

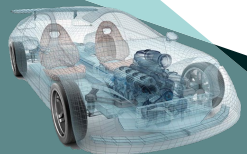
**Efficient Structural Optimization Techniques using the Incremental
Equivalent Static Load Method for ANSYS LS-DYNA**

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Nov 2023

Outline

- Review of ESL Method and Previous Work
- New Improved Incremental ESL method
- ESLDYNA Implementations
- Examples
- Summary

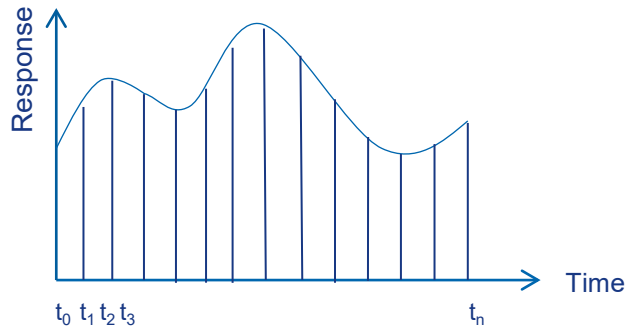


Review of ESL Method and Previous Work



■ ESL method (G. J. Park)

- The ESL is defined as the static loads in the linear static analysis that produce the same response field (displacement) as the nonlinear analysis.

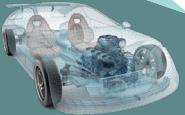
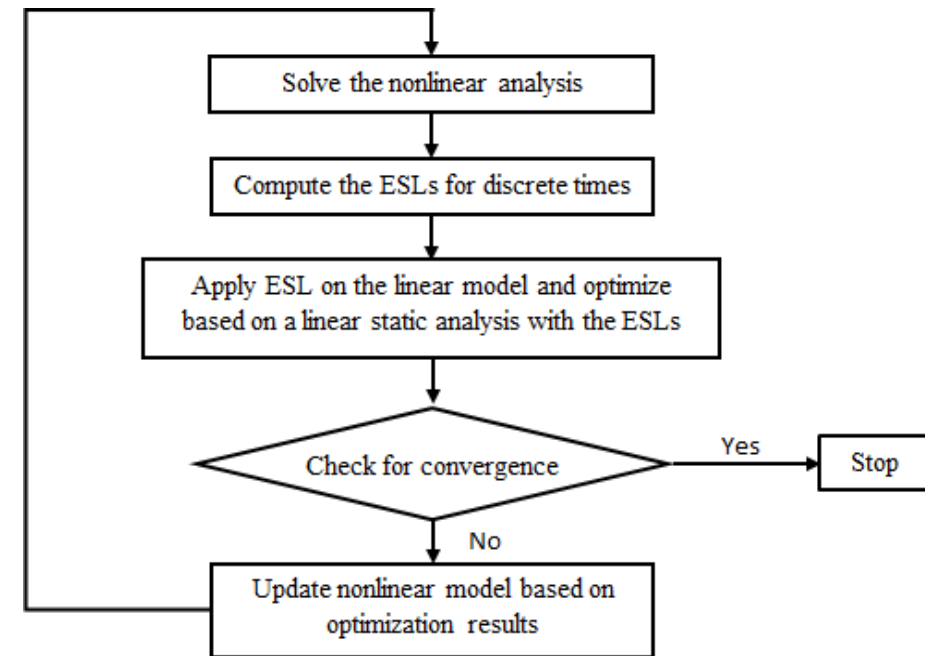


$$f_{ESL}(t_a) = K_L Z_N(t_a)$$

■ Difference-Based ESL method (J. Triller)

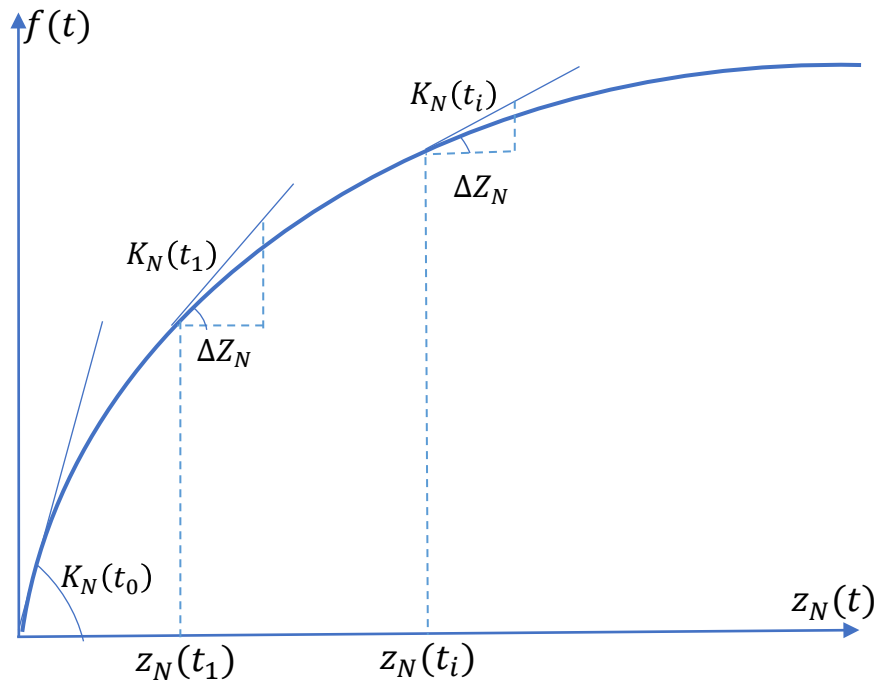
- Proposes a difference-based ESL method to improve the approximation quality
- Need to modify the mesh and use multi-model

■ ESL Workflow

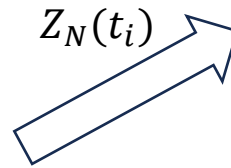


ESL Method vs. Incremental ESL Method

$$M\ddot{z}_N(t) + C\dot{z}_N(t) + K_N(z_N(t))z_N(t) = f(t)$$



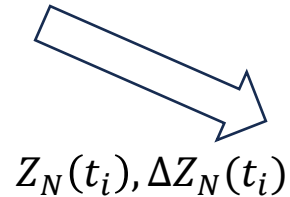
Nonlinear Analysis Domain



Standard ESL Method

$$f_{ESL}(t_i) = K_L(t_0)Z_N(t_i)$$

Linear Stiffness matrix is constant



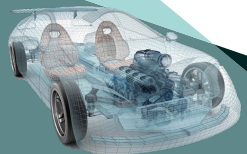
Incremental ESL Method

$$f_{ESL}(t_i) = K_L(t_{i-1})\Delta Z_N(t_i)$$

$$\Delta Z_N(t_i) = z_N(t_i) - z_N(t_{i-1})$$

Linear Stiffness matrix is a function of nonlinear displacement

Linear Analysis Domain

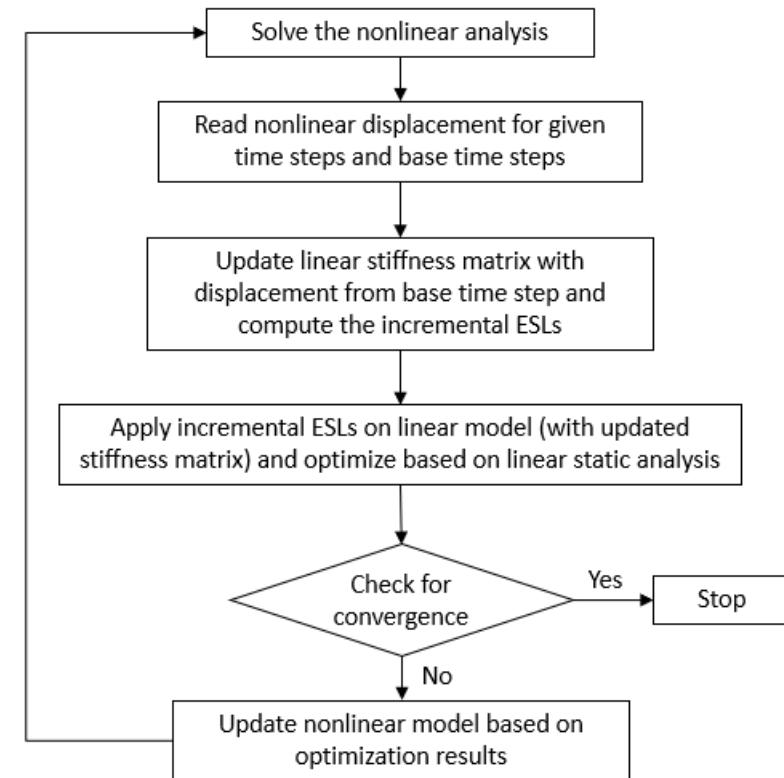


Incremental ESL for LS-DYNA Implementations in GENESIS



- New incremental option
 - Solution control data: ESLBASE
 - Stiffness matrix is updated using displacement specified on ESLBASE
 - ESL loads is computed as
 - $K_L(Disp_{ESLBASE}) * (Disp_{ESLOAD} - Disp_{ESLBASE})$
 - Where $K_L(Disp_{ESLBASE})$ is the updated stiffness matrix using displacement from base time step

- Workflow for optimization with incremental ESL method



Responses

- Displacement for each incremental ESL loadcase

$$\text{disp} = \Delta z_N^j \quad (j=1,2,\dots, i)$$

- Total displacement up to time t_i

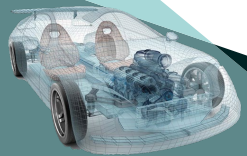
$$\text{total_disp} = z_N(t_i) = \sum_{j=1}^i \Delta z_N^j$$

- Strain energy for each incremental ESL loadcase

$$\text{strain_energy} = \frac{1}{2} \left(f_{ESL}(t_j) \right)^T * \Delta z_N^j \quad (j=1,2,\dots, i)$$

- Sum of strain energy up to time t_i

$$\text{strain_energy_sum} = SE_N(t_i) = \sum_{j=1}^i \frac{1}{2} \left(f_{ESL}(t_j) \right)^T * \Delta z_N^j$$

