Meshless Methods in LS-DYNA: An Overview of EFG and SPH

Yong Guo

Livermore Software Technology Corporation

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Outline

1. Introduction to Meshless Methods
2. EFG and SPH in LS-DYNA
3. EFG Applications
4. SPH Applications
5. Conclusions

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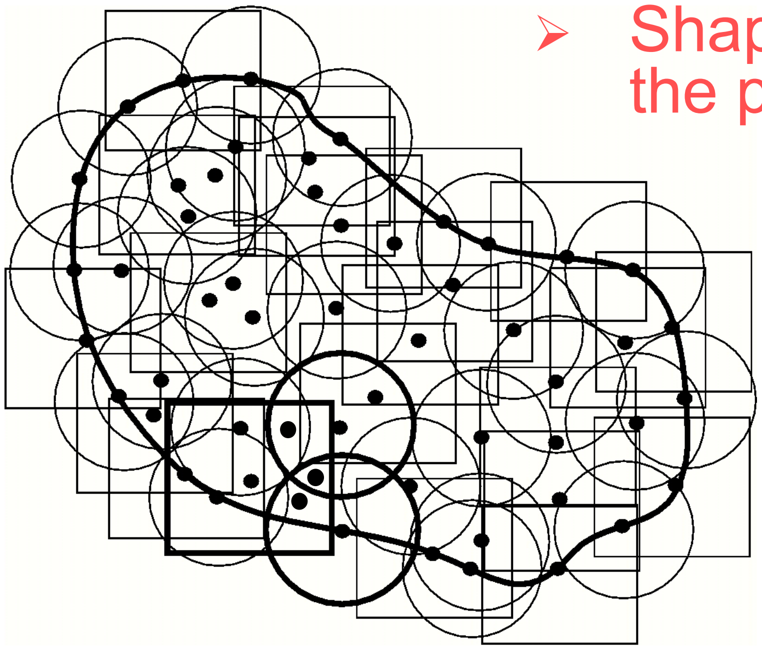
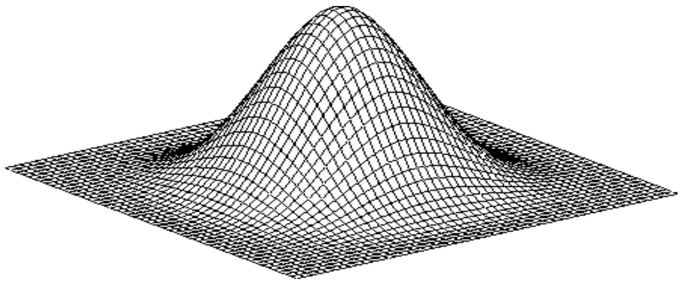
1. Introduction to Meshless Methods

What is the Meshfree/Meshless/ Particle Method?

- Physical domain is discretized with particles.
- Approximation solution is solved at the particles.
- Shape functions are constructed from the particles; no mesh required.





Meshfree Shape Function

History and Research Trend

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- **Meshfree Method**
  - Meshfree Collocation Method
    - **Smooth Particle Hydrodynamics (SPH) [Lucy1977, Monaghan 1980, Libersky1993]**
    - Finite Point Method [Onate et al.1996]
  - Meshfree Galerkin Method
    - **Element Free Galerkin (EFG) [Belytschko et al. 1994]**
    - **Reproducing Kernel Particle Method (RKPM) [Liu et al. 1995]**
    - Partition of Unity Method [Babuska and Melenk 1995]
    - HP-Clouds [Duarte and Oden 1996]
    - Free-Mesh Method [Yagawa et al. 1996]
    - Natural Element Method [Sukumar et al.1998]
    - Meshless Local Petrov-Galerkin Meshfree Method(MLPG) [Atluri et al.1998]
    - Local Boundary Integral Equation (LBIE) [Atluri et al. 1998]
    - Finite Sphere Method [Bathe 1998], **Particle Finite Element Method [Idelsohn et al.2004]** …
  - Meshfree Least Square Method, …
- **(FEM, Control Volume, BEM …) + Meshfree Method**
    - **Coupled FEM/Meshfree Method [1995]**
    - Extended FEM Method [1999]
    - Finite Particle Method [1999]

Classification of Particle Methods

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Particle Method

- **Continuum**
  - **Implicit Meshfree Galerkin RKPM, EFG, …**
  - **Explicit; Hydrocode**
    - Meshfree Collocation SPH, Finite point ..
    - Meshfree Galerkin RKPM, EFG, …
- **Discrete**
  - Molecular Dynamics
  - Others: Lattice Boltzmann, Discrete Element, …

Classification of Transient Dynamic Code

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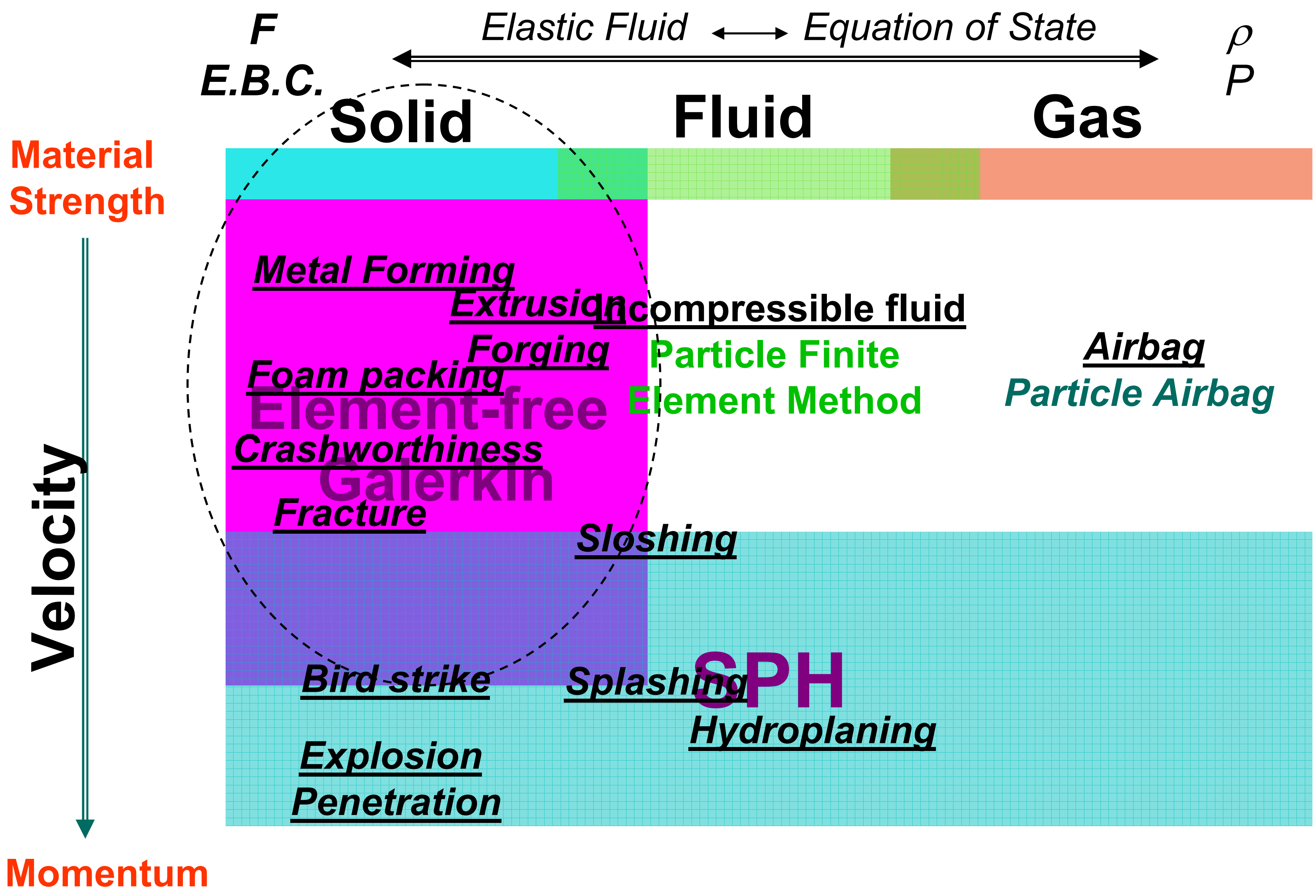
Hydrocode

- **Lagrangian Hydrocode**
  - **FEM explicit**
    (LS-DYNA, PAM-CRASH, ABAQUS …) structure
  - **Smooth Particle Hydrodynamics (SPH)**
    (LS-DYNA, PAM-CRASH, PRONTO3D …)
    structure, fluid, fluid-structure
  - **Mesh-free Galerkin Explicit Method**
    (LS-DYNA, TAHOE, DYNA) structure
- **Semi-Lagrangian (Eulerian) Hydrocode; Adaptivity**
  - **Mesh-free Galerkin Explicit Method**
    (LS-DYNA) structure, fluid, fluid-structure, metal forming adaptivity
- **Arbitrary Lagrangian-Eulerian Hydrocode**
  (LS-DYNA, MSC/DYTRAN,ALE3D,CALE …) fluid-structure interaction
- **Eulerian Hydrocode**
  (LS-DYNA, MSC/DYTRAN, ALE3D, DYSMAS …) fluid flow

Meshfree Application Range

"Meshfree Solution looking for problems"



Problem Looking for Meshfree Solution

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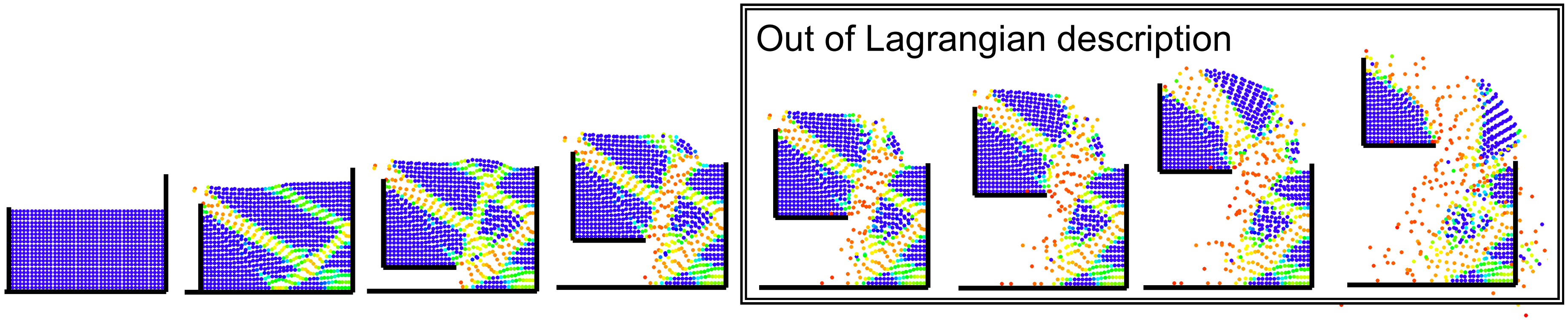
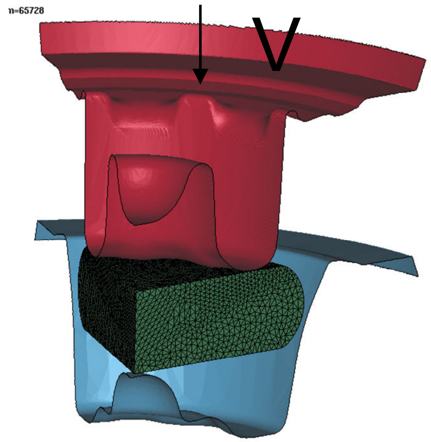
Multi-Physics : *shear band* + ***history dependent large deformation*** + *failure*

