Review of Solid Element Formulations in LS-DYNA

Properties, Limits, Advantages, Disadvantages

LS-DYNA Forum 2011 > Entwicklerforum
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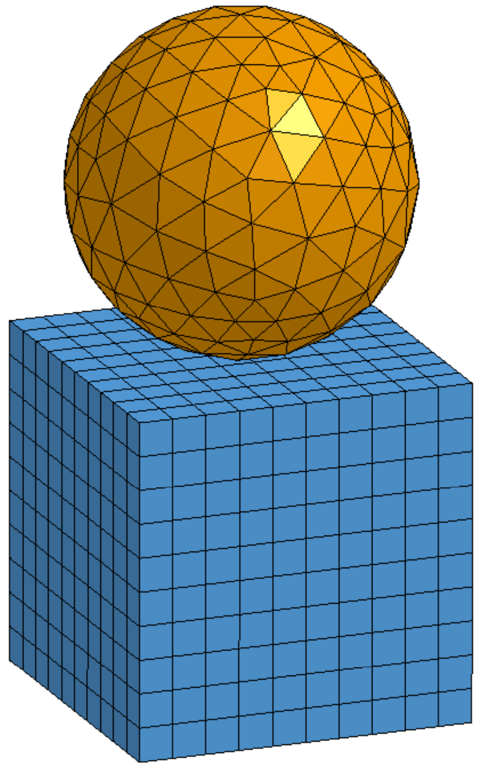
Motivation

Solid elements are three-dimensional finite elements that can model solid bodies and structures without any a priori geometric simplification.

- No geometric, constitutive and loading assumptions required.
- Boundary conditions treated more realistically (compared to shells or beams).
- FE mesh visually looks like the physical system.



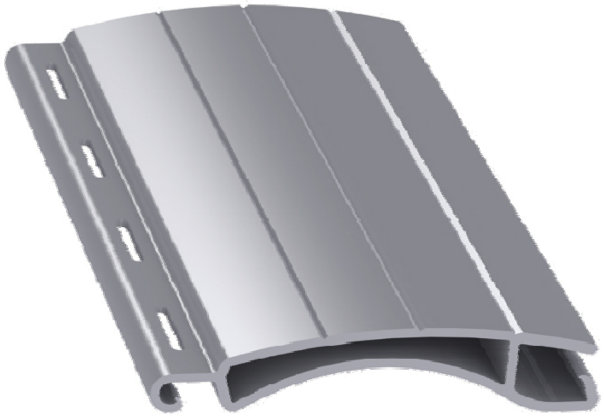
but…

- Higher effort: mesh preparation, CPU time, post-processing, …
- Expensive mesh refinement: Curse of dimensionality.
- Often poor performance for thin-walled structures (locking problems).

Motivation

Applications

- Foam structures
- Rubber components
- Cast iron parts
- Solid barriers
- Plastic parts
- Bulk forming
- Thick metal sheets
- Elastic tools
- Impact analysis
- ...



Overview

LS-DYNA User's manual: *SECTION_SOLID, parameter ELFORM

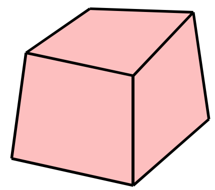
EQ. -2: fully integrated S/R solid intended for elements with poor aspect ratio, accurate formulation

EQ. -1: fully integrated S/R solid intended for elements with poor aspect ratio, efficient formulation

EQ. 1: constant stress solid element (default)

EQ. 2: fully integrated S/R solid

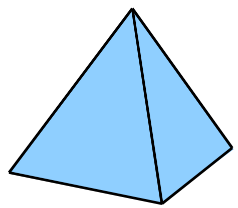
EQ. 3: fully integrated quadratic 8 node element with nodal rotations



EQ. 4: S/R quadratic tetrahedron element with nodal rotations

EQ. 10: 1 point tetrahedron

EQ. 13: 1 point nodal pressure tetrahedron

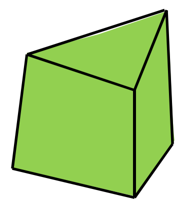


EQ. 15: 2 point pentahedron element

EQ. 16: 4 or 5 point 10-noded tetrahedron

EQ. 17: 10-noded composite tetrahedron

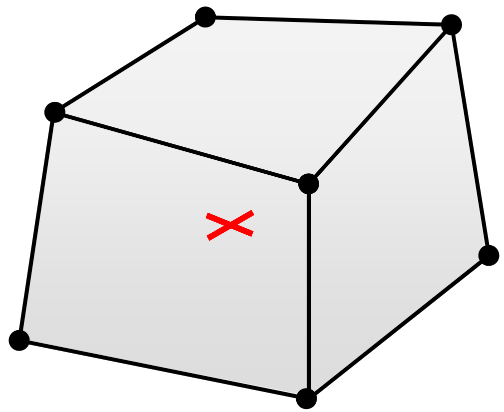
EQ. 115: 1 point pentahedron element with hourglass control



Hexahedra elements in LS-DYNA

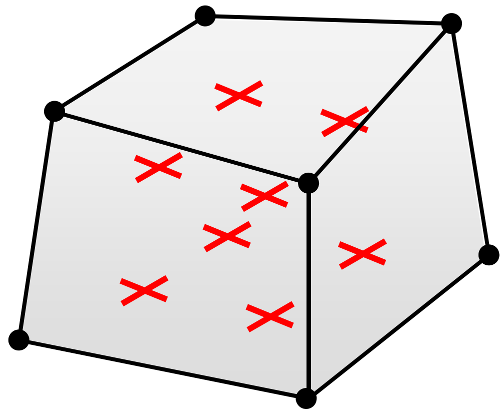
ELFORM = 1

- underintegrated constant stress
- efficient and accurate
- even works for severe deformations
- needs hourglass stabilization:
  choice of hourglass formulation
  and values remains an issue



ELFORM = 2

- selective reduced integrated brick element
  (volumetric locking alleviated)
- no hourglass stabilization needed
- slower than ELFORM=1
- too stiff in many situations,
  especially for poor aspect ratios (shear locking)
- more unstable in large deformation applications

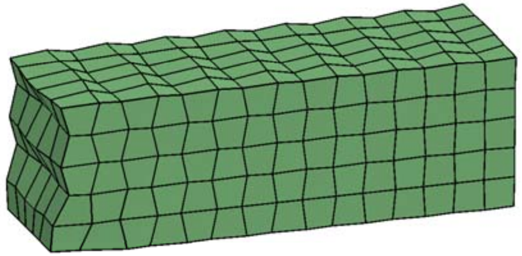


Hourglass control for ELFORM=1

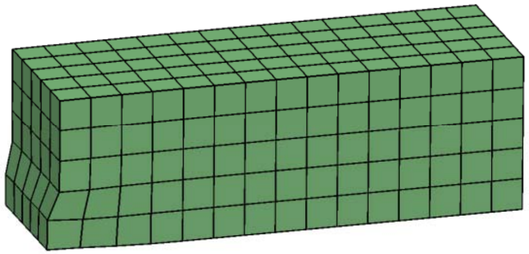
*HOURGLASS: IHQ = 1…5

- viscous form (1,2,3) for higher velocities
- stiffness form (4,5) for lower velocities
- exact volume integration recommended (3,5)

*HOURGLASS: IHQ = 6



- the QBI (Quintessential Bending Incompressible) hourglass control by Belytschko and Bindeman
- hourglass stiffness uses elastic constants
- **recommended in most situations**
- sometimes modified QM makes sense (watch hourglass energy)



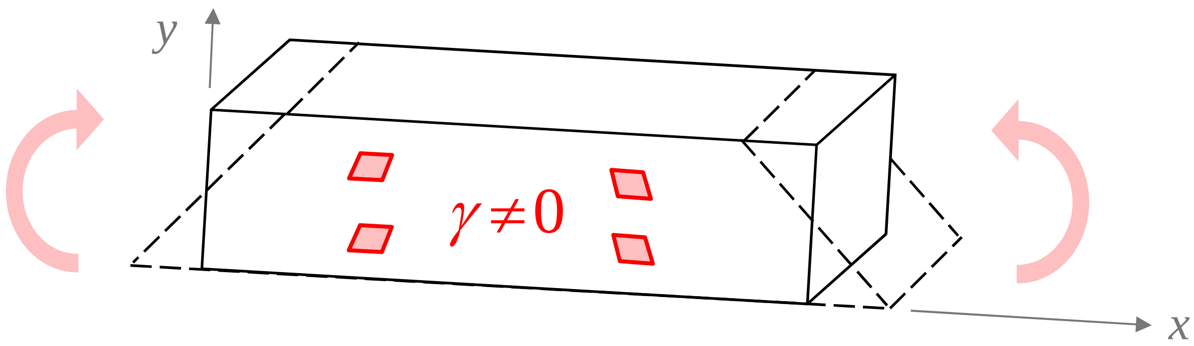
*HOURGLASS: IHQ = 7/9

- similar to type 6, but less experience
- type 7 uses total deformation instead of updated
- type 9 should provide more accurate results for distorted meshes

Property of ELFORM=2

Shear locking

- pure bending modes trigger spurious shear energy
- getting worse for poor aspect ratios



$$\varepsilon_{xx} = 2\xi_y / l_x,\ \varepsilon_{yy} = 0,\ \gamma_{xy} = \xi_x / l_y$$

- **Alleviation possibility 1:** under-integration → ELFORM = 1
- **Alleviation possibility 2:** enhanced strain formulations modified Jacobian matrix

$$\varepsilon_{xx} = 2\xi_y / l_x,\ \varepsilon_{yy} = 0,\ \gamma_{xy} = \ldots = \xi_x / l_x$$

ELFORM = -1 / -2

Solid element types -1 and -2

NEW: ELFORM = -1 / -2

- Thomas Borrvall: "*A heuristic attempt to reduce transverse shear locking in fully integrated hexahedra with poor aspect ratio*", Salzburg 2009

- Modification of the Jacobian matrix: reduction of spurious stiffness without affecting the true physical behavior of the element

$$J_{ij}^{\text{orig}} = \frac{\partial x_i}{\partial \xi_j} = x_{Ii} \frac{1}{8}\left(\xi_j^I + \xi_{jk}^I \xi_k + \xi_{jl}^I \xi_l + \xi_{123}^I \xi_k \xi_l \right)$$

aspect ratios between dimensions

$$J_{ij}^{\text{mod}} = x_{Ii} \frac{1}{8}\left(\xi_j^I + \xi_{jk}^I \xi_k \kappa_{jk} + \xi_{jl}^I \xi_l \kappa_{jl} + \xi_{123}^I \xi_k \kappa_{jk} \xi_l \kappa_{jl} \right)$$

- **Type -2:** accurate formulation, but higher computational cost in explicit

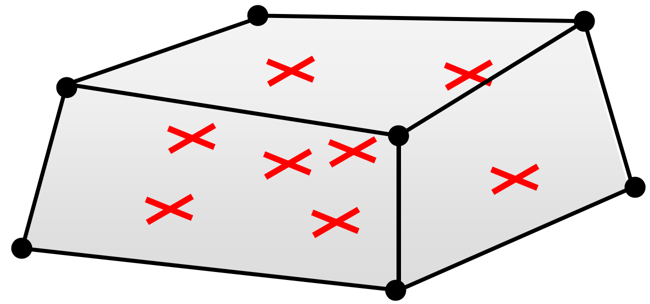
- **Type -1:** efficient formulation

- CPU cost compared to type 2: ~1.2 (type -1), ~4 (type -2)

New hexahedra elements in LS-DYNA

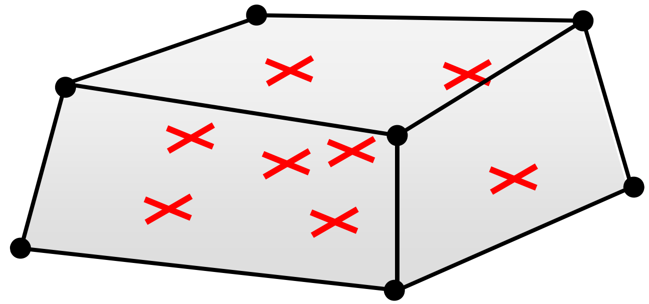
ELFORM = -1

- identical with type 2, but accounted for poor aspect ratio on order to reduce shear locking
- „efficient formulation“
- sometimes hourglass tendencies