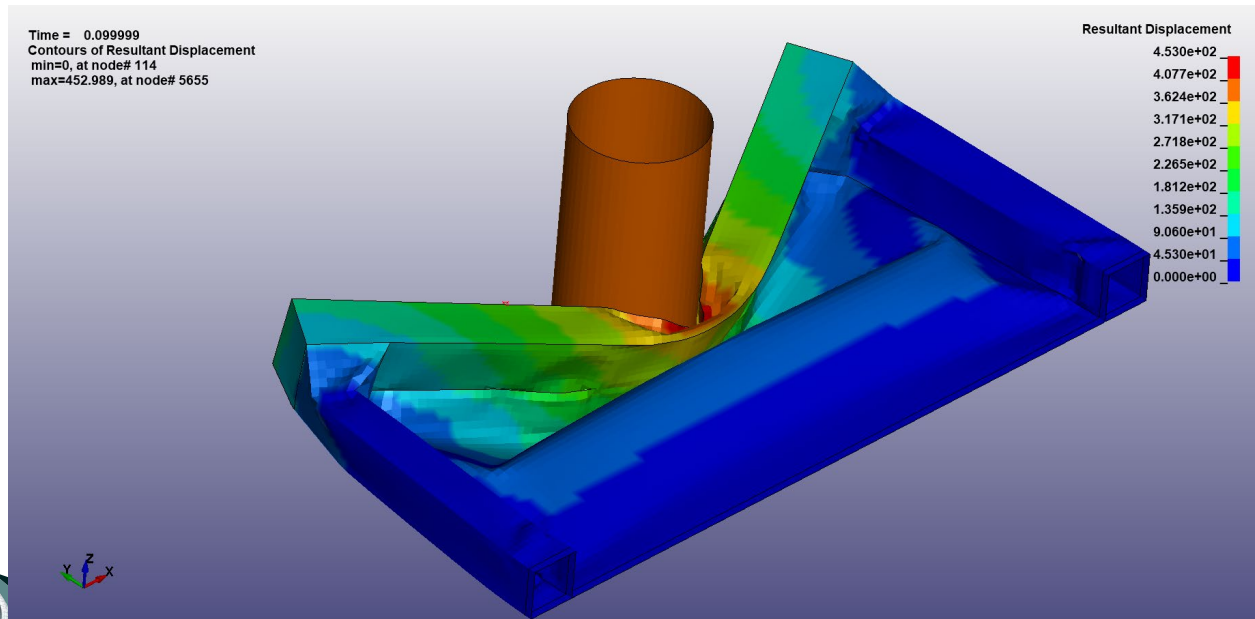


Sizing optimization of sill and floor under dynamic impact



■ LS-DYNA Loading conditions

- Initial velocity: 8e3 mm/s
- End time: 0.1 s



ESL Loadcases and Sizing Design

■ 10 ESL loadcases

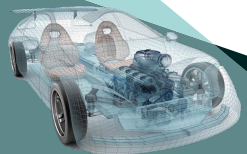
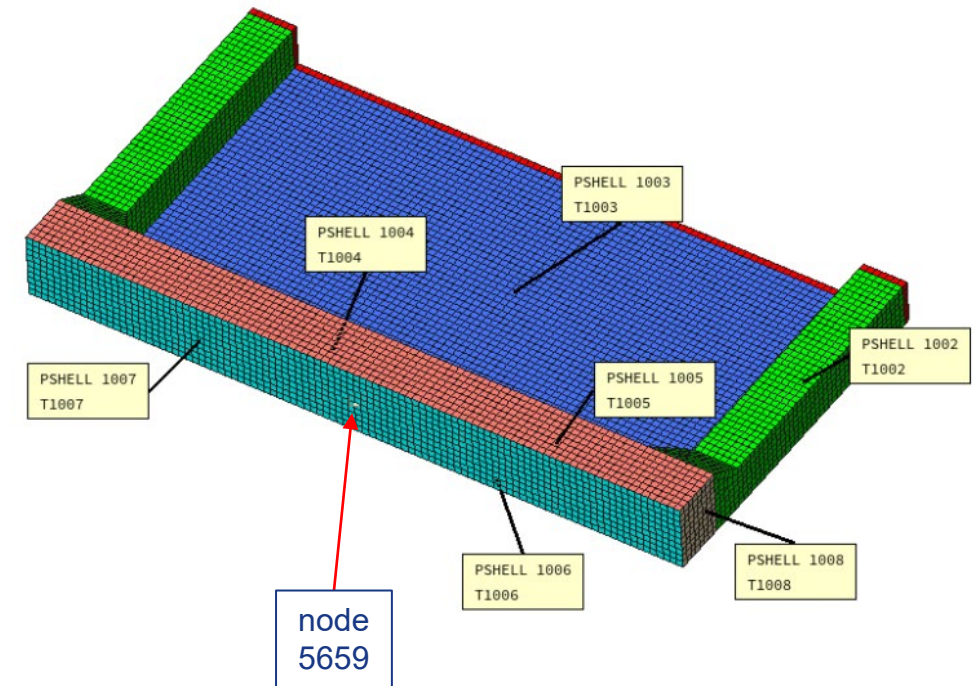
ESL loadcase	1	2	3	4	5	6	7	8	9	10
Base time	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
Load time	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1

■ Sizing design

- 7 design variables – thickness of the shell
- Design variable range: 0.5 to 3 mm
- Initial thickness: 0.8 mm

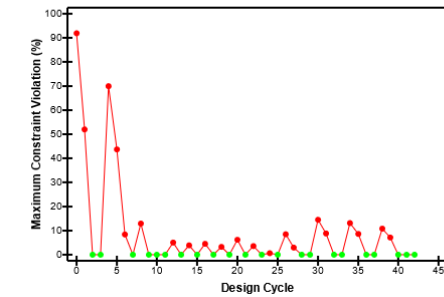
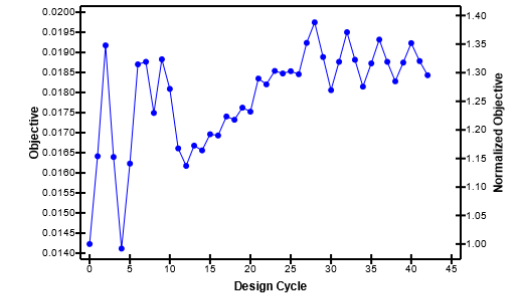
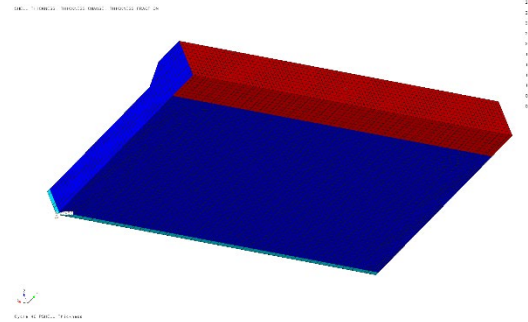
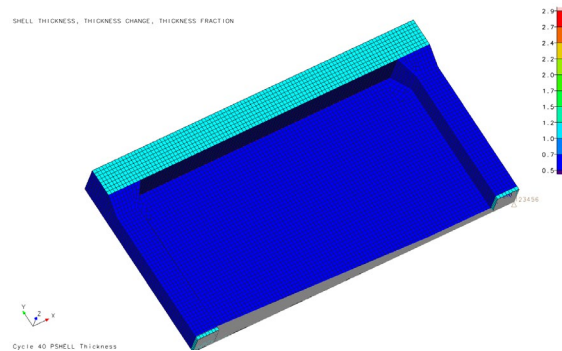
■ Optimization

- Min mass
- Displacement Y (sum) > -220 (for node 5659)

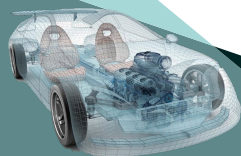


Optimization result with Standard ESL Method

- Optimal mass = 19.2 kg

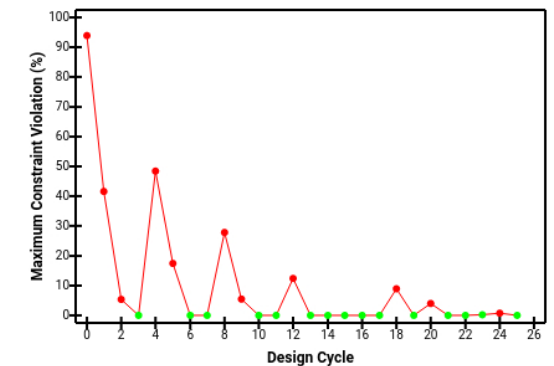
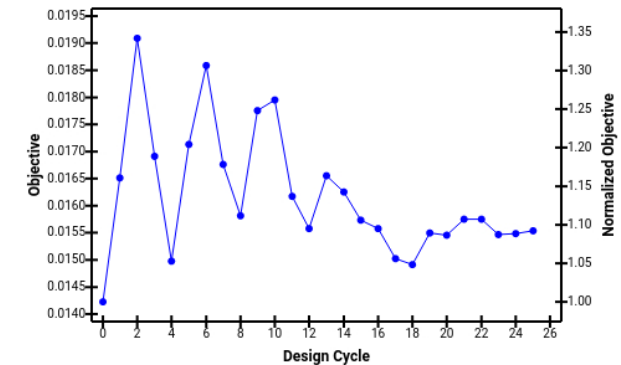
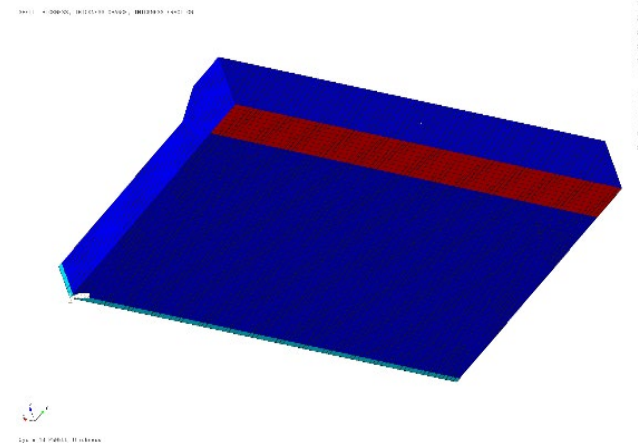
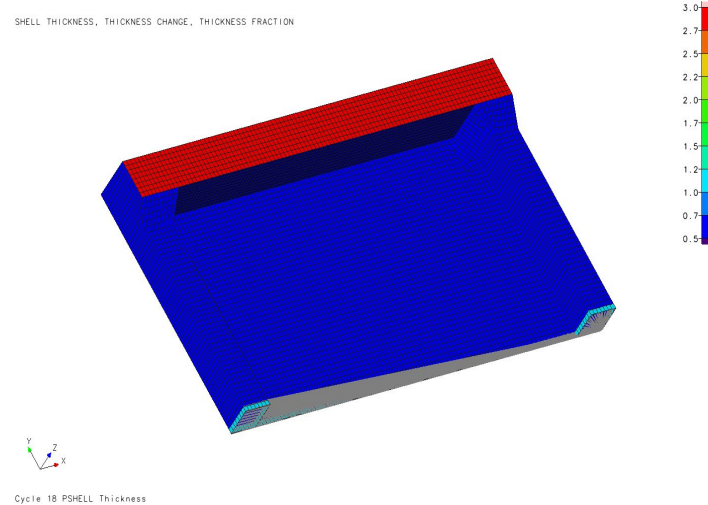


Design variable	T1002	T1003	T1004	T1005	T1006	T1007	T1008
value	5.3696E-01	7.4124E-01	5.0001E-01	1.0857E+00	2.9172E+00	2.9172E+00	5.5034E-01



Optimization Result with Incremental ESL Method

■ Optimal mass = 15.4 kg



Design variable	T1002	T1003	T1004	T1005	T1006	T1007	T1008
value	5.000046E-01	5.000046E-01	5.000046E-01	2.840044E+00	2.813525E+00	5.000000E-01	5.000000E-01

Similar answer is obtained by both response surface method (with more than 200 LS-DYNA analysis) and by difference-based ESL method in Triller's paper [2]

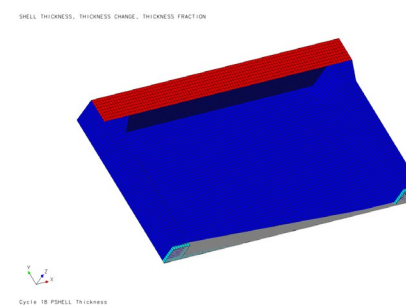
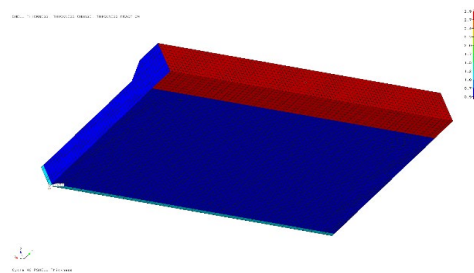
Compare optimization results with/without incremental option

