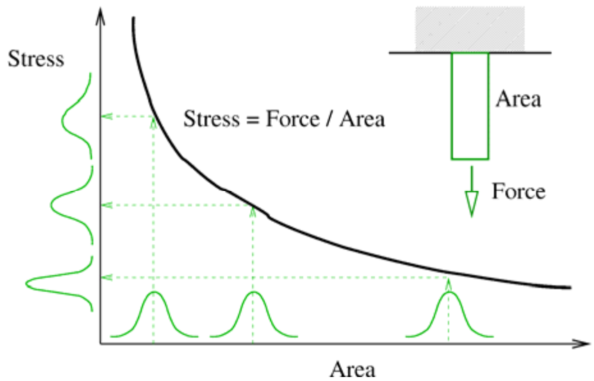
Robust Parameter Design

Search for a flat part of design space



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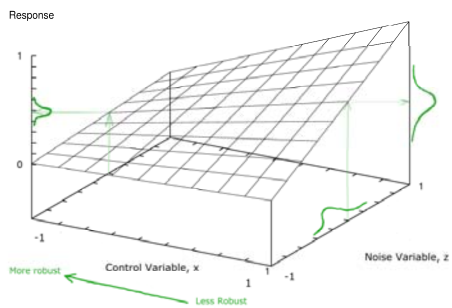
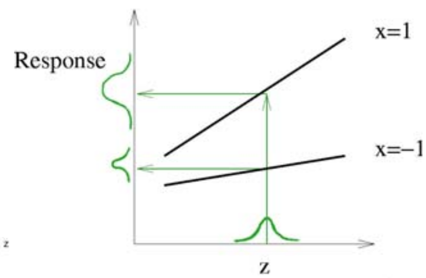
Robust Parameter Design

Control variables are adjusted to minimize the effect of the noise variables.





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Robust Parameter Design

LS-OPT Command

composite 'var x11' noise 'x11'

Experimental design

Interaction terms must be included.

Use multi-criteria design to described bigger-is-better etc.

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Tutorial II

RBDO Tutorial

RBDO

Robustness Tutorial

ROBUST_PARAMETER_DESIGN

Probabilistic Analysis

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STRUCTURAL MECHANICS

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Probabilistic Analysis

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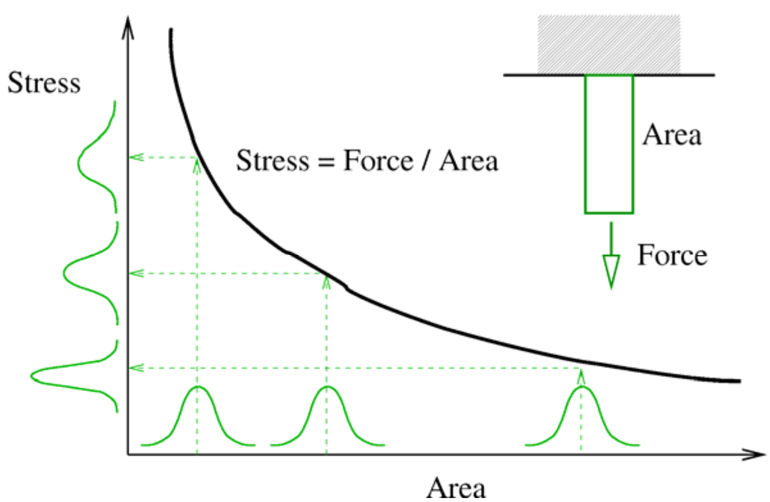
Deterministic Variation



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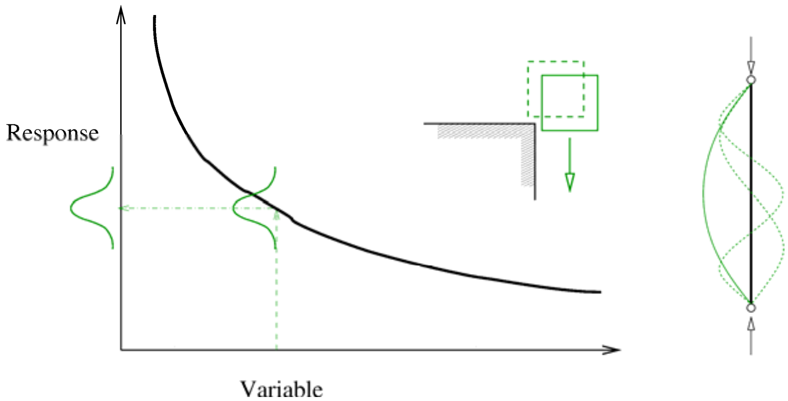
Process Variation



