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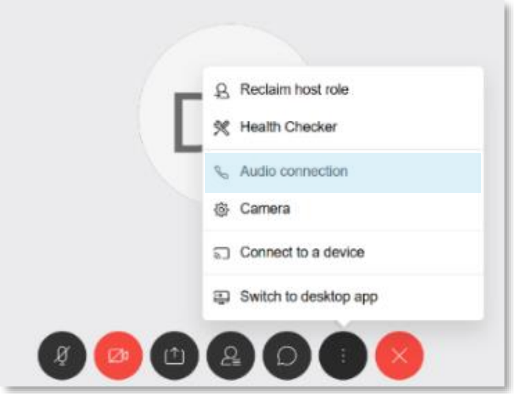

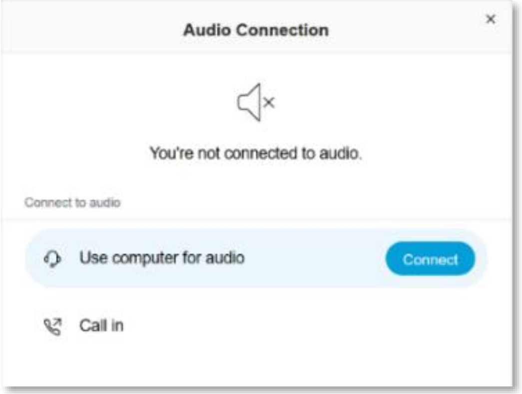

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Beyond FEM:
The Method of Smoothed Hydrodynamics (SPH)

Maik Schenke

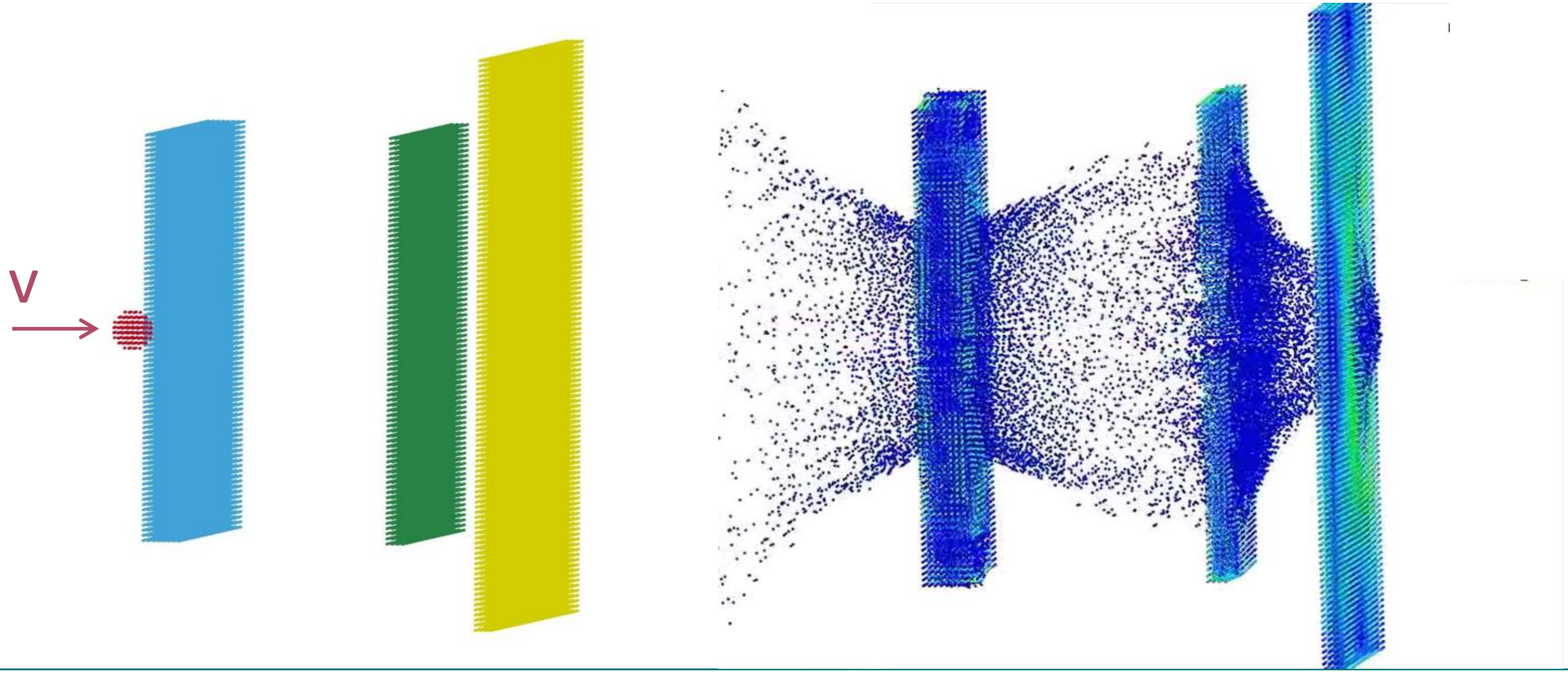
17 March 2021

Outline

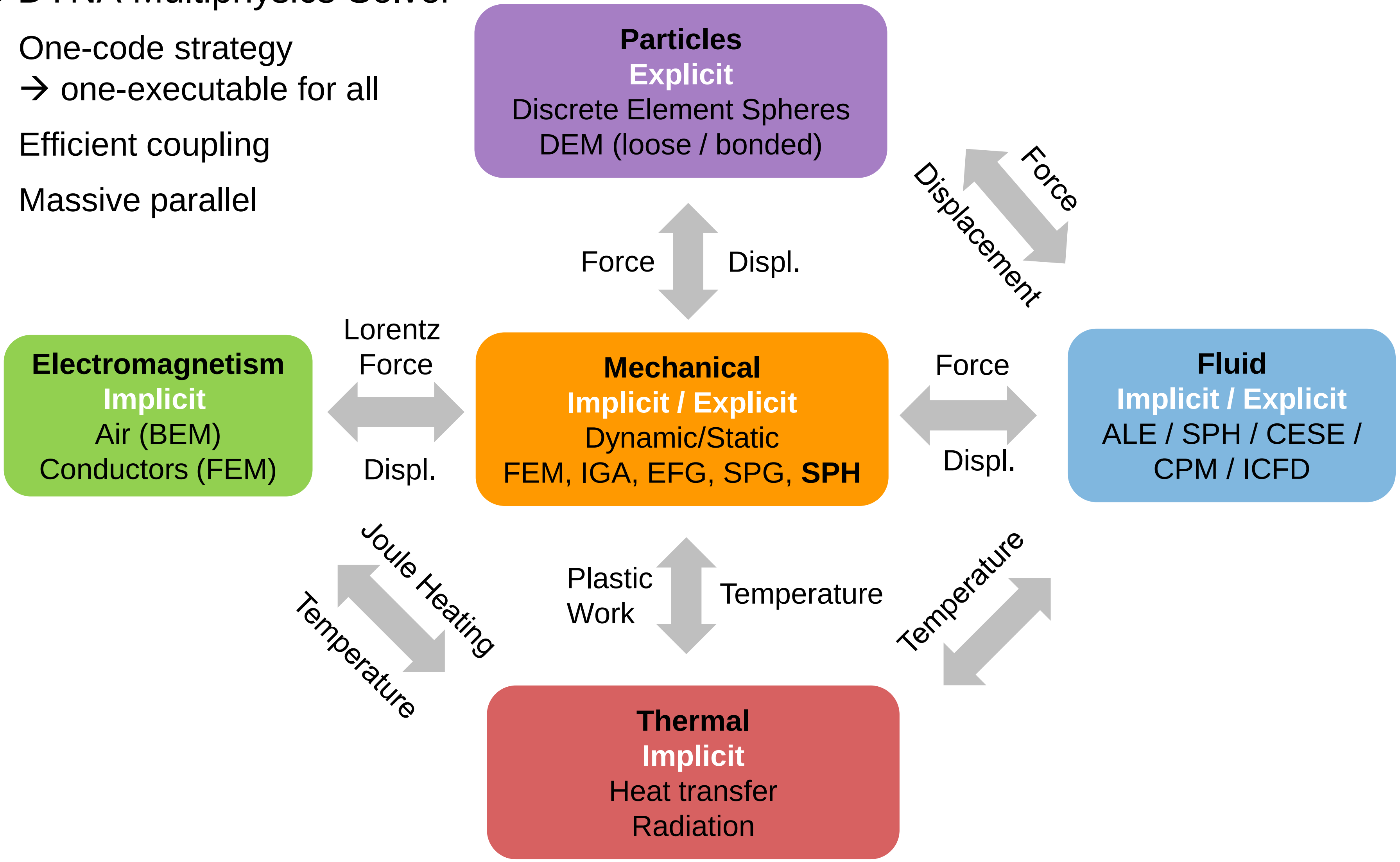
- Introduction
- Smoothed Particle Hydrodynamics (SPH)
- SPH with LS-DYNA
- Recent SPH Updatés
- Summary

