*DEFINE_PRESSURE_TUBE

A pressure tube sensor for pedestrian crash simulation

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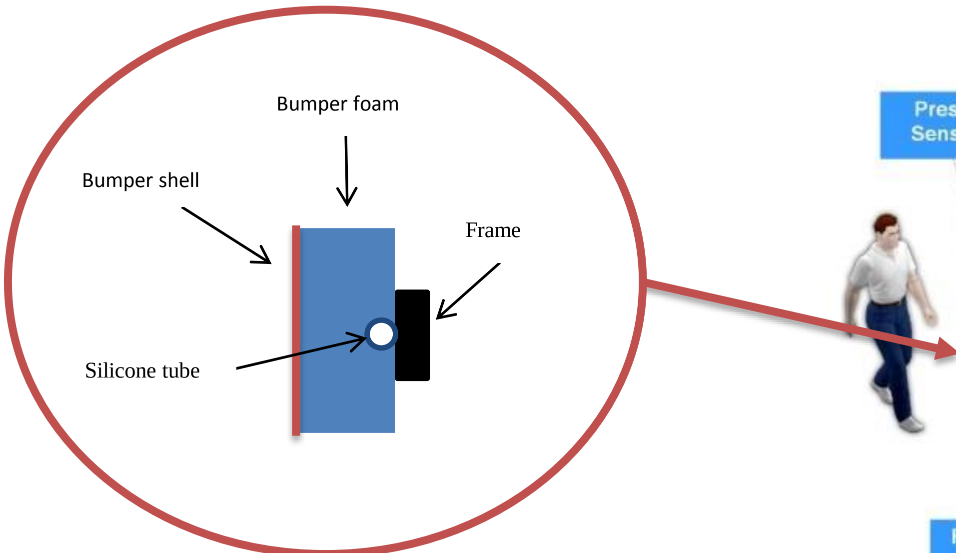
Introduction

- Motivated by the need to model pressure-based sensor systems designed to detect collisions with pedestrians
- Pressure-based sensor system consists of
  - Air filled silicone tube embedded in front bumper foam
  - Pressure sensors located at tube ends to detect collision and activate protective systems



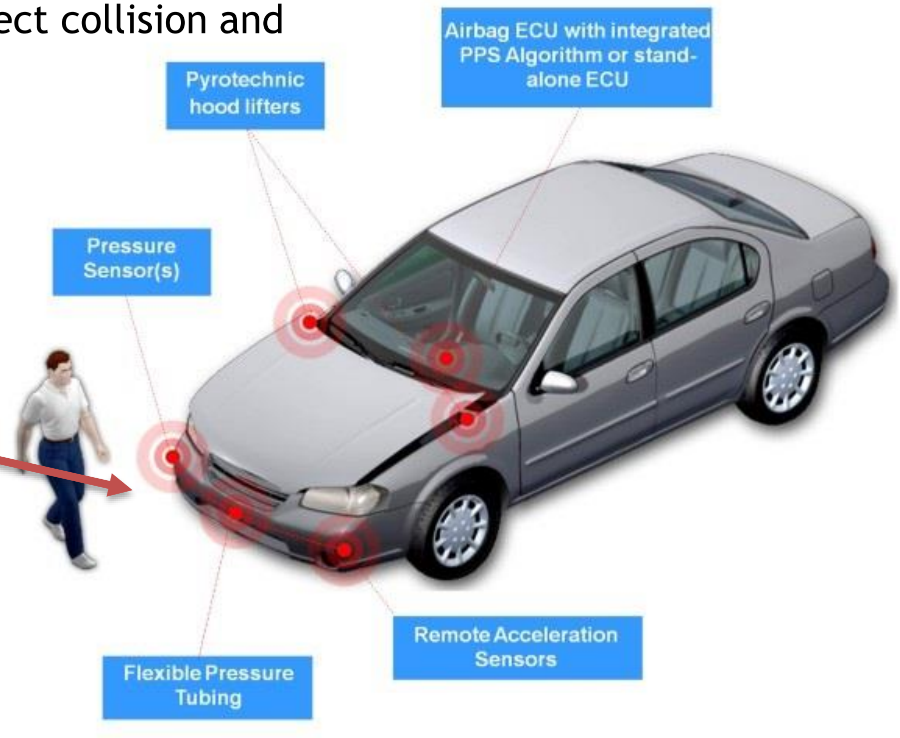


Photo: ZF TRW.

