The influence of ondulation in fabric reinforced composites on dynamic properties in a mesoscopic scale

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Abstract

Structural mechanic properties of fiber reinforced plastics depend on the single components' properties, namely matrix and fiber [5]. Simple micromechanic homogenization theories reach a limit when a laminate consists of fabric reinforced layers instead of unidirectional layers. The ondulations of warp and fill yarn caused by the textile semi-finished product are the reason why the mesoscopic scale, which is in between the microscopic and the macroscopic scale, has to be taken into account when mechanically characterizing fabric reinforced composites [3]. In this scale a mesomechanic kinematic can be derived analytically. Especially, when considering free damped vibrations of structures the repeated acting of the kinematic correlation significantly affects the damping behaviour to higher values compared to theoretically predicted damping ratios. The model is investigated using Finite-Element-Analyses and basically validated experimentally.

1 Introduction

Simplified theoretical approaches for fiber reinforced plastics often presume a layup of only unidirectional reinforced layers and homogenization approaches for the prediction of structural properties. However, different kinds of fabrics are often applied as reinforcements in the layup of structural parts. The mesoscopic geometry of fabric reinforcements, however, is distinctively different compared to unidirectional layers. The effect of the repeated ondulations of the yarns in warp and fill direction is supposed to influence structural mechanical properties.
The aim of this paper is to introduce a theoretical approach towards the kinematics in a mesoscopic scale considering the ondulation in fabric reinforced composites. Therefore investigations on a continuous mathematical model based on a sinusoid curve are carried out on a representative sequence of one complete ondulation. The achieved results are the basis for formulating a more appropriate mechanical model of fabric reinforced plastics.

2 Mesoscopic approach

The phenomena of ondulation in fabric reinforced composites can be examined on the mesoscopic scale. The mesoscopic scale which is in between the scales of micromechanics and macromechanics. Micromechanics for example provide rules of mixtures for structural mechanical properties whereas macromechanics provide predictions of idealized bearing structures.
The ondulation is an effect of a fabric as a textile semi-finished product. The warp yarns perpendicularly cross the fill yarns alternating at its top and at its bottom. The geometrical dimensions depend distinctively on the yarns used and the type of the fabric construction. A very simple construction is a plain view fabric built up out of equal yarns in warp and fill direction. A plain view geometry is presumed for the following investigations. Different types of fabrics with non-equilibrated yarns in warp and fill direction requires modifications of the model.

2.1 Mathematical model and presumptions

The aim of this section is to introduce a mathematical model for the ondulation in composites with fabric reinforcements. In order to reduce the complexity down to a one-dimensional problem, only the

centerline of an ondulated yarn is considered. The obtained mesoscopic geometry can thus be described by a sine curve with arbitrary amplitude [5].
The one-dimensional mathematical model is formulated in order to obtain a representative sequence of one complete ondulation. Because of the ondulation the total deformation is presumed to consist of the mechanic strain and additionally a purely geometrical deformation that contributes to the total deformation behavior [5], [7]. In a mathematical approach the ondulated yarn is assumed not to lengthen or shorten due to strain or compression but to remain constant in length. Mechanically this presumption can be stated in terms of an infinite high YOUNG's modulus in longitudinal direction

$$E_l \to \infty \tag{1}$$

and a flexural modulus transverse to it that is presumed to be zero

$$E_b \to 0 \tag{2}$$

so that an ideally stiff and at the same time ideally flexible yarn is indicated.
Under the previously stated presumptions a kinematic can be lead back to geometrical constraints only. The previously stated presumptions lead to two different effects in the model when positively and negatively defined deformations are considered, respectively. Positively defined deformations lead to a smoothing or flattening. The amplitude decreases. In this case the maximum in elongation is reached when the yarn gets completely flattened. The amplitude increases for a negative deformation applied. In this case the shift in amplitude reaches no boundary value.

