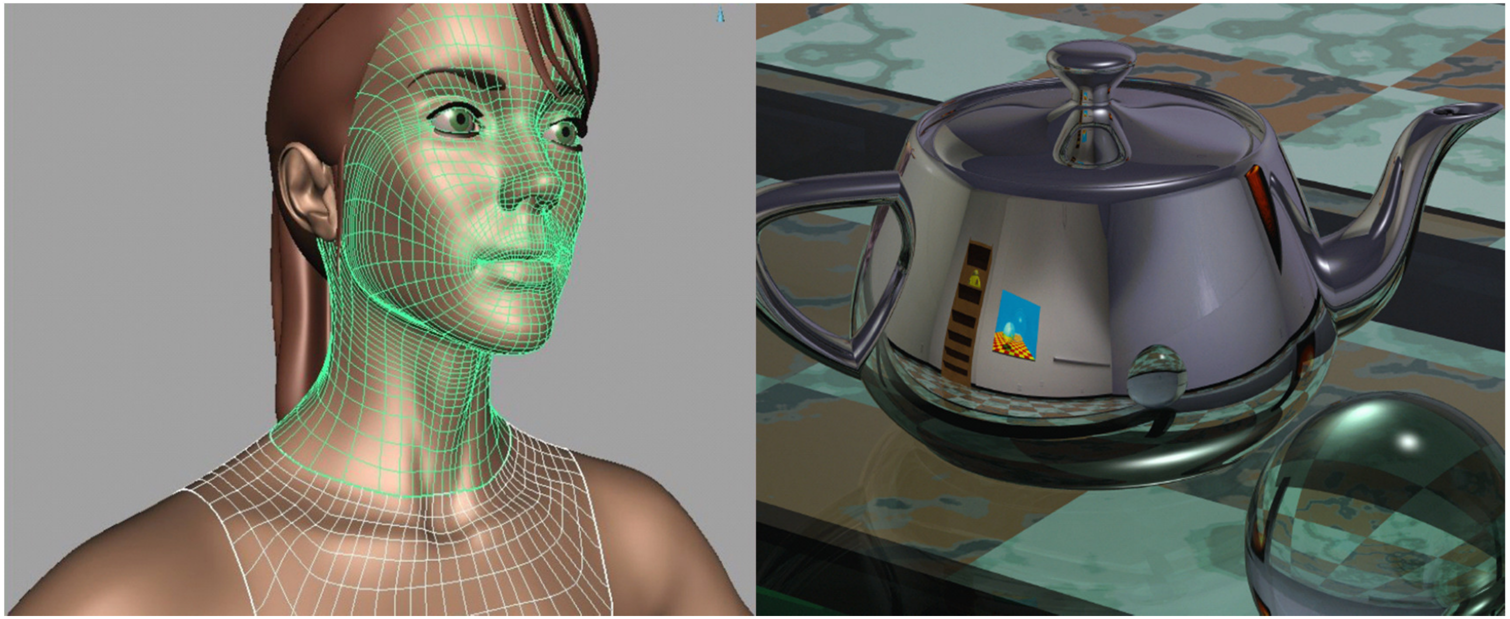
An Introduction to Isogeometric Elements in LS-DYNA



T.J.R. Hughes

Infoday
24th November 2010

Stefan Hartmann

With slides from
T.J.R. Hughes: Professor of Aerospace Engineering and Engineering Mechanics, University of Texas at Austin
D.J. Benson: Professor of Applied Mechanics, University of California, San Diego

Outline

- ISOGEOMETRIC Analysis
  - Definition / Motivation / History
- From B-Splines to NURBS (T.J.R. Hughes)
  - Shape-Functions / Control-Net / Refinement
- FEA with NURBS
  - Patch / Elements / Continuity
- Present capabilities in LS-DYNA (D.J. Benson)
  - *ELEMENT_NURBS_PATCH_2D / Interpolation-Nodes / Interpolation-Elements
- Examples (T.J.R. Hughes and D.J. Benson)
  - Vibrationanalysis / Buckling / Sheet metal forming
- Summary and Outlook

ISOGEOMETRIC Analysis - Definition

- ISOPARAMETRIC (FE-Analysis)

  Use the same approximation for the geometry and the deformation
  (mainly low order Lagrange polynomials like linear elements in LS-DYNA)

GEOMETRY	←→	DEFORMATION

- ISOGEOMETRIC (CAD - FEA)

  Use the same approximation/description for the geometry in the design (CAD) and in the analysis (FEA)

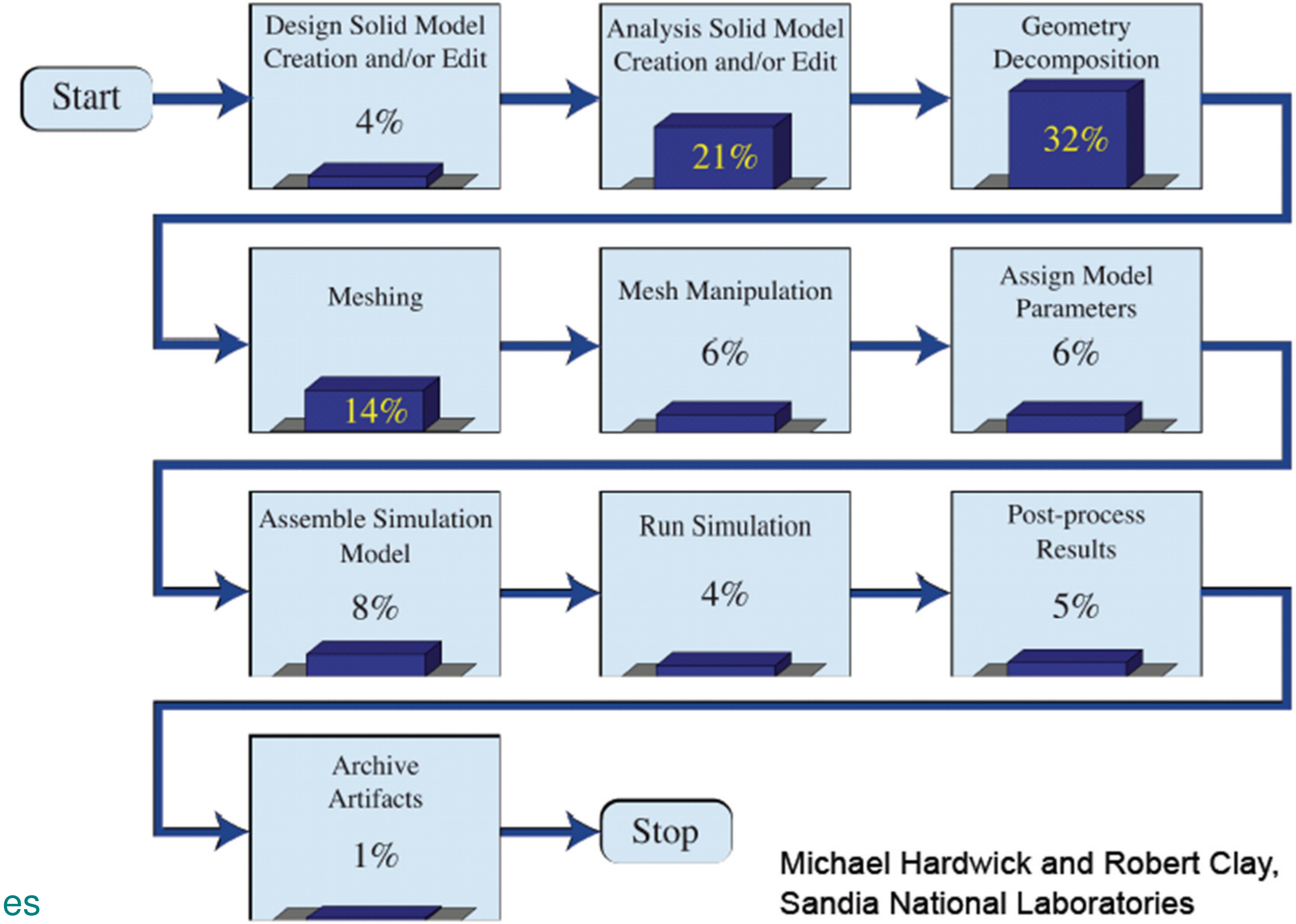
CAD	←→	FEA

- Descriptions for the geometry in CAD

  - NURBS (Non-Uniform Rational B-Splines) → Most widely used
  - T-Splines → Generalization of NURBS
  - Subdivision Surfaces → Mainly in animation industry
  - and others

ISOGEOMETRIC Analysis – Motivation (originally)

- Reduction of time cost for meshing (transfer of geometry)



Michael Hardwick and Robert Clay,
Sandia National Laboratories

T.J.R. Hughes

ISOGEOMETRIC Analysis - History

- Beginning 2003
  - Summer: Austin Cotrell starts as PhD Student of Prof. T.J.R. Hughes (University of Texas, Austin)
  - Autumn: First NURBS based FE-Code for linear, static problems provides good results and the name „ISOGEOMETRIC“ is used the first time.

- 2004 until now: Many research activities in the field on different topics
  - nonlinear structural analysis
    - → shells with and without rotational degrees of freedom
    - → implicit gradient enhanced damage
    - → XFEM
  - shape and topology optimization
  - efficient numerical integration
  - turbulence and fluid-structure-interaction (FSI)
  - accustics
  - efficient mesh-refinement algorithms
  - …

- January 2011: First international workshop on „Isogeometric Analysis“
  - “Isogeometric Analysis 2011: Integrating Design and Analysis“, University of Texas at Austin

From B-Splines to NURBS

- B-Spline Basis Functions
  - constructed recursively
  - are always positiv (in contrast to Lagrangian polynomials)
  - shape of basis function depends on knot-vector and polynomial order
  - knot-vector: sequence of (positive, ascending) parametric coordinates
  - generally $C^{(P-1)}$-continuity

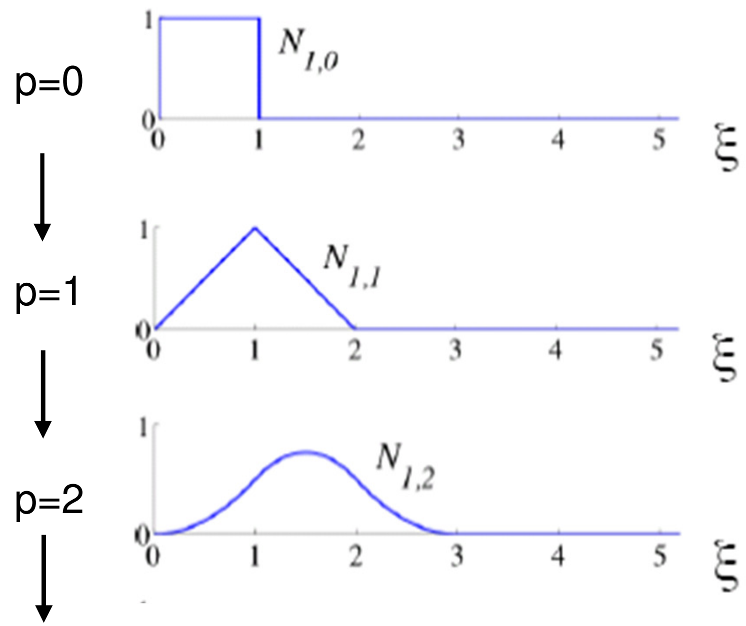
  → e.g. lin. / quad. / cub. / quart. Lagrange: → C^0 / C^0 / C^0 / C^0