Introduction, contd.

- Two new keywords:
  - *DEFINE_PRESSURE_TUBE
  - *DATABASE_PRTUBE
- Still in experimental/development stage
  - Currently undergoing full-scale testing
- Models pressure waves in a (closed) gas filled tube
  - Uses tubular beam elements
  - Approximation of 1D compressible Euler equations
  - Uses variation in tube cross section area over time
  - Uncoupled from tube deformation
- Output through “binout” or “prtube” ascii-file

Keyword input/output

- *DEFINE_PRESSURE_TUBE

Card	1	2	3	4	5	6	7	8
Variable	PID	WS	PR					
Type	I	F	F					
Default	0	0.0	0.0					

  - PID: Tube consists of all beam elements in this part. Must be a unique PID for each card.
  - WS: Wave propagation speed
  - PR: Initial gas pressure

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- *DATABASE_PRTUBE
  - Cross section area
  - Pressure
  - Velocity
  - Density (currently not independent variable)

Keyword input/output, contd.

- *SECTION_BEAM
  - Only ELFORM=1,4,5,11 with CST=1, i.e. hollow circular beams
  - Initial tube area set to inner beam area
- Geometric constraints
  - Each set of joint beam elements in a part will model a separate closed tube
  - Different parts used in *DEFINE_PRESSURE_TUBE cards may not share beam nodes
  - No junctions allowed
- MPP
  - All elements in a part referenced by *DEFINE_PRESSURE_TUBE will be on same processor
  - Recommended to only have beam elements in such parts

Euler equations

- 1D compressible Euler equations
  - Inviscid ideal gas in chemical and thermal equilibrium
  - Fluid density $\rho(x,t)$, velocity $u(x,t)$, energy per unit volume $E(x,t)$, and pressure $p(x,t)$
- Conservation of mass: $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$
- Conservation of momentum: $\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2 + p) = 0$
- Conservation of energy: $\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\big(u(E+p)\big) = 0$

  where total energy = kinetic + internal: $\mathrm{E} = \frac{1}{2}\rho u^2 + \rho e$
- Equation of state: $e = e(p, \rho)$
  - Ideal gas: $p = R\rho T$ and $e = c_v T$ gives the EOS $\mathrm{e} = \frac{\mathrm{c_v p}}{R\rho}$
- Allows non-smooth solutions, e.g. shocks from supersonic flow

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- Conservation of momentum: $\frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u^2 A + pA) = p\frac{\partial A}{\partial x}$
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- Allows non-smooth solutions, e.g. shocks from supersonic flow
- Varying area $A(x,t)$

Acoustic approximation

- Euler equations with varying area

$$\frac{\partial(\rho A)}{\partial t} + \frac{\partial}{\partial x}(\rho u A) = 0,$$
$$\frac{\partial}{\partial t}(\rho u A) + \frac{\partial}{\partial x}(\rho u^2 A + pA) = p\frac{\partial A}{\partial x}$$

- Variation around mean

$$\rho(x,t) = \rho_0 + \delta\rho(x,t),$$
$$p(x,t) = p_0 + \delta p(x,t),$$
$$u(x,t) = u_0 + \delta u(x,t).$$

- Linearization

$$\frac{1}{c_0^2}\frac{\partial(A\delta p)}{\partial t} + \rho_0\frac{\partial(A\delta u)}{\partial x} = -\rho_0\frac{\partial A}{\partial t},$$
$$\rho_0\frac{\partial(A\delta u)}{\partial t} + \frac{\partial(A\delta p)}{\partial x} = \delta p\frac{\partial A}{\partial x}$$

- Does not allow shock formation
- Constant area gives wave equation: $\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} = 0$
- Constant area in time gives Webster's equation: $\frac{1}{c_0^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial}{\partial x}\big(ln(A)\big)\frac{\partial p}{\partial x} - \frac{\partial^2 p}{\partial x^2} = 0$