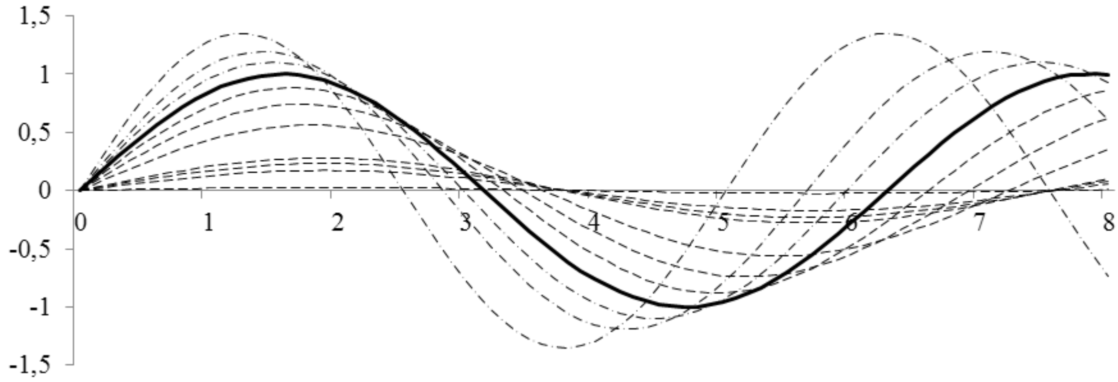

Fig. 1: Shift of the amplitude against the respective degree of deformation.

Fig. 1 shows the obtained sine-waves for an originally presumed sinusoidal function as a bold solid line, the elongated yarns as dashed lines and the shortened yarns as dash-dotted lines. For reasons of simplification the function is normalized by setting the amplitude to $A = 1$ and the originally considered interval to $x = [0, 2\pi]$.
The arc length of one complete sinusoidal ondulation with arbitrary amplitude A can be calculated by solving an elliptic integral of second kind and considering a diminution factor [2]. To fulfill the presumptions the arc length has to remain constant under the applied degree of deformation u_{rel} introducing

$$u_{rel} = \frac{\Delta l}{l_0} = \frac{\Delta l}{2\pi} = \frac{l_1 - l_0}{l_0} = \frac{l_1}{l_0} - 1 \tag{3}$$

that is positive for elongation and negative for shortening.
Thus for selected degrees of deformations a governing equation leads to the respective amplitudes and so to a shift in the amplitude ΔA. Considering the theoretical range of possible degrees of deformation u_{rel} the correlation is distinctively nonlinear. A direct and linear coupling between deformation and shape of the ondulation in fabric reinforced plastics can be derived by considering much smaller and thus a more relevant range $u_{rel} = -0{,}001 \ldots + 0{,}001$ [5]. For further investigations it is reasonable to introduce the relative shift of the amplitude

$$w_{rel} = \frac{\Delta A}{A_0} = \frac{A_1 - A_0}{A_0} = \frac{A_1}{A_0} - 1 \tag{4}$$

in order to receive a relative dimension. So the relative shift of the amplitude w_{rel} can be plotted against the relative degree of deformation u_{rel} and a w_{rel}-u_{rel}-diagram results.

2.2 Graphical illustration of the kinematic considering real geometric dimensions

Fig. 2 exemplarily shows a w_{rel}-u_{rel}-diagram for the amplitudes $A_{01} = 0.04$, $A_{02} = 0.07$ and $A_{03} = 0.1$ over one complete wavelength on the domain of definition $[0, 2\pi]$ for the argument in the trigonometric function. The gradient depends on the sign of the deformation. Elongation as positively defined deformation leads to a significantly higher diminution of the amplitude A. The selected values can be assumed to appear as real amplitudes due to ondulation in fabric reinforced layers [1], [4]. The applied degrees of deformation u_{rel} have been selected in relevant ranges for structural dynamic problems. The diagram shows the before mentioned behavior in the range $u_{rel} = -0{,}001 \ldots + 0{,}001$, i. e. in a range of ± 1 ‰ in change of the degree of deformation. In this range an almost linear correlation between degree of deformation u_{rel} and relative change in amplitude w_{rel} can be identified. A direct and linear coupling between deformation and shape of the ondulation in fabric reinforced plastics can be stated.