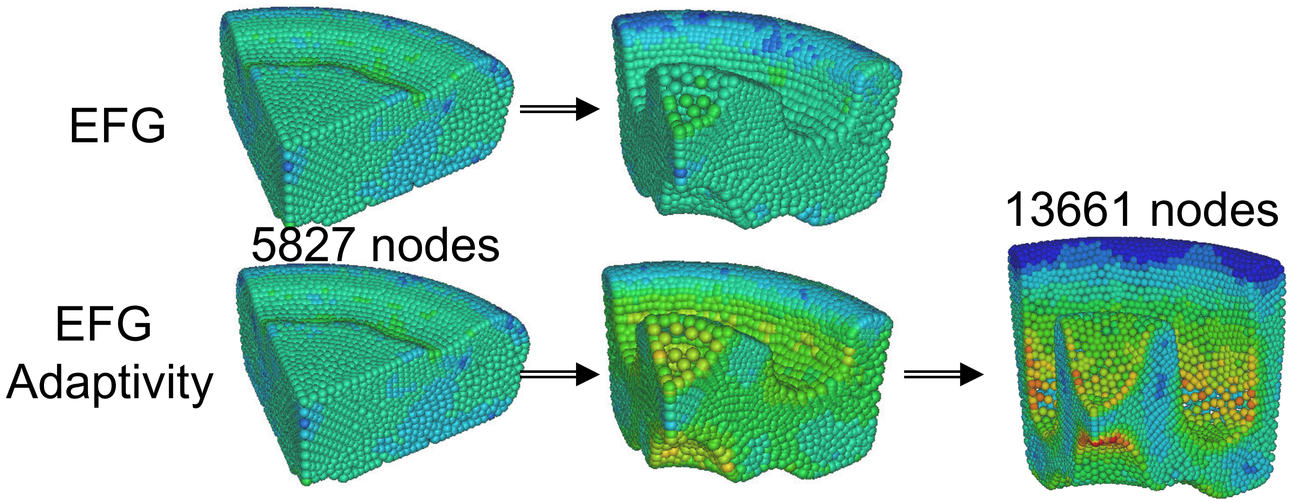


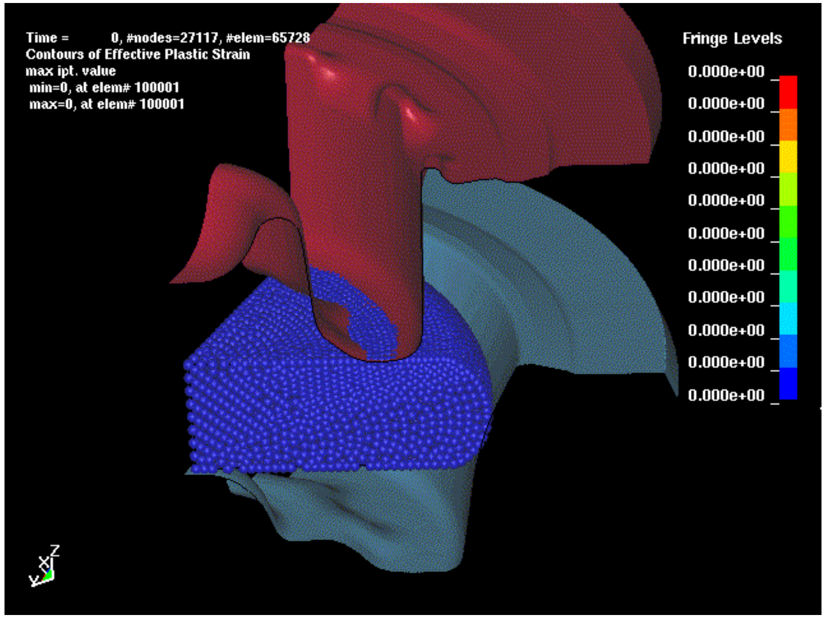
Numerical : *multi-resolution* + ***avoid mesh tangle*** + *failure mechanics*

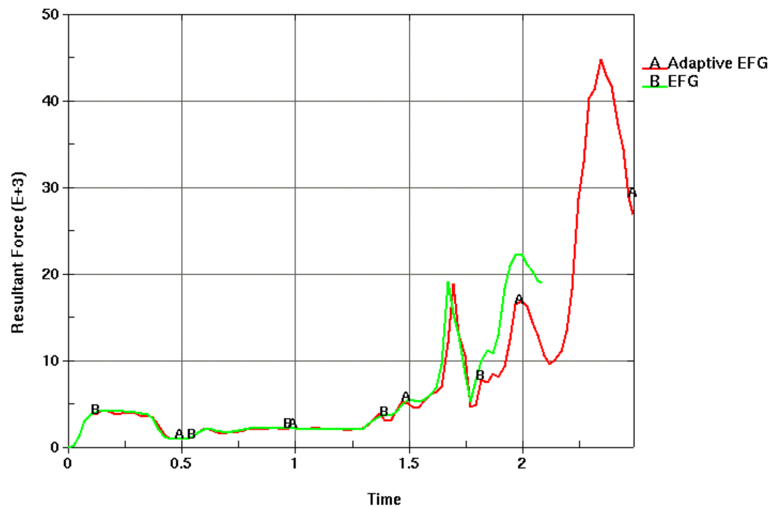
Spectral element method	ALE	**Damage mechanics**
The variationl multiscale method	Eulerian	**Cohesive model**
Partition of unity method	**Adaptivity**	Discrete element method
(strong discontinuity)	**Mesh-free**	

Large Deformation Simulation









Force

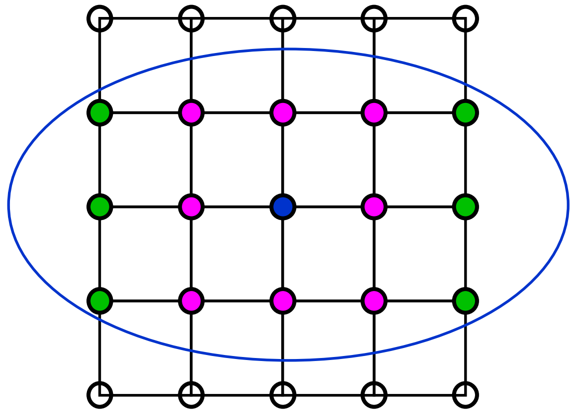
Overview on Element Free Galerkin Method (EFG)

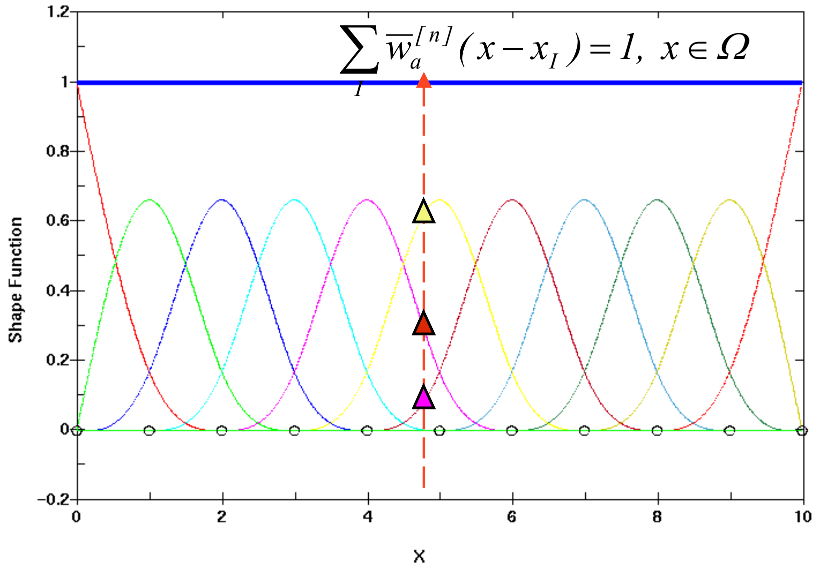
$$u^h(x) = \sum_{I=1}^{NP} \overline{w}_a^{[n]}(x - x_I)\, u(x_I)\, \Delta x_I$$

$$\overline{w}_a^{[n]}(x - x_I) = \overbrace{\underbrace{\boldsymbol{H}^{[n]^T}(0)\boldsymbol{M}^{[n]^{-1}}(x)\boldsymbol{H}^{[n]}(x - x_I)}_{n-th\text{ order completeness}}\ \underbrace{w_a(x - x_I)}_{\text{weighting function}}}^{\text{Moving Least-Squares approximation or Reproducing Kernel approximation}}$$

$$\overline{w}_{aI}^{[n]}(x_J) \neq \delta_{IJ}$$

$$\boldsymbol{A}^{-T}\boldsymbol{M}\boldsymbol{A}^{-1}\Delta\ddot{\boldsymbol{d}} + \boldsymbol{A}^{-T}\boldsymbol{K}\boldsymbol{A}^{-1}\Delta\boldsymbol{d} = -\boldsymbol{A}^{-T}\boldsymbol{R}$$





- Higher-order approximation
- More neighboring nodes
- Complicated domain integration
- Special treatment on B.C.
- Special treatment in nearly incompressible limit

Basic SPH Equation of Motion

Strong Form

$$\frac{d\rho}{dt} = -\rho \frac{\partial v^{\beta}}{\partial x^{\beta}}$$

$$\frac{dv^{\alpha}}{dt} = -\frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^{\beta}}$$

$$\frac{dE}{dt} = -\frac{\sigma^{\alpha\beta}}{\rho}\frac{\partial v^{\alpha}}{\partial x^{\beta}}$$

$$v(x) = Tu = \int_{-\infty}^{\infty} w_a(x-s)u(s)ds$$

Kernel approximation ⟹

Weak Form

$$\frac{d\rho_i}{dt} = \rho_i \sum_j \frac{m_j}{\rho_j}\left(v_i^{\beta} - v_j^{\beta}\right)W_{ij,\beta} \quad ✔$$

$$\rho_i = \sum_j m_j W_{ij}$$

$$\frac{dv_i^{\alpha}}{dt} = -\sum_j m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right)W_{ij,\beta} \quad ✔$$

$$\frac{dv_i^{\alpha}}{dt} = -\sum_j \frac{m_j}{\rho_i \rho_j}\left(\sigma_i^{\alpha\beta} \pm \sigma_j^{\alpha\beta}\right)W_{ij,\beta}$$

$$\frac{dE_i}{dt} = \frac{\sigma_i^{\alpha\beta}}{\rho_i^2}\sum_j m_j \left(v_i^{\alpha} - v_j^{\alpha}\right)W_{ij,\beta} \quad ✔$$

✔ in LS-DYNA 960, 970, 971

Comparison of SPH and EFG

SPH	EFG
Explicit Lagrangian Collocative method	Explicit/implicit Lagrangian/Eulerian Galerkin method
Impact/penetration compressible flow	Manufacturing Crashworthiness Fracture
2D and 3D	2D, 3D and shell
Efficient Difficult Boundary condition	Accurate Slow

Computational Challenges

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Advantages of Using Meshfree Method

- ✓ Large material distortion, e.g., crashworthiness, hyper-velocity impact
- ✓ Moving boundaries, free surface, e.g., fluid and structure interaction
- ✓ Adaptive procedure,e.g., forging and extrusion
- Multiple-scale phenomenon, e.g., shear band
- ✓ Moving discontinuities, e.g., crack propagation

Disadvantages of Using Meshfree Method

- High CPU and memory in implicit/explicit analysis (EFG)
- Complicated in parallel (EFG)
- Tensile instability and zero-energy mode (SPH)
- Difficult essential boundary condition treatment (SPH)
- Does not pass *Patch Test* (most mesh-free methods); Dispersed wave properties in coarse model

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2. EFG and SPH in LS-DYNA

Element-Free Galerkin Method in LS-DYNA

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- ❑ Applied to solids, shell and fluid (trial version)
- ❑ Fully coupled with finite element model
- ❑ Easy change from finite element formulation to EFG formulation
- ❑ Various formulations for industrial applications
- ❑ More effort spent on improving efficiency
- ❑ Available in SMP and MPP; Explicit/Implicit solver

Representative EFG Applications

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EFG Basic Features

1. Smoother stress and strain
2. Less sensitive to the discretization
3. No hourglass control
4. Higher accuracy
5. Natural in adaptivity
6. Higher CPU
7. More memory
8. Difficult in parallel
9. More difficult in theory
10. More developments and refinements on theory

- **Solid**
  EFG Plane strain #43
  EFG Axisymmetric #44
  EFG 3D solid #41
  #42
  - Rubber industry
  - Highly compressible foam
  - Defense and safety design
  - Human dummy and barrier
  - Adaptive forging simulation
  - Fracture simulation
- **Shell**
  EFG shell #41
  EFG shell #42
  - Metal Forming
  - Crashworthiness
- **Fluid**
  EFG 3D fluid #41
  (limited version)
  - Compressible fluid flow

Current EFG Formulations for Industrial Applications

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- Metal materials in Forging/Extrusion analysis: **Adaptive formulation**
- Foam materials: **Semi-Lagrangian kernel formulation**
- Rubber materials: **Lagrangian kernel formulation**

} **Stabilized Method**

- Meshfree Shell: **Lagrangian kernel, adaptivity**
- Quasibrittle material fracture: **Strong discontinuities formulation**
- E.O.S. materials: **Eulerian kernel formulation (trial version)**

Current Practice in Crashworthiness

- CPU time

  RI FEM: SR FEM : Meshfree (8 I.P.) = 1: 4: 10

- Stabilized Meshfree formulation (1 I.P.) +

  Switch to fully integrated (8 I.P.)

