# SIMPLIFIED CONCRETE MODELING

# WITH

# *MAT_CONCRET_DAMAGE_REL3

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**Summary:**

(The Geotechnical & Structures Laboratory of the US Army Engineering Research & Development Center (ERDC) has a well characterized 45.6 MPa unconfined compression strength concrete, that is commonly used as the 'standard concrete' in many numerical simulations. The purpose of this manuscript is to compare the laboratory material characterization of this standard concrete with the corresponding material responses from the so called K&C Concrete Model, as implemented in LS-DYNA Version 971 Release 5266 as *Mat_Concrete_Damage_Rel3 , i.e. Release III of the K&C Concrete Model. A key aspect of this comparison is that the model's default parameters, for an unconfined compressive strength of 45.6 MPa, are used in all the material characterization simulations. Thus the constitutive model inputs are trivial, yet the complex response for many different types of material characterization tests are adequately reproduced. Where the constitutive model response differs significantly from the laboratory characterization suggestions are provided for how model users could improve the comparison via additional model parameter calibration.

**Keywords:**

Concrete, Mat_72R3, Mat_Concrete_Damage_Rel3, Version 971, Laboratory data

# 1 Introduction

Concrete is a common construction material in both civil and military applications. Although there are essentially infinite types of concrete, the majority of common concretes can be characterized by a single parameter called the uniaxial unconfined compressive strength, often denoted in the Civil Engineering literature as $f_c'$. While at first this single parameter characterization appears to be similar to metals, where the yield strength can be used as a single parameter to characterize metals of a similar class, e.g. steels, it needs to be noted that the yield strength only characterizes the elastic response of the metal. For concretes, the unconfined compressive strength parameter not only describes the elastic response, but the inelastic (plastic) response including the shear failure envelope, compressibility (compaction), and tensile failure.

Obviously a single parameter cannot accurately characterize all aspects of all concretes. However, frequently engineers are asked to perform analyses involving concrete where little or no information is available to characterize the concrete besides $f_c'$. Further complicating this lack of material characterization knowledge, it is often the case that the engineers performing the analyses have had no formal training in concrete material response and characterization. For these reasons it is practical to have available to the analyst a concrete material model that requires minimal input, but provides a robust representation of the many response characteristics of this complex material, including damage and failure.

The default parameter K&C Concrete Model[1] has been calibrated using a well characterized concrete for which uniaxial, biaxial, and triaxial test data in tension and compression were available, including isotropic and uniaxial strain data. In addition, that original calibration was modified or completed via generally accepted relationships, such as those giving the tensile strength (or modulus of elasticity) as a function of compressive strength. The objective of this paper is to compare the generated data based on the original well-characterized concrete (and the generally accepted relationships) with a new, different, but also well-characterized concrete. This comparison allows for an estimate of the potential variations in representation when the new concrete is only defined by its compressive strength (note that these variations can be due to different physical responses of the different concretes under various stress paths, or to the new concrete not following the general concrete relationships). If the new concrete is itself well characterized, the model parameter can be changed to provide a closer fit for all the stress paths studied.

The remainder of this manuscript consists of:

- •A brief introduction to the K&C Concrete Model, with a view to providing the new or novice user with the essentials of concrete response and the corresponding model parameters,
- •Comparison of the default parameter version of the K&C Concrete Model with various material characterization tests for a well characterized 45.6 MPa concrete.
- •A description of the minimal user supplied inputs required to use *Mat_Concrete_Damage_Rel3 with its default parameter generation.

# 2 Basic Concrete Reponses

Concrete is a porous and brittle material. The porosity gives rise to a nonlinear compaction response, i.e. pressure versus volume strain; in contrast to metals, where the bulk modulus, i.e. the slope of the pressure versus volume strain response is a constant. The brittle nature of concretes, and other geomaterials, provides for markedly different strengths in tension and compression; also in contrast to metals where it is often (correctly) assumed that the yield strength is the same in uniaxial tension or compression. A final difference between metals and concretes is the shear strength of concretes increase with increasing amounts of confinement (mean stress); for example, in seismic areas, transverse reinforcement is added to columns to provide both increased shear capacity and concrete confinement, which, in turn, significantly increase the column ductility.

[1] Optionally known as *Mat_072R3 in the LS-DYNA Version 971User's Manual.

 

## 2.1 Compaction

Figure 1 shows the pressure versus volume strain response measure for the example 45.6 MPa concrete under isotropic compression. This laboratory test is usually performed on right circular cylinders of concrete where load (pressures) are applied independently to the top and lateral surfaces and the axial and lateral strains are measured on the outer surface of the specimen. For this isotropic (hydrostatic) compression test the applied axial and lateral pressures are equal. The pressure versus volume strain response has three general phase:

1. An initial elastic phase as low pressure and volume strains; the slope of this portion of the response curve is the elastic bulk modulus.
2. A large amount of straining as the voids in the concrete are collapsed while the pressure increases less dramatically (lower slope)
3. The final phase of compaction is reached when all the voids have been collapsed and the material response stiffens (this is not depicted in Figure 1 for the tested pressure range).
4. The slope of unloading path shown in Figure 1 provides an estimate of the bulk modulus of the fully compacted concrete.

