The Development of XFEM Fracture and Mesh-free Adaptivity

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Ulm

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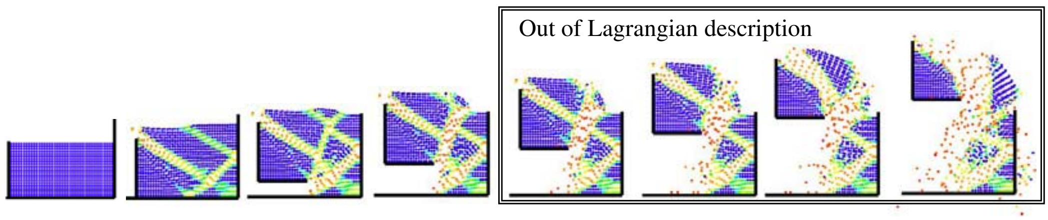
Problem Looking for Solution LS-DYNA

Multi-Physics : *shear band* + ***history dependent large deformation*** + *failure*



Numerical : *multi-resolution* + ***avoid mesh tangle*** + *failure mechanics*

multi-resolution	avoid mesh tangle	failure mechanics
Spectral element method	ALE	Damage mechanics
The variationl multiscale method	Eulerian	Cohesive model
Partition of unity method	Mesh-free	Discrete element method
	Adaptivity	Interface Element, **XFEM** …

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Adaptive Mesh-free Method

LS-DYNA

■ **Motivation**

Improve the large deformation analysis that is beyond the Lagrangian description.

■ **Current Practice**

Solving the forging and extrusion problems.

■ **Formulation**

Adaptive Mesh-free Method = EFG Fast Transformation Method

+ Adaptive Lagrnagian Particles with Eulerian Kernel

+ Consistent Mesh-free Interpolation for State Variable Transfer

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1. EFG Fast Transformation Method

LS-DYNA

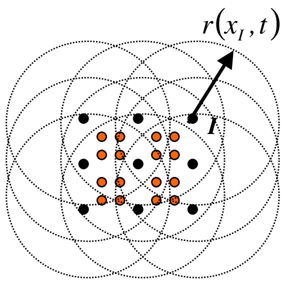
- Momentum equation
  $\rho\,\dot{\boldsymbol{v}} = \nabla_x \cdot \boldsymbol{\sigma} + \boldsymbol{b}$
- Continuity equation
  $\dot{\rho} = -\rho\,\nabla_x \boldsymbol{v}$

$$\int_\Omega \delta\boldsymbol{v} \cdot \rho\,\dot{\boldsymbol{v}}\,d\Omega = -\int_\Omega \nabla\delta v : \boldsymbol{\sigma}\,d\Omega + \int_\Omega \delta\boldsymbol{v}\cdot\boldsymbol{b}\,d\Omega + \int_\Gamma \delta\boldsymbol{v}\cdot\boldsymbol{\tau}\,d\Gamma$$

$$m_I \dot{\boldsymbol{v}}_I = -\sum_I \nabla_x \Phi_I(\boldsymbol{x}_s)\cdot\boldsymbol{\sigma}_s V_s$$

$$\dot{\rho}_s = -\rho_s \sum_I \boldsymbol{v}_I \cdot \nabla_x \Phi_I(\boldsymbol{x}_s)$$

$$u^h_{\bar{\Omega}}(\boldsymbol{x}) = \sum_{\substack{I\in\bar{\Omega}\\ \boldsymbol{x}\in\Omega}} \Phi_I^{[n]}(\boldsymbol{x})\cdot\sum_{J\in\Omega}\hat{\Psi}_J^{[m]}(\boldsymbol{x}_I)\boldsymbol{x}_J \equiv \sum_{\substack{I\in\bar{\Omega}\\ \boldsymbol{x}\in\Omega}} \overline{\Psi}_I^{[m]}(\boldsymbol{x})\boldsymbol{x}_I$$



● Particle
● Stress point

Direct variables computation

Particles	Stress points
Acceleration	Density
Velocity	Strain rate
Position	Stresses
	Internal energy

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2. Adaptive Lagrangian Particles with Eulerian Kernel