  → e.g. lin. / quad. / cub. / quart. B-Spline: → C^0 / C^1 / C^2 / C^3

example of a uniform knot-vector: $\Xi = \{0,1,2,3,4,\ldots\}$

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$



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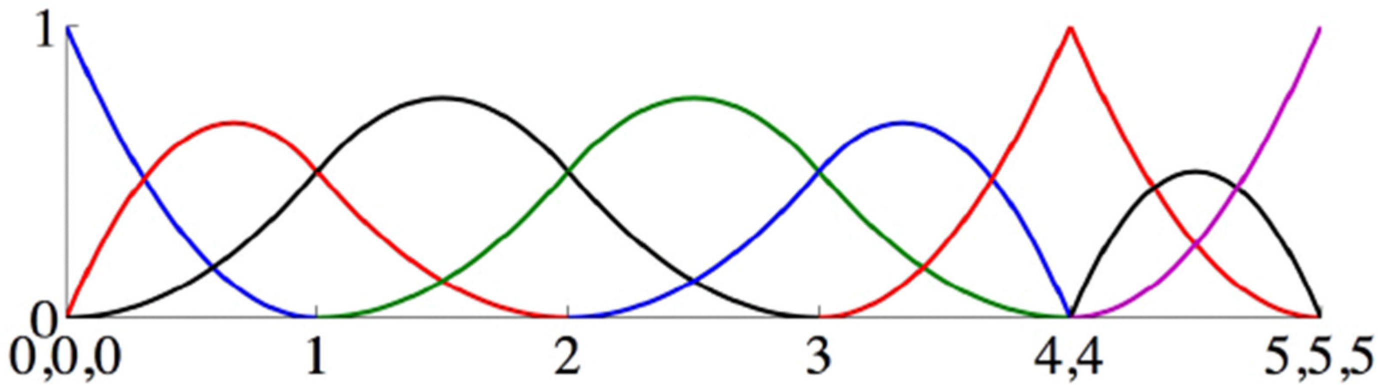
From B-Splines to NURBS

- B-Spline Basis Funktionen



Quadratic (p=2) basis functions for an

open, non-uniform knot vector:

$$\Xi = \{0,0,0,1,2,3,4,4,5,5,5\}$$

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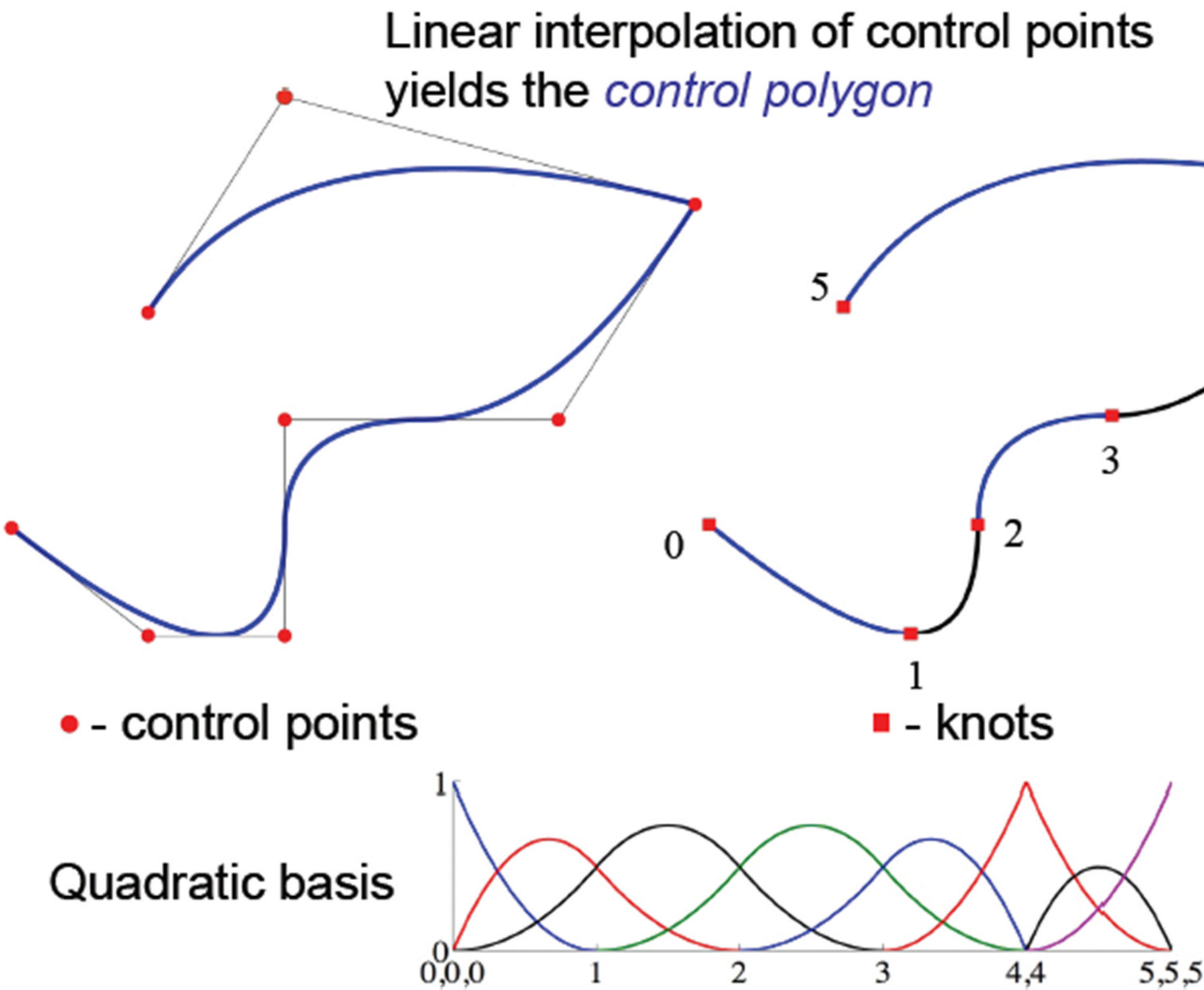
From B-Splines to NURBS

- B-Splines
  - Control-Points (-Nodes) / Control-Polygon (-Net)
  - Knots



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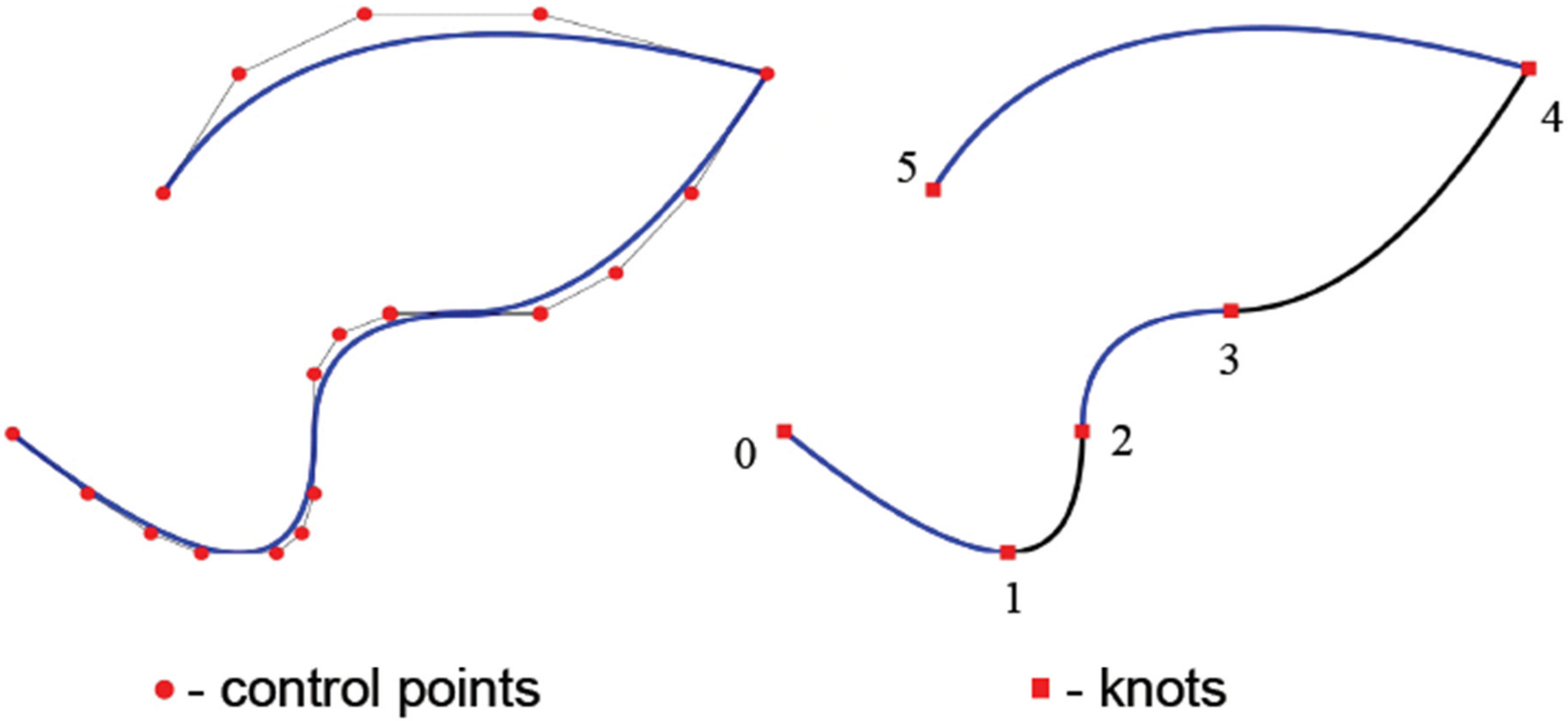
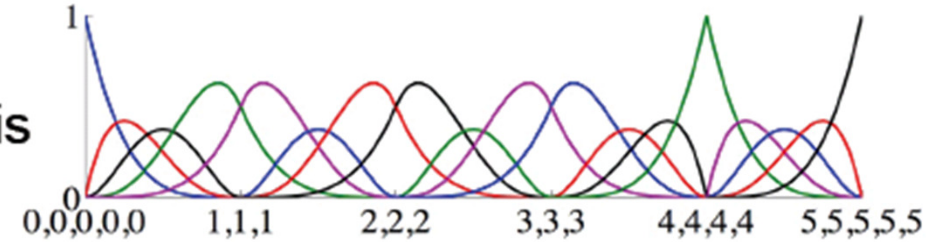
From B-Splines to NURBS

