4. Compute the scale factor using the yield strength at time $n+1$:

$$m = \frac{\sigma_y^{n+1}}{s^*}$$

5. Radial return the deviatoric stresses to the yield surface:

$$s_{ij}^{n+1} = m\,{}^* s_{ij}^{n+1}$$

Material Model 11: Elastic-Plastic With Thermal Softening

Steinberg and Guinan [1978] developed this model for treating plasticity at high strain rates (10^5 s^{-1}) where enhancement of the yield strength due to strain rate effects is saturated out.

Both the shear modulus G and yield strength σ_y increase with pressure but decrease with temperature. As a melt temperature is reached, these quantities approach zero. We define the shear modulus before the material melts as

$$G = G_0 \left[1 + bpV^{1/3} - h\left(\frac{E - E_c}{3R'} - 300\right)\right] e^{-\frac{fE}{E_m - E}} \qquad (19.11.1)$$

where G_0, b, h, and f are input parameters, E_c is the cold compression energy:

$$E_c(X) = \int_0^x p\,dx - \frac{900\,R' \exp(ax)}{(1-X)^{2\left(\gamma_o - a - \frac{1}{2}\right)}}, \qquad (19.11.2)$$

where

$$X = 1 - V\,, \qquad (19.11.3)$$

and E_m is the melting energy:

$$E_m\ (X) = E_c\ (X) + 3R'T_m\ (X) \qquad (19.11.4)$$

which is a function of the melting temperature $T_m\ (X)$:

$$T_m(X) = \frac{T_{mo}\exp(2aX)}{(1-X)^{2\left(\gamma_o - a - 1/3\right)}} \qquad (19.11.5)$$

and the melting temperature T_{mo} at $\rho = \rho_0$. The constants γ_0 and a are input parameters. In the above equation, R' is defined by

$$R' = \frac{R\rho_o}{A} \tag{19.11.6}$$

where R is the gas constant and A is the atomic weight. The yield strength σ_y is given by:

$$\sigma_y = \sigma_0' \left[1 + b' p V^{1/3} - h \left(\frac{E - E_c}{3R'} - 300 \right) \right] e^{-\frac{fE}{E_m - E}} \tag{19.11.7}$$

If E_m exceeds E_i. Here, σ_0' is given by:

$$\sigma_0' = \sigma_0 \left[1 + \beta \left(\gamma_i + \bar{\varepsilon}^p \right) \right]^n \tag{19.11.8}$$

where γ_1 is the initial plastic strain, and b' and σ_0' are input parameters. Where σ_0' exceeds $\sigma_{\max}$, the maximum permitted yield strength, σ_0' is set to equal to $\sigma_{\max}$. After the material melts, σ_y and G are set to zero.

LS-DYNA fits the cold compression energy to a ten-term polynomial expansion:

$$E_c = \sum_{i=0}^{9} EC_i \eta^i \tag{19.11.9}$$

where EC_i is the ith coefficient and $\eta = \frac{\rho}{\rho_o}$. The least squares method is used to perform the fit [Kreyszig 1972]. The ten coefficients may also be specified in the input.

Once the yield strength and shear modulus are known, the numerical treatment is similar to that for material model 10.

Material Model 12: Isotropic Elastic-Plastic

The von Mises yield condition is given by:

$$\phi = J_2 - \frac{\sigma_y^2}{3} \tag{19.12.1}$$

where the second stress invariant, J_2, is defined in terms of the deviatoric stress components as

$$J_2 = \frac{1}{2} s_{ij} s_{ij} \tag{19.12.2}$$