ELFORM = -2

- identical with type 2, but accounted for poor aspect ratio on order to reduce shear locking
- „accurate formulation“
- higher computational cost than type -1



Implicit elastic bending

- clamped plate of dimensions 10x5x1 mm^3
- subjected to 1 Nm torque at the free end
- E = 210 GPa
- analytical solution for end tip deflection:
  0.57143 mm
- convergence study
  with aspect ratio 5:1 kept constant

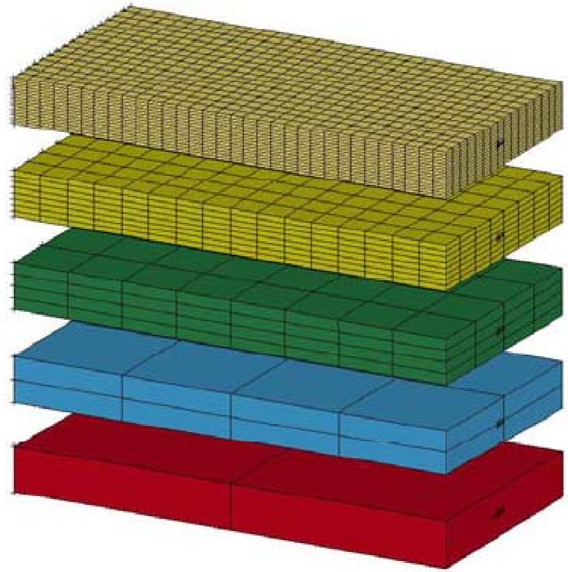
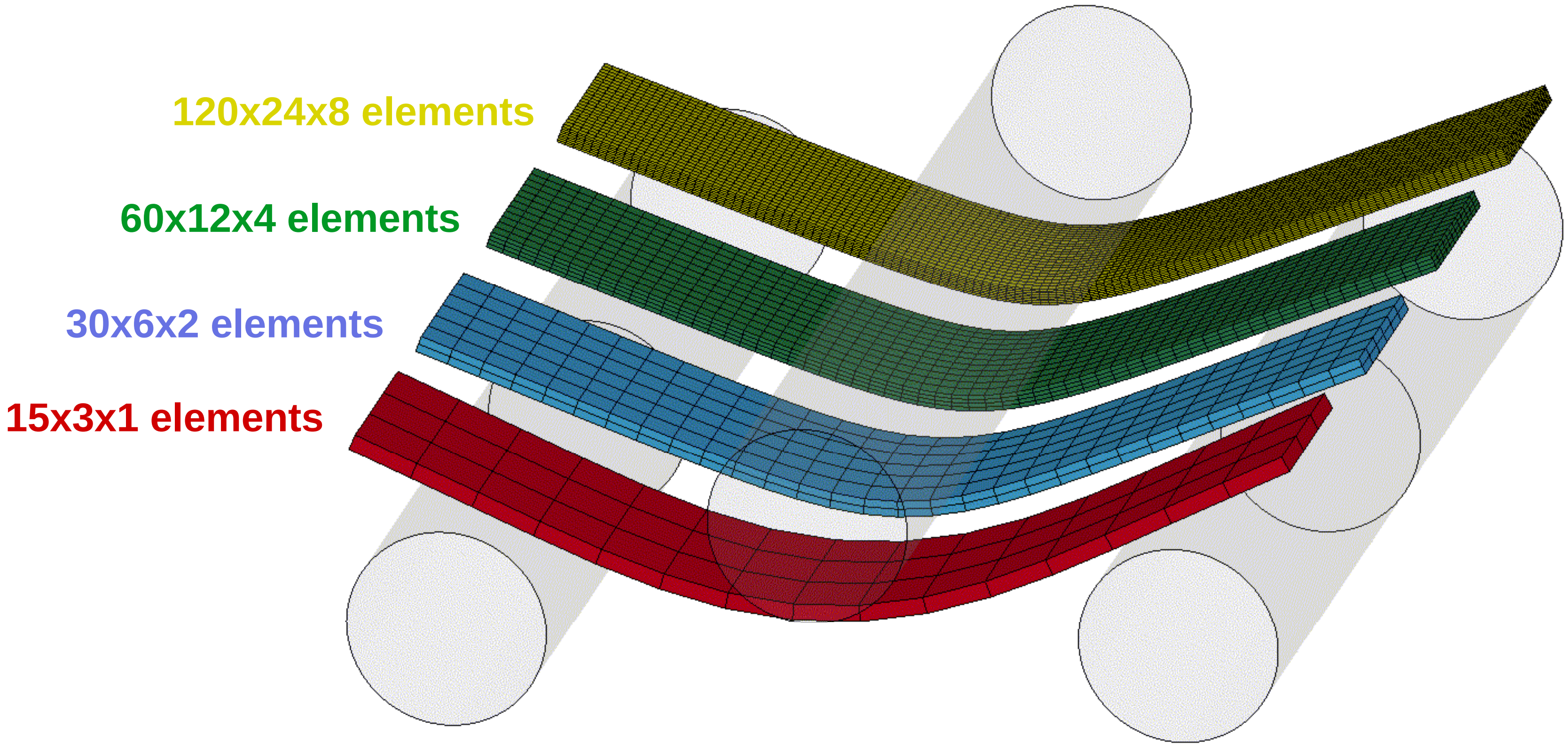


Table 1 End tip deflection for different mesh discretizations and element types, error in parenthesis.

Discretization	Solid element type 2	Solid element type -2	Solid element type -1
2x1x1	0.0564 (90.1%)	0.6711 (17.4%)	0.6751 (18.1%)
4x2x2	0.1699 (70.3%)	0.5466 (4.3%)	0.5522 (3.4%)
8x4x4	0.3469 (39.3%)	0.5472 (4.2%)	0.5500 (3.8%)
16x8x8	0.4820 (15.7%)	0.5516 (3.5%)	0.5527 (3.3%)
32x16x16	0.5340 (6.6%)	0.5535 (3.1%)	0.5540 (3.1%)

Plastic bending

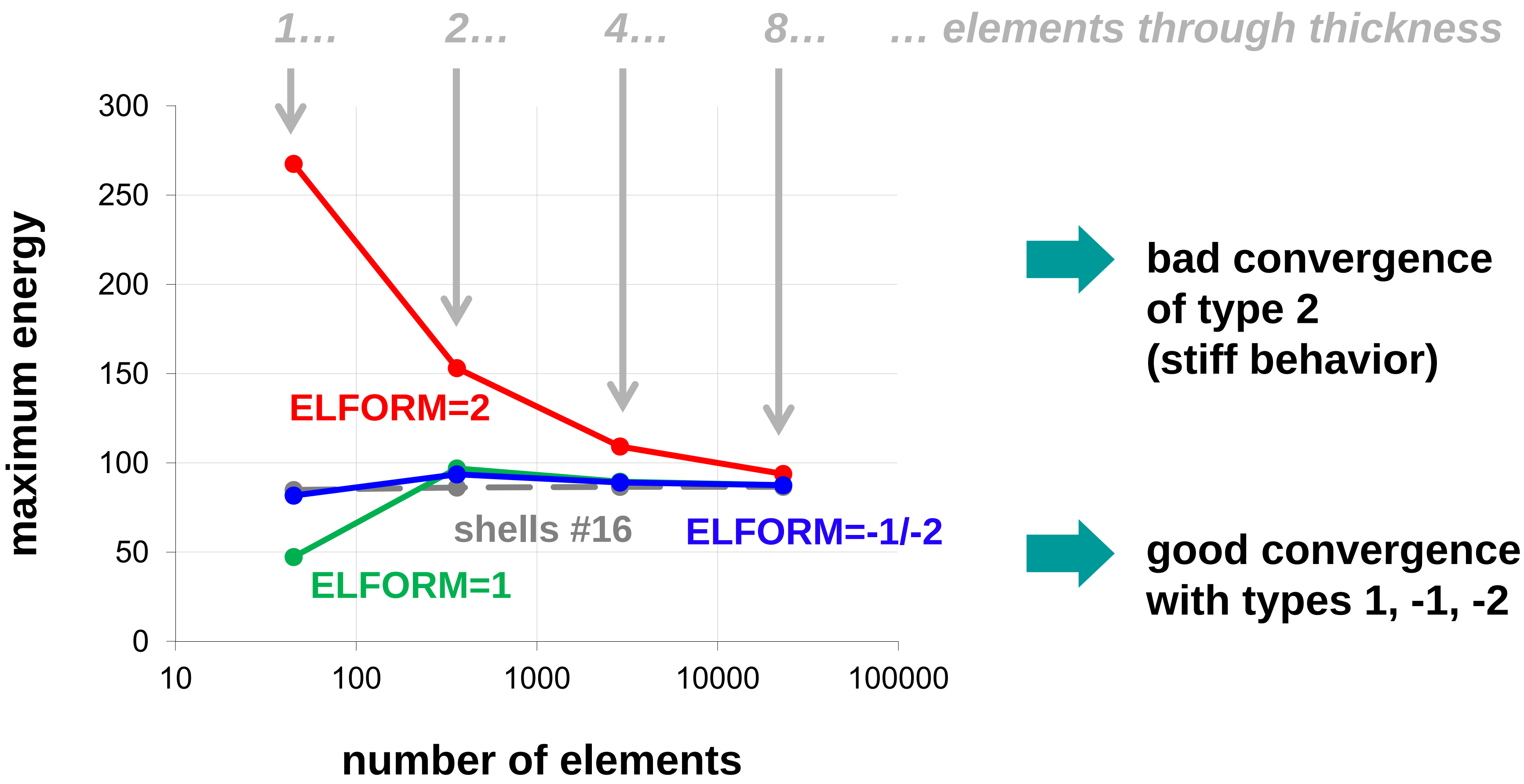
- explicit plastic 3 point bending (prescribed motion)
- plate of dimensions 300x60x5 mm^3
- *MAT_024 (aluminum)
- convergence study - aspect ratio 4:1 kept constant



Plastic bending

Results

- maximum energy (internal + hourglass)



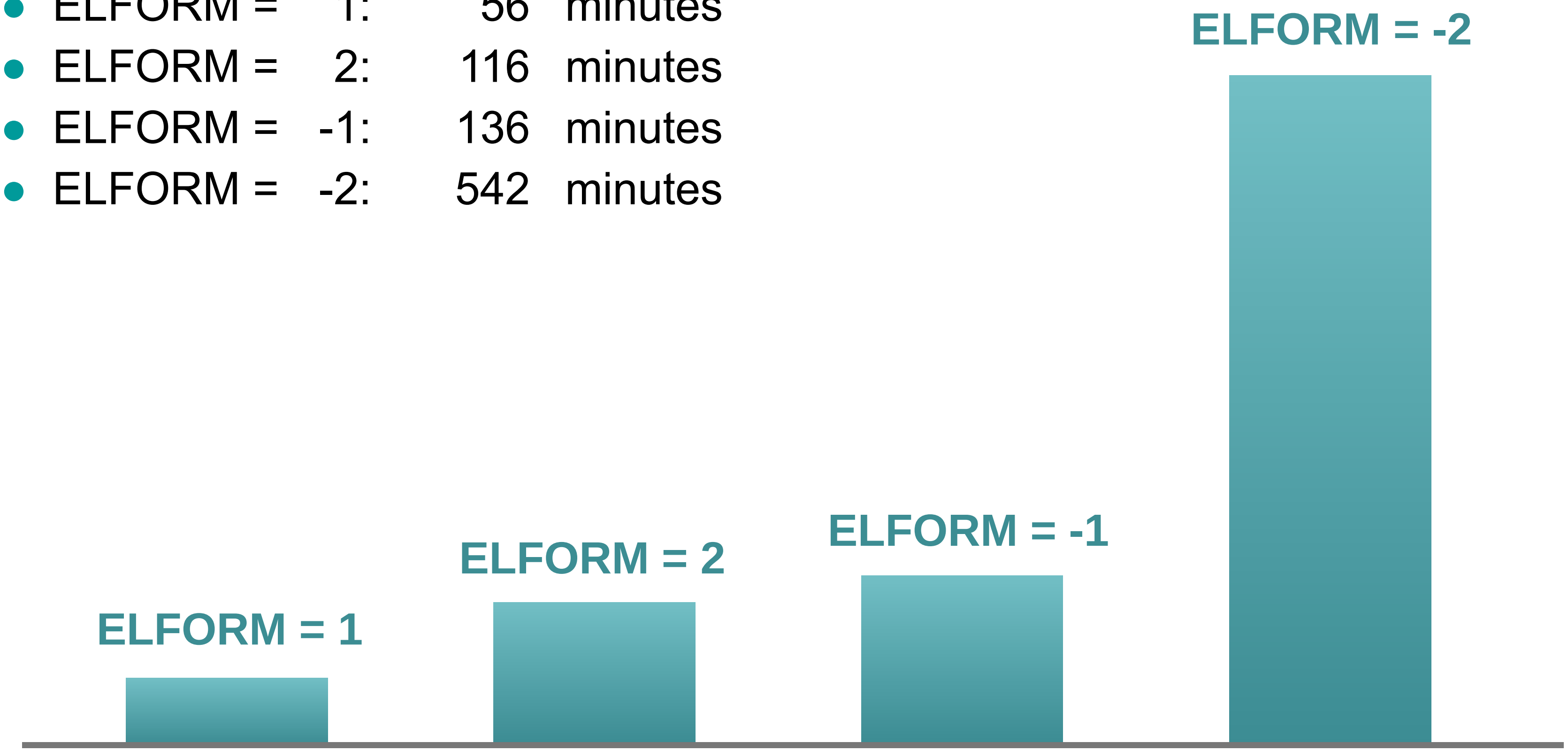
➡ **bad convergence of type 2 (stiff behavior)**

