Simulation of metal forming processes under consideration of imprecise probabilities

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Abstract:

The design of metal forming processes is an ambitious task in order to ensure a high quality of the subsequent products. The assessments of designs, neglecting the data uncertainty, can result in fallacious prognosis and hence lead to false decisions. Therefore, the consideration of uncertainties in the design process has been brought forward in the recent past. Until now merely probabilistic uncertainty models are applied, solely allowing to model information with the characteristic randomness. This is insufficient for engineering applications reasoning that available information are dubious, incomplete, or fragmentary. To model those information appropriately enhanced uncertainty models on the basis of imprecise probabilities have been developed, e.g., the data model fuzzy randomness. This enables the assessment of alternative design variants on the basis of truly available information. The numerical realization is performed by means of generic optimization algorithms and fuzzy stochastic structural analysis. Both are introduced in this paper and their applicability is demonstrated by an example.

Keywords:

imprecise probabilities, fuzzy randomness, reliability based optimization, imprecise optimal design, metal forming

1 Introduction

The design of a metal forming process is focused on the accuracy of products and the minimization of forming failure such as fracture, wrinkling and excessive thickness reduction. The aim is to identify the best configuration of adjustable design quantities, e.g. draw bead forces and binder forces, to obtain high quality products. The quality of a product is strongly influenced by various further process quantities such as the anisotropic behavior of the material, the initial sheet thickness, the initial trimming of the blank, the friction coefficient, etc. Due to several production conditions the description of these quantities by crisp values may yield erroneous conclusions. In order to perform a realistic analysis the uncertainty of the process quantities must be appropriately considered within the numerical simulation. A proper consideration and treatment of uncertainty basically enable a reliability assessment of a metal forming process to ensure the subsequent quality of the product and hence provide a basis to evaluate several design variants.

A popular classification of uncertainty, with respect to its sources, distinguishes between aleatory and epistemic uncertainty. Aleatory uncertainty is characterized by randomness and is primarily associated with objectivity. Epistemic uncertainty may be comprised of substantial amounts of both objectivity and subjectivity separately. This appears for instance due to a lack of information, which impedes the specification of a unique probabilistic model and unclear generation schemes of observations that deviate from a pure random nature. The problems of accounting for subjective uncertainty and specifying a unique probabilistic model are often addressed in literature, e.g., "In real situations, it mostly turns out that the needed information is lacking. Moreover ... even small errors in probabilistic data may lead to large errors in estimating probabilities of failure" [5].

The probabilistic concept is a well known and widespread method in the engineering practice [13] accounting for aleatory uncertainty. In order to model stochastic quantities appropriately, e.g., with a probability distribution function and its parameters, some boundary conditions have to be met. The importance once are a sufficiently large sample size, the i.i.d. paradigm and the absence of imprecision. The compliance of those conditions is all but impossible for practical applications.

Uncertainty models incorporating epistemic uncertainty have been developed in the recent past [9]. They increasingly attract attention in solving practice relevant problems. Methods capable of mathematically describing and quantifying objective and subjective uncertainty are, e.g., probability theory including subjective probability and Bayesian approach, interval mathematics, evidence theory, concepts of imprecise probabilities and fuzzy randomness. These developments do not only deal with the analysis and reliability assessment of structures under uncertainty but also include novel approaches for robustness assessment and structural design [2].

The approach presented in this paper utilizes the uncertainty model fuzzy randomness. Uncertain input quantities are taken into account as fuzzy random quantities, which include real random quantities and fuzzy quantities as special cases. They are processed simultaneously within a fuzzy stochastic analysis [8] to obtain uncertain structural responses and uncertain reliability levels that reflect both objective and subjective information.

A viable method to determine the best design variant is an optimization task. While traditional optimization tasks incorporate crisp parameter values, in the recent past optimization tasks are upgraded to consider uncertain parameter values. Depending on the respective aim of the investigation, reliability measures within an reliability based optimization, robustness measures within a robustness based optimization [4] or both of them within a reliability based robust optimization [7] may be evaluated. Based on those approaches the optimization task can be extended to incorporate fuzzy random quantities.

The benefit of utilizing advanced uncertainty models in the metal forming design process is demonstrated by way of an example. Thereby the data analysis, the modeling of fuzzy random input quantities and the formulation of an appropriate optimization task are exemplarily shown.

2 Uncertainty models for imprecise data

In the engineering practice the available information are generally affected by uncertainty. The selection of an appropriate uncertainty model depends on the respective underlying nature. A rough distinction may be done between aleatory uncertainty due to a pure random nature, detected with a sufficient size of sample elements, and epistemic uncertainty due to imprecise, fluctuating, vague, incomplete, linguistic or expert evaluated information. The available data for practical problems are frequently characterized by uncertainty that comprises incomplete objective data. Simultaneously these information are dubious and imprecise and may be adjusted by subjective evaluations. An uncertainty quantification of those information with non-probabilistic methods only neglects the content of available information. Otherwise utilizing pure probabilistic methods suggests knowledge which is absent. Thus, an uncertainty model comprising objective as well as subjective information is required.

A commonly used probabilistic based method is the Bayesian approach [3]. This approach processes subjective information on the basis of subjective probability using prior distributions. The shortcoming of this approach arises from the description of subjective information with the aid of objective uncertainty measures. Thus, a separation of objectivity and subjectivity is impeded. For instance, failure probabilities, determined with the aid of the Bayesian approach, are represented by crisp values, without reflecting the source of the respective uncertainty.

A rather appropriate modeling of uncertainty differs between aleatory and epistemic uncertainty utilizing different uncertainty measures. This general approach is denoted by imprecise probabilities [16]. The main idea is to measure the probability of an event within a lower and a upper bound.