- B-Splines
  - Refinements (h-/p-/k-refinement)



Quartic basis



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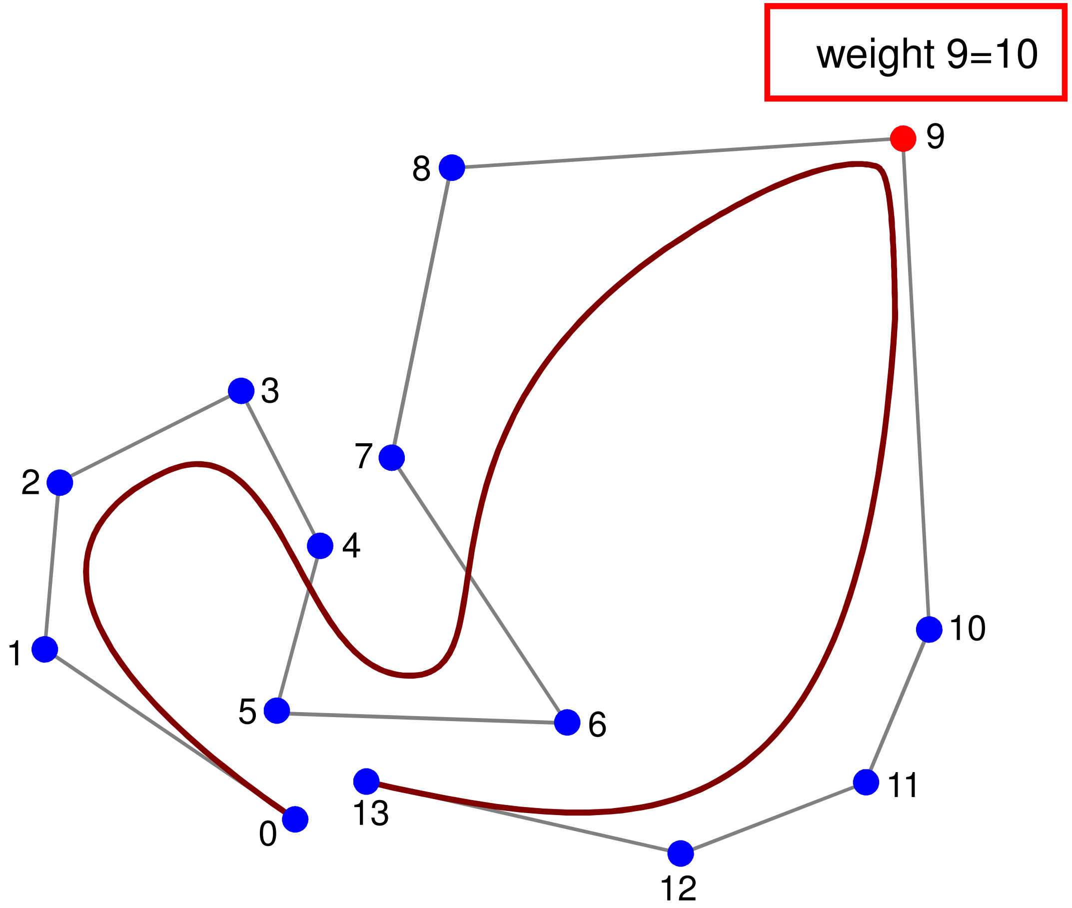
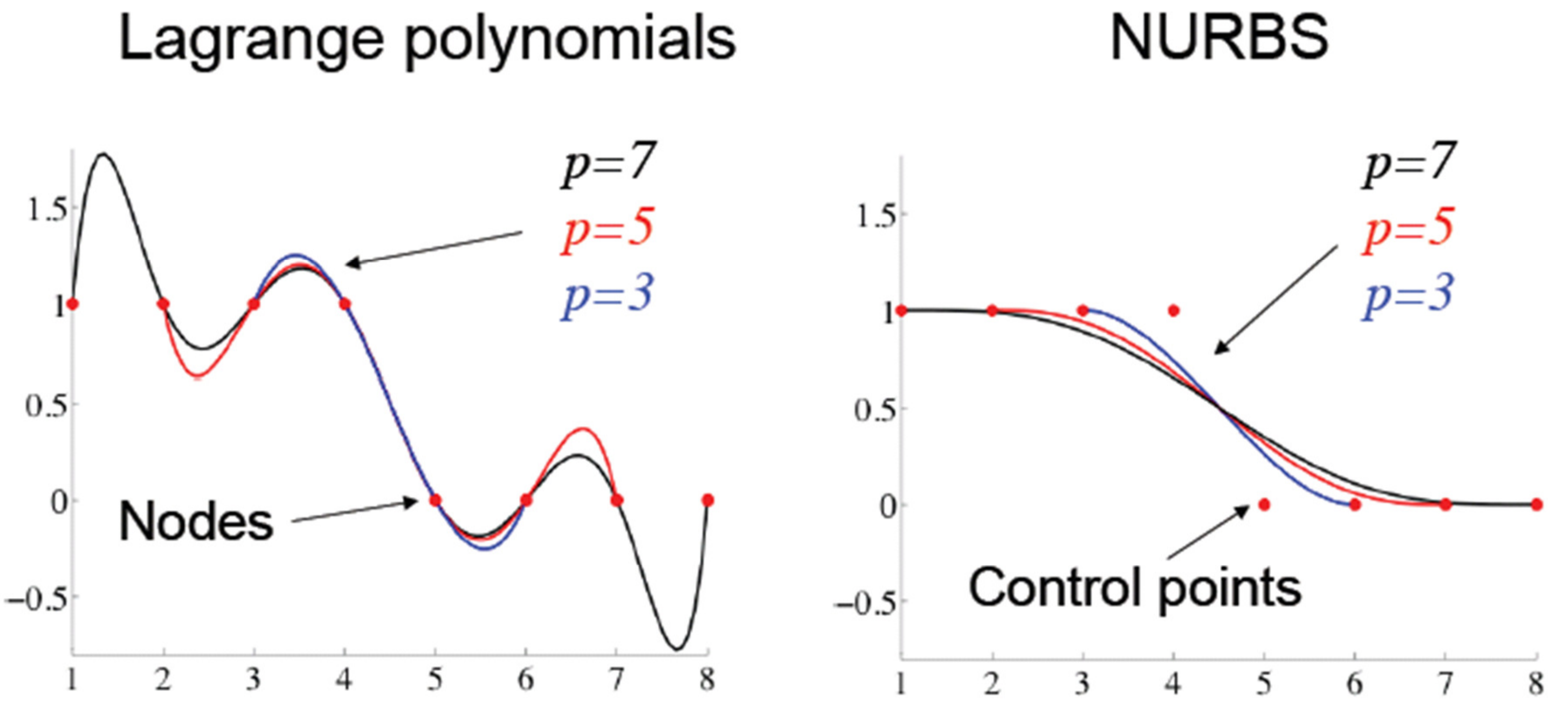
From B-Splines to NURBS

- NURBS – Non-Uniform Rational B-Splines
  - weights at the control-points allow a detailed control over the shape of the curve



From B-Splines to NURBS

- Smoothness of Lagrange polynomials vs. NURBS



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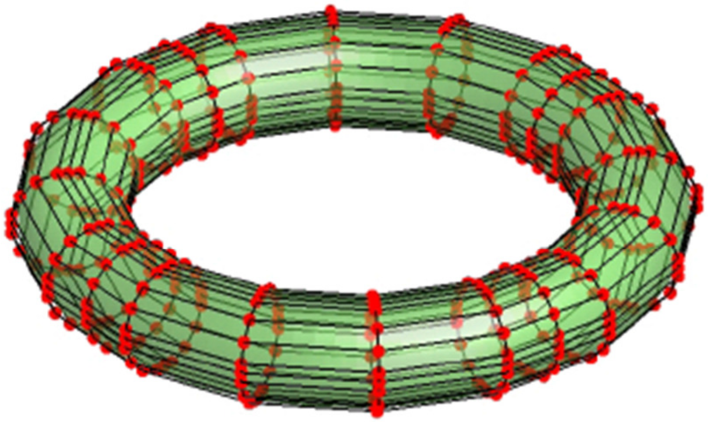
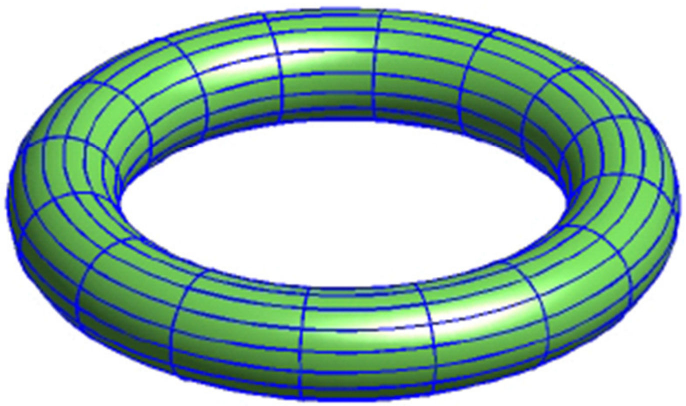
From B-Splines to NURBS

- NURBS – 2D-surfaces (tensor-product of 1D-shape functions)

Further *h*-refined Surface



Control net



Mesh

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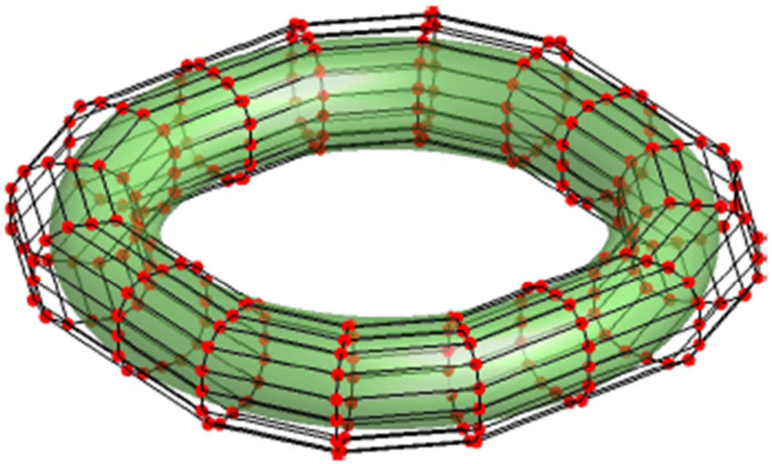
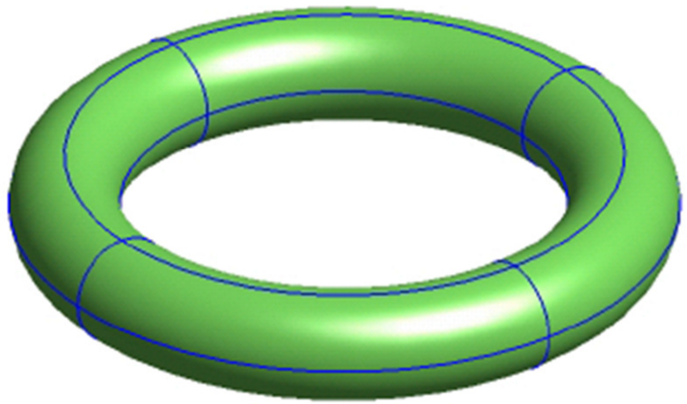
From B-Splines to NURBS