Process is seeded by stochastic fields

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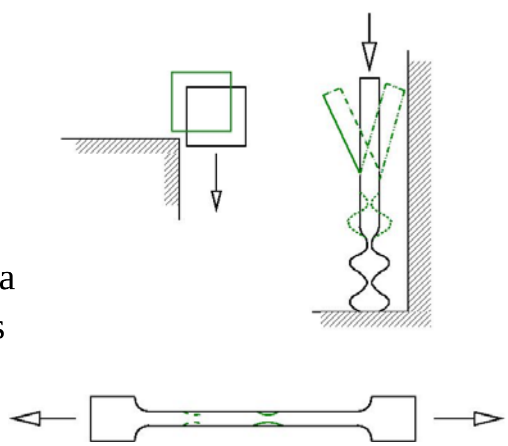
Mechanics of stochastic processes

- Physical
  - Instability
  - On-Off Impact
- Numerical
  - Discretization
  - Convergence criteria
  - Different computers
  - Different compilers



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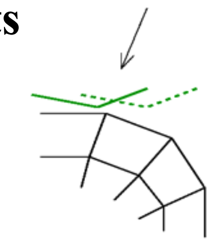
Process Variation

Discretization (modeling) effects

- Smoothness of FE mesh



- Postprocessing: time step size and filter selection
- Selection of node/element to monitor

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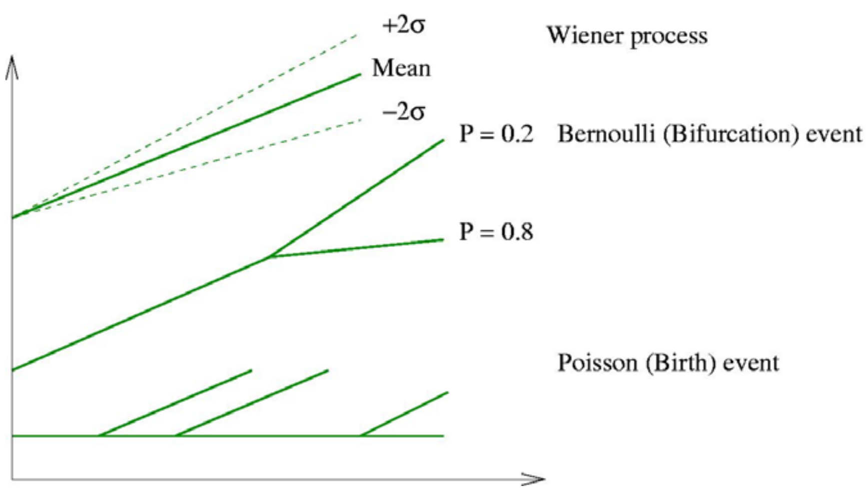
Stochastic Processes



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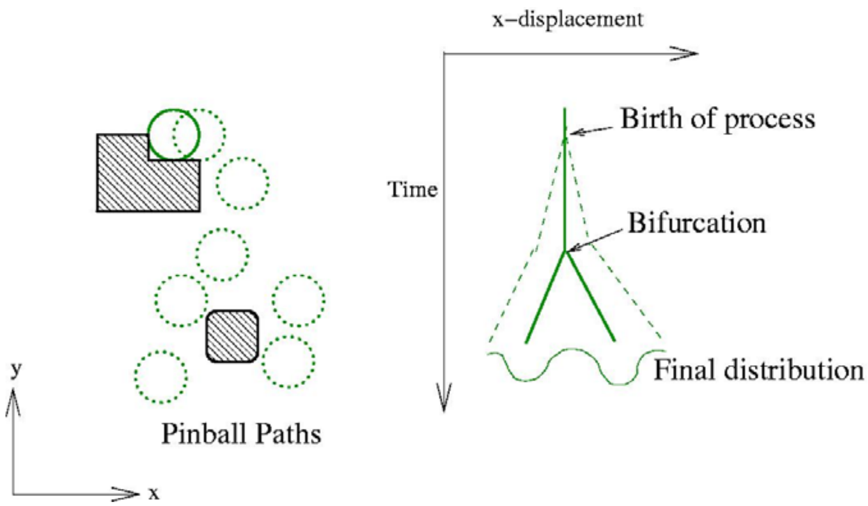
Stochastic Process Example