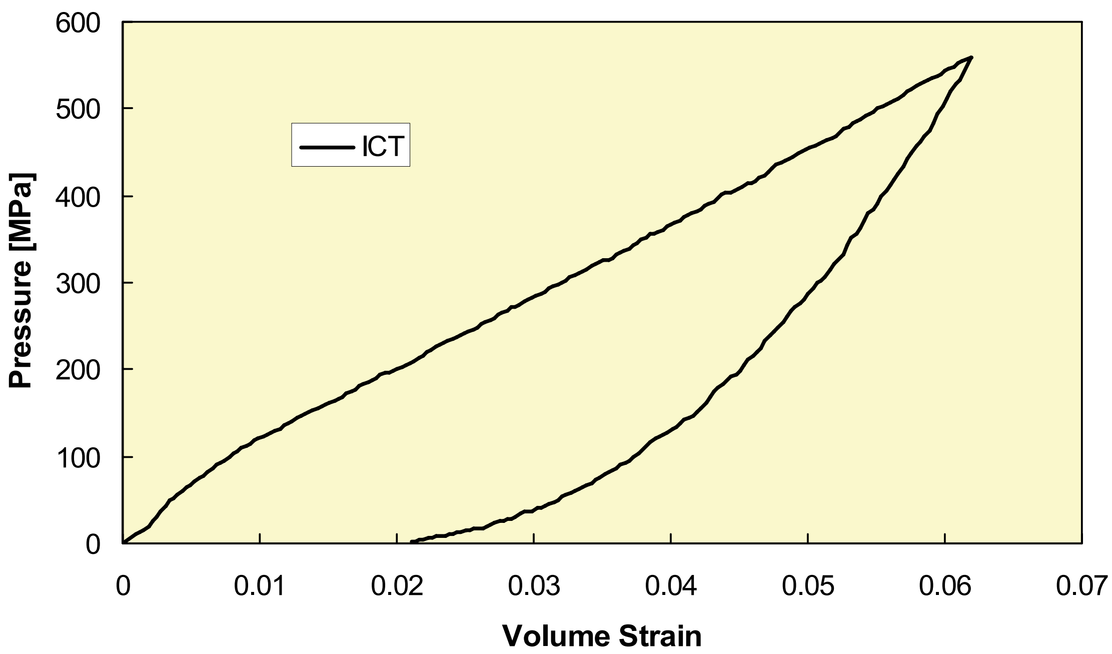


*Figure 1 Pressure versus volume strain compaction response of a 45.6 MPa concrete.*

## 2.2 Compressive Shear Strength

The compressive shear strength of concrete is measured by a laboratory test referred to as a *triaxial compression test*[2]. The specimen and testing procedure are as described above for the compaction test with the exception that the axial and lateral loads do not remain equal. In a typical triaxial compression test the specimen is load hydrostatically, i.e. equal axial and lateral pressures, until a desired confining pressure is attained, e.g. 20 MPa as shown in Figure 2. After that point in the load history the lateral pressure is held constant and the axial pressure is increased until the specimen fails, usually catastrophically. The end results (failure points) of several of these tests, conducted at different confining pressures, are plotted as shown in Figure 2 as a regression fit[3] to the discrete data. The abscissa of this plot is the mean stress, $\sigma_{kk}/3$, and the ordinate is the stress difference,

[2] The word 'triaxial' is a bit of a misnomer, as two of the three stress are equal.
[3] The odd 'bump' in the failure surface is not typical, but is thought to be an artifact of the data and its regression fit.

$\sigma_{axial} - \sigma_{lateral}$ ; it is easily shown for this stress state that the stress difference is the same as the effective stress also known as the von Mises stress.

As a point of reference, if this was a perfectly plastic metal, with a 400 MPa yield strength, the failure surface for in Figure 2 would be a horizontal line with intercept 400 MPa, i.e. the yield strength of metals is not affected by confinement (mean stress).