LS-DYNA

Convective velocity $\boldsymbol{C}$ due to Eulerian kernel

$$\boldsymbol{v}^- = \frac{\partial \boldsymbol{x}\ (\boldsymbol{X}, t_{n+1})}{\partial t}\Big|_X \ (material\ time\ frame)$$

$$\boldsymbol{v}^+ = \frac{\partial \boldsymbol{x}\ (\boldsymbol{\chi}, t_{n+1})}{\partial t}\Big|_\chi \ (reference\ time\ frame)$$

$$\boldsymbol{C} = \boldsymbol{v}^- - \boldsymbol{v}^+$$

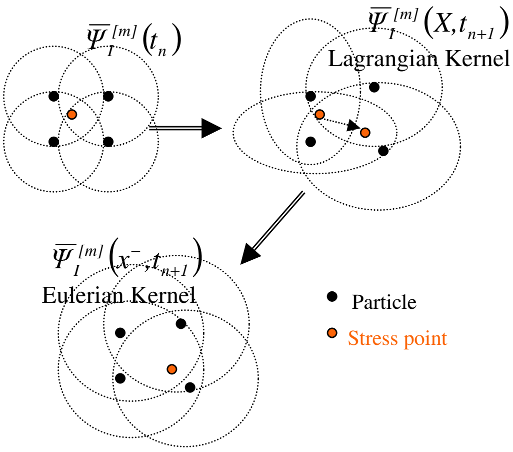
$$\dot{f} = \frac{\partial f}{\partial t}\Big|_\chi + (\boldsymbol{C}\cdot\nabla)f$$

- Momentum equation

$$\rho\frac{\partial v_i}{\partial t}\Big|_\chi + \rho C_j\frac{\partial v_i}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j} + b_i$$

- Continuity equation

$$\frac{\partial \rho}{\partial t}\Big|_\chi + C_j\frac{\partial \rho}{\partial x_j} = -\rho\frac{\partial v_j}{\partial x_j}$$



LS-DYNA

- Operator Split

1. Lagrnagian phase : $\Psi_J^{[m]}(\boldsymbol{x}, t_n) = \bar{\Psi}_J^{[m]}$

2. Transport phase : $\frac{\partial f}{\partial t}\Big|_\chi + (\boldsymbol{C}\cdot\nabla)f = 0$

$$\nabla f = \sum_J \frac{\partial \Psi_J^{[m]}}{\partial \boldsymbol{x}}\tilde{f}_I;\ f_I = \sum_J \Psi_J^{[m]}(\boldsymbol{x}_I, t_n)\tilde{f}_J$$

A →
$$f^+ \approx f^- + \Delta t\boldsymbol{C}\cdot\sum_J \frac{\partial \Phi_J}{\partial \boldsymbol{x}}\Big|_{x=x_J^\theta}\tilde{f}_J^-;\ \boldsymbol{x}^\theta = \boldsymbol{x}(\boldsymbol{x}^n, t_n) + \theta\Delta t(\boldsymbol{v}^+ - \boldsymbol{v}^-)$$

: **Ponthot and Belytschko 1998**

B →
$$\Psi_J^{[m]}(\boldsymbol{x}^+) = \Psi_J^{[m]}(\boldsymbol{x}^-) - (\boldsymbol{x}^+ - \boldsymbol{x}^-)\cdot\frac{\partial \Psi_J^{[m]}}{\partial \boldsymbol{x}}\Big|_{x^-} + \cdots$$

$$f^+ \approx \sum_J \Psi_J^{[m]}(\boldsymbol{x}^+)\tilde{f}_J$$

: TB stress recovery scheme – conservative
– consistent
is *[m]* order accurate; Currently *m*=1

Remapped value ↑ Nodal value ↑

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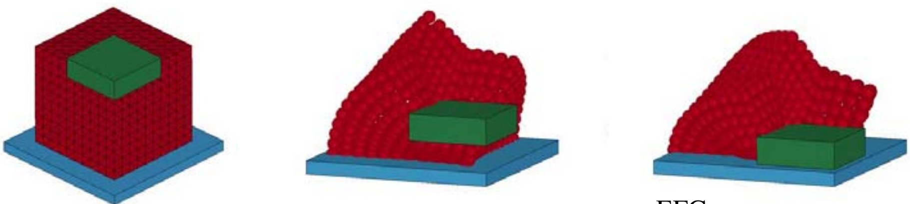
Current EFG Methodology for Large Deformation Analysis