Introduction

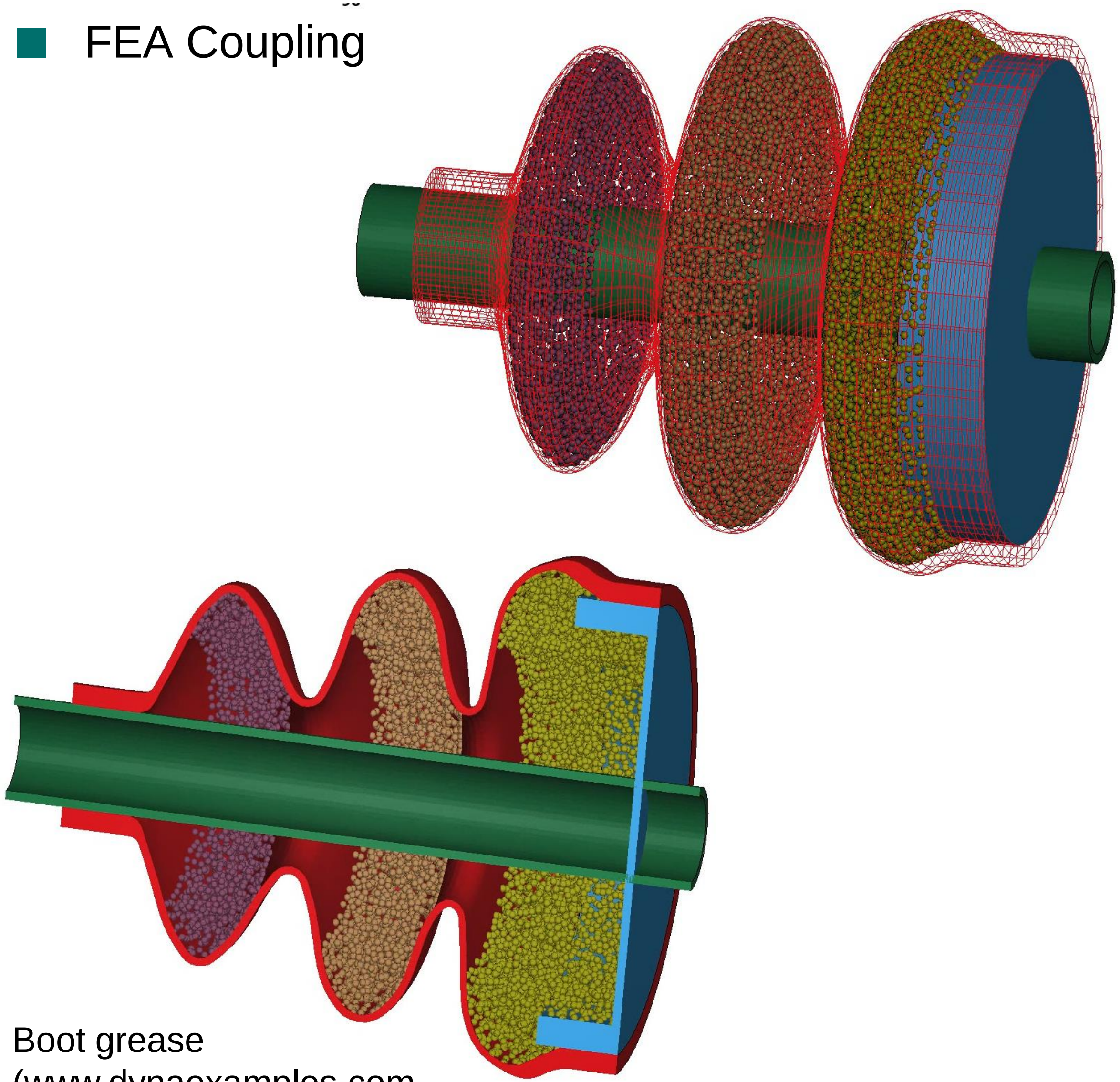
- Meshfree method exploiting so-called to Kernel functions to a assemble approximations
- Suitable for
  - Large deformations
  - Material separation and mixing
  - Intrinsic mimicking of free surfaces, e. g. water surface



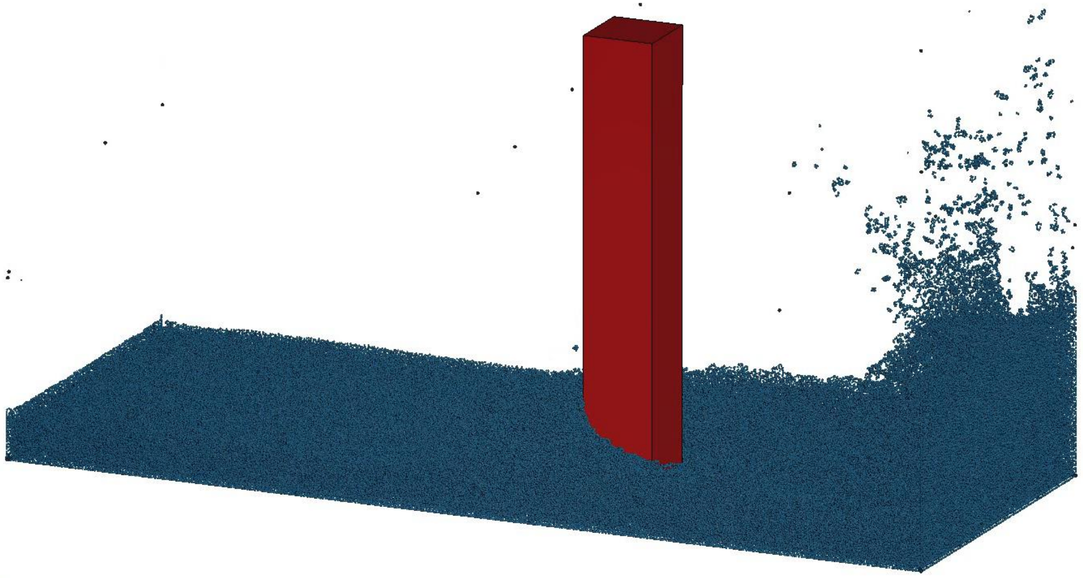
- LS-DYNA Multiphysics Solver
  - One-code strategy
    → one-executable for all
  - Efficient coupling
  - Massive parallel