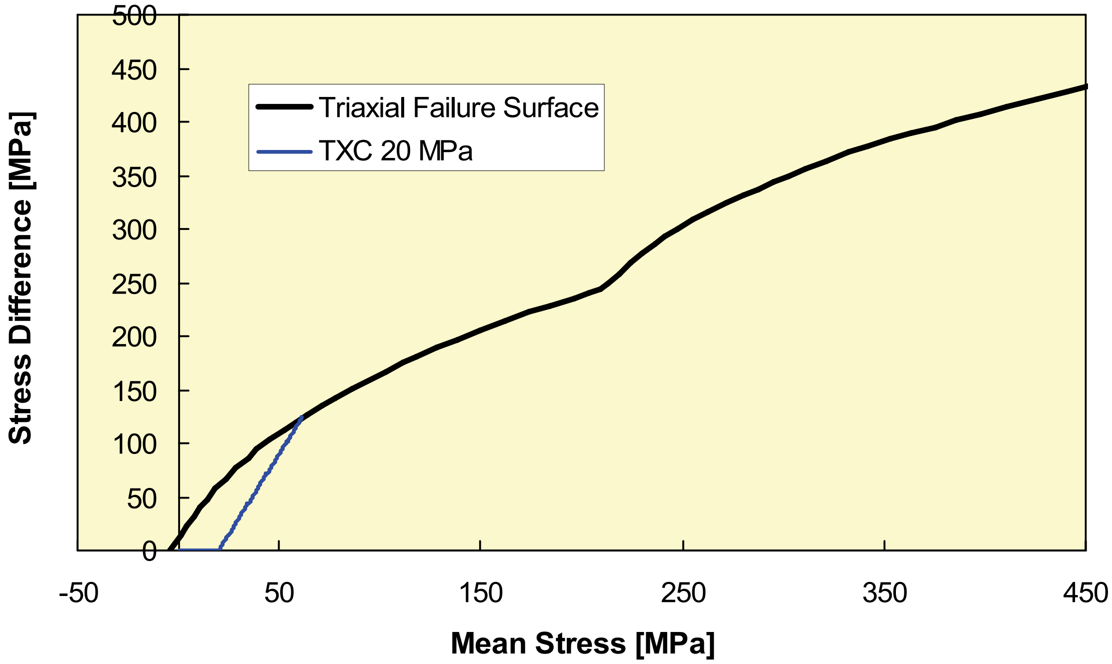


*Figure 2 Compressive failure surface for 45.6 MPa concrete.*

### 2.3 Extension and Tension

The above described triaxial compression tests are characterized by the relation $\sigma_{axial} \geq \sigma_{lateral}$. When the order of the stresses is reversed in a triaxial test, i.e. $\sigma_{lateral} \geq \sigma_{axial}$ the test is called a *triaxial extension test*. The word extension does not imply the sample is in tension but rather that the axial strains become less compressive during the test, i.e. they tend to increase (extend) towards zero. In other words, the concrete is first subjected to triaxial compression, then the vertical compression is relaxed.

There is an interesting relation between triaxial compression test and triaxial extension tests for all geomaterials, at the same mean stress, samples are observed to fail at lower levels of stress difference in triaxial extension than in triaxial compression. So in Figure 2, if the triaxial extension failure surface had been measured in the laboratory, it would look similar to the shown triaxial compression failure surface but be closer to the abscissa.

Perhaps a convenient ‘mental model’ for why this occurs is to think of a unit cube under triaxial loading. In triaxial compression the load on one surface is greater than the other two surfaces, i.e. $\sigma_1 \geq \sigma_2 = \sigma_3$, but in a triaxial extension test the load on two surfaces is greater than the load on the remaining surface, i.e. $\sigma_2 = \sigma_3 \geq \sigma_1$.

When the tensile strength of concrete has not been measured, use can be made of one of the many standard concrete relations [CEB-FIP Model Code 1990] that are based on the unconfined compression strength, $f_c'$

 

$$f_t' = 1.58 \left( \frac{\left(f_c'\right)^2}{a_0} \right)^{1/3} \quad (1)$$

where $f_t'$ is the unconfined tension strength of the concrete, and $a_0$ is a unit conversion factor: unity for $f_c'$ measured in standard English stress units of psi, and 145 for MPa metric units of stress. In the present case, $f_c' = 45.6$ MPa the corresponding uniaxial tensile strength is $f_t' = 3.8$ MPa or a factor of about 12-to-1 compression-to-tension in uniaxial stress strength. This very low tensile strength is what severely limits the tensile pressure region (left portion of the abscissa) shown in Figure 2.

## 3 Basics of the K&C Concrete Model (*Concrete_Damage_Rel3)

The presentation of the K&C Concrete Model in this section is very abbreviated, as the goal is to present the reader with sufficient information to use the model successfully, without becoming a concrete modeling expert. Additional reference material is available: an open source reference, that precedes the parameter generation capability, is provided in Malvar et al. [1997]. A workshop proceedings reference, Malvar et al. [1996], is useful, but may be difficult to obtain. More recent, but *limited distribution* reference materials, e.g. Malvar et al. [2000], may be obtained by contacting Karagozian & Case (www.kcse.com).

### 3.1 Compaction

The isotropic compression portion of the K&C Concrete Model consists of pairs of pressure and corresponding volume strain representing a piecewise linear description of the response in Figure 1. The *Mat_Concrete_Damage_Rel3 model uses an Equation-of-State[4] to provide the pressure and volume strain pairs; for this, and some other concrete models, using an Equation of State is a convenient way to input pressure versus volume strain data. The default parameter model pressure versus volume strain for a 45.6 MPa concrete is shown in Figure 3, along with the laboratory data, previously shown in Figure 1, for comparison. In this case, the default *Mat_Concrete_Damage_Rel3 model significantly under estimates the post-elastic bulk modulus of the example 45.6 MPa concrete, i.e. the slope of the pressure versus volume strain response in Figure 3 is too low. If the user has access to the pressure versus volume strain data for the particular concrete of interest, the default *Mat_Concrete_Damage_Rel3 model response could be replaced, via an Equation of State description, with the available data, and thus provided a more accurate representation of the particular concrete. To this end, the model creates an input file with all the parameters generated from $f_c'$, which can be modified and used as an input.