- NURBS – 2D-surfaces (tensor-product of 1D-shape functions)

Quartic *p*-refined Surface



Control net



Mesh

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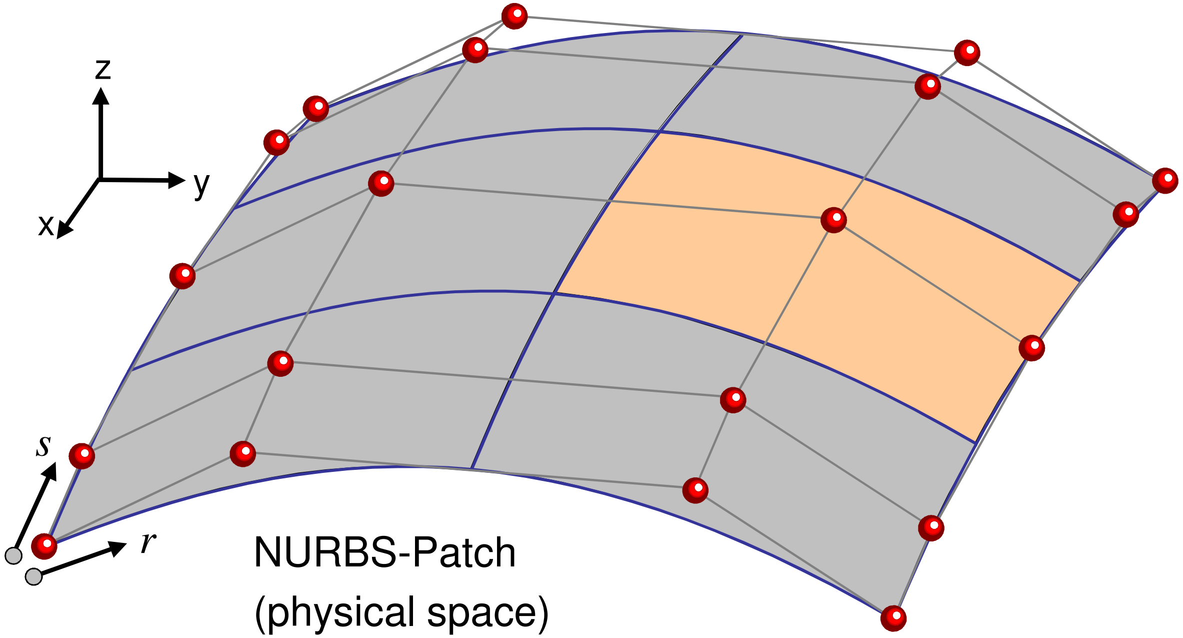
From B-Splines to NURBS - Summary

- B-Spline shape functions
  - recursiv
  - dependent on knot-vector and polynomial order
  - normally $C^{(P-1)}$-continuity
  - „Partition of unity“
  - h-/p-/k-refinement without changing the geometry → adaptivity
  - control-points are (in general) NOT part of the real surface

- NURBS
  - B-Spline shape functions + control-points with weigths
  - all properties for B-Splines apply to NURBS

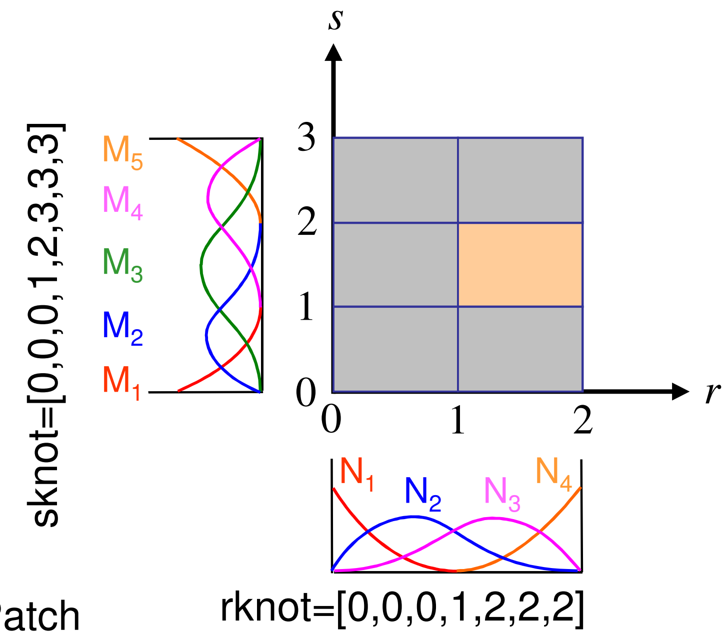
FEA with NURBS

- A typical NURBS-Patch and the definition of elements
  - elements are defined through the knot-vectors (interval between different values)
  - shape functions for each control-point

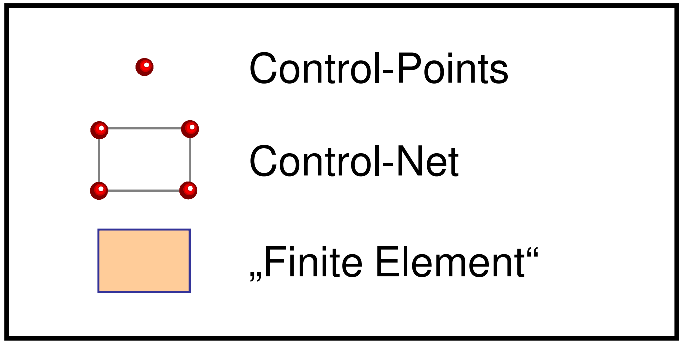


NURBS-Patch
(physical space)

polynomial order:
- quadratic in r-direction (pr=2)
- quadratic in s-direction (ps=2)



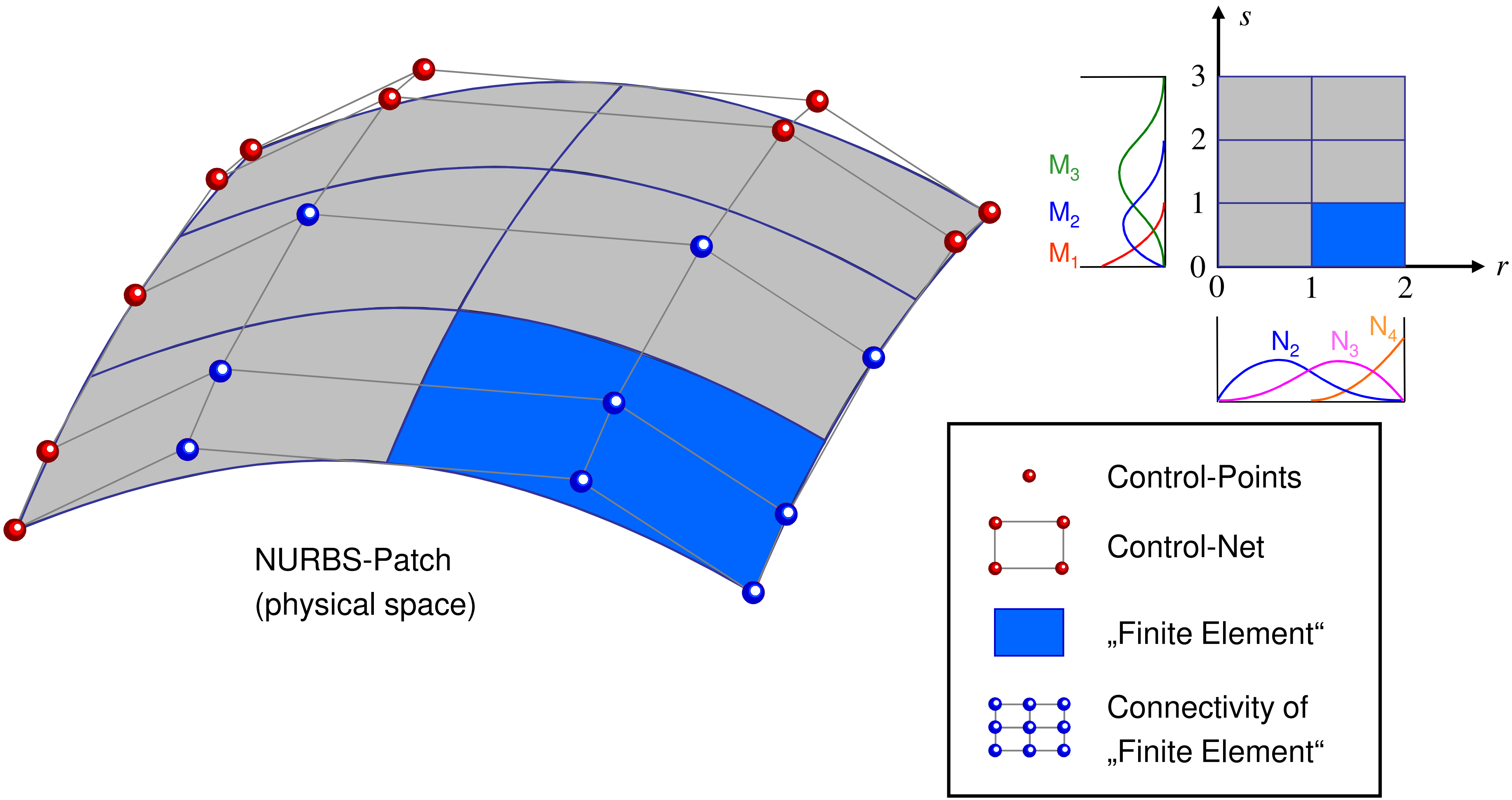
NURBS-Patch
(parameter space)



FEA with NURBS

- A typical NURBS-Patch – Connectivity of elements
  - Possible „overlaps“ (→ higher continuity!)
  - Size of „overlap“ depends on polynomial order (and on knot-vector)