➡ **good convergence with types 1, -1, -2**

Plastic bending

CPU times

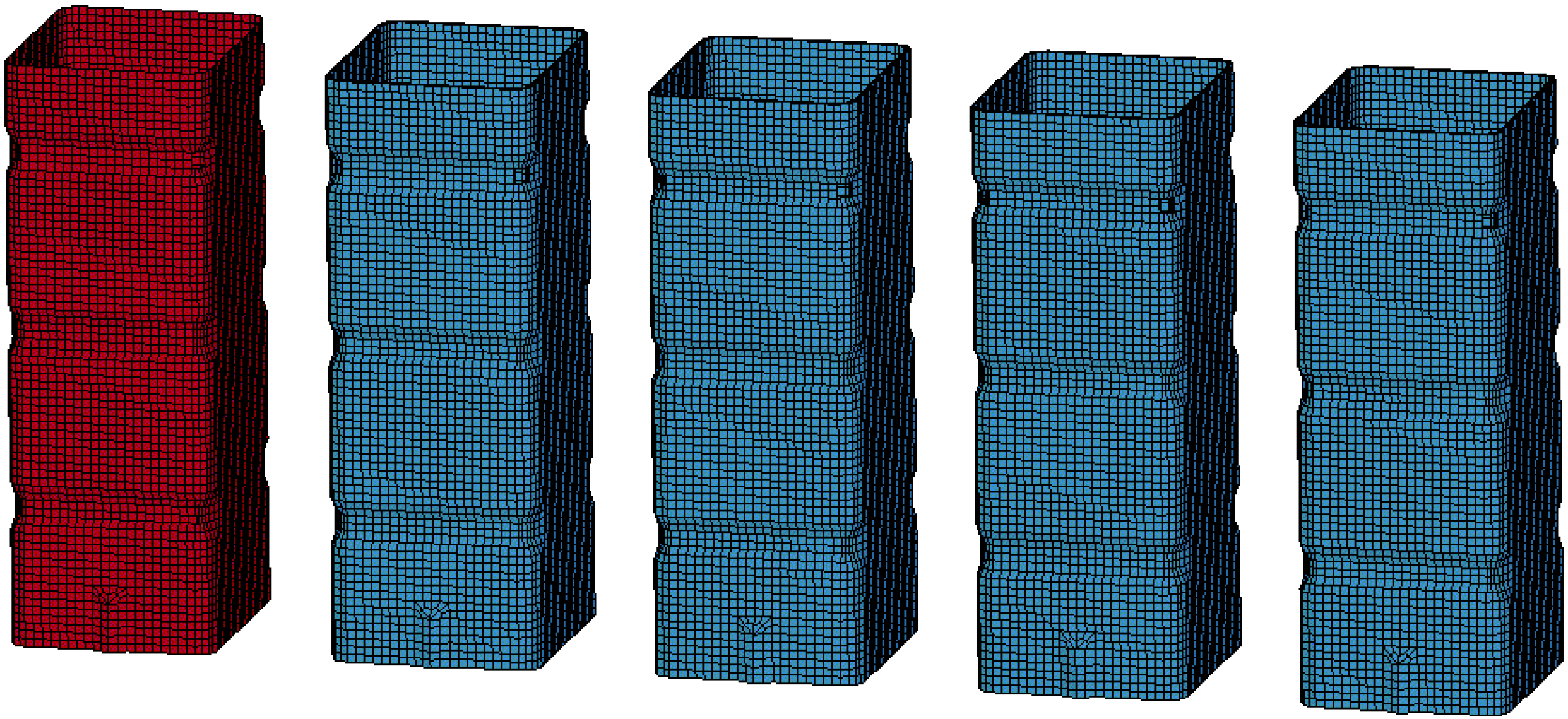
- ELFORM = 1: 56 minutes
- ELFORM = 2: 116 minutes
- ELFORM = -1: 136 minutes
- ELFORM = -2: 542 minutes



ELFORM = -2 not efficient, ELFORM = -1 comparable to 2

Tube crash

element size: 3.5 mm
thickness: 2 mm



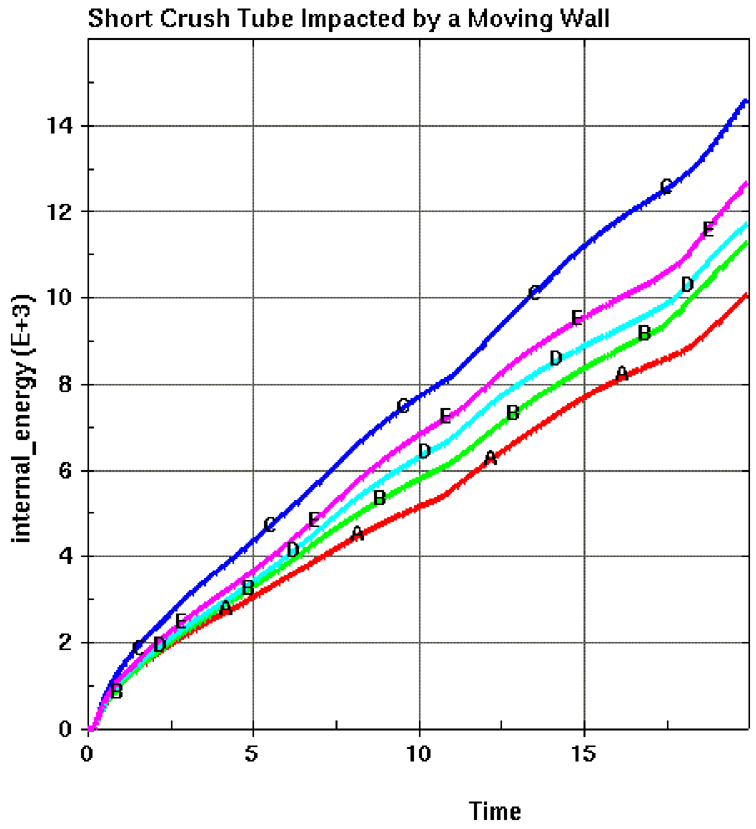
shells type 16	solids type 1 (t_{CPU}=1.0)	solids type 2 (t_{CPU}=5.5)	solids type -1 (t_{CPU}=5.2)	solids type -2 (t_{CPU}=8.3)

Tube crash

Results

contact force

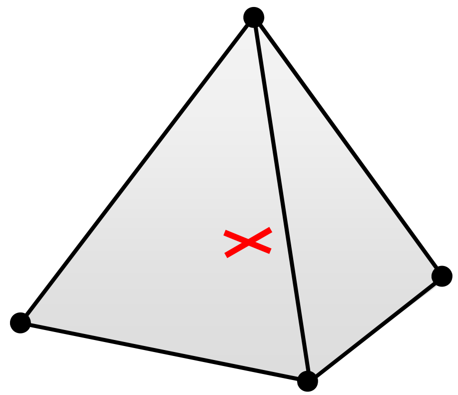
internal energy



Tetrahedra elements in LS-DYNA

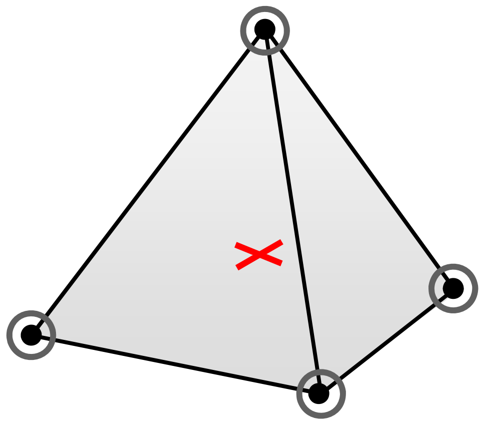
ELFORM = 10

- 1 point constant stress
- volumetric locking – stiff behavior
- only applicable for foams with $\nu = 0$ (not recommended in general)
- often used for transitions in meshes (ESORT=1)



ELFORM = 13

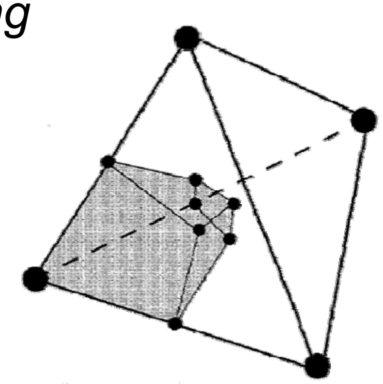
- 1 point constant stress with nodal pressure averaging
- alleviated volumetric locking
- better performance than ELFORM=10 if Poisson's ratio $\nu > 0$ (metals, rubber, …)
- implemented for common materials: 1,3,6,24,27,77,81,82,91,92,106,120,123,124,128,129,181,183,224,225,244



Theoretical background

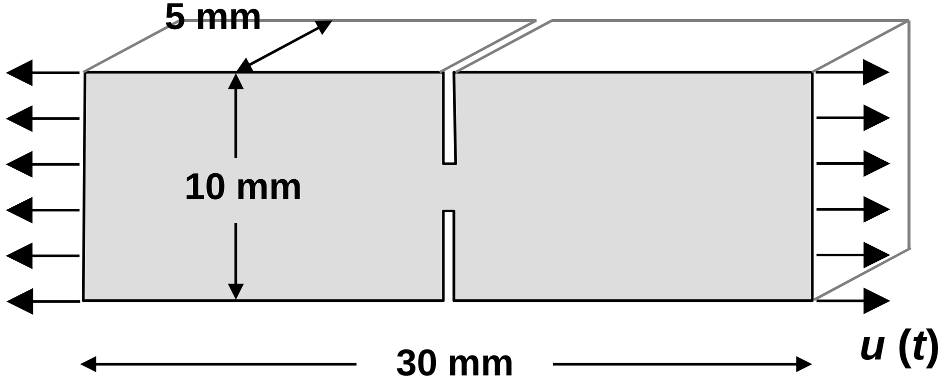
- **manual:** *"1 point nodal pressure tetrahedron for bulk forming"*
- **paper:** J. Bonet & A.J. Burton. A simple average nodal pressure tetrahedral element for incompressible dynamic explicit applications. Comm. Num. Meth. Engrg. 14: 437-449, 1998

➡ *"... the element **prevents volumetric locking** by defining nodal volumes and evaluating average nodal pressures in terms of these volumes ...*
*... it can be used in explicit dynamic applications involving (nearly) **incompressible material behavior***
(e.g. rubber, ductile elastoplastic metals) ..."