- FEA Coupling

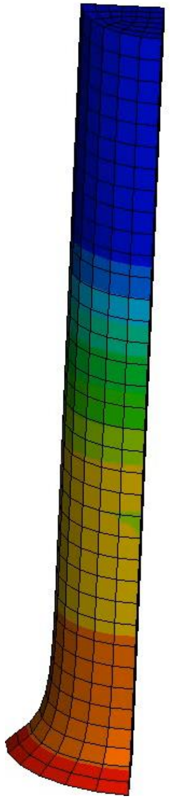


Boot grease
(www.dynaexamples.com

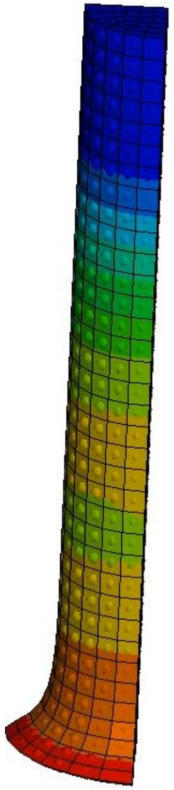


Dam break (www.dynaexamples.com)

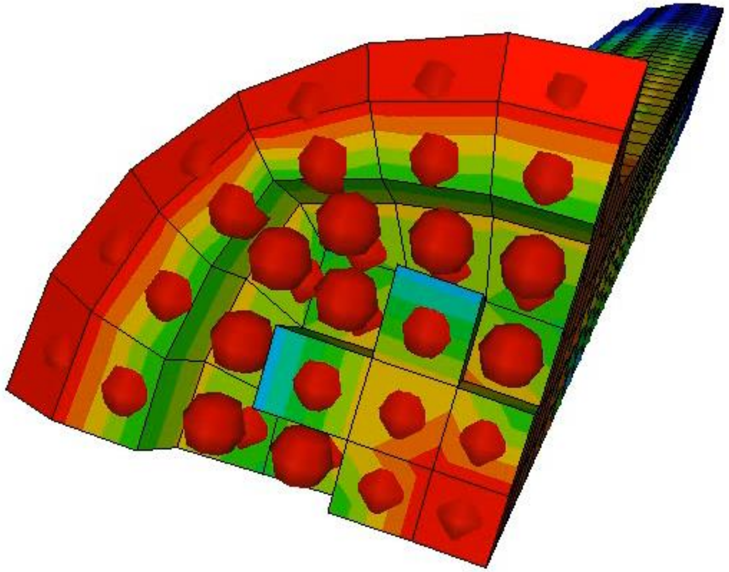
- FEA Coupling - Automatic Finite-Element (FE) to SPH Conversion
  - Suitable for failure analysis → failed FE elements will act as debris in the simulation
  - Taylor-beam example



Pure FEA

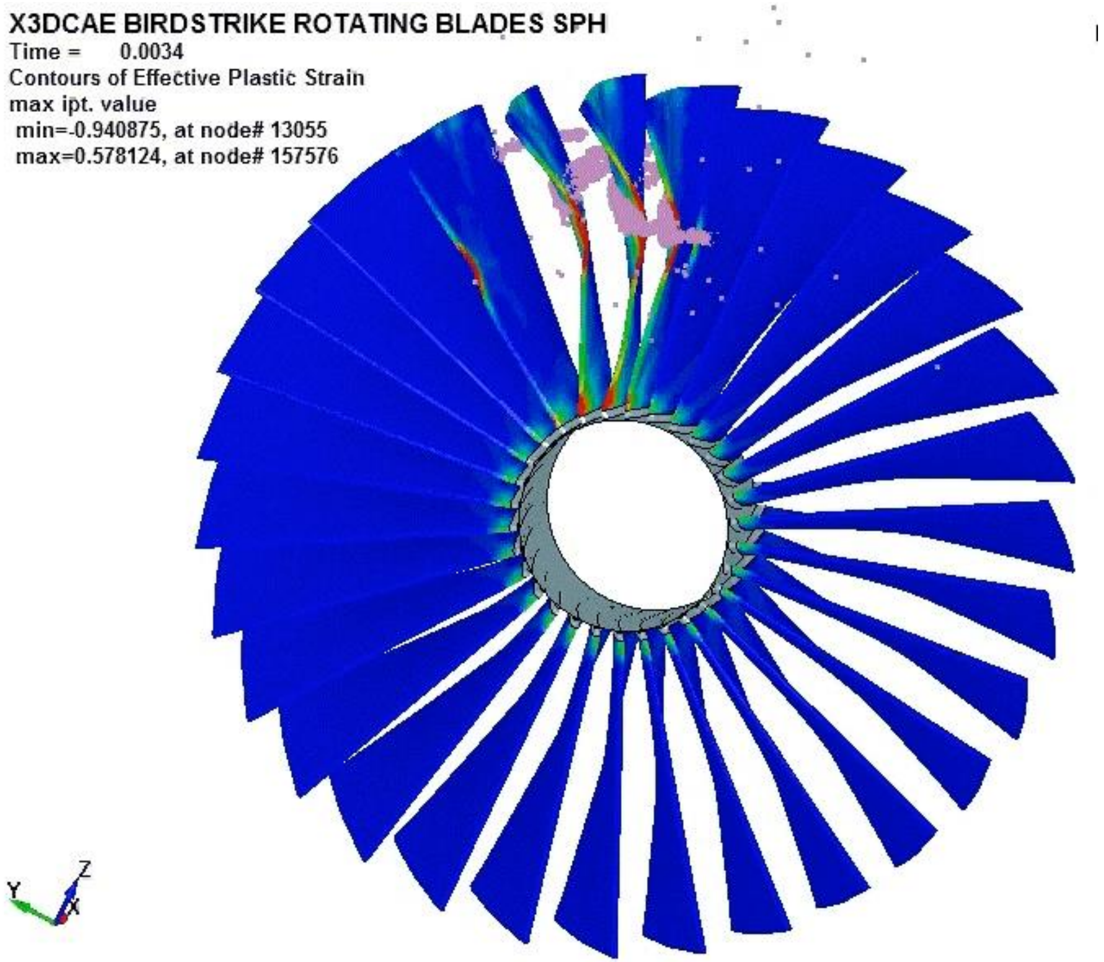


FEA-SPH coupling

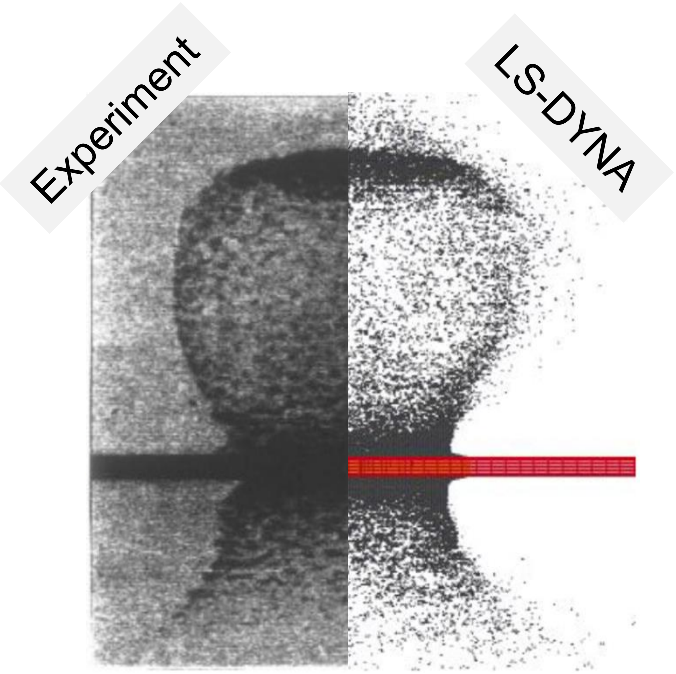


FEA-SPH coupling
(bottom view)