LS-DYNA

- **Rubber materials : Lagrangian kernel** (ls970, ls971)
- **Foam materials : Semi-Lagrangian and Eulerian kernel** (ls971)
- **Metal materials : Adaptivity + Eulerian kernel** (ls971)
- **Fluid materials and E.Q.S. materials : Eulerian kernel (trial version)** (ls971)



EFG

EFG +
Semi-Lagrangian Kernel

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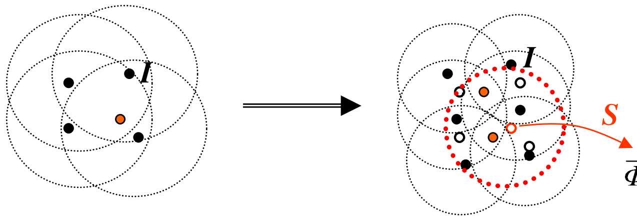
3. Mesh-free Interpolation for Data Transfer

LS-DYNA

Current variable update : $f_s^{n+1} \approx A_{\alpha s}^{n+1} \tilde{f}_\alpha = A_{\alpha s}^{n+1} A_{\alpha\beta}^{n^{-1}} f_\beta^-$

$A_{IJ} = \overline{\Phi}_I^{[m]}(x_J)$

$\overline{\Psi}_I^{[m]}(x^-, t_{n+1})$ $\overline{\Psi}_I^{[m]}(x^+, t_{n+1})$



$\overline{\Phi}_S^{[m]}(x^+, t_{n+1})$

- Particle
- Stress point

- Old Particle
- Old Stress point
- New Particle
- New Stress point

Stress recovery scheme is conservative, consistent and monotonic !

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Interpolation of the State Variable in 1-D

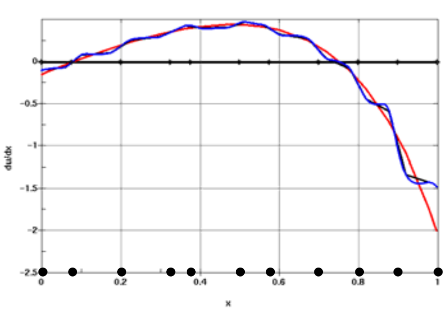
LS-DYNA

$$-\frac{d}{dx}\left(\frac{du}{dx}\right)+u=f \qquad x\in(0,1)$$

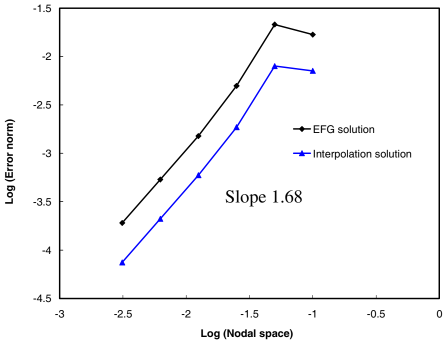
$$u(0)=u(1)=0$$

$$f=x^2-2+\frac{15\sinh 4x}{\sinh 4}$$

$$u=x^2-\frac{\sinh 4x}{\sinh 4}$$



Analytical Solution
EFG Solution
Interpolation Solution



Convergence in derivative of solution

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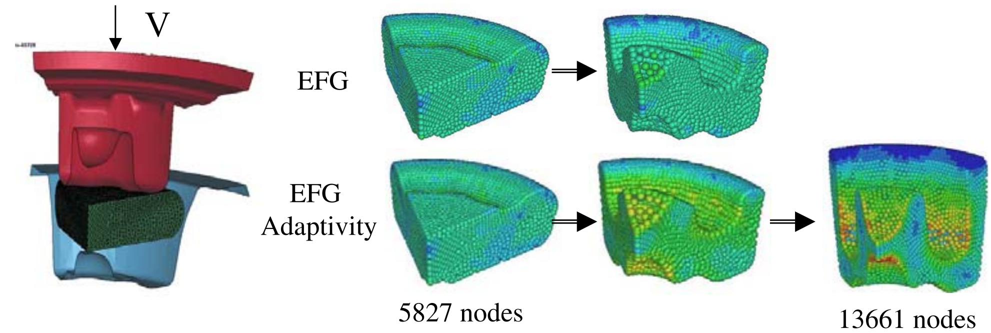
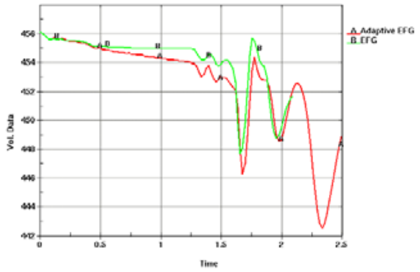
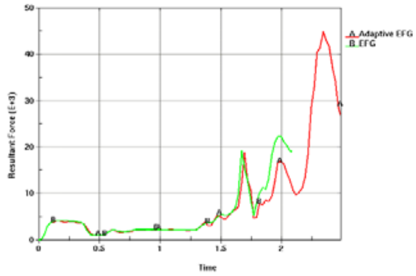
Wheel Forging Simulation