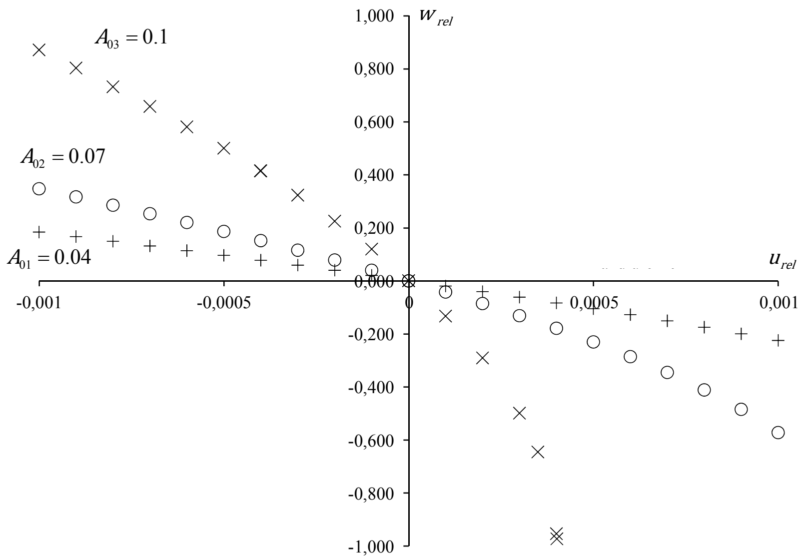

Fig. 2: Relative shift of the amplitude w_{rel} against the applied respective degree of deformation $u_{rel} = -0{,}001 \ldots + 0{,}001$ for the selected amplitudes $A_{01} = 0.04$, $A_{02} = 0.07$ and $A_{03} = 0.1$.

The linearity reaches a limit when the applied positively defined degree of deformation approaches the limits of the purely mathematical model. This singularity represents the state when the former sinusoidal curve gets completely flattened. Furthermore the sensitivity to the selected amplitude is distinctively high. Yet in the range of deformation $u_{rel} = -0{,}001 \ldots + 0{,}001$ the correlation reaches a nonlinearity for the lowest amplitude $A_{01} = 0.04$ whereas the higher amplitudes $A_{02} = 0.07$ and $A_{03} = 0.1$ still follow linear correlations, as shown in Fig. 2.

3 Investigations by Finite-Element-Analyses

In order to further investigate the influence of geometric kinematic correlations calculations with the Finite-Element-Analyses have been carried out. The aim is to verify the considered geometric kinematic. Further the elastic parts that have been neglected by the strongly simplifying presumptions in the mathematical model are identified. The simulation is carried out in LS-DYNA under application of an explicit solver.

3.1 Plain representative sequence of a fabric reinforced layer

Based on MATSUDA et al. [4] the geometry and the dimensions of the representative volume element have been chosen as shown in Fig. 3. The height of the volume element has been modified in order to avoid too small elements and the resulting long computing time. The chosen dimensions are all indicated in mm.

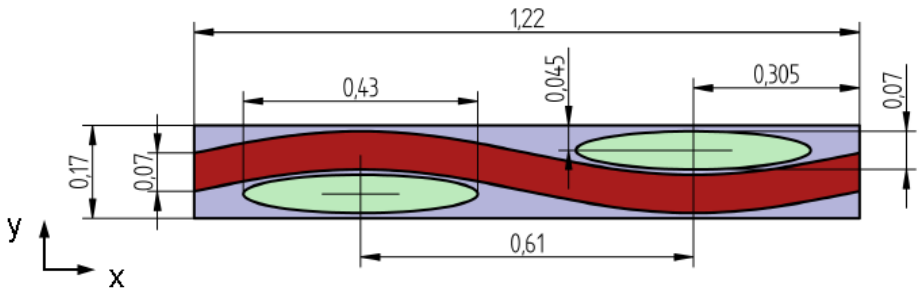

Fig. 3: Exemplarily selected cross section for a representative sequence according to MATSUDA et. al [4] used for the investigations in the Finite-Element-Analyses: Warp yarn (red), Weft yarn (green), matrix (blue).