NURBS-Patch
(parameter space)

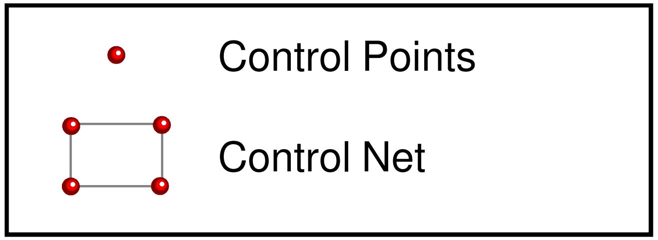


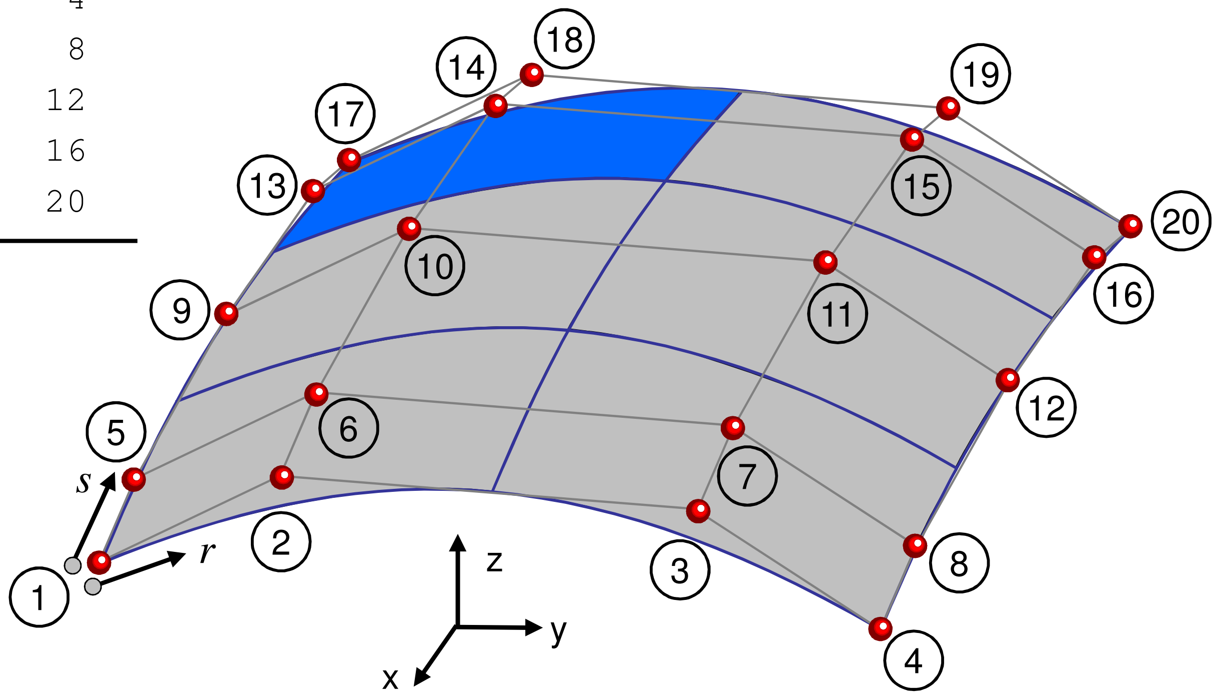
Present capabilities in LS-DYNA

- New Keyword: *ELEMENT_NURBS_PATCH_2D
  - definition of NURBS-surfaces
  - 4 different shell formulations with/without rotational degrees of freedom
- Pre- and Postprocessing
  - work in progress for LS-PrePost … current status (lspp3.1beta)
    → Visualization of 2D-NURBS-Patches
    → import IGES-format and construct *ELEMENT_NURBS_PATCH_2D
    → Modification of 2D-NURBS geometry
    → … much more to come!
- Postprocessing and boundary conditions (i.e. contact) currently with
  - Interpolation nodes
  - Interpolation elements
- Analysis capabilities
  - implicit and explicit time integration
  - eigenvalue analysis
  - other capabilities (e.g. geometic stiffness for buckling) implemented but not yet tested
- LS-DYNA material library available (including umats)

Present capabilities in LS-DYNA

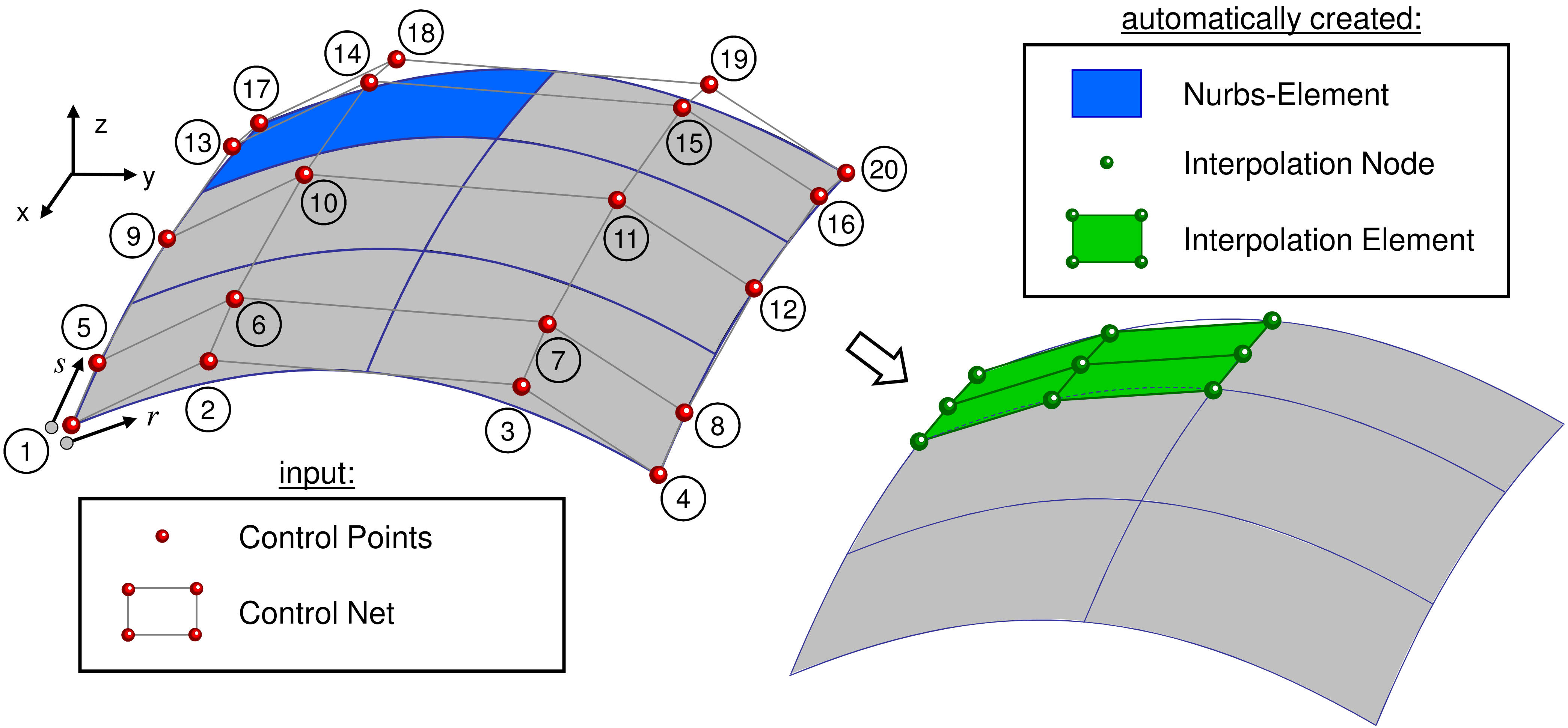
```
*ELEMENT_NURBS_PATCH_2D
$---+--EID----+--PID----+--NPR----+---PR----+--NPS----+---PS----+----7----+----8
        11        12         4         2         5         2
$---+--WFL----+-FORM----+--INT----+-NISR----+-NISS----+IMASS----+----7----+----8
         0         0         1         2         2         0
$rk-+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
       0.0       0.0       0.0       1.0       2.0       2.0       2.0
$sk-+----1----+----2----+----3----+----4----+----5----+----6----+----7----+----8
       0.0       0.0       0.0       1.0       2.0       3.0       3.0       3.0
$net+---N1----+---N2----+---N3----+---N4----+---N5----+---N6----+---N7----+---N8
         1         2         3         4
         5         6         7         8
         9        10        11        12
        13        14        15        16
        17        18        19        20
```



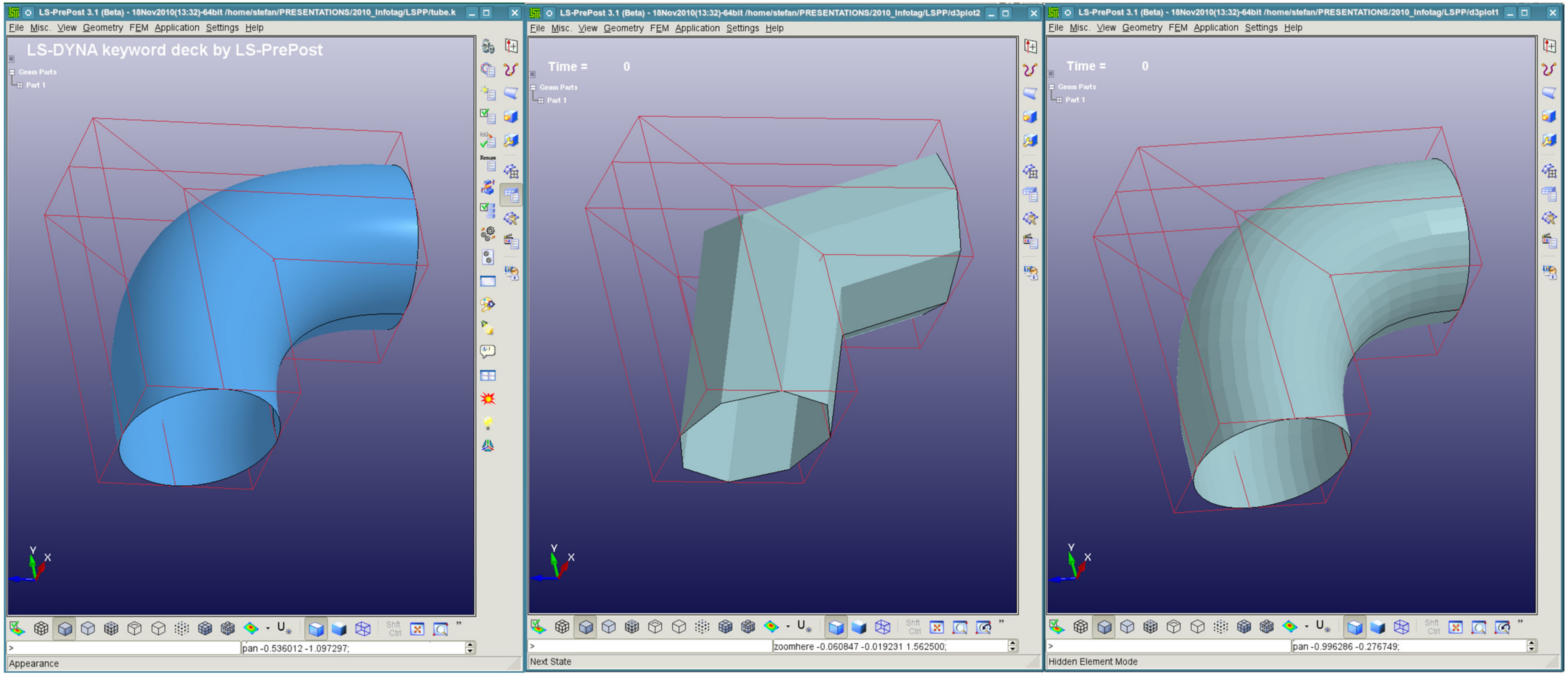


Present capabilities in LS-DYNA

```
...
$---+--WFL----+-FORM----+--INT----+-NISR----+-NISS----+IMASS
          0         0         1         2         2         0
...
```