One suitable mathematical model to evaluate imprecise probabilities is served by the approach of interval probabilities. Thereby, with the aid of interval mathematics bounds of probability are specified. This approach reflects the fact, that the observation of an random event can only be described imprecise. On this account the probability of an uncertain quantity is not bounded to a crisp probability but rather to an interval of probabilities. This dispose the potential to consider imprecise random quantities.

The advancement of the traditional probabilistic uncertainty model enables the additional consideration of epistemic uncertainty. Thereby, epistemic uncertainty is associated with human cognition, which is not limited to a binary measure. Contrary to this, interval mathematics are limited to a binary assessment. Advanced concepts allow a gradual assessment of intervals. This extension can be realized with the uncertainty characteristic fuzziness, quantified on the basis of fuzzy set theory [17, 1]. Hence, the uncertainty model presented in this paper, fuzzy randomness, contains the model of interval probabilities as special case.

The uncertainty model fuzzy randomness considers aleatory and epistemic uncertainty simultaneously. It combines both components without mixing. In the results of a fuzzy random analysis the source of uncertainty is always observable. In accordance to [9], fuzzy randomness may also be interpreted as an imprecise probabilistic model considering simultaneously all possible probabilistic models that are relevant to describe a problem. Additional, the appropriateness of each probabilistic model is assessed by a gradual measure. The combination of fuzzy methods and probabilistic methods contains fuzziness and randomness as special cases. As fuzzy randomness may account for aleatory and/or epistemic uncertainty in an overall manner, it can be denoted as a *generalized uncertainty model*.

First ideas and definitions of fuzzy random quantities $\underline{\tilde{X}}$ have been discussed in [6]. The enhancement of describing fuzzy random quantities with α-level sets $\underline{\tilde{X}}_\alpha \in \underline{\tilde{X}}$ in [12] was the precursor for elaborating basic concepts for engineering applications [8].

The definition of fuzzy random quantities are developed on the basis of the probabilistic concept. The probability space is thereby extended by the dimension of fuzziness. Starting on the basis of real random quantities, a fuzzy random quantity can be derived. As in probabilistic the space of random elementary events Ω and the fundamental set $\underline{\mathbf{X}} = \mathbb{R}^n$ describes a hyperplane $\Omega \times \underline{\mathbf{X}}$. Considering that in real world applications realizations $\underline{x} \in \mathbb{R}^n$ of an elementary event $\omega \in \Omega$ can only be observed imprecise, a membership scale may be added perpendicular to the hyperplane to assign fuzzy realizations $\underline{\tilde{x}}(\omega) = (\tilde{x}_1, \ldots, \tilde{x}_n) \subseteq \mathbb{R}^n$ to each elementary event $\omega \in \Omega$, see Fig. 1.

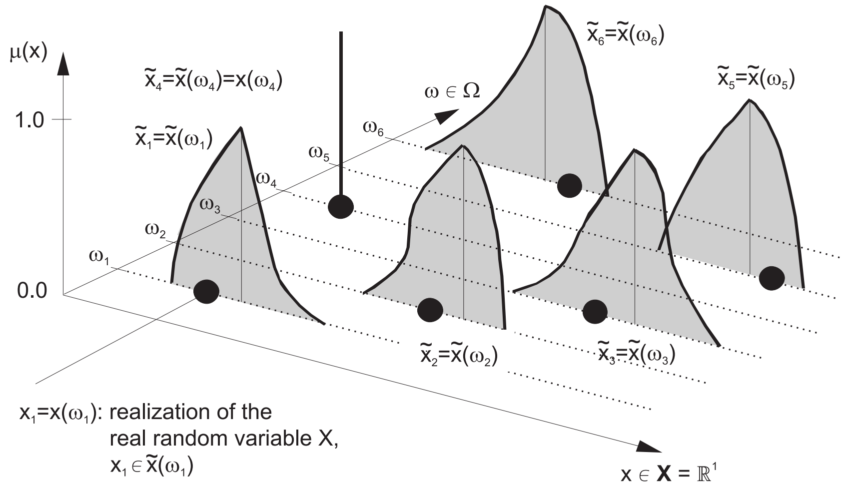

Figure 1: Fuzzy realizations $\tilde{x}$ of a fuzzy random quantity $\underline{\tilde{X}}$

A fuzzy random quantity $\underline{\tilde{X}}$ is the fuzzy result of the mapping

$$\underline{\tilde{X}} : \Omega \rightarrow \mathbf{F}(\underline{\mathbf{X}}) \quad (1)$$

with $\mathbf{F}(\underline{\mathbf{X}})$ being the set of all fuzzy quantities $\underline{\tilde{x}}$ on $\mathbb{R}^n$.

Based on this definitions a fuzzy probability measure has been developed. A fuzzy probability distribution function $\tilde{F}(\underline{x})$ is introduced as the set of probability distribution functions $F_j(\underline{x})$ gradual assessed with membership $\mu(F_j(\underline{x}))$. Since each $F_j(\underline{x})$ describes precisely one trajectory, i.e. one probabilistic distribution function, the bunch of trajectories contained in $\tilde{F}(\underline{x})$ comprises *all possible probability models*, see also Fig. 2.