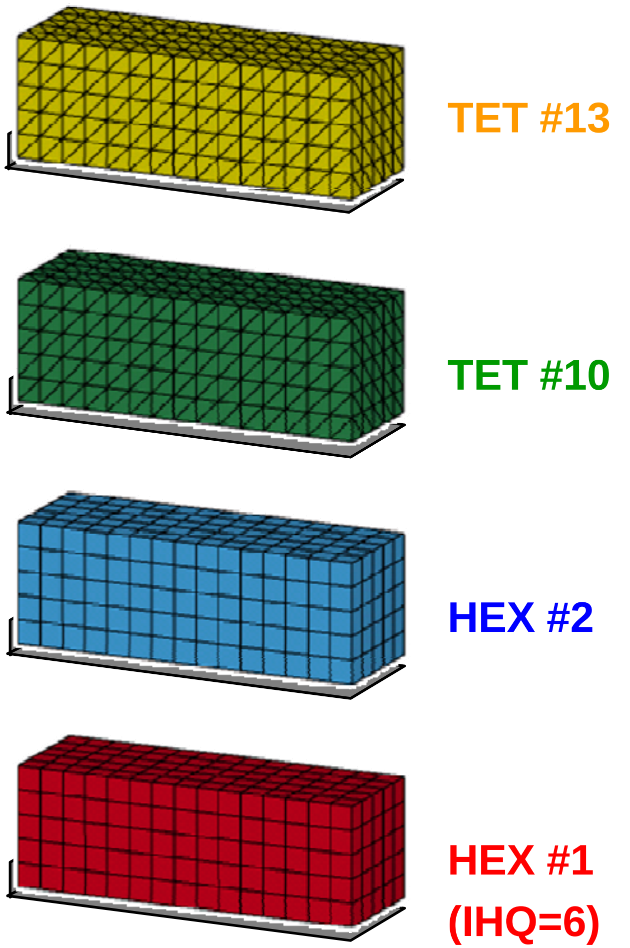


➡ **TET #13 = TET #10 + averaging nodal pressures**
= TET #10 – volumetric locking

Notched steel specimen



discretized quarter system:



*MAT_PIECEWISE_LINEAR_PLASTICITY

E = 206.9 kN/mm^2

v = 0.29

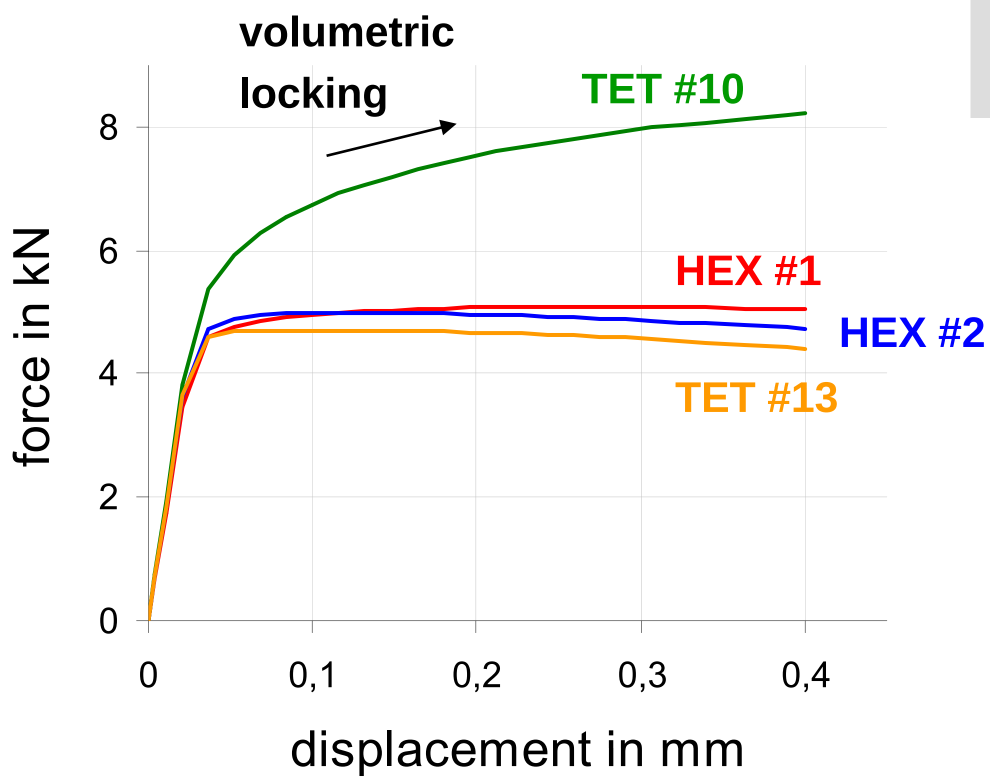
σ_y = 0.45 kN/mm^2

E_t = 0.02 kN/mm^2 (nearly ideal plastic)

➡ **isochoric plastic flow**

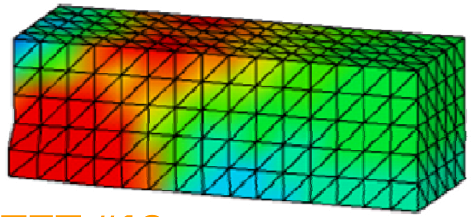
Notched steel specimen

load-displacement curve:
should show a limit force

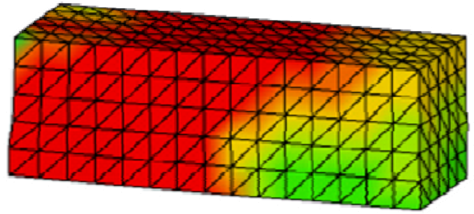


von Mises
stresses

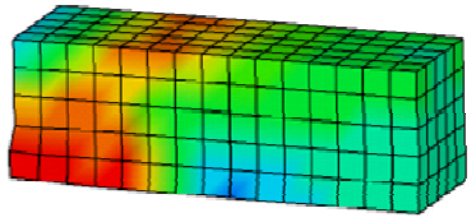
0 - 480
N/mm^2



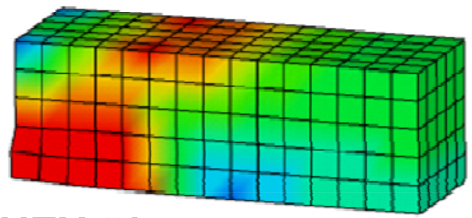
TET #13



TET #10

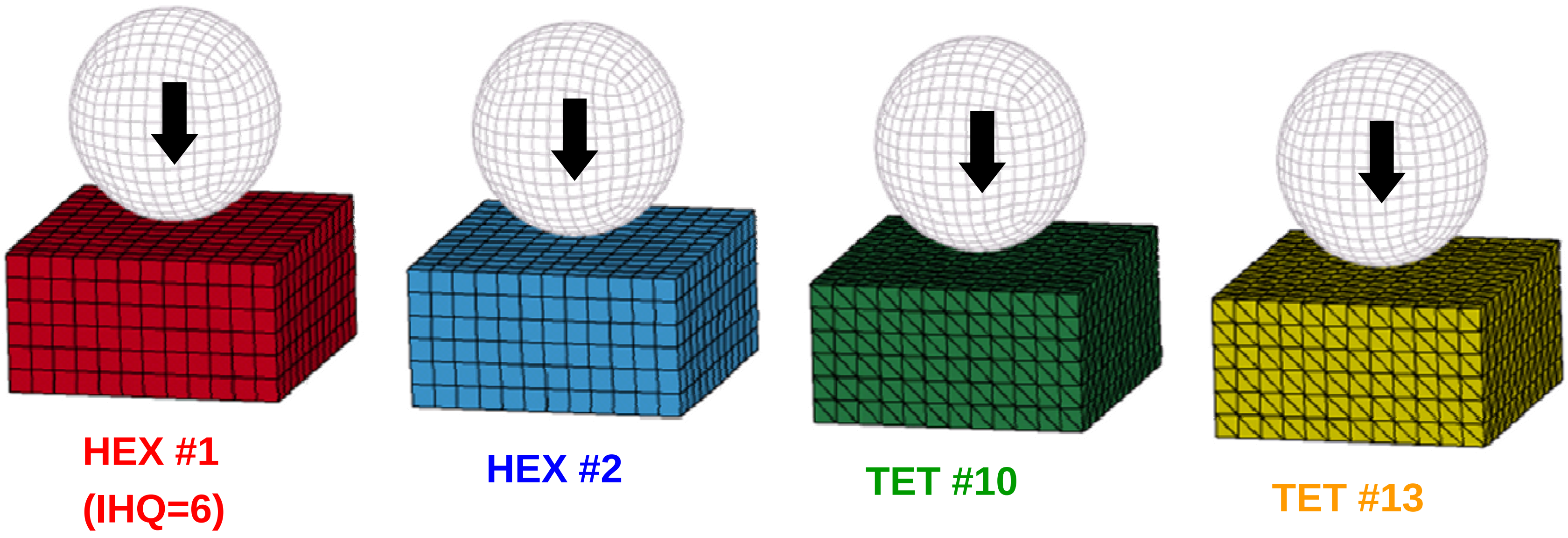


HEX #2



HEX #1
(IHQ=6)

Rubber block compression



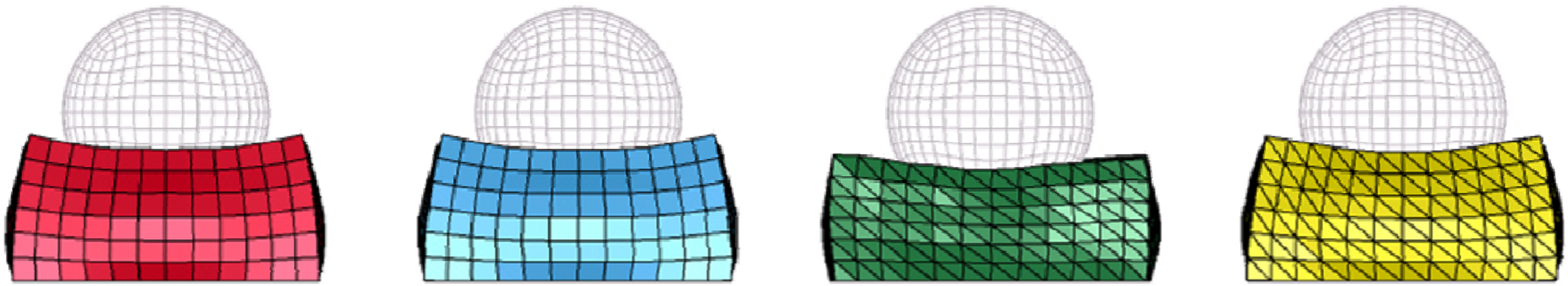
*MAT_MOONEY-RIVLIN_RUBBER

A = 4.0 N/mm^2
B = 2.4 N/mm^2
v = 0.499
ρ = 1.5E-06 kg/mm^3

→ **nearly incompressible material**

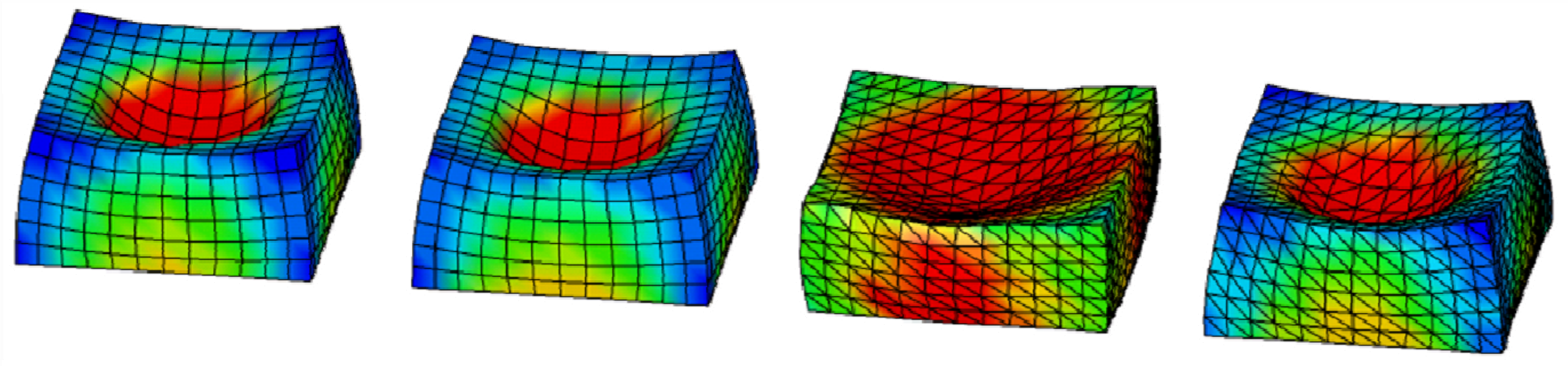
Rubber block compression

deformation



HEX #1 (IHQ=6) | **HEX #2** | **TET #10** | **TET #13**

von Mises stresses (0 – 1.2 N/mm²)