Present capabilities in LS-DYNA



LSPP: Preprocessing
- control-net
- nurbs surface

LSPP: Postprocessing
- Interpolation nodes/elements

nisr=niss=2

nisr=niss=10

Examples - Vibrationanalysis

- Vibration of a Finite Elastic Rod with Fixed Ends

Comparison of C^0 FEM and C^{p-1} NURBS – Frequency Errors

Problem:

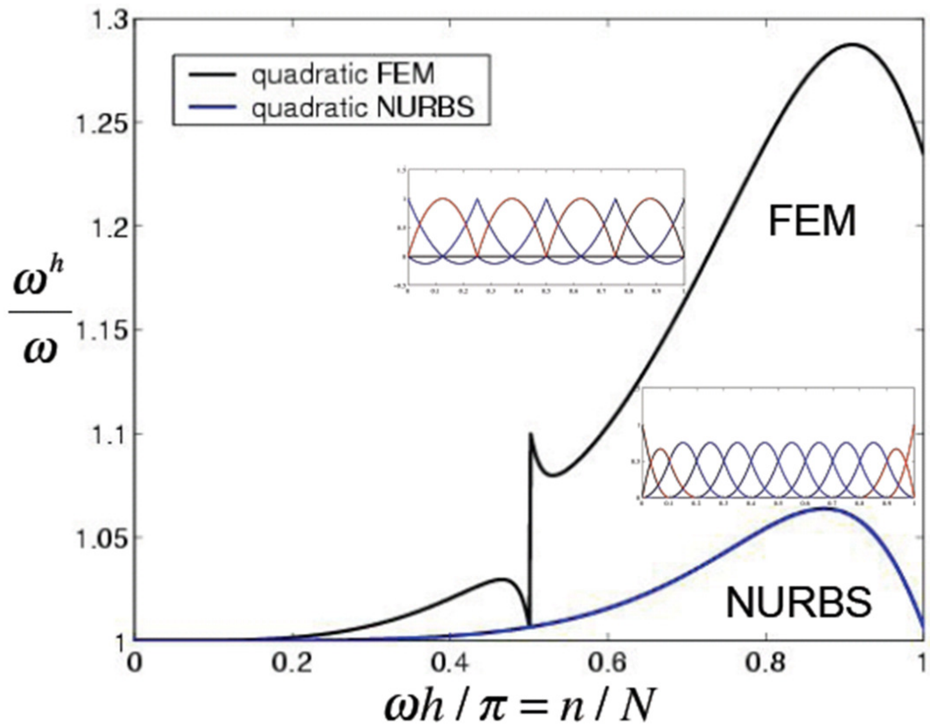
$$\begin{cases} u_{,xx} + \omega^2 u = 0 & \text{for} \quad x \in (0,1) \\ u(0) = u(1) = 0 \end{cases}$$

Natural frequencies:

$$\omega_n = n\pi, \quad \text{with} \quad n = 1,2,3,\ldots$$

Frequency errors:

$$\omega_n^h / \omega_n$$



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Examples - Vibrationanalysis

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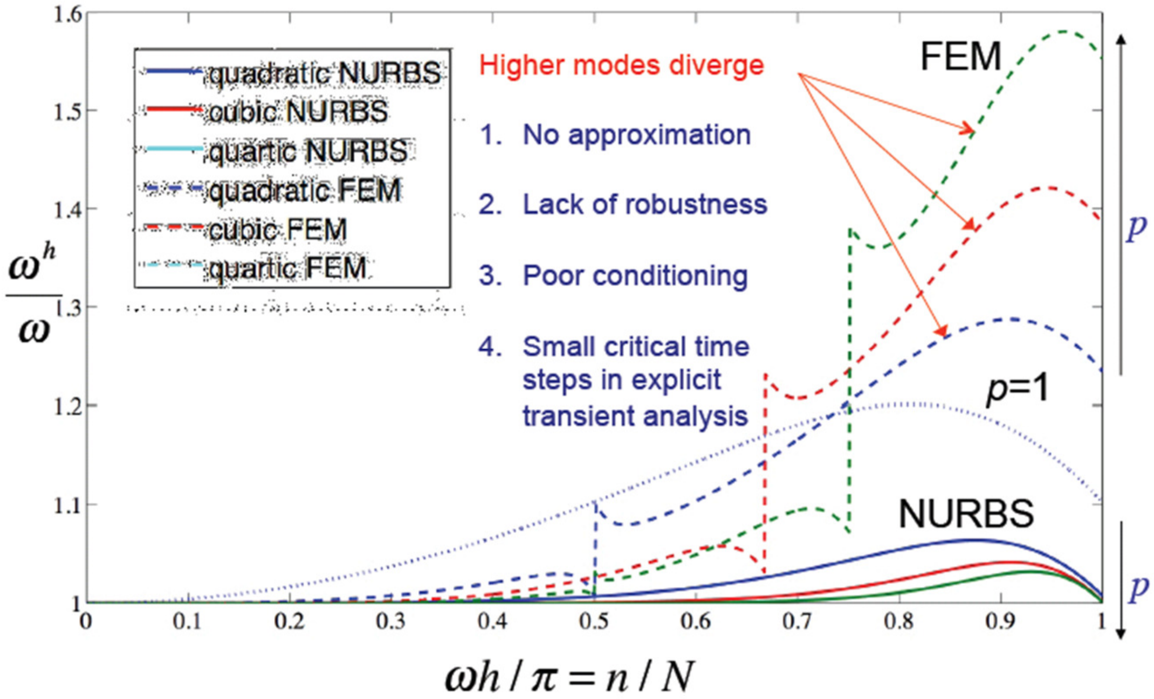

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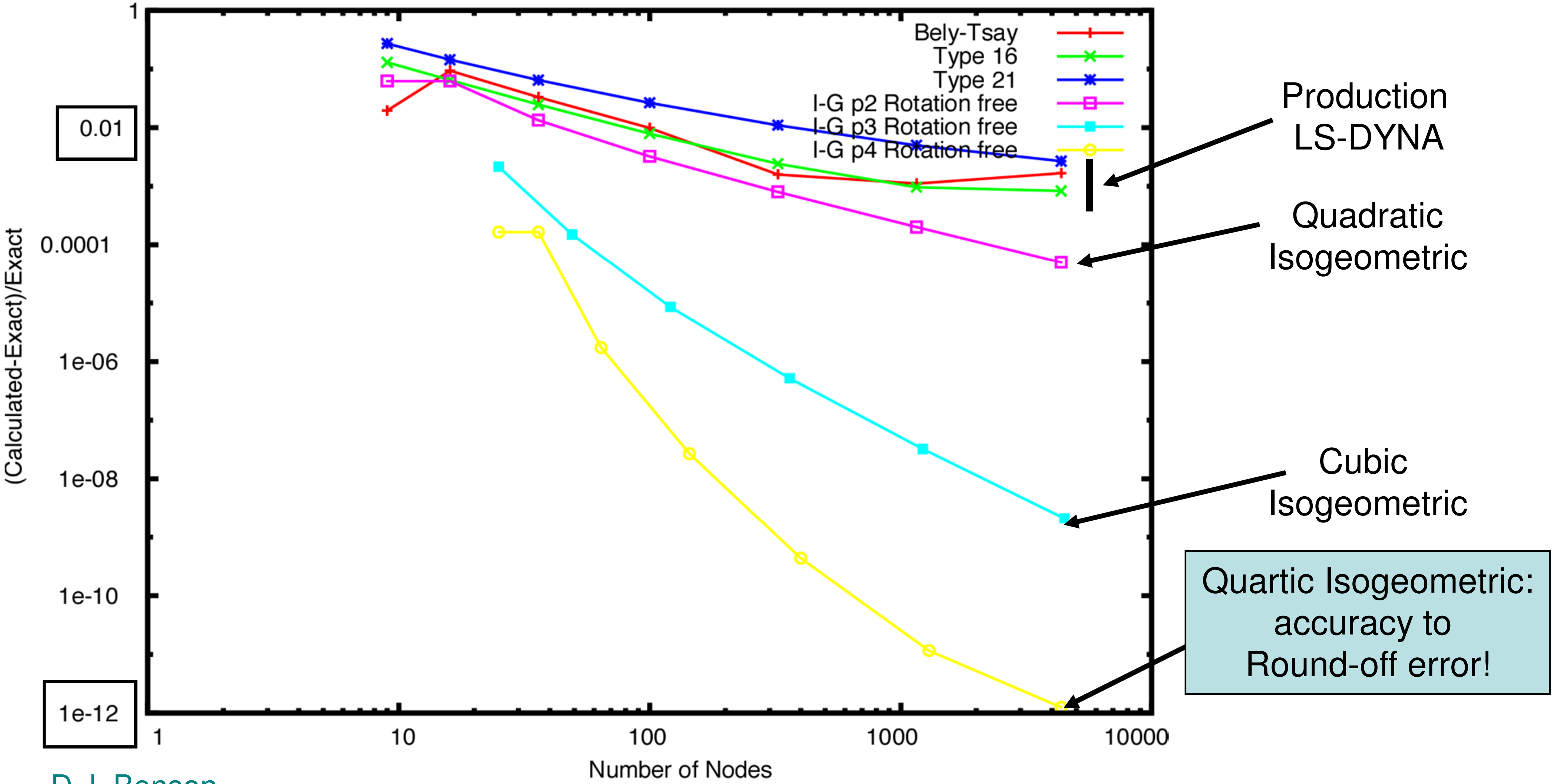
Examples - Vibrationanalysis