- Design variable value, mass and no. ESL cycles

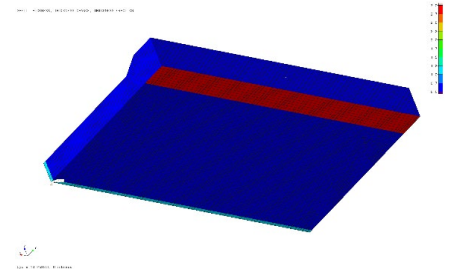
Method	Design variable							Mass (kg)	No. ESL cycles (LS-DYNA analyses)
	T1002	T1003	T1004	T1005	T1006	T1007	T1008		
ESL	0.53696	0.74124	0.50001	1.0857	2.9172	2.9172	0.55034	19.2	20
Incremental ESL	0.50000	0.50000	0.50000	2.84004	2.81353	0.50000	0.50000	15.4	13



ESL

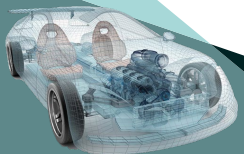


Incremental ESL



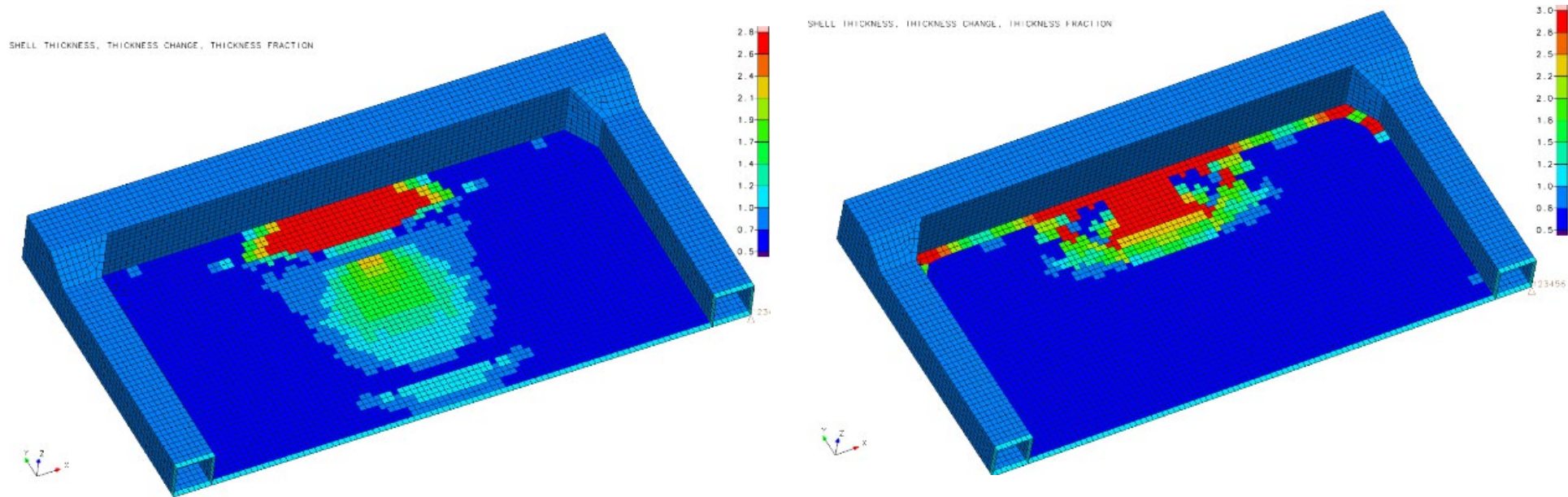
Topometry optimization of sill and floor structure under dynamic impact

- Same LS-DYNA loading conditions as in previous example
- Topometry optimization
 - Design region: element thickness of the floor panel
 - Objective
 - Max sum of the displacement (along negative y direction) for all esl loadcases
 - Constraint
 - $\text{Mass} < 1.0 * \text{Initial_Mass}$

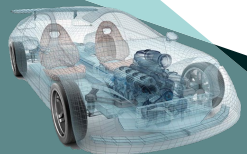


Topometry Optimization Results

■ Element thickness results



Thickness distribution result with the original ESL method (left) and with the incremental ESL method (right)

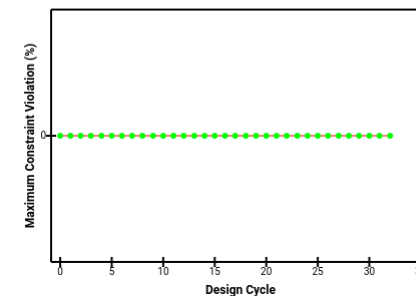
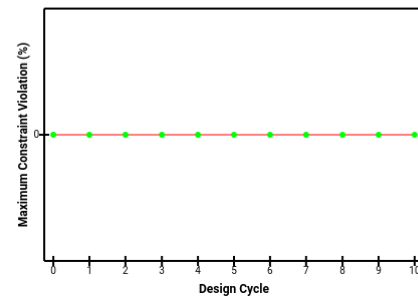
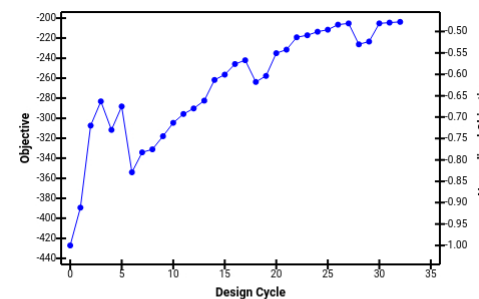
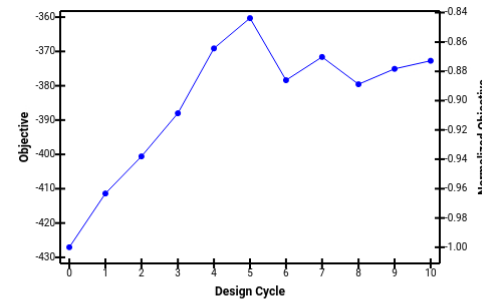


Topometry Optimization Results

■ Optimized intrusion

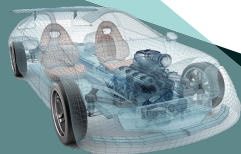
Method	Maximum intrusion at node 5659 (mm)	Mass (kg)	No. ESL cycles (LS-DYNA analyses)
ESL	382.9	14.2	4
Incremental ESL	221.8	14.2	15

■ Design history



ESL

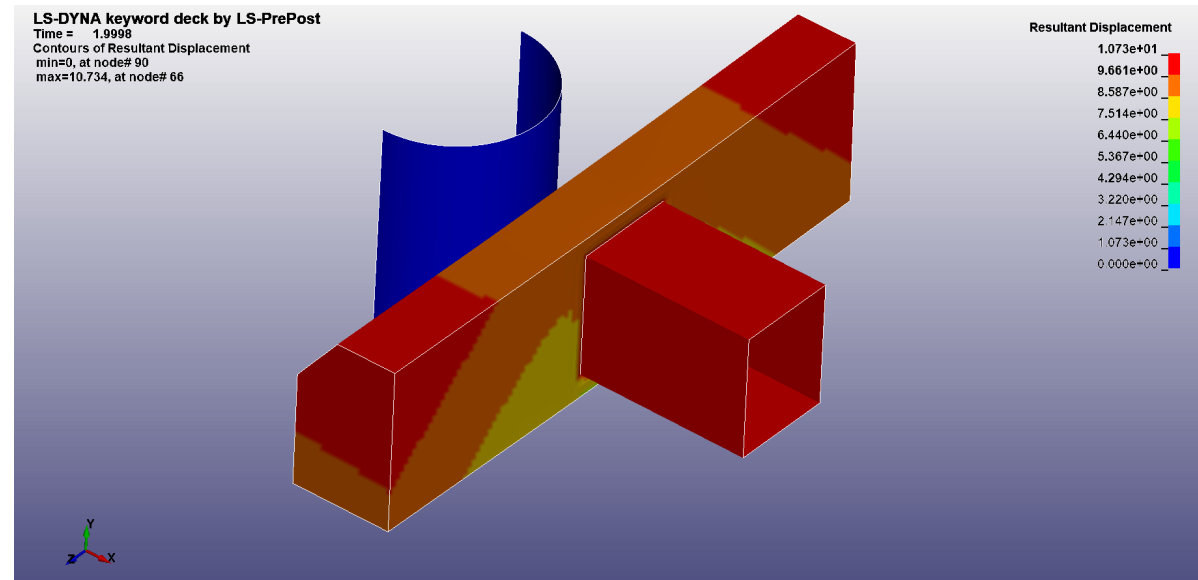
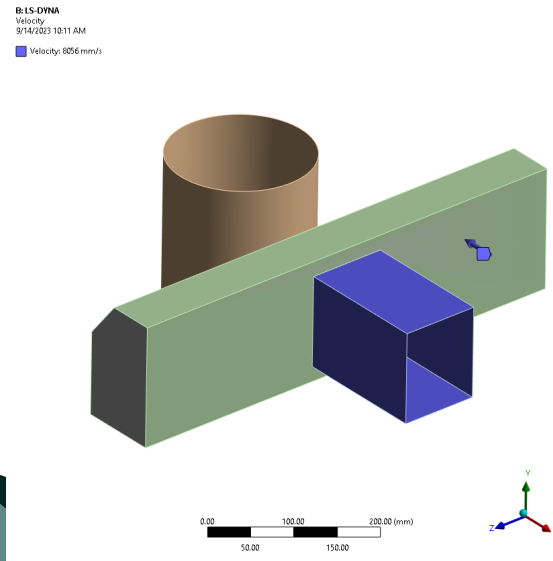
Incremental ESL



Design of Rocker Profile with Topology and Sizing Optimization

■ LS-DYNA Loading conditions

- Initial velocity: 8e3 mm/s
- End time: 30 milli seconds
- Piecewise plasticity material



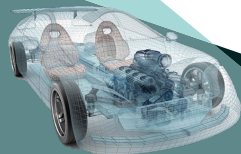
ESL Loadcases and Topology Optimization

■ ESL loadcases

ESL loadcase	1	2	3	4	5	6
Base time	0.0	1.0	2.0	3.0	4.0	5.0
Load time	1.0	2.0	3.0	4.0	5.0	6.0