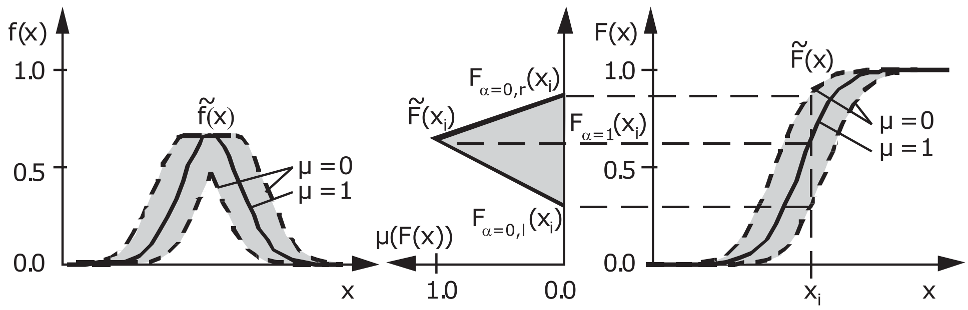

Figure 2: Fuzzy probability density function and fuzzy cumulative distribution function

The basis for a computational handling of fuzzy random quantities $\underline{\tilde{X}}$ is the α-discretization [8] and the bunch parameter representation [14], i.e. applied for a fuzzy probability distribution function

$$\underline{\tilde{F}}(\underline{x}) = \underline{F}(\tilde{\underline{s}},\underline{x}) = \{(\underline{F}_\alpha(\underline{x});\ \mu(\underline{F}_\alpha(\underline{x}))\ \mid \underline{F}_\alpha(\underline{x}) = [\underline{F}_{\min,\alpha}(\underline{x});\ \underline{F}_{\max,\alpha}(\underline{x})];\mu(\underline{F}_\alpha(\underline{x})) = \alpha,\ \forall\ \alpha \in (0,1]\}\,.$$

As the concept of fuzzy randomness contain the traditional probabilistic concept as special case, the whole framework of the probabilistic concept may be adopted for the fuzzy random approach. Thus, a fuzzy random function $\underline{\tilde{X}}(\underline{t})$ is defined as the fuzzy result of the mapping $\underline{\tilde{X}}(\underline{t}) : \underline{\mathbf{T}} \times \Omega \rightarrow \mathbf{F}(\underline{\mathbf{X}})$. The parameter vector $\underline{t} = \left(\underline{\theta},\, \tau,\, \underline{\varphi}\right)$ of the parameter space $\underline{\mathbf{T}} \subseteq \mathbb{R}^m$ represents spatial coordinates $\underline{\theta} = (\theta_1,\, \theta_2,\, \theta_3)$, the time τ, and occasionally further parameters $\underline{\varphi} = (\varphi_1,\, \varphi_2,\, \ldots)$.

3 Uncertainty in structural analysis

3.1 General procedure

If the uncertainty of the input quantities of a structural analysis is described with the aid of fuzzy random functions, the following problem is then to be solved for a crisp mapping model

$$\mathrm{F_{FSA}}:\ \underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}}) \rightarrow \underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}). \tag{2}$$

The fuzzy random functions (structural input quantities) $\underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}})$ are mapped onto the fuzzy random functions (structural responses) $\underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}})$. As fuzzy vectors and real random vectors are special cases of fuzzy random functions, these uncertainty models are also covered by Eq. (2). The mapping according to Eq. (2) is the symbolic representation of a fuzzy stochastic structural analysis.

3.2 Numerical realization

It is intended to compute the structural responses $\underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}})$ as fuzzy random vectors $\underline{\tilde{\mathrm{Z}}}_{\mathrm{t_r}} = \underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}_{\mathrm{r}})|\ r = 1,\ldots,q_1$ with the fuzzy probability distribution functions $\tilde{\mathrm{F}}_{\mathrm{t_r}}(\underline{\mathrm{z}}) = \tilde{\mathrm{F}}(\mathrm{z},\underline{\mathrm{t}}_{\mathrm{r}}) = \mathrm{F}(\underline{\tilde{\sigma}}_{\mathrm{r}},\underline{\mathrm{z}},\mathrm{t_r})$ at q_1 points $\underline{t}_r$ in the parameter space $\mathbf{T}$. For this purpose q_1 fuzzy bunch parameter vectors $\underline{\tilde{\sigma}}_r$ are to be determined, which comprise a total of m_1 bunch parameters $\tilde{\sigma}_1,\ldots,\tilde{\sigma}_{m_1}$. The m_1 fuzzy bunch parameters are combined in the fuzzy vector $\underline{\tilde{\sigma}}$. The fuzzy stochastic structural analysis characterized by Eq. (2) has thus been transformed into the mapping

$$\mathrm{F_A}:\ \underline{\tilde{\mathrm{s}}} \rightarrow \underline{\tilde{\sigma}}. \tag{3}$$

An α-discretization [8] of the fuzzy bunch parameters $\tilde{\mathrm{s}}_1,\ldots,\tilde{\mathrm{s}}_{\mathrm{n}_1}$ belonging to fuzzy probability distribution functions $\mathrm{F}(\underline{\tilde{\mathrm{s}}}_{\mathrm{i}},\underline{\mathrm{x}},\mathrm{t_r})$ yields the α-level sets $\mathrm{S}_{1,\alpha_k},\ldots,\mathrm{S}_{\mathrm{n}_1,\alpha_k}$ for the level α_{k} (Fig. 3). These α-level sets together with the α-level sets $\mathrm{S}_{\mathrm{r},\alpha_k}\mid \mathrm{r} = \mathrm{n}_1+1,\ldots,\mathrm{n}$ form the n-dimensional crisp subspace $\underline{\mathrm{S}}_{\alpha_k}$. If one element is selected from each α-level set, one crisp vector $\underline{\mathrm{s}}$ in the subspace $\underline{\mathrm{S}}_{\alpha_k}$ is obtained.