[4] Typically the *EOS_Tabulated_Compaction is used to input the isotropic compression data.

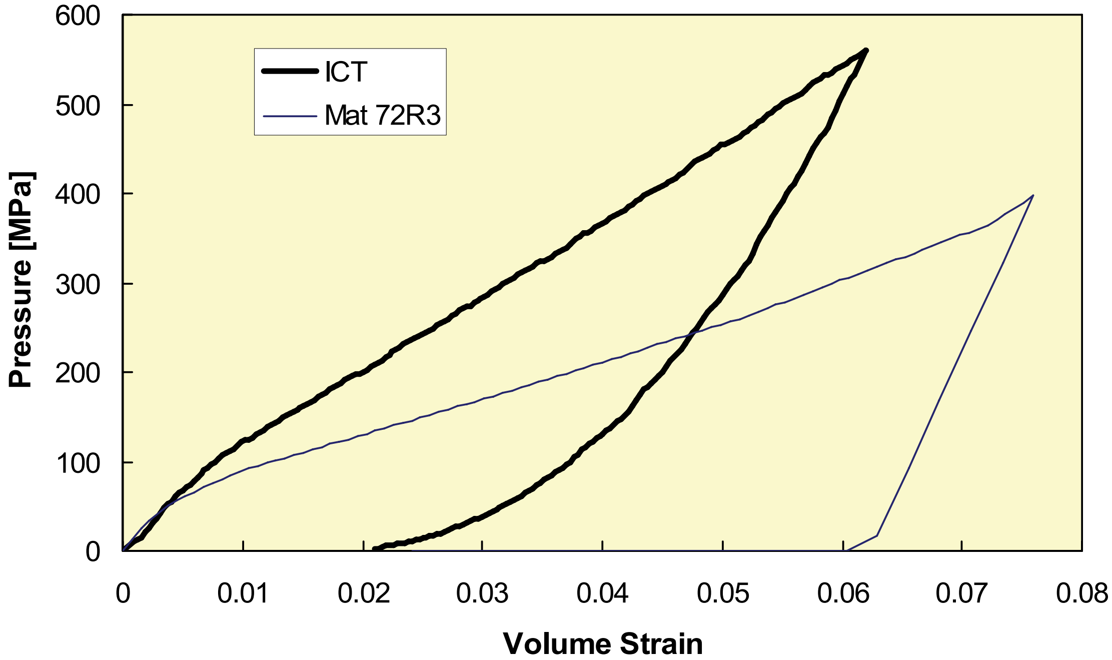


*Figure 3 Comparison of laboratory and default Mat 72R3 isotropic compression response of a 45.6 MPa concrete.*

### 3.2 Compressive Shear Strength

The *Mat_Concrete_Damage_Rel3 model uses a three parameter function to represent the variation of compressive shear strength with mean stress of the form

$$SD = a_0 + \frac{P}{a_1 + a_2 P} \tag{2}$$

Where $SD$ is the stress difference and $P$ is the mean stress in a triaxial compression failure test, and the parameters $(a_0, a_1, a_2)$ are determined by a regression fit of Equation (2) to the available laboratory data. The default parameter model stress difference versus mean stress for a 45.6 MPa concrete is shown in Figure 3, along with the laboratory data, previously shown in Figure 2, for comparison. As shown in Figure 4, the default parameter model provides a good average fit to the laboratory (regression fit) up to about a mean stress of 300 MPa (it smoothes out the unexpected “bump” in the failure surface around 200 MPa), and then under predicts the failure strength. Again, if the user has access to triaxial compression failure data for a particular concrete of interest, the default Mat 72R3 input parameters $(a_0, a_1, a_2)$ can be replaced to provide a more accurate description of the particular concrete of interest and within the range of interest.