The odulation of the fiber bundles in warp direction is described by a sine curve defined by

$$y(x) = 0.04\,mm \sin\left(\frac{2\pi}{1.22\,mm}x\right) \tag{5}$$

where the amplitude of the centreline of the warp yarn is $A_0 = 0.04$.
The representative volume element is discretized by 3521 nodes into 3352 elements. Under the presumption of a plain strain condition in the x - y -plane, shell elements of the element formulation ELFORM 13 and shell thickness of $t = 0$ are chosen to approximate the strongly simplifying presumptions of the mathematical model. Fig. 4 shows the discretization of the representative volume element with the defined boundary conditions.

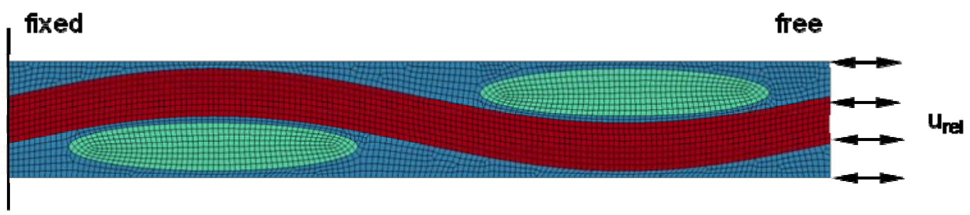

Fig. 4: Modeled cross section, discretization and boundary conditions for the selected representative sequence according to MATSUDA et. al [4] used for the investigations by the FE-Analyses.

The clamped edge is defined on the left hand side. On the right hand side the free edge is defined where the relative displacements are applied. Both definitions on the boundaries allow a contraction of the cross-section due to POISSON effects. Thus a free contraction of the cross section over the complete length of the representative element is allowed. A relative displacement in the range of $u_{rel} = -0{,}0001\ldots+0{,}0001$ is applied on the free edge on the right hand side in selected steps.
Basalt fibers and carbon fibers have been presumed as reinforcement fibers in order to compare two different materials. Basalt fibers have been considered as isotropic, with a YOUNG's modulus in longitudinal direction $E_{f,Basalt} = 89\,\mathrm{GPa}$ whereas carbon fibers have been considered as transversally isotropic with a YOUNG's modulus in longitudinal direction $E_{f,Carbon} = 235\,\mathrm{GPa}$. The results are the basis for a basically comparison of the influence of different longitudinal stiffness of the fibers. Therefore both types of fiber reinforcements have been presumed to be embedded in the same epoxy matrix system with an Young's modulus $E_{Matrix} = 2.8\,\mathrm{GPa}$. The respective rules of mixture according

to CHAMIS [5], [6], [7] are applied in order to calculate the properties of the compound. Therefore a fiber volume content of 60 % has been presumed.
In order to apply the material properties to the FE-model in LS-DYNA an orthotropic material model *MAT_002 has been chosen. The center line of the warp yarn follows the sine curve indicated by equation (5). Therefore the material directions are defined by the local material axes applied by the function AOPT 0 and appropriate node numbering. A manual optimization of the element axes has been necessary.
As a first approach an idealized contact is obtained by coincident nodes, where failure mechanisms and friction effects are neglected.

3.2 Results of the FE-calculations

Fig. 5 shows the w_{rel}-u_{rel}-diagram obtained for the two selected fiber reinforcements. They are considered to be embedded in the same thermoset matrix system in order to identify only the influence of the different fiber reinforcement. The slope for the carbon fibers is bigger than the slope for basalt fibers. The effect can be traced back to the differences in stiffness in longitudinal direction of the fibers. Carbon fibers show nearly three times higher stiffness in longitudinal direction as basalt fibers. So carbon fibers correspond more to the strongly simplifying mathematical presumptions of an ideally stiff and at the same time ideally flexible yarn. A linear correlation for both types of fiber reinforcements can be identified. Yet the relative shift in amplitude w_{rel} as formerly indicated in the purely mathematical approach is smaller by approximately three decades.