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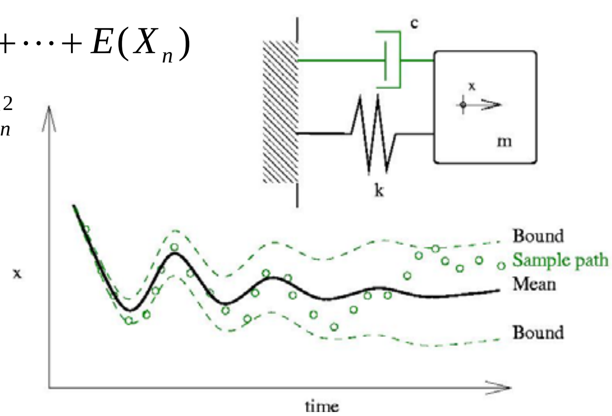
Statistics of Stochastic Process

One event after another.

$$E(X) = E(X_1) + \cdots + E(X_n)$$

$$\sigma^2 = \sigma_1^2 + \cdots + \sigma_n^2$$



LS-OPT

Stochastic Fields

Our stochastic capability consists of:

- Stochastic fields and
- Stochastic variables.

Random variables are part of LS-OPT ® while stochastic fields are part of LS-DYNA ®.

LS-OPT

Stochastic Fields

Stochastic process is seeded by a stochastic field.

A stochastic field allows a property (e.g. shell thickness) to vary over a part.

LS-DYNA keywords:

*PERTURBATION_NODE

*PERTURBATION_SHELL_THICKNESS

Methods

- Harmonic Fields
- Import DYNA displacements or eigen mode

Probabilistic Analysis

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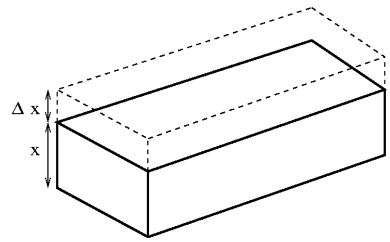
Random fields vs. variables

Random Fields

*PERTURBATION in LS-DYNA ®, valid for geometry, shell thickness, and some material parameters.

Correlation function defined as part of the keyword.

Random Variables



*PARAMETER, valid for any number in the input file.

Statistical distribution assigned in LS-OPT ®

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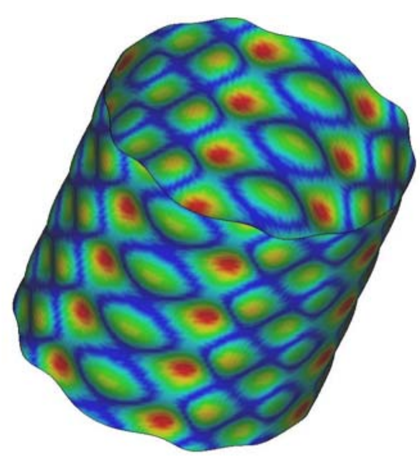
Stochastic Field: Geometric imperfection

*PERTURBATION_NODE



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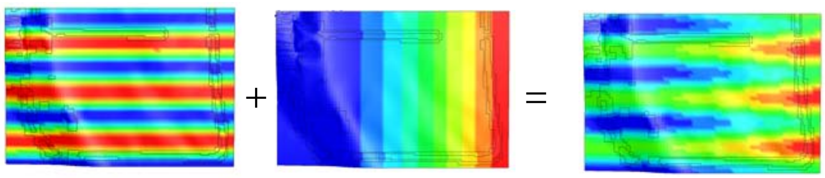
Stochastic Field: Blank Thickness

Variation due to rolling of steel.