LS-DYNA





Volume change (1.7%)



Force

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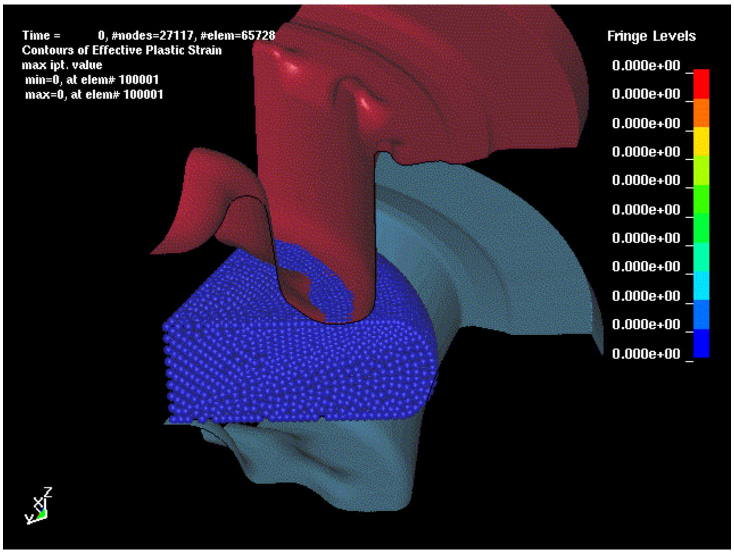
LS-DYNA

Wheel Forging Simulation (Explicit)



Movie

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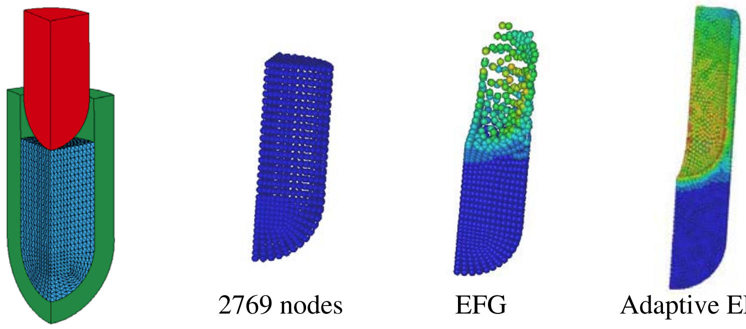
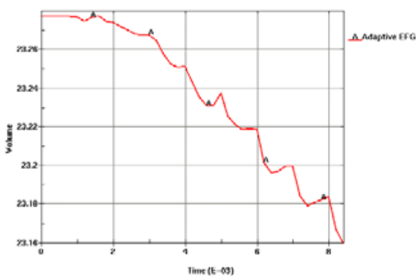
LS-DYNA

Extrusion Simulation

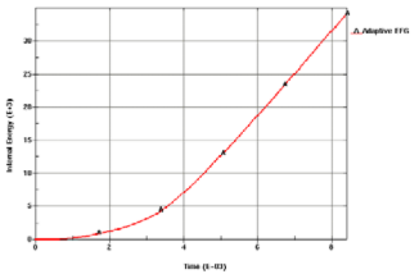


10410 nodes

2769 nodes EFG Adaptive EFG



Volume change (0.6%)