  RI FEM: SR FEM : Meshfree (8 I.P.) = 1: 4: 3~5

- 99% > Compression > 85% requires

  Formulation change to Eulerian kernel + data remapping or

  Smooth meshfree approximation

Current Practice in Adaptive EFG

- CPU time

  FEM: Meshfree = 1: 2~3

- Global refinement behaves more robust than local refinement

- Adaptivity can be controlled by fixed frequency or interactively activated by distortion triggers.

- Mass scaling is allowable.

- Element erosion is allowed and surface reconstructed for metal cutting.

Smooth Particle Hydrodynamics in LS-DYNA

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- A Lagrangian collocative method – explicit
- Efficient
- Choices of formulations to improve accuracy
- Applied for Impact/Penetration, In/compressible Flow
- Most material laws and all E.O.S are available
- Coupled with Finite Elements through 3 contacts or hybrid element
- Implemented in MPP version

IFORM : Particle approximation theory

0 : standard formulation (default)

1 : renormalized formulation

2 : symmetric formulation

3 : symmetric formulation with renormalization

4 : elliptical formulation

5 : fluid formulation

6 : fluid formulation with renormalization

3. EFG Applications

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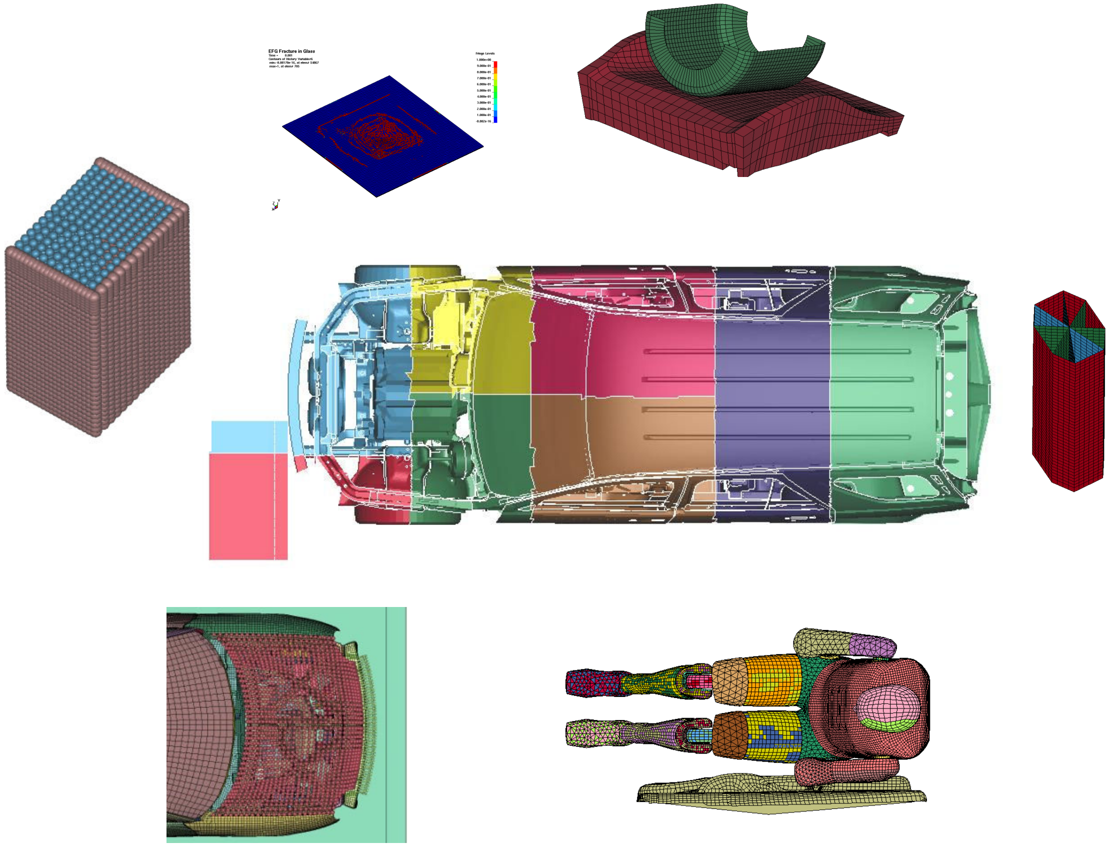
Robustness > Efficiency > Accuracy ?

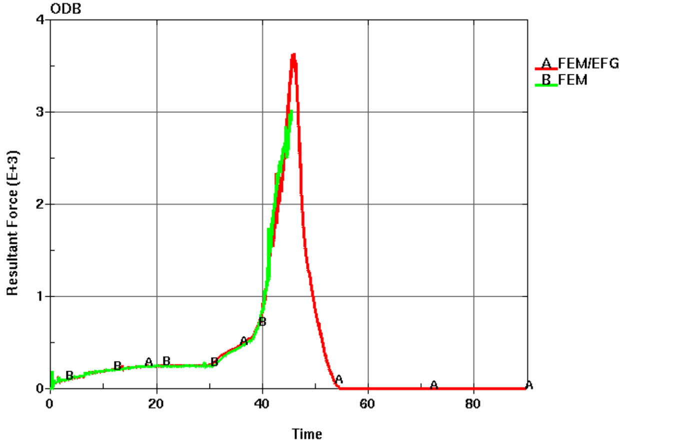
Meshfree Components in Crashworthiness Model

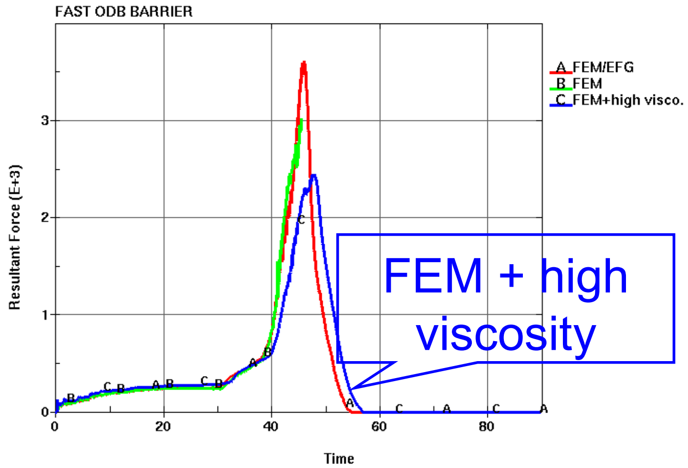
- Barriers; bumpers
- Car seats
- Human dummies
- Crush tubes
- Windshields
- Fuel slashing
- …