- Linear Vibration of a Square Plate (simply supported)



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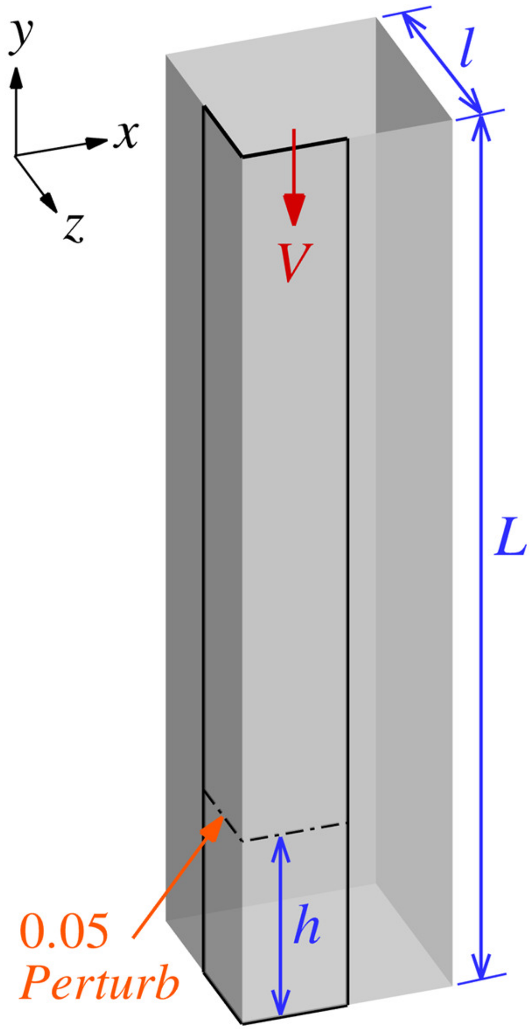
Examples - Buckling

- Square Tube Buckling



- Standard benchmark for automobile crashworthiness
- Quarter symmetry to reduce cost
- Perturbation to initiate buckling mode
- J_2 plasticity with linear isotropic hardening
- Mesh:
  - 640 quartic (P=4) elements
  - 1156 control points
  - 3 integration points through thickness

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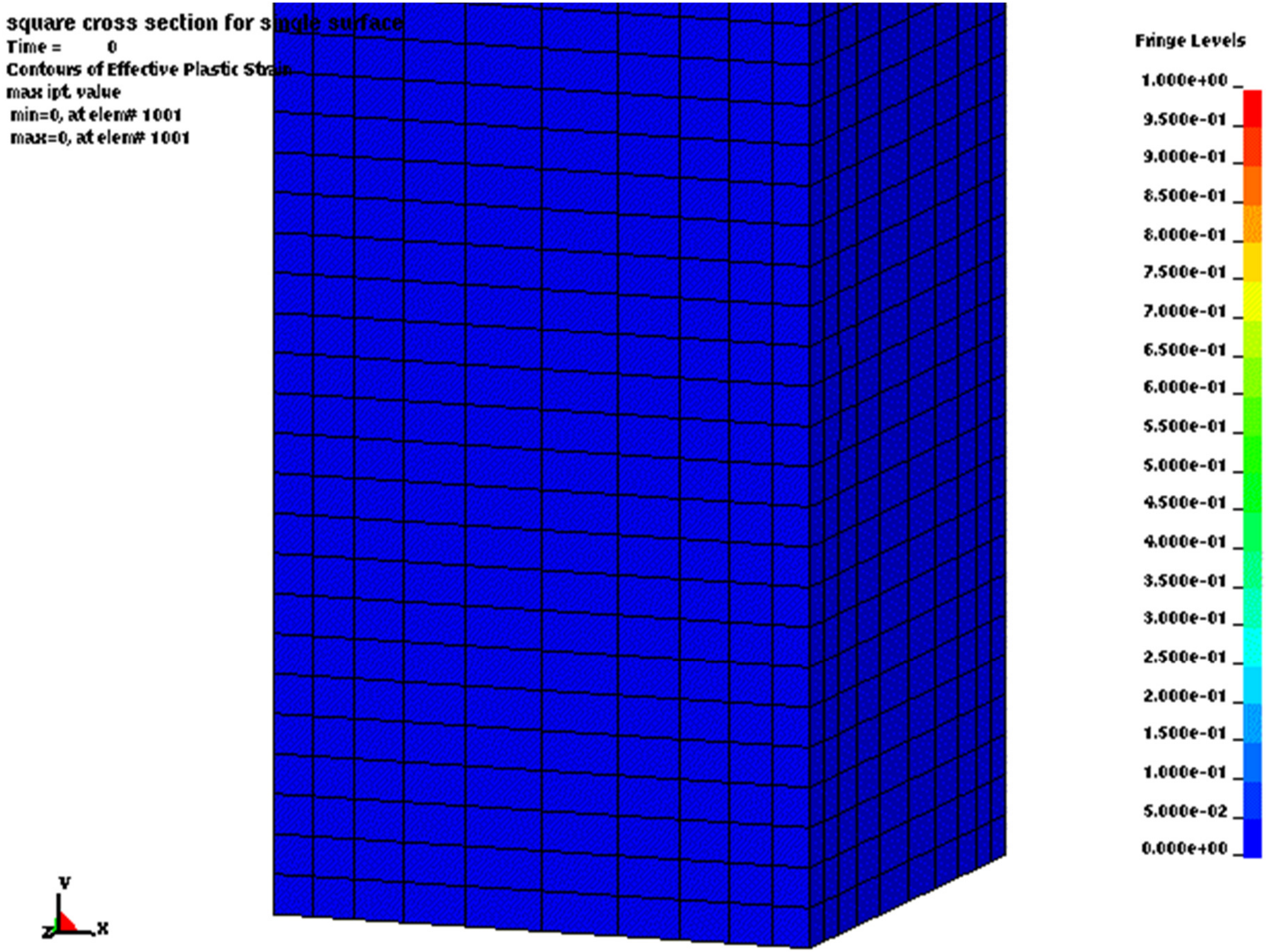
Square Tube Buckling (NURBS-Elements: p=4)



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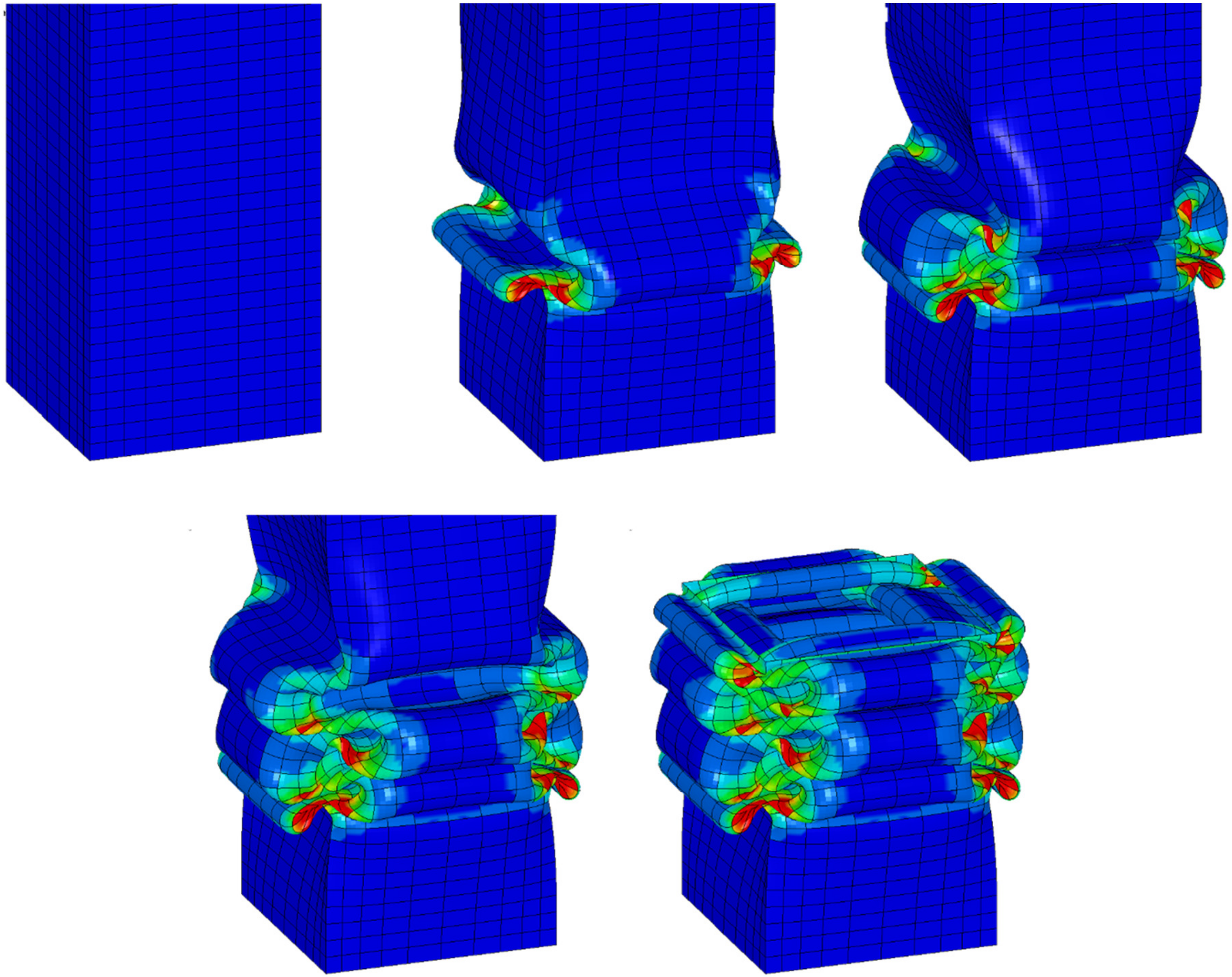
- Square Tube Buckling (NURBS-Elements: p=4)