Internal energy

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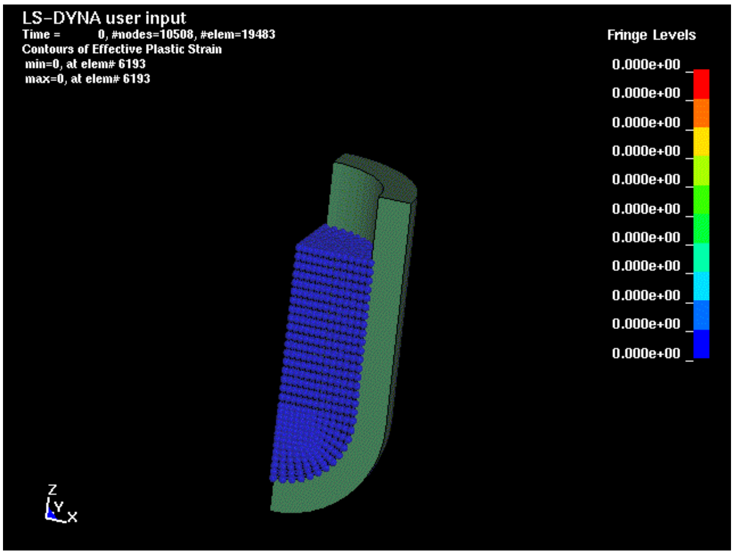
Extrusion Simulation

LS-DYNA



Movie

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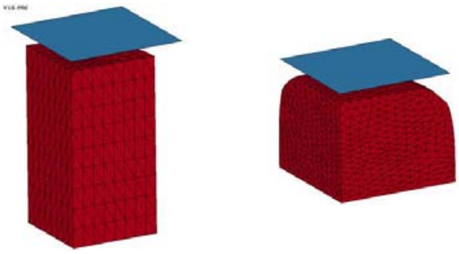
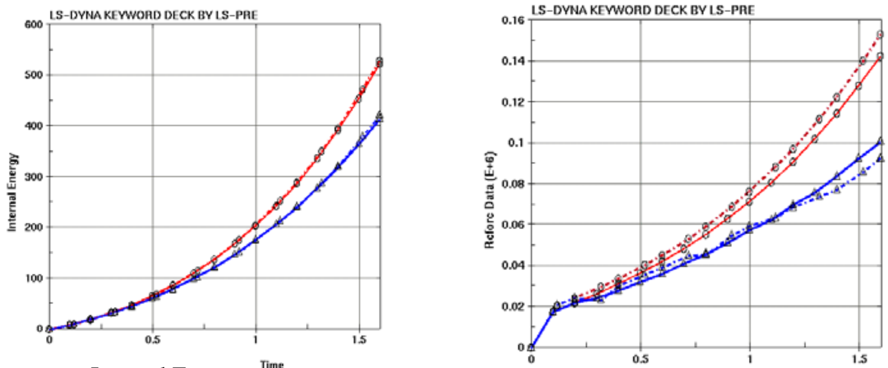
The Upsetting Process with Thermal

LS-DYNA



Initial Temperature 20 ^{0}C
Low carbon steel Ck105
No heat transfer to the environment

EFG Explicit w/o Adaptivity
EFG Explicit Adaptivity
EFG Implicit w/o Adaptivity
EFG Implicit Adaptivity



Internal Energy Force

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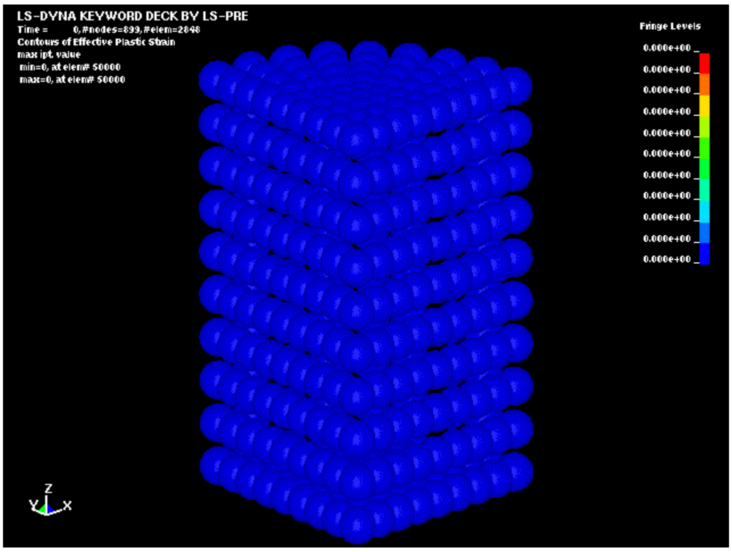
The Upsetting Process (Implicit with Thermal) LS-DYNA



Plastic Strain

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XFEM Fracture LS-DYNA

- **Motivation**

Modeling arbitrary dynamic cracks.