With each set of crisp elements $\mathrm{s}_{1,\mathrm{j}} \in \mathrm{S}_{1,\alpha_k},\ldots,\mathrm{s}_{\mathrm{n}_1,\mathrm{j}} \in \mathrm{S}_{\mathrm{n}_1,\alpha_k}$ constituting the vector $\underline{\mathrm{s}}_{\mathrm{j}} \in \mathrm{S}_{\alpha,\mathrm{k}}$ precisely one trajectory $\mathrm{F}_{\mathrm{t_i},\mathrm{j}}(\underline{\mathrm{x}}) \in \mathrm{F}_{\mathrm{t_i},\alpha_k}(\mathrm{x})\ \mid\ \mathrm{i} = 1,\ldots,\mathrm{p}_1$ with the membership value $\mu(\mathrm{F}_{\mathrm{t_i},\mathrm{j}}(\underline{\mathrm{x}})) = \mu(s_j) \geq \alpha_{\mathrm{k}}$ is simultaneously selected from each of the p_1 fuzzy probability distribution functions (Fig. 3). The trajectories are $\mathrm{F}_{\mathrm{t_i},\mathrm{j}}(\underline{\mathrm{x}})$ real-valued probability distribution functions. Each α-function set $\mathrm{F}_{\mathrm{t_i},\alpha_k}(\underline{\mathrm{x}})$ comprises all trajectories of the fuzzy probability distribution function $\tilde{\mathrm{F}}_{\mathrm{t_i}}(\underline{\mathrm{x}})$ at the state $\underline{\mathrm{t}}_{\mathrm{i}} \in \underline{\mathrm{T}}$ for the level α_{k}.

Selecting one crisp point $\mathrm{s_j}$ from the subspace $\underline{\mathrm{S}}_{\alpha_k}$ one real probability distribution function (trajectory) is known for each fuzzy random vector $\underline{\tilde{\mathrm{X}}}_{\mathrm{t_i}} = \underline{\tilde{\mathrm{X}}}(\mathrm{t_i})$. Moreover, one element from the respective α-level set (for the same level α_k) of each bunch parameter belonging to the fuzzy vectors, fuzzy fields, real random vectors, and fuzzy covariances is to be selected. One stochastic structural analysis may now be carried out for the crisp bunch parameter vector $\underline{\mathrm{s}}_{\mathrm{j}} \in \underline{\mathrm{S}}_{\alpha_k}$.

The trajectories of the fuzzy probability distribution functions of $\mathrm{F}(\underline{\tilde{\sigma}},\underline{\mathrm{z}},\underline{\mathrm{t}}_{\mathrm{r}})$ the result vectors $\underline{\tilde{\mathrm{Z}}}_{\mathrm{t_r}} = \underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}_{\mathrm{r}})$ $\mid \mathrm{r} = 1,\ldots,\mathrm{q}_1$ are designated by $\mathrm{F}_{\mathrm{t_r},\mathrm{j}}(\underline{\mathrm{z}}) \mid \mathrm{r} = 1,\ldots,\mathrm{q}_1$. For each defined α-level α_{k} these are elements of the assigned α-function sets $\mathrm{F}_{\mathrm{t_r},\mathrm{j}}(\underline{\mathrm{z}}) \in \mathrm{F}_{\mathrm{t_r},\alpha_k}(\underline{\mathrm{z}})$. For determining the α-function sets $\mathrm{F}_{\mathrm{t_r},\alpha_k}(\underline{\mathrm{z}})$ the following functional relationship concerning the trajectories may then be stated

$$(\mathrm{F}_{\mathrm{t_r},\mathrm{j}}(\underline{\mathrm{z}})\ \mid\ \mathrm{r} = 1,\ldots,\mathrm{q}_1) = \mathrm{g}(\mathrm{F}_{\mathrm{t_r},\mathrm{j}}(\underline{\mathrm{x}})\ \mid\ \mathrm{r} = 1,\ldots,\mathrm{q}_1)\ . \tag{4}$$

The solution of Eq. (4) may be obtained with the aid of the Monte Carlo simulation (MCS). Based on the trajectories $\mathrm{F}_{\mathrm{t_i},\mathrm{j}}(\underline{\mathrm{x}})$ sample vectors are generated. Each sample vector represents a crisp input vector for one deterministic structural analysis. The deterministic fundamental solution may be an arbitrary, linear or nonlinear structural analysis, e.g. a Finite Element code.

The application of MCS results in a sample of result values for each trajectory $\mathrm{F}_{\mathrm{t_r},\mathrm{j}}(\underline{\mathrm{z}})$ of the fuzzy random result vectors $\underline{\tilde{\mathrm{Z}}}_{\mathrm{t_r}} = \underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}_{\mathrm{r}})$. Statistical evaluation of these samples yields the trajectories in bunch parameter representation. For each α-level α_{k} the obtained bunch parameters are elements of the assigned α-level sets $\sigma_{1,\mathrm{j}} \in \sigma_{1,\alpha_k},\ldots,\sigma_{\mathrm{m},\mathrm{j}} \in \sigma_{\mathrm{m}_1,\alpha_k}$ of the fuzzy bunch parameters $\tilde{\sigma}_1,\ldots,\tilde{\sigma}_{\mathrm{m}_1}$ constituting the fuzzy vector $\underline{\tilde{\sigma}}$. Once the smallest and largest elements of the α-level sets $\sigma_{1,\alpha_k},\ldots,\sigma_{m_1,\alpha_k}$ have been