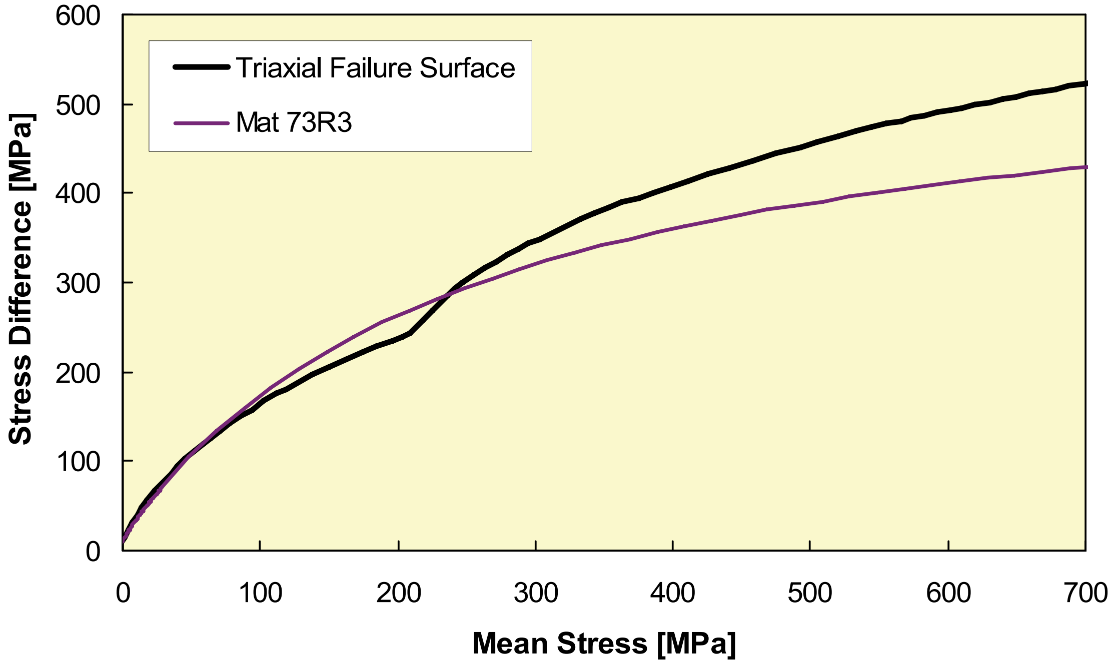


*Figure 4 Comparison of laboratory and default Mat 72R3 triaxial compression failure surface of for a 45.6 MPa concrete.*

### 3.3 Uniaxial Strain Compression Test

Another interesting laboratory test is called a uniaxial strain test. In this test the lateral surface *strains* are maintained at zero while the axial stress is increased. This stress trajectory combines compaction with shearing, and as shown in Figure 5, the stress trajectory parallels the shear failure surface and eventually intersects the shear failure surface. This laboratory test is a good check on the material model parameters because it combines compaction and shear, but typically none of the model parameters are calibrated to the test result, i.e. it serves as a verification that the material model and parameters are correct.

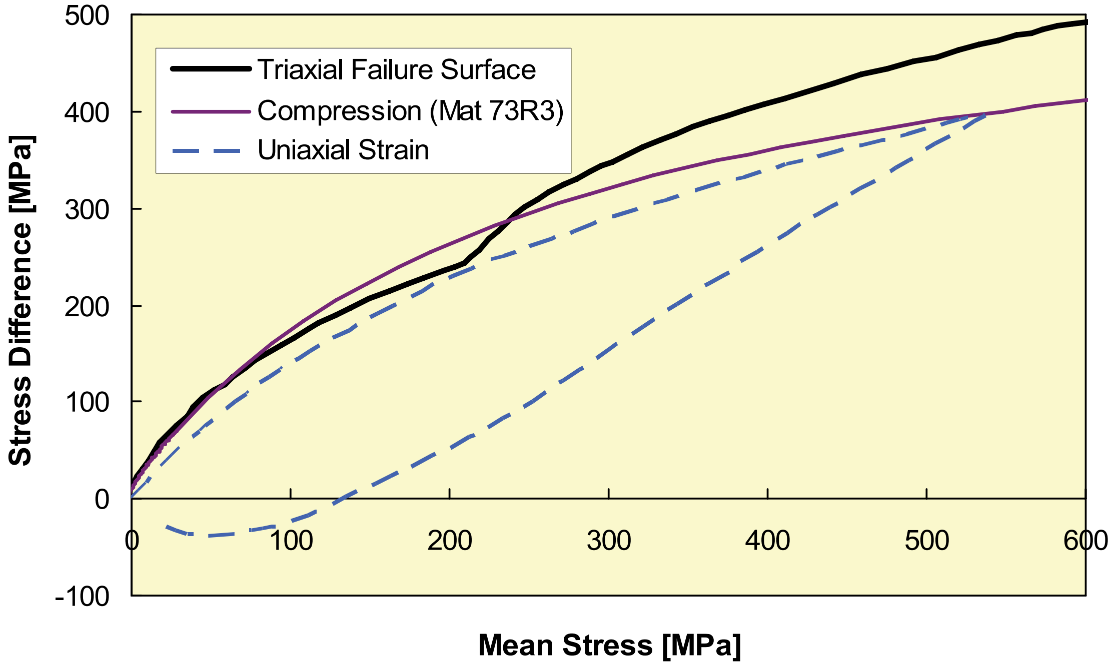


*Figure 5 Illustration of a laboratory stress trajectory for a 45.6 MPa concrete compared with the laboratory and Mat 72R3 triaxial failure surfaces.*

Figure 6 compares the laboratory and *Mat_Concrete_Damage_Rel3 model stress trajectories for the uniaxial strain test of a 46.5 MPa concrete. This comparison indicated that the default Mat 72R3 parameters provide for a very accurate simulation of the uniaxial strain test during the loading phase of the test. The unloading portion of the uniaxial strain test is not reproduced as well by the model, but could be if non-linear elasticity during unloaded was included in the model.