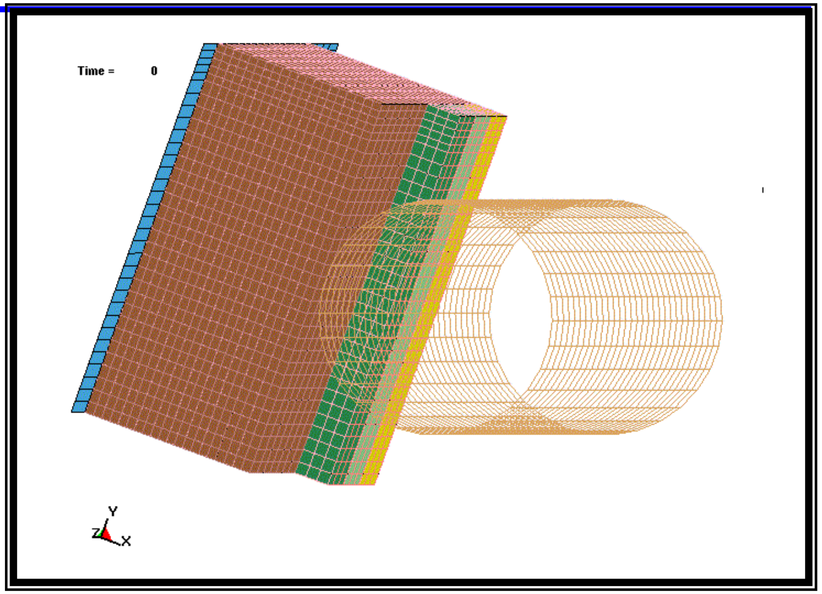
ODB Simulation

Impact Force

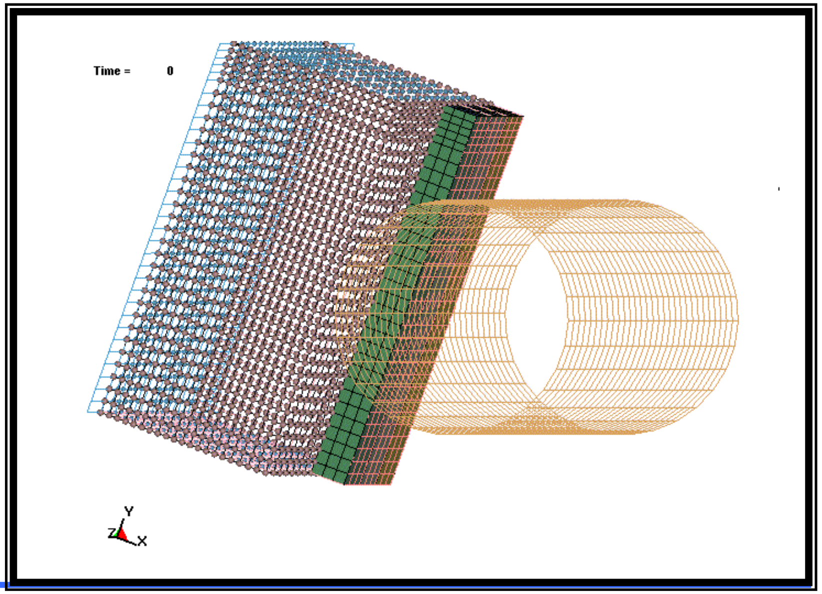




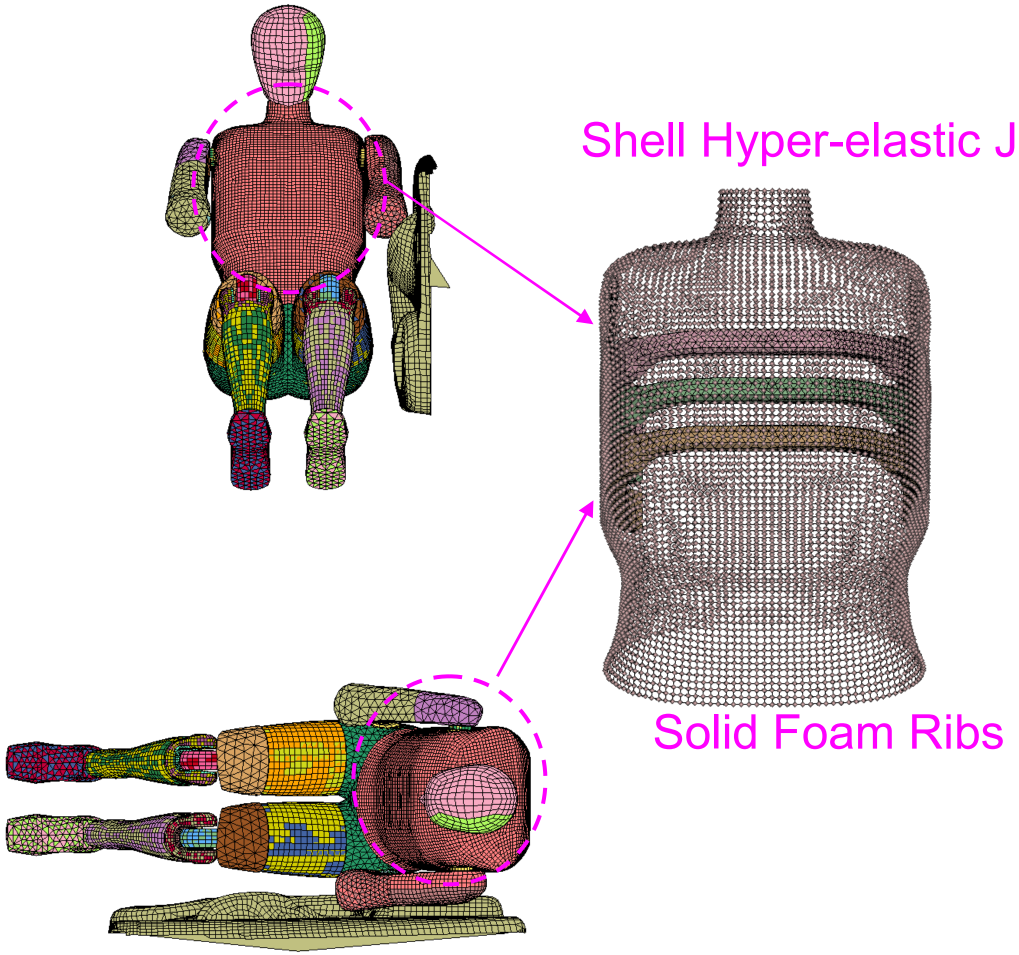
FEM

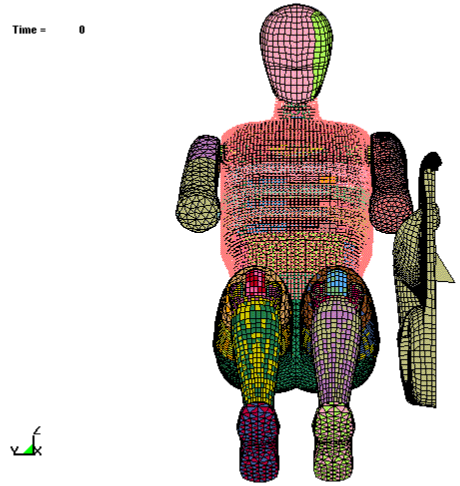


FEM/Meshfree



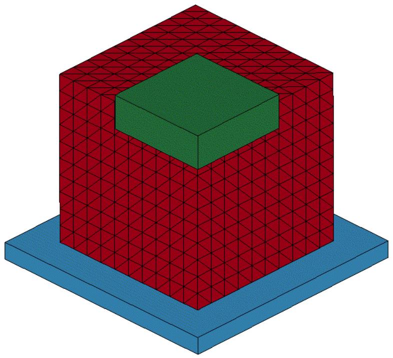
Dummy with Side Impact

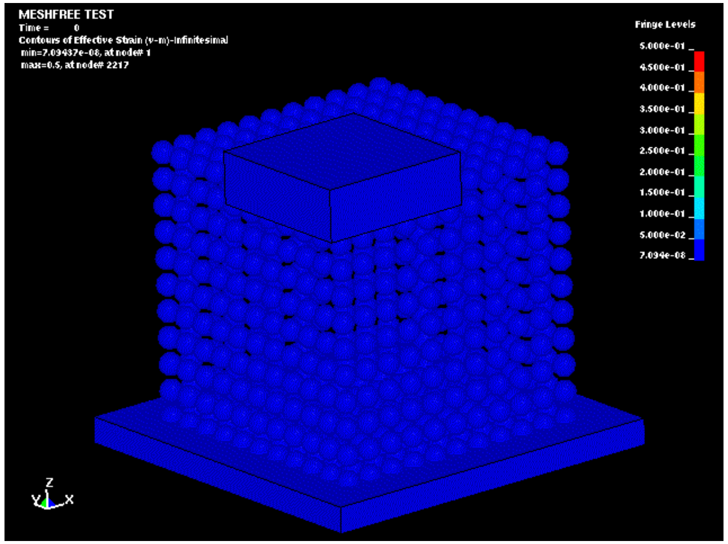


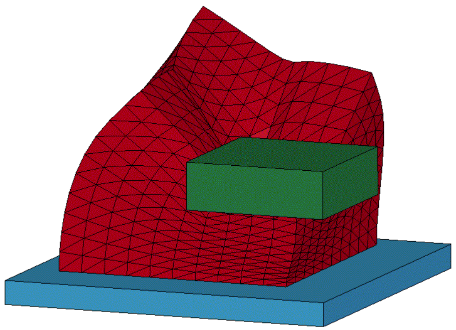


Courtesy of GM

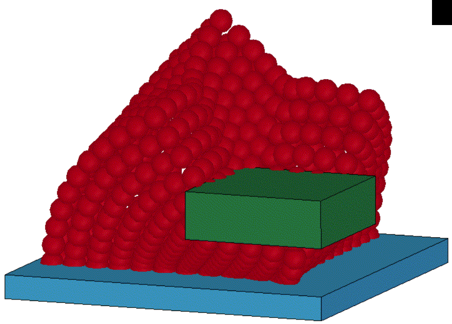
Foam Compression Simulation



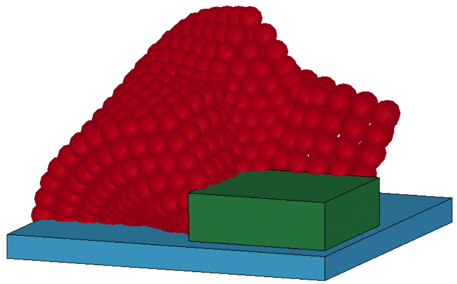




FEM



EFG



EFG +
Semi-Lagrangian Kernel

Foam materials : Semi-Lagrangian kernel

Rubber Bushing Analysis
using Stabilized EFG Method

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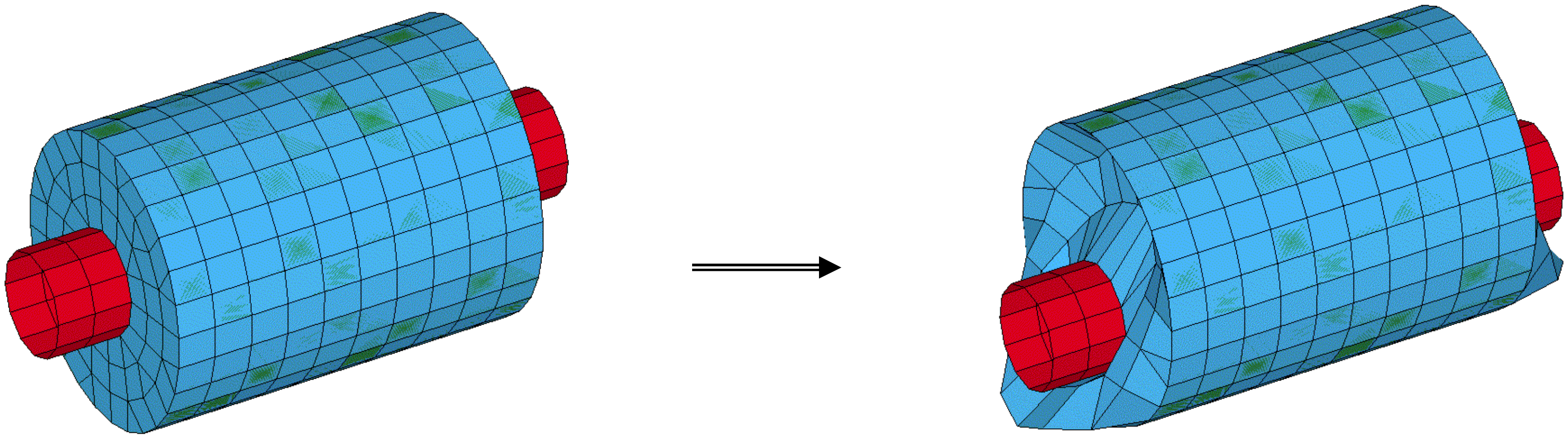
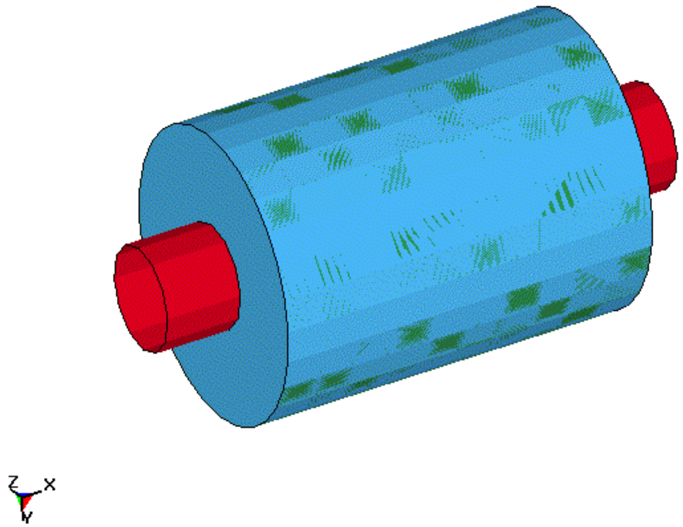


Time = 0

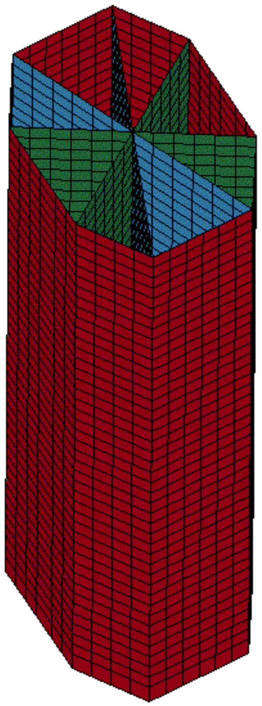


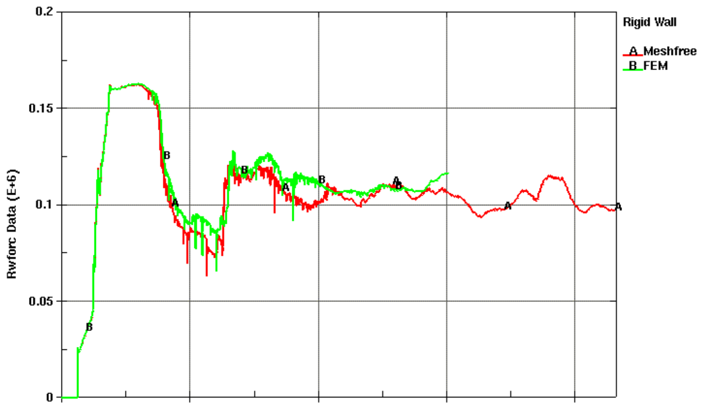
Mooney-Rivlin Rubber
Poisson's =0.4995
Stabilized EFG explicit analysis
Switched to full integration at t=100
Completion at t=150