***PERTURBATION_SHELL_THICKNESS**



Mill chatter   Non-uniform roll contact

See also Reliability Based Design Optimization with LS-OPT for a Metal Forming Application by Müllerschön, Lorenz & Roll

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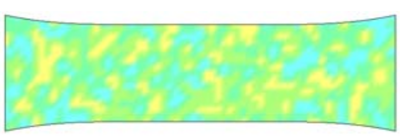
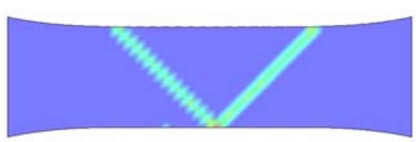
*PERTURBATION_MAT

This allows the spatial variation of material properties such as the yield stress.

It is being developed. It should be available for MAT024 in LS980.



Yield Strength Variation



Resulting Plastic Strain just before rupture.

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Correlation function

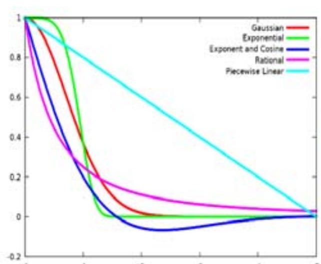
A random field (or process) is characterized by its correlation function.

$$R(\tau) = E[X(t+\tau)X(t)]$$

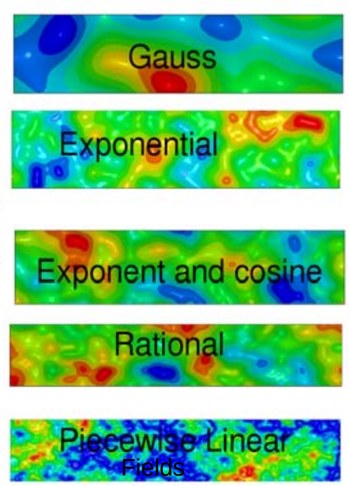
A description of the experimentally measured correlation function allows us to create many random fields in LS-DYNA with the same properties. We have predefined functions for which the user must define the parameters.

The methodology works for very large problems (millions of nodes).

Effect of correlation functions type

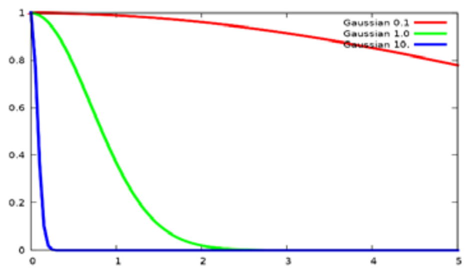


Different Functions Types

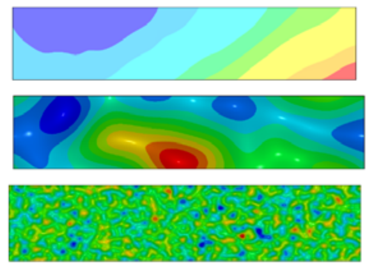


Fields

Effect of parameter values



Gaussian functions with different values of the function parameter



Fields

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Alternative methods of specifying stochastic fields

User defined fields: A file containing the stochastic field can be specified. This allows, for example, scaled values of the eigen modes to be used as perturbations in a buckling analysis.

Sinusoidal perturbations: A sine expansion describing the perturbation can be specified.

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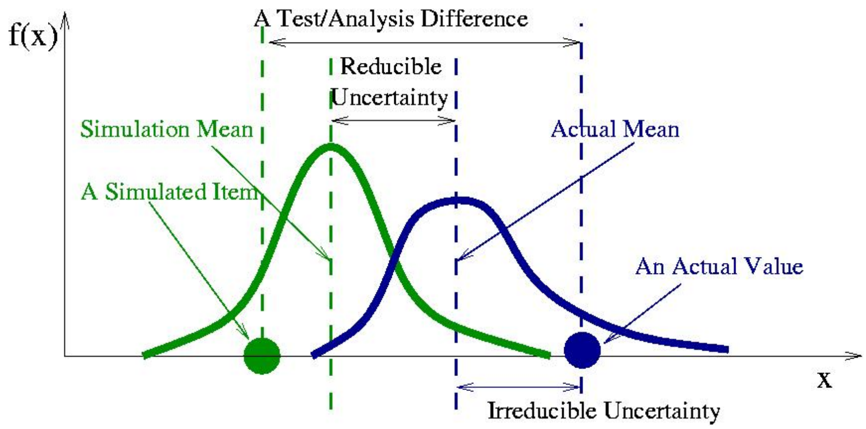
Test vs. Analysis