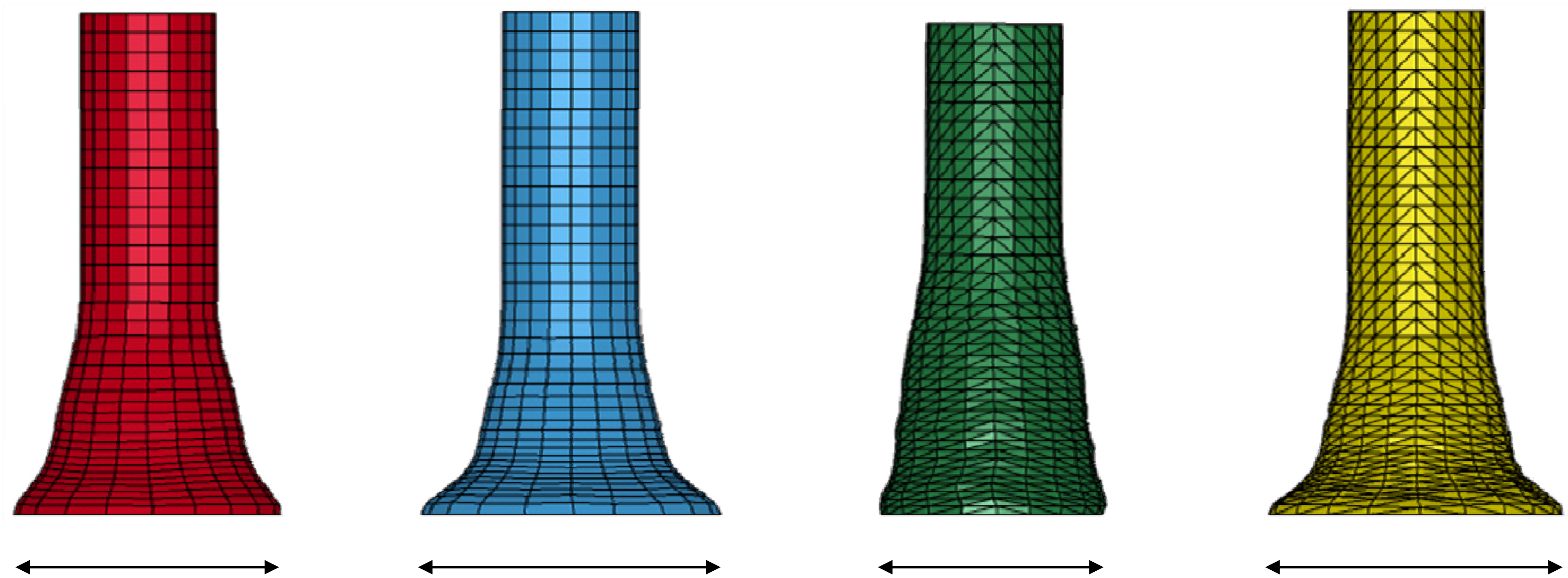
Taylor bar impact

***MAT_PIECEWISE_LINEAR_PLASTICITY:**

ρ = 8930 kg/m³, E = 117 kN/mm², v = 0.35, σ_y = 0.4 kN/mm², E_t = 0.1 kN/mm²

deformation

HEX #1 (IHQ=6) | **HEX #2** | **TET #10** | **TET #13**

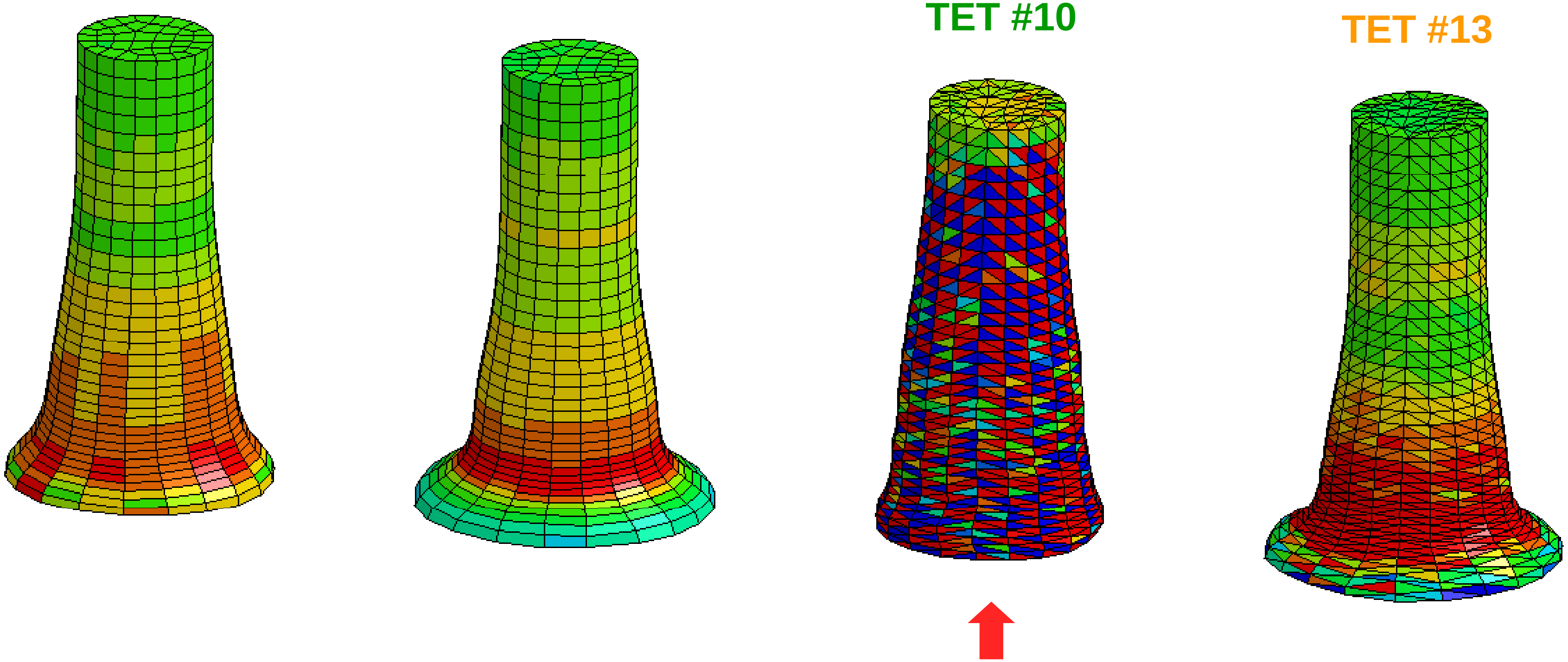


12.6 mm (QM=0.10)
13.8 mm (QM=0.01) | **14.1 mm** | **10.4 mm** | **13.9 mm**

Taylor bar impact

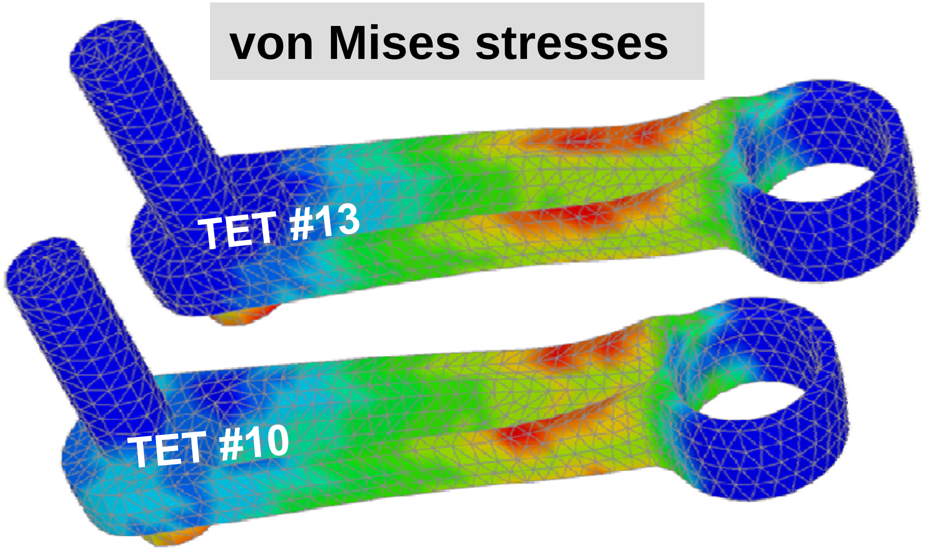
pressure (-300 – 300 N/mm²)