- **Current Practice**

Solving 2D plain-strain problems.

- **Formulation**

XFEM Fracture = Extended FEM (Partition of Unity) with Level Set Theory

+ Phantom Nodes Approach

+ Initially-Rigid Cohesive Law

- Ted Belytschko *et al*. (Northwestern University), Pablo D. Zavattieri (GM R&D)

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Modeling Material Failure and Delamination/Debonding LS-DYNA

1. **Weak Discontinuities :** discontinuous deformation gradients
   - Continuum damage constitutive equation + **Nonlocal strain smoothing** + Material erosion
   - Implicit Cracks: Crack is an assumed width
   - Polynomial basis is inadequate to represent the fine scale.
   - Time step tents to be very small in explicit analysis with fine mesh

2. **Strong Discontinuities :** discontinuous displacement
   - Cohesive model + (Interface element, or elemental enrichment **EFEM**, or nodal enrichment **XFEM**, or **EFG**)
   - Explicit Cracks : remove the influence of mesh size and orientation
   - No direct correlation between the strain softening and critical energy release rate.
   - Time step:

$$\Delta t \le \frac{2}{\omega_{\max}}; \omega_{\max} = \sqrt{\frac{2k}{\rho h}}$$

3. **Weak + Strong Discontinuities**

- loss of uniqueness as a criterion for changing from continuum damage mechanics to cohesive law

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Literatures in Strong Discontinuities LS-DYNA

- Tie-break interface
  (force/stress-based failure + spring element, rigid rods, or other constraints)

- **Cohesive Interface Element**
  (**Cohesive Zone model** + **Interface element**, or contact forces)
  Tvergaard, V. and Hutchinson, J. W. (1993)
  Needlemam, A. (1997)
  Ortoz, M. and Pandolfi, A. (1999)
  Borg, R., Nilsson, L. and Simonsson, K. (2002)
  Espinos, H. D. and Zavattieri, P. D. (2003)

- **XFEM** (**Cohesive Zone model** + level sets + extended finite element)
  Belystchko, T., Moes, S., Usui, S. and Parimi, C. (2001)
  Sukumar, Huang, Z., Prevost, J. H. and Suo, Z. (2004)
  Hettich, T. and Ramm, E. (2006)

- **EFG (Cohesive Zone model +** Moving least-square **+** EFG visibility**)**
  Rabczul, T. and Belytschko, T. (2004)
  Simonsen, B. C. and Li S. (2004)
  Brighenti, R (2005)

- Others (Virtual Crack Closure technique …)

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1. Extended Finite Element with Level Set

LS-DYNA

Signed distance function

$$f_\alpha(\mathbf{x}) = \min_{\bar{\mathbf{x}} \in \Gamma_\alpha} \|\mathbf{x} - \bar{\mathbf{x}}\| \, sign(\mathbf{n} \cdot (\mathbf{x} - \bar{\mathbf{x}}))$$

$$f_\alpha(\mathbf{x}) = \sum_I f_{\alpha I} N_I(\mathbf{x})$$

Discontinuity

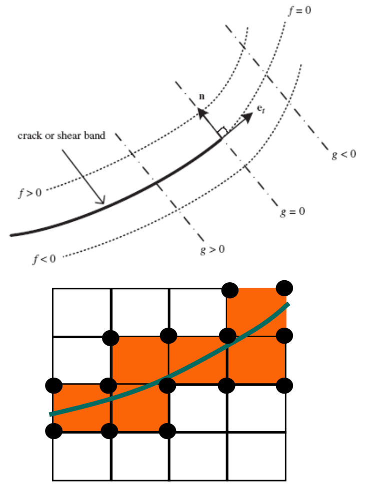
$$\boldsymbol{X} \in \Gamma_\alpha^0 \;\; if \;\; f_\alpha(\boldsymbol{X}) = 0 \;\; and \;\; g(\boldsymbol{X}, t) > 0$$

Approximation of crack in element

$$\boldsymbol{u}^h(\boldsymbol{X}, t) = \sum N_I(\boldsymbol{X}) [\boldsymbol{u}_I(t) + \boldsymbol{q}_I(t) H(f_\alpha(\boldsymbol{X}))]$$