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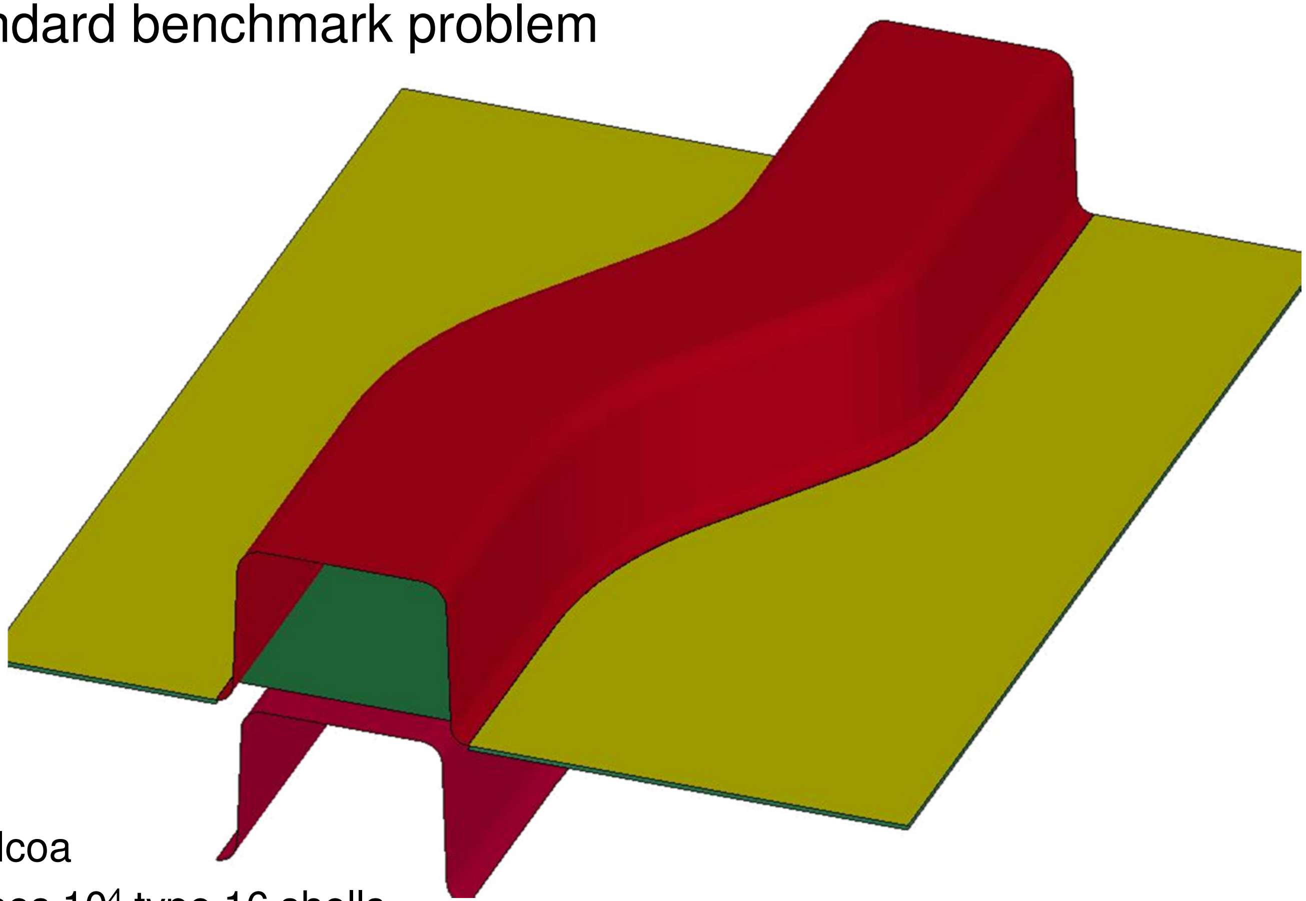
Examples – Sheet metal forming

- NUMISHEET: Standard benchmark problem



- Data
  - provided by R. Dick, Alcoa
  - Benchmark solution uses 10^4 type 16 shells

- No changes made to input except to replace blank with *isogeometric shells*

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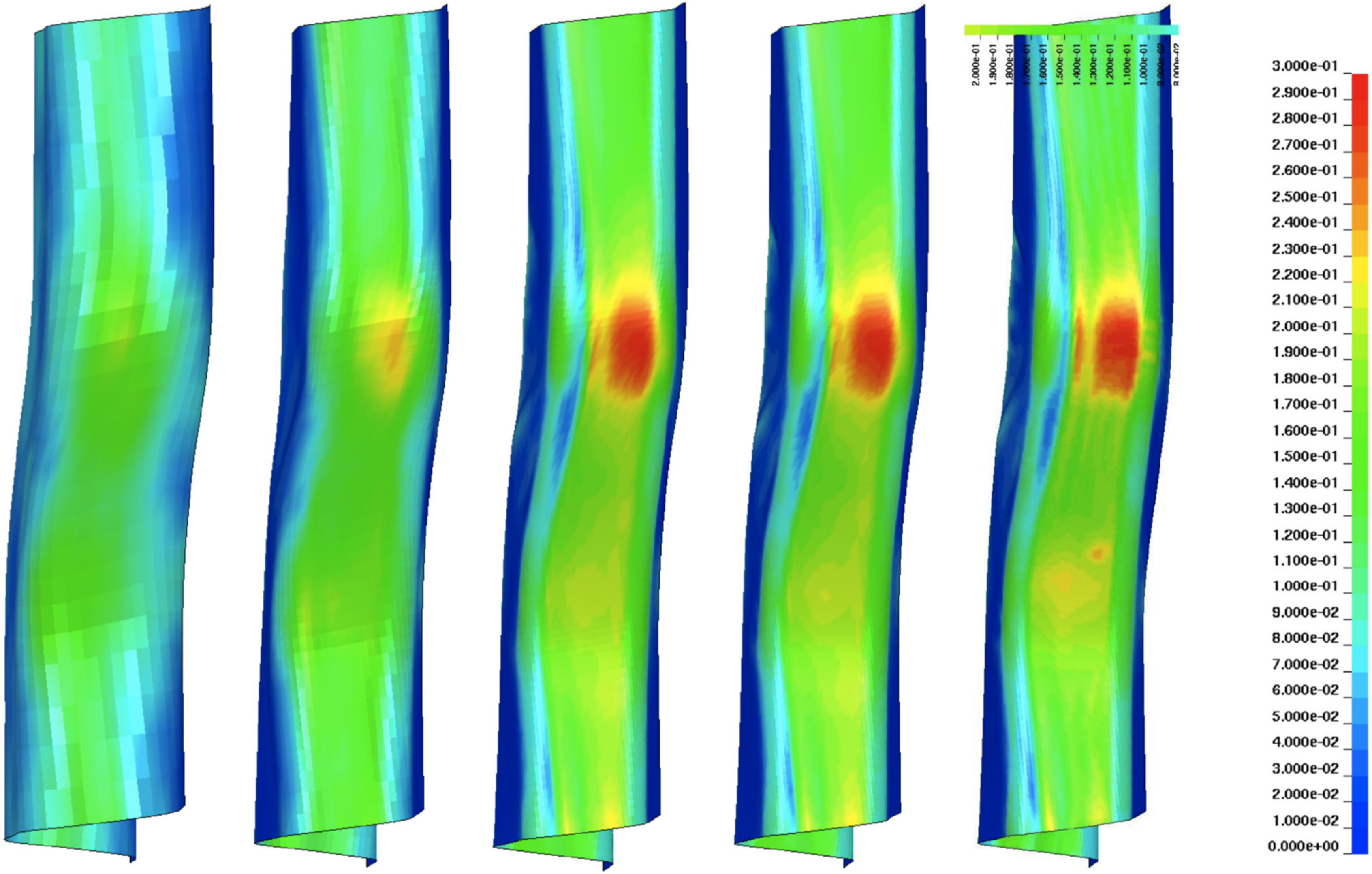
- Plastic Strain: Rotation-Free Shell vs. Reference Solution



240 NURBS | 1092 NURBS | 3840 NURBS | 7680 NURBS | 10000 Type 16

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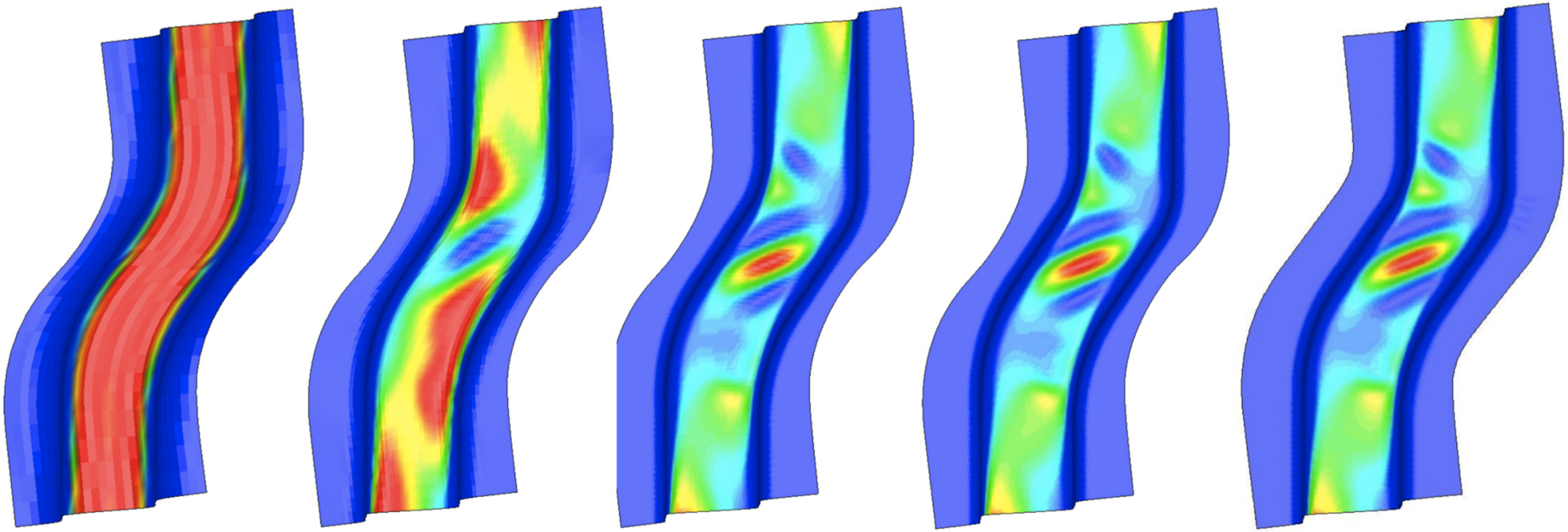
z-Displacements: Rotation Free Shell vs. Reference Solution



240 NURBS	1092 NURBS	3840 NURBS	7680 NURBS	10000 Type 16

Wrinkling mode is the right shape but inverted in comparison to others.

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Summary

- Higher order accurate isogeometric analysis can be cost competitive even in explicit dynamics
- Shell formulations without rotational DOF can be cost competitive to conventional formulations
  - Fewer DOF → faster computation
  - implicit → eliminate convergence problems with rotational DOF
- Accuracy is excellent
- Very robust

Outlook

- Further implementation
  - use NURBS for contact
  - make pre- and post-processing more user-friendly
  - … much more

Thank you!