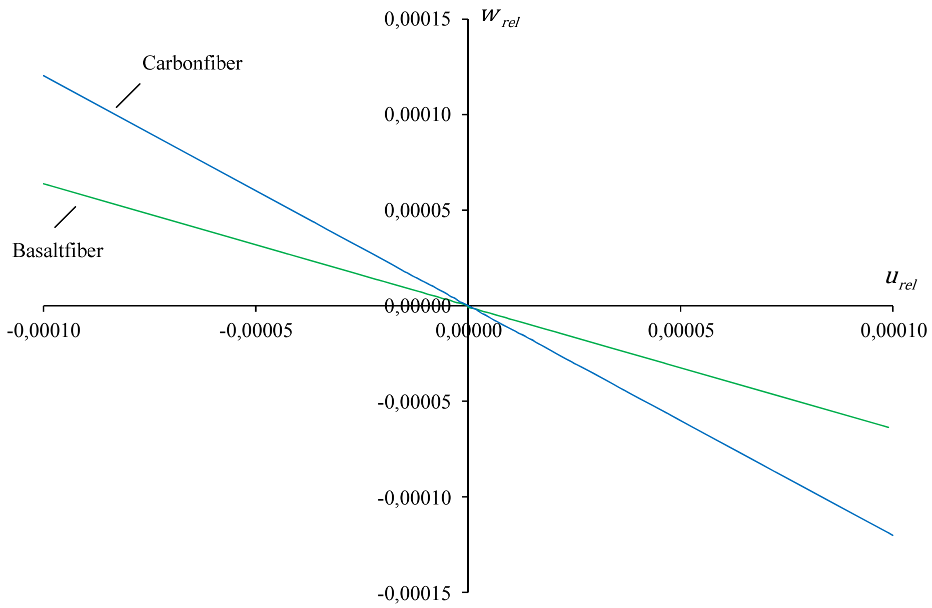

Fig. 5: w_{rel}-u_{rel}-diagram obtained by different fiber reinforcements in the same matrix system: Blue: Carbon fiber; Green: Basalt fiber.

4 Results and Discussion

The obtained results of both the mathematical model and the FE-calculations are compared to each other. A possible mechanism is derived that could significantly affect the damping behavior to higher values in fabric reinforced plastics.

4.1 Comparable illustration of the obtained results

A direct comparison of the mathematical results and the results obtained by the Finite-Element-Analyses is not reasonable in a linear equidistantially scaled coordinate system. Because of the big differences between the mathematical results and the FE-results of approximately three decades a logarithmic scale is chosen for the relative shift of the amplitude w_{rel} on the ordinate. Therefore the sign of the shift in amplitude has to be neglected. The relative degree of deformation u_{rel} on the abscissa remains linear equidistantially scaled. Fig. 6 shows the single-logarithmically scaled w_{rel}-

u_{rel}-diagram. The considered amplitude of the mathematical model is $A_{01} = 0.04$, which results are shown as a red bold line. The FE-results for the relative shift in amplitude obtained by the FE-calculations are shown as solid lines, blue for carbon fiber reinforcement and green for basalt fiber reinforcement. For later explained reasons the respective amount of contraction or compression effects due to POISSON's ratio in a unidirectional reinforced layer is plotted against the degree of deformation. The corresponding value ν_{13} is calculated by the relation after MAXWELL-BETTI of the reciprocal work theorem [5], [6], [7].