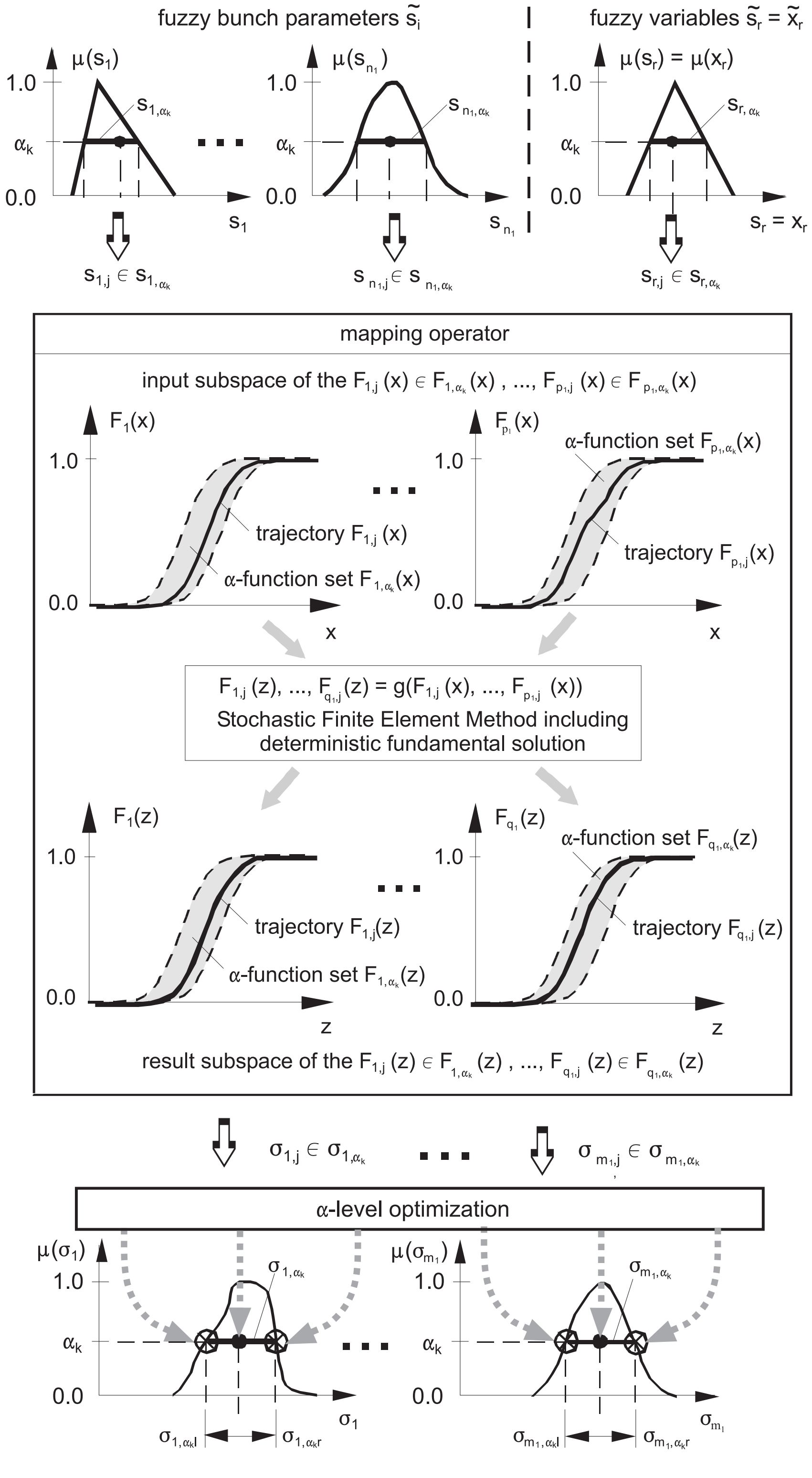

Figure 3: Numerical realization of the fuzzy stochastic analysis

determined for each α-level α_k, the fuzzy bunch parameter vectors $\tilde{\sigma}_1, \ldots, \tilde{\sigma}_{q_1}$ and hence the fuzzy probability distribution functions $\tilde{F}(\underline{z}, t_r) = F(\underline{\tilde{\sigma}}, \underline{z}, t_r)$ are then known. The search for the smallest and largest elements of the bunch parameters is realized by applying α-level optimization, see Fig. 3, [8, 10].

4 Optimization with fuzzy random quantities

On the basis of [2, 4, 7], the reliability based optimization is enhanced by the concept of imprecise probabilities, utilizing the generalized uncertainty model fuzzy randomness [8]. Therefore, an optimization task is formulated under consideration of fuzzy random quantities, yielding to both an fuzzy random objective function and fuzzy random constraints. Applying information reducing methods, e.g., determining failure probabilities, the task is transferred into a nonlinear fuzzy optimization task.

The parameters of an optimization task are subdivided in design parameters $x_{k,d}, k = 1,\ldots,n_{x,d}$ and a-priori parameters $x_{l,t}, l = 1,\ldots,n_{x,t}$. While the quantities in the space of design parameters $\mathbb{X}_d$ are freely selectable within user-specified ranges, the quantities in the space of a-priori parameters $\mathbb{X}_t$ are prescribed and unchangeable. All parameters may be deterministic or uncertain as well as time-dependent. Considering imprecise data the parameters $\underline{x}_d$ and $\underline{x}_t$ become fuzzy random variables $\underline{\tilde{X}}_d$ and $\underline{\tilde{X}}_t$.

The numerical treatment of such an optimization task requires an affine transformation of the design parameters. The transformation reads $\underline{\tilde{X}}_d = \underline{x}_{d_1} \cdot \underline{\tilde{E}}_d + \underline{x}_{d_2}$ postulating fixed fuzzy parameters. Assuming, that especially for engineering applications $\underline{x}_{d_1}$ and $\underline{x}_{d_2}$ are correlated, the transformation can be performed with a unique parameter $\underline{x}_d$. Furthermore, $\underline{\tilde{E}}_d$ is invariant and thus becomes an a-priori parameter vector $\underline{\tilde{X}}_E = (\,\tilde{X}_{1,t}\,\ldots\,\tilde{X}_{n_{x,t},t}\,,\tilde{E}_{1,d}\,\ldots\,\tilde{E}_{n_{x,d},d})$.

The application of sampling schemes to incorporate fuzzy random a-priori quantities is enabled utilizing a bunch parameter representation [14] $\underline{\tilde{X}}_E = \underline{X}_E(\underline{\tilde{s}})$. Comprising fuzzy bunch parameters $\underline{\tilde{s}}$ and the fuzzy parameters $\underline{\tilde{x}}_E$ the fuzzy vector $\underline{\tilde{s}}_E = (\underline{\tilde{s}}, \underline{\tilde{x}}_E)$ is constituted.