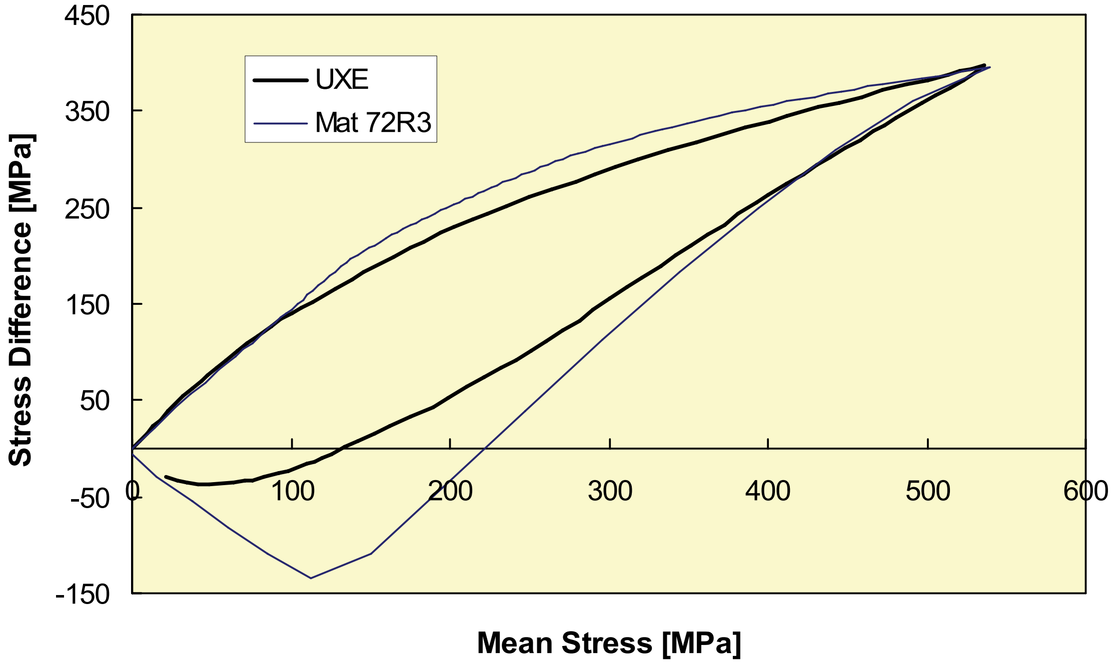


*Figure 6 Comparison of laboratory uniaxial strain test stress trajectory with default parameter response of Mat 72R3 for a 45.6 MPa concrete.*

### 3.4 Extension and Tension

Although there currently is no triaxial extension test data available for the 45.6 MPa concrete, Figure 7 shows a comparison of the laboratory triaxial compression failure surface, along with the default *Mat_Concrete_Damage_Rel3 model triaxial compression and triaxial extension failure surfaces; the laboratory and model triaxial compression surfaces were shown previously in Figure 4. An interesting feature of the Mat 72R3 default triaxial compression and extension surfaces are they merge together at a means stress of about 370 MPa. This reflects laboratory observations for other concretes that the strength difference between triaxial compression and extension decreases with increasing mean stress.

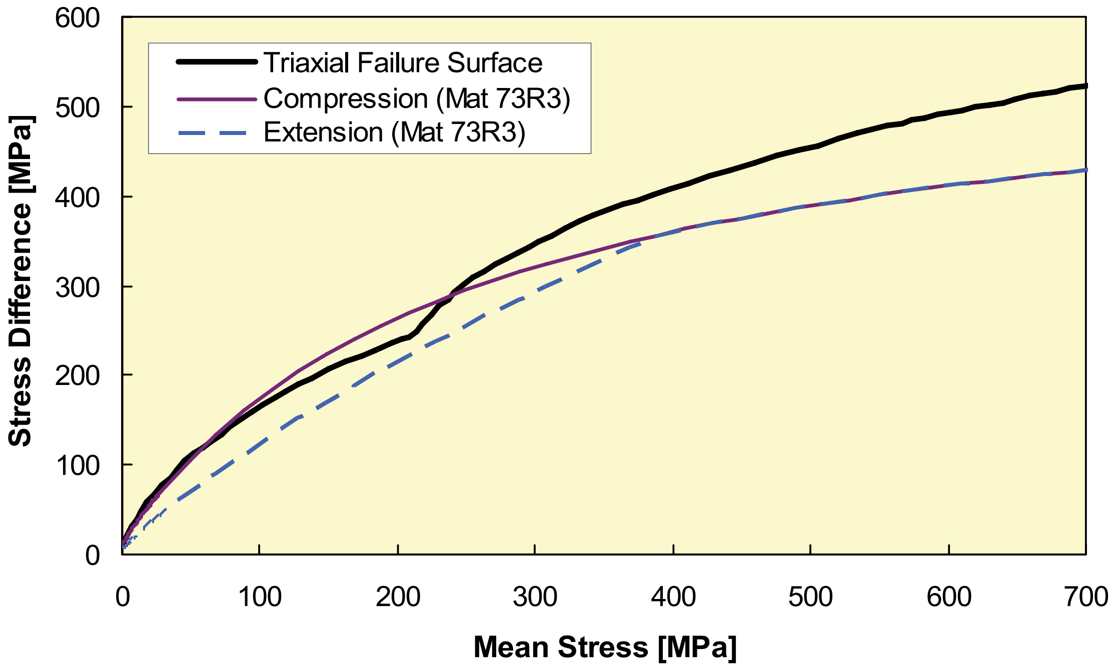


*Figure 7 Comparison of laboratory triaxial compression surface with default Mat 72R3 triaxial compression and extension surfaces.*