- Bird strike

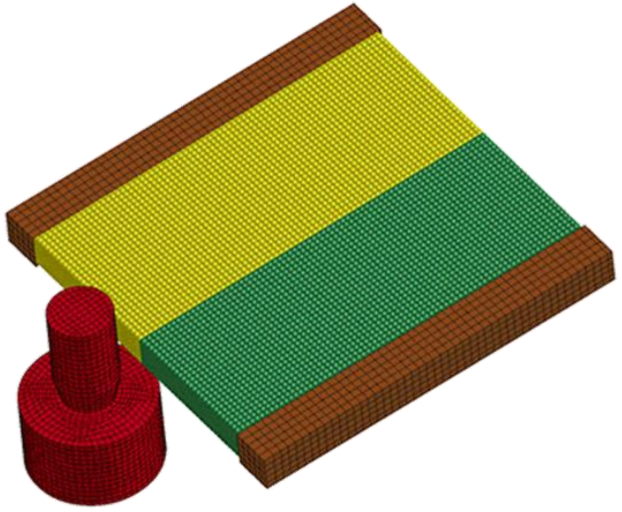


- High-velocity impacts

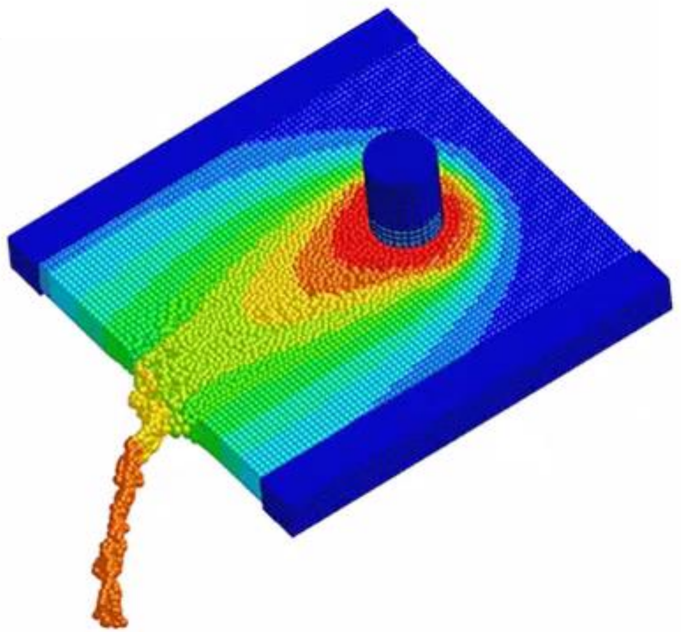


- Projectile
  - Material: 304 L Steel
  - Velocity: 5530 m/s
  - Geometry: sphere, r = 5 mm
- Target
  - Material: 6061-T651 Al
  - Thickness: 2.85 mm

- Friction-stir welding (FSW)
  - Double sided FSW @ 600 RPM, 1200 mm/min
  - Plastic work and friction energy to heat



Material mixing



Temperature evolution

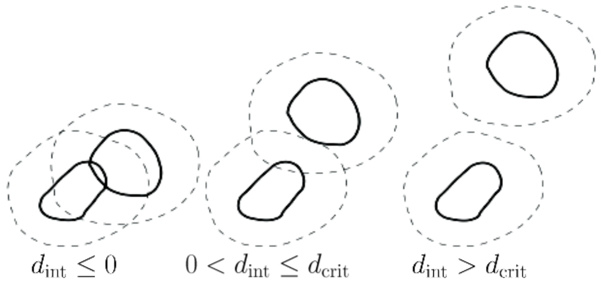
Smoothed Particle Hydrodynamics (SPH)

- **Numerical method** are needed **to solve space and time-dependent partial differential equations**, e. g. the momentum balance in structural mechanics

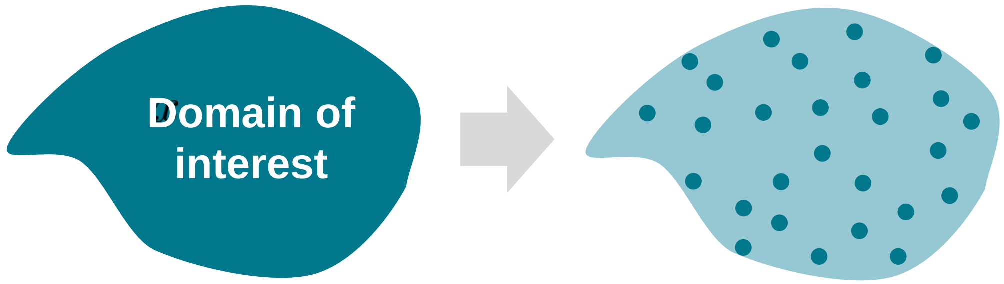
$$\rho \ddot{\mathbf{u}} = \operatorname{div} \mathbf{T}(\mathbf{u})$$

- In general, numerical methods are **based on space- and time-discretization schemes**
  - Space discretization accounts for spatial changes
  - Time discretisation accounts for temporal changes
- Spatial discretisation methods, e. g.,
  - Finite-Element Method (FEM)
  - **Smoothed Particle Hydrodynamics (SPH)**
  - Element-Free Galerkin (EFG) Method
- Time discretization methods, **explicit or implicit**, e. g.
  - Central Difference Method
  - Newmark Scheme

- Finite-Element Analysis (FEA) fails at large element distortions
  → Use of **particle-based (mesh-free) spatial discretization methods for continua**
  - **Smoothed Particle Hydrodynamics (SPH)**
  - Element-Free Galerkin (EFG) Method
  - Smooth Petrov Galerkin (SPG) Method
- SPH is a **continuum- and particle-based spatial discretization method**
- Discrete particle method (DEM) versus continuum-based particle method (CBPM)



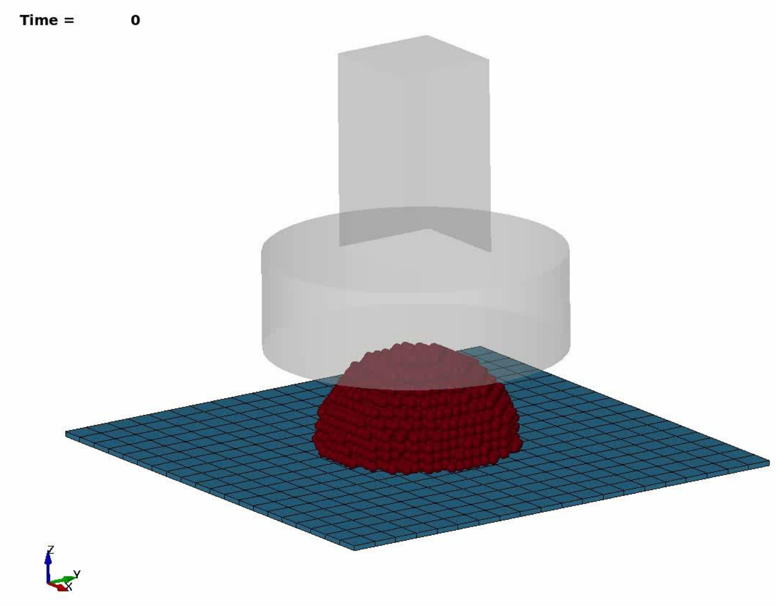
In DEM, discrete particles interact via contact laws