Incorporating fuzzy random quantities $\underline{\tilde{X}}_E$ within an optimization task the deterministic objective function $z = f_z(\underline{x}_d, \underline{x}_t)$ is transferred into a fuzzy random objective function $\tilde{Z} = f_z(\underline{x}_d, \underline{\tilde{X}}_E)$ and the deterministic constraints $\underline{g} = \underline{f}_g(\underline{x}_d, \underline{x}_t) \leq \underline{g}^*$ are transferred into fuzzy random constraints $\underline{\tilde{G}} = \underline{f}_g(\underline{x}_d, \underline{\tilde{X}}_E) \leq \underline{\tilde{G}}^*$.

4.1 Imprecise optimal design

If the information content of randomness is summarized with the aid of failure probabilities the objective function and the constraints are obtained as $\tilde{z} = f_z(\underline{x}_d, \underline{\tilde{s}}_{E_z})$ and $\underline{\tilde{g}} = \underline{f}_g(\underline{x}_d, \underline{\tilde{s}}_{E_z}, \underline{\tilde{s}}_{E_g}) \leq \underline{\tilde{g}}^*$. In consequence, the fuzzy stochastic optimization problem is transferred in a pure fuzzy optimization problem. Thereby, the minimum is defined by

$$\begin{aligned}\tilde{z}_{min} \;=\;& \Big\{(z_{min}, \mu(z_{min})) \\ & \mid z_{min} = \min_{\underline{x}_d \in \mathbb{X}_d} \; f_z(\underline{x}_d, \underline{s}_{E_z}),\; \underline{f}_g(\underline{x}_d, \underline{s}_{E_g}, \underline{s}_{E_z}) \leq \underline{g}*,\; \mu(z_{min}) = \mu(\underline{s}_E) \\ & \forall \underline{s}_E \in \underline{\tilde{s}}_E,\; \underline{g}^* \in \underline{\tilde{g}}^* \Big\} \quad . \end{aligned} \tag{5}$$

The respective optimal design $\underline{\tilde{x}}_{d,min}$ is simultaneously obtained with

$$\underline{\tilde{x}}_{d,min} = \left\{(\underline{x}_d, \mu(\underline{x}_d)) \mid \underline{x}_d = f_z^{-1}(z), \mu(\underline{x}_d) = \mu(z) \;\forall\, z \in \tilde{z}_{min}\right\} \quad . \tag{6}$$

Due to the fact, that the mapping $x \rightarrow z$ is just unique and not one to one, $\underline{\tilde{x}}_{d,min}$ is obtained as a non convex fuzzy quantity. Actually, the treatment of non convex fuzzy quantities is not state of the art.

4.2 Application of fuzzy optimization

The application of fuzzy optimization methods aims on the determination of crisp design quantities with respective result quantities to enable a decision making. For each design $\underline{x}_d \in \underline{\tilde{x}}_{d,min}$ the fuzzy result $z \in \underline{\tilde{z}}_{min}$ is permissible, evaluated with respect to the constraints, with a particular membership $\mu(z)$. This is proper from a mathematical point of view. But for engineering applications it can be worthwhile to assign a unique fuzzy result quantity $\tilde{z}$ and statements about the permissibility to a discrete design vector $\underline{x}_d$. The challenge of this modified optimization task is on the one hand to assess the permissibility of a design and on the other hand to define relations between two fuzzy quantities due to alternative design variants. This can be performed by utilizing information reducing methods, which are inherently user-defined and problem-dependent.

For the solution of nonlinear industry-sized problems on the field of civil and automotive engineering it is suggested to use three criteria to assess alternative designs:

- permissibility, e.g. $\underline{\tilde{g}} - \underline{\tilde{g}}^* \mid \underline{\tilde{g}} = \underline{f}_g(\underline{x}_d, \underline{\tilde{s}}_{E_g}, \underline{\tilde{s}}_{E_z}) \rightarrow \min$, to comply with the constraints. This presuppose a metric to determine the distance between fuzzy quantities and furthermore measures, to evaluate the obtained distance.
- minimization, requiring defuzzification methods, e.g. $\int z \cdot \mu(z)dz \cdot (\int \mu(z)dz)^{-1} \rightarrow \min$
- robustness assessment utilizing robustness measures, e.g. $\int (z - \bar{z})^2 \cdot \mu(z)dz \cdot (\int \mu(z)dz)^{-1} \rightarrow \min$

Utilizing these three criteria the optimization task is converted in a multi-criteria optimization task, while the quoted order suggests a weighting.

5 Design of a metal forming process

In this example an appropriate design of a metal forming process, see Fig. 4, should be determined. The aim is to identify a setting of design quantities whose results comply with reliability requirements in a best possible manner. Among all input quantities, sixteen are indicated to be sensitive to the