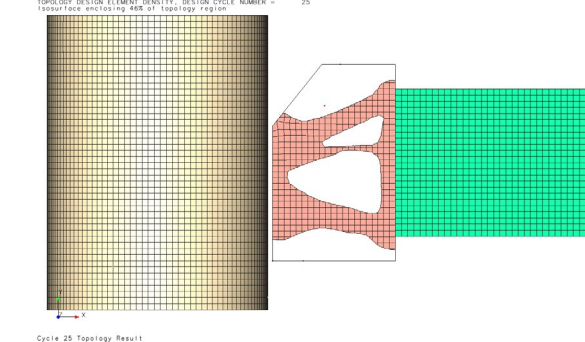
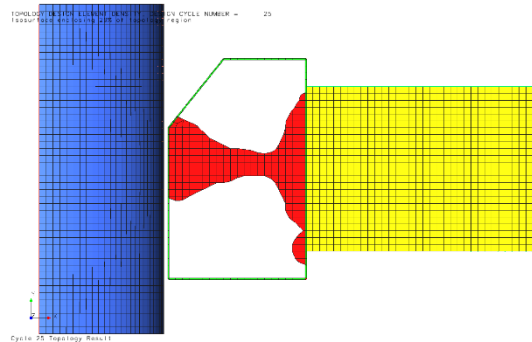
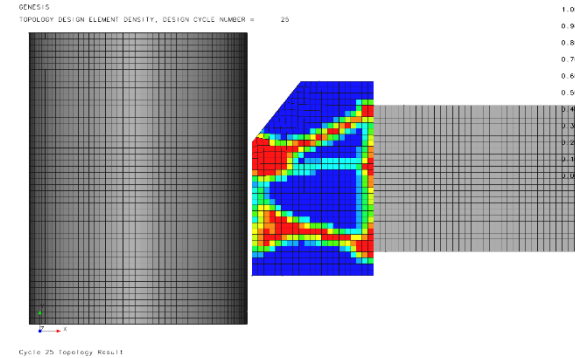
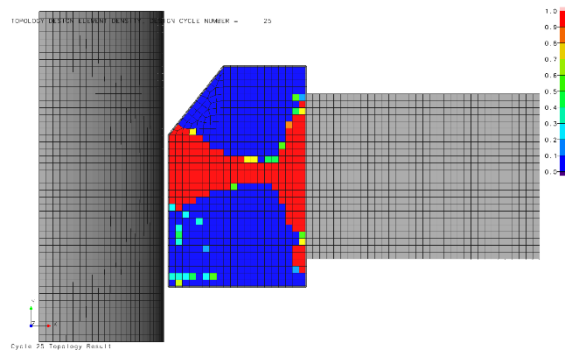
■ Topology Optimization

- Design region: the inner solid of the rocker beam
- Min sum of strain energy from all ESL loadcases
- Mass fraction constraints 30%



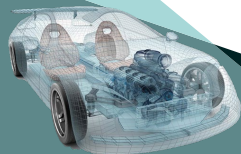
Topology Results

- Mass fraction = 30%



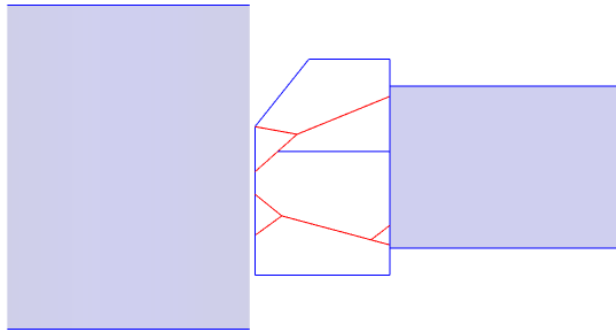
Next Step: Sizing optimization to further refine the thickness of ribs

Topology result with the original ESL method (left)
and with the incremental ESL method (right)

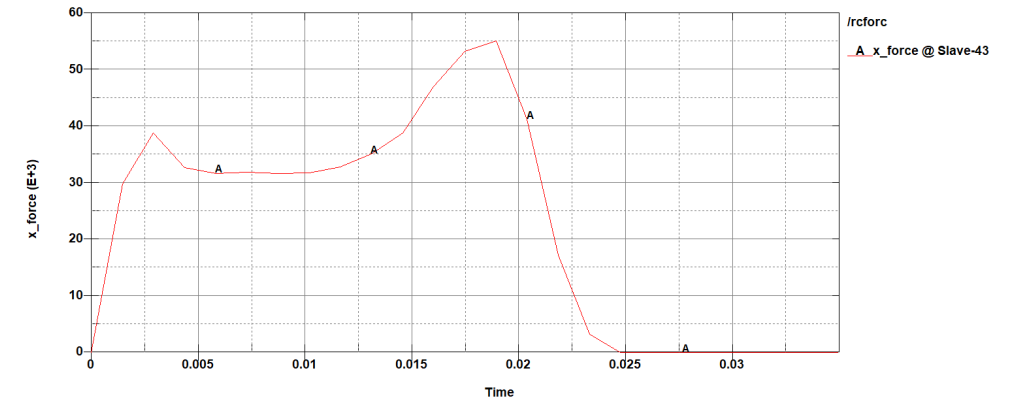
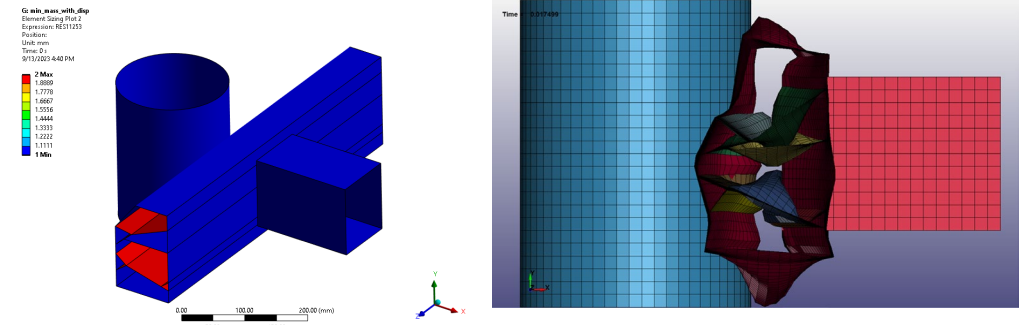


Interpretation of the Topology Result with Shell QUEST™

■ Interpreted shell model and analysis results



Initial thickness: red = 2mm, blue = 1mm

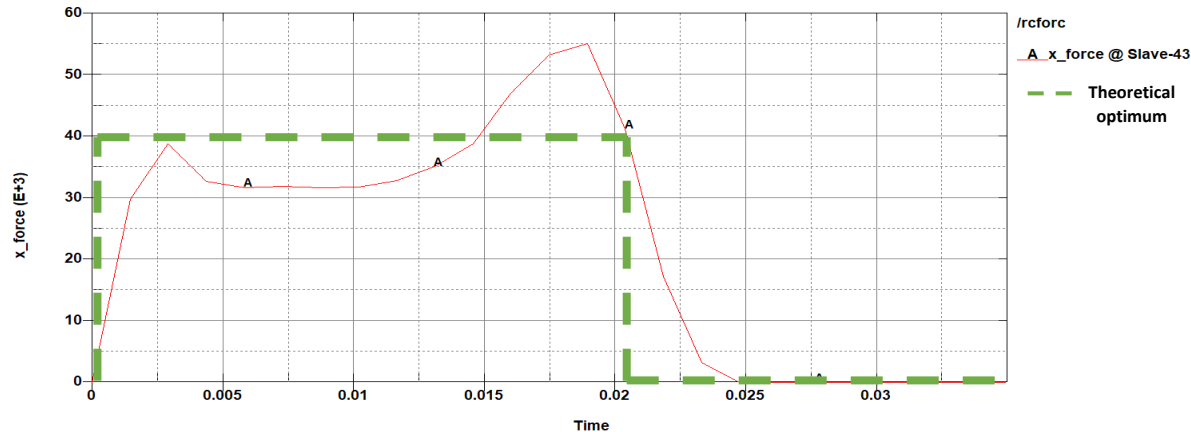


Force vs. time (A: initial design), max force = 55.2 kN

Design Requirement for Rocker Profile

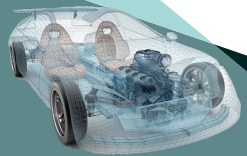
■ Requirement

- Input kinetic energy = 2.77 kJ
- Intrusion < 70 mm
- Reaction force < 45 kN



Theoretical optimal reaction force =
kinetic energy/max intrusion \sim 40 kN

Although reaction force cannot be used as constraints directly in ESDYNA, the mass, displacement and strain energy can be optimized to control the reaction force.



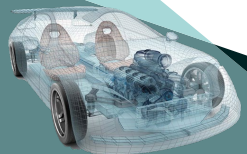
Sizing design to fine tune the shell thickness

■ ESL Loadcases

ESL loadcase	1	2	3	4	5	6	7	8	9	10
Base time	0.0	1.5	3	4.5	6	9	12	15	18	21
Load time	1.5	3	4.5	6	9	12	15	18	21	24

■ Sizing optimization

- Outer shell is not designed (thickness = 1 mm)
- Inner ribs designed; range of the thickness is from 1.0 to 3.5 mm

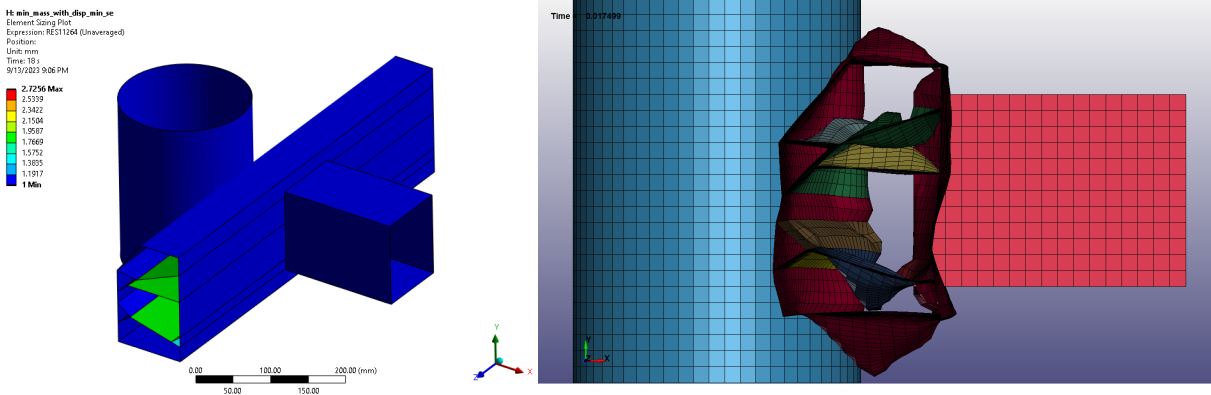


Sizing Optimization

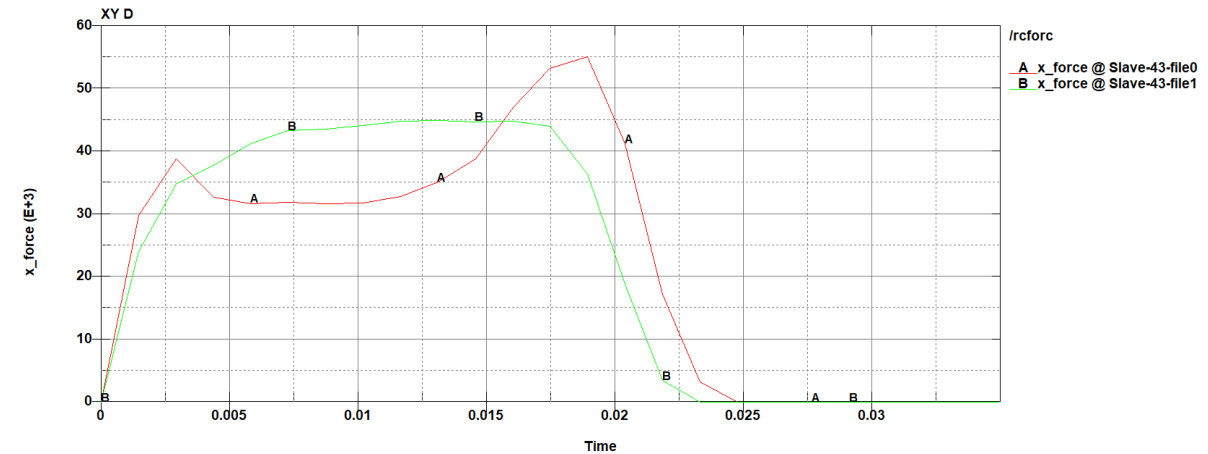
■ Case 1

- Min mass with displacement < 70 mm, min sum of strain energy (weighting factor 0.1)