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Probabilistic Analysis

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METHODOLOGIES II

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Probabilistic Analysis

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Probabilistic Analysis

- Monte Carlo
- Monte Carlo using Metamodels
- Reliability Based Design Optimization
- Robust Parameter Design
- Outlier/bifurcation investigation
- Metal Forming

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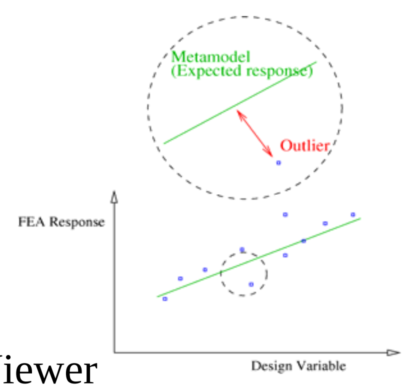
Outlier Analysis

Unexpected events:

- Buckling
- Modeling



Investigate:

- Accuracy Plots in Viewer
- Visualize on FE model in LS-PrePost

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Metal Forming

Visualization of metal forming results requires:

- Consider adaptivity (meshes differ for designs),
- Map to geometric location (not node),
- Map to base design (run *iteration*.1).

Finer meshes give better accuracy.

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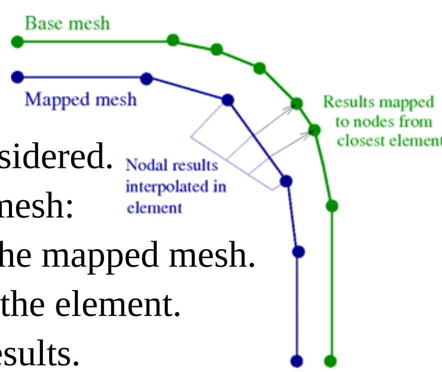
LS-OPT

Metal Forming Results Interpolation



Only the work piece is considered.

For each node in the base mesh:

Find closest element in the mapped mesh.

Find the closest point in the element.

Use the element nodal results.

Interpolated linearly between the nodes.

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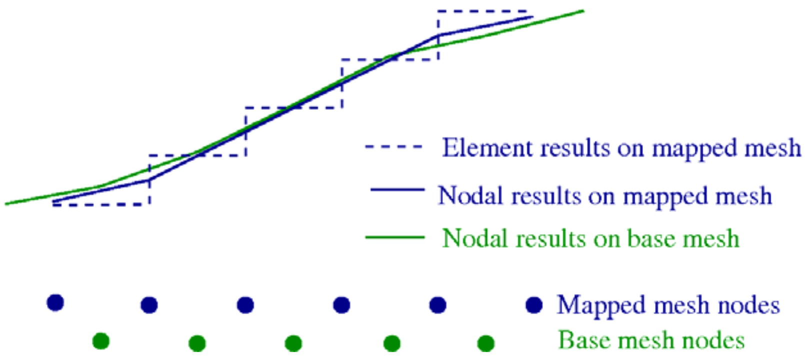
LS-OPT

Metal Forming Results Interpolation

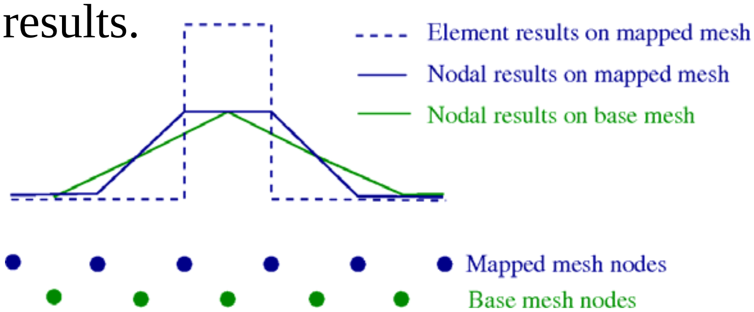
Element results are transformed to nodal averaged as in LS-PREPOST.