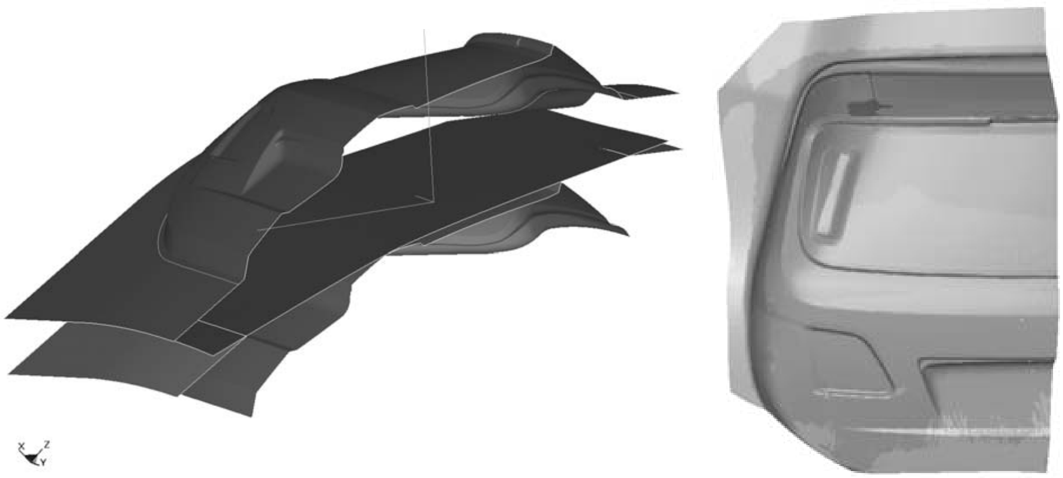

Figure 4: Metal forming device and component part

result quantities and thus to influence the reliability predominantly, elucidated in detail in [11]. These are material parameters for the blank (DCO6(1.0873)) to describe the yield strength, the elasto-plastic hardening and anisotropic effects. In special, these are the parameters of the swift law R_p, n, K and the anisotropic coefficients r_0, r_{45}, r_{90}. Furthermore, variations of the manufacturing process parameters, like the friction coefficient μ, the draw bead forces and the binder forces, affect the performance of the deep drawing process. At least spatial perturbations of the initial shell thickness, caused by the production process of the blank itself, have to be considered. They are modeled with the aid of fuzzy fields [8].

On account of a design process, a distinction of the input quantities in design quantities $\underline{x}_d$, which may be freely selected during the design process, and invariant a-priori input quantities $\underline{x}_{l,t}$, which are prescribed and non-alterable, have to accomplished. Design quantities $\underline{x}_d$ are the mean value of the binder force $x_{1,d}$ and the mean values of the draw bead forces $x_{2,d}, \ldots, x_{7,d}$, see also 4. The respective design ranges are predefined with intervals as follows: $x_{1,d} = [1400, 2400]$, $x_{2,d} = x_{3,d} = x_{7,d} = [20, 200]$, $x_{4,d} = [50, 120]$, $x_{5,d} = [60, 120]$, $x_{6,d} = [70, 130]$. A-priori input quantities are the invariant a-priori parameters itself and the remaining uncertainty of the design quantities. The quantification of the uncertain input quantities is accomplished under consideration of the respective source of uncertainty. Hence, the generalized uncertainty model fuzzy randomness [8] is utilized, see Table 1. Thereby, it is assumed that the randomness in respective input parameters can be modeled with a normal distribution.

Table 1: Fuzzy random input quantities

fuzzy random quantity		normal distribution mean value	standard deviation
yield strength f_y	$\tilde{f}(x_1)$	0.14	$< 0.0067; 0.008; 0.01 >$
strength coefficient K	$\tilde{f}(x_2)$	0.55	$< 0.0367; 0.044; 0.055 >$
hardening exponent n	$\tilde{f}(x_3)$	$< 0.23; 0.275; 0.3 >$	0
friction coefficient μ	$\tilde{f}(x_4)$	$< 0.05; 0.075; 0.1 >$	0
perturbation longitudinal p_1	$\tilde{f}(x_5)$	$< -0.005; 0.0; 0.005 >$	0
perturbation lateral p_2	$\tilde{f}(x_6)$	$< -0.005; 0.0; 0.005 >$	0
material parameter r_0	$\tilde{f}(x_7)$	2.25	$< 0.0833; 0.1; 0.125 >$
material parameter r_{45}	$\tilde{f}(x_8)$	1.7	$< 0.1; 0.12; 0.15 >$
material parameter r_{90}	$\tilde{f}(x_9)$	2.85	$< 0.167; 0.14; 0.175 >$
draw bead force 1	$\tilde{f}(x_{10})$	$x_{2,d}$	$< 4.0; 5.0; 6.0 >$
draw bead force 2	$\tilde{f}(x_{11})$	$x_{3,d}$	$< 4.0; 5.0; 6.0 >$
draw bead force 3	$\tilde{f}(x_{12})$	$x_{4,d}$	$< 4.0; 5.0; 6.0 >$
draw bead force 4	$\tilde{f}(x_{13})$	$x_{5,d}$	$< 4.0; 5.0; 6.0 >$
draw bead force 5	$\tilde{f}(x_{14})$	$x_{6,d}$	$< 4.0; 5.0; 6.0 >$
draw bead force 6	$\tilde{f}(x_{15})$	$x_{7,d}$	$< 4.0; 5.0; 6.0 >$
binder force	$\tilde{f}(x_{16})$	$x_{1,d}$	$< 40; 50; 60 >$

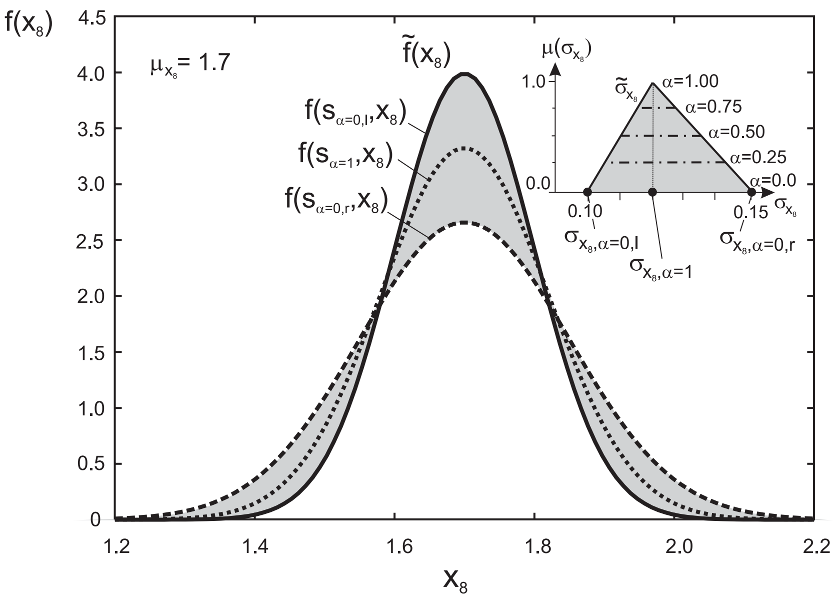

Figure 5: Fuzzy probability density function $\tilde{f}(x_8)$

The optimization under consideration of fuzzy random quantities is performed by means of generic optimization algorithms, fuzzy structural analysis, and direct Monte Carlo simulation. The application of those methods requires a high numerical effort. Hence, methods to improve the numerical efficiency are inevitable. Here, a response surface approximation on the basis of artificial neural networks [15] is applied.