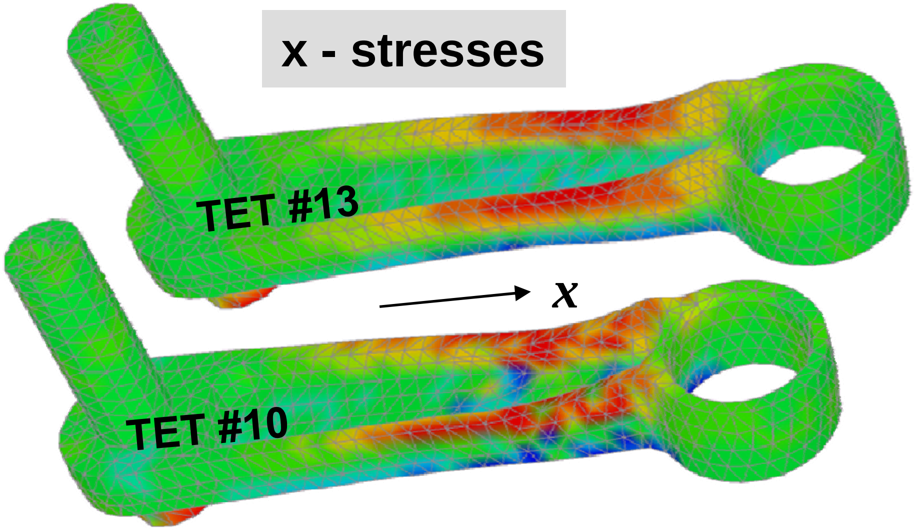
HEX #1 (IHQ=6) HEX #2 TET #10 TET #13

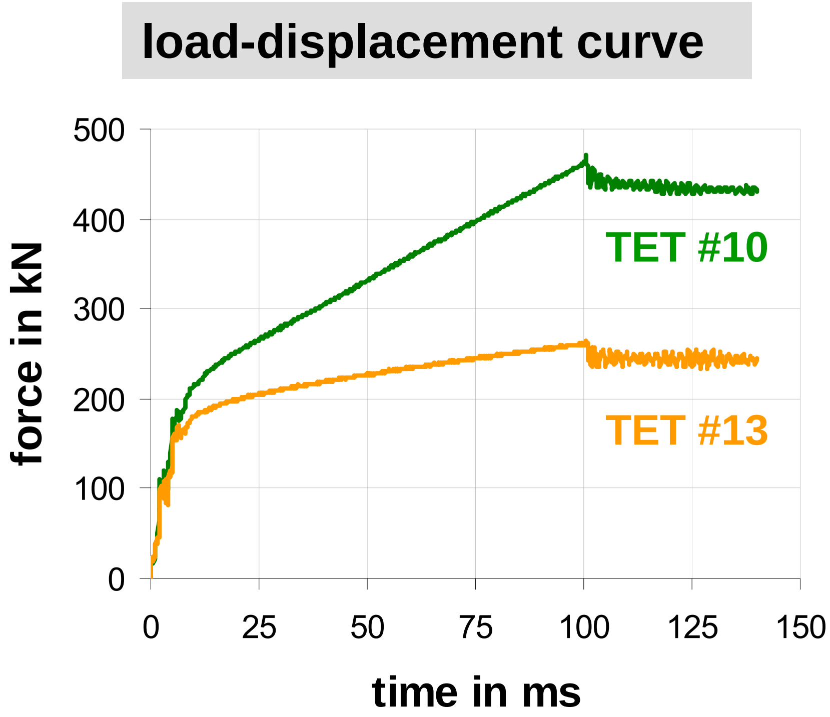


checkerboard mode

Structural component



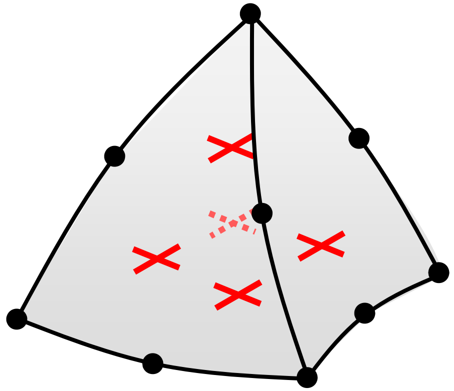




Higher order tets in LS-DYNA

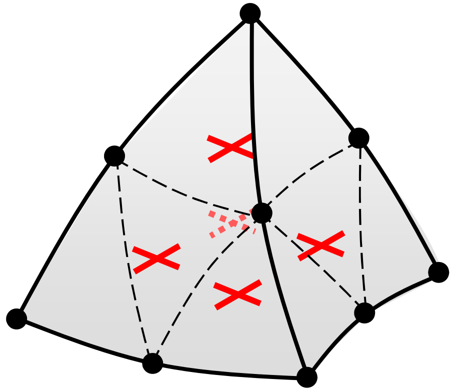
ELFORM = 16

- 4(5) point 10-noded tetrahedron
- good accuracy for moderate strains
- high cpu cost
- observe the node numbering
- use *CONTACT_AUTOMATIC_... With PID
- easy conversion of 4-noded tets via *ELEMENT_SOLID_TET4TOTET10
- full output with TET10=1 on *CONTROL_OUTPUT



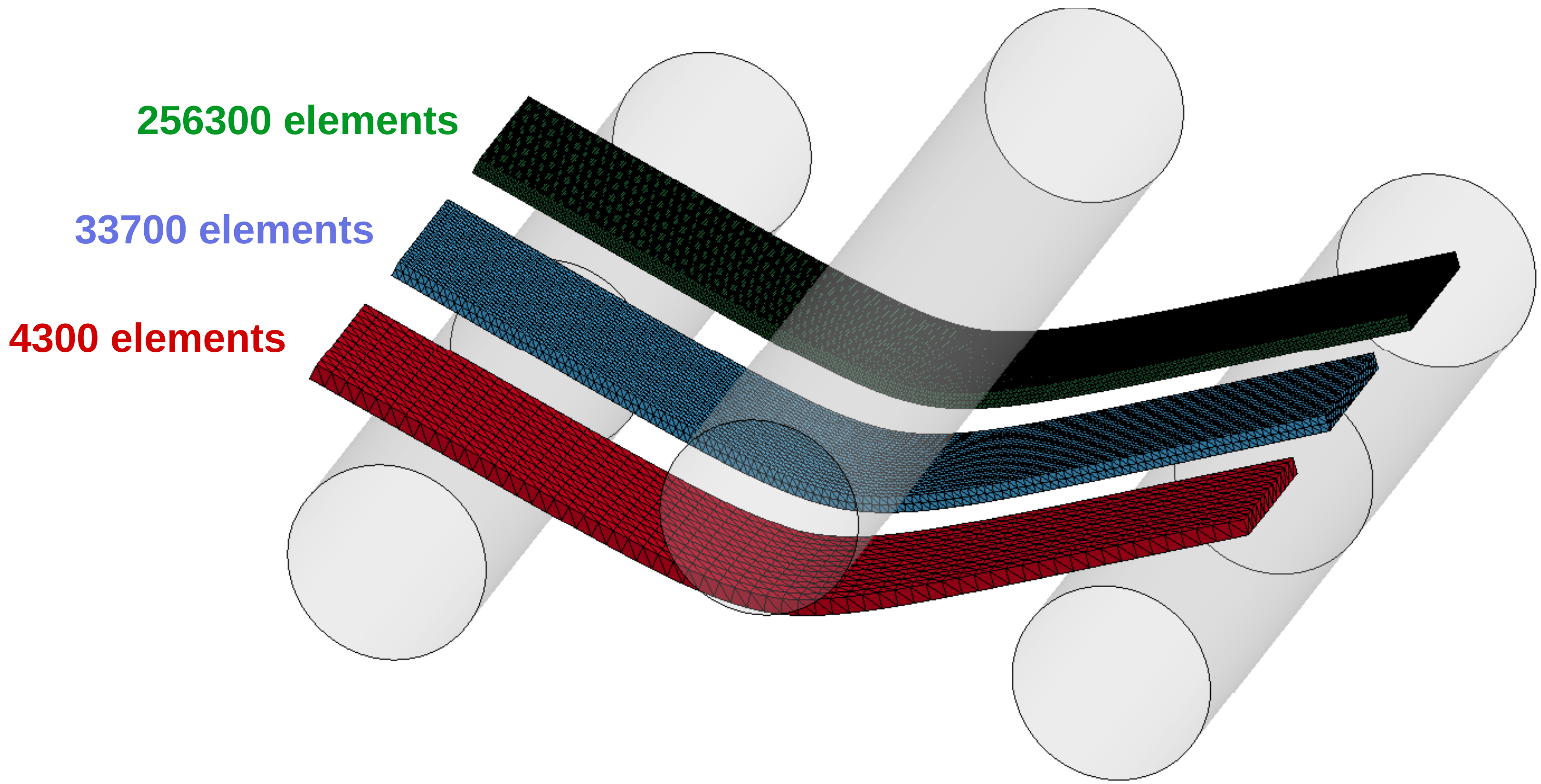
ELFORM = 17

- 4(5) point 10-noded „composite“ tetrahedron (12 linear sub-tetrahedrons)
- properties similar to type 16
- correct external force distribution



Plastic bending

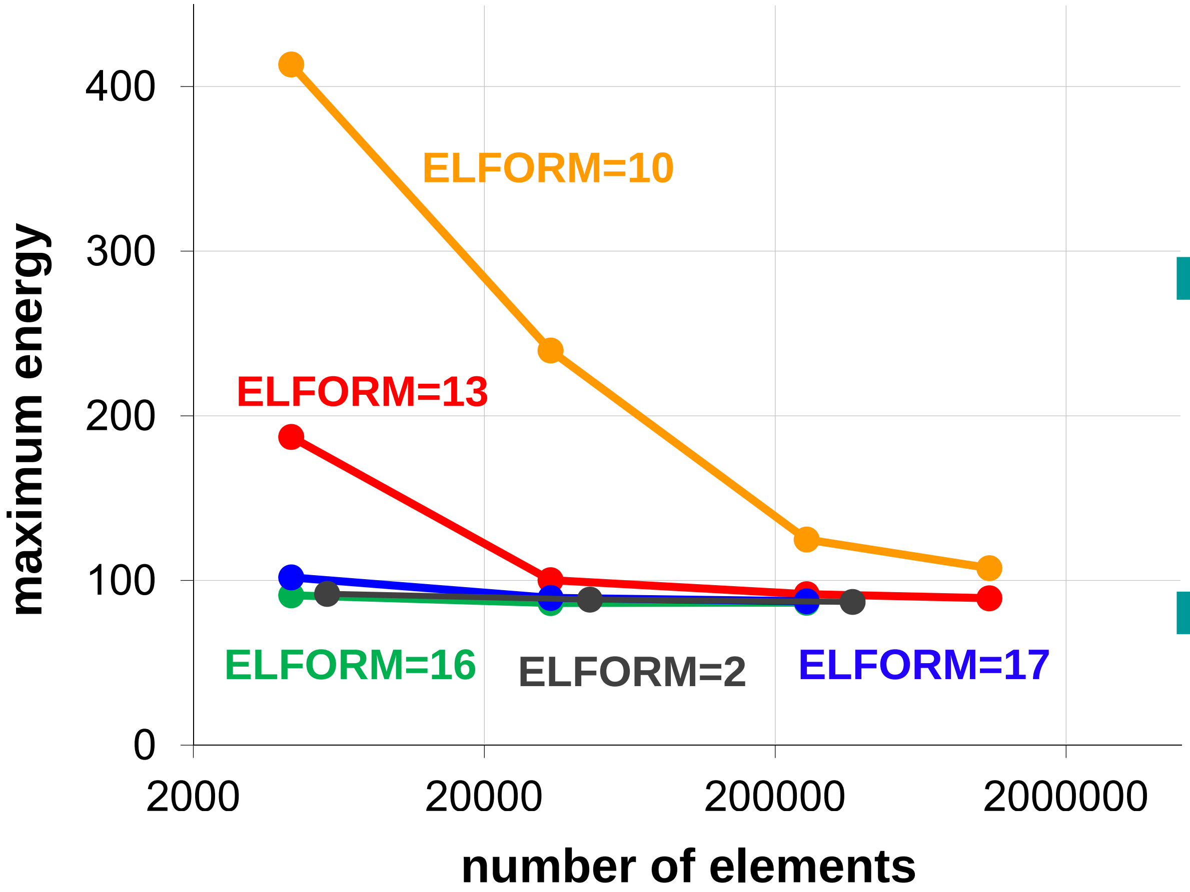
- Explicit plastic 3 point bending (prescribed motion)
- plate of dimensions 300x60x5 mm^3
- *MAT_024 (aluminum)



Plastic bending

Results

- maximum energy (internal)



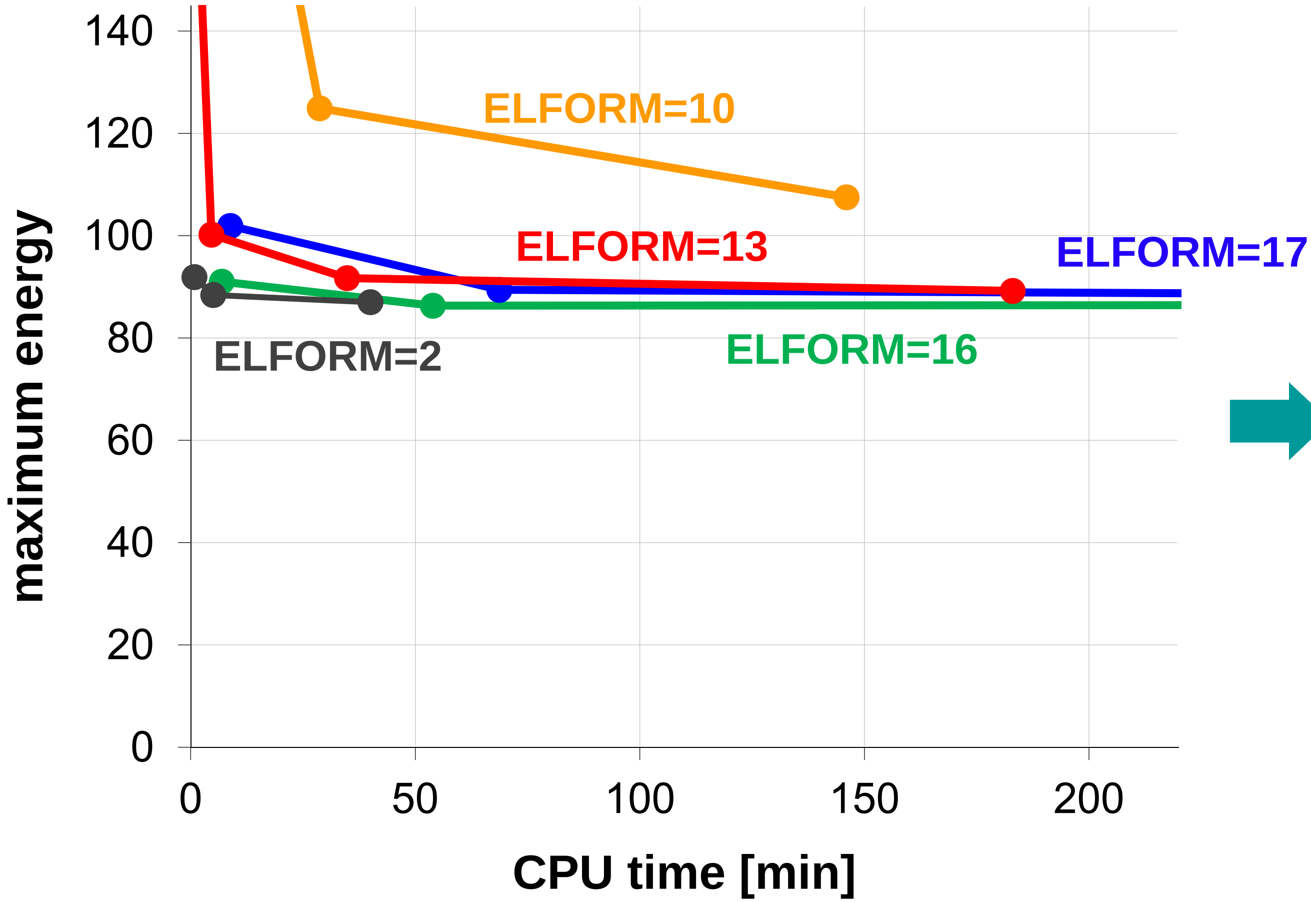
bad convergence of type 10 (stiff behavior)