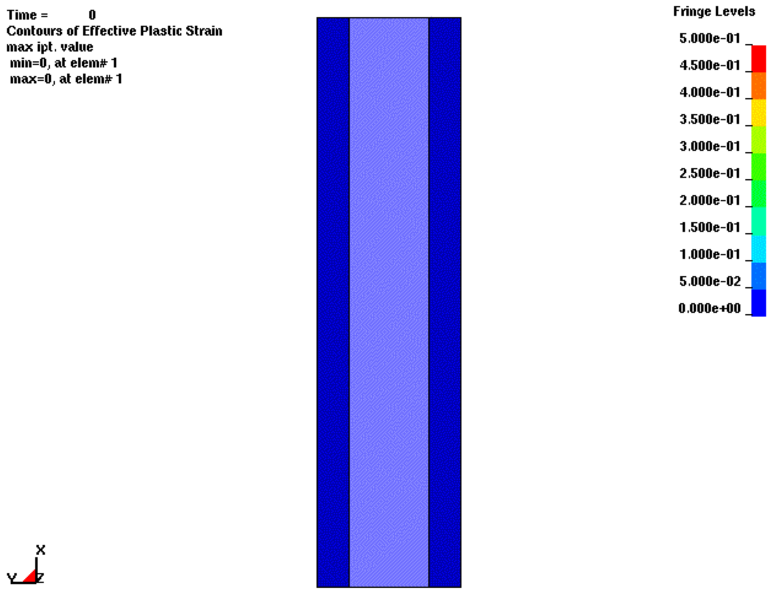
CPU comparison at t=50

Methods	S-FEM(#1)	F-FEM(#2)	EFG	S-EFG
CPU	1.0	4.1	5.4~12.9	2.6

Crushing Tube

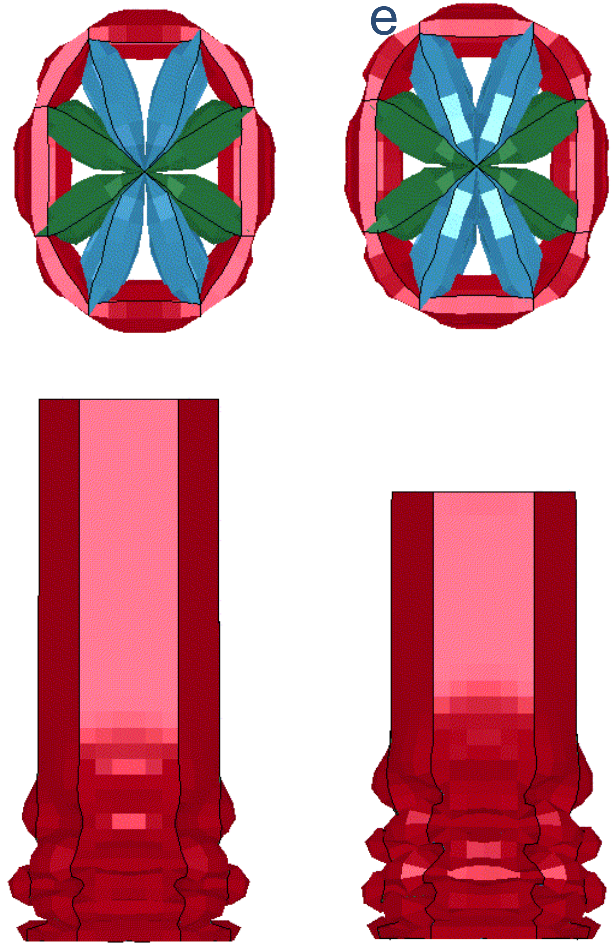






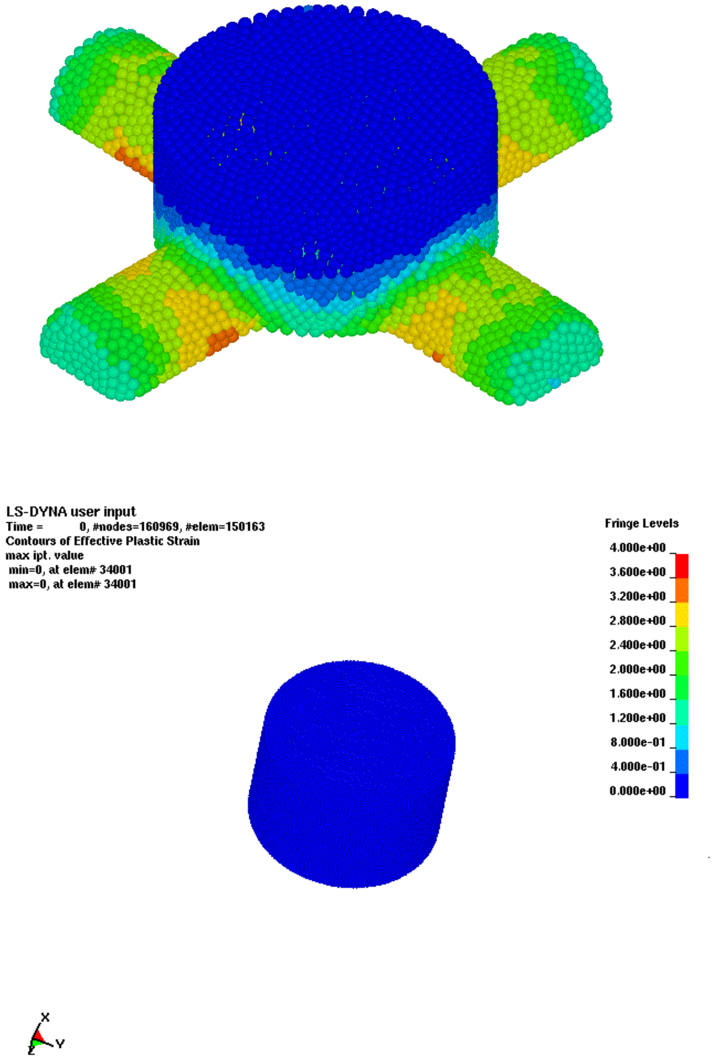
FEM

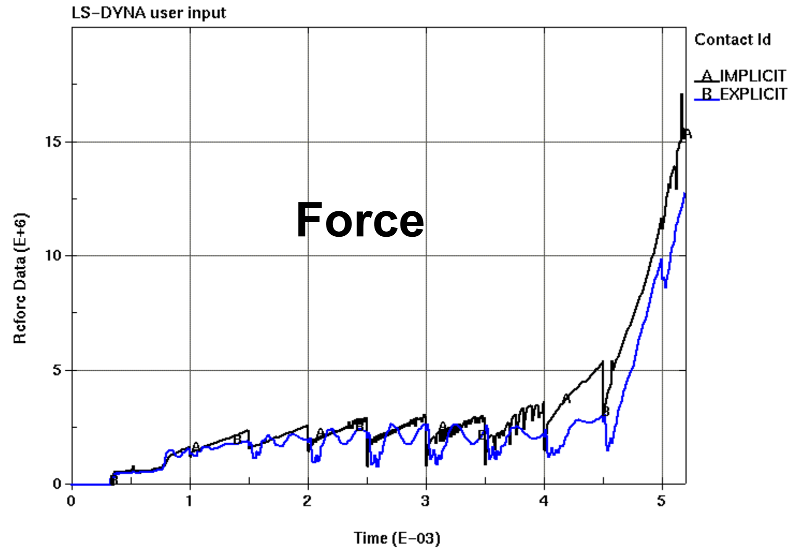
Meshfree

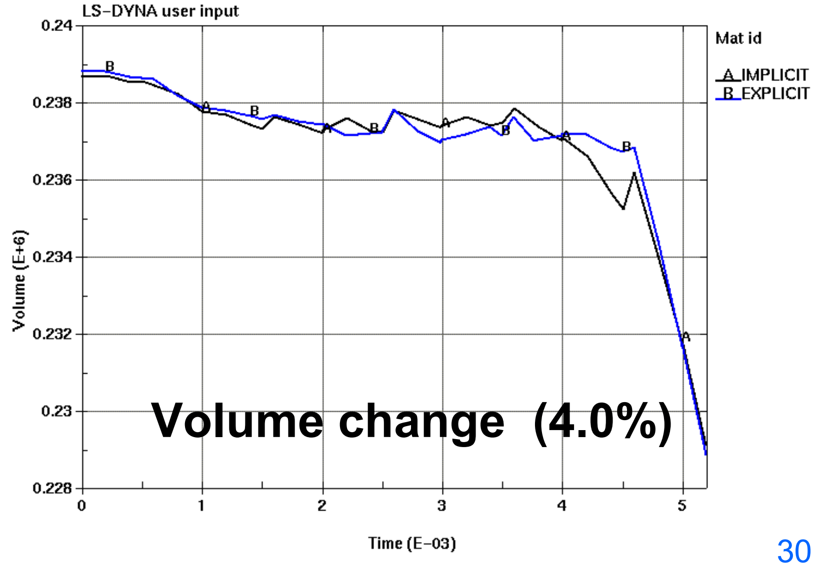


Cross Joint Forging

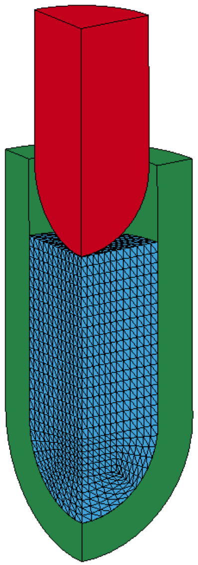
Comparisons of Implicit and Explicit Analysis

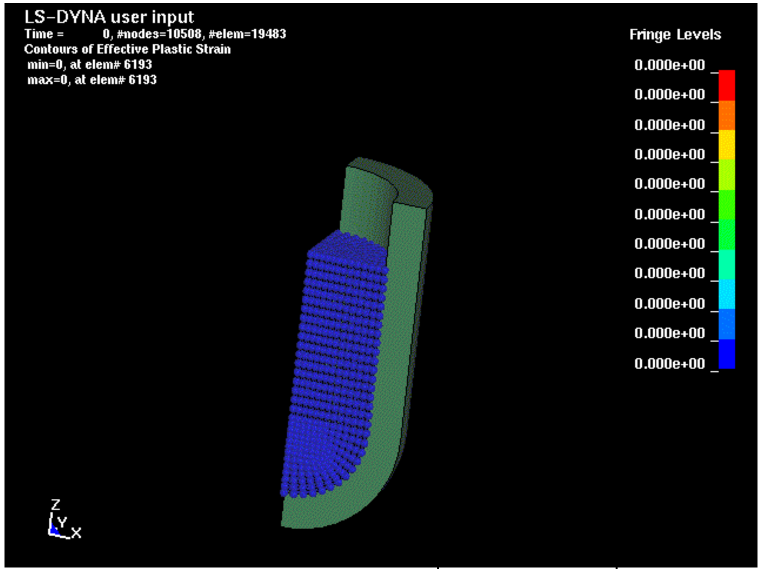


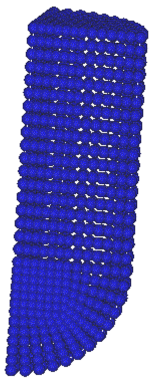


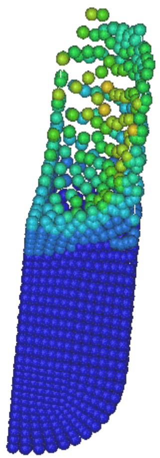


Metal Extrusion









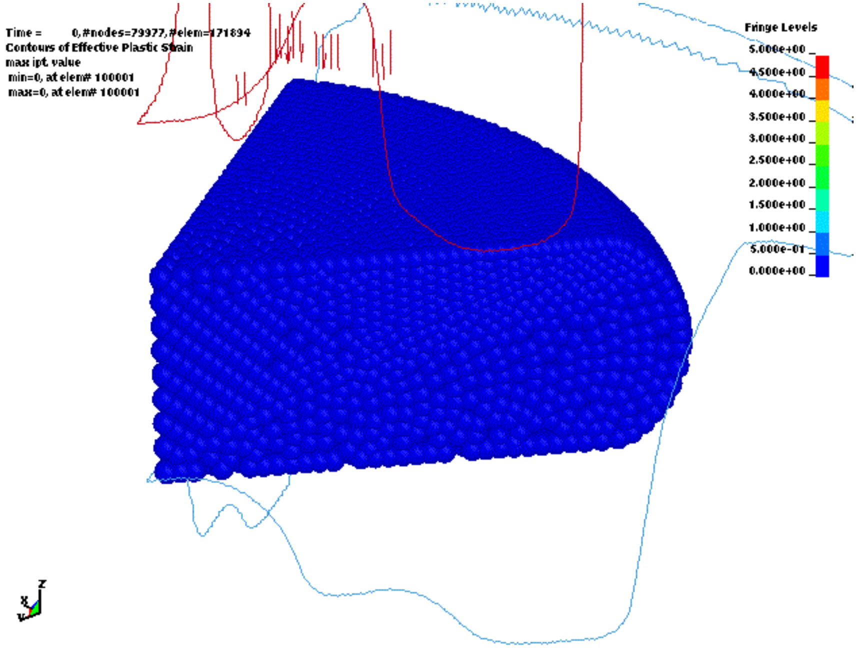
10410 nodes

2769 nodes

EFG

Adaptive EFG

Wheel Forging Simulation
Interactive Adaptivity



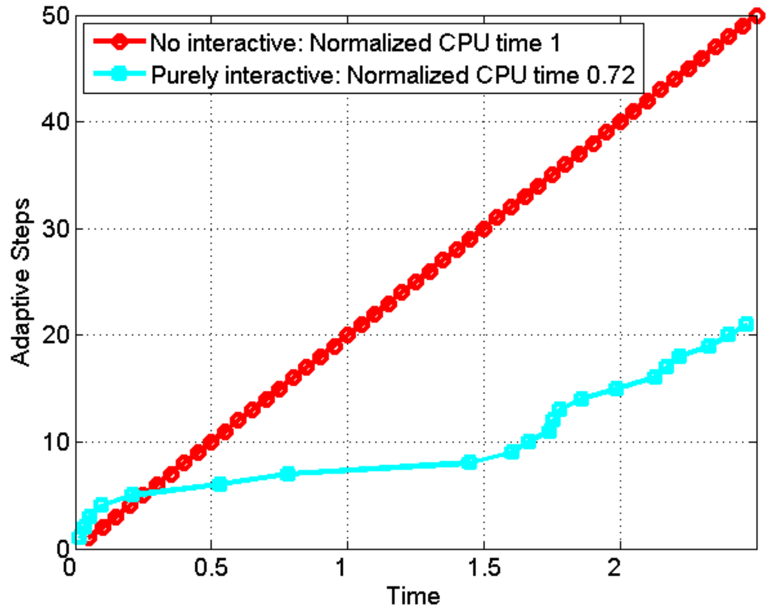

```
*CONTROL_ADAPTIVE
        2.5 …
  …
*CONTROL_REMESHING_EFG
        0.15        0.30
                     3           1
        0.20        3.5        0.80
```