As a result of each fuzzy stochastic analysis for a crisp design $\underline{x}_d$, under consideration of $\underline{\tilde{X}}_E$, the maximal shell thickness reduction $\underline{\tilde{Z}}$ is evaluated. The maximal shell thickness reduction represents in a crude way a result to appraise the permissibility of a design. Specifying results of more than $20\,\%$ (normalized) as failure, a fuzzy failure probability $\tilde{P}_f$ can be evaluated.

On the basis of the suggested 3 criteria optimization (see Sec. 4.2) the assessment of alternative design variants is enabled. In the absence of design constraints the objective is to minimize the largest possible failure probability $P_{f,\alpha=0,r}$. Thereby, an inappropriate robustness, evaluated with $\int_{P_f} \mu(P_f)dP_f$, is penalized.

In the process of optimization the fuzzy failure probability $\tilde{P}_f$ decreases, see Fig. 6.

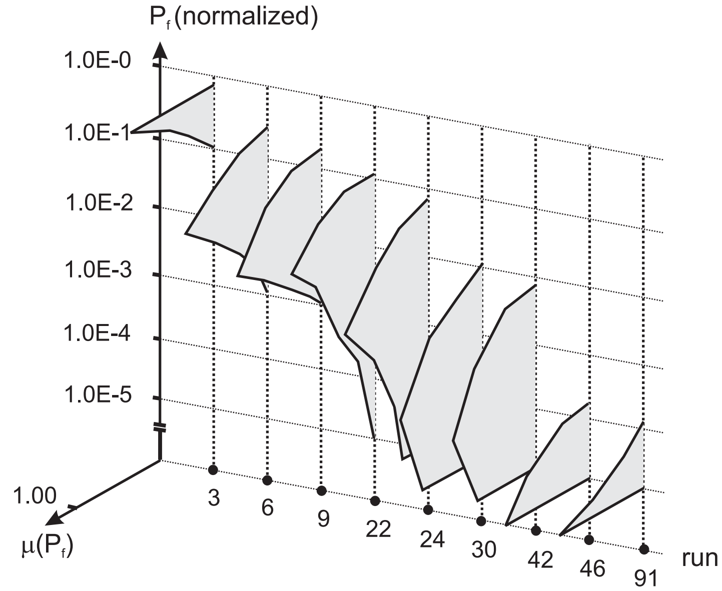

Figure 6: Fuzzy failure probabilities in selected optimization states

The optimal design is determined with $\underline{x}_{d,opt} = [25.23; 200.0; 50.0; 60.0; 77.5; 20.0; 1414.81]$. The respective result of the fuzzy stochastic analysis is depicted in Fig. 7 typified by the fuzzy cumulative distribution function $\tilde{F}(\underline{z})$ and the fuzzy failure probability $\tilde{P}_f$.

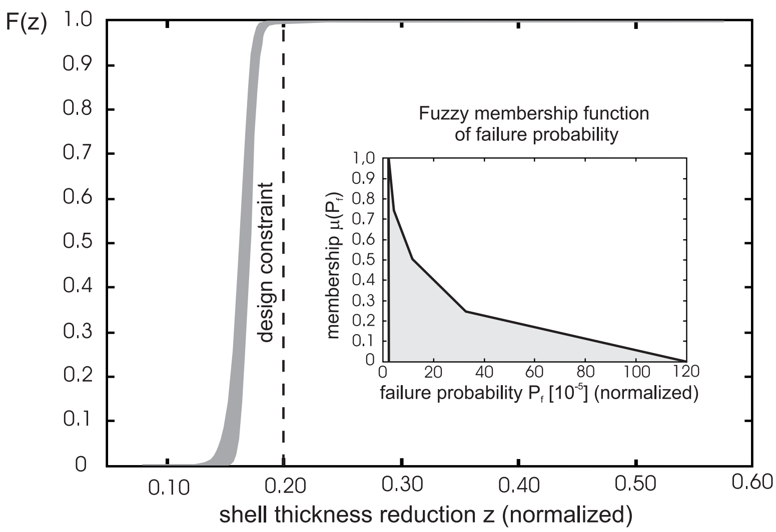

Figure 7: Result of optimal design; fuzzy cumulative distribution function and fuzzy failure probability

The result shows, that imprecisions in specifying input quantities consequently end in imprecisions in determining probabilities of failure. The data model fuzzy randomness enable the consideration of those imprecisions within the numerical simulation.

6 Conclusions

In this paper a computational algorithm for the design of metal forming processes incorporating fuzzy random quantities is presented. Thereby, the applied generalized uncertainty model fuzzy randomness is capable to represent objective information as well as dubious, incomplete, fluctuating, and fragmentary information. The numerical realization with the aid of fuzzy stochastic analysis provides an appropriate computational method to take account of uncertainty of different characteristics. Randomness, fuzziness and fuzzy randomness can be considered simultaneously. The presented concept is generally applicable in combination with arbitrary computational models. The application of the uncertainty model fuzzy randomness within a design process for industry-sized problems enables a better insight into the specific problem. In consequence the results and prognosis bases on truly available information and ease the respective decision making to ensure a high quality of the subsequent product. The applicability is demonstrated by way of example for a reliability assessment of a metal forming process.

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