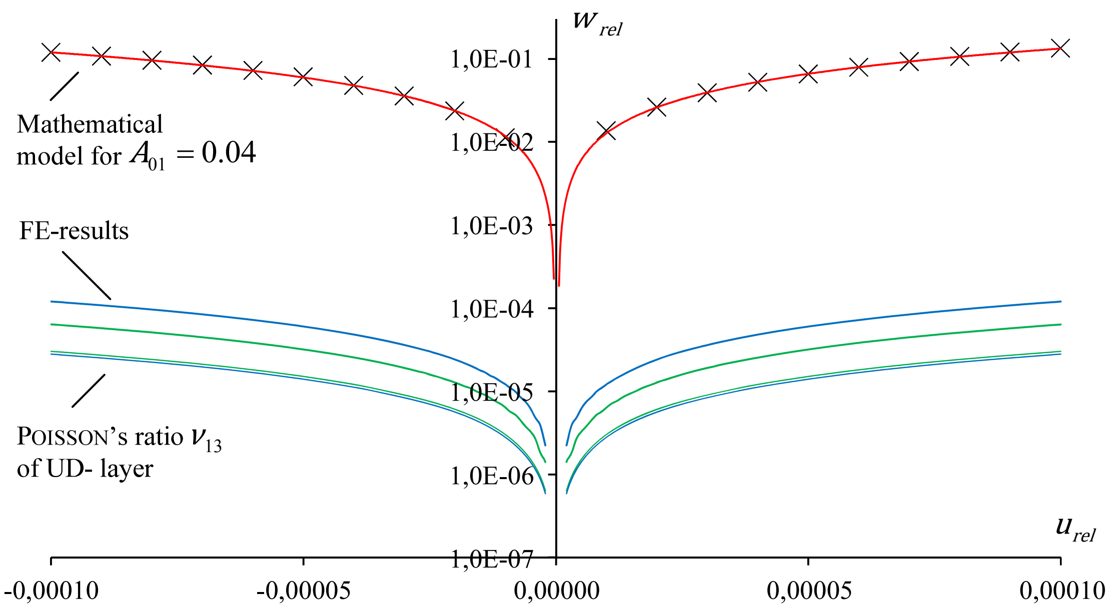

Fig. 6: Single-logarithmically scaled w_{rel}-u_{rel}-diagram obtained for the different investigations: Red solid line: Mathematical model; Upper blue solid line: FE-results for carbon fiber; Upper green solid line: FE-results for basalt fiber; Lower blue solid line: Amount of POISSON's ratio for carbon fiber; Lower green solid line: Amount of POISSON's ratio for basalt fiber.

4.2 Discussion

The big difference of approximately three decades between the mathematical model and the results of the FE-calculations can be explained by a very elastic response of the FE-model. It considers the elasticity of the yarn, the perpendicularly orientated fill yarns and the surrounding matrix. In contrast the mathematical model presumes an ideally stiff and at the same time an ideally flexible yarn without any elastic support on surrounding matrix or fill yarns.

The obtained shifts in amplitude of the FE-model must be evaluated further. The FE-model considers the complete shift in amplitude $w_{rel(z)}$. It contains the effects due to Poisson's ratio with an amount $w_{rel(Poisson)}$ and the geometrically induced kinematic amount $w_{rel(geo)}$ so that it can be stated

$$w_{(z)} = w_{(\mathrm{Material})} + w_{(\mathrm{geometrisch})} = w_{(\mathrm{Querkontraktion})} + w_{(\mathrm{Kinematik})} \quad (6)$$

In order to distinguish its respective part, the FE-results have to be corrected by the amount of POISSON'S effect. Therefore equation (6) has to be solved for $w_{rel(geo)}$. So the amount $w_{rel(geo)}$ can be identified as the area included by the curves $w_{rel(z)}$ and $w_{rel(Poisson)}$ in Fig. 6. The amount of the geometrically induced kinematic depends on the fiber reinforcement. It is bigger for carbon fibers as they show high stiffness in longitudinal direction and a low POISSON's ratio $\nu_{13,Carbon} = 0.28$. The amount is smaller for basalt fibers with an approximately three times lower stiffness in longitudinal direction and an approximately three times higher POISSON's ratio $\nu_{13,Basalt} = 0.302$, compared to carbon fibers.