good convergence with types 13,16,17

Plastic bending

Results

- maximum energy (internal)

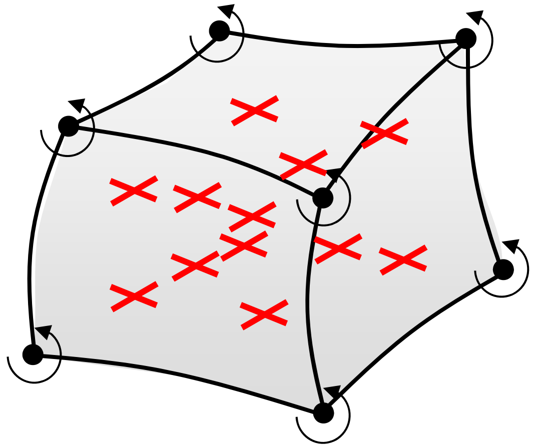


Tet type 13 comparable to types 16 / 17

Hex and Tet with nodal rotations

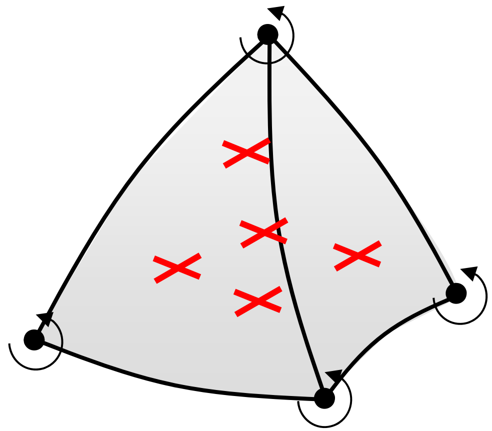
ELFORM = 3

- quadratic 8 node hexahedron with nodal rotations, i.e. 6 DOF per node
- derived from 20 node hexahedron
- full integration (12-point)
- well suited for connections to shells
- good accuracy for small strains
- tendency to volumetric locking



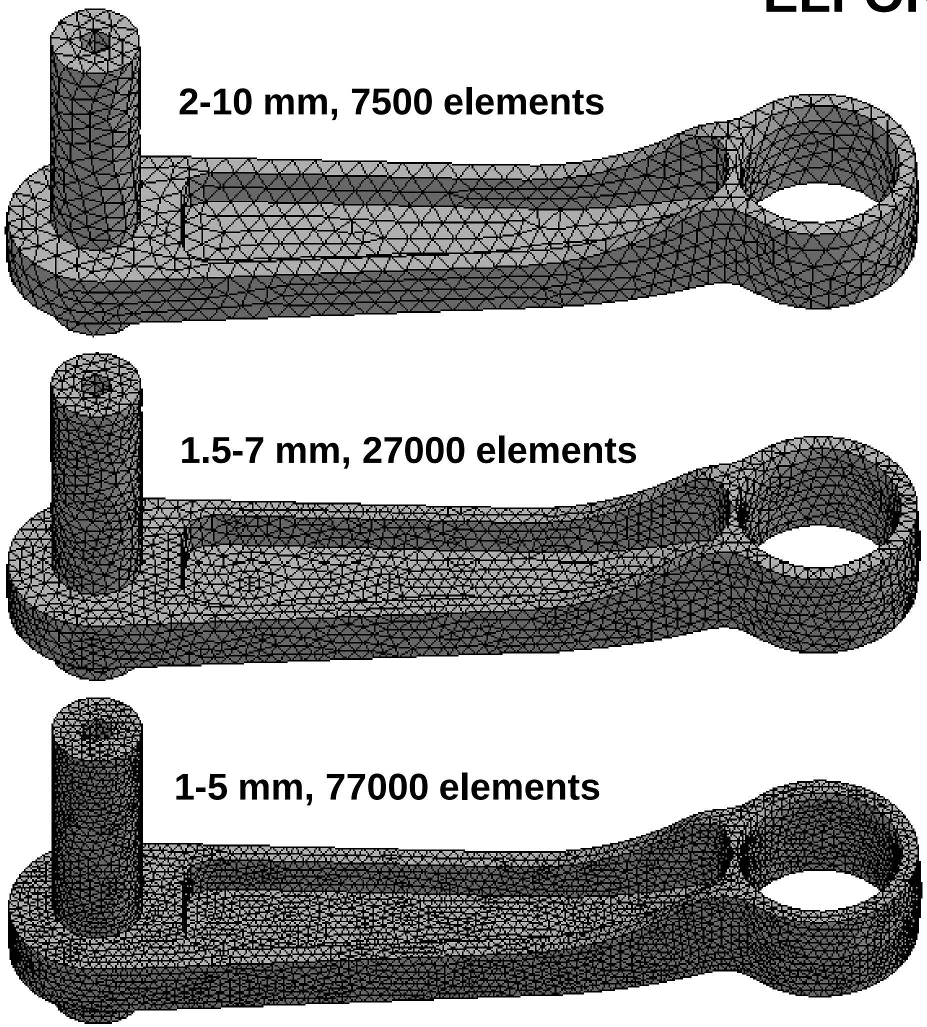
ELFORM = 4

- quadratic 4 node tetrahedron with nodal rotations, i.e. 6 DOF per node
- derived from 10 node tetrahedron
- S/R integration (5-point)
- well suited for connections to shells
- good accuracy for small strains
- tendency to volumetric locking