LS-OPT

Metal Forming Results Interpolation

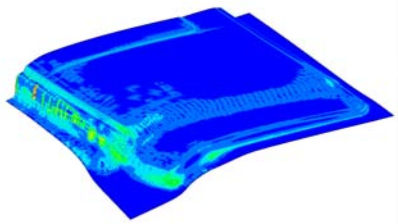
Sudden jumps, especially over a single element, are filtered. Prediction of failure should be preserved in the mapped results.

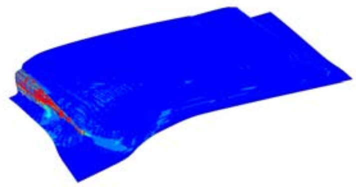


Probabilistic Analysis

LS-OPT

Statistic of plastic strain





Mean

Standard deviation

Red = 0.5

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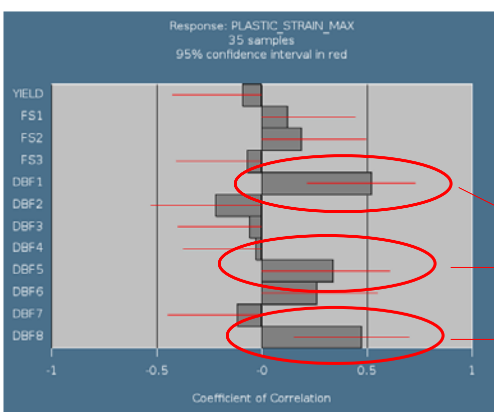
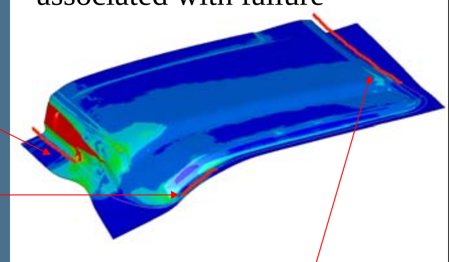
Probabilistic Analysis

LS-OPT

Maximum plastic strain: Sources of variation



Draw bead variation associated with failure



Coefficient of correlation plot

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Probabilistic Analysis

LS-OPT

EXAMPLES II

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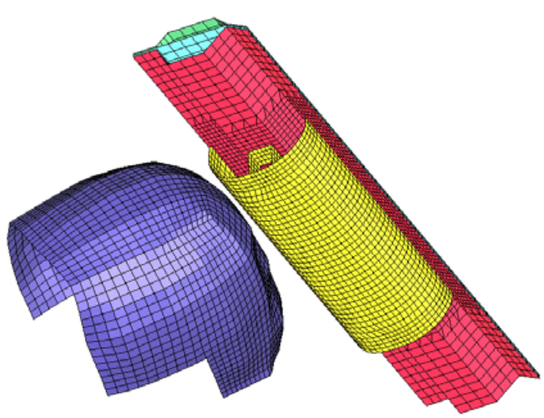
Probabilistic Analysis

LS-OPT

Head Impact Problem



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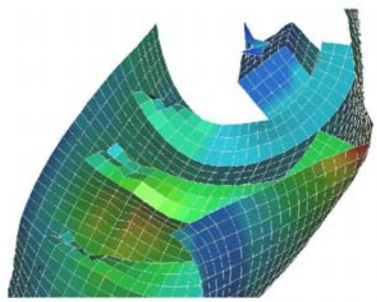
LS-OPT

Outlier Analysis

Some displacements may be:

- Unrelated to a design variable change
- Not repeatable



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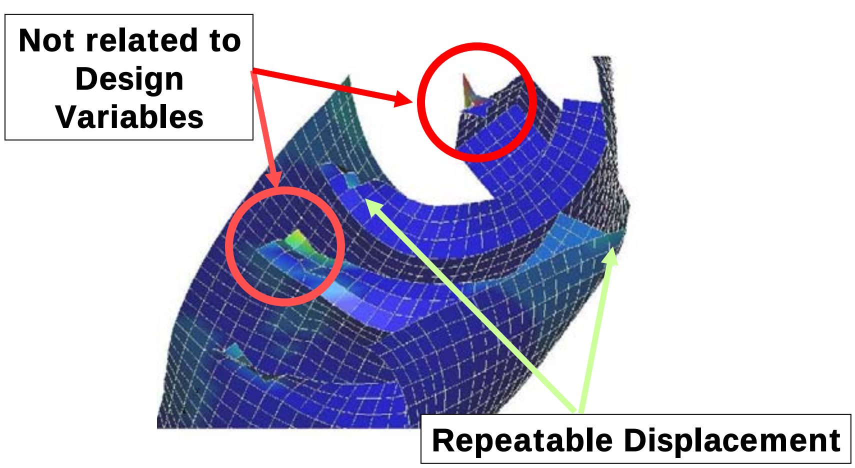
LS-OPT

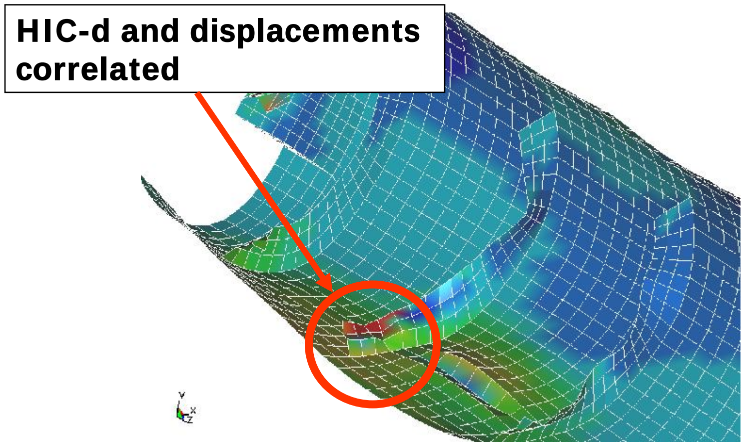
Outliers Variation



Probabilistic Analysis

LS-OPT

Correlation



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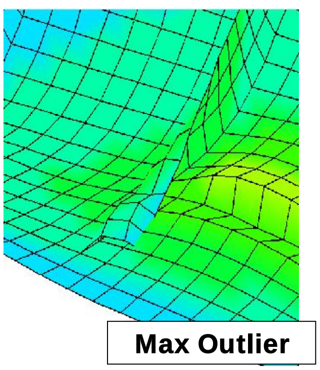
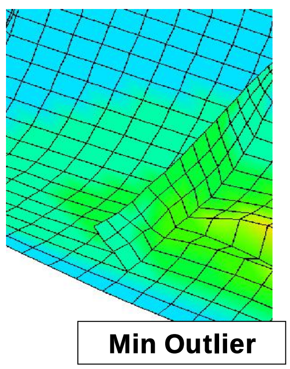
Probabilistic Analysis

LS-OPT

Investigate Outliers

Different buckling modes





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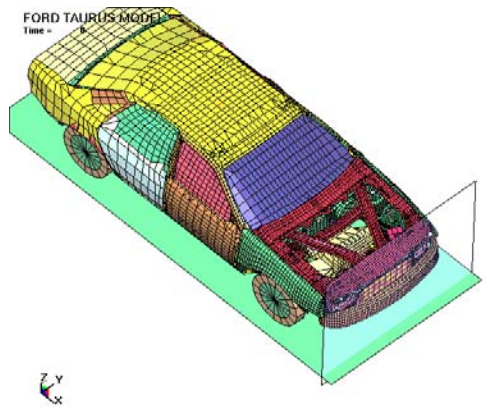
Probabilistic Analysis