* **SMP with 6 CPUs**

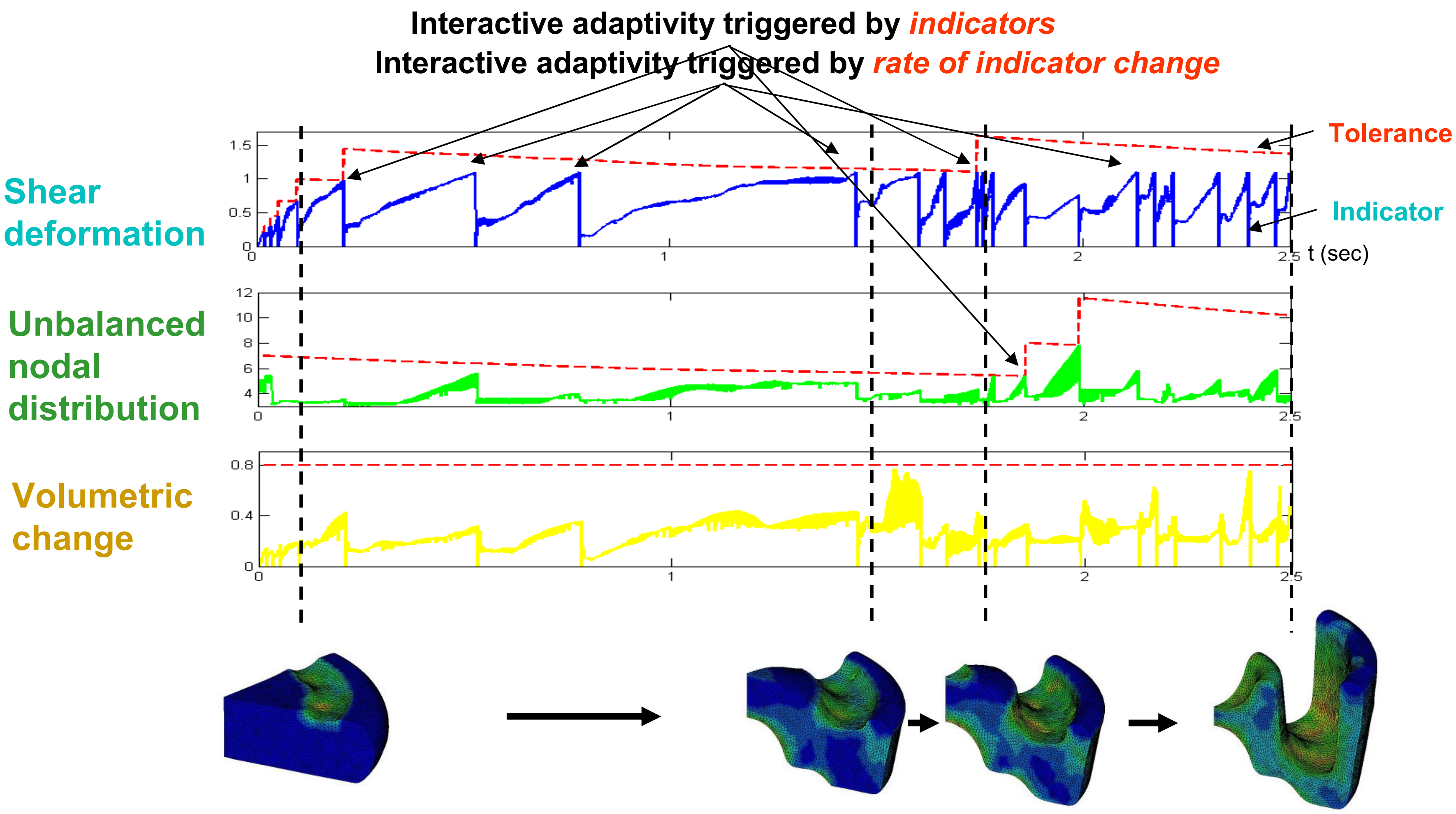
Traditional adaptivity

IAT	**0**	**3**
Normalized CPU time	**1.0**	**0.72**
# of adaptive steps	**50**	**22**