An important portion of the triaxial extension surface is the portion in the tensile mean stress (pressure) region especially where the concrete's tensile strength, from a uniaxial tension test, intersects the triaxial extension surface. Figure 8 shows again the same three failure surfaces shown previously in Figure 7 with the addition of the stress trajectory for a uniaxial tension test simulation of a 45.6 MPa concrete; in this figure the scales of the plot have been changed to emphasize the region near the origin. The uniaxial tension test (direct pull test) stress trajectory intersects the triaxial extension surface, rather than the triaxial compression surface, because this special case of triaxial loading satisfies the relationship $\sigma_{lateral} \geq \sigma_{axial}$ since $\sigma_{lateral} = 0 \geq \sigma_{axial} = -f_t'$.

Figure 9 shows the axial stress versus axial strain measured in 5 laboratory direct pull (uniaxial tension tests) on a 45.6 MPa concrete, and the corresponding *Mat_Concrete_Damage_Rel3 model result. The average axial tensile strength of the five laboratory measurements was -2.86 MPa with a standard deviation of $\pm 3.45$ MPa. The default Mat 72R3 axial tensile strength for this material is -3.84 MPa, see Equation (1), which falls slightly outside of the standard deviation range of the laboratory measurements. This just means that the example 45.6 MPa concrete does not exactly follow the generally accepted relationship depicted in Equation (1). Again, if the user has access to uniaxial tension failure data for a particular concrete of interest, the default Mat 72R3 input parameters $f_t'$ can be replaced to provide a more accurate description of the particular concrete of interest.

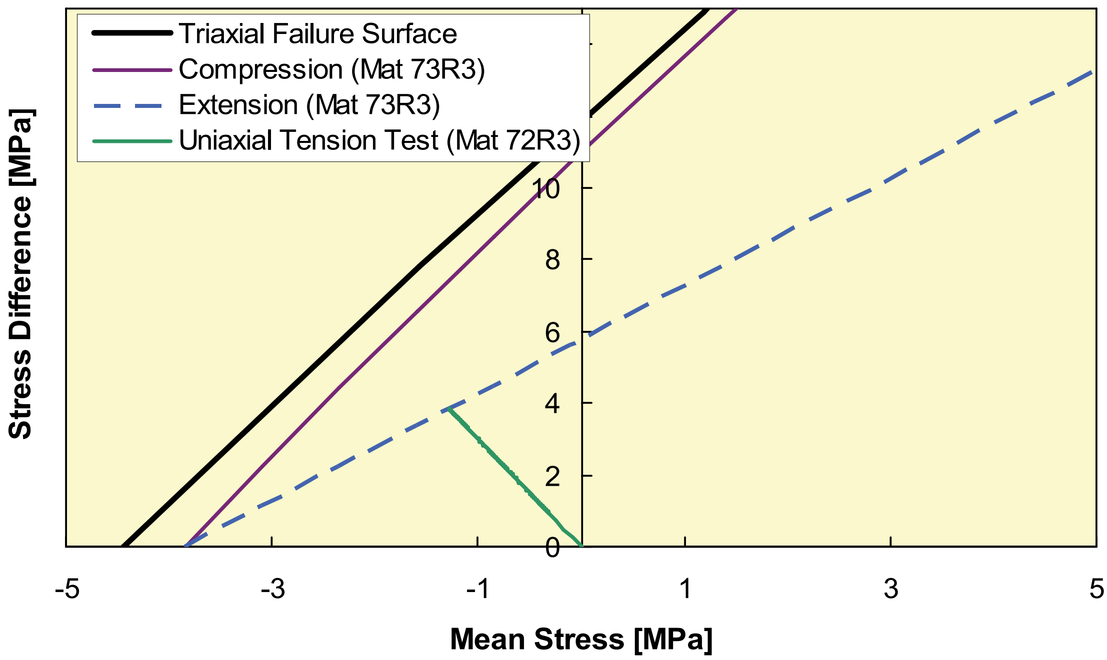


*Figure 8 Illustration of the stress trajectory for a uniaxial tension test simulation of a 45.6 MPa concrete.*