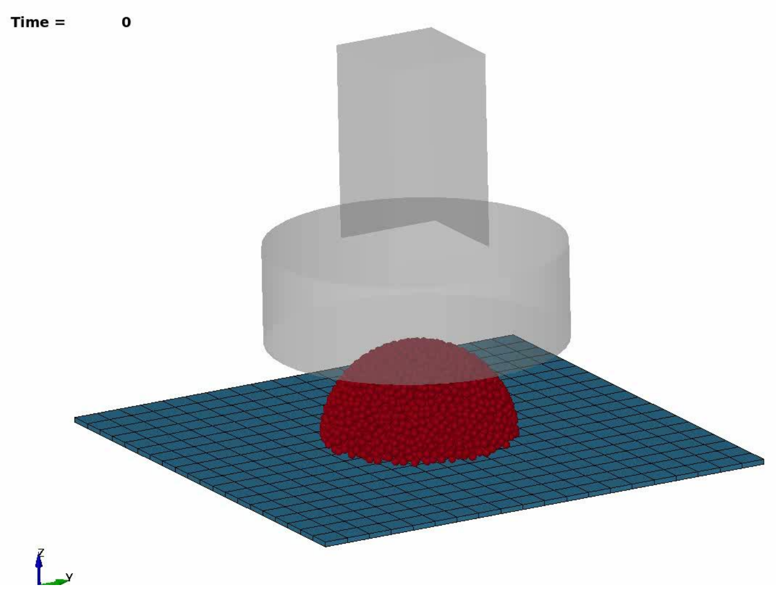


In CBPM, particles serve as computational points for field variables inside of a continuum

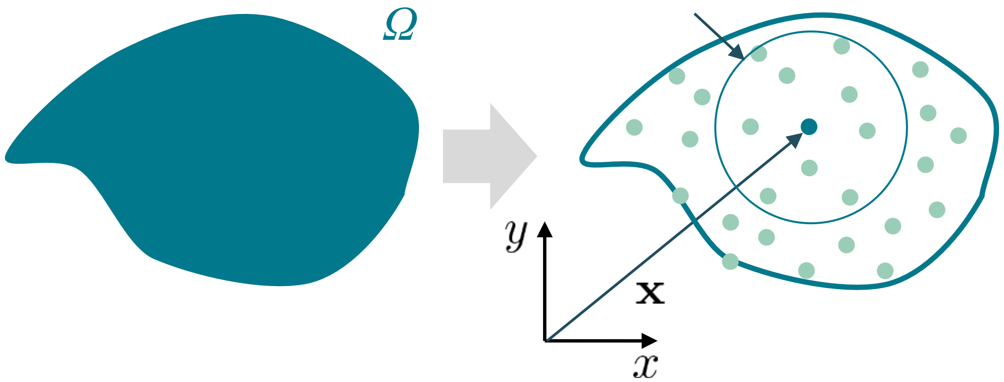
- Smooth Particle Hydrodynamics
  - Each nodal point represents a chunk of material
  - Low order approximation
  - Collocation method
  - Fluid / EOS (Diffusive)

- Discrete Element Method
  - Each particle may represent an individual particle of various sizes or a chunk of material both interacting through bonds and contacts
  - Particle dynamics
  - Granular material (Discrete)





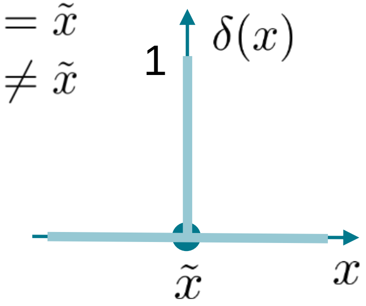
- **Basic idea**
  - Replacement of the continuum by a set of particles
  - Construction of shape functions without a mesh but based on kernel functions associated with the individual particles instead [*Lucy* 1977, *Gingold & Monaghan* 1977, *Liu* 2003]



- **Approximation of the field variables via the Kernel function (also smoothing or interpolating kernel)**
  - Dirac delta function $\delta(x-\tilde{x})$

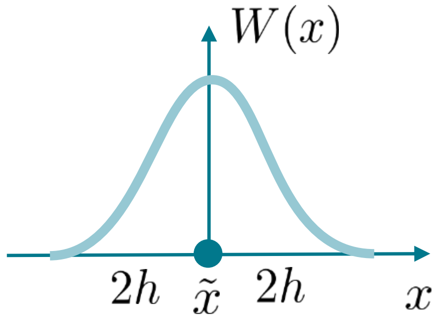
$$u(x) = \int_\Omega u(\tilde{x})\,\delta(x-\tilde{x})\,\mathrm{d}\tilde{x}$$

where
$$\delta(x-\tilde{x}) = 1 \quad \forall\; x = \tilde{x}$$
$$\delta(x-\tilde{x}) = 0 \quad \forall\; x \neq \tilde{x}$$
$$\int_\Omega \delta(x-\tilde{x})\,\mathrm{d}\tilde{x} = 1$$



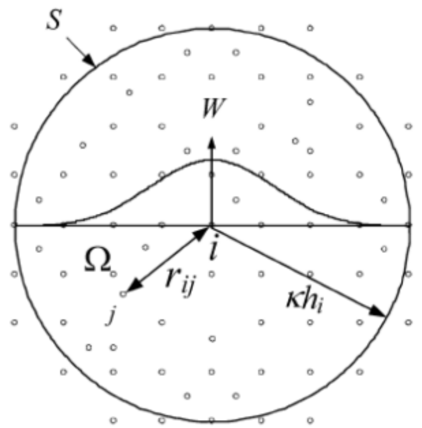
  - Continuous and differentiable kernel $W(x-\tilde{x},\,h)$

$$u^h(x) = \int_\Omega u(\tilde{x})\,W(x-\tilde{x},\,h)\,\mathrm{d}\tilde{x} \quad \text{where} \quad \int_\Omega W(x-\tilde{x},h)\,\mathrm{d}\tilde{x} = 1$$



- Approximation of the field variables, e. g. displacements

$$u^h_\alpha(\mathbf{x}_i) = \sum_j \frac{m_j}{\rho_j} u_\alpha(\mathbf{x}_j)\, W_{ij} \quad \text{with} \quad \mathbf{u}^h = u^h_\alpha\, \mathbf{e}_\alpha \quad \forall\, \alpha = 1, 2, 3$$



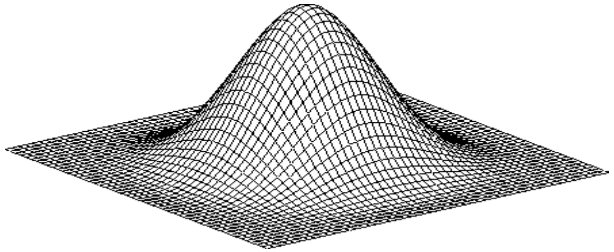
with $W_{ij} = W_i(r_{ij}, h_i) = \frac{1}{h_i^3}\, \Theta\!\left(\frac{r_{ij}}{h_i}\right)$

$$\begin{cases} r_{ij} & = & |\mathbf{x}_i - \mathbf{x}_j| \\ 2h_i & : & \text{smoothing length} \\ m_i & : & \text{particle mass} \\ \rho_i & : & \text{density} \end{cases}$$