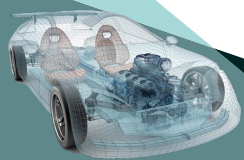
Thickness



Optimized: ESL cycle 9, Mass = 1.92 kg, max intrusion = 67.3 mm



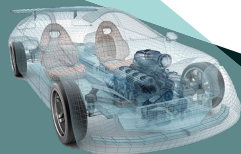
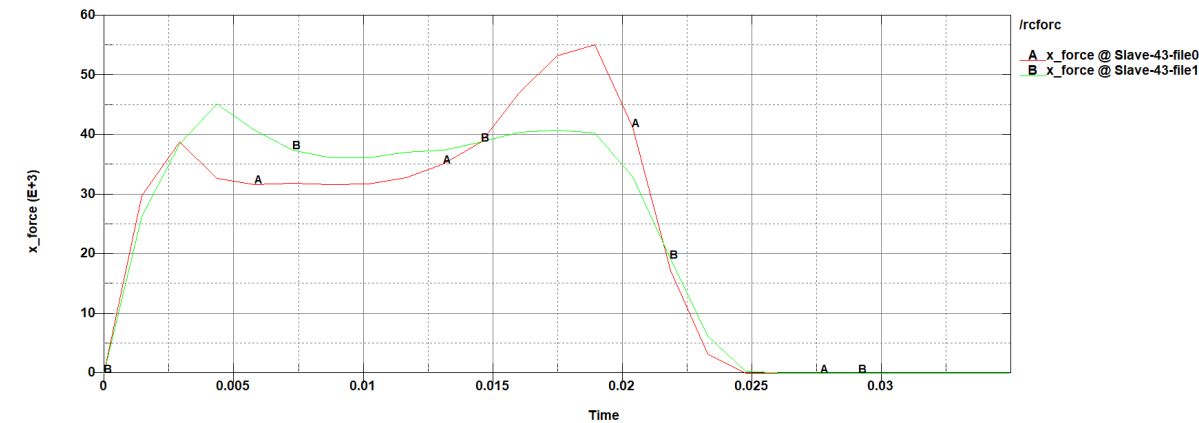
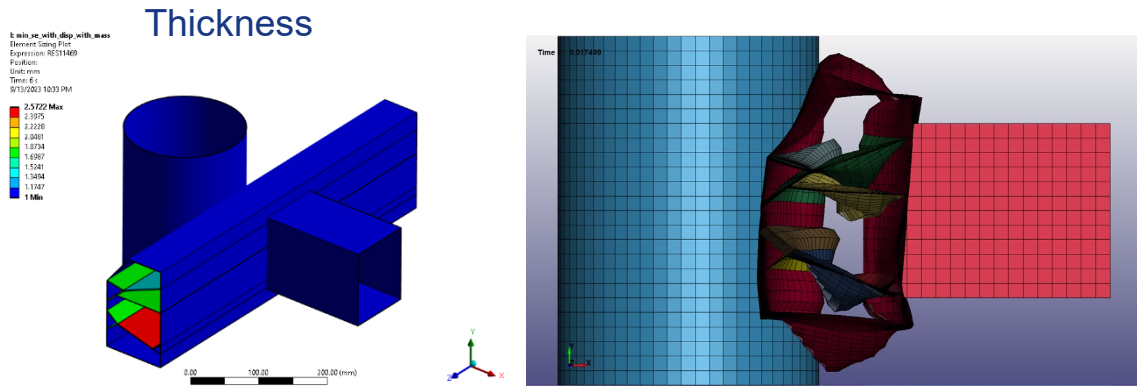
Force vs. time (A: original design, B: optimized ESL cycle 9), max force = 44.8 kN



Sizing Optimization

Case 2

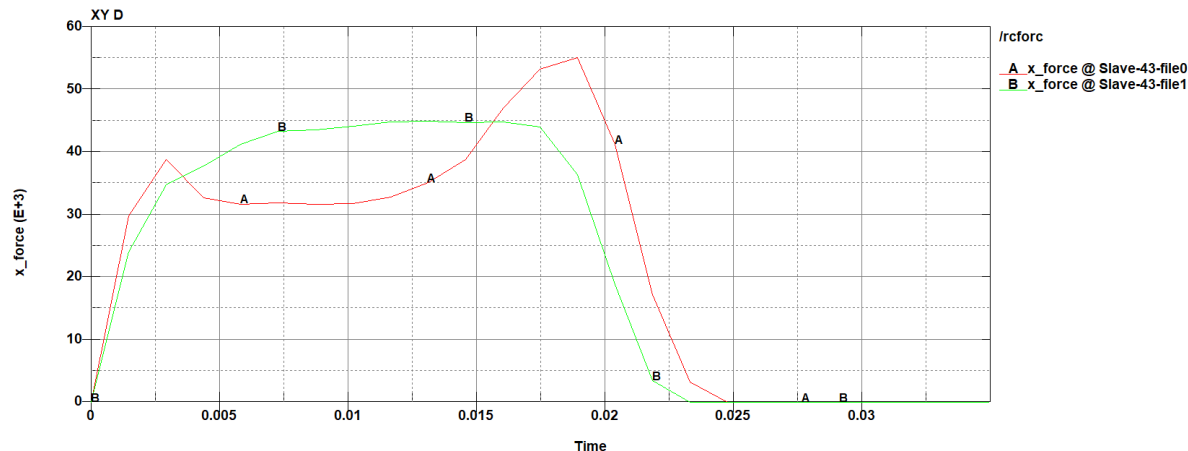
- Min sum of strain energy with mass $\leq 1.0 \times$ initial mass, and with displacement < 70 mm



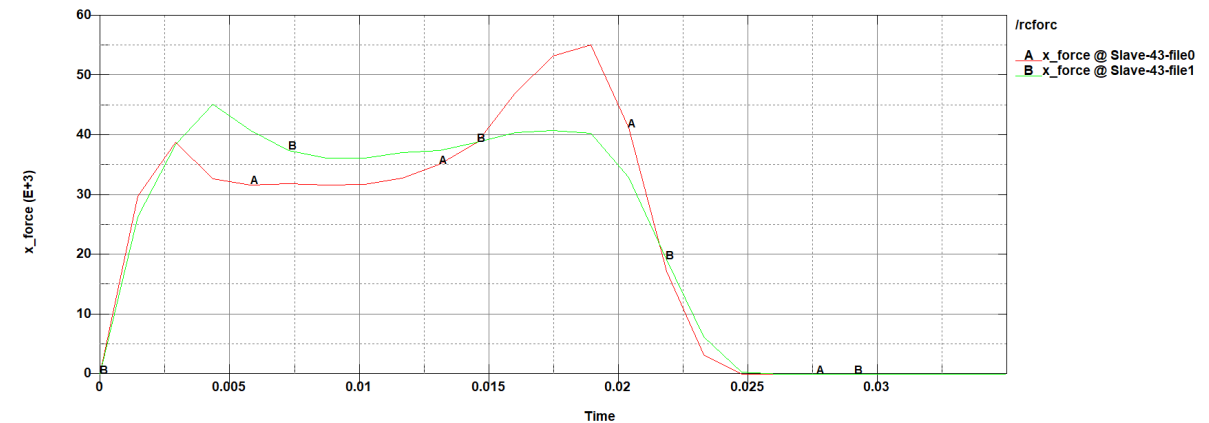
Sizing Optimization -- Summary

Comparison

Problem formulation		Max reaction force (kN)	Intrusion (mm)	Mass (kg)
Initial		55.2	75.0	1.95
Case 1	Min mass (wt = 1.0) Min sum of strain energy (wt = 0.1) with disp < 70	44.8	67.3	1.92
Case 2	Min sum of strain energy with mass < 1.0*initial_mass with disp < 70	45.2	67.4	1.95



Case 1



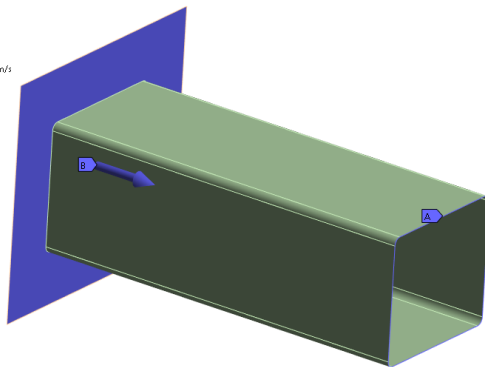
Case 2

Topography optimization of a tube under crush

■ Loading conditions

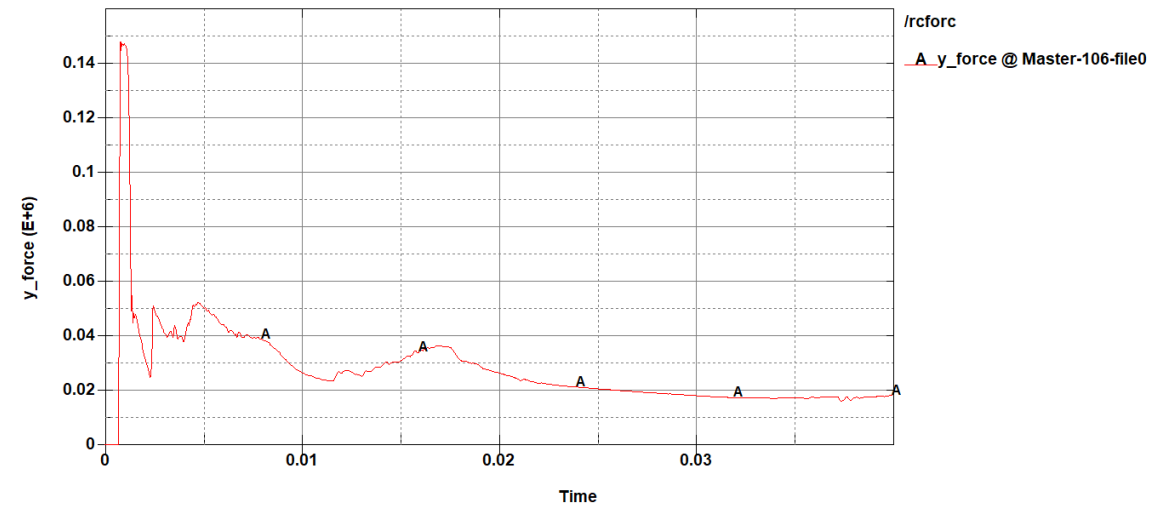
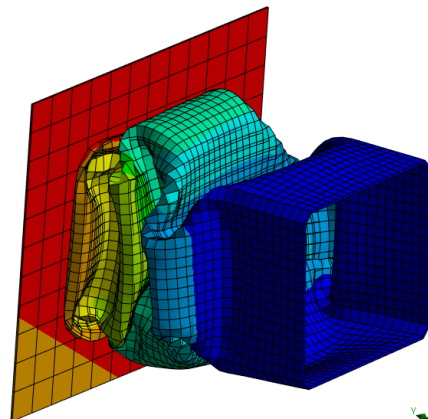
- Initial velocity: 8e3 mm/s
- End time: 0.04 s

A: LS-DYNA
Fixed Support
Time: 3.e-002 s
12/21/2022 10:38 AM



A: LS-DYNA
Total Deformation
Type: Total Deformation
Unit: mm
Time: 4.e-002 s
10/6/2023 11:11 AM