Numerics

- Continuous Galerkin on system

$$\frac{\partial p}{\partial t} + \frac{p_0}{A}\frac{\partial y}{\partial x} + \frac{\partial \ln A}{\partial t} p = 0,$$
$$\frac{\partial y}{\partial t} + \frac{c_0^2}{p_0} A \frac{\partial p}{\partial x} = 0$$

  where $y = Au$.
- Semi-discretization

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M} \end{bmatrix} \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{y} \end{bmatrix} + \begin{bmatrix} \boldsymbol{M}_A(t) & p_0 \boldsymbol{K}_A(t) \\ \frac{c_0^2}{p_0} \boldsymbol{K}_B(t) & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{p} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$$

- Strictly hyperbolic i.e. distinct real eigenvalues

$$\lambda_{1,2}(t) = \frac{\Delta x}{2}\frac{\partial \ln A}{\partial t} \pm \sqrt{\left(\frac{\Delta x}{2}\frac{\partial \ln A}{\partial t}\right)^2 + c_0^2},$$

- CFL condition

$$\Delta t(t) < \frac{\Delta x}{\max(\lambda_1(t), \lambda_2(t))} \leq \frac{\Delta x}{\Delta x \left|\frac{\partial \ln A}{\partial t}\right| + c_0}$$

Numerics, contd.

- Continuous Galerkin on system (with artificial diffusion)

$$\frac{\partial p}{\partial t} + \frac{p_0}{A}\frac{\partial y}{\partial x} + \frac{\partial \ln A}{\partial t} p = \epsilon \frac{\partial^2 p}{\partial x^2},$$
$$\frac{\partial y}{\partial t} + \frac{c_0^2}{p_0} A \frac{\partial p}{\partial x} = \epsilon \frac{\partial^2 y}{\partial x^2}$$

  where $y = Au$.

- Semi-discretization

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$$\lambda_{1,2}(t) = \frac{\epsilon}{\Delta \mathrm{x}} + \frac{\Delta x}{2}\frac{\partial \ln A}{\partial t} \pm \sqrt{\left(\frac{\epsilon}{\Delta \mathrm{x}} + \frac{\Delta x}{2}\frac{\partial \ln A}{\partial t}\right)^2 + c_0^2},$$

- CFL condition

$$\Delta t(t) < \frac{\Delta x}{\max(\lambda_1(t), \lambda_2(t))} \leq \frac{\Delta x}{\frac{\epsilon}{\Delta \mathrm{x}} + \Delta x \left|\frac{\partial \ln A}{\partial t}\right| + c_0}$$

Numerics, contd.

- Heun's method (RK2): $y' = f(x,y) \implies$

$$\tilde{y}_{n+1} = y_n + \Delta t f(x_n, y_n),$$
$$y_{n+1} = y_n + \frac{\Delta t}{2}\left(f(x_n, y_n) + f(x_{n+1}, \tilde{y}_{n+1})\right),$$

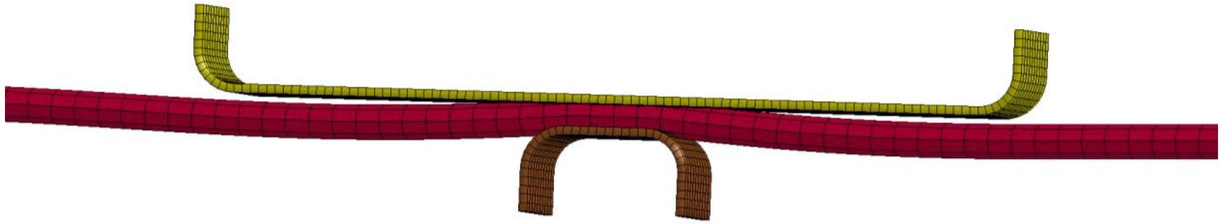
- CFL-condition fulfilled by substepping
  - Only performed for tube elements – does not affect global step
  - Substep changes in time depending on $\dfrac{\partial \ln A}{\partial t}$
- Tube algorithm uses initial beam element length only