- Approximation of the spatial derivatives (*CONTROL_SPH), e. g. gradient of displacements

$$\operatorname{grad} \mathbf{u}^h(\mathbf{x}_i) = \frac{\mathrm{d}u^h_\alpha(\mathbf{x}_i)}{\mathrm{d}x_\beta} = \sum_j \frac{m_j}{\rho_j} \left[u_\alpha(\mathbf{x}_j)\, W_{ij,\beta} - u_\alpha(\mathbf{x}_i)\, W_{ji,\beta}\right]$$

with $W_{ij,\beta}(r_{ij}, h_i) = \frac{1}{h_i^4}\, \frac{\mathrm{d}}{\mathrm{d}x_\beta} \Theta\!\left(\frac{r_{ij}}{h_i}\right)$



kernel function θ

- Discretisation of the strong forms (collocation method)
  - Kernel discretisation

$$u^h(x) = \int_\Omega u(\tilde{x})\, W(x - \tilde{x},\, h)\, \mathrm{d}\tilde{x}$$

volume of the particle i

$$u^h(x) \approx \sum_{i=1}^{N} \frac{m}{\rho_i} u(x_i)\, W(x - x_i,\, h)$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho\, \mathrm{div}\, \mathbf{v} \quad \Rightarrow \quad \frac{\mathrm{d}\rho_i}{\mathrm{d}t} = -\rho_i \sum_j \frac{m_j}{\rho_j} \left[v_\alpha(\mathbf{x}_j) - v_\alpha(\mathbf{x}_i) \right] W_{ij,\beta}$$

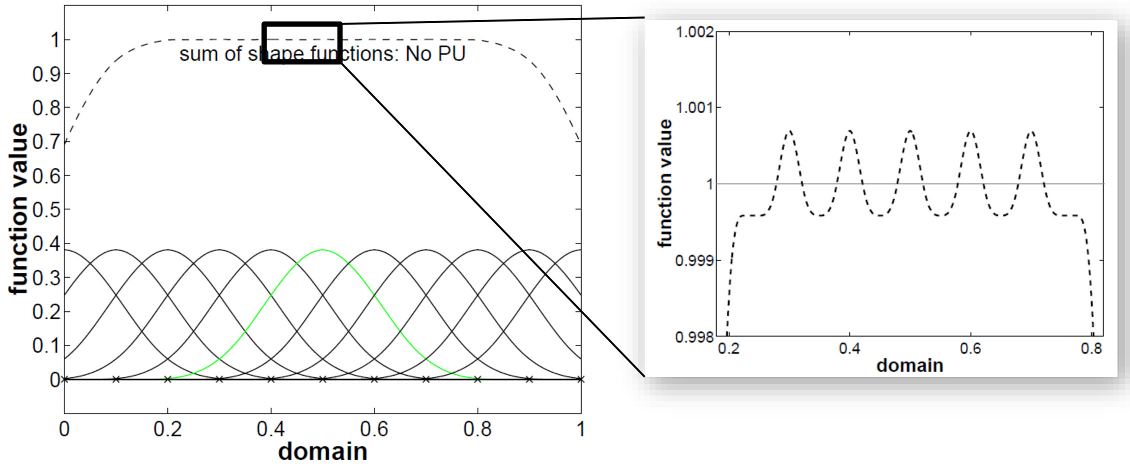
$$\text{with} \begin{cases} \mathbf{v} = v_\alpha\, \mathbf{e}_\alpha \quad \forall\ \alpha = 1,2,3 \\ \rho_i = \rho(\mathbf{x}_i) = \sum_j m_j\, W_{ij} \end{cases}$$

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{1}{\rho}\, \mathrm{div}\, \mathbf{T} \quad \Rightarrow \quad \frac{\mathrm{d}v_\alpha(\mathbf{x}_i)}{\mathrm{d}t} = \sum_j m_j \left[\frac{t_{\alpha\beta}(\mathbf{x}_j)}{\rho_j^2} W_{ji,\beta} - \frac{t_{\alpha\beta}(\mathbf{x}_i)}{\rho_i^2} W_{ij,\beta} \right]$$

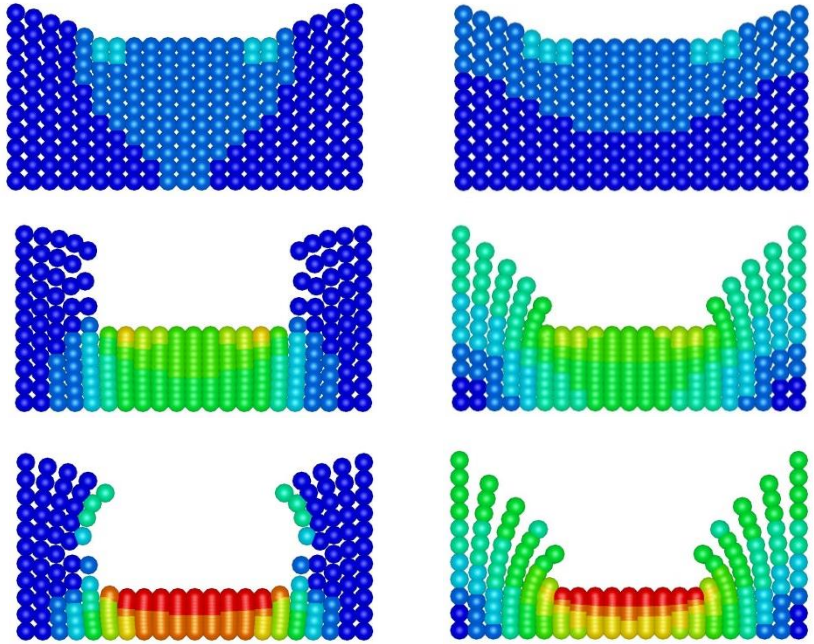
$$\text{with} \quad \mathbf{T} = t_{\alpha\beta}\, \mathbf{e}_\alpha \otimes \mathbf{e}_\beta \quad \forall\ \alpha, \beta = 1,2,3$$

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{1}{\rho}\, \mathbf{T} \cdot \mathrm{grad}\, \mathbf{v} \quad \Rightarrow \quad \frac{\mathrm{d}\varepsilon_i}{\mathrm{d}t} = \frac{t_{\alpha\beta}(\mathbf{x}_i)}{\rho_i^2} \sum_j m_j \left[(v_\alpha(\mathbf{x}_i) - v_\alpha(\mathbf{x}_j) \right] W_{ij,\beta}$$

- Drawbacks of SPH method
  - Treatment of essential boundary conditions is difficult
  - Lack of consistency, i. e. shape functions do not form a partition of unity (PU)
    → renormelization of kernel function necessary
  - Tensile instability



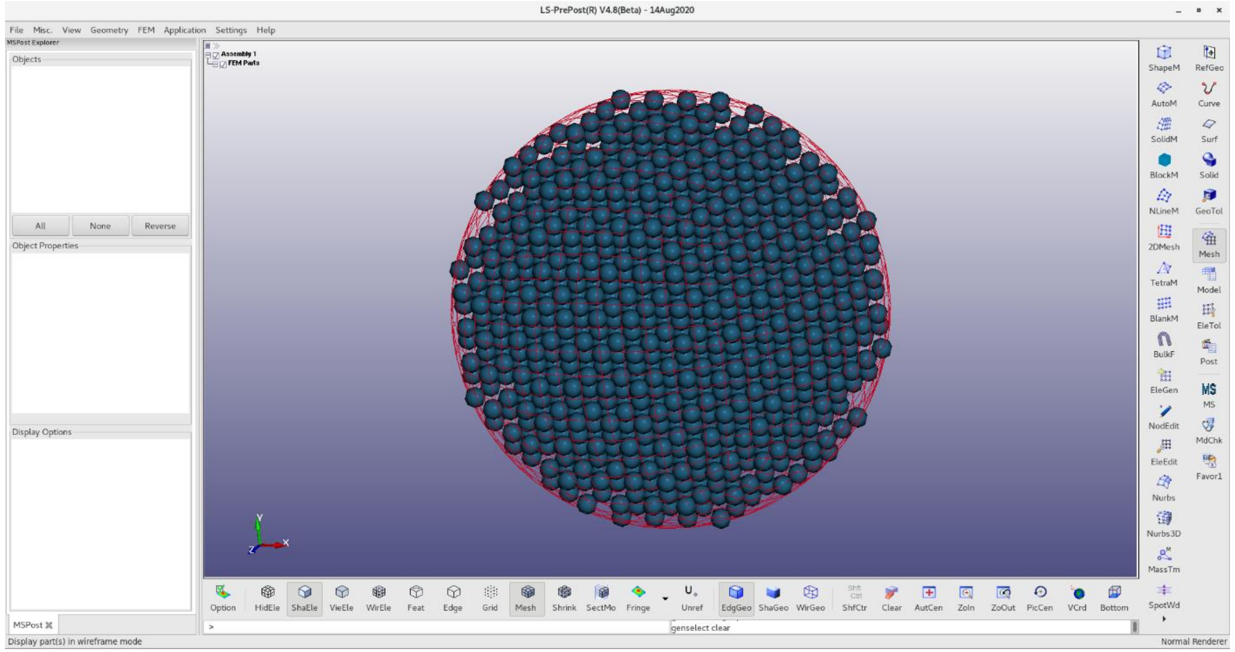
Boundary effects and lack of consistency [*Fries* et *al.* 2004]