4.3 Experimental results for a basic validation

Experimental results basically show the aforementioned behavior. Flat basalt fiber reinforced specimens have been investigated. The layup of the specimens is 0° unidirectional reinforcement and $0°/90°$ twill weave (2/2) reinforced, all of them with a fiber volume content of approx. $58\ \% \pm 2\ \%$. Their beam-like dimensions of $250\ \mathrm{mm}$ x $25\ \mathrm{mm}$ x $\mathrm{approx.}\ 2\ \mathrm{mm}$ allow the one-dimensional consideration of the problem as formerly stated by the mathematical model and the FE-investigations. The specimens have been clamped at one end over $50\ \mathrm{mm}$. The resulting $200\ \mathrm{mm}$ can so be considered as a cantilever. The specimens have been deflected at the free end and so excited to free damped vibrations. The cantilever specimens have been measured with a laser-vibrometer. The measured points are five equidistantially points over the length of the specimen that are centric respective to the width.
The analyses of the measured data shows two significant tendencies. The behavior in the frequency domain is only slightly affected. The natural frequencies of the unidirectional reinforced specimens are higher of only approximately $5\ \%$ compared to the twill-weave reinforced specimen. Namely the natural frequency of the first mode shape is $f_{1,uni} = 47\,\mathrm{Hz}$ for the unidirectional reinforced specimen whereas the natural frequency is $f_{1fabrici} = 44\,\mathrm{Hz}$ for the twill-weave-reinforced one. The reason therefore is that frequency and mode shape basically depends on geometric and physical properties, such as area of the cross section, moment of inertia of the cross section, length of the beam, stiffness and density.
In contrast the damping behavior is significantly different for the two kinds of specimens. The logarithmic decrement of the vibrations for the twill-weave-reinforced specimens is higher of approximately $50\ \%$ compared to the unidirectional reinforced specimens. Namely the vibration of the first mode shape decays with a logarithmic decrement $\Lambda_{fabric} = 0.015$ for the twill-weave-reinforced specimens whereas the logarithmic of the unidirectional reinforced ones is $\Lambda_{uni} = 0.010$.
Analog investigations for carbon fiber reinforced specimens with both unidirectional reinforcement and fabric reinforcement have to be carried out in order to validate the sensitivity to the kind of fiber reinforcement. Nevertheless the experimentally determined results generally show the previously stated influence of kinematic correlations in a mesoscopic scale on the damping behavior of fiber reinforced plastics. The repeated acting of the kinematic correlation significantly affects the damping behavior.

5 Conclusions

The kinematics in fiber reinforced composites caused by the ondulation in the mesoscopic scale can be described by a purely mathematical model. On the presumed ideally stiff and at the same time ideally flexible yarn a certain degree of deformation is applied. In the applied range of relative deformation u_{rel} a distinctively linear behavior can numerically be determined.
A kinematic due to geometric constraints can be determined. A linear direct coupling between applied degree of deformation and the shift in amplitude can be shown within the model's limits. The carried out FE-calculations lead to a similar direct coupling between applied deformation and obtained shift in amplitude. Due to huge elastic parts that are neglected in the mathematical model but are considered in the FE-analyses the results differ in approximately three decades. As in linear-elastically presumed models POISSON's ratio acts coupled effects in perpendicularly orientated directions act. Though the obtained results in the FE-analyses have to be corrected by the amount of contraction due to POISSON effects. The remaining amount $w_{rel(geo)}$ can be determined as the difference between the FE-results and the POISSON's ratio ν_{13} for unidirectional reinforced single layer.
The amount $w_{rel(geo)}$ can be stated to describe the potential of the different kinds of fibers to contribute to the damping behavior of the material. The contributions of the fibers and especially the ondulation of fabric reinforcements caused on the mesoscopic scale are supposed to affect the damping behavior as a structural dynamic's property.

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6 Literature

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