157.27 Max
139.79
122.32
104.84
87.37
69.896
52.422
34.940
17.474
0 Min

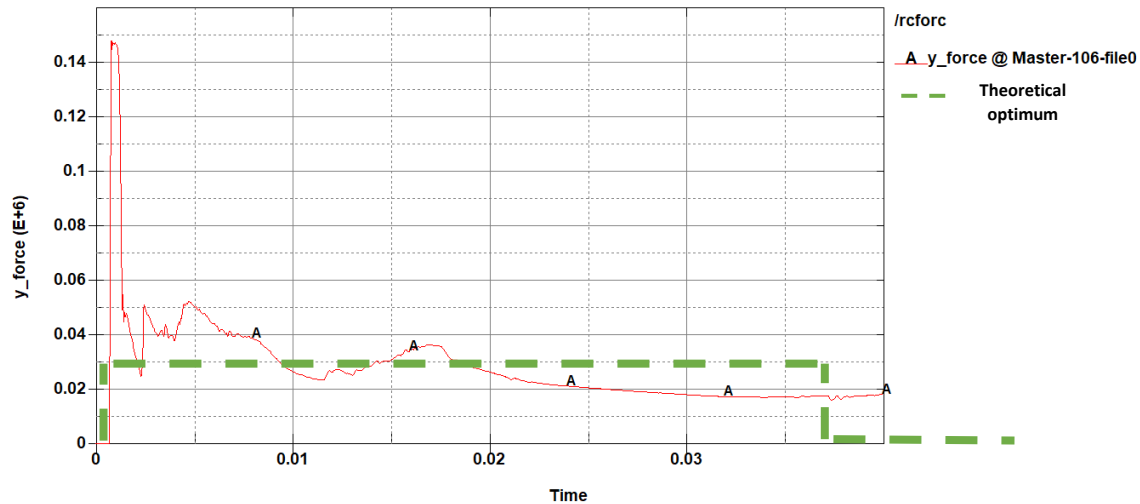


Force vs. time (A: initial design), max force = 147 kN

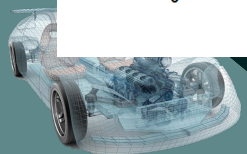
Design Requirement for Crush Tube

■ Requirement

- Input kinetic energy = 4.92 kJ
- Intrusion < 160 mm
- Reaction force < 50 kN



Theoretical optimal reaction force =
kinetic energy/max intrusion \approx 30 kN



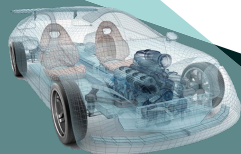
ESL Loadcases and Topography Optimization

■ 10 ESL loadcases

ESL loadcase	1	2	3	4	5	6	7	8	9	10
Base time	0.0	0.004	0.008	0.012	0.016	0.02	0.024	0.028	0.032	0.036
Load time	0.004	0.008	0.012	0.016	0.02	0.024	0.028	0.032	0.036	0.04

■ Optimization

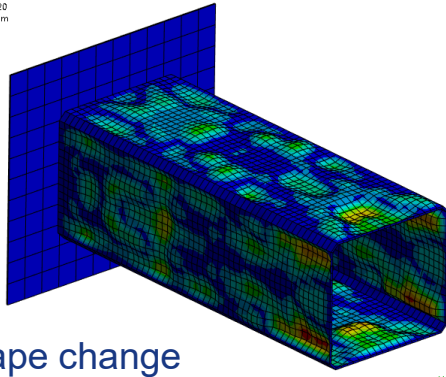
- Objective
 - Min sum of strain energy for all loadcases
- Constraints
 - Displacement along Y > -160 mm
- Max perturbation
 - 10 mm



Topography Optimization (free)

B: topography with esl
Shape Change Plot 2
Expression: RES12300
Position: Top/Bottom
Unit: mm
Time: 18 s
10/6/2023 2:41 PM

7.1202 Max
6.3291
5.538
4.7468
3.9557
3.1645
2.3734
1.5823
0.79114
0 Min

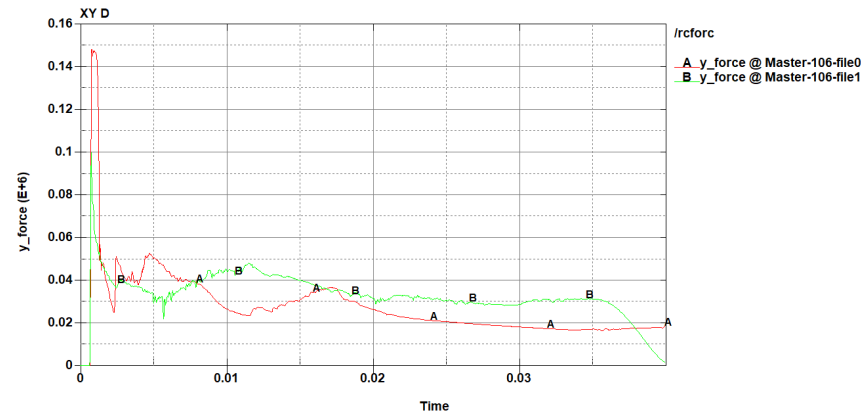


Shape change

0.00 50.00 100.00 (mm)
25.00 75.00



ESL

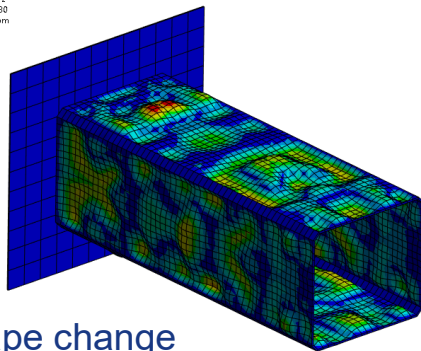


A: original design,
B: optimized

Optimization	Max Contact Force	Max Intrusion
Baseline	147 kN	161 mm
Topography with ESL method	100 kN	145 mm
Topography with Incremental ESL method	64 kN	155 mm

C: topography with incremental esl
Shape Change Plot 2
Expression: RES12330
Position: Top/Bottom
Unit: mm
Time: 18 s
10/6/2023 2:42 PM

7.6666 Max
6.8147
5.9629
5.1111
4.2592
3.4074
2.5555
1.7037
0.85184
0 Min

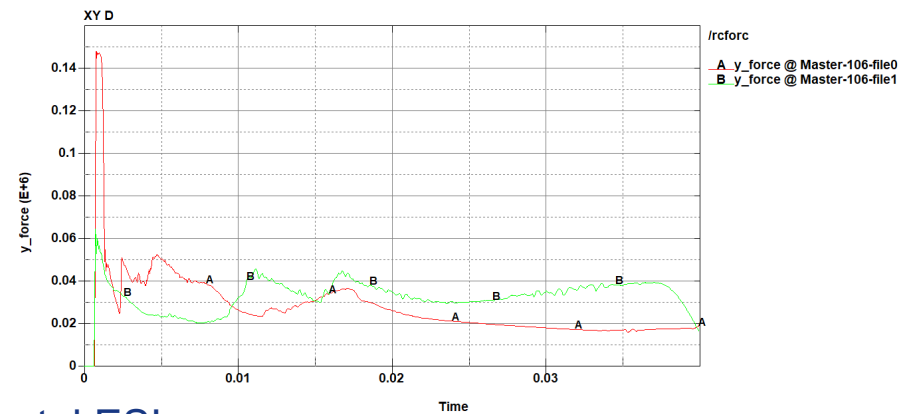


Shape change

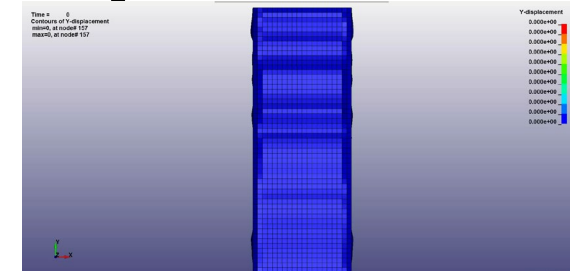
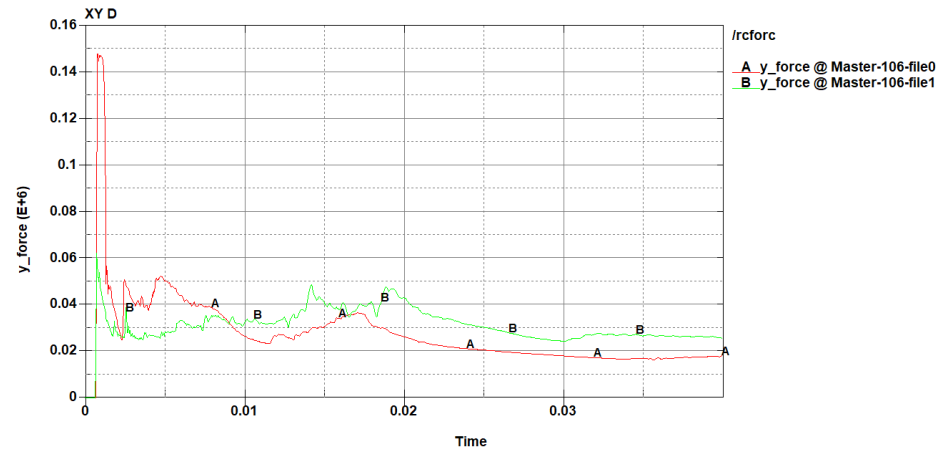
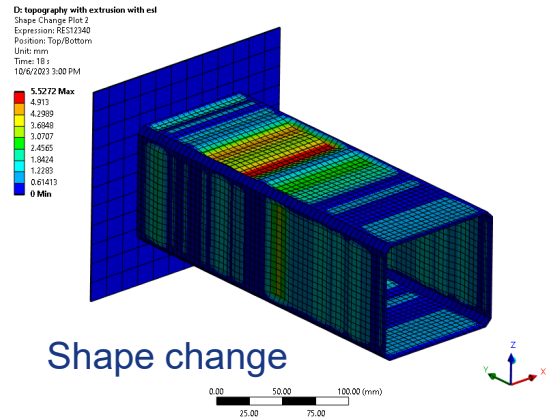
0.00 50.00 100.00 (mm)
25.00 75.00



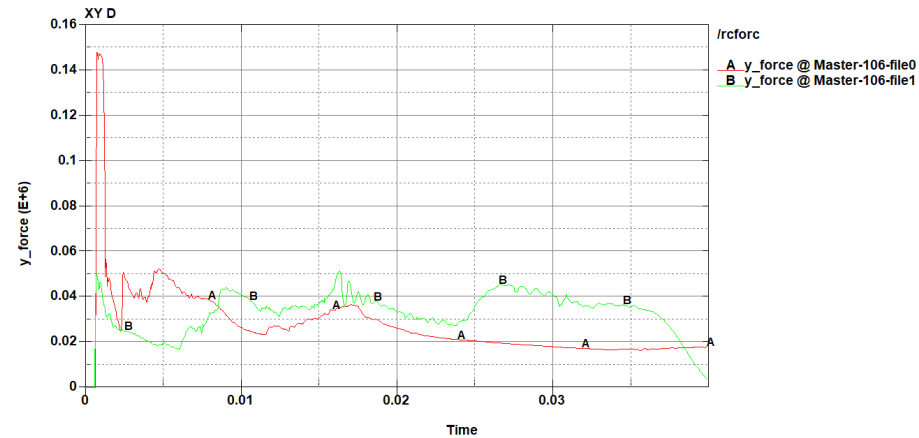
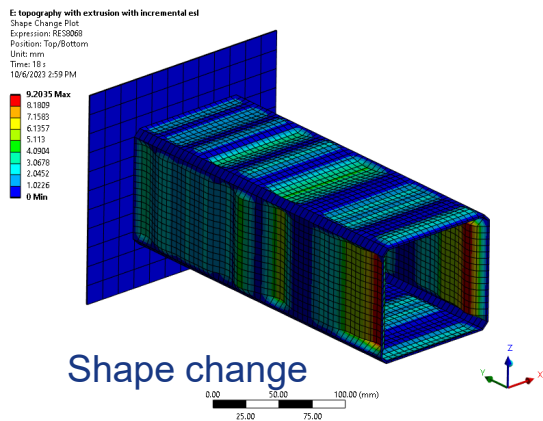
Incremental ESL