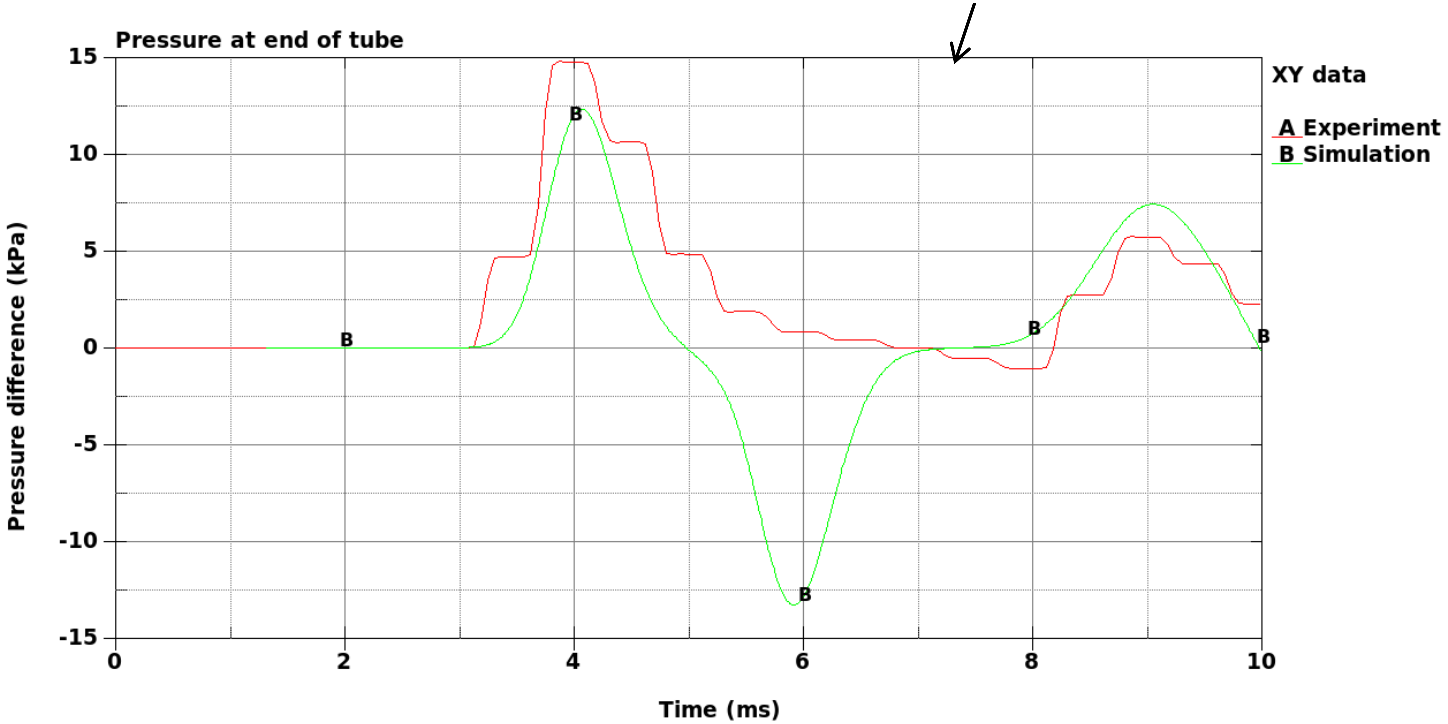
Numerical example

- Silicone tube of length 1.7m, inner diameter 4mm and outer diameter 8mm

```
*DEFINE_PRESSURE_TUBE
$#      pid      sndspd init_prsr
          6        340.      1.e-4
*DATABASE_PRTUBE
0.01,1
*PART
Pressure tube
$#      pid     secid         mid     eosid      hgid      grav    adpopt      tmid
          6         6           3         0         0         0         0         0
*SECTION_BEAM
$#    secid   elform        shrf   qr/irid         cst     scoor       nsm
          6         1         1.0         2           1       0.0       0.0
$#      ts1       ts2         tt1       tt2       nsloc     ntloc
        8.0       8.0         4.0       4.0         0.0       0.0
*MAT_ELASTIC_TITLE
Silicone
$#      mid        ro           e        pr          da        db  not used
          32.30000E-6         1.0       0.2         0.0       0.0         0
```

Numerical example, contd.





Summary

- Pros
  - Very simple to use
  - Extremely efficient (3D simulations with CPM/ALE/CESE are significantly slower without any success so far)

- Cons
  - Pressure solely dependent on area (area changes needs to be modeled accurately)
  - Inaccurate mechanical response in beam thickness direction (contact stiffness only)
  - 1D acoustic approximation may not be sufficient

- Enhancements
  - Include shell geometry around beam or a phenomenological model for accurate area calculation and mechanical response
  - Solve full 1D Euler equations with e.g. Discontinuous Galerkin
  - Other boundary conditions

Thank you!

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