LS-OPT

Vehicle Crash

Vary angle of impact

25 FE Runs



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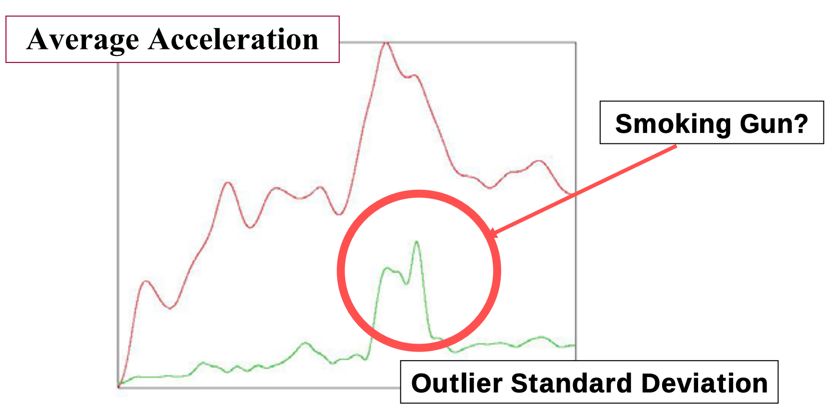
Probabilistic Analysis

LS-OPT

History Variation



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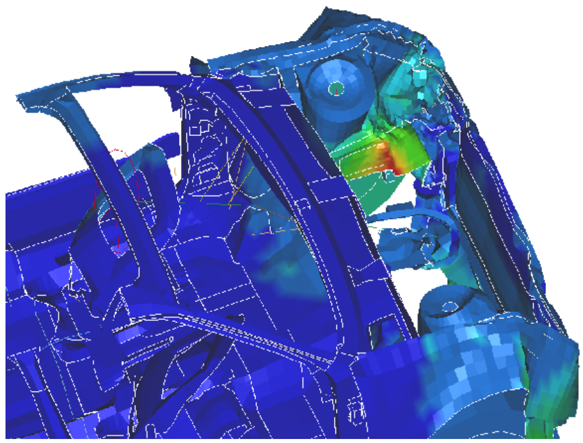
Probabilistic Analysis

LS-OPT

Displacement Variation



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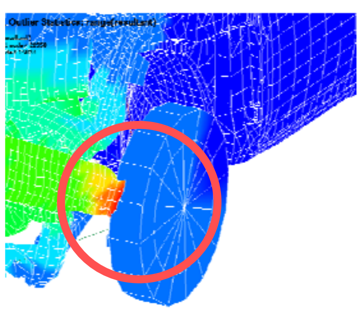
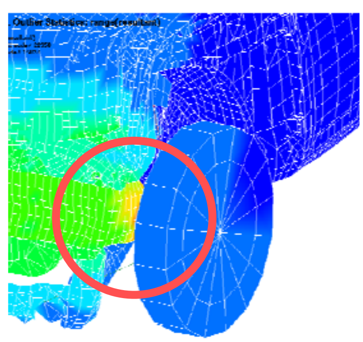
Probabilistic Analysis

LS-OPT

Impact





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LS-OPT

Tutorial III

Bifurcation/Outlier Analysis

- OUTLIER

Metal Forming

- METALFORMING_RELIABILITY

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LS-OPT

PROBABILITY

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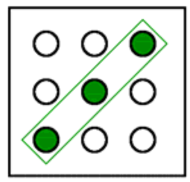
Probabilistic Analysis

LS-OPT

Basic Probability

Probability of subset of events

B



A

Count: P(A=B) = 3/9 = 1/3

Need probabilistic rules for general case

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Probabilistic Analysis

LS-OPT

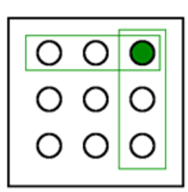
AND Rule

$P(A \cap B) = P(A)\ P(B \mid A)$

Independent: $P(A \cap B) = P(A)\ P(B)$

Probability of event A AND event B

B



A

$$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

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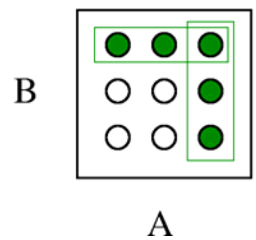
OR Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Mutually exclusive: $P(A \cup B) = P(A) + P(B)$

Probability of event A OR event B



$$\frac{1}{3} + \frac{1}{3} - \frac{1}{9} = \frac{5}{9}$$

Probability Example

P[Failure due to A]=0.6

P[Failure due to B]=0.4

P[Failure] = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

= 0.6 + 0.4 - 0.6*0.4

= 0.76

LS-OPT

The End

<<EOF>>