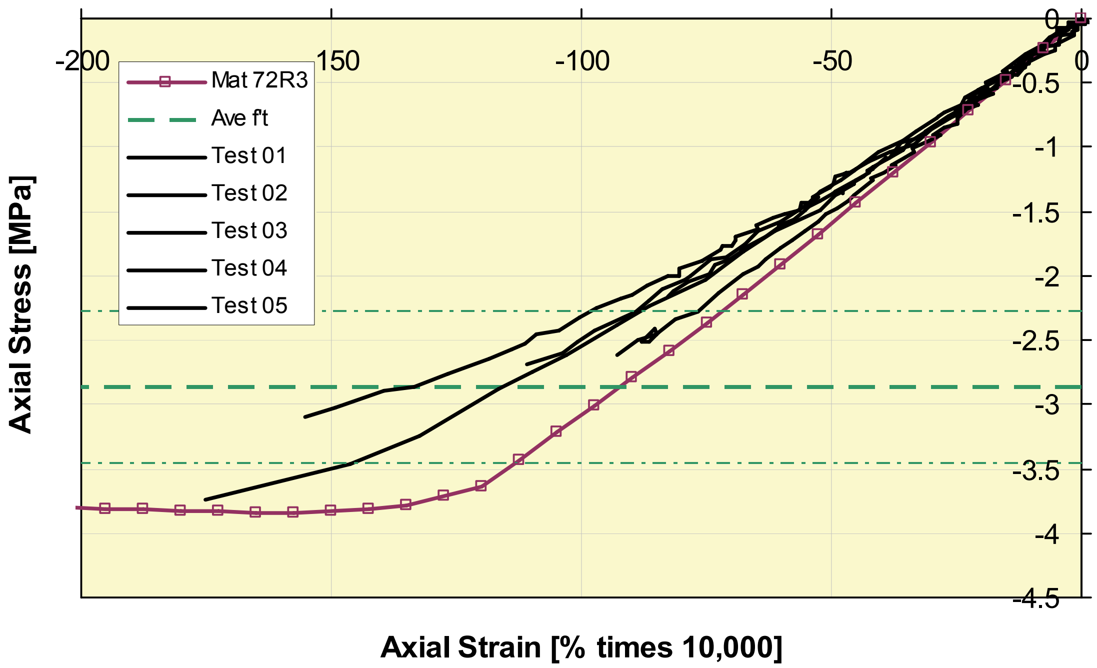


*Figure 9 Comparison of axial stress versus axial strain for five direct pull tests measurements with the simulation result for a 45.6 MPa concrete.*

## 4 Simplified User Input for *Mat_Concrete_Damage_Rel3

All of the above 45.6 MPa concrete results were computed with the *Mat_Concrete_Damage_Rel3 default parameters and a single 8-noded (unit cube) solid element using one-point integration. Appropriate boundary conditions are applied for the various test simulations, and all simulations include symmetry conditions on the three ordinal Cartesian planes, with the single element located in the positive octant. The other load and boundary conditions are:

1. Isotropic Compression – equal prescribed displacement on the three (non-symmetry) faces of the cube,
2. Triaxial Compression or Extension – equal prescribed stress on the three (non-symmetry) faces of the cube, up to the confining pressure and then increasing/decreasing axial stress for a compression/extension test. For unconfined compression or uniaxial tension, the lateral stresses are always zero.
3. Uniaxial Strain – prescribed zero displacement on the two (non-symmetry) lateral surfaces and increasing prescribed displacement in the axial direction.

To use the *Mat_Concrete_Damage_Rel3 default model parameter generation feature requires the user to specify *only* the unconfined compressions strength, i.e. $f_c' = 45.6$ MPa, and if a transient (dynamic) analysis is desired the concrete density[5] must also be specified. Finally, because the metric system of units is used to specify the concrete strength, two additional conversion parameters need to be specified in the input, e.g. the conversion for user units of stress (MPa) to standard English units (psi) and the conversion for user units of length (mm) to standard English units (inches). These conversions are necessary as the internal parameter generation is performed using relations originally derived for standard English units, e.g. Equation (1). For the present example of a 45.6 MPa concrete the non-zero user inputs are

- Card 1 – `RO` – concrete density (2.17$\times 10^{-3}$ g/mm$^3$)
- Card 2 – `A0` – negative of the unconfined compressive strength (-45.6 MPa)
- Card 3 –`RSIZE & UCF` – conversion factors for length 3.972$\times 10^{-2}$ for inches –to-millimeters and 145 for psi-to-MPa.

all of the other parameters are blank, or zero.

As mentioned above, if the user has some laboratory information to be integrated into the material model description, this laboratory data can be interlaced with the default model parameters. The suggested procedure is to run the model for one time step with the above described minimum input parameters, e.g. density, strength, and stress & length conversion factors. The default parameters for this minimum input are written to the LS-DYNA "`messag`" file. The Mat73R3 input from the `messag` file, along with the Equation-of-State parameters, also written to the `messag` file should then be copied to the user's input file, and any then edit the generated default parameters to include available laboratory data.

## 5 Summary

The new *Mat_Concrete_Damage_Rel3, a.k.a. Mat 72R3,i.e. Release III of the K&C Concrete model provides an excellent material model for modeling the complex behavior of concrete when only the most minimal information about the concrete, i.e. its unconfined compression strength, is known.

## 6 References

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[5] A nominal concrete density is 2400 kg/m$^3$.

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