Punch test with (left) and without **tensile instability** (right)

SPH with LS-DYNA

- SPH particle generation in LS-PrePost
  - Either by converting FE nodes to SPH particles or by filling a hull of shell elements with particles
  - FEM → Element and Mesh → SPH generation
  - Choose “Method”, e. g. Shell Volume, and select the volume
  - Set the particle pitch in „Num Particles Definition“ via PitX, PitY and PitZ
  - Click „Set Params“ button
  - Click „Create“



SPH keyword

```
*ELEMENT_SPH
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$#      nid       pid      mass
          1         1     2.e-7
          2         1     2.e-7
          3         1     2.e-7
          …
```

nid	Node ID and Element ID are the same for the SPH option
pid	Part ID to which this node (element) belongs to
mass	GT.0: mass of the particle LT.0: volume of the particle

SPH Control (performance- and stability-affecting parameters)

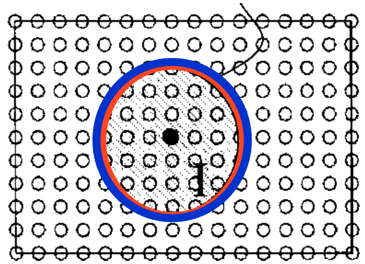
```
*CONTROL_SPH
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$#    ncbs     boxid        dt      idim    memory      form     start      maxy
         1        42
$#    cont     deriv       ini     ishow     ierod     icont     iavis     isymp
       1.3       1.3       1.3                             3         2
```

ncbs	Number of time steps between particle sorting
boxid	SPH approximations are computed inside a specified BOX. When a particle has gone outside the BOX, it is deactivated. This will save computational time by eliminating particles that no longer interact with the structure.
dt	Death time. Determines when the SPH calculations are stopped.
start	start time for particle approximation. Particle approximations will be computed when time of the analysis has reached the value defined in start.
maxv	Particles with a velocity greater than maxv are deactivated. A negative maxv will turn off the velocity checking.

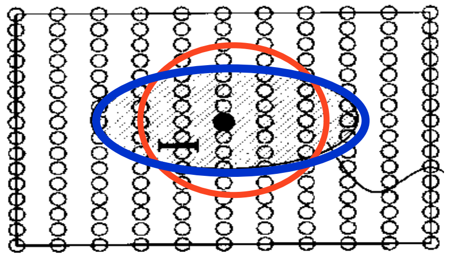
SPH Control (accuracy-related parameters)

```
*CONTROL_SPH
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$#    ncbs     boxid        dt      idim    memory      form     start      maxy
         1        42                                      12
$#    cont     deriv       ini     ishow     ierod     icont     iavis     isymp
```

form — Particle approximation theory
EQ.0: Eulerian formulation (default)
EQ.1: Eulerian renormalization approximation
…
EQ.6: Eulerian fluid particle with renormalization
EQ.8: Lagrangian formulation with renormalization
…
EQ.10: Lagrangian renormalized adaptive SPH (ASPH)
EQ.12: Moving least-squares based (MLS) formulation enhanced fluid formulation
…
EQ.16: Enhanced Eulerian fluid formulation with renormalization



initial configuration



deformed configuration

SPH in LS-DYNA (accuracy-related parameters)

```
*CONTROL_SPH
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$#      ncbs     boxid        dt      idim    memory      form     start      maxy
           1        42
$#      cont     deriv       ini     ishow     ierod     icont     iavis     isymp
           1                                                           1
```

`iavis` Defines artificial viscosity formulation for SPH elements
EQ.0: Monaghan type artificial viscosity formulation is used.
EQ.1: Standard type artificial viscosity formulation from solid element is used (recomended for fluids)

Note that artificial viscousity is always applied
→ energy dissipates to avoid numerical high-frequency noise

For fluids minimize impact via

- `iavis = 1`
- `Q1 = 0.01` and `Q2 = 1e-12` in `*CONTROL_SOLID`

SPH Contact

```
*CONTROL_SPH
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$#      ncbs     boxid        dt      idim    memory      form     start      maxy
           1        42
$#      cont     deriv       ini     ishow    ierod      icont     iavis     isymp
           1
```

cont — Defines the computation of the particle approximation between different SPH parts:

EQ.0: Particle approximation is defined (default)

EQ.1: Particle approximation is not computed. Different SPH materials will not interact with each other and penetration is allowed unless *DEFINE_SPH_TO_SPH_COUPLING is defined. Combined with *SECTION_SPH_INTERACTION, a partial interaction between SPH parts through normal interpolation method and partially interact through the contact option can be realized. See *SECTION_SPH_INTERACTION.

icont — Controls contact behavior for deactivated SPH particles:

EQ.0: Any contact defined for SPH remains active for deactivated particles.

EQ.1: Contact is inactive for deactivated particles.

SPH Section

```
*SECTION_SPH
$---+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
$#    secid      cslh      hmin      hmax    sphini     death     start
          1       1.2       0.2       2.0
```

`secid` section ID

`cslh` Constant applied to the smoothing length of the particles. **Default values are good values**. However, values between 1.05 and 1.3 are commonly used.
The higher the better from an accuracy point of view for the trade-off of higher computational costs

`hmin` Scale factor for the minimum smoothing length

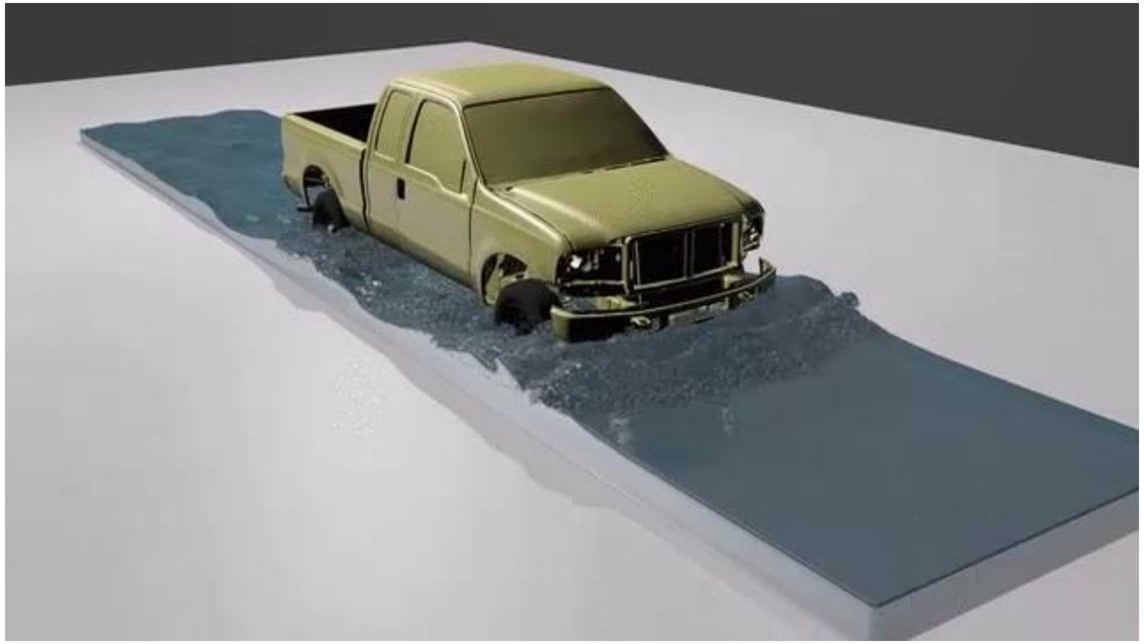
`hmax` Scale factor for the maximum smoothing length