Structural component

3 different discretizations

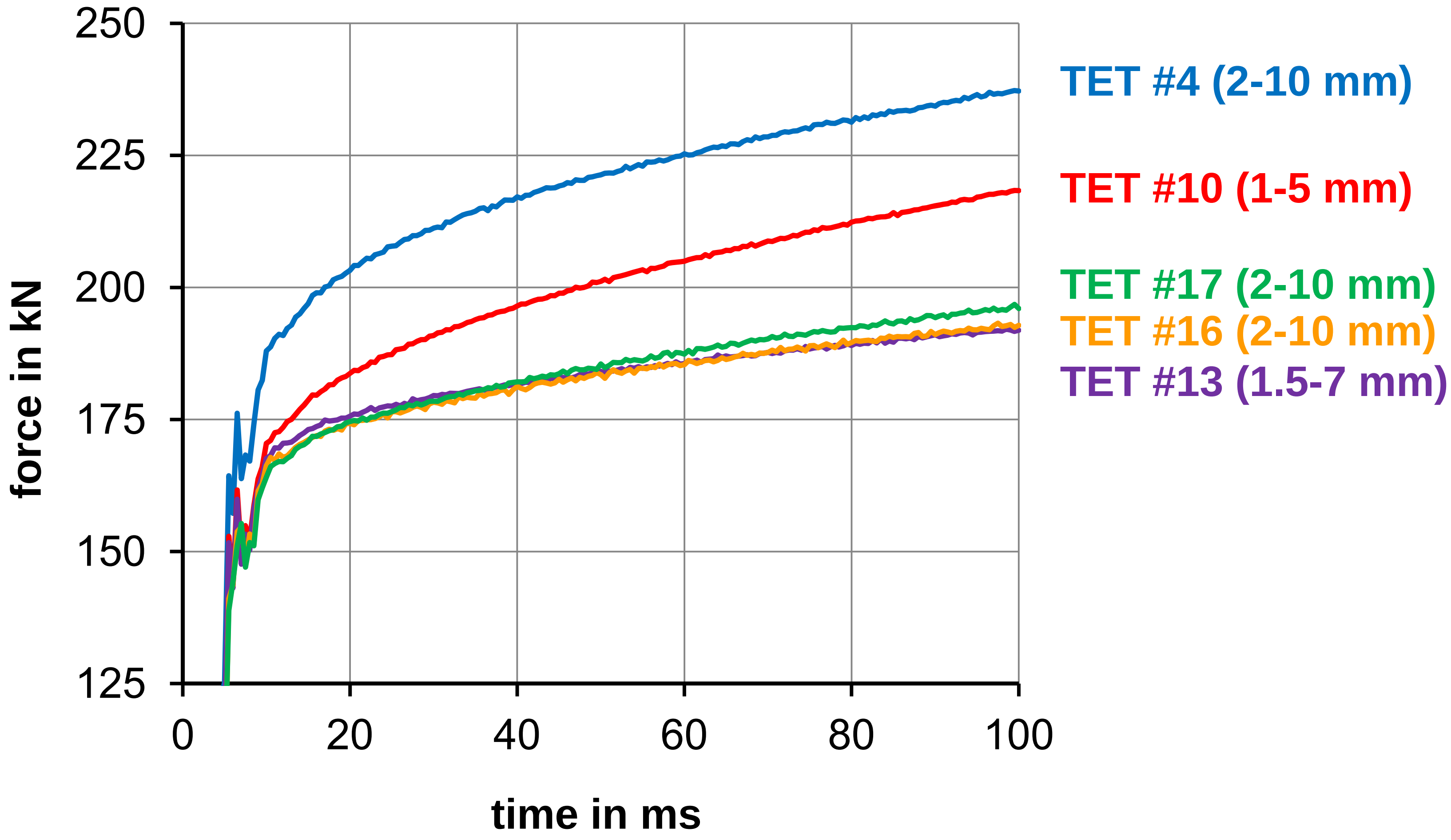


ELFORM =	4	10	13	16	17
	18	3	4	39	66
	97	14	22	212	344
	403	64	98	945	1529

CPU times in minutes

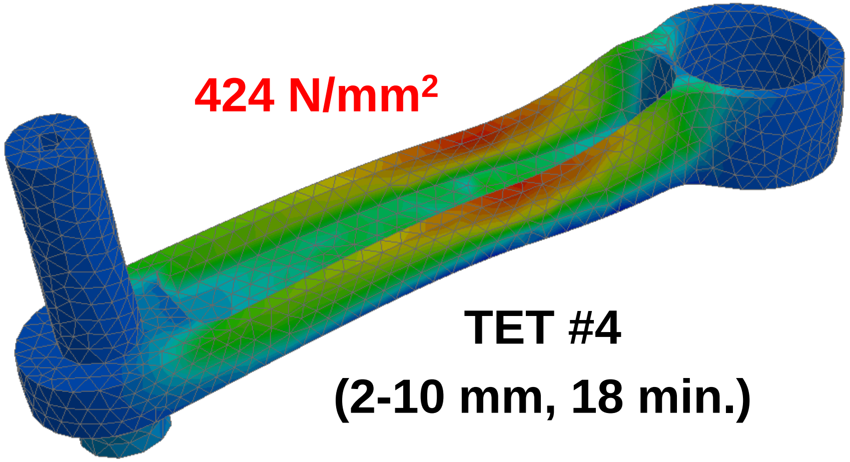
Structural component

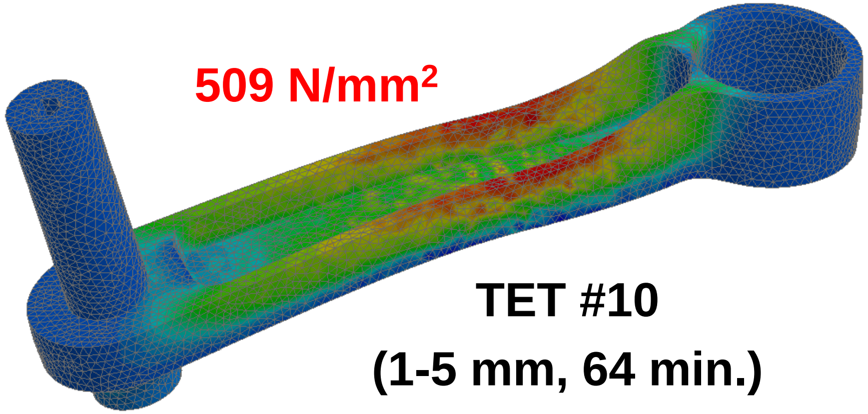
load-displacement curve

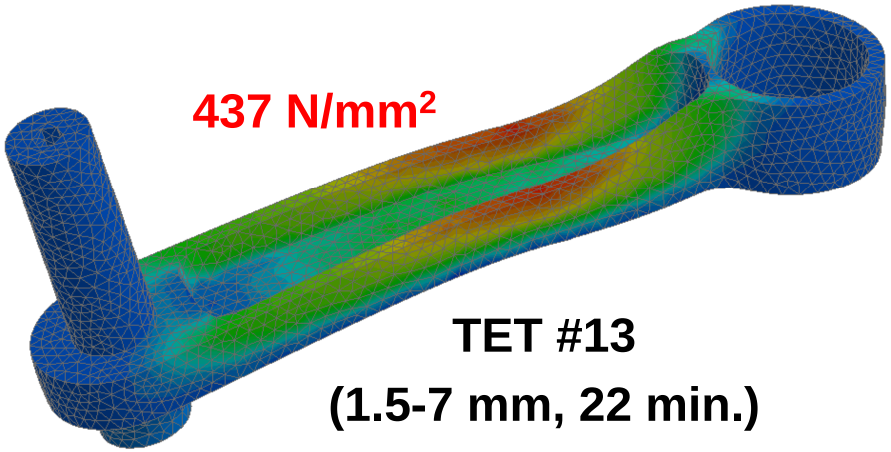


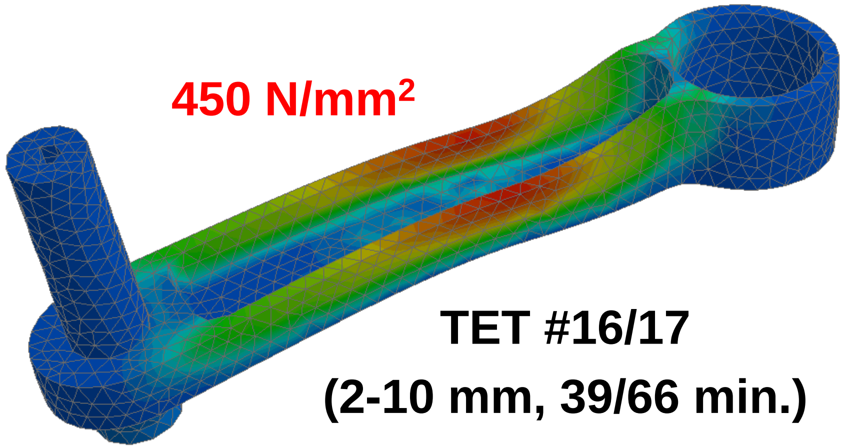
Structural component

Maximum principal stress (-50.0 – 450.0 N/mm²)





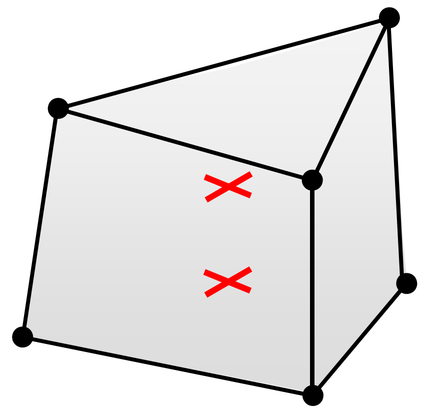




Pentahedra elements in LS-DYNA

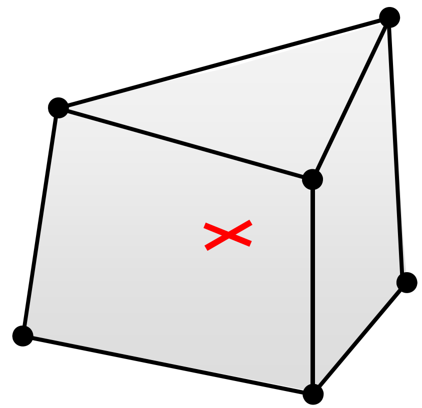
ELFORM = 15

- 2 point selective reduced integration
- needs hourglass stabilization for twist mode (recent improvement → next official versions)
- often used as transition element (ESORT=1)



ELFORM = 115 (new in next official versions)

- 1 point reduced integration
- needs hourglass stabilization (analogue to hexahedron element type 1 with Flanagan-Belytschko hourglass formulation)

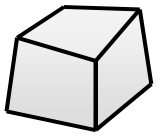


Time step control

- **critical time step:** $\Delta t_e = \dfrac{L_e}{Q + (Q^2 + c^2)^{1/2}} \approx \dfrac{L_e}{c}$

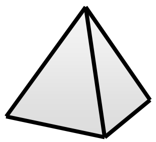
- **adiabatic sound speed:** $c = \sqrt{\dfrac{E(1-\nu)}{(1+\nu)(1-2\nu)\rho}} = \sqrt{\dfrac{K + \frac{4}{3}G}{\rho}}$

- **characteristic element length**



ELFORM = 1 / 2 / 3 / -1 / -2: $L_e = V/A_{max}$

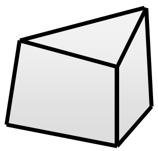
ELFORM = 4: $L_e = 0.85\, h_{min}$



ELFORM = 10 / 13: $L_e = h_{min}$

ELFORM = 16: $L_e = 0.3889\, h_{min}$

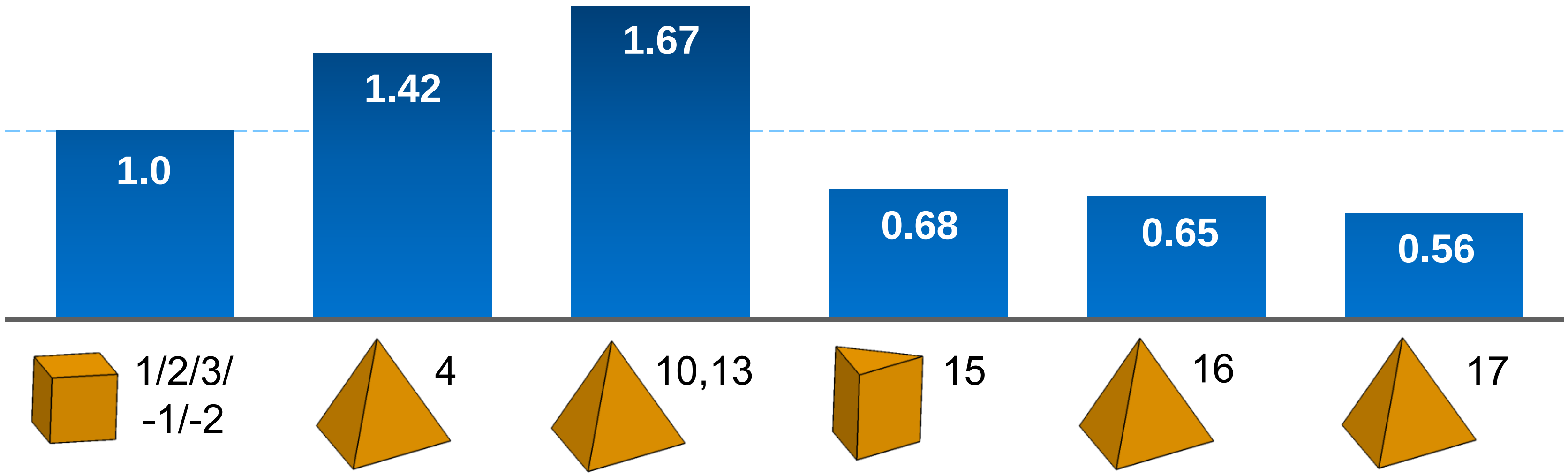
ELFORM = 17: $L_e = V/A_{max}$



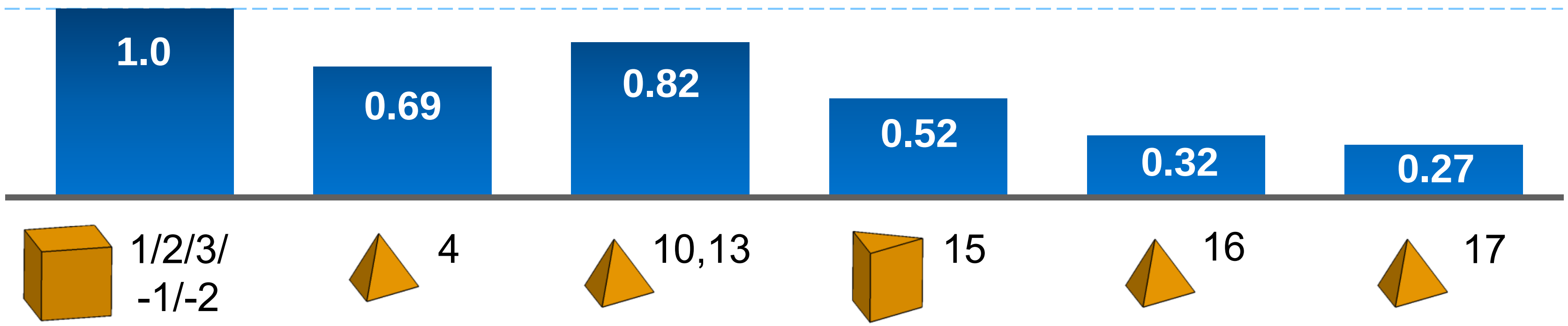
ELFORM = 15: $L_e = 1/\sqrt{B_{ij}B_{ij}}$

Time step control

- **Example 1: Time step for solid elements with same volume**



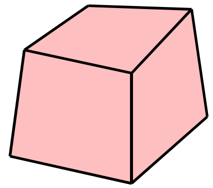
- **Example 2: Time step for solid elements with same edge length**



Conclusions & Remarks

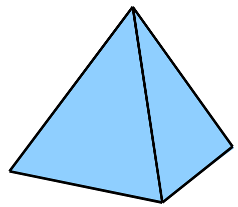
- Always set ESORT = 1 on *CONTROL_SOLID

- Use hexahedron elements if possible (regular solid bodies)
  - ELFORM = 1 with IHQ = 6 or ELFORM = 2, 3
  - ELFORM = -1 or -2 for „flat“ hexas



- For complex solid structures, use tetrahedrons type 4, 13, 16, or 17
  - ELFORM = 16/17 are the most accurate tets, but not suited for large strains
  - ELFORM = 13 needs finer mesh, well suited even for large strains
    (check if your material is supported)

- For metals or plastics (moderate strains), use tet type 4, 13, 16, or 17



- For rubber materials (incompressible, large strains) use tet type 13

- For bulk forming problems, use ELFORM = 13 and r-adaptivity

- Pentahedrons 15/115 should only be used as transition elements