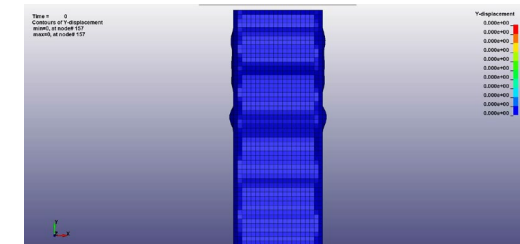
Topography Optimization (uniform)



ESL



Optimization	Max Contact Force	Max Intrusion
Baseline	147 kN	161 mm
Topography with ESL method (extrusion)	60 kN	150 mm
Topography with Incremental ESL method (extrusion)	50 kN	150 mm

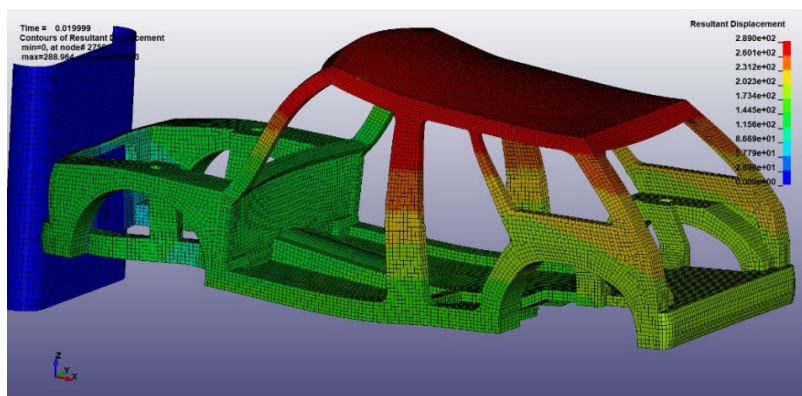


Incremental ESL

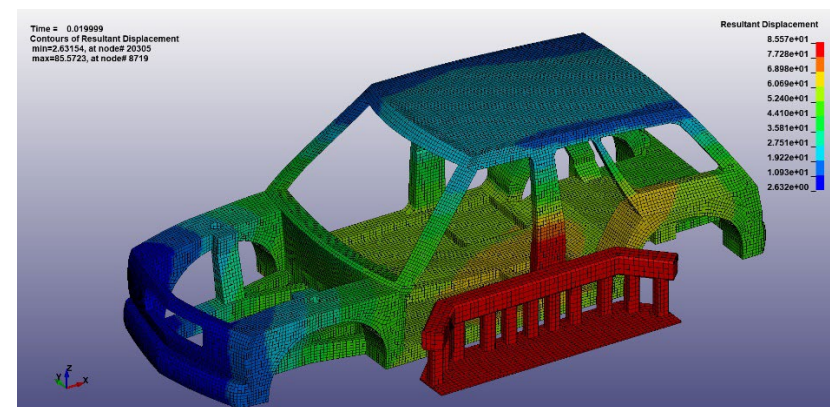
A: original design,
B: optimized

Topology optimization of a car body with multiple dynamic loading conditions

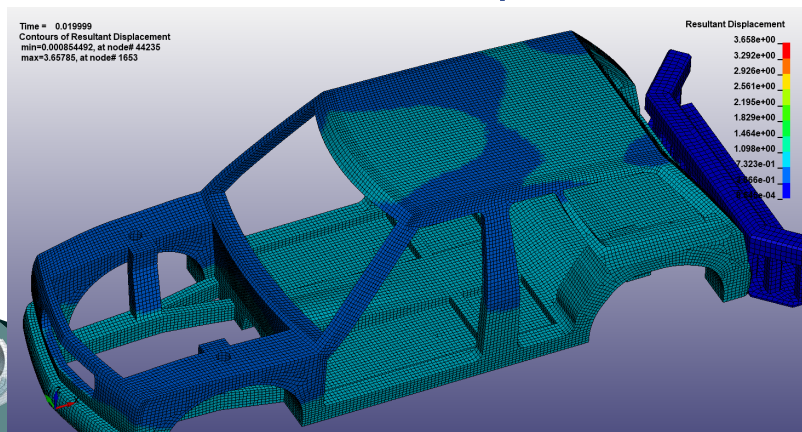
■ LS-DYNA loading conditions



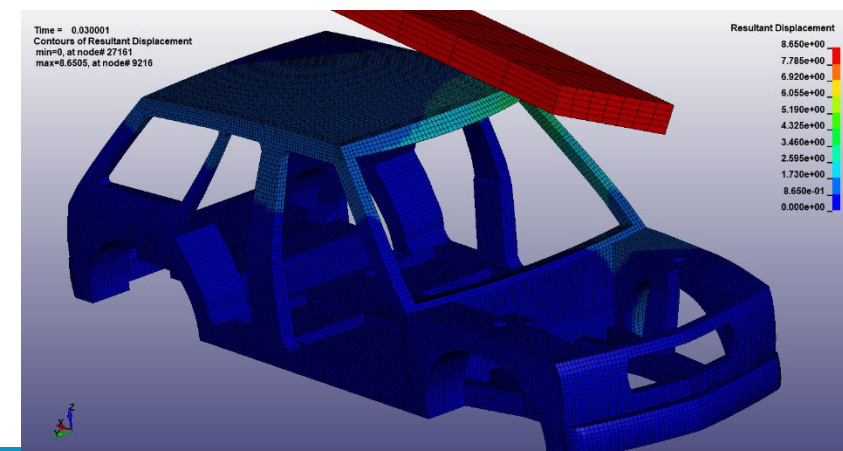
Frontal impact



Side impact



Rear impact



Roof crush

ESL Loadcases and Topology Optimization

■ ESL loadcases

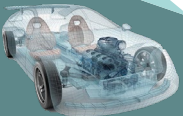
Impact condition	ESL loadcases					
frontal	Loadcase no.	1	2	3	4	5
	Base time (s)	0.0	0.005	0.01	0.015	0.02
	Load time (s)	0.005	0.01	0.015	0.02	0.025
side	Loadcase no.	6	7	8	9	10
	Base time (s)	0.0	0.005	0.01	0.015	0.02
	Load time (s)	0.005	0.01	0.015	0.02	0.025
rear	Loadcase no.	11	12	13	14	15
	Base time (s)	0.0	0.005	0.01	0.015	0.02
	Load time (s)	0.005	0.01	0.015	0.02	0.025
Roof	Loadcase no.	16	17			
	Base time (s)	0.0	0.025			
	Load time (s)	0.025	0.03			

■ Topology optimization

- Objective: min. normalized strain energy for each ESL Loadcase (17 total)
- Constraints: mass fraction $\leq 30\%$
- Topology region: symmetric about ZX plane, with minimum member size as $1.0 * \text{average_element_size}$

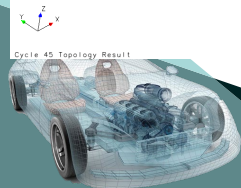
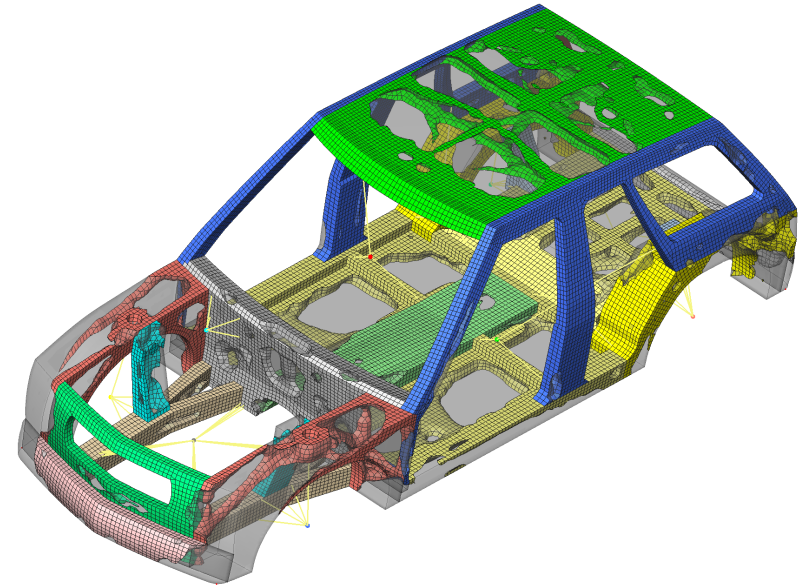
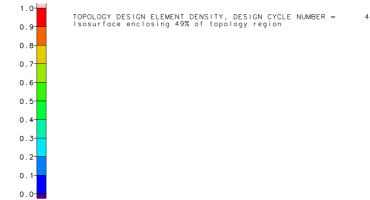
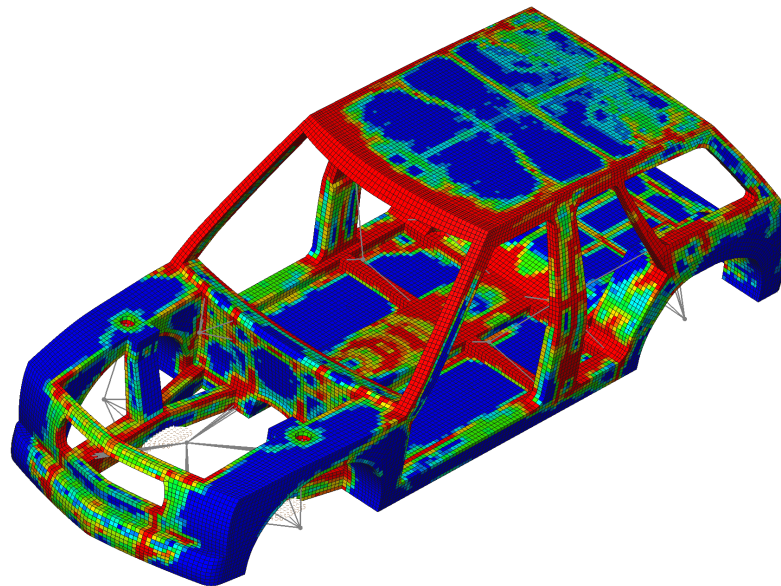
■ ESL cycles

- Maximum number of ESL cycles (LS-DYNA simulation) is set as 10
- GENESIS optimization will restart for every 5 cycles to run LS-DYNA analysis for checking convergence