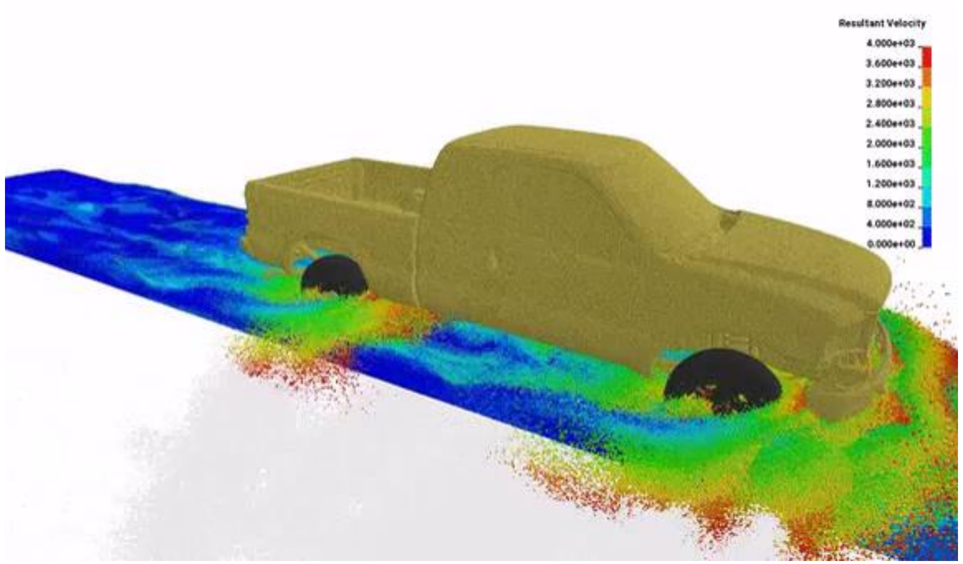
`death` Time imposed SPH approximation is stopped (see also `dt` in CONTROL_SPH).

`start` Time imposed SPH approximation is activated (see also `start` in CONTROL_SPH).

Recent updates in SPH

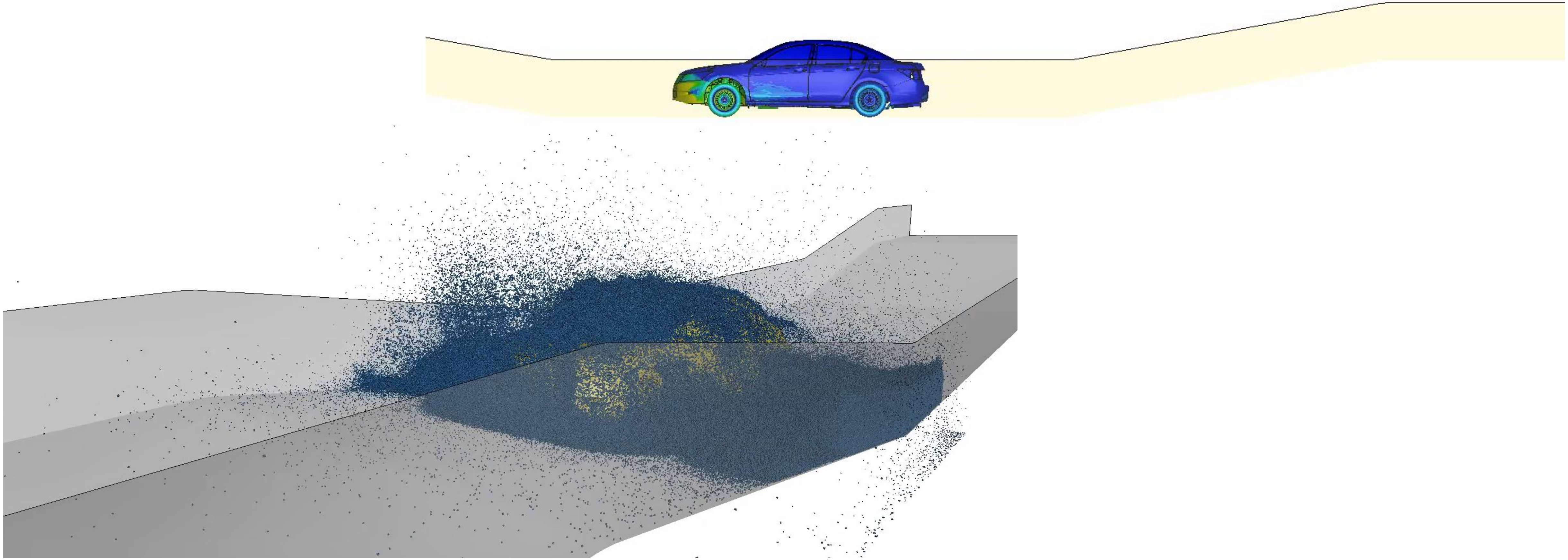
- Implicit SPH, e. g. wading simulation
  - Activated by `form = 13` in *`CONTROL_SPH`
  - Allows larger time-step size
  - Tailored for wading-type problems





Recent updates in SPH

- Implicit SPH, e. g. wading simulation
  - Wetness indicator



Summary

- SPH is a meshfree method
- Superior to FEA when it comes to
  - Large deformations
  - Material separation and mixing
  - Intrinsic mimicking of free surfaces, e. g. water surface
- However, it
  - Is more computational expensive
  - Has accuracy issues with the more efficient implementations

Thank you for your attention!

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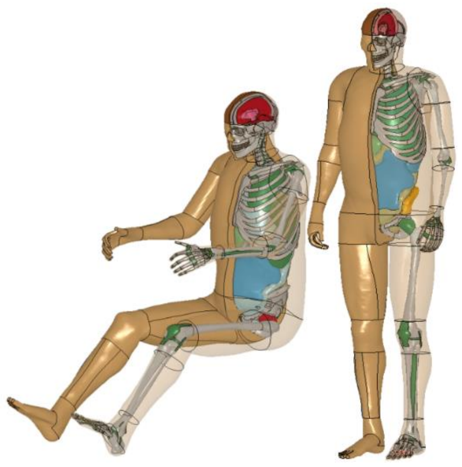

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and our **YouTube-Channel for the session recordings**.

If you have questions, feel free to contact us at support@dynamore.de
or take a look at

- LS-DYNA examples (www.dynaexamples.com)
- Papers/Conference proceedings (www.dynalook.com)
- Hints, FAQ,… (www.dynasupport.com)
- Manuals (http://lstc.com/download/manuals)