In one-dimension

$$u = u_1 N_1 + u_2 N_2 + q_1 N_1 H + q_2 N_2 (H - 1)$$



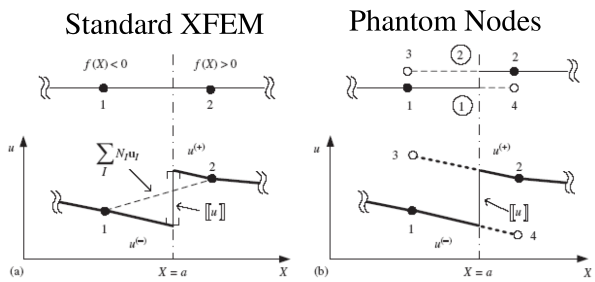
2. Phantom Nodes Approach

LS-DYNA

$$u = (u_1 + q_1) N_1 H + u_1 N_1 (1 - H) + (u_2 - q_2) N_2 (1 - H) + u_2 N_2 H$$

$$element\ 1 \begin{cases} u_1^1 = u_1 \\ u_2^1 = u_2 - q_2 \end{cases} ; element\ 2 \begin{cases} u_1^2 = u_1 + q_1 \\ u_2^2 = u_2 \end{cases}$$

$$u = u_1^1 N_1 (1 - H(X - a)) + u_2^1 N_2 (1 - H(X - a)) + u_1^2 N_1 H(X - a) + u_2^2 N_2 H(X - a)$$



- **Ted Belytschko *et al.*** (2006)

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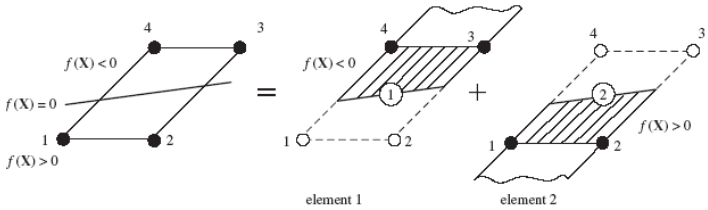
LS-DYNA

In Two-dimension



$$\boldsymbol{f}^{kin} = \boldsymbol{f}^{int} - \boldsymbol{f}^{ext} + \boldsymbol{f}^{coh}$$

$$\boldsymbol{f}_e^{kin} = \int_{\Omega_0^e} \rho_0 \boldsymbol{N}^T \boldsymbol{N} H((-1)^e f(\boldsymbol{X})) d\Omega_0^e \ddot{\boldsymbol{u}}$$

$$\boldsymbol{f}_e^{int} = \int_{\Omega_0^e} \boldsymbol{B}^T \sigma H((-1)^e f(\boldsymbol{X})) d\Omega_0^e$$

$$\boldsymbol{f}_e^{ext} = \int_{\Omega_0^e} \rho_0 \boldsymbol{N}^T \boldsymbol{b} H((-1)^e f(\boldsymbol{X})) d\Omega_0^e + \int_{\Gamma_{0,t}^e} \boldsymbol{N}^T \boldsymbol{t} H((-1)^e f(\boldsymbol{X})) d\Gamma_{0,t}^e$$

$$\boldsymbol{f}_e^{coh} = (-1)^e \int_{\Gamma_{0,t}^e} \boldsymbol{N}^T \boldsymbol{\tau}^c \boldsymbol{n}_0 d\Gamma_{0,t}^e$$

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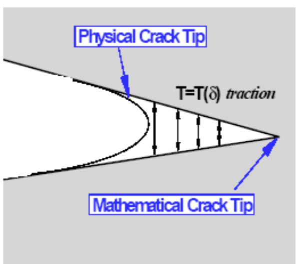
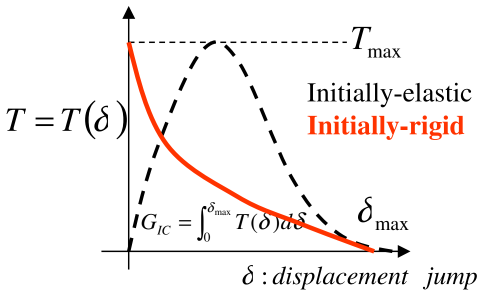
3. Initially-Rigid Cohesive Zone Model

LS-DYNA



– **Dugdale** (1960), **& Barenblatt** (1962)



• The potential crack propagation plane is idealized as a *cohesive zone* and is assumed to support a traction field ***T***.