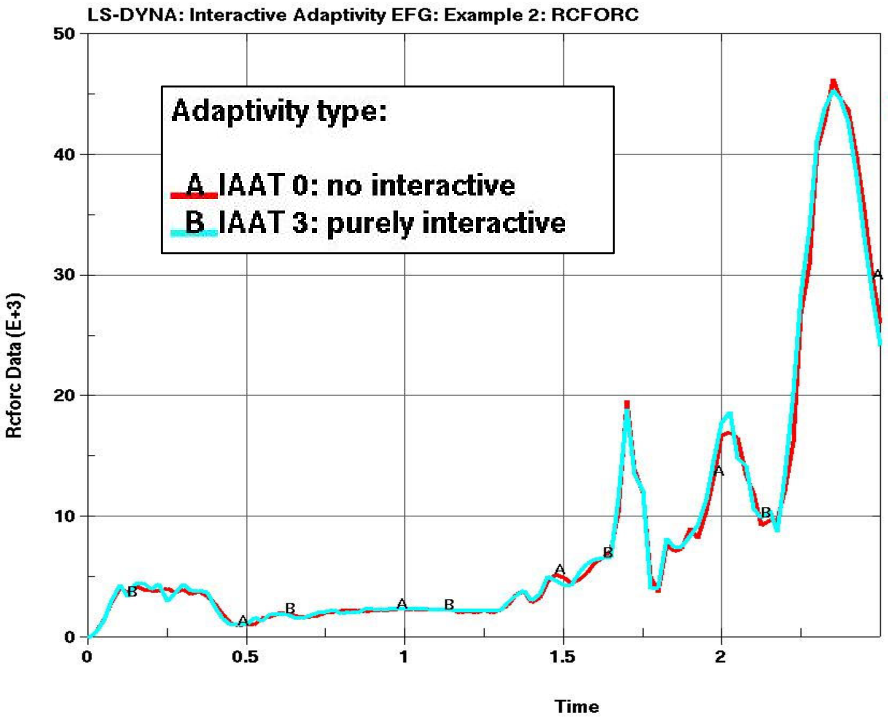
Purely interactive adaptivity



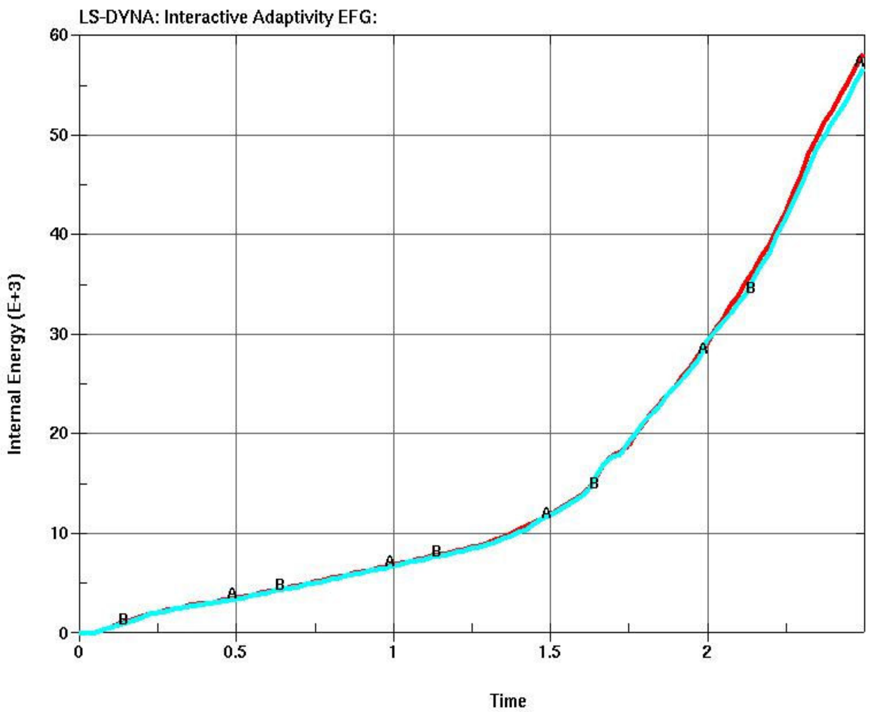
Wheel Forging Simulation



Wheel Forging Simulation

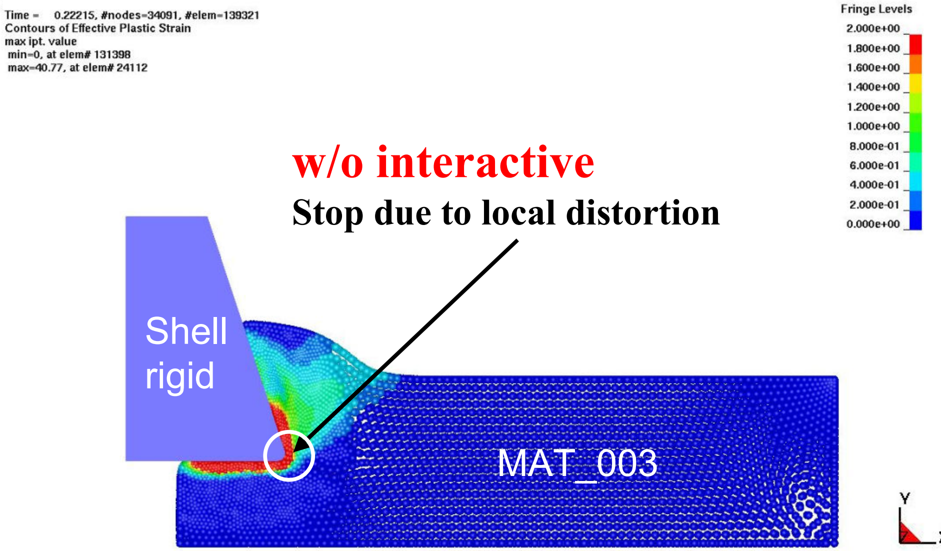


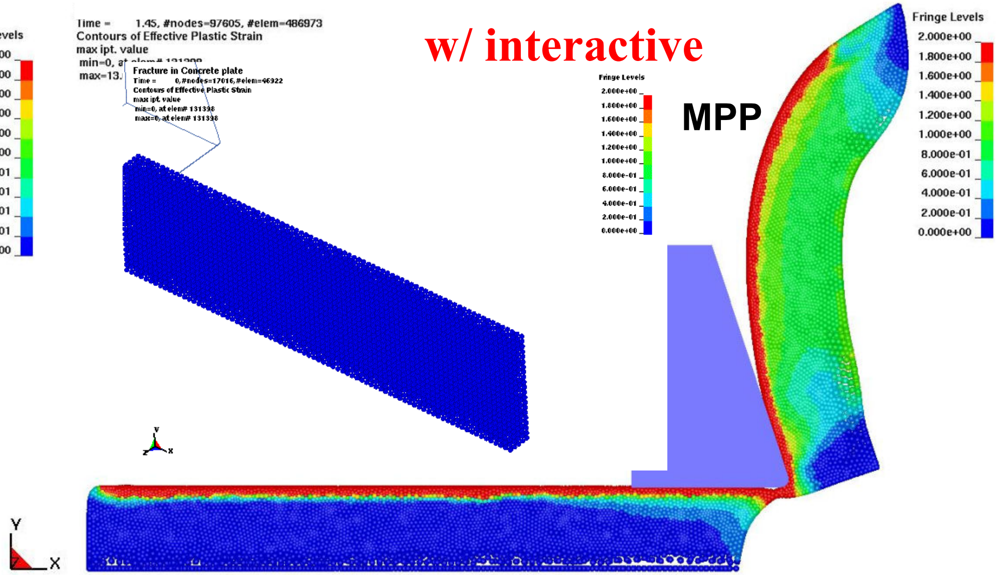
Contact force

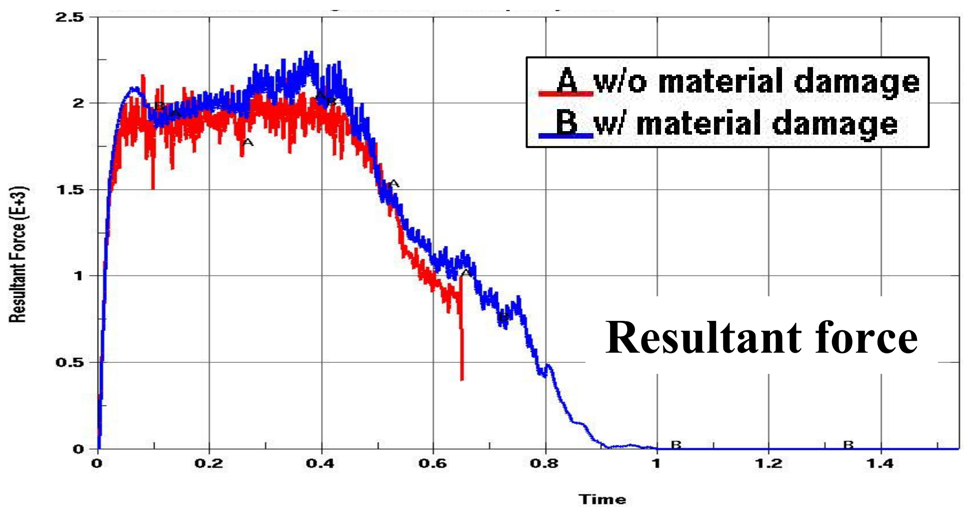


Internal energy

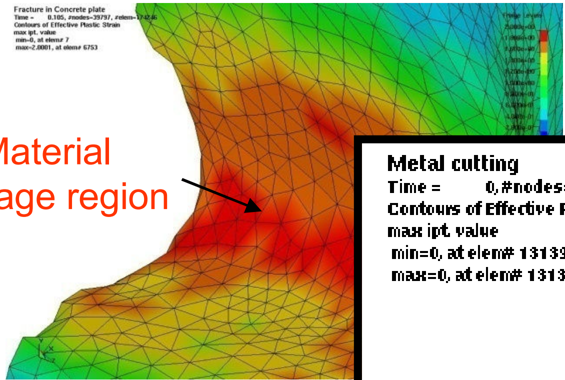
Metal Cutting Simulation



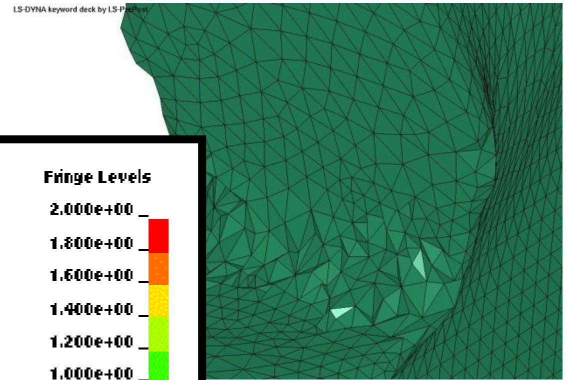




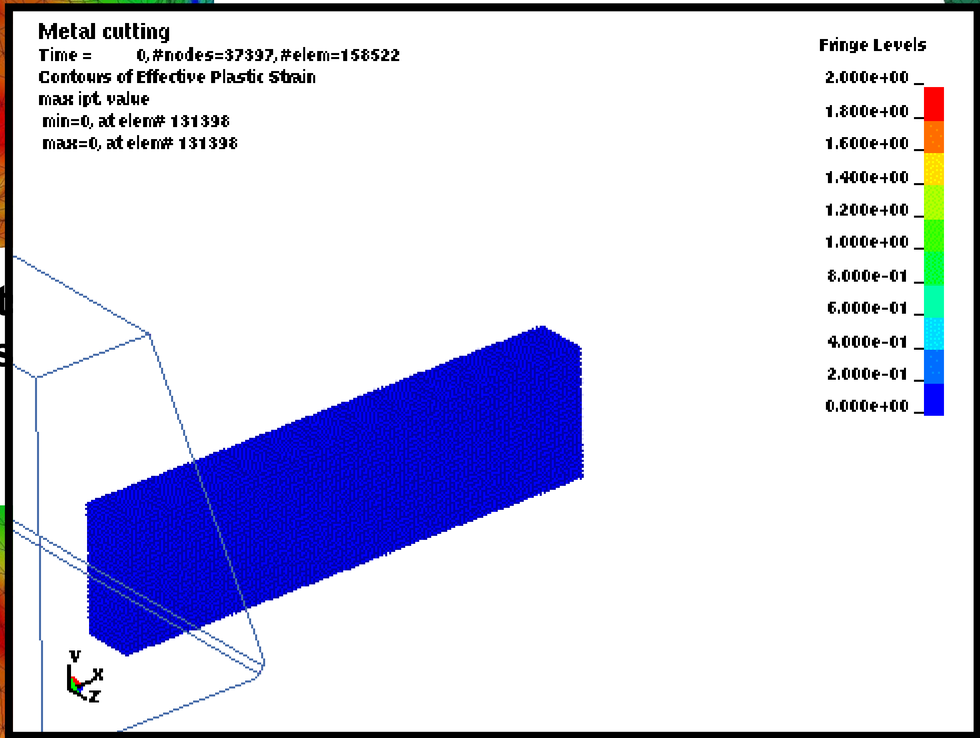
Metal Cutting Simulation

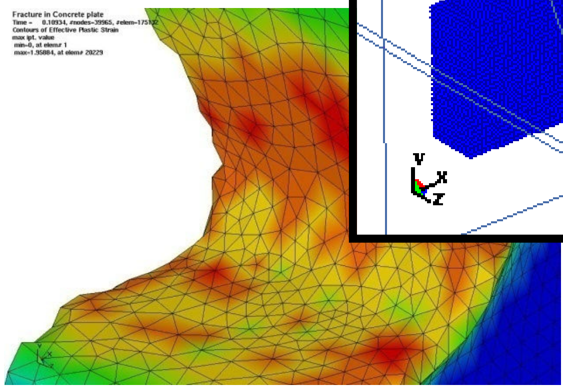


'Damaged' element
adaptive re-mes



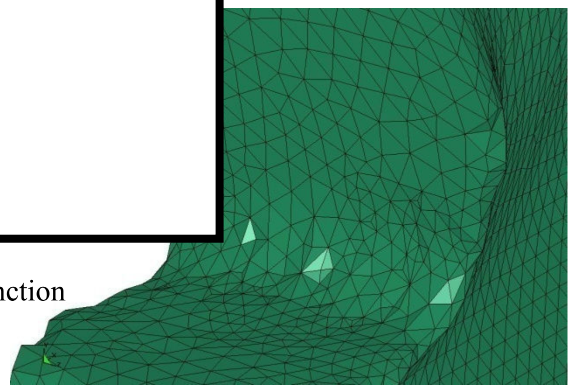
ent erosion





Global remeshing and remapping

$\psi_i(\mathbf{X})$: Local smoothing function



Local surface smoothing and reconstruction

4. SPH Applications

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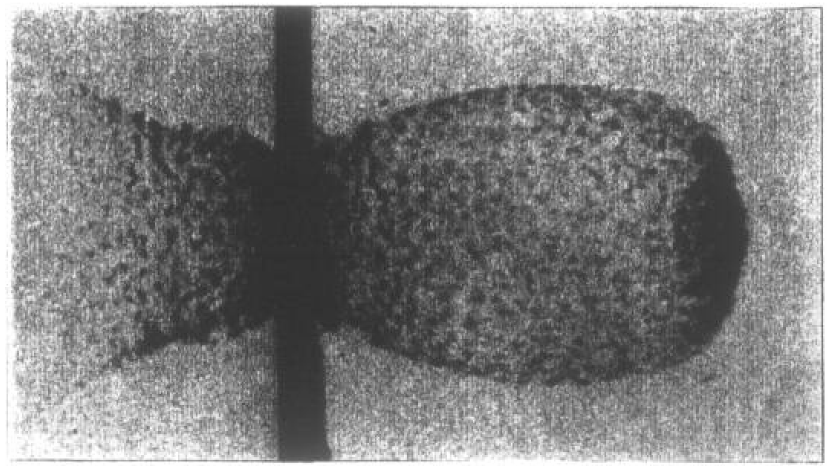
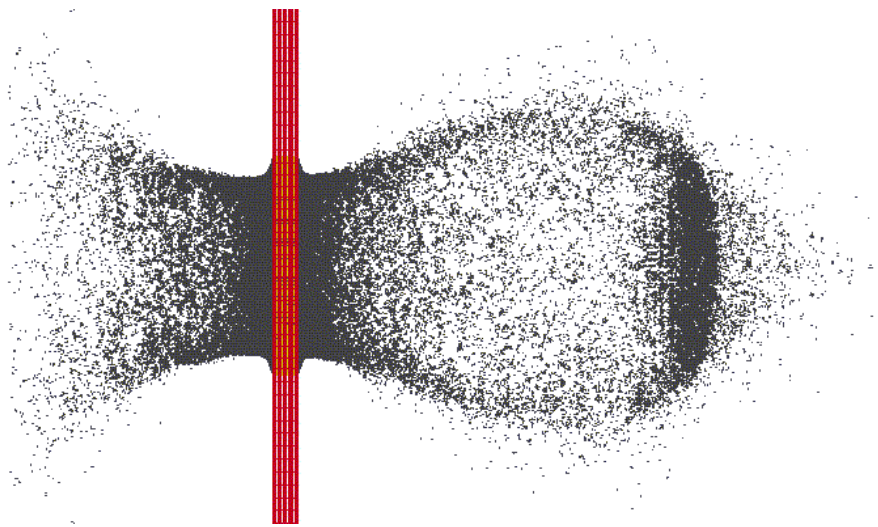
CONFIGURATION:

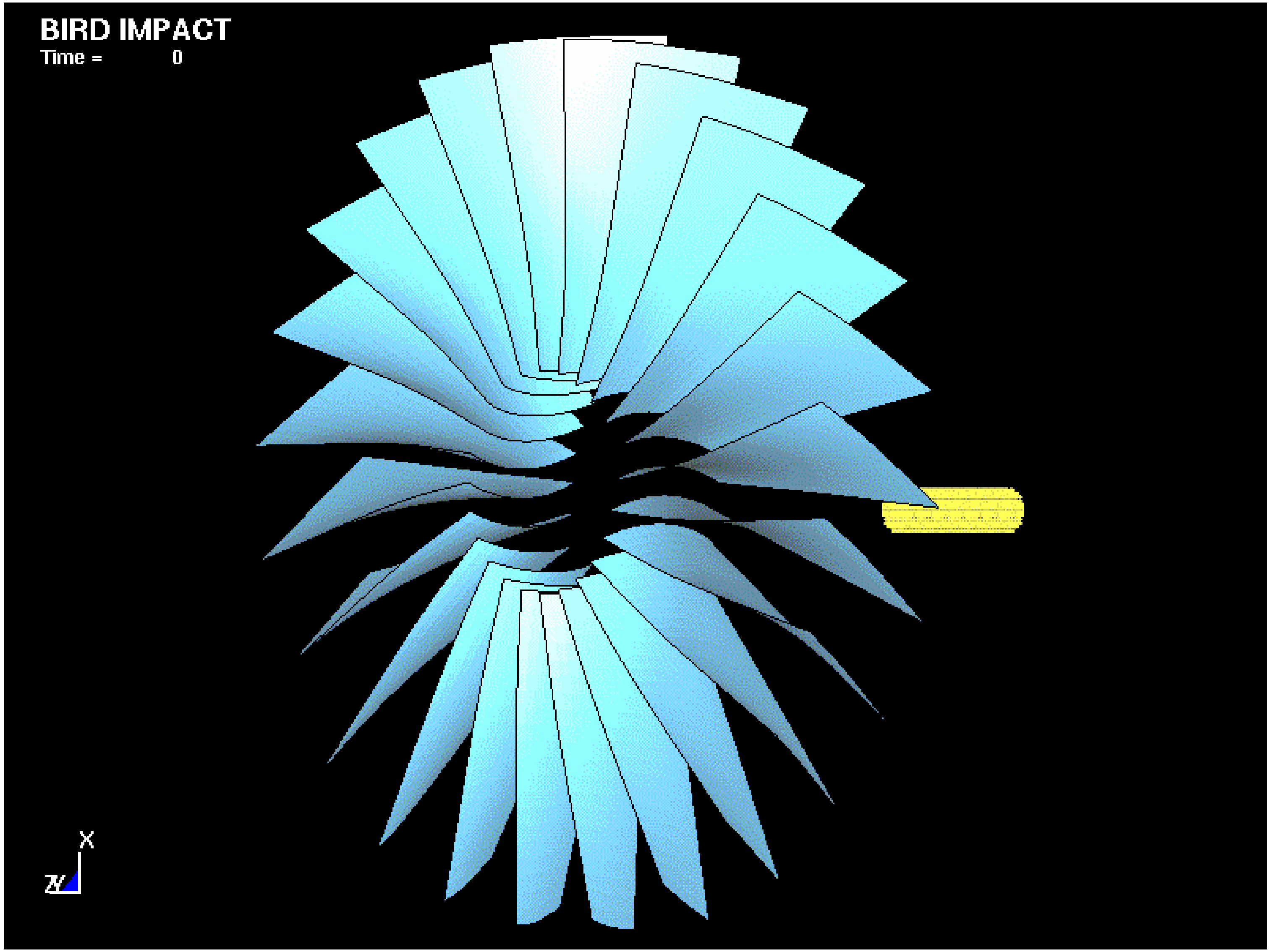
Projectile:
- material: 304 L Steel
- velocity: 5530 m/s
- geometry :sphere,φ = 5 mm