Topology results

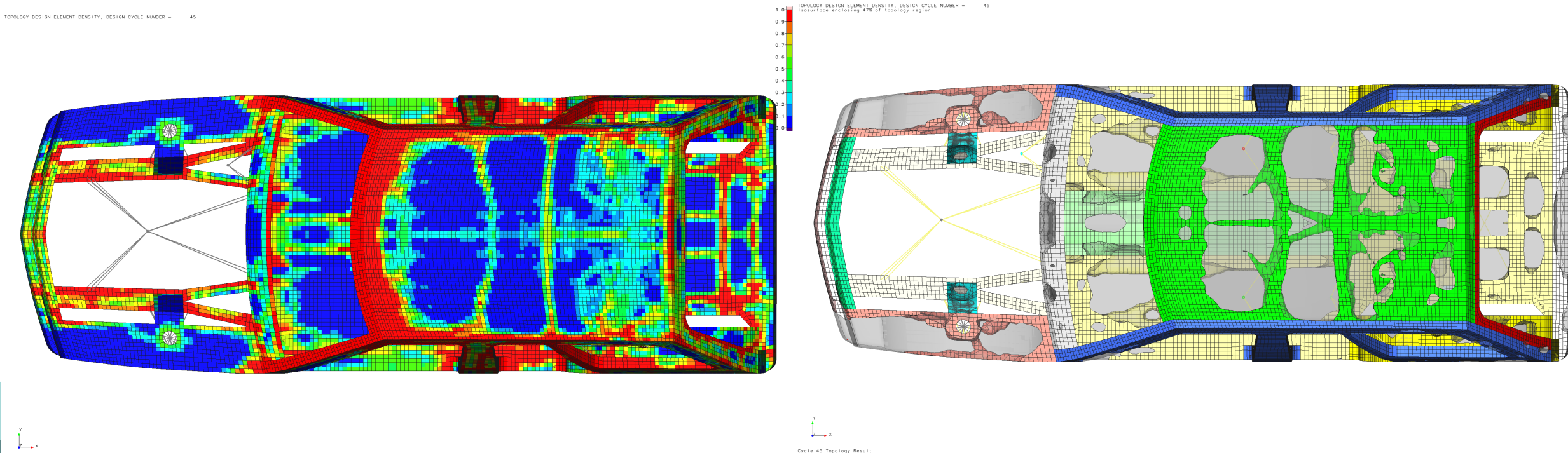
■ Element density and isosurface results (ISO view)



Topology element density and isosurface result with the incremental ESL method

Topology results

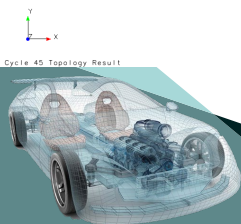
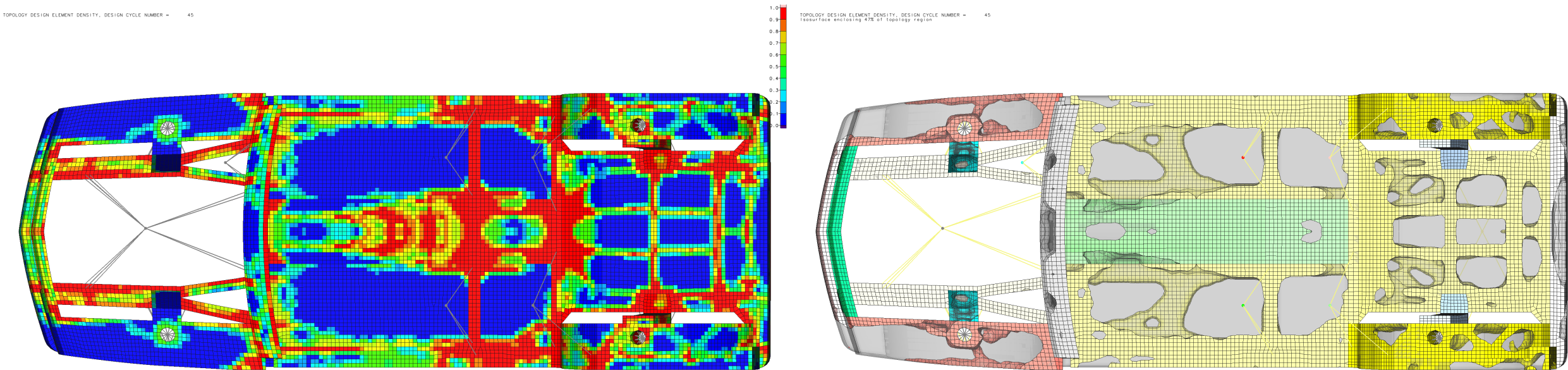
■ Element density and isosurface results (top view)



Topology element density and isosurface result with the incremental ESL method

Topology results

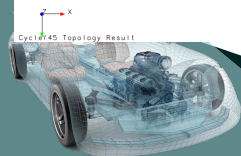
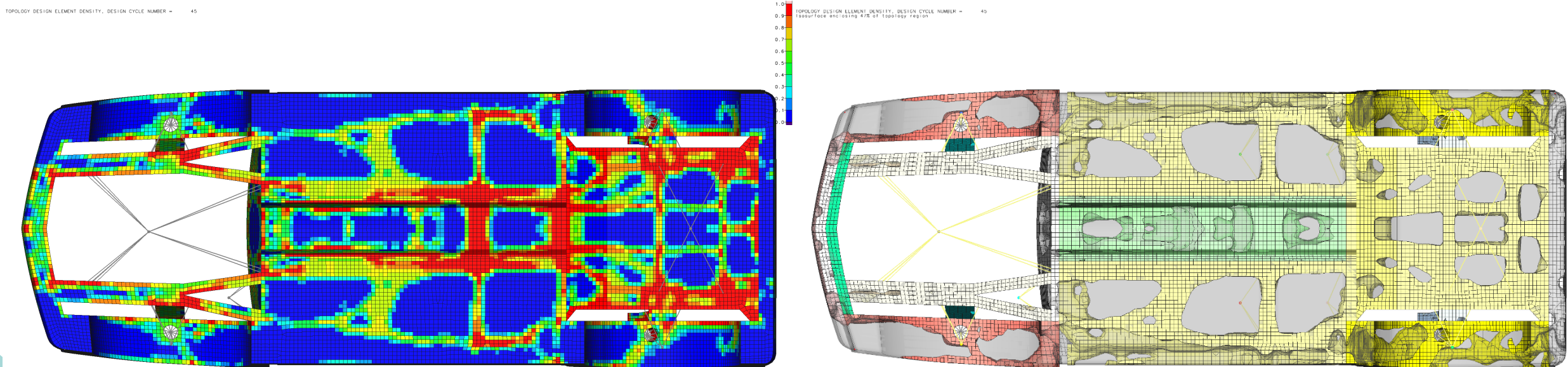
- Element density and isosurface results (top view with roof hiding)



Topology element density and isosurface result with the incremental ESL method

Topology results

■ Element density and isosurface results (bottom view)

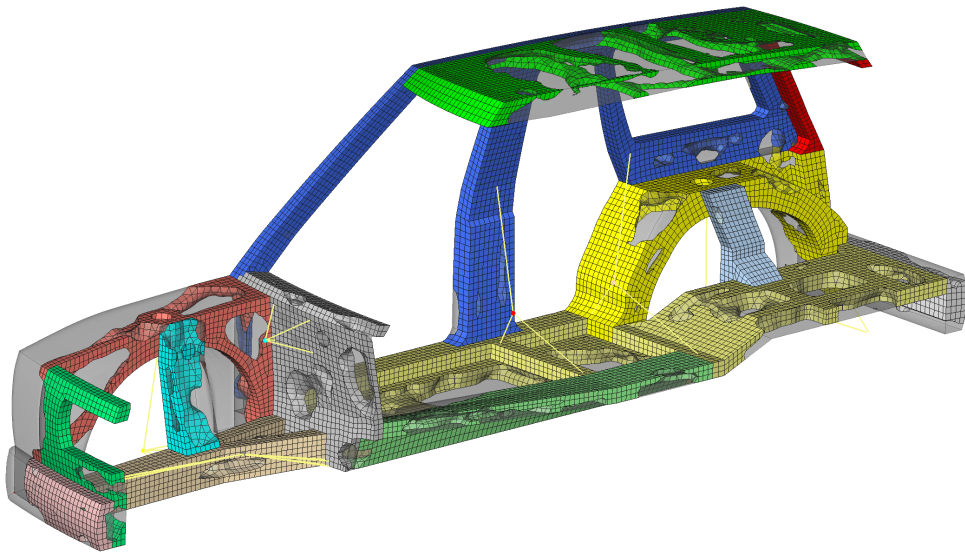


Topology element density and isosurface result with the incremental ESL method

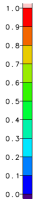
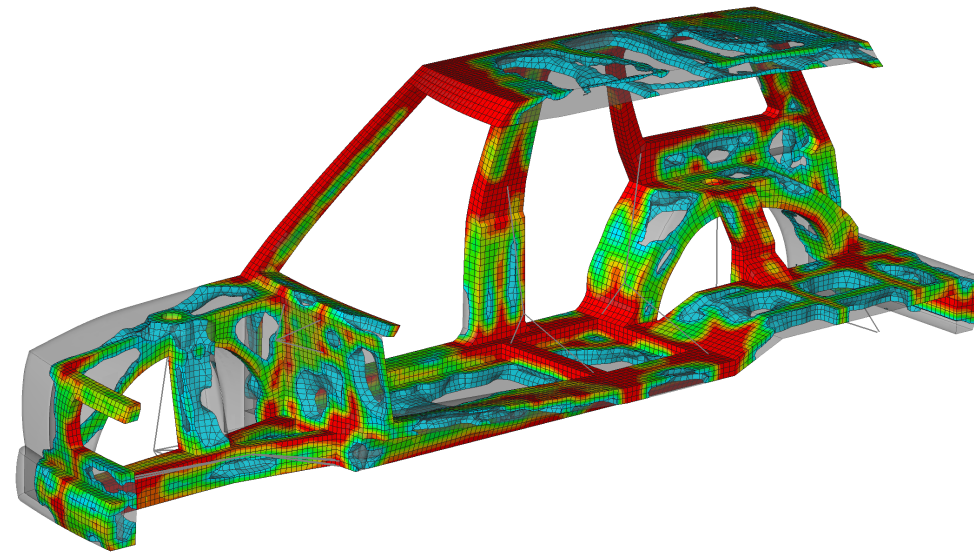
Topology results

■ Isosurface results (cut-away view)

TOPOLOGY DESIGN ELEMENT DENSITY, DESIGN CYCLE NUMBER = 45
Isosurface enclosing 47% of topology region



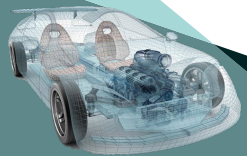
TOPOLOGY DESIGN ELEMENT DENSITY, DESIGN CYCLE NUMBER = 45
Isosurface enclosing 47% of topology region



Topology element density and isosurface result with the incremental ESL method

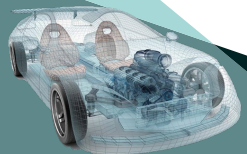
Summary

- ESL with incremental option produces more accurate approximation for nonlinear responses
 - Therefore, more accurate sensitivity for optimization
- ESL with incremental option potentially takes less number of ESL cycles (LS-DYNA analysis) to converge
- ESL with incremental option takes longer time in each GENESIS analysis because each ESL loadcase with base time needs to assemble a new stiffness matrix



Reference

- G. J. Park, Technical Overview of The Equivalent Static Loads Method for Non-Linear Static Response Structural Optimization. Structural and Multidisciplinary Optimization, 43(3), 319-337, 2011.
- Triller, J., Immel, R., Timmer, A., and Harzheim, L., Topology optimization using difference-based equivalent static loads. Structural and Multidisciplinary Optimization. 65:8, Aug 2022.



Questions?

- Thanks for attending
- Contact
 - jp@omniquest.com