• The mechanical response of the cohesive interface is described through a constitutive law relating the traction field ***T*** with a separation parameter.

$$\boldsymbol{T} = \frac{\partial \phi(\delta_n, \delta_t, \boldsymbol{q})}{\partial \boldsymbol{\delta}}$$

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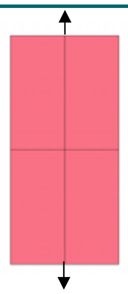
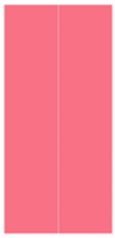
Mode-I Failure

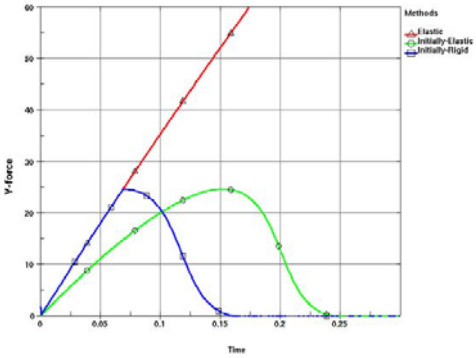
LS-DYNA



Initially-elastic Cohesive element



Initially-rigid XFEM



Size: 5.0 X 10.0
Displacement-control (1.2 in 0.3 sec)
Tvergaard-Law I cohesive law

$T_{\max} = 5.0$
$\alpha = 1.0$
$\delta_n = 1.0$
$\delta_t = 1.0$
$\lambda_{cr} = 1./3.$
$\rho = 1.0e-6; E = 167.0; \nu = 0.3$

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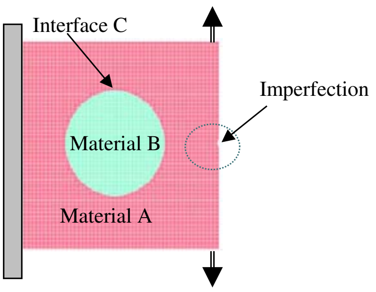
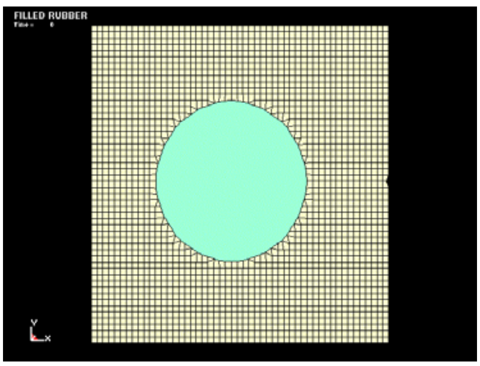
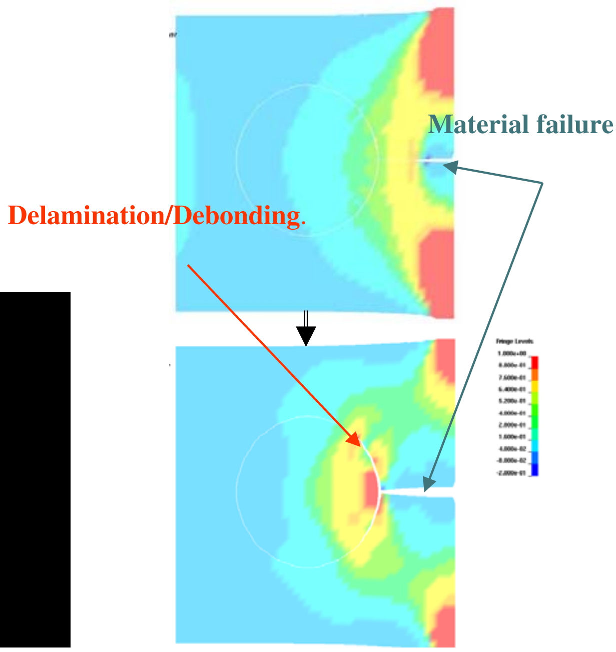
Failure in Particle Reinforced Rubber

LS-DYNA







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Ongoing Works

LS-DYNA

- Include local refinement and error estimator/indicator for solid adaptivity.
- Study the stability for XFEM
- Consider crack closure (contact) and mass scaling in XFEM.
- Extend XFEM into three dimensional case.
- Implement MPP version.

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