Target :
- material: 6061-T651 Al
- Thickness : 2.85 mm

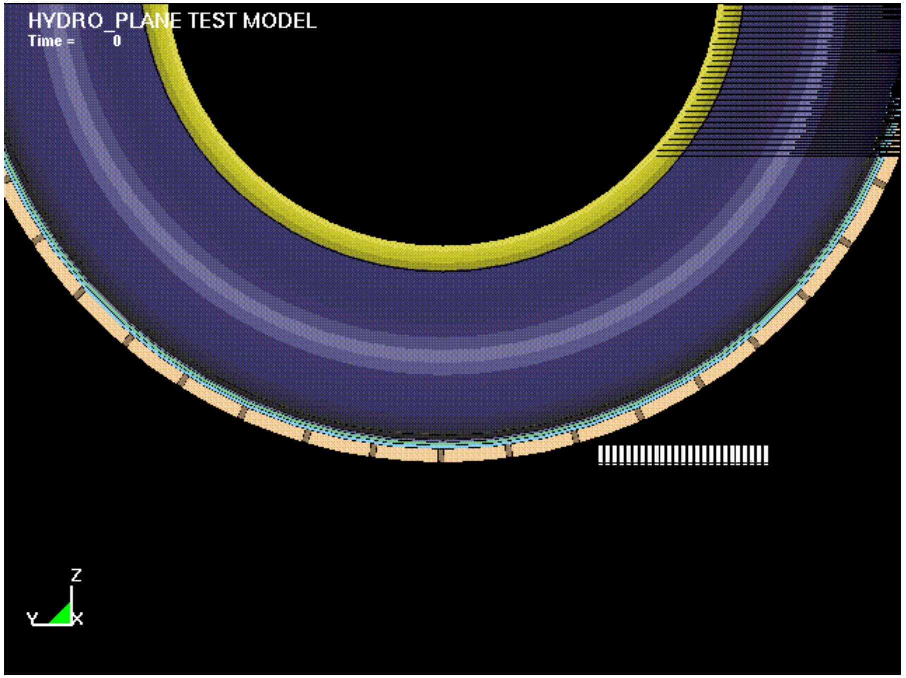


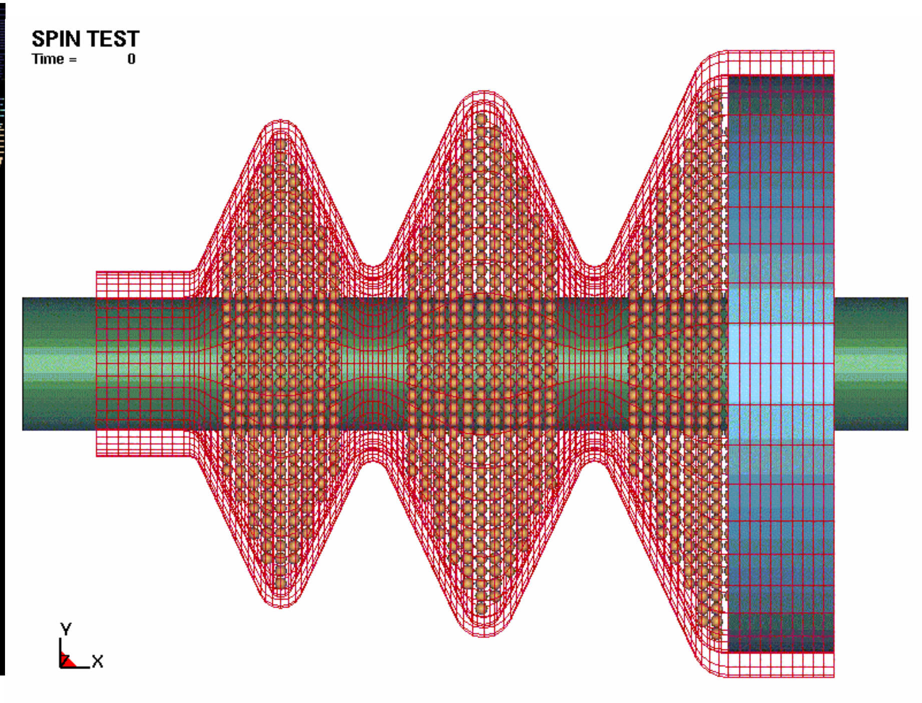


Bird Strike



Automotive Applications





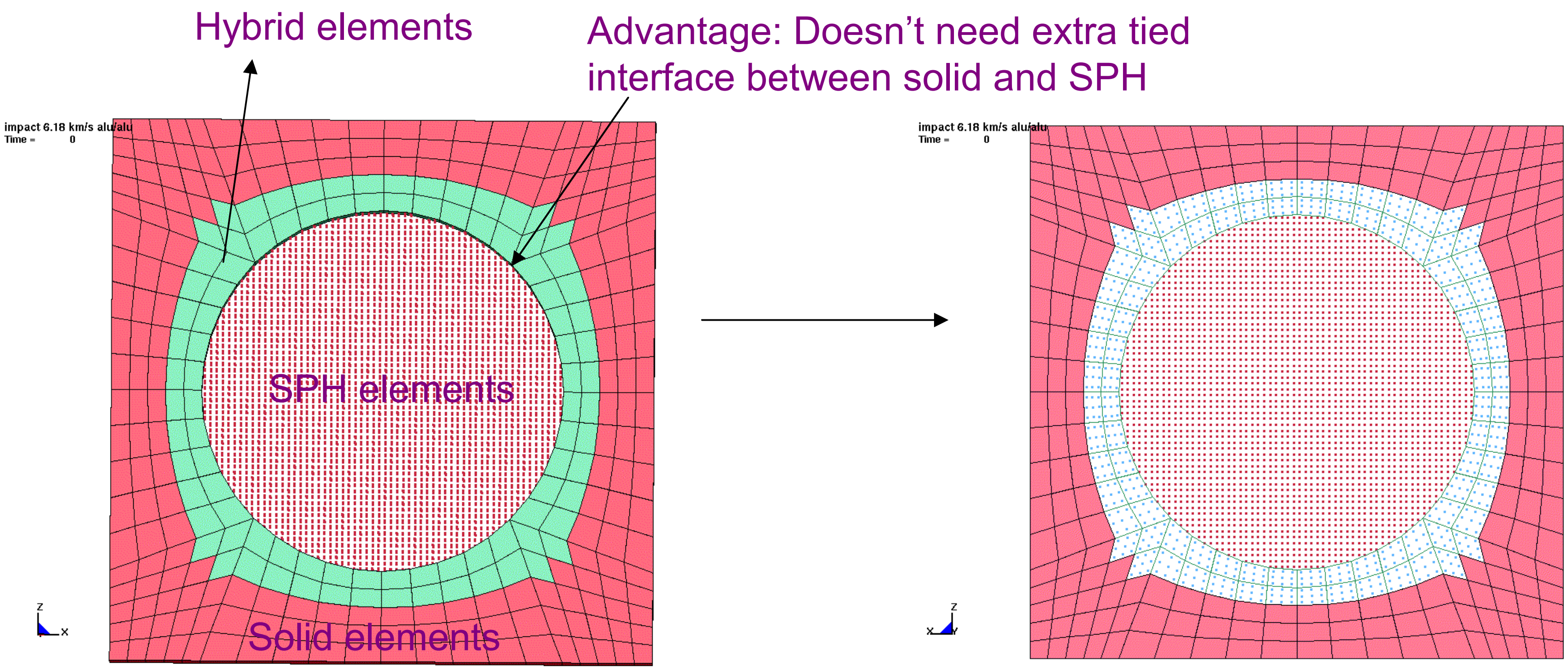
Hydro-plane

Spin test

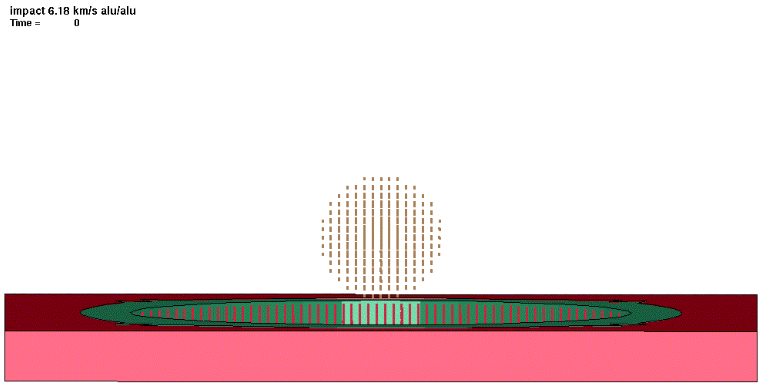
Hybrid Element Coupling SPH with Solid

Hybrid element: Solid elements constrain SPH nodal locations. SPH elements provide "penalty force" against solid nodal motion. Hybrid elements are used as transit layers between SPH elements and solid elements. (shared part ID)

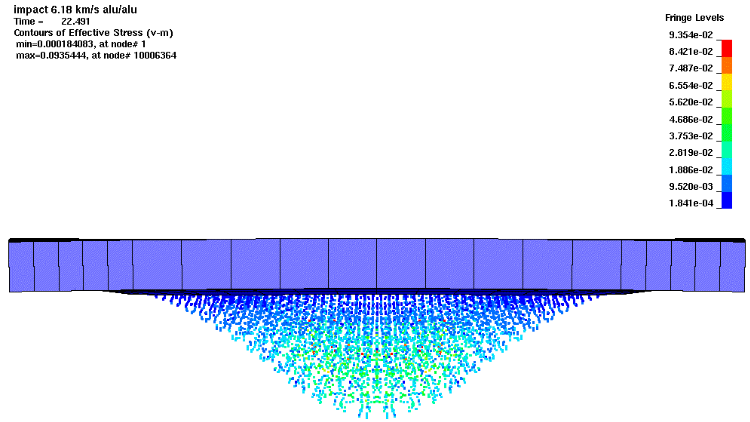
Advantage: We have the SPH formulation which can endure quite large deformation and at the same time we have the solid mesh which clearly describes the material interface.



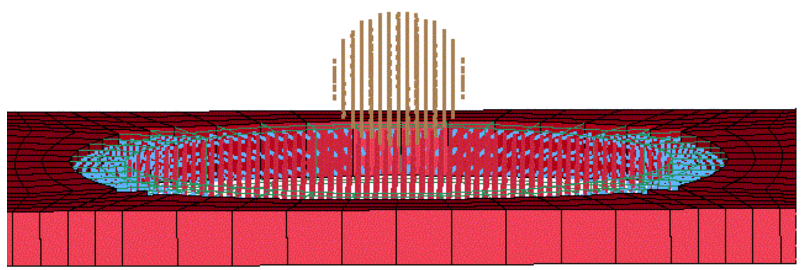
Impact Example



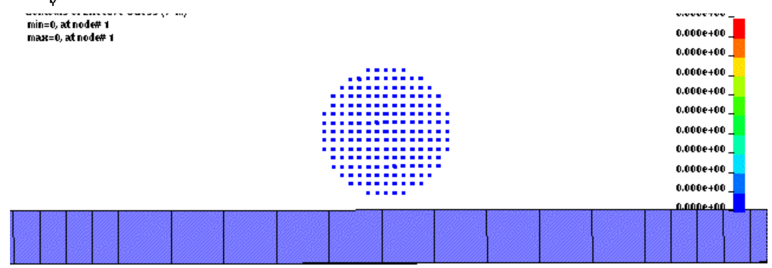
Set up



Final shape



Coupling layers



Effective stress

5. Conclusions

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- **Meshfree methods can solve problems that finite element methods have difficulties.**
- **EFG in LS-DYNA provides engineers a powerful tool with robustness, efficiency and accuracy.**
- **EFG has been successfully applied to crashworthiness, metal forging/extrusion, and can be used in metal cutting and fracture analysis.**
- **SPH in LS-DYNA is an efficient tool for high velocity impact, penetration and can simulate